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# Risk sharing in an asymmetric environment<sup>☆</sup>



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## ABSTRACT

We study the effect of an asymmetric environment on risk sharing. In our model, entrepreneurs consider undertaking risky projects in the real sector as well as selling part of their projects to investors. To capture the idea of an asymmetric environment, the returns on the alternative risk-free investment are allowed to differ between the entrepreneurs and the investors, i.e., agents have different opportunity costs of participating in the risky projects. We first show that the presence of asymmetric options establishes links between the risk-free and risky sectors as well as between the real and financial sectors. In particular, an asymmetric environment implies that the amount of risk sharing depends on the risk-free rates and the expected return of the risky project. Moreover, the level of real investment also depends on the risk-free rates. Second, we show how different risk-free rates may encourage or discourage risk sharing, and even prevent risk sharing altogether.

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## 1. Introduction

Uncertain and risky events are ubiquitous in society. While economic agents cannot eliminate all of the exogenous sources of risk, they can exercise a certain control over the amount of risk that they face through the market process. Specifically, markets and prices allocate resources to different risky activities, and among different agents. For instance, when an entrepreneur undertakes a risky project in the real sector, the size of the project as well as the share of risk borne by each shareholder depend on market forces in both the real and financial sectors. In particular, the choice and allocation of risk depend on the prices of goods in the real sector as well as the prices of financial instruments. These prices depend, in turn, on the preferences of agents, the alternative assets of the shareholders, the market structure, and the exogenous source of risk.

The link between real and financial decisions has been previously studied in Mirman and Santugini (2013) in the case of a monopoly firm owned by a risk-averse entrepreneur who shares the risk with risk-averse investors.<sup>1</sup> The interaction of agents (i.e., entrepreneurs and investors) in a competitive financial market and the determination of the financial price are shown to have a profound

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<sup>1</sup> For the behavior of risk-averse firms maximizing the expected utility of profit in the absence of financial markets, see Baron (1970) and Sandmo (1971) for the competitive firm, and Baron (1971) for an imperfectly competitive market. Leland (1972) provides a general treatment of a risk-averse firm facing demand uncertainty under both perfect and imperfect competition. See also Hawawini (1978) for a geometric exposition using the mean–variance framework. In this literature, while firms take account of risk, i.e., decision-making is influenced by the riskiness of profits, risk sharing via the financial market is absent.

effect on the behavior of the monopoly in the real sector. However, in their model, the agents have access to the same financial markets and the opportunity cost of buying shares of the risky asset is the same across the agents. Assuming away an asymmetric environment severs links between sectors. Indeed, the decision to share the risk of the risky asset and the level of investment associated with the risky asset are unaffected by the opportunity cost (i.e., the risk-free rate) in their model. Moreover, the ownership structure among entrepreneurs and investors depends solely on risk aversion and not the underlying risk of the real sector (i.e., the distribution of the real payoffs).

In general, agents do not have access to the same types of investment opportunities. To study how asymmetric outside options alter risk sharing and investment levels, we focus on risk sharing when opportunity costs for the risky assets sold on financial markets differ across agents. To that end, we present a microstructure model in which risk-averse entrepreneurs consider undertaking risky projects in the real sector as well as selling part of their projects to risk-averse investors in the financial sector. In addition to the entrepreneurs' risky investment, all agents have access to an alternative investment. To capture the idea of an asymmetric environment, the alternative investment's returns are allowed to differ between the entrepreneurs and the investors.

After characterizing the unique equilibrium, we study the effect of an asymmetric environment on the comparative analysis and the entrepreneurs' ability to share risk with the investors. We first show that the presence of asymmetric options establishes links between the risk-free and risky sectors, as well as between the real and financial sectors. More precisely, with equal risk-free rates (i.e., equal outside options), the degree of risk sharing and the level of real investment in the risky project are both independent of the risk-free rate. Moreover, the degree of risk sharing is independent of the expected payoff of the risky project. When risk-free rates differ, the comparative analysis is richer. Indeed, the share of the risky project retained by the entrepreneurs decreases in the risk-free rate offered to the entrepreneurs, increases in the risk-free rate offered to the investors, and may increase or decrease in the expected payoff of the risky project depending on the ordering of the risk-free rates. Finally, the level of real investment in the risky projects increases in the risk-free rate offered to the entrepreneurs but decreases in the risk-free rate offered to the investors.

