8-2021

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Aug 2021

Paper No. 08-2021

Efficient Bilateral Trade via Two-Stage Mechanisms under One-Sided Asymmetric Information*

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This Version: August 2021

Abstract

This paper considers a bilateral-trade model with one-sided asymmetric information in which one agent (seller) initially owns an indivisible object and is fully informed of its value, while the other agent (buyer) intends to obtain the object whose value is unknown to himself. As Jehiel and Pauzner (2006) show that no mechanisms can generally result in efficient, voluntary bilateral trades, we aim to overturn this impossibility result by employing two-stage mechanisms (Mezzetti (2004)) in which first, the outcome (e.g., allocation of the goods) is determined, then the agents observe their own outcome-decision payoffs, and finally, transfers are made. We show that the generalized two-stage Groves mechanism induces efficient, voluntary bilateral trades. On the contrary, we also show by means of an example that the generalized two-stage Groves mechanism fails to achieve efficient, voluntary trades in a two-sided asymmetric information setup in which both parties have private information and each party's valuation depends on the other's information in the same way.

JEL Classification: C72, D78, D82.

Keywords: bilateral trades, one-sided asymmetric information, two-stage mechanisms

*We thank Shurojit Chatterji and Takuro Yamashita for useful comments.

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1 Introduction

This paper investigates efficient, voluntary bilateral trades in an interdependent values environment with one-sided asymmetric information in which one agent (seller) initially owns an indivisible object and is fully informed of its value, while the other agent (buyer) intends to obtain the object whose value is unknown to himself. Efficiency adopted in this paper is an ex post notion, which requires that (i) there be a trade of the good if and only if the buyer's valuation for the good is at least as high as the seller's valuation (decision efficiency) and (ii) whatever the buyer pays be always exactly what the seller receives (budget balance). Voluntary trades mean that each agent of every type has a weak incentive to participate in the mechanism (interim individual rationality). By the well-known revelation principle, efficient, voluntary trades are implementable if there exists a *direct revelation* mechanism satisfying decision efficiency (EFF), interim individual rationality (IIR), and ex post budget balance (BB) in which each agent is asked to announce his own type and telling the true type profile constitutes a Bayesian Nash equilibrium (i.e., Bayesian incentive compatibility (BIC)). Unfortunately, we know from Proposition 1 of Jehiel and Pauzner (2006) that when the single crossing condition fails, there are no mechanisms satisfying BIC, EFF, BB, and IIR in the bilateral trade model with one-sided asymmetric information. In our context, the single crossing property means that the seller's valuation increases faster than the buyer's valuation as the seller's signal increases. The reader is referred to Example 1 of Mezzetti (2004) for an example in which the single crossing property fails. Samuelson (1984) also acknowledges the impossibility of achieving efficient, voluntary trades in a one-sided asymmetric information situation as in this paper. He therefore provides some characterizations of the second-best (or optimal) mechanism.

To overcome this negative message in our one-sided asymmetric information environment, we seek more positive results by looking at *two-stage* mechanisms (Mezzetti (2004)) in which first, the outcome (e.g., allocation of the goods) is determined, then the agents observe their own outcome-decision payoffs, and finally, transfers are made. In his Proposition 1, Mezzetti (2003) establishes the generalized revelation principle, which says that it entails no loss of generality to focus on the following two-stage generalized revelation mechanisms: in the first stage, agents are asked to report their type and the allocation of the good is determined on the type reports; after agents observe their allocation payoff, they are asked to report their realized allocation payoff in the second stage; and finally, the monetary transfers are finalized on the reports of both stages. By this generalized revelation principle, we call a two-stage generalized revelation mechanism simply a two-stage mechanism. Appealing to the generalized revelation principle, we need to modify the notion of Bayesian incentive compatibility: a two-stage mechanism satisfies BIC if there exists a *perfect Bayesian equilibrium* of that two-stage mechanism in which all agents tell the truth in both stages.

The assumption behind the use of two-stage mechanisms can be justified. Imagine that two parties in a bilateral trade setup invite a trusted mediator (a third party) to their contractual relationship: the mediator asks both agents to put a large sum of money as a deposit in the mediator's account and the mediator pays back the remaining deposit to each agent after the two-stage mechanism is played out. We can replace the mediator by a *smart contract* based on the blockchain technology as a commitment device that prevents agents from reneging the contract terms (See, for example, Matsushima and Noda (2020)). Even without a trusted mediator or smart contracts, we can sometimes implement a two-stage mechanism by a long-term relationship. For example, in the context of a labor market, employers learn the quality of the workers after employing them and after both the employer and the worker find out the extent to which the worker is qualified for the job, the worker's contract is revised. Of course, it goes without saying that the power of two-stage mechanisms may well be compromised in some other scenarios.

This paper considers a bilateral trade model with the following features: (i) the seller is fully informed of the quality of the good, while the buyer is uninformed of it. This is related to a market for lemons considered by Akerlof (1970); (ii) each agent's valuation depends upon not only his own type but also the type of other agent (i.e., interdependent values). So, our setup is different from that considered by Myerson and Satterthwaite (1983) who consider a private values environment with two-sided asymmetric information; and (iii) utilities are quasilinear and so, utilities consist of the sum of a payoff from an outcome decision and a monetary transfer.

Section 3 introduces the *generalized two-stage Groves* mechanism. Mezzetti (2004) shows that the generalized two-stage Groves mechanism always satisfies BIC, EFF, and BB in a general mechanism design setup including our bilateral trade model. The main result of this section shows that the generalized two-stage Groves mechanism with lump-sum transfers satisfies BIC, EFF, BB, and IIR in the bilateral trade model with one-sided asymmetric information (Theorem 1). This exhibits a contrast with Jehiel and Pauzner (2006) who establish a general impossibility result within the class of one-stage mechanisms.

In Section 4, we ask whether our Theorem 1 can be extended to a *two-sided* asymmetric information setup in which agents' information is ex ante symmetric, i.e., both parties have private information and each party's valuation depends on the other's information in the same way. It turns out that this extension is not possible. We show this by focusing on a stylized model in which each agent's type is chosen from the uniform distribution over $[0, 1]$ and each agent i's valuation for the object is represented by a linear function, i.e., $\tilde{u}_i(\theta_i, \theta_j) = \theta_i + \gamma_i \theta_j$, where γ_i is considered the degree of interdependence of preferences for agent i. In this context, the single crossing property requires that $\gamma_i < 1$ for each agent i. We show that the generalized two-stage Groves mechanism never satisfies IIR (Proposition 1).

The rest of the paper is organized as follows. In Section 2, we introduce the general notation and basic concepts for the paper and go over some key important results in the literature to benchmark our paper. In Section 3, we show that the generalized two-stage Groves mechanism satisfies BIC, EFF, BB, and IIR (Theorem 1). Section 4 introduces a highly stylized interdependent values model of bilateral trade with two-sided asymmetric information setup in which both parties have private information and each party's valuation depends on the other's information in the same way. In this stylized model, we show that the generalized two-stage Groves mechanism always violates IIR. Section 5 concludes the paper with final remarks. In the Appendix, we provide all the proofs of the results omitted from the main text of the paper.

