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Flipped Classroom for Linear Algebra at Undergraduate Level

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***Abstract*—In this article, we describe our experience in developing an undergraduate Linear Algebra course tailored to highlight its relevance and applicability in Computer Science. Over the course of three years, the course transitioned from a traditional direct-instruction format to a flipped-classroom design, resulting in positive student learning outcomes. This article covers the course design philosophy, its syllabus, learning objectives, and the incorporation of both quantitative and qualitative student feedback in shaping the course. Furthermore, the article shares the insights gleaned from our experience, which can serve as best practices for instructors aiming to deliver a successful Linear Algebra course for undergraduate students in Computer Science. Our findings demonstrate the effectiveness of the flipped-classroom approach, with carefully created lecture videos and appropriate in-class exercises, in engaging students and enhancing their understanding of the mathematical concepts by incorporating innovative teaching methodologies and emphasizing the practical applications of Linear Algebra within the context of Computer Science education. From our study of our students' performance in their final summative assessments, we establish that their learning is enhanced using the flipped-classroom methodology. In addition, from their quantitative and qualitative feedback, we conclude that the flipped classroom fosters strong in-class engagement and overall satisfaction with the course.**

***Index Terms*—Linear Algebra, Course Design, Computer Science Curriculum, Student Learning Experience.**

I. Introduction

Linear Algebra plays a crucial role in many aspects of modern technology. Because of its "unreasonable effectiveness," it is part of the undergraduate mathematics experience [1] in STEM curricula. When it comes to Computer Science (CS), Linear Algebra is an essential competency. Although a critical subject for undergraduate CS students, Linear Algebra is often taught as a separate course in the mathematics department, as a pure theoretical course, not fully integrated with the CS curriculum. The traditional approach to teaching Linear Algebra includes lectures, problem sets, and perhaps a project or two. However, to keep CS students engaged and motivated, it is important to focus on the computational aspects and applications of Linear Algebra.

In this article, we present our experience in designing and iteratively improving a Linear Algebra course specifically tailored for CS students. Our course has an overarching pedagogical goal of emphasizing the applicability and usefulness of Linear Algebra in CS. We aim to teach the mathematical foundations of Linear Algebra and illustrate their relevance to CS and its applications. The course also prepares students for advanced numerical methods in computing, particularly in machine learning and data analytics. Despite the focus on applications, Linear Algebra is still a branch of pure mathematics and in a first course introducing it, it is challenging to avoid the purely theoretical aspects. Therefore, the course requires the development of some theoretical results, with proofs and consequences that employ a high level of mathematical rigor, algebraic manipulation, geometric insights as well as numerical techniques.

The course began with a traditional structure, where direct instruction through lectures was followed by homework and assignments. Later, it adopted the flipped-classroom model. This innovative approach reverses the typical educational framework by offering instructional content via online videos, shifting activities traditionally considered homework into the classroom. Such preparatory work sets the stage for students to apply their learned knowledge in class through activities such as problem-solving, projects, and discussions.

The main purpose of our analysis is to answer two questions based on our experience with the design of the course:

- **RQ1**: Do students learn and perform better in a flipped-classroom setting?
- **RQ2**: Does the flipped-classroom delivery method result in increased engagement and higher satisfaction levels among students?

It should be noted that our study is based on this linear algebra course specifically for undergraduate computer science students. While our expectation is that the conclusions from this study will generalize to other advanced courses in mathematics, it is not established in this article.

In the rest of the article, after a brief overview of related pedagogical theories behind mathematics courses, we will provide the details of the current design of the course. We will then summarize the evolution of the course design over the past three years, focusing on the student experience as reflected in their feedback, both quantitative instructor/course ratings and their textual comments. Finally, we will conclude with the insights we gleaned while designing our successful Linear Algebra course for computer science students over three years.

II. Related Work

Several pedagogical theories are commonly implemented and employed in courses teaching Linear Algebra, and mathematics in general. Although it is difficult to rank them based on their popularity at the undergraduate level, according to recent trends in the literature, an approximate ranking of the theories in terms of popularity can be listed as the following:

1) Direct Instruction focuses on the teacher imparting knowledge to the students through clear explanations and examples. Although it is considered debunked [2], this so-called empty-vessel theory of learning is still widely used at the undergraduate level, particularly in introductory and foundational-level courses, where students are introduced to new mathematical concepts and techniques.
2) Problem-Based Learning [3] emphasizes the use of real-world problems and scenarios to help students understand mathematical concepts and develop problem-solving skills. It is becoming increasingly popular at the undergraduate level, as it encourages students to connect mathematical concepts to real-world problems and applications.
3) Collaborative Learning [4], which encourages students to work in small groups to solve problems and share their understanding of mathematical concepts, is also gaining popularity at the undergraduate level. In the latest iteration of our course, we implemented a project component, to facilitate collaborative learning.
4) The Flipped Classroom, a blend of constructivism and discovery learning, has been widely adopted at the undergraduate level. As highlighted in the works of Bergmann and Sams [5], it enables students to watch video lectures and read materials as homework, and then apply what they have learned in class. This approach has the potential to enhance student learning as reported in Love et al. [6]. In a recent study [7], Liao et al. describe the need for active learning (through peer instruction, problem-based or project-based learning, or through flipped-classroom pedagogy) in order to improve student retention rates in undergraduate CS programs.
5) Constructivism [8] emphasizes that students should actively construct their own understanding of mathematical concepts through hands-on activities and problem-solving. It is also used at the undergraduate level, particularly in more advanced and specialized courses.
6) Discovery Learning suggests that students learn best when they are allowed to discover concepts and ideas through exploration and experimentation. This pedagogy is less common at the undergraduate level mathematics courses, as it requires more time and resources to implement and may not be suitable for all students.
7) Realistic Mathematics Education (RME) is an approach based on the belief that students learn mathematics best when it is presented in a realistic context and is connected to their everyday experiences. This approach is also less common at the undergraduate level, but it is being increasingly adopted as it helps students to connect mathematical concepts to real-world situations and to see the relevance of math. However, Dafid et al. [9] argue that RME does not compare favorably against laboratory-based approach.

The pedagogical theories listed above are not mutually exclusive, and educators often use a combination of them in their teaching. For instance, a teaching method called "Extreme Apprenticeship" combines several of these theories [10], [11]. The flipped-classroom, when combined with problem-based learning and other collaborative-learning strategies can improve the learning motivation and outcome of students [12].

The choice of the pedagogical approach is critical in Linear Algebra because it uses multiple systems of representations, such as algebraic and geometric views as well as abstract multi-dimensional spaces. These aspects make it a difficult course at the undergraduate level [13]–[15], requiring a high degree of "cognitive flexibility."

In our design of the "Linear Algebra for Computer Science" (LA4CS), we incorporated aspects of several of the pedagogical theories listed above. Furthermore, we evolved from the traditional direct instruction to an experiential, flipped-classroom approach, although it is believed to be a daunting challenge [16]. In this sense, the iterative redesign and continuous improvement of LA4CS can be viewed as an instance of design-based research [17], [18], which has become popular in exploring and investigative several aspects of mathematics education [19].

III. Course Context and Design

Linear Algebra for Computer Science (LA4CS) is a mandatory course for the undergraduate CS students at our school. Our undergraduate degree in CS runs over four years, and the students are exposed to a wide range of core-curriculum courses in their first year. Each class of LA4CS is three hours long and students have one class per week for a total of 12 classes over a 13-week term, with no instruction during the seventh week. The LA4CS course was offered to our CS students over three years, as shown in Table I. The table also summarizes the demographic and contextual information about the course and the cohorts.

TABLE I: Contextual Information of the Course

	2019	2021	2022
Cohort Year	First	Second	Second
# Students (CS Students)	52 (49)	130 (125)	103 (99)
# Classes	1	3	3
Students/Class	52	43, 44, 43	38, 32, 33
Male:Female Ratio	28:24	80:50	67:36
Instruction	Lecture only	Lecture only	Flipped classroom
Delivery Mode	In-peson	Online	In-peson

A. Syllabus and Course Topics

In a recent survey [1], Andrews-Larson et al. provide a list of topics in undergraduate Linear Algebra courses, categorizing them as universally covered, often covered and sometimes covered. Since our course is designed to provide students with the Linear Algebra skills necessary for a successful career in CS, it deviates slightly from the results of the survey. Our 13-week LA4CS course is organized into four parts, with topics for each week as listed below:

Part I. *Numerical Computations*
In the first part, we cover the basics of Linear Algebra as typically taught in an undergraduate curriculum. We work with vectors and matrices represented by arrays of numbers and their basic operations, with the following weekly topics:

1) Functions, Equations, and Linearity
2) Vectors, Matrices, and Operations
3) Transposes and Determinants

Part II. *Algebraic View*
In this second part, we view matrices as encoding systems of linear equations, and teach ways of solving them. The topics for this two-week part are:

4) Gaussian Elimination
5) Ranks and Inverses of Matrices

Part III. *Geometric View*
In the third part, we explore the elegant geometry that arises from vectors and matrices, from the perspective of the spaces they define. During these five weeks, we cover the following topics:

6) Vector Spaces, Basis and Dimensions
7) Change of Basis, Orthogonality, and Gram-Schmidt
8) Recess week
9) The Four Fundamental Spaces
10) Projection, Least Squares, and Linear Regression

Part IV. *Advanced Topics*
In the final part of the course, we discuss advanced topics such as eigenvalue and singular value decompositions, their significance and applications, especially in CS, with the following weekly topics:

11) Eigenvalue Decomposition and Diagonalization
12) Special Matrices, Similarity, and Algorithms
13) Singular Value Decomposition

While Linear Algebra cannot be easily divided into distinct categories such as Numerical, Algebraic, and Geometric, organizing the topics in this way addresses some of the challenges outlined in [13], related to "Cognitive Flexibility." The last two topics, although important for CS, were considered too challenging for an undergraduate course and made optional.

B. Learning Outcomes

Upon the successful completion of this course, students should be able to perform the following (both manually and in software tools such as SageMath [20]):

- Determine the existence and uniqueness of the solution of a system linear equations, and find all solutions by choosing an effective method such as Gaussian elimination, factorization, or diagonalization.
- Test for linear independence and orthogonality of vectors and vector spaces. Determine the rank, determinant, inverse, perform Gram-Schmidt orthogonalization, and different factorizations of a matrix.
- Visualize and compute the four fundamental spaces of a matrix, identify their relation to systems of linear equations, and find their dimensions and bases.
- Identify special properties of a matrix, such as symmetry, positive definiteness, etc., and use this information to compute matrix characteristics.
- Describe the use of mathematical techniques from Linear Algebra as applied to computing applications.
- Compute eigenvalues and eigenvectors of a matrix, use them for diagonalizing it, taking its powers, and solving advanced problems.
- Describe the use of Singular Value Decomposition and Principal Component Analysis in data science algorithms.

IV. Evolution of the Course

While the main objectives and the syllabus of the course remained unchanged, we modified the topic flow as well as the emphasis, along with the instruction methodologies over the successive iterations of the course based on our understanding of the appropriateness of pedagogical theories and in response to the student feedback and performance.

A. First Iteration

We first offered this course in 2019 to our incoming CS students. As seen in Table I, three students from other programs also took the course as an elective. As is common in STEM courses, we had more male than female students: a feature that became more prominent in subsequent cohorts.

The initial design of the LA4CS course followed the traditional, direct instruction mode [2]. Consistent with the seminar-style, experiential pedagogy that we practice at our university, we designed LA4CS to have an interactive classroom, keeping the students engaged and participating in class activities. To achieve this, we decided to make the course application-focused and created labs to illustrate the utility and relevance of Linear Algebra in CS. We prepared several labs on SageMath for the students to perform numerical computations and to see Linear Algebra in action in image processing, perspective correction, etc. It is worth noting that a recent study by Rensaa et al. [14] argues that a digital tool is not necessary in teaching Linear Algebra.

B. Second Iteration

The design we implemented in the first run of the course did not work as well as we expected. It turned out that the first-year students were ill-prepared for a course of this complexity. They also lacked the necessary skills in Python to work with our SageMath labs. Based on our observations and the student feedback, we moved the course to their second year and made some realignments of the topics. We also made the labs optional and introduced a project component.

In addition, to facilitate student learning, we prepared a bespoke textbook, specifically written for this LA4CS course. Additionally, we made some of the more advanced topics, such as eigenvalue and singular value decomposition, optional. These topics were taught in class but were not included in the assessment. Due to the COVID-19 pandemic, the lectures were delivered online during the second run of the course.

From a pedagogical perspective, we attempted to incorporate elements of collaborative learning [4] into our course design by introducing a mini-project, which was of a problem-solving kind. The students selected a tutorial-type problem from a list provided and attempted to teach it to the rest of the class.

C. Third Iteration

In the current iteration of LA4CS, we decided to move to the flipped-classroom model. We prepared about 50 videos of 10-20 minutes in duration and carefully created playlists for each weekly session. This course redesign using flipped classroom [21] was an attempt to incorporate several aspects of active learning pedagogy [7]. Promising results were reported [22] in a larger scale redesign of CS curriculum along similar lines.

In the context of the flipped-classroom model, students are expected to come to class fully prepared by watching instructional videos and reviewing lecture notes, which are made available approximately one week prior to the class session. The classroom time is then utilized for a quick recap, interactive quizzes, and in-class exercises (ICE). This structure followed for the first ten weekly classes. The 11^{th} class is devoted to a mock-final exam, while the 12^{th} class is designated for project presentations.