We then show that asymmetries in the risk-free sector have an effect on risk sharing in the risky sector. When there is equal access to the risk-free asset, risk sharing always occurs as the entrepreneurs participate in the financial market to reduce risk. The reduction in risk is achieved at the expense of an unprofitable sale of shares, i.e., the return from selling a share is less than the payoff from retaining it. With different risk-free rates, the entrepreneurs' opportunity cost of retaining a share of the risky asset is different than the investors' opportunity cost of buying one, which may encourage or discourage risk sharing. On the one hand, when the investors' risk-free rate is the lowest, the entrepreneurs are able to exploit the differences in the returns of the risk-free asset to make a profitable sale of the risky asset to investors who value the risky asset more than the entrepreneurs. In some cases, the price that the investors are willing to pay is high enough that the entrepreneurs sell the entire project, completely removing any exposure to risk. On the other hand, when the entrepreneurs' risk-free rate is the lowest, the entrepreneurs might decide not to participate in the financial market at all, and thus asymmetries in the outside options prevent risk sharing. The reason is that the reduction of risk via risk sharing with the investors is too costly.

The paper is organized as follows. Section 2 presents the model and characterizes the equilibrium. The effect of an asymmetric environment on the comparative analysis and risk sharing are studied in Sections 3 and 4, respectively. Section 5 contains the final remarks.

## 2. Model and equilibrium

### 2.1. Model

#### 2.1.1. Preliminaries

We present a model with a continuum of entrepreneurs and a continuum of investors, each of mass one. The objective of each agent is to maximize the expected utility of final wealth. We assume that agents' preferences over final wealth exhibit constant absolute risk aversion with coefficient of absolute risk aversion  $a_e > 0$  for an entrepreneur and  $a_i > 0$  for an investor. In other words, the utility functions for final wealth  $x$  are exponential:  $u(x; a) = -e^{-ax}$ ,  $a \in \{a_e, a_i\}$ .

Entrepreneurs undertake costly projects that generate random profits. Moreover, they sell claims tied to the random profits. The proceeds of the sale are invested in a risk-free asset with rate of return  $R_e > 0$ . The investors, on the other hand, do not have entrepreneurial prospects, but have some initial wealth and may purchase claims to the entrepreneurs' profits or invest in a risk-free asset with rate of return  $R_i > 0$ . Due to asymmetric outside options, the returns of the risk-free asset that each type of agent has access to are potentially different:  $R_e \neq R_i$ .<sup>2</sup>

#### 2.1.2. Entrepreneurs

Each entrepreneur chooses the level of real investment  $q$ . The payoff to the investment is  $\tilde{\pi} = (\theta + \tilde{\varepsilon})q$ , where  $\theta > 0$  is the expected payoff of one unit of investment and  $\tilde{\varepsilon} \sim N(0, \sigma^2)$  is a shock. For each unit of investment, an entrepreneur incurs a cost of effort  $c > 0$ . The cost, unlike the payoff, cannot be shared with investors, and is borne entirely by the entrepreneurs.

In addition to choosing the level of investment, each entrepreneur decides the ownership structure of his investment. Specifically, an entrepreneur retains the payoff from  $\omega q$  units of investment, while selling the payoff of the remaining  $(1 - \omega)q$  units, where  $\omega \in [0, 1]$  is the entrepreneur's level of ownership. In other words, as an entrepreneur produces  $q$  units of investment,

<sup>2</sup> A bank might offer a preferential treatment to either the entrepreneurs or the investors depending on the group with which the bank has a closer business relationship.

