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Abstract: Although urban rail transit systems play a crucial role in urban mobility, they frequently suffer from unexpected disruptions due to power loss, severe weather, equipment failure, and other factors that cause significant disruptions in passenger travel and, in turn, socioeconomic losses. To alleviate the inconvenience of affected passengers, bus bridging services are often provided when rail service has been suspended. Prior research has yielded various methodologies for effective bus bridging services; however, they are mainly based on the strong assumption that passengers must follow predetermined bus bridging routes. Less attention is paid to passengers' path choice behaviors, which could affect the performance of the bus bridging services deployed. In this paper, we specifically take passengers' path choice behaviors into account and address the bus bridging optimization problem under urban rail transit disruptions. Incorporating a PS-logit model to estimate the probabilities of passenger path choices, we propose a mixed-integer nonlinear programming model to simultaneously determine the selection of bus bridging routes and vehicle deployment on selected bridging routes, with the objective of minimizing the cost associated with passenger travel time and unsatisfied demand. To solve this computationally challenging large-scale nonlinear model, we design a customized variable neighborhood search algorithm framework. A case study based on the Shanghai rail transit system is conducted to demonstrate the applicability and feasibility of the proposed approach. The results indicate that our approach can provide an effective bus bridging scheme that considers passenger path choice, which facilitates rapid response to rail disruptions. Our scheme substantially outperforms the current bridging designs that do not consider passenger path choice behaviors by significantly reducing the number of unserved passengers.

Keywords: Urban rail transit disruption, Bus bridging design, Path-size logit choice, Variable neighborhood search

1. Introduction

With the development of urban rail transit systems, urban rail service has become increasingly important for urban mobility, and especially in mega-cities. For instance, the rail transit network in Shanghai accounts for over 60% of the city's public transportation volume. However, urban rail transit systems frequently suffer from service disruptions. In February 2023, Shanghai's rail transit system experienced at least three significant service disruptions lasting 1∼5 h. In Melbourne, unexpected train disruptions may occur as often as once a week (Chen and An, 2021). Various factors induce rail disruptions, which can be classified as unplanned or planned based on the cause. Unplanned disruptions are primarily caused by natural disasters, terrorist attacks, accidents, medical emergencies, severe weather conditions, and equipment failures. Planned disruptions arise from strikes, major special events, facility maintenance, and upgrades.

If not properly addressed, rail service disruptions can lead to a range of adverse consequences. For instance, the decrease in railway system capacity causes travel delays and disruptions for passengers, and consequent station overcrowding increases the risk to public safety. Worse, it can rapidly degrade the quality of service and even paralyze the entire transportation system (Pender et al., 2013), which may erode public confidence in rail transit and public transportation services. In addition, lower service level of public transportation will lead to more serious inequity in the distribution of accessibility (Qin and Liao, 2022). Therefore, when rail disruptions occur, authorities need to swiftly identify effective response measures, ensure coordination among public agencies for resource allocation, and synchronously provide relevant information to passengers to restore travel, and maintain smooth urban traffic, and thus mitigate the negative impacts.

The most common response to rail disruptions is to provide bus bridging services (Pender et al., 2013), deciding temporary bus line alignments and deploying available buses, as an alternative to the disrupted rail transit services for passengers. Rail disruptions result in inadequate rail capacity to meet passengers' travel demand, and in turn, cause passengers to be stranded at stations. Due to buses' advantages in terms of high capacity and relatively flexible scheduling, they can efficiently evacuate passengers and satisfy their travel demand, and thereby reduce disruptions to passengers, economic losses, and the risk to public safety.

A survey conducted by Currie and Muir (2017) in Melbourne revealed that more than two-thirds of passengers tend to wait for bus services during unplanned disruptions, and thus authorities usually implement temporary bus bridging schemes as an alternative to rail transit services. In cases in which the affected rail and bus transit systems are operated by different companies, rail authorities should ensure the availability of buses via agreements or contracts with bus companies (Zhang and Lo, 2020). The design of temporary bus bridging services includes selecting bus bridging routes and allocating bus resources to the selected routes. Due to the large number of affected origin–destination (OD) pairs of passengers and candidate bus bridging routes, this is a large-scale mathematical programming problem.