Studies, such as those conducted by Johnston et al. [23] and Nasir et al. [24], have indicated that students generally prefer courses that employ the flipped-classroom method and exhibit improved learning outcomes. Building on these insights, we developed extensive problem sets in addition to the exercises already provided in the textbook. To support students' learning, the teaching team (comprising the professor, an additional instructor, and a teaching assistant) is available to offer assistance whenever required. After two hours of interactive work and guided practice, the remaining hour is dedicated to demonstrating solutions and addressing any remaining questions.

In order to further enhance the active and collaborative learning aspects of the course, we revamped the project component. In this third iteration of the LA4CS course, we provided a list of suggested topics related to the applications of Linear Algebra in CS. This helped students appreciate the significance of what they learn in class for their future careers as computer scientists. From their feedback, we could see that the students found the project component really useful.

V. Student Learning Experience

In addition to making use of pedagogical theories, we also paid careful attention to the student performance redesigning the course. This iterative process is especially important for a new course such as LA4CS. Although it is not easy to compare student learning across cohorts without making heuristic assumptions, we take their raw scores in their final exam as representative of their knowledge in the course. The final grades are not a true measure because of the statistical moderation [25] applied to conform to a prescribed grade distribution. But the raw scores can be used to quantify the knowledge transfer in the course over the iterations.

Fig. 1 shows the performance of our students in their final exams. We can see that they performed better in the latest iteration of the course. Based on this observation, we can confirm that the students do learn linear algebra better when taught in the flipped-classroom setting, and the answer to our first question (**RQ1**) is positive.

More insights into the students' learning experience can be gleaned from their feedback. As is customary in universities, we collect feedback from students at the end of the term and use it to improve our course offerings. It should be highlighted, however, that student feedback alone cannot be thought of as a measure of how much they learn, or how well we teach [26]. We, therefore, report the student-satisfaction levels, as reflected in their feedback, as an independent dimension of their learning experience.

Quantitative Feedback: In our university, student feedback includes both closed and open-ended questions. For the closed questions, the students rate the instructor and the course separately on a Likert-type scales [27] from 1 to 7. The

TABLE II: Student performance in the Course

	2019	2021	2022
Mean	54.9	56.3	64.5
Standard Deviation	14.2	15.0	13.9

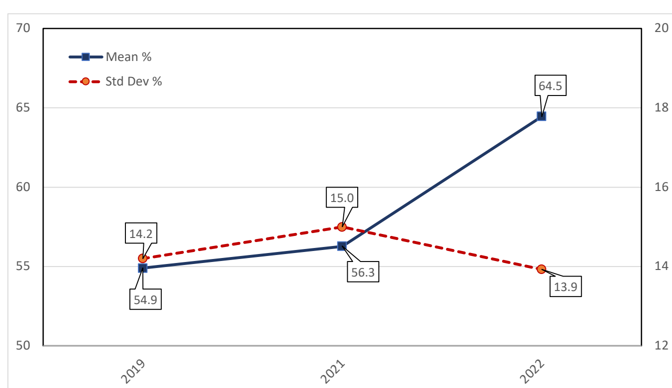

Fig. 1: The student performance as quantified by their raw scores in the final exam of the course over the three iterations of the course, as tabulated in Table II.

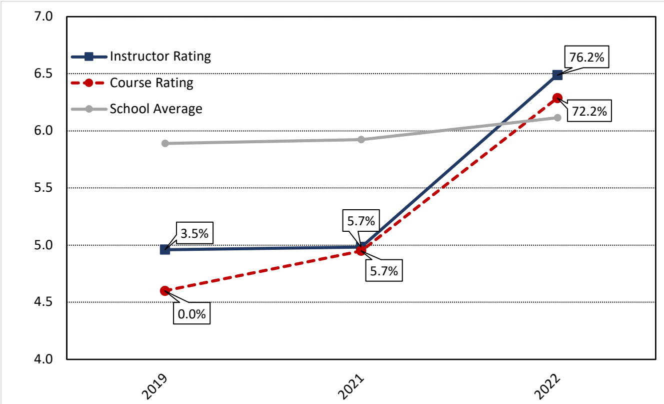

Fig. 2: The overall ratings of the instructor and the LA4CS course on a 1 to 7 scale. More information is provided in the Summary Table III. The data labels shown on the chart are the percentiles in the school. Also shown is the school average of combined instructor and course ratings.