$q$  shares of a risky claim are issued of which  $\omega q$  shares are retained and  $(1 - \omega)q$  shares are sold. By selling a share of the risky claim at price  $P$ , which is then invested into a risk-free asset with a rate of return  $R_e$ , an entrepreneur earns  $R_e P$  but relinquishes  $\theta + \tilde{\varepsilon}$ . Hence, an entrepreneur's final wealth (net of the cost of effort) is

$$\widetilde{W}'_e = \omega(\theta + \tilde{\varepsilon})q + R_e P(1 - \omega)q - cq. \tag{1}$$

Given CARA preferences and a normally distributed shock, there exists a closed-form characterization of each agent's certainty equivalent as well as a strictly monotonic relation between utility and the certainty equivalent. Hence, maximizing the certainty equivalent is equivalent to maximizing the expected utility of final wealth. The certainty equivalent approach is used throughout the paper. The certainty equivalent of an entrepreneur is

$$CE_e(q, \omega, P) = \omega\theta q + R_e P(1 - \omega)q - cq - a_e \sigma^2 \omega^2 q^2 / 2. \tag{2}$$

2.1.3. Investors

Each investor diversifies his initial wealth  $W_i > 0$  by buying  $m$  shares of the risk-free asset and  $z$  shares of the risky claims on profits. Given the budget constraint  $W_i = m + Pz$ , an investor's final wealth is

$$\widetilde{W}'_i = R_i(W_i - Pz) + (\theta + \tilde{\varepsilon})z, \tag{3}$$

where  $R_i(W_i - Pz)$  is the return on investing  $W_i - Pz$  in a risk-free asset with a rate of return  $R_i$ , and  $(\theta + \tilde{\varepsilon})z$  is the payoff from buying  $z$  shares from the entrepreneurs. The certainty equivalent of an investor is

$$CE_i(z, P) = R_i W_i + (\theta - R_i P)z - a_i \sigma^2 z^2 / 2. \tag{4}$$

2.2. Equilibrium

Since there is a continuum of entrepreneurs and investors, the financial sector is perfectly competitive, i.e., in the trading equilibrium entrepreneurs and investors take the price of the risky asset as given. The trading equilibrium consists of the entrepreneurs' investment and ownership decisions  $\{q^*, \omega^*\}$ , the investors' quantity demanded for the risky asset  $z^*$ , and the financial price  $P^*$ . While Definition 1 refers to a trading equilibrium, there is also a no-trading equilibrium in which the entrepreneurs do not share risk (including the outcome that no investment is made at all). The no-trading equilibrium simply refers to a constrained maximization problem for the entrepreneurs.<sup>3</sup>

**Definition 2.1.** The tuple  $\{q^*, \omega^*, z^*, P^*\}$  is an equilibrium if

1. Given  $P^*$ ,

(a) For any entrepreneur,

$$\{q^*, \omega^*\} = \operatorname{argmax}_{q>0, \omega \in (0,1]} CE_e(q, \omega, P^*). \tag{5}$$

(b) For any investor,

$$z^* = \operatorname{argmax}_{z>0} CE_i(z, P^*). \tag{6}$$

2. Given  $q^*, \omega^*$ , and  $z^*, P^*$  clears the market, i.e.,  $(1 - \omega^*)q^* = z^*$ .

**Proposition 2.2** provides the equilibrium value of investment,  $q^*$ . Differences in risk-free rates have an effect on the existence of the project (i.e., whether investment is positive or zero) as well as the level of investment.

**Proposition 2.2.** In equilibrium, the level of investment is

$$q^* = \frac{\max\{\theta - c, 0\}}{a_e \sigma^2} + \frac{\max\{\theta - R_i c / R_e, 0\}}{a_i \sigma^2}. \tag{7}$$

**Proof.** See Appendix A. □

**Proposition 2.3** states the different cases for risk sharing and the equilibrium values of  $\omega^*$  and  $P^*$ . Each entrepreneur may either sell his entire project to the investors, retain full ownership, or share risk with investors, depending on the differences in the risk-free rates. In particular, the absence of risk sharing is due to a combination of differential risk-free rates and unsharable cost.

<sup>3</sup> If there is no trading,  $z^*$  and  $P^*$  are not defined since there is no market. Further,  $\omega^* = 1$  so that, from Eq. (2),  $q^*$  is the solution to  $\max_{q \geq 0} (\theta - c)q - a_e \sigma^2 q^2 / 2$ .