2 Preliminaries

We consider an economy with two agents. Agent 1 is a seller who initially owns one indivisible object for sale and agent 2 is one potential buyer for the object. For each agent $i \in \{1, 2\}$, Θ_i denotes the set of types, each of which corresponds to agent i's private information about the value of the object. We assume that Θ_1 is a nonempty compact subset of $\mathbb{R}^{m_1}$ where m_1 is a positive integer and $\Theta_2 = \{\bar{\theta}_2\}$ is a singleton. Therefore, we consider a bilateral trade problem with interdependent values in which the seller is fully informed of the value of the object, while the buyer is uninformed of it. This is a typical Akerlof's lemons market (Akerlof (1970)). For $i \in \{1, 2\}$, let $F_i : \Theta_i \to [0, 1]$ be the cumulative distribution function. Types are drawn independently across agents. Note that F_2 is a degenerate distribution function, as Θ_2 is assumed to be a singleton. In what follows, we write $\Theta = \Theta_1 \times \Theta_2$ and $\Theta_{-i} = \Theta_j$ where $j \neq i$ with a generic element θ_{-i}.[1]

Let $q \in Q = [0, 1]$ denote the probability that the good is sold to the buyer,

[1]Similar notation will be used for products of other sets.

or trading probability for short. Preferences of each agent $i \in \{1, 2\}$ are given by $U_i : Q \times \Theta \times \mathbb{R} \to \mathbb{R}$, which depends on the trading probability q, the type profile θ and agent i's monetary transfer t_i:

$$\begin{aligned} U_1(q, \theta, t_1) &= u_1(q, \theta) + t_1 = (1 - q)\tilde{u}_1(\theta) + t_1; \\ U_2(q, \theta, t_2) &= u_2(q, \theta) + t_2 = q\tilde{u}_2(\theta) + t_2, \end{aligned}$$

where $u_i(q, \theta)$ is agent i's allocation payoff and $\tilde{u}_i(\theta)$ is agent i's valuation for the object in state $\theta \in \Theta$. We introduce the following categories to facilitate our analysis. By a *trivial* case we mean that it is always efficient to trade, i.e., $\tilde{u}_2(\theta_1, \theta_2) > \tilde{u}_1(\theta_1, \theta_2)$ for any $(\theta_1, \theta_2) \in \Theta$. We call any other case a *nontrivial case.* We summarize $(\Theta_i, F_i, \tilde{u}_i(\cdot))_{i \in \{1,2\}}$ as a bilateral trade environment and assume that this environment is commonly known between agents.

2.1 One-Stage Mechanisms

In this section, we introduce a notion of one-stage mechanisms (or simply mechanisms) which are differentiated from two-stage mechanisms discussed later. We define a *one-stage* mechanism as (M, x, t) where $M = M_1 \times M_2$ such that

- M_i is agent i's message space;
- $x : M \to [0, 1]$ is the decision rule specifying the trading probability; and
- $t : M \to \mathbb{R}^2$ is the transfer rule specifying the monetary transfer for both agents.

The mechanism works as follows: after observing his own type, each agent i sends a message m_i from M_i to the mechanism. Then, the good is allocated according to the decision rule $x(m)$ and the monetary transfers are finalized as $t(m) = (t_1(m), t_2(m))$ where $m = (m_1, m_2)$ is the reported message profile.

By the revelation principle, we can restrict attention to *direct revelation* mechanisms in which we set $M_i = \Theta_i$ for each agent $i \in \{1, 2\}$. From now on, we call a direct revelation mechanism simply a mechanism. We introduce the properties we want our mechanisms to satisfy.

Definition 1. A mechanism (Θ, x, t) satisfies *Bayesian incentive compatibility* (BIC) if truth-telling constitutes a Bayesian Nash equilibrium; that is, for each agent $i \in \{1, 2\}$ and each type $\theta_i \in \Theta_i$, the equilibrium report is θ_i.

We also assume that each agent has the option of not participating in the mechanism (Θ, x, t) and let $U_i^O(\theta_i)$ be the expected utility of agent i with type θ_i from non-participation. To be specific,

$$U_1^O(\theta_1) = \int_{\Theta_2} \tilde{u}_1(\theta_1, \theta_2) dF_2(\theta_2) \text{ for all } \theta_1 \in \Theta_1$$

and

$$U_2^O(\theta_2) = 0 \text{ for all } \theta_2 \in \Theta_2.$$

We introduce the following individual rationality constraint:

Definition 2. A mechanism (Θ, x, t) satisfies *interim individual rationality* (IIR) if, for all $\theta_1 \in \Theta_1$,

$$\int_{\Theta_2} (u_1(x(\theta_1, \theta_2), \theta_1, \theta_2) + t_1(\theta_1, \theta_2)) \, dF_2(\theta_2) \geq U_1^O(\theta_1),$$

and for all $\theta_2 \in \Theta_2$,

$$\int_{\Theta_1} (u_2(x(\theta_1, \theta_2), \theta_1, \theta_2) + t_2(\theta_1, \theta_2)) \, dF_1(\theta_1) \geq U_2^O(\theta_2).$$

Next, we require that trade occur if and only if there are gains from trade from the ex post point of view.

Definition 3. A mechanism (Θ, x, t) satisfies *decision efficiency* (EFF) if, for all $(\theta_1, \theta_2) \in \Theta$,

$$x(\theta_1, \theta_2) \in \arg\max_{q \in Q} (u_1(q, \theta_1, \theta_2) + u_2(q, \theta_1, \theta_2)).$$

In what follows, we denote by x^* the efficient decision rule. We further require that what the seller receives be exactly the same as what the buyer pays.

Definition 4. A mechanism (Θ, x, t) satisfies *ex post budget balance* (BB) if, for all $(\theta_1, \theta_2) \in \Theta$,

$$t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) = 0.$$

To have a good benchmark, we discuss a result of Jehiel and Pauzner (2006). We assume that for all $i, j \in \{1, 2\}$, $\tilde{u}_i(\theta_1, \theta_2)$ is differentiable in both θ_1 and θ_2 and $\tilde{u}_{i,j} \equiv \partial \tilde{u}_i(\theta_i, \theta_j)/\partial \theta_j > 0$ (i.e., i's utility is strictly increasing in agent j' type). We say that the valuation functions $(\tilde{u}_i(\cdot))_{i \in \{1,2\}}$ satisfies the *single crossing* condition if, for all $i, j \in \{1, 2\}$ with $i \neq j$, we have $\tilde{u}_{i,i} > \tilde{u}_{j,i}$.

In their Proposition 1, Jehiel and Pauzner (2006) show that if the valuation functions $(\tilde{u}_i(\cdot))_{i \in \{1,2\}}$ do not satisfy the single crossing condition, then, no mechanisms satisfy BIC, IIR, EFF and BB. Thus, there is no general hope for efficient, voluntary bilateral trade. This largely explains why Jehiel and Pauzner (2006) and Samuelson (1984) among others rather focus on characterizing the optimal (second-best) mechanisms in the bilateral trade model with one-sided asymmetric information. On the contrary, we adopt the notion of *two-stage* mechanisms, proposed by Mezzetti (2004), to investigate whether the first-best solution is achieved in the same environment.

2.2 Two-Stage Mechanisms

Following Mezzetti (2004), we define a *two-stage* mechanism as a quadruple (M^1, M^2, δ, τ) such that

- M_i^1 is agent i's message space in the first stage and M_i^2 is agent i's message space in the second stage, respectively;
- $\delta : M^1 \to [0, 1]$ is the decision rule specifying the trading probability; and
- $\tau : M^1 \times M^2 \to \mathbb{R}^2$ is the transfer rule specifying the monetary transfer for both agents.