Specifically, passenger path choice behaviors are critical. Passengers with sufficient autonomy choose suitable paths based on their preferences. Therefore, the distribution of passengers in a public transportation network is jointly determined by the bus bridging scheme and passenger path choice behaviors. For a given bridging scheme, unlike typical network flow problems, passenger flows are not distributed according to a centralized optimization method but instead must depend on their path choice behaviors. Neglecting passenger path choice would cause significant discrepancies between expected and actual demand for the bus bridging routes, and thereby influence bus deployment and trigger inconsistencies between demand and resource allocation. Hence, passenger path choice behaviors must be taken into consideration. Moreover, there are also constraints on the number of available buses, which renders it an even more complex problem.

In this research, we develop a path-choice-constrained bus bridging design (PCBBD) model in response to service disruptions in urban rail transit. We make three main contributions to the literature.

First, we establish a mixed-integer nonlinear programming (MINLP) model that incorporates a path-size logit model to estimate the probabilities of passenger path choices. The objective is to minimize the sum of passenger travel time cost and the penalties associated with unserved passengers. The selection of bus bridging routes and the allocation of available buses are simultaneously determined. Unlike many prior studies that allocate passenger demand as centralized network flows, we consider individual passenger path choice behaviors and propose a choice model to estimate passenger flows. It yields bridging routes that cater to passenger preferences and determines bus allocation to match actual demand.

Second, we design a customized variable neighborhood search (VNS) algorithm to solve the large-scale MINLP model; this entails complex nonlinear constraints and computational requirements.

Finally, a case study based on the Shanghai rail transit system demonstrates that the proposed method can provide effective bus bridging schemes within an acceptable timeframe and significantly reduce the number of unserved passengers compared with solutions that do not consider passenger path choice behaviors.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the path-choice-constrained bus bridging design problem and proposes a mathematical model. A solution approach for solving the model is presented in Section 4. Section 5 conducts a case study to demonstrate the applicability and feasibility of the proposed method. Section 6 summarizes our work and discusses promising directions for future research.

2. Literature review

Research on rail disruptions can be divided into operation-oriented and passenger-oriented studies (Leng et al., 2018). In this section, we mainly review research related to the design of bus bridging services from an operational standpoint. Passenger-oriented studies primarily analyze passenger behaviors to design more satisfactory service solutions. Prior passenger-oriented research mainly focuses on modeling and analyzing passenger behaviors related to whether to continue the journey and the choice of transportation mode (Pnevmatikou et al., 2015; Shires et al., 2019; Mo et al., 2022b). In this paper, we specifically consider passenger behaviors in terms of their path choices under bus bridging services. We review relevant studies regarding bus bridging design and passenger path-size logit choice models.

2.1. Bus bridging design

Bus bridging design during rail transit disruptions includes the selection of bridging routes and the allocation of vehicles. Kepaptsoglou and Karlaftis (2009) propose a framework that decomposes the problem into two subproblems: (1) designing bus bridging routes and operating frequencies and (2) bus dispatching. They first design bus bridging routes by determining the route layout and choosing frequencies, then allocate vehicle resources by assigning idle buses or extracting vehicles from existing bus routes. The objective is to maximize passenger welfare, which depends on the capacity provided and passenger travel time. The generation of bus routes is modeled as a transit route network design problem (TRNDP), and a shortest path algorithm is used to generate the candidate bridging route set. Based on criteria such as unsatisfied demand, a heuristic algorithm is proposed to select the optimal set of bridging routes. Jin et al. (2016) also employ this two-step framework. To account for commuter demand during disruptions, a column generation (CG) algorithm is proposed to generate candidate bus bridging routes and a multi-commodity flow network model is formulated to select bridging routes that minimize the sum of all passenger delays. Simultaneously, they design a time–space network optimization model to determine optimal headways and the allocation of vehicles. This approach is considered to be applicable to both planned and unplanned disruptions. Liang et al. (2019) further consider the uncertainty in bus travel time and develop a robust optimization model. They set the objective function to minimize the sum of passenger cost and bus bridging operation cost, in which passenger cost includes rail and bus travel time cost, penalties for unsatisfied demand, and transfer cost. Wang et al. (2023) also develop a CG algorithm and tackle the dual variables using specialized techniques. The methods used to generate feasible bus bridging routes in these four studies include shortest path algorithms and CG algorithms. In addition, Gu et al. (2018) allow buses to flexibly serve different bridging routes and develop a two-stage model. Similarly, Wang et al. (2019) formulate the problem as a vehicle routing problem (VRP). They develop a multi-objective optimization model that considers dynamic passenger flows in designing the bus bridging scheme and use the NSGA-II algorithm to solve the model. Song and Shao (2023) also propose flexible bus bridging services and bus dispatching with uneven headway is decided. Dou et al. (2019) generate candidate bridging routes through a brute-force search model and create an MILP model to simultaneously select bridging routes, deploy buses, and allocate terminal berths. Chen and An (2021) also develop a brute-force search model to generate all candidate bridging routes. They formulate an MILP model to simultaneously determine route selection, bus deployment, and bus timetables while taking time-varying travel demands into consideration, and propose a tabu search algorithm to solve the MILP model.