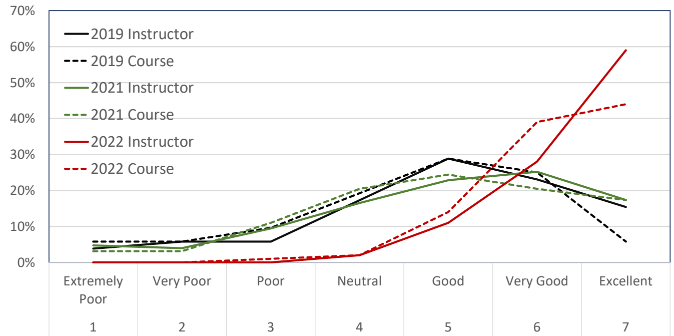

Fig. 3: The distribution of the ratings of the instructor and the LA4CS course on a 1 to 7 scale, as tabulated in Table V.

quantitative scores from these closed questions are tabulated in Table IV, which shows the various aspects on which the instructor and course are evaluated and the average scores on a per-year basis.

The overall ratings of the instructor and the course are tabulated separately in Table III. In order to provide context for the quantitative feedback on the instructor and course, we also furnish their percentile standing among the instructor-course pairs offered in our school for the respective years as the "Percentile Base" in Table III. The values in the Table are also charted in Fig. 2 for easy visualization. It is worth noting the significant jump from 2021 to 2022, particularly in the percentile standing. Also noteworthy is that the first time the course was offered in 2019, the percentile was 0%, indicating that it was the worst-received course in our school.

From the quantitative feedback reported in Tables III to V and displayed in Figures 2 and 3, we conclude that delivery of LA4CS in the flipped-classroom setting is well-received by our students, answering our second question (**RQ2**) in the affirmative.

TABLE III: Summary of Quantitative Student Evaluation

	2019	2021	2022
Number of Students (Feedbacks)	52 (52)	130 (127)	103 (100)
Instructor Rating (Percentile)	4.96 (3.5%)	4.98 (5.7%)	6.49 (76.2%)
Course Rating (Percentile)	4.60 (0.0%)	4.95 (5.7%)	6.29 (72.2%)
Percentile Base	58	89	85

The first row lists the number of students in the cohort (and the number of respondents). The second and third row are the instructor and course ratings, respectively, along with the percentile (the fraction of instructor-course combinations that had a rating lower than our course). The last row shows the number of instructor-course pairs based on which the percentile is computed.

Qualitative Feedback: In addition to the quantitative rating, we also include open-ended questions in the evaluation exercise. These questions solicit both positive and negative comments. The student responses to them provide a more comprehensive feedback on the course and the instructor. We elicit these comments through the following questions in our end-of-term student feedback exercise:

1) **[*Instructor: Positive*]** What are the strengths of the instructor's teaching?
2) **[*Instructor: Negative*]** What suggestions do you have to improve the instructor's teaching?
3) **[*Course: Positive*]** What elements of the course most contributed to your learning?
4) **[*Course: Negative*]** What suggestions do you have to improve the course?

The responses to these questions are summarized below, in five themes per question, with the numbers of comments or suggestions contributing to the theme indicated in parentheses. In order to present as unbiased a summary as possible, we used an online AI platform [28] to create the initial versions of these summaries, which were further edited for brevity and anonymization.

A. 2019: First Iteration

[*Instructor: Strengths*]

Patience and Engagement (15 comments): Demonstrates patience and engages students by providing a big picture, relating concepts, and using interactive methods such as Kahoot quizzes. They ensure everyone understands by giving recaps and clarifying difficult topics.

Clarity in Teaching (9 comments): Speaks clearly and makes an effort to explain complex concepts thoroughly. Uses video links, lab assignments, and visualizations to aid understanding.

Knowledge and Relevance (6 comments): Has a deep knowledge of the subject matter and can apply it to real-world examples, making the relevance of the content apparent.

TABLE IV: Details of the Instructor and Course Ratings and Their Evolution

Question	2019	2021	2022
Instructor's preparation and organisation	4.82	5.37	6.60
Instructor's clarity and understandability	4.40	4.91	6.31
Instructor's stimulation of interest in content	5.14	4.93	6.44
Instructor's encouragement and openness	5.50	5.26	6.55
Instructor's availability and helpfulness	5.38	4.80	6.52
Instructor's presentation and speaking skills	5.26	5.22	6.55
Instructor's enthusiasm for the subject	6.02	5.76	6.72
Instructor's fairness	5.60	5.39	6.60
Instructor's concern for students	5.46	5.05	6.44
Overall rating of the instructor	**4.96**	**4.98**	**6.49**
The learning experience in this course	4.42	4.81	6.19
The clarity of objectives and requirements	4.38	5.09	6.34
Quality and frequency of feedback	4.66	4.84	6.37
Quality and value of the course material	4.46	5.10	6.35
Quality and usefulness of course assignments/projects	4.74	5.24	6.28
Degree to which the course was participative and interactive	4.74	5.26	6.46
Overall rating of the course	**4.60**	**4.95**	**6.29**

The first column (Question) shows the aspect of the instructor or course that the students evaluated. The next three columns (2019, 2021 and 2022) list the average of the student rating for the respective year and the aspect of evaluation. The overall ratings (highlighted in bold) are summarized in Table III and displayed in Fig. 2.