The two-stage mechanism works as follows: in the first stage, after observing his own type, each agent i chooses a message m_i^1 from M_i^1 and send it to the mechanism. Then, the good is allocated according to the decision rule $\delta(m^1)$ where m^1 is the reported message profile in the first stage. In the second stage, after the decision $\delta(m^1)$ is implemented and each agent i observes his realized allocation payoff, he is asked to choose a message m_i^2 from M_i^2 and send it to the mechanism. Finally, the monetary transfers $\tau(m^1, m^2)$ are determined based on the reports of both stages $(m^1, m^2) \in M^1 \times M^2$.

We denote by $\widehat{Q} = \{0, 1\}$ the final decision outcome after randomization is resolved and by $\Pi_i = \{u_i(q, \theta) | \; q \in \widehat{Q}, \theta \in \Theta\}$ the range of agent i's allocation payoffs. We further denote by $r_i = (r_i^1, r_i^2)$ agent i's strategy such that $r_i^1 : \Theta_i \to M_i^1$ is his strategy in the first stage and $r_i^2 : \widehat{Q} \times \Theta_i \times \Pi_i \to M_i^2$ is his strategy in the second stage.

We can justify the use of two-stage mechanisms. Imagine that two parties in a bilateral trade setup invite a trusted mediator (a third party) to their contractual relationship: the mediator asks both agents to put a large sum of money as a deposit in the mediator's account and the mediator pays back the remaining deposit to each agent after the two-stage mechanism is fully played out. As we argue in the introduction, this contractual premise of a trusted mediator can sometimes be replaced by either a smart contract or a long-term relationship.

2.3 Generalized Revelation Mechanisms

For any two-stage mechanism (M^1, M^2, δ, τ), we define the *generalized revelation mechanism* (Θ, Π, x, t) in which we set $M_i^1 = \Theta_i$ and $M_i^2 = \Pi_i$, i.e., the agents are asked to report their own type in the first stage and realized allocation payoffs in the second stage; the decision rule $x : \Theta \to [0, 1]$ is given by the composite function $x(\theta) = \delta(r^1(\theta))$ and the transfer rule $t : \Theta \times \Pi \to \mathbb{R}^2$ is given by the composite function $t_i(\theta, \pi) = \tau_i(r^1(\theta), r^2(\delta(r^1(\theta)), \theta, \pi))$. Since each agent i's allocation payoff $u_i(x(\theta_i, \theta_{-i}), \theta_i, \theta_{-i})$ depends on the whole type profile, then the second-stage reports in the generalized revelation mechanism indeed provide extra information about the type profile, while there is a loss of generality in assuming that the designer only uses the standard "one-stage" revelation mechanisms.

Following Mezzetti (2003), we adopt *perfect Bayesian equilibrium* as a solution concept in extensive form games and appeal to the following generalized revelation principle, the counterpart of revelation principle for one-stage mechanisms.[2]

Lemma 1 (The Generalized Revelation Principle in Mezzetti (2003))**.** Any perfect Bayesian equilibrium outcome of any two-stage mechanism can be implemented as a perfect Bayesian equilibrium of the generalized revelation mechanism in which reporting his true allocation payoff in the second stage and reporting his true type in the first stage is an equilibrium strategy for each player.

From now on, by the generalized revelation principle, we call a generalized revelation mechanism simply a two-stage mechanism. We now discuss the main properties we want our two-stage mechanisms to satisfy. We denote by (θ_1^r, θ_2^r) the first-stage report profile and (u_1^r, u_2^r) the second-stage report profile in a two-stage mechanism, respectively. By the generalized revelation principle, we first extend the notion of Bayesian incentive compatibility to two-stage mechanisms.

Definition 5. A two-stage mechanism (Θ, Π, x, t) satisfies *Bayesian incentive compatibility* (BIC) if truth-telling in both stages constitutes an equilibrium strategy of each agent in a perfect Bayesian equilibrium; that is, for each agent $i \in \{1, 2\}$ and each type profile $(\theta_i, \theta_{-i}), (\theta_i^r, \theta_{-i}^r) \in \Theta$, agent i's equilibrium second-stage report is $u_i^r = u_i(x(\theta_i^r, \theta_{-i}^r), \theta_i, \theta_{-i})$ and equilibrium first-stage report is $\theta_i^r = \theta_i$.

BIC implies that, given the first-stage report, each agent reports his realized allocation payoff truthfully in the second stage. BIC further implies that, on the

[2]For perfect Bayesian equilibrium, for example, the reader is referred to Osborne and Rubinstein (1994, pp.232-233).

equilibrium path, each agent reports his true type in the first stage and for any type profile $(\theta_1, \theta_2) \in \Theta$, $u_i(x(\theta_1, \theta_2), \theta_1, \theta_2)$ is agent i's true allocation payoff.

We next adapt the individual rationality constraint to two-stage mechanisms:

Definition 6. A two-stage mechanism (Θ, Π, x, t) satisfies *interim individual rationality* (IIR) if, for all $\theta_1 \in \Theta_1$,

$$\int_{\Theta_2} (u_1(x(\theta_1, \theta_2), \theta_1, \theta_2) + t_1(\theta_1, \theta_2; u_1, u_2))\, dF_2(\theta_2) \geq U_1^O(\theta_1),$$

and for all $\theta_2 \in \Theta_2$,

$$\int_{\Theta_1} (u_2(x(\theta_1, \theta_2), \theta_1, \theta_2) + t_2(\theta_1, \theta_2; u_1, u_2))\, dF_1(\theta_1) \geq U_2^O(\theta_2),$$

where $u_1 = u_1(x(\theta_1, \theta_2), \theta_1, \theta_2)$ and $u_2 = u_2(x(\theta_1, \theta_2), \theta_1, \theta_2)$.

Third, we adapt the ex post budget balance to two-stage mechanisms:

Definition 7. A two-stage mechanism (Θ, Π, x, t) satisfies *ex post budget balance* (BB) if, for all $(\theta_1, \theta_2) \in \Theta$,

$$t_1(\theta_1, \theta_2; u_1, u_2) + t_2(\theta_1, \theta_2; u_1, u_2) = 0,$$

where $u_1 = u_1(x(\theta_1, \theta_2), \theta_1, \theta_2)$ and $u_2 = u_2(x(\theta_1, \theta_2), \theta_1, \theta_2)$.

We consider the generalized two-stage Groves mechanism proposed by Mezzetti (2004) and adapt it to our bilateral trade model.

Definition 8. A two-stage mechanism (Θ, Π, x^*, t^G) is called the *generalized two-stage Groves mechanism* if, for each agent $i \in \{1, 2\}$, type report $(\theta_i^r, \theta_{-i}^r) \in \Theta$ and payoff report $(u_i^r, u_{-i}^r) \in \Pi_i \times \Pi_{-i}$, agent i's transfer is determined as follows:

$$t_i^G(\theta_i^r, \theta_{-i}^r; u_i^r, u_{-i}^r) = u_{-i}^r - h_i(\theta_i^r, \theta_{-i}^r),$$

where

$$\begin{aligned} 2h_i(\theta_i^r, \theta_{-i}^r) \quad &= \quad \sum_{j=1}^{2} u_j\left(x^*(\theta^r), \theta^r\right) - \mathbb{E}_{\theta_{-i}}\left(\sum_{j=1}^{2} u_j\left(x^*(\theta_i^r, \theta_{-i}), \theta_i^r, \theta_{-i}\right)\right) \\ &\quad + \mathbb{E}_{\theta_{-(i+1)}}\left(\sum_{j=1}^{2} u_j\left(x^*(\theta_{i+1}^r, \theta_{-(i+1)}), \theta_{i+1}^r, \theta_{-(i+1)}\right)\right), \end{aligned}$$

where $\mathbb{E}_{\theta_{-i}}$ denotes the expectation operator over θ_{-i} and

$$i + 1 = \begin{cases} i + 1 & \text{if } i = 1 \\ 1 & \text{if } i = 2. \end{cases}$$

The result below is already proved by Mezzetti (2004, Proposition 2) in a general mechanism design setup including this paper's model.