Given a candidate set of bus bridging routes, Van der Hurk et al. (2016) propose a model for selecting bridging routes and operating frequencies under budget constraints for planned closures in high-frequency urban transit networks. The model sets minimal frequency constraints on any operated route, and the objective is to minimize the inconvenience cost to passengers; this includes transfer and frequency-related waiting time cost. A path-reduction process is introduced to reduce the problem's complexity, which enables the model to handle a large number of origin–destination (OD) pairs. Luo and Xu (2021) generate alternative travel paths for commuters using a multimodal k-shortest path model and develop a two-stage stochastic model to identify the optimal bus bridging solution while accounting for uncertainty in commuting demand and the available capacity of remaining rail transit and existing bus routes. In the first stage, they determine the optimal selection of bridging routes and frequencies to minimize the sum of expected unsatisfied demand and the total number of bridging routes under uncertainty. In the second stage, commuter flows are allocated on feasible paths to minimize total unsatisfied demand. To compute passenger waiting time more accurately, Mo et al. (2023) use a simulation method to linearize the objective function and conduct robust optimization under passenger demand uncertainty.

Recent research tends to focus on passenger characteristics in bus bridging design. Tan et al. (2020) consider the heterogeneous risk-taking behaviors of affected passengers, as well as the uncertainty of disruption recovery time. Zheng et al. (2022) emphasize the heterogeneity of passengers and try to balance the benefit of affected rail passengers and conventional bus passengers. Furthermore, Mo et al. (2022a) develop an accident analysis framework based on automatic fare collection (AFC) and automatic vehicle location (AVC) data for rail transit service disruptions. In the case study, they find instances in which passengers did not choose the expected optimal paths; instead, they followed their own preferences when choosing alternative paths. This highlights the importance of understanding passenger behaviors and preferences when designing bus bridging services. Van der Hurk et al. (2018) consider scenarios in which passengers have the flexibility to choose their paths. They impose certain rules with uncertainty to estimate whether passengers would accept the recommended paths by authorities. However, most prior literature has paid little attention to passenger subjectivity. In practice, passengers may not necessarily choose the path that is optimal from the system's perspective, which leads to suboptimal outcomes. This aspect should not be overlooked, which is the focus of this paper.

2.2. The path-size logit choice model

As we have demonstrated, passenger path choice behaviors cannot be ignored in the bus bridging design problem. After disruptions occur, authorities design and provide bus bridging services to resume passenger travel, in which case passengers may have multiple path options to choose from. Because the distribution of flows on various passenger paths depends on individual preferences rather than adhering to theoretically optimal flow allocations derived from centralized algorithms, it is necessary to analyze passenger path choice behaviors. Otherwise, there may be discrepancies between actual flows and the allocation of passengers, which leads to misalignment in resource allocation and other problems. In recent studies, the path-size logit (PS-Logit) model has been widely used to estimate path choice behaviors (Marra and Corman, 2020).