**Proposition 2.3.** In equilibrium,

- For  $\theta > c$  and  $\theta \leq R_i c / R_e$ , there is a positive level of investment (i.e.,  $q^* > 0$ ) but there is no trading of assets and entrepreneurs retain full ownership (i.e.,  $\omega^* = 1$ ).
- For  $\theta > R_i c / R_e$ , assets are traded at price

$$P^* = \frac{c}{R_e} \tag{8}$$

- and entrepreneurs retain a fraction  $\omega^* \in [0, 1)$  of the investments' profits, where

$$\omega^* = \begin{cases} \frac{a_i(\theta - c)}{a_i(\theta - c) + a_e(\theta - R_i c / R_e)}, & \theta > c \\ 0, & \theta \leq c \end{cases} \tag{9}$$

**Proof.** See Appendix A. □

Because the effect of differential risk-free rates is related to the allocation of the profit claims, it is convenient to study the equilibrium from the viewpoint of the allocation of shares between entrepreneurs and investors. Formally, define  $x \equiv \omega q$  to be the number of shares retained by an entrepreneur and  $y \equiv (1 - \omega)q$  to be the number of shares sold to the investors, so that  $q \equiv x + y$  and  $\omega \equiv x / (x + y)$ .

**Proposition 2.4.** In equilibrium, the number of profit claims retained by an entrepreneur is

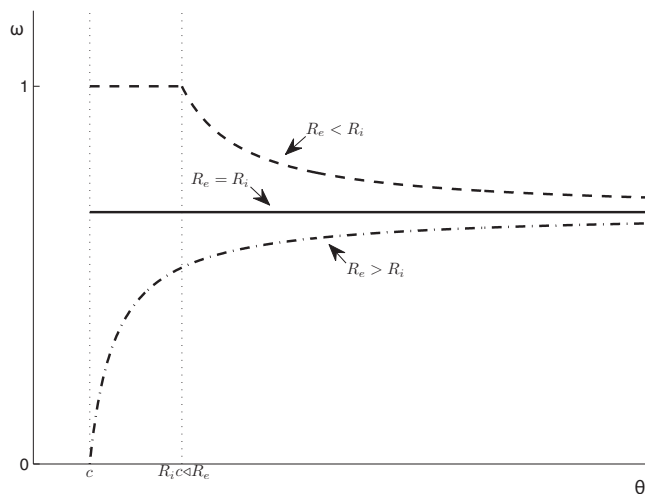
$$x^* = \frac{\max\{\theta - c, 0\}}{a_e \sigma^2}, \tag{10}$$

while the number of claims sold to investors is

$$y^* = \frac{\max\{\theta - R_i c / R_e, 0\}}{a_i \sigma^2}. \tag{11}$$

**Proof.** See Appendix A. □

In Propositions 2.2 to 2.4, the level of investment and risk sharing depends not only on the expected payoff of the investment relative to the marginal cost, but also on the expected payoff relative to the risk-free rate adjusted cost of the investment,  $R_i c / R_e$ . This relationship, which we will see throughout the results, is explored in more detail in the next sections as we study the effect of differential risk-free rates on the comparative analysis and on the trading of assets and risk sharing.



**Fig. 1.** The effect of  $\theta$  on ownership.

### 3. Comparative analysis

In this section, we show that differential risk-free rates have an effect on the comparative analysis. We consider first the benchmark case of no asymmetries in the risk-free asset as studied in Mirman and Santugini (2013) for the case of a monopoly firm sharing risk with many investors. Proposition 3.1 states that when risk-free rates do not differ the allocation of the risky asset and the level of real investment are independent of the common risk-free rate. Moreover, the allocation of the risky asset is immune to changes in its mean return.

**Proposition 3.1.** Suppose that  $\theta > c$  and  $R \equiv R_e = R_i$ . Then, from Eqs. (7) and (9),

1.  $q^*$  is independent of  $R$ , and
2.  $\omega^*$  is independent of  $\theta$  and  $R$ .

Proposition 3.2 states that the ownership structure depends on the expected payoff of the risky asset,  $\theta$ , when the entrepreneurs and the investors face different risk-free rates.

**Proposition 3.2.** Suppose that entrepreneurs and investors have different risk-free rates. Then, from Eq. (9),  $\omega^*$  increases in  $\theta$  if and only if  $R_e > R_i$ .