Lemma 2. The generalized two-stage Groves mechanism always satisfies BIC, EFF, and BB.

Jehiel and Moldovanu (2001) show that there are no one-stage mechanisms satisfying BIC and EFF in a generic interdependent values environment where the type space is a multi-dimensional Euclidean subspace. So, the above result is a lot more permissive than the one-stage mechanism counterpart. However, it is not clear whether the generalized two-stage Groves mechanism also satisfies IIR. We will show in the next section that the generalized two-stage Groves mechanism does satisfy IIR in the bilateral trade model with one-sided asymmetric information.

3 Main Results

To illustrate the power of two-stage mechanisms, Mezzetti (2004, Example 1) argues by means of an example that it is possible to construct a two-stage mechanism satisfying BIC, IIR, EFF and BB, even if the single crossing condition is violated. The main objective of this paper is to extend this insight to its full generality. That is, we go beyond this example and show that one can construct a two-stage mechanism satisfying all the four properties in the bilateral trade model with one-sided asymmetric information.

We are ready to state our main result.

Theorem 1. In the bilateral trade model with one-sided asymmetric information, the generalized two-stage Groves mechanism with lump-sum transfers always satisfies BIC, EFF, BB, and IIR in all nontrivial cases.

Remark: Recall that a trivial case means that it is always efficient to trade, i.e., $\tilde{u}_2(\theta_1, \theta_2) > \tilde{u}_1(\theta_1, \theta_2)$ for any $(\theta_1, \theta_2) \in \Theta$. We can easily show that if we adapt the *shoot-the-liar* mechanism, proposed by Mezzetti (2007) in an auction setup, to our setup and that the modified shoot-the-liar mechanism satisfies BIC, EFF, BB, and IIR in any trivial case. The modified shoot-the-liar mechanism roughly works as follows: first, based on the first-stage type reports, the good is always delivered to the buyer. Second, the seller receives a transfer from the buyer; this transfer, which can be made before the buyer observes his allocation payoff, is equal to the true value of the good to the buyer, $\tilde{u}_2(\theta_1, \bar{\theta}_2)$. Third, the buyer reports his realized allocation payoff in the second reporting stage. If this report is inconsistent with the type report made by the seller in the first stage, then the seller is imposed severe fines. On the equilibrium path, the seller will not lie about his type in the first stage.

Proof. We know from Lemma 2 that the generalized two-stage Groves mechanism always satisfies BIC, EFF and BB. It thus suffices to show that the generalized two-stage Groves mechanism also satisfies IIR. The proof is completed by the following three steps.

Step I: We show that the seller's worst-off types from participating in the generalized two-stage Groves mechanism are those where it is efficient not to trade and that the expected loss of his worst-off type, denoted by L_1, is

$$L_1 = \frac{1}{2}\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right]. \tag{1}$$

Proof. The proof is in the Appendix. □

Step II: We show that the buyer's expected loss from participating in the generalized two-stage Groves mechanism is a constant, which equals the following:

$$L_2 = -\frac{1}{2}\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right]. \tag{2}$$

Proof. The proof is in the Appendix. □

Step III: We verify that $L_1+L_2 = 0$ and thus, by Proposition 3 of Mezzetti (2003), the generalized two-stage Groves mechanism with lump-sum transfers satisfies IIR without violating BIC, EFF and BB.

Proof. It follows from Proposition 3 of Mezzetti (2003) that the generalized two-stage Groves mechanism with lump-sum transfers satisfies IIR without violating BIC, EFF and BB if and only if $L_1 + L_2 \leq 0$. Since

$$L_1 + L_2 = \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right) - \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j\left(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2\right)\right) = 0,$$

we conclude that the generalized two-stage Groves mechanism with lump-sum transfers satisfies IIR. This completes the proof of Step III. □

This completes the proof of the theorem. □

4 Two-Sided Asymmetric Information

We know from Theorem 1 that the generalized two-stage Groves mechanism always satisfies BIC, EFF, BB, and IIR under one-sided asymmetric information. In this section, we will show by means of an example that this result cannot be extended to two-sided asymmetric information environments.

We specialize in the following environment: both agents' types are uniformly distributed on the unit interval $[0, 1]$ and for each type profile $(\theta_1, \theta_2) \in [0, 1]^2$, their valuation functions are $\tilde{u}_1(\theta_1, \theta_2) = \theta_1 + \gamma_1\theta_2$ and $\tilde{u}_2(\theta_1, \theta_2) = \theta_2 + \gamma_2\theta_1$ where $\gamma_1, \gamma_2 > 0$. Suppose that the single crossing condition is satisfied, which implies that $\gamma_1 < 1$ and $\gamma_2 < 1$. In the proposition below, we show that the generalized two-stage Groves mechanism violates IIR in this example. For this result, we even allow for lump-sum transfers in addition to the original transfers in the generalized two-stage Groves mechanism. This simply makes our result stronger.

Proposition 1. The generalized two-stage Groves mechanism (Θ, Π, x^*, t^G) with lump-sum transfers violates IIR in this example.

Remark: We have the impossibility result for one-stage mechanisms, as we can apply Theorem 5 of Fieseler, Kittsteiner, and Moldovanu (2003) to this example so that there are no one-stage mechanisms satisfying BIC, EFF, BB, and IIR.

Proof. Since $\tilde{u}_2(\theta_1, \theta_2) - \tilde{u}_1(\theta_1, \theta_2) = (1 - \gamma_1)\theta_2 - (1 - \gamma_2)\theta_1$ for each $(\theta_1, \theta_2) \in \Theta$, then we have that $\tilde{u}_2(\theta_1, \theta_2) > \tilde{u}_1(\theta_1, \theta_2)$ if and only if $\theta_2 > (1 - \gamma_2)\theta_1/(1 - \gamma_1)$. Hence, the efficient decision rule dictates that, for each $(\theta_1, \theta_2) \in \Theta$,

$$x^*(\theta_1, \theta_2) = \begin{cases} 1 & \text{if } \theta_2 > (1 - \gamma_2)\theta_1/(1 - \gamma_1) \\ 0 & \text{otherwise.} \end{cases}$$

There are two cases we consider: (i) $0 < \gamma_2 \leq \gamma_1 < 1$ and (ii) $0 < \gamma_1 < \gamma_2 < 1$. The following two figures illustrate the decision at different type profiles in the two cases, respectively; in particular, the shaded regions represent $\Theta^* = \{(\theta_1, \theta_2) \in \Theta : x^*(\theta_1, \theta_2) = 1\}$, which exhausts all the type profiles in which trade occurs. Note that Θ^* in each of the two cases results in a different set, leading to different integration results when we compute the agents' expected losses.

We divide our proof into two cases. In each case, the proof is completed by three steps. We first consider Case 1: $0 < \gamma_2 \leq \gamma_1 < 1$. We complete the proof by the following three steps.