Table 1
Comparison of related studies on bus bridging types.

Bridging type	Publications
Line bridging	Gu et al. (2018), Dou et al. (2019), Wang et al. (2019), Hu et al. (2020), Bojic et al. (2021), Chen and An (2021), Wang et al. (2021), Li et al. (2022), Zheng et al. (2022) and Zhang et al. (2023)
Line and network bridging	Jin et al. (2014, 2016) and Wang et al. (2023)

Ben-Akiva and Bierlaire (1999) propose the classical PS-Logit discrete choice model, which incorporates a path size factor in the multinomial logit (MNL) model to account for correlations among path choices. The utility of a path for individual n can be represented as

$$U_{in} = V_{in} + \beta_{PS} \ln(PS_{in}) + \epsilon_{in}, \tag{1}$$

where V_{in} denotes the deterministic utility of path i for individual n, β_{PS} represents the path-size coefficient, ϵ_{in} represents the error component, and PS_{in} can be represented as

$$PS_{in} = \sum_{a \in \Gamma_i} \frac{l_a}{L_i} \frac{1}{\sum_{j \in C_n} \frac{1}{\delta_{aj}}}, \tag{2}$$

where Γ_i represents the set of arcs in path i, C_n denotes the path choice set, l_a represents the length of arc a, L_i represents the length of path i, and the dummy parameter δ_{aj} indicates whether arc a is covered by path j.

The PS-Logit model has been widely applied in the field of transportation. For instance, it has been used to analyze path choices for bicycle trips (Khatri et al., 2016; Liu et al., 2019) and taxi customer search (Tang et al., 2020). The PS-Logit model can be applied to analyze path choices in public transportation. Tan et al. (2015) consider the uniqueness of the public transportation network and propose a path size formulation for the PS-Logit path choice model in public transportation networks, and find that the proposed path size formulation outperformed other models in terms of estimation and prediction. Marra and Corman (2020) develop and validate an algorithm for generating passenger choice sets in public transportation based on the estimated PS-Logit model. Arriagada et al. (2022) use the PS-Logit model to understand the path choice behaviors of public transportation passengers based on classification and synthesis strategies. In this paper, for the case of rail service disruptions and the bus bridging design, we will also use the PS-Logit model to estimate the probabilities that passengers will choose various feasible paths, which enables us to calculate a more realistic and accurate distribution of passenger flows and facilitates the improved design of bus routes and the allocation of buses.

3. A path-choice-constrained bus bridging design problem

In this section, we first provide a detailed description of a bus bridging design problem that considers passenger path choice behaviors under rail transit disruptions, highlight the challenges associated with this problem, and present the general outline of our work. Next, we introduce the methodologies we use to generate bus bridging routes and passenger paths. Finally, we develop an MINLP model that incorporates passenger path choice behaviors, which simultaneously determines bridging route selection and vehicle allocation.

3.1. Problem description

We consider a scenario of a bidirectional segment disruption on a single rail line, in which all stations within the segment are disconnected. Taking the hypothetical rail network shown in Fig. 1 as an example, the service disruption occurs between stations S2 and S5, which means that the links between S2 and S5 are down and the affected stations must be suspended. In this situation, the rail network in the target area will be unable to satisfy travel demand, and cause a large number of passengers to be stranded at stations.

In practical operations, a standard bridging route is typically established along the disrupted segment with buses making stops at each station, as in bus bridging route 1 in Fig. 1. However, this type of design does not consider the specific traffic demand pattern and lacks flexibility and specificity, which may lead to suboptimal performance. Therefore, it is worth considering the design of alternative bus bridging routes that align with the pattern of traffic demand. This can include reducing the number of stops between stations with high passenger demand and establishing express routes—e.g., bridging routes 2 to 4. Further, stations outside the disrupted segment can also be served by bus transfer routes, such as bridging route 5. We refer to this type of route as a network bridging route. Supposing there is high travel demand from station S2 to S7, it may be more effective to establish a direct route instead of having passengers take a bus from S2 to S5 and then transfer to the rail transit to reach S7. Moreover, bus bridging routes that only serve the disrupted segment are referred to as line bridging routes. Table 1 compares the different types of bus bridging approaches in practice that are studied in the literature.