TABLE V: Evolution of instructor and course rating distributions

Score	Description	2019		2021		2022	
		Instructor	Course	Instructor	Course	Instructor	Course
1	Extremely Poor	2 (3.8%)	3 (5.8%)	6 (4.7%)	4 (3.1%)	0 (0.0%)	0 (0.0%)
2	Very Poor	3 (5.8%)	3 (5.8%)	5 (3.9%)	4 (3.1%)	0 (0.0%)	0 (0.0%)
3	Poor	3 (5.8%)	5 (9.6%)	12 (9.4%)	14 (11.0%)	0 (0.0%)	1 (1.0%)
4	Neutral	9 (17.3%)	10 (19.2%)	21 (16.5%)	26 (20.5%)	2 (2.0%)	2 (2.0%)
5	Good	15 (28.8%)	15 (28.8%)	29 (22.8%)	31 (24.4%)	11 (11.0%)	14 (14.0%)
6	Very Good	12 (23.1%)	13 (25.0%)	32 (25.2%)	26 (20.5%)	28 (28.0%)	39 (39.0%)
7	Excellent	8 (15.4%)	3 (5.8%)	22 (17.3%)	22 (17.3%)	59 (59.0%)	44 (44.0%)

The instructor and the course are evaluated on a scale of 1 to 7. The first two columns above show the rating score and its meaning. The next three columns (2019, 2021 and 2022) list the number (and the fraction in parentheses) of students who gave the respective rating. This evolution of the fraction of students is also charted in Fig. 3 for easy visualization.

Approachability (6 comments): Approachable, kind, and willing to repeat until students understand. Encourages questions and provide precise answers.

Fairness (3 comments): Ensures fair class participation by using random student selection. Creates a comprehensive syllabus and provide recaps to facilitate understanding.

[*Instructor: Suggestions to Improve*]

Teaching Approach (12 suggestions): Be more receptive to feedback, avoid excessive repetition, improve lesson structure, prioritize student learning, and clarify assessment structure.

Learning Materials (10 suggestions): Improve organization and usefulness of learning materials, enhance slides with graphics and examples, address issues with animations and accessibility, and provide clearer explanations.

Class Engagement (9 suggestions): Encourage more in-class exercises, practice questions, and student discussions. Consider alternative platforms for class participation and provide guidance rather than direct answers.

Course Coherence (6 suggestions): Enhance the coherence of course material, clarify the connections between topics, and ensure a logical order of presentation.

Additional Resources (6 suggestions): Provide more examples, tutorials, exercises, and practice questions for better understanding and preparation.

[*Course: Positive Elements*]

Videos (10 comments): Highly beneficial for learning and understanding concepts.

Lab Exercises (6 comments): Provided real-world applications and helped in applying the learned concepts.

Assignments (5 comments): Useful for hands-on practice and reinforcing understanding.

SageMath (4 comments): Aided in learning and eliminating arithmetic errors.

Labs (4 comments): Demonstrated real-world applications and immediate practicality to students.

[*Course: Suggestions to Improve*]

Structure and Delivery (15 suggestions): Provide clearer explanations, include more practical applications, and organized slides.

Additional Resources (14 suggestions): Include examples, mock papers, tutorials, and practice questions.

Time Management (13 suggestions): Improve the pace of the class, and teaching methods, with clearer course structure, more practice papers, and better explanation of concepts.

Clarify of Expectations (4 suggestions): Specify requirements for assignments and assessments, including grading criteria and more examples.

Scaffolding (3 suggestions): Help students with no programming background, particularly in SageMath.

Action Items: Based on the feedback, we decided to:

- Create a bespoke textbook for the course, with weekly sessions as chapters and practice exercises.
- Make the advanced topics like eigenvalue and singular value decompositions optional, and untested.
- Introduce a project component for problem solving.