Figure 1: when $0 < \gamma_2 \leq \gamma_1 < 1$

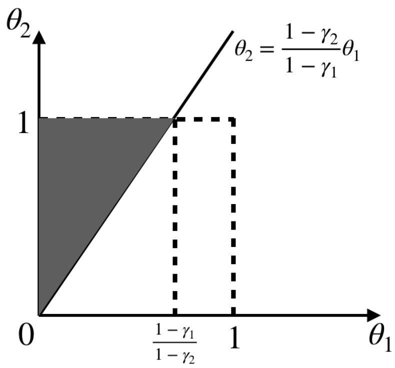
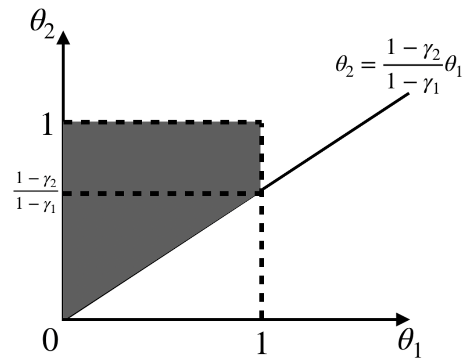


Figure 2: when $0 < \gamma_1 < \gamma_2 < 1$



Step 1-1: We show that the seller's worst-off type after participation is $\theta_1^w = 1$ and that the expected loss of his worst-off type, denoted by L_1, is

$$L_1 = \frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12}\frac{(1-\gamma_1)^2}{1-\gamma_2}. \tag{3}$$

Proof. The proof is in the Appendix. □

Step 1-2: We show that the buyer's worst-off type after participation is $\theta_2^w = 0$ and that the expected loss of his worst-off type, denoted by L_2, is

$$L_2 = -\frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12}\frac{(1-\gamma_1)^2}{1-\gamma_2}. \tag{4}$$

Proof. The proof is in the Appendix. □

Step 1-3: We verify that $L_1 + L_2 > 0$ in this case and thus, by Proposition 3 of Mezzetti (2003), the generalized two-stage Groves mechanism violates IIR.

Proof. The proof is in the Appendix. □

We complete the proof in Case 1.

Next we consider Case 2: $0 < \gamma_1 < \gamma_2 < 1$. We complete the proof by the following three steps, as in Case 1.

Step 2-1: We show that the seller's worst-off type after participation is $\theta_1^w = 1$ and that the expected loss of his worst-off type, denoted by L_1, is

$$L_1 = -\frac{1}{2}\frac{(\gamma_2-\gamma_1)^2}{1-\gamma_1} + \frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1}. \tag{5}$$

Proof. The proof is in the Appendix. □

Step 2-2: We show that the buyer's worst-off type after participation is $\theta_2^w = 0$ and that the expected loss of his worst-off type, denoted by L_2, is

$$L_2 = -\frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1}. \tag{6}$$

Proof. The proof is in the Appendix. □

Step 2-3: We verify that $L_1 + L_2 > 0$ in this case and thus, by Proposition 3 of Mezzetti (2003), the generalized two-stage Groves mechanism violates IIR.

Proof. The proof is in the Appendix. □

This completes the proof in Case 2. Therefore, the generalized two-stage Groves mechanism violates IIR. □

5 Concluding Remarks

This paper considers a bilateral trade model with interdependent values in which one agent (seller) initially owns an indivisible object and is fully informed of its value, while the other agent (buyer) intends to obtain the object whose value is unknown to himself. The main objective of this paper is to overturn the impossibility results on standard mechanisms (which we call one-stage mechanisms) by employing two-stage mechanisms (Mezzetti (2004)) in which first, the final outcome (e.g., allocation of the goods) is determined, then the agents observe their own outcome-decision payoffs, and finally, transfers are made. We show that the generalized two-stage Groves mechanism with lump-sum transfers satisfies BIC, EFF, BB, and IIR. This exhibits a stark contrast with the one-stage mechanism counterpart: Jehiel and Pauzner (2006) show that when the single-crossing condition fails, there are no one-stage mechanisms satisfying BIC, EFF, BB, and IIR. We also show by means of an example that this possibility result via two-stage mechanisms cannot be extended to a two-sided asymmetric information setup in which agents' information is ex ante symmetric (i.e., both parties have private information and each party's valuation depends on the other's information in the same way). This leaves us an open question of whether there exist other two-stage mechanisms satisfying BIC, EFF, BB, and IIR in a two-sided asymmetric information environment, even though the generalized two-stage Groves mechanism fails to satisfy those properties. We fully investigate this very question in Kunimoto and Zhang (2021).

6 Appendix

In the Appendix, we provide all the proofs omitted from the main text of the paper.

6.1 Proof of Theorem 1

6.1.1 Proof of Step I

Proof. On the equilibrium path in which both agents report truthfully in both stages, agent i of type θ_i receives the following expected transfer:

$$\mathbb{E}_{\theta_{-i}}\left[t_i^G(\theta_i,\theta_{-i};u_i,u_{-i})\right]=\mathbb{E}_{\theta_{-i}}\left[u_{-i}(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})-h_i(\theta_i,\theta_{-i})\right],$$

where $u_i=u_i(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})$ and $u_{-i}=u_{-i}(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})$. Note

$$\begin{aligned}\mathbb{E}_{\theta_{-i}}\left[h_i(\theta_i,\theta_{-i})\right] &= \frac{1}{2}\mathbb{E}_{\theta_{-i}}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right]-\frac{1}{2}\mathbb{E}_{\theta_{-i}}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right]\\ &\quad+\frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right],\end{aligned}$$

which can be simplified as follows:

$$\mathbb{E}_{\theta_{-i}}\left[h_i(\theta_i,\theta_{-i})\right]=\frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right],$$

where $\mathbb{E}$ denotes the expectation over (θ_i,θ_{-i}). Then, we obtain the expected transfer of agent i:

$$\mathbb{E}_{\theta_{-i}}\left[t_i^G(\theta_i,\theta_{-i};u_i,u_{-i})\right]=\mathbb{E}_{\theta_{-i}}\left[u_{-i}(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right]-\frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right].$$

We compute the expected utility of agent i of type θ_i as follows:

$$\begin{aligned}U_i^G(\theta_i) &= \mathbb{E}_{\theta_{-i}}\left[u_i(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})+t_i^G(\theta_i,\theta_{-i};u_i,u_{-i})\right]\\ &= \mathbb{E}_{\theta_{-i}}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right]-\frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2}u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right].\end{aligned}$$

Consider the seller of type θ_1. The seller receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}U_1^G(\theta_1) &= \mathbb{E}_{\theta_2}\left[\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right]-\frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right]\\ &= \sum_{j=1}^{2}u_j(x^*(\theta_1,\bar\theta_2),\theta_1,\bar\theta_2)-\frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\bar\theta_2),\theta_1,\bar\theta_2)\right),\end{aligned}$$

where the second equality follows because the buyer's type space $\Theta_2 = \{\bar{\theta}_2\}$ is a singleton. Then, we can derive the worst-off type θ_1^w of the seller from participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}\theta_1^w &\in \arg\min_{\theta_1 \in \Theta_1} \left[U_1^G(\theta_1) - U_1^O(\theta_1)\right] \\ &= \arg\min_{\theta_1 \in \Theta_1} \left[\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2) - \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right) - U_1^O(\theta_1)\right].\end{aligned}$$

Since the second term is a constant, the above expression can be rewritten as follows:

$$\theta_1^w \in \arg\min_{\theta_1 \in \Theta_1} \left[\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2) - U_1^O(\theta_1)\right] = \arg\min_{\theta_1 \in \Theta_1} \left[\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2) - \tilde{u}_1(\theta_1, \bar{\theta}_2)\right].$$