In addition to selecting bus bridging routes, bus bridging service design entails determining the number of buses to allocate to each route. Bus allocation is dependent on travel demand for bus bridging routes, which in turn is influenced by anticipated passenger flows. Unlike typical network flow problems, passenger flows do not stem from a centralized optimization problem, but instead are related to passengers' path choice behaviors. In the network shown in Fig. 1, we focus on passenger OD pairs (S2, S5),

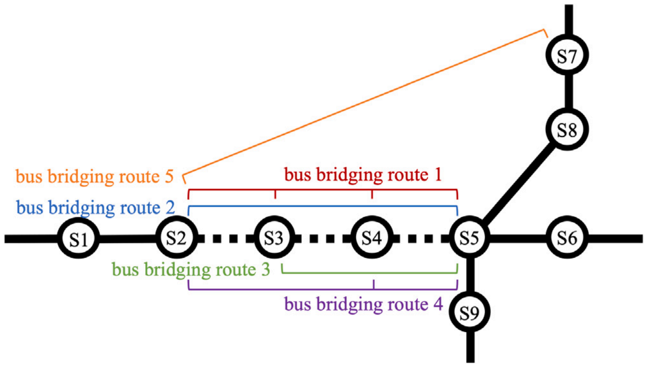

Fig. 1. Example of a transit rail network disruption and responsive bus bridging routes.

(S2, S7), and (S2, S8), with respective demand of q_{25}, q_{27}, and q_{28}. Assume that bus bridging routes 2 and 5 are selected in this scenario. In this case, passengers in OD (S2, S5) and (S2, S7) would choose routes 2 and 5, respectively, since those direct paths have the shortest travel time and do not require transfers, which makes them the clear preference for most passengers.

However, passengers traveling from S2 to S8 have two feasible paths: Path 1 involves taking bridging route 2 from S2 to S5 and then transferring to the rail transit from S5 to S8; Path 2 involves taking bridging route 5 from S2 to S7 and then transferring to the rail transit from S7 to S8. Let t_1 and t_2 represent the travel time for the two paths and $t_1 > t_2$. Without considering passengers' path choice behaviors, the optimal solution would allocate all passengers of OD (S2, S8) to Path 2. Thus, the demand for bridging routes 2 and 5 would be q_{25} and $q_{27} + q_{28}$, respectively. However, if the difference between t_1 and t_2 is small, since both paths have the same number of transfers, passengers may still have a certain probability of choosing Path 1. Consequently, the actual demand for bridging routes 2 and 5 could be $q_{25} + \lambda q_{28}$ and $q_{27} + (1 - \lambda)q_{28}$, respectively, where $0 \leq \lambda \leq 1$, which may induce significant discrepancy between actual and expected demand. Allocating buses without considering passengers' path choices could result in a mismatch between actual demand and vehicle resource allocation; this would lead to a shortage of bus seats on bridging route 2 and a surplus of seats on bridging route 5. It would also induce crowded conditions for passengers of OD pairs (S2, S5) and (S2, S8) and result in some passengers' being stranded. Hence, passenger path choice behaviors are critical in order to avoid misalignment in vehicle resource allocation.

For the PCBBD problem, we first generate a set of candidate bus bridging routes and, based on this set, generate all potential paths for each OD pair of passengers to complete their journeys. To cover various possible schemes, the size of the candidate bus bridging route set should be large; hence, it is necessary to establish a mathematical model to determine the optimal solution. We propose an MINLP model that simultaneously selects bus bridging routes and allocates buses, and the objective is to minimize the sum of the penalty cost associated with unserved passengers and passenger travel time cost. We will use the PS-Logit model to estimate the probability of passenger path choices, which will enable us to infer the number of passengers choosing to use a particular bus bridging route and facilitate the allocation of vehicle resources.

We make the following assumptions: (1) Each rail station has one virtual bus station combined from all bus stations parallel to it, which serves as a potential station for bus bridging. (2) Passengers walk when transferring within the rail system or between the rail and bus systems. (3) Rail and bus travel time and walking time are all known and deterministic. (4) Boarding and alighting time for buses are negligible. (5) Passengers are unaware of waiting time and bus congestion levels when choosing their paths.