Fig. 1 graphically shows the effect of  $\theta$  on the entrepreneurs' level of ownership.<sup>4</sup> In the benchmark case, when risk-free rates do not differ, entrepreneurial ownership is independent of  $\theta$ . Consider now the case in which the entrepreneurs have the highest risk-free rate. When  $\theta$  is close to the marginal cost, the entrepreneurs retain ownership over only a very small portion of the project. As the expected payoff of the risky asset increases, entrepreneurial ownership increases and converges to the level of ownership in the benchmark case. Consider next the case in which the investors face the best risk-free rate. When the expected payoff of the risky asset is close to the marginal cost, the entrepreneurs retain the entire ownership of the project. As the expected payoff increases, it becomes more profitable to expand the investment through the financial market and to sell part of the project. Similarly, entrepreneurial ownership converges to the level of ownership in the benchmark case as  $\theta$  increases.

Proposition 3.3 states that both the ownership structure and the level of real investment depend on the risk-free rates of return when the entrepreneurs and the investors face different risk-free rates.

**Proposition 3.3.** Suppose that entrepreneurs and investors have different risk-free rates. Then, from Eqs. (7) and (9),

1.  $q^*$  increases in  $R_e$  and decreases in  $R_i$ , and
2.  $\omega^*$  decreases in  $R_e$  and increases in  $R_i$ .

Differential risk-free rates have an effect on the ownership structure and the level of real investment solely through the number of claims sold to investors. Indeed, from Eq. (10), the number of shares retained by the entrepreneurs is always independent of the rates of return of the risk-free assets. However, from Eq. (11), the number of shares sold depends positively on the ratio  $R_e/R_i$ . If entrepreneurs face a higher risk-free rate, selling a share of the risky asset and investing the proceeds in the risk-free asset yields a higher return. However, as the investors' risk-free rate increases, investors' willingness to pay for the risky asset decreases, which, in turn, lowers the entrepreneurs' net revenues. Hence, as the entrepreneurs' risk-free rate increases, entrepreneurs increase the number of shares sold to investors in order to take advantage of a better risk-free rate. As the investors' risk-free rate increases, entrepreneurs reduce the number of shares sold as investors are willing to pay less for the risky asset. Since  $q^* = x^* + y^*$  and  $\omega^* = x^*/(x^* + y^*)$ , both the level of investment and ownership of the entrepreneurs depends on the risk-free rates, solely through the financial market, in the way stated in Proposition 3.3.

### 4. Risk sharing

Under a common risk-free rate, entrepreneurs always access the financial market and share the risk with investors when undertaking a project. The single motivation for accessing the financial market is to reduce risk as entrepreneurs are unable to make a profitable sale: net revenue  $RP^* - \theta$  from selling a share is always negative. In other words, entrepreneurs accept a lower expected final wealth in order to reduce the risk premium. Consistent with Mirman and Santugini (2013) for the case of a monopoly firm sharing risk with investors, Proposition 4.1 states that when an entrepreneur decides to undertake a risky project, the absence of asymmetries in the outside options allows the financial market to exist and shares to be traded as long as the expected return is higher than the cost of the project.

**Proposition 4.1.** Suppose that  $R \equiv R_e = R_i$ . Then, from Eqs. (7) and (9), there is risk sharing if and only if there is a positive level of investment, i.e., if and only if  $\theta > c$ . Further, from Eq. (8),  $RP^* - \theta = c - \theta < 0$ .

With differential risk-free rates, however, entrepreneurs do not always share risk with investors. Moreover, entrepreneurs' net revenues from selling a share can be positive.

<sup>4</sup> Fig. 1 is generated with  $\{a_e, a_i, R_e, R_i, c\} = \{1, 2, 1.3, 1.1, 0.9\}$ .

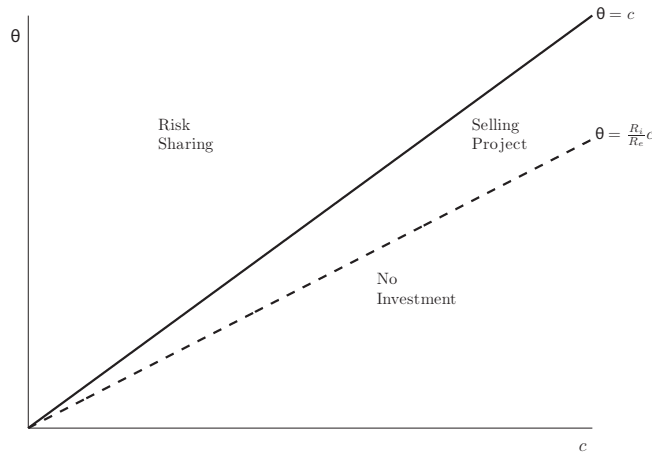


Fig. 2.  $R_e > R_i$ .