B. 2021: Second Iteration

[*Instructor: Strengths*]

Engagement and Interactivity (18 comments): Uses Kahoot quizzes, breaks, and class participation to make the class fun and interactive, encouraging discussions and questions.

Clarity and Explanation (15 comments): Provides clear explanations, numerous examples, and emphasizes conceptual understanding.

Knowledge and Passion (13 comments): Demonstrates deep knowledge and enthusiasm for the subject and has relevant professional experience, and shows passion in teaching.

Approachability (12 comments): Patient, kind, and generous, and creates resources like textbooks, lecture videos, and additional materials.

Organization (9 comments): The classes are well-organized, structured, with clear summaries, and ample practice opportunities.

[*Instructor: Suggestions to Improve*]

Grading Clarity (25 suggestions): Have clear grading guidelines, consistent grading standards, and concise explanations during lectures.

Engagement and Interaction (20 suggestions): Increase student engagement and interaction. Allow time for questions, encourage active participation, and sharing of answers and insights.

Course Materials (15 suggestions): Ensure accuracy, clarity, and consistency in materials, provide timely feedback on assignments and solutions for exercises.

Teaching Approach (14 suggestions): Provide clearer explanations, relatable examples, and a balanced pace of content delivery with more effective time management during lectures and a focus on essential concepts.

Visual Learning (14 suggestions): Incorporate more visual aids, such as whiteboard illustrations and animations. Include practical examples and real-world applications to demonstrate the relevance of concepts.

[*Course: Positive Elements*]

Assignments (14 comments): Key element in contributing to learning, as they required students to solve problems and apply what they had learned.

Lectures (13 comments): Significant contributor to learning, especially with off-topic examples to further understanding.

Quizzes (10 comments), In-class and Kahoot quizzes were useful in testing understanding and identifying misconceptions.

Course Materials (9 comments): Textbook and online resources, were helpful in providing additional support for learning and revising concepts.

Labs and Assignments (8 comments): Effective in reinforcing understanding and teaching students how to apply concepts to real-world problems.

[*Course: Suggestions to Improve*]

Additional Resources (26 suggestions): Release tutorial questions and answers, offer more examples and real-world applications, and provide additional practice sets and math problems.

Course Structure (23 suggestions): Improve organization, release materials earlier, space out assignments, and provide clearer rubrics.

Clarity and Engagement (22 suggestions): Enhance clarity of notes and content, incorporate visuals, relate the material to computer science, and introduce interactive elements.

Feedback and Assessment (14 suggestions): Provide timely feedback on assignments, adjust quiz format for fairness, and base it on previous week's content.

Textbook and Learning Materials (14 suggestions): Address errors and revisions in the textbook, release solutions earlier, and provide more polished learning resources.

Action Items: Based on the feedback, we decided to:

- Redesign the course using the flipped-classroom methodology.
- Create additional problem sets for in-class exercises with instructor scaffolding.
- Reinstate eigenvalue decomposition as a tested topic.
- Make the project component about applications of Linear Algebra in CS by providing a list of topics.

C. 2022: Third Iteration

[*Instructor: Strengths*]

Knowledge and Engagement (47 comments): Exhibits proficiency and clarity in teaching. Actively involves students in the learning process.

Enthusiasm and Passion (30 comments): Simplifies complex concepts makes the course interesting and accessible with examples and clear explanations. Passionate about the subject.

Flipped Classroom (26 comments): Videos for revision are effective and helpful for clarifying doubts. Students value Prof's availability for questions and his willingness to create a safe space for learning.

Openness to Feedback (18 comments): Dedication to improvement, and regular review of student reflections contribute to a nurturing and supportive learning environment, tailoring the course based on feedback and providing additional materials.

Approachability (17 comments): Patient and willing to help students understand concepts, even when questions are slightly off-topic, creating a positive learning experience.

[*Instructor: Suggestions to Improve*]

Workload and Pace (10 suggestions): Be mindful of students' workload, adjust teaching pace, allocate more time in class for detailed content coverage, provide more numerical examples, and avoid unnecessary complexity.

Clarity and Conciseness (8 suggestions): Enhance clarity and conciseness of explanations, provide step-by-step instructions, use layman's terms, and adapt explanations to different learning styles.

Flipped Classroom (8 suggestions): Integrate Wooclap questions, provide more review questions, reduce time spent on reflections, offer in-class teaching for challenging topics, and provide additional support and clarification in pre-class videos.

General Suggestions (8 suggestions): Allow personalized reference notes (cheat sheets) in the final exam, and release assignments earlier.

Teaching Materials (7 suggestions): Improve video quality with clear visuals and examples, use more diagrams to aid understanding, incorporate visualizations on the iPad, and work through tutorial examples in videos.