3.2. Generation of the candidate bus bridging set

Generating a diverse yet appropriately sized candidate bus bridging route set is crucial, because a comprehensive bus bridging route set that includes all possible combinations would be unnecessarily large and impractical for decision-making. On the other hand, including too few candidate bridging routes could lead to missing the optimal solution and significant reductions in solution quality. Our method, with careful trade-off, is described as follows.

First, divide all the rail stations into major and minor for the sake of bus bridging definition and subsequent bridging scheme updating. Terminal stations of the interrupted segment are defined as major stations, and other stations are classified based on their travel flows. Then, define $V_{BS} = \{i_{(1)}, \dots, i_{(|V_{BS}|)}\}$ as the set of bus stations parallel to major rail stations in the disrupted segment. $V_I = \{j_{(1)}, \dots, j_{(|V_I|)}\}$ represents the set of bus stations parallel to major stations that are not disrupted. Let $V_{BT} = V_{BS} \cup V_I = \{i_{(1)}, \dots, i_{(|V_{BS}|)}, j_{(1)}, \dots, j_{(|V_I|)}\}$, defined as major bus stations, and all other bus stations are defined as minor bus stations. For the single direction of a bus bridging route, specify that the start terminal is selected from V_{BS} and the end terminal is selected from V_{BT}. Start and end terminals must be different. Fig. 2 illustrates an example of a complete bus bridging route with both up- and downstream. It will travel from the start terminal to the end terminal (i.e., upstream) and then back to the start terminal (i.e., downstream).

Bus bridging routes can be categorized based on the stations they serve in line and network bridging routes. Line bridging routes run parallel to the disrupted railway segment and only serve bus stations within the disrupted segment, and network bridging

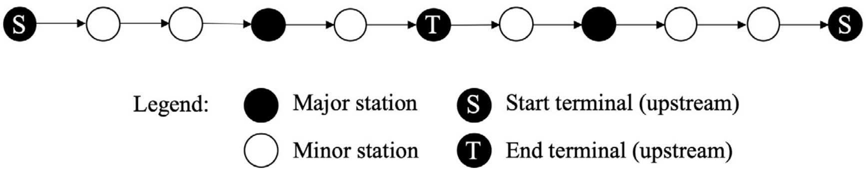

Fig. 2. Example of the bus bridging route (up- and downstream).

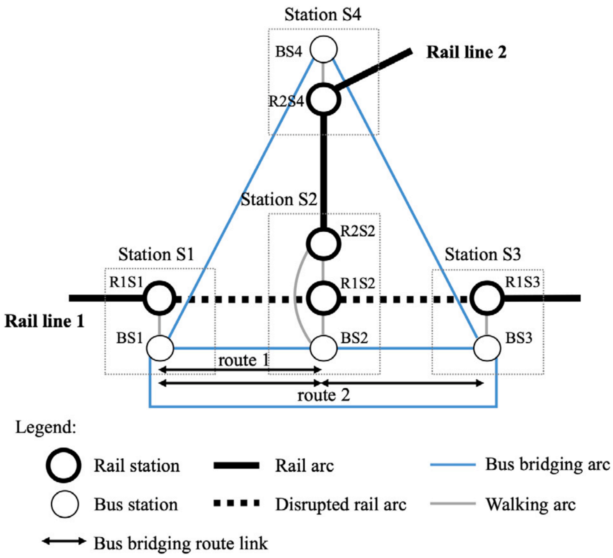

Fig. 3. Integrated rail-bus network.

routes also serve stations outside the disrupted segment. In this research, line bridging routes are generated through enumeration, and network bridging routes are generated using a k-shortest path algorithm (Yen, 1971). After selecting the start and end terminals, we calculate the shortest path. An increment is predetermined to keep the travel time of the bus bridging routes within reasonable limits. And if the travel time of the generated kth shortest path differs from the shortest by less than the increment, it is considered a feasible bridging route and included in the set.