**Proposition 4.2.** Suppose that entrepreneurs and investors have different risk-free rates. Then, from Eq. (9), each entrepreneur

1. sells the entire project (i.e.,  $\omega^* = 0$ ) if  $\theta < c$  and  $\theta > \frac{R_i}{R_e}c$ .
2. shares risk (i.e.,  $0 < \omega^* < 1$ ) if  $\theta > c$  and  $\theta > \frac{R_i}{R_e}c$ .
3. retains the entire project (i.e.,  $\omega^* = 1$ ) if  $\theta > c$  and  $\theta < \frac{R_i}{R_e}c$ .

Further, from Eq. (8), entrepreneurs make a profit when selling the entire project, i.e.,  $R_e P^* > \theta$  if  $\theta < c$  and  $\theta > \frac{R_i}{R_e}c$ .

With differential risk-free rates, the entrepreneurs may make a profitable sale or reduce risk or both. A profitable sale is due to differential risk-free rates, which create an arbitrage investment opportunity between the entrepreneurs and the investors. Entering the financial market for arbitrage, rather than risk sharing, occurs only when the entrepreneurs face the best risk-free rate and are encouraged to sell shares of the risky asset because of a relatively high return in the risk-free asset. Moreover, the relatively low risk-free rate available to the investors induces them to pay a higher price for the risky asset. In particular, when  $R_e > R_i$ , for some quantity of shares sold  $R_e P^* > \theta$ . In fact, in the same cases, each entrepreneur sells his entire project because the arbitrage opportunity outweighs any benefits from risk sharing. Consequently, the entrepreneurs prefer to sell the entire project as they not only rid themselves of all risk, which reduces the risk premium to zero, but also increase their expected final wealth because of the high price paid by investors. From Fig. 2, each entrepreneur sells the entire project with a net real benefit  $R_e P^* - \theta > 0$  when  $\theta \in [R_i c/R_e, c]$ ,  $R_e > R_i$ .

As much as differential risk-free rates give entrepreneurs an arbitrage incentive to enter the financial market, it might also prevent them from selling shares due to too high a cost. This might occur when the entrepreneurs' alternative risk-free investment is worse than the investors'. In that case, entrepreneurs might be unable to obtain from the investors a price of the risky asset high enough to induce them to participate in the financial market. Consequently, the entrepreneurs retain ownership because the payment for risk sharing is greater than the benefits from the reduction in the risk borne. Retaining the entire project occurs when  $\theta \in [c, R_i c/R_e]$ ,  $R_e < R_i$ , as shown in Fig. 3. Note that the entrepreneurs never sell the entire project if faced with a worse risk-free rate than the investors.

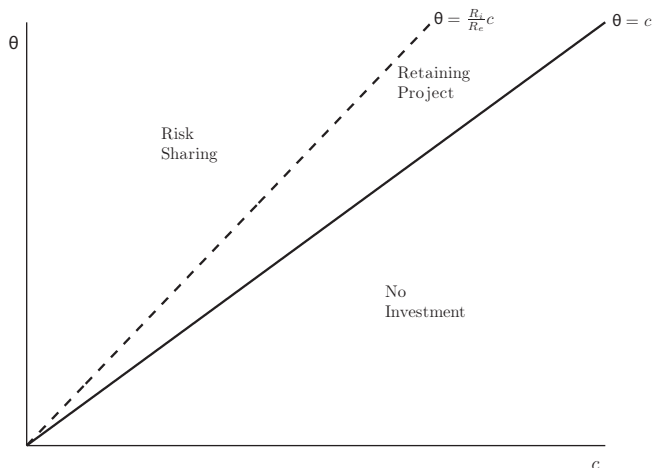


Fig. 3.  $R_e < R_i$ .

Finally, with differential risk-free rates, the entrepreneurs might also share the risk with investors. This occurs when  $\theta > \max\{c, R_e c/R_e\}$ , as shown in Figs. 2 and 3. However, unlike the case of a common risk-free rate, accessing the financial market might also yield a positive net revenue from selling a share. In other words, entrepreneurs might have more than one reason to access the financial market, i.e., they might want to risk share, and, at the same time, make the sale of shares profitable.