[*Course: Positive Elements*]

Flipped Classroom (19 comments): Appreciated for its effectiveness in grasping concepts and clarifying doubts.

Assignments and In-Class Exercises (17 comments): Valuable for reinforcing understanding and consolidating knowledge.

Course Textbooks, Videos, and Resources (14 comments): Played a significant role in supporting learning.

Quizzes, and Wooclap (13 comments): Promoted student engagement, tested their understanding, and kept them on track with the course content.

In-Class Activities (12 comments): Group projects and Q&A sessions, helped clarify concepts, and connected the course content with real-life examples.

[*Course: Suggestions to Improve*]

Teaching Methods and Materials (18 suggestions): Improve slides for better clarity, include more examples and visuals in lectures, and provide additional videos explaining tutorial questions and concepts ensuring thorough explanations for answers and reasoning.

Workload and Course Content (17 suggestions): Reduce the number and length of assignments, quizzes, and projects. Make the course content lighter and more manageable, provide clearer project requirements, and focus more on solutions and explanations.

Flipped Classroom (12 suggestions): Revert to a more traditional format. A semi-flipped classroom model was also suggested.

Assessment and Grading (10 suggestions): Extend assignment deadlines or allow online submission. Reduce the weightage of quizzes and final exams, provide more time for topic revision, offer more examples for better understanding, and release problem set solutions for reference.

Miscellaneous (10 suggestions): Remove or modify the group project, provide supplementary classes for tutorials, focus on topics more relevant to computer science, and align the order of concepts in videos with course notes.

Action Items: Based on the feedback, we decided to:

- Continue with the flipped-classroom methodology for the course.
- Create additional problem sets for practicing in the classroom with instructor help.

D. Discussion and Limitations

In the two questions with which we started our article, namely the effect of flipped classroom on student learning (**RQ1**) and the level of its acceptance and engagement among students (**RQ2**), we find evidence indicating affirmative responses to both queries.

One limitation of our study is that as we fine-tuned our course design, we lacked a reliable measure of student learning. Instead, we assumed that their raw score in their final summative assessment (before any form of moderation) could be used and a yardstick of their learning experience. This assumption can be challenged on the basis of the variability in the cohort strength and instructor competence, as well as the difficulty level of the test itself.

Secondly, we used student feedback as a proxy measure of the success of our design. Although subject to year-on-year variations, we could assume comparability in the subjective ratings because we used the percentile for our purposes.

Typically, it can be challenging to receive positive ratings from students in a technically demanding course. But, the third iteration of the LA4CS course has been remarkably successful, as reflected in both the quantitative and qualitative feedback from students. We had expected the second iteration to be an improvement over the first. However, it was likely impacted by the COVID-19 pandemic, and showed a disappointingly poor level of acceptance among the students.

We also note that the aggregation is done (in Table IV, for instance) using arithmetic mean. The rating distributions are highly skewed and arithmetic mean is perhaps not the best measure of central tendency. However, to the extent that we are using it for comparison across cohorts, rather than as an absolute measure, it is still a valid measure.

VI. Conclusion

This paper presents our experience in developing a Linear Algebra course tailored for undergraduate students of Computer Science. After initially following a traditional direct instruction approach, we evolved the course over three years into a flipped-classroom model centered on active learning through in-class exercises. This design resulted in the best learning experience for the students, as reflected both in their performance and feedback in our specific context.

Our findings indicate that while a small minority of students expressed dissatisfaction with the flipped-classroom approach and video usage, the vast majority responded positively, with some displaying notable enthusiasm. Most students appreciated the convenience and flexibility of accessing videos and learning at their own pace, while the negative sentiments primarily revolved around a preference for face-to-face instruction and a perceived lack of engagement. Quantitative ratings of the instructor and course further supported these observations.

The inclusion of the project component aimed at emphasizing the relevance of Linear Algebra in Computer Science proved to be a crucial aspect of the course, fostering students' recognition of the material's significance. We discovered that while digital tools like SageMath can facilitate numerical computations, they are not indispensable for the success of the course. Instead, the emphasis must lie on practicing problem-solving through in-class exercises, coupled with timely availability of instructor guidance.

Managing student expectations by effectively communicating the challenging yet rewarding nature of the course is paramount. Highlighting the interconnectedness of Linear Algebra concepts with their applications in Computer Science, supported by both theoretical arguments and anecdotal examples, enhances student comprehension, retention and engagement.

For those interested, the course materials, including the textbook and videos, can be obtained from the course website `https://LA4CS.com`, or from the author upon request.

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