3.3. Generation of reasonable passenger path sets

Given a set of bus bridging routes, we can generate all reasonable paths for passengers in each OD pair to reach their destinations. First, the integrated rail-bus network is represented by the graph $G(V, A)$ (Fig. 3). V denotes the set of nodes, which includes both rail and bus stations, and $V = V_R \cup V_B$. Taking Station S1 as an example, the rail station named S1 on rail line 1 is denoted R1S1, and the bus station named S1 is denoted BS1. A denotes the set of arcs, which consist of walking arcs (A_W), rail arcs (A_R), and bus arcs (A_B). Walking arcs represent transfers between different rail lines or between parallel rail and bus stations, rail arcs connect non-transfer rail transit stations, and bus arcs connect bus stations. A complete passenger path includes a sequence of stations and the arcs between adjacent stations. For example, suppose there is a bus bridging route 1 that travels from BS1 to BS2 and a bus bridging route 2 that sequentially serves BS1, BS2, and BS3 in Fig. 3. For OD pair (S1, S4), a passenger walks from rail station R1S1 to bus station BS1, then takes bus bridging route 1 to bus station BS2, walks to rail station R2S2, and finally takes rail line 2 to rail station R2S4. This path can be represented as R1S1-(w)-BS1-(b1)-BS2-(w)-R2S2-(r2)-R2S4, where (x) describes the arcs between adjacent stations, and $x \in \{$w, bn, rm$\}$ represents walking, taking bus bridging route n, and taking rail line m, respectively.

To generate sets of reasonable passenger paths, similar to the generation of network bridging routes, we use the k-shortest path algorithm to generate the sequence of passing-by stations for each OD pair. Note that the station sequence and the path are not always one-to-one mapping. In the network, there may be multiple bridging routes that cover the same arcs, which means that there can be multiple sets of possible bus arcs. In the example illustrated above, the station sequence in the path is R1S1, BS1, BS2, R2S2, and R2S4. The sequence can also be achieved through another path R1S1-(w)-BS1-(b2)-BS2-(w)-R2S2-(r2)-R2S4 using bus bridging route 2, differs from the previous sequence and is a reasonable alternative. Therefore, it is necessary to enumerate the connections between stations in the station sequence. In addition, the maximum transfer constraint can be included to obtain the final set of reasonable passenger paths.

Table 2
Notation defined in the PCBBD model.

Notation	Description
Sets	
K	Set of all affected OD pairs
R	Set of candidate bus bridging routes
P^k	Set of potential paths from origin to destination for OD pair k
Parameters	
t_{ij}	Travel time for arc (i,j)
t_{kp}	Travel time for path $p \in P^k$
t_r	Travel time for bus bridging route r
c_t	Cost per unit travel time (yuan per unit of time)
c_k	Penalty cost for one unserved passenger of OD pair k (yuan)
Q_k	Travel demands of OD pair k
f_{min}, f_{max}	Lower and upper bounds of the operated frequency of the selected bus bridging route (trip per unit of time)
C_B	Bus capacity
N_B	Available bus fleet size
N_R	Planned number of bus bridging routes to select
$\alpha^k_{ij,p} \in \{0,1\}$	1 if arc $(i,j) \in A$ is covered by path $p \in P^k$ and 0 otherwise
$\beta^k_{ij,r,p} \in \{0,1\}$	1 if bus stations i and j, $(i,j) \in A_B$, are connected by bus bridging route r in path $p \in P^k$ and 0 otherwise
$\gamma^k_{rp} \in \{0,1\}$	1 if passenger path $p \in P^k$ uses bus bridging route r and 0 otherwise
n_{kp}	Number of bus bridging routes included in passenger path $p \in P^k$
$\theta_1, \theta_2, \theta_3, \theta_4, \theta$	parameters of PS-Logit model
$\tau^{in-bus}_{kp}, \tau^{in-train}_{kp}, \tau^{walking}_{kp}$	Time spent on bus, train, and walking, respectively, in path $p \in P^k$
$n^{transfer}_{kp}$	Number of transfers (interchange between different rail transit lines or between rail transit and bus) required in passenger path $p \in P^k$
V_{kp}	Utility of choosing path $p \in P^k$, $V_{kp} = \theta_1 \tau^{in-bus}_{kp} + \theta_2 \tau^{in-train}_{kp} + \theta_3 \tau^{walking}_{kp} + \theta_4 n^{transfer}_{kp}$
e	A small constant
Decision variables	
$y_r \in \{0,1\}$	1 if bus bridging route r is selected and 0 otherwise
v_r	Number of buses allocated to bus bridging route r
Intermediate variables	
x_{kp}	Number of passengers of OD pair k choosing path $p \in P^k$
Pr_{kp}	Probability of passengers from OD pair k who choose path $p \in P^k$
u_k	Unsatisfied demand of OD pair k
$z_{kp} \in \{0,1\}$	1 if path $p \in P^k$ is feasible—i.e., all bus bridging routes used in p are selected—and 0 otherwise
PS_{kp}	Path-size factor for path $p \in P^k$ in the PS-Logit model