Note that for all $\theta_1 \in \Theta_1$,

$$\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2) - \tilde{u}_1(\theta_1, \bar{\theta}_2) = \begin{cases} \tilde{u}_2(\theta_1, \bar{\theta}_2) - \tilde{u}_1(\theta_1, \bar{\theta}_2) & \text{if } x^*(\theta_1, \bar{\theta}_2) = 1 \\ \tilde{u}_1(\theta_1, \bar{\theta}_2) - \tilde{u}_1(\theta_1, \bar{\theta}_2) & \text{otherwise.} \end{cases}$$

Since $x^*(\theta_1, \bar{\theta}_2) = 1$ implies that the buyer has a higher valuation than the seller, i.e., $\tilde{u}_2(\theta_1, \bar{\theta}_2) > \tilde{u}_1(\theta_1, \bar{\theta}_2)$, then the seller's worst-off types are those where it is efficient not to trade. Recall that we call it a trivial case if it is always efficient to trade. So, the existence of the seller's worst-off types is guaranteed because we only consider nontrivial cases in this theorem. At the worst-off type θ_1^w, the seller's expected loss from participating in the generalized two-stage Groves mechanism, denoted by L_1, is

$$\begin{aligned}L_1 &= U_1^O(\theta_1^w) - U_1^G(\theta_1^w) \\ &= -\left[\sum_{j=1}^{2} u_j(x^*(\theta_1^w, \bar{\theta}_2), \theta_1^w, \bar{\theta}_2) - \tilde{u}_1(\theta_1^w, \bar{\theta}_2)\right] + \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right) \\ &= -\left[\tilde{u}_1(\theta_1^w, \bar{\theta}_2) - \tilde{u}_1(\theta_1^w, \bar{\theta}_2)\right] + \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right),\end{aligned}$$

which can be further simplified as follows:

$$L_1 = \frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \bar{\theta}_2), \theta_1, \bar{\theta}_2)\right).$$

This completes the proof of Step I. □

6.1.2 Proof of Step II

Proof. Recall that agent i of type θ_i receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$U_i^G(\theta_i) = \mathbb{E}_{\theta_{-i}}\left[\sum_{j=1}^{2} u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2} u_j(x^*(\theta_i,\theta_{-i}),\theta_i,\theta_{-i})\right].$$

Consider the buyer of type θ_2. The buyer receives the following expected utility after participation:

$$U_2^G(\theta_2) = \mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right].$$

Since the buyer's type space is a singleton, i.e., $\Theta_2 = \{\bar{\theta}_2\}$, then the buyer's expected utility is rewritten as follows:

$$\begin{aligned} U_2^G(\bar{\theta}_2) &= \mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\bar{\theta}_2),\theta_1,\bar{\theta}_2)\right] - \frac{1}{2}\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\bar{\theta}_2),\theta_1,\bar{\theta}_2)\right] \\ &= \frac{1}{2}\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\bar{\theta}_2),\theta_1,\bar{\theta}_2)\right]. \end{aligned}$$

Since the buyer has no private information at the beginning of the first stage, the buyer's expected loss is a constant, which is

$$L_2 = U_2^O(\bar{\theta}_2) - U_2^G(\bar{\theta}_2) = -\frac{1}{2}\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j\left(x^*(\theta_1,\bar{\theta}_2),\theta_1,\bar{\theta}_2\right)\right),$$

where the last equality follows because the buyer's outside option utility $U_2^O(\bar{\theta}_2)$ equals zero. This completes the proof of Step II. □

6.2 Proof of Proposition 1

6.2.1 Proof of Step 1-1

Proof. Recall that the seller of type θ_1 receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$U_1^G(\theta_1) = \mathbb{E}_{\theta_2}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right].$$

Then, we can derive the worst-off type θ_1^w of the seller from participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}
\theta_1^w &\in \arg\min_{\theta_1\in\Theta_1} \left[U_1^G(\theta_1) - U_1^O(\theta_1)\right] \\
&= \arg\min_{\theta_1\in\Theta_1} \left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - U_1^O(\theta_1)\right].
\end{aligned}$$

Since the second term is a constant, the above expression can be rewritten as follows:

$$\begin{aligned}
\theta_1^w &\in \arg\min_{\theta_1\in\Theta_1} \left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - U_1^O(\theta_1)\right] \\
&= \arg\min_{\theta_1\in\Theta_1} \mathbb{E}_{\theta_2}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2) - \tilde{u}_1(\theta_1,\theta_2)\right].
\end{aligned}$$

Note that for all $\theta_1 \in \Theta_1$,

$$\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2) - \tilde{u}_1(\theta_1,\theta_2) = \begin{cases} \tilde{u}_2(\theta_1,\theta_2) - \tilde{u}_1(\theta_1,\theta_2) & \text{if } x^*(\theta_1,\theta_2) = 1, \\ \tilde{u}_1(\theta_1,\theta_2) - \tilde{u}_1(\theta_1,\theta_2) & \text{otherwise.} \end{cases}$$

Since $x^*(\theta_1,\theta_2) = 1$ implies that the buyer has a higher valuation than the seller, i.e., $\tilde{u}_2(\theta_1,\theta_2) > \tilde{u}_1(\theta_1,\theta_2)$, then the seller's worst-off types are those where it is always efficient not to trade. Indeed, in Case 1, when the seller has type $\theta_1 = 1$, it is always efficient not to trade. Therefore, the seller's worst-off type is $\theta_1^w = 1$. At the worst-off type $\theta_1^w = 1$, the seller's expected loss from participating in the generalized two-stage Groves mechanism, denoted by L_1, is

$$\begin{aligned}
L_1 &= U_1^O(\theta_1^w) - U_1^G(\theta_1^w) \\
&= -\mathbb{E}_{\theta_2}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1^w,\theta_2),\theta_1^w,\theta_2) - \tilde{u}_1(\theta_1^w,\theta_2)\right] + \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) \\
&= \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right).
\end{aligned}$$

Note that for each $\theta_2 \in \Theta_2$,

$$\begin{aligned}
&\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) \\
=\ & \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2} \tilde{u}_2(\theta_1,\theta_2)d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2}^{1} \tilde{u}_1(\theta_1,\theta_2)d\theta_1 \\
=\ & \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2} (\gamma_2\theta_1+\theta_2)d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2}^{1} (\theta_1+\gamma_1\theta_2)d\theta_1 \\
=\ & \frac{1}{2}\gamma_2\left(\frac{1-\gamma_1}{1-\gamma_2}\theta_2\right)^2 + \frac{1-\gamma_1}{1-\gamma_2}(\theta_2)^2 + \frac{1}{2}\left(1-\left(\frac{1-\gamma_1}{1-\gamma_2}\theta_2\right)^2\right) + \gamma_1\theta_2\left(1-\frac{1-\gamma_1}{1-\gamma_2}\theta_2\right) \\
=\ & \frac{1}{2} + \gamma_1\theta_2 + \frac{1}{2}\frac{(1-\gamma_1)^2}{1-\gamma_2}(\theta_2)^2. \qquad (7)
\end{aligned}$$

Then, we compute the expected losss for the seller's worst-off type $\theta_1^w = 1$:

$$\begin{aligned}
L_1 &= \frac{1}{2}\mathbb{E}_{\theta_2}\left[\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\right] \\
&= \frac{1}{2}\int_0^1 \left[\frac{1}{2} + \gamma_1\theta_2 + \frac{1}{2}\frac{(1-\gamma_1)^2}{1-\gamma_2}(\theta_2)^2\right] d\theta_2 \\
&= \frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12}\frac{(1-\gamma_1)^2}{1-\gamma_2}. \qquad (8)
\end{aligned}$$