## 5. Final remarks

In this paper, we have presented a microstructure model in which risk-averse entrepreneurs decide whether to undertake risky projects and how much risk to share with risk-averse investors. When the environment is asymmetric, i.e., when the entrepreneurs and investors have different opportunity costs of investing, risk sharing depends on the risk-free rates and the expected return of the risky project, unlike in the case when the environment is symmetric. Moreover, we show how different risk-free rates may encourage or discourage risk sharing and even prevent risk sharing altogether. We have abstracted from one important aspect, namely we have assumed that all agents have the same information about the distribution of the returns for this project. In fact, asymmetric information is ubiquitous among shareholders. In a dynamic setting learning would occur as the price of the risky asset would be used by the uninformed investors as a signal of the expected payoff. We leave such an extension to future work.

Differential risk-free rates are not the only source of asymmetries among entrepreneurs and investors. Future work could consider other sources of asymmetries such as investment barriers across countries. Investment across national boundaries, particularly FDI, can suffer from barriers such as restrictions on the share of equity held by foreigners and limits on foreign personal and operational freedom (Golub, 2003).<sup>5</sup> Some other examples of investment barriers include corruption (Wei, 2000), lack of property rights protection and contract enforcement (Du, Lu, & Tao, 2008), differing tax rates (Mishra & Ratti, 2014), and the difficulty that foreigners have in obtaining information about foreign stocks, differences in the depth and quality of financial reporting, and a reluctance to deal with foreigners (Jorion & Schwartz, 1986; Kim & Song, 2010).

## Appendix A. Proofs

We provide a combined proof of Propositions 2.2, 2.3, and 2.4. To that end, we consider the entrepreneur's maximization problem from the point of view of the allocation of the profit claims. Formally, let  $x \equiv \omega q$  be the number of shares retained by the entrepreneur and  $y \equiv (1 - \omega)q$  be the number of shares sold to the investor so that  $q \equiv x + y$  and  $\omega \equiv x/(x + y)$ . Using Eq. (2), an entrepreneur's maximization problem is rewritten as  $\max_{x,y \geq 0} \{(\theta - c)x + (R_e P - c)y - a_e \sigma^2 x^2/2\}$ .

For interior solutions, the first-order condition with respect to  $x$  yields

$$x^* = \frac{\theta - c}{a_e \sigma^2}. \quad (12)$$

In perfect competition, it must be that  $R_e P = c$  yielding Eq. (8). In order to determine  $y^*$ , we solve for  $z^*$ . Using Eq. (4), the first-order condition with respect to  $z$  yields

$$z^* = \frac{\theta - R_i P^*}{a_i \sigma^2}. \quad (13)$$

Plugging Eq. (8) into Eq. (13) and using the market-clearing condition  $y^* = z^*$  yields

$$y^* = z^* = \frac{R_e \theta - R_i c}{a_i \sigma^2 R_e}. \quad (14)$$

From Eqs. (12) and (14), interior solutions for  $x^*$  and  $y^*$  exist when  $\theta > c$  and  $R_e \theta > R_i c$ , i.e., entrepreneurs share the investment's profits with investors. If the expected payoff is less than the marginal cost, i.e.,  $\theta \leq c$ , then  $x^* = 0$ . Similarly, if the marginal revenue of selling a share is always less than the marginal cost, i.e.,  $R_e \theta \leq R_i c$ , then  $y^* = 0$ . Therefore, due to the corner solutions, three types of outcomes with no risk sharing are possible. First, each entrepreneur undertakes the project and retains ownership of it, i.e.,  $x^* > 0$  and  $y^* = 0$ . Second, each entrepreneur proceeds with the investment but sells the entire project, i.e.,  $x^* = 0$  and  $y^* > 0$ . Finally, the investment does not take place when  $x^* = 0$  and  $y^* = 0$ . Combining the interior and corner solutions yields Eqs. (10) and (11), or, equivalently, Eqs. (7) and (9).

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<sup>5</sup> Several large, important economies such as China, Russia, Mexico, Japan, and India scored poorly on the 2012 OECD FDI Regulatory Restrictiveness Index, which measures statutory restrictions on foreign direct investment.



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