3.4. Mathematical model

Notation in this model is defined in Table 2.

The path-choice-constrained bus bridging service design model is formulated as follows:

$$\text{minimize} \quad c_t \sum_{k\in K} \sum_{p\in P^k} t_{kp} x_{kp} + \sum_{k\in K} c_k u_k, \tag{3}$$

subject to

$$\sum_{p\in P^k} x_{kp} + u_k = Q_k, \quad \forall k \in K, \tag{4}$$

$$y_r f_{min} \le \frac{v_r}{t_r} \le y_r f_{max}, \quad \forall r \in R, \tag{5}$$

$$\sum_{k\in K} \sum_{p\in P^k} \beta^k_{ij,r,p} x_{kp} \le C_B \frac{v_r}{t_r}, \quad \forall r \in R, \forall (i,j) \in A_B, \tag{6}$$

$$\sum_{r\in R} v_r \le N_B, \tag{7}$$

$$\sum_{r\in R} y_r = N_R, \tag{8}$$

$$z_{kp} \le y_r + 1 - \gamma^k_{rp}, \quad \forall r \in R, \forall p \in P^k, \forall k \in K, \tag{9}$$

$$z_{kp} \ge 1 - (n_{kp} - \sum_{r\in R} \gamma^k_{rp} y_r), \quad \forall p \in P^k, \forall k \in K, \tag{10}$$

$$x_{kp} \le Q_k Pr_{kp} + e, \quad \forall p \in P^k, \forall k \in K, \tag{11}$$

$$Pr_{kp} \sum_{q\in P^k} z_{kq} \exp(V_{kq} + \theta \ln(PS_{kq})) = z_{kp} \exp(V_{kp} + \theta \ln(PS_{kp})), \quad \forall p \in P^k, \forall k \in K, \tag{12}$$

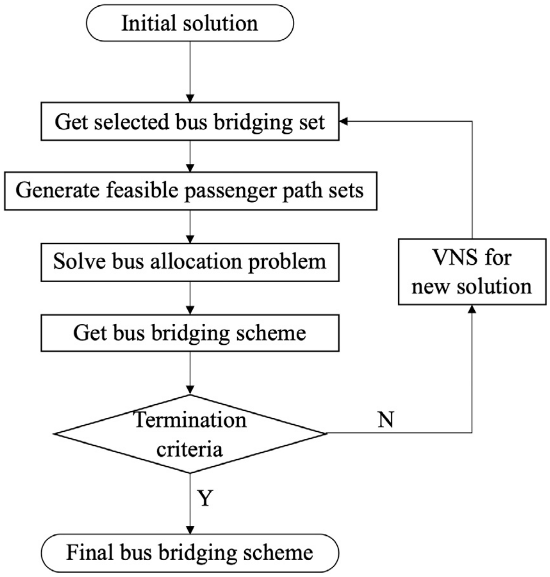

Fig. 4. Flow chart of the computational framework for the PCBBD.

$$PS_{kp} = \sum_{(i,j)\in A} \frac{\frac{\alpha^k_{ij,p} t_{ij}}{t_{kp}}}{1 + \sum_{q\in P^k\setminus\{p\}} z_{kq}\alpha^