This completes the proof of Step 1-1. □

6.2.2 Proof of Step 1-2

Proof. Recall that the buyer of type θ_2 receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$U_2^G(\theta_2) = \mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right].$$

Then, we can derive the worst-off type θ_2^w of the buyer from participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}
\theta_2^w &\in \arg\min_{\theta_2\in\Theta_2}\left[U_2^G(\theta_2) - U_2^O(\theta_2)\right] \\
&= \arg\min_{\theta_2\in\Theta_2}\left[\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\right],
\end{aligned}$$

where the equality follows because the buyer's outside option utility $U_2^O(\theta_2)$ is always zero for any $\theta_2 \in \Theta_2$. Since the second term is a constant, the above expression can be rewritten as follows:

$$\theta_2^w \in \arg\min_{\theta_2 \in \Theta_2} \left[\mathbb{E}_{\theta_1} \left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \theta_2), \theta_1, \theta_2) \right) \right] = \arg\min_{\theta_2 \in \Theta_2} \left[\frac{1}{2} + \gamma_1 \theta_2 + \frac{1}{2} \frac{(1-\gamma_1)^2}{1-\gamma_2} (\theta_2)^2 \right],$$

where the equality follows from (7). It is easy to see that the buyer's worst-off type is $\theta_2^w = 0$. At the worst-off type $\theta_2^w = 0$, the buyer's expected loss from participating in the generalized two-stage Groves mechanism, denoted by L_2, is

$$\begin{aligned} L_2 &= U_2^G(\theta_2^w) - U_2^O(\theta_2^w) \\ &= -\mathbb{E}_{\theta_1} \left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \theta_2^w), \theta_1, \theta_2^w) \right) + \frac{1}{2} \mathbb{E} \left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \theta_2), \theta_1, \theta_2) \right) \\ &= -\frac{1}{2} + \frac{1}{2} \mathbb{E} \left(\sum_{j=1}^{2} u_j(x^*(\theta_1, \theta_2), \theta_1, \theta_2) \right), \end{aligned}$$

which can be simplified as follows:

$$L_2 = -\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12} \frac{(1-\gamma_1)^2}{1-\gamma_2} = -\frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12} \frac{(1-\gamma_1)^2}{1-\gamma_2},$$

where the first equality follows because of (8). This completes the proof of Step 1-2. □

6.2.3 Proof of Step 1-3

Proof. It follows from Proposition 3 of Mezzetti (2003) that the generalized two-stage Groves mechanism with lump-sum transfers satisfies IIR without violating BIC, EFF and BB if and only if $L_1 + L_2 \leq 0$. We compute:

$$\begin{aligned} L_1 + L_2 &= \frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12} \frac{(1-\gamma_1)^2}{1-\gamma_2} - \frac{1}{4} + \frac{1}{4}\gamma_1 + \frac{1}{12} \frac{(1-\gamma_1)^2}{1-\gamma_2} \\ &= \frac{1}{2}\gamma_1 + \frac{1}{6} \frac{(1-\gamma_1)^2}{1-\gamma_2}. \end{aligned}$$

Since $\gamma_1 > 0$ and $0 < \gamma_2 < 1$, we conclude $L_1 + L_2 > 0$, implying that the generalized two-stage Groves mechanism with lump-sum transfers violates IIR in Case 1. This completes the proof of Step 1-3. □

6.2.4 Proof of Step 2-1

Proof. Recall that the seller of type θ_1 receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$U_1^G(\theta_1) = \mathbb{E}_{\theta_2}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right].$$

Then, we can derive the worst-off type θ_1^w of the seller from participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}\theta_1^w &\in \arg\min_{\theta_1\in\Theta_1}\left[U_1^G(\theta_1) - U_1^O(\theta_1)\right] \\ &= \arg\min_{\theta_1\in\Theta_1}\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - U_1^O(\theta_1)\right].\end{aligned}$$

Since the second term is a constant, the above expression can be rewritten as follows:

$$\begin{aligned}\theta_1^w &\in \arg\min_{\theta_1\in\Theta_1}\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - U_1^O(\theta_1)\right] \\ &= \arg\min_{\theta_1\in\Theta_1}\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - \int_{\Theta_2}\tilde{u}_1(\theta_1,\theta_2)dF_2(\theta_2)\right].\end{aligned}$$

Note that

$$\begin{aligned}&\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) \\ =& \int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}\tilde{u}_1(\theta_1,\theta_2)d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}^{1}\tilde{u}_2(\theta_1,\theta_2)d\theta_2 \\ =& \int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}(\theta_1+\gamma_1\theta_2)d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}^{1}(\gamma_2\theta_1+\theta_2)d\theta_2 \\ =& \frac{1-\gamma_2}{1-\gamma_1}(\theta_1)^2 + \frac{1}{2}\frac{\gamma_1(1-\gamma_2)^2}{(1-\gamma_1)^2}(\theta_1)^2 + \gamma_2\theta_1\left(1-\frac{1-\gamma_2}{1-\gamma_1}\theta_1\right) + \frac{1}{2}\left[1-\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^2(\theta_1)^2\right] \\ =& \frac{1}{2}+\gamma_2\theta_1+\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}(\theta_1)^2. \qquad (9)\end{aligned}$$

Then, we compute the seller's worst-off type as follows:

$$\begin{aligned}\theta_1^w &\in \arg\min_{\theta_1\in\Theta_1}\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^{2} u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) - \int_{\Theta_2}\tilde{u}_1(\theta_1,\theta_2)dF_2(\theta_2)\right] \\ &= \arg\min_{\theta_1\in\Theta_1}\left[\frac{1}{2}+\gamma_2\theta_1+\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}(\theta_1)^2 - \theta_1 - \frac{1}{2}\gamma_1\right],\end{aligned}$$

which can be rewritten as follows:

$$\theta_1^w \in \arg\min_{\theta_1\in\Theta_1} \left[\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}\left(\theta_1 - \frac{1-\gamma_1}{1-\gamma_2}\right)^2\right].$$

Since $0<\gamma_1<\gamma_2<1$ implies $(1-\gamma_1)/(1-\gamma_2)>1$, the seller's worst-off type is $\theta_1^w = 1$. At the worst-off type $\theta_1^w = 1$, the seller's expected loss from participating in the generalized two-stage Groves mechanism, denoted by L_1, is

$$\begin{aligned}
L_1 &= U_1^O(\theta_1^w) - U_1^G(\theta_1^w) \\
&= -\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^2 u_j(x^*(\theta_1^w,\theta_2),\theta_1^w,\theta_2)\right) - \int_{\Theta_2}\tilde{u}_1(\theta_1^w,\theta_2)dF_2(\theta_2)\right] + \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) \\
&= -\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}\left(1-\frac{1-\gamma_1}{1-\gamma_2}\right)^2 + \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right).
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{1}{2}\mathbb{E}\left(\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) &= \frac{1}{2}\mathbb{E}_{\theta_1}\left[\mathbb{E}_{\theta_2}\left(\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\right] \\
&= \frac{1}{2}\int_0^1\left[\int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}\tilde{u}_1(\theta_1,\theta_2)d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}^1 \tilde{u}_2(\theta_1,\theta_2)d\theta_2\right]d\theta_1 \\
&= \frac{1}{2}\int_0^1\left[\frac{1}{2}+\gamma_2\theta_1+\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}(\theta_1)^2\right]d\theta_1 \text{ (recall (9))} \\
&= \frac{1}{4}+\frac{1}{4}\gamma_2+\frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1}. \qquad (10)
\end{aligned}$$

Then, the seller's expected loss from participating in the generalized two-stage Groves mechanism is

$$\begin{aligned}
L_1 &= -\frac{1}{2}\frac{(1-\gamma_2)^2}{1-\gamma_1}\left(1-\frac{1-\gamma_1}{1-\gamma_2}\right)^2 + \frac{1}{2}\mathbb{E}\left(\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right) \\
&= -\frac{1}{2}\frac{(\gamma_2-\gamma_1)^2}{1-\gamma_1}+\frac{1}{4}+\frac{1}{4}\gamma_2+\frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1}.
\end{aligned}$$

This completes the proof of Step 2-1. □

6.2.5 Proof of Step 2-2

Proof. Recall that the buyer of type θ_2 receives the following expected utility after participating in the generalized two-stage Groves mechanism:

$$U_2^G(\theta_2) = \mathbb{E}_{\theta_1}\left[\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right] - \frac{1}{2}\mathbb{E}\left[\sum_{j=1}^2 u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right].$$

Then, we can derive the worst-off type θ_2^w of the buyer from participating in the generalized two-stage Groves mechanism:

$$\begin{aligned}\theta_2^w &\in \arg\min_{\theta_2\in\Theta_2}\left[U_2^G(\theta_2)-U_2^O(\theta_2)\right]\\ &= \arg\min_{\theta_2\in\Theta_2}\left[\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)-\frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\right],\end{aligned}$$

where the equality follows because the buyer's outside option utility $U_2^O(\theta_2)$ is always zero for any $\theta_2\in\Theta_2$. Since the second term is a constant, the above expression can be rewritten as follows:

$$\theta_2^w \in \arg\min_{\theta_2\in\Theta_2}\left[\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\right].$$

We identify the worst-off type for the buyer by the following cases.

Case (i): $0<\theta_2<(1-\gamma_2)/(1-\gamma_1)$

$$\begin{aligned}\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2}u_j(x^*(\theta_2,\theta_1),\theta_2,\theta_1)\right] &= \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2}\tilde{u}_2(\theta_1,\theta_2)d\theta_1+\int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2}^{1}\tilde{u}_1(\theta_1,\theta_2)d\theta_1\\ &= \frac{1}{2}+\gamma_1\theta_2+\frac{1}{2}\frac{(1-\gamma_1)^2}{1-\gamma_2}(\theta_2)^2 \text{ (recall (7))}.\end{aligned}$$

It is easy to see that $\theta_2=0$ achieves its minimum, which is $1/2$.

Case (ii): $(1-\gamma_2)/(1-\gamma_1)\le\theta_2\le 1$

$$\mathbb{E}_{\theta_1}\left[\sum_{j=1}^{2}u_j(x^*(\theta_2,\theta_1),\theta_2,\theta_1)\right]=\int_0^1\tilde{u}_2(\theta_1,\theta_2)d\theta_1=\int_0^1(\gamma_2\theta_1+\theta_2)d\theta_1=\frac{1}{2}\gamma_2+\theta_2.$$

Clearly, $\theta_2=(1-\gamma_2)/(1-\gamma_1)$ achieves its minimum, which is $\gamma_2/2+(1-\gamma_2)/(1-\gamma_1)$.

Since

$$\frac{1}{2}-\left[\frac{1}{2}\gamma_2+\frac{1-\gamma_2}{1-\gamma_1}\right]=-\frac{(1-\gamma_2)(1+\gamma_1)}{2(1-\gamma_1)}<0,$$

then the buyer's worst-off type is $\theta_2^w=0$. At the worst-off type $\theta_2^w=0$, the buyer's expected loss from participating in the generalized two-stage Groves mechanism, denoted by L_2, is

$$\begin{aligned}L_2 &= U_2^O(\theta_2^w)-U_2^G(\theta_2^w)\\ &= -\mathbb{E}_{\theta_1}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2^w),\theta_1,\theta_2^w)\right)+\frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right)\\ &= -\frac{1}{2}+\frac{1}{2}\mathbb{E}\left(\sum_{j=1}^{2}u_j(x^*(\theta_1,\theta_2),\theta_1,\theta_2)\right),\end{aligned}$$

which can be simplified as follows:

$$L_2 = -\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1} = -\frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1},$$

where the first equality follows because of (10). This completes the proof of Step 2-2. □

6.2.6 Proof of Step 2-3

Proof. It follows from Proposition 3 of Mezzetti (2003) that the generalized two-stage Groves mechanism with lump-sum transfers satisfies IIR without violating BIC, EFF and BB if and only if $L_1 + L_2 \leq 0$. We compute

$$\begin{aligned} L_1 + L_2 &= -\frac{1}{2}\frac{(\gamma_2-\gamma_1)^2}{1-\gamma_1} + \frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1} - \frac{1}{4} + \frac{1}{4}\gamma_2 + \frac{1}{12}\frac{(1-\gamma_2)^2}{1-\gamma_1} \\ &= \frac{1}{2(1-\gamma_1)}\left[\gamma_2(1-\gamma_1) - (\gamma_2-\gamma_1)^2\right] + \frac{1}{6}\frac{(1-\gamma_2)^2}{1-\gamma_1}. \end{aligned}$$

Since $\gamma_2 > \gamma_2 - \gamma_1$ and $1 - \gamma_1 > \gamma_2 - \gamma_1$, we obtain $L_1 + L_2 > 0$, implying that the generalized two-stage Groves mechanism with lump-sum transfers violates IIR in Case 2. This completes the proof of Step 2-3. □

References

[1] Akerlof, G., "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, vol. 84, (1970), 488-500.

[2] Fieseler, K., T. Kittsteiner, and B. Moldovanu, "Partnerships, Lemons, and Efficient Trade," *Journal of Economic Theory*, vol. 113, (2003), 223-234.

[3] Kunimoto, T. and C. Zhang, "Efficient Bilateral Trade with Interdependent Values: The Use of Two-Stage Mechanisms," *Working Paper*, 2021.

[4] Jehiel, P., and A. Pauzner, "Partnership dissolution with interdependent values," *The RAND Journal of Economics*, vol. 37, (2006), 1-22.

[5] Jehiel, P., and B. Moldovanu, "Efficient design with interdependent valuations," *Econometrica*, vol. 69, (2001), 1237-1259.

[6] Matsushima, H., and S. Noda, "Mechanism Design with Blockchain Enforcement," *Working Paper*, 2020.

[7] Mezzetti, C., "Auction Design with Interdependent Valuations: The Generalized Revelation Principle, Efficiency, Full Surplus Extraction and Information Acquisition," *Working Paper*, 2003.

[8] Mezzetti, C., "Mechanism Design with Interdependent Valuations: Efficiency," *Econometrica*, vol. 72, (2004), 1617-1626.

[9] Mezzetti, C., "Mechanism Design with Interdependent Valuations: Surplus Extraction," *Economic Theory*, vol. 31, (2007), 473-488.

[10] Myerson, R.B. and M.A. Satterthwaite, "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, vol. 29, (1983), 265-281.

[11] Samuelson, W., "Bargaining under Asymmetric Information," *Econometrica*, vol. 52, (1984), 995-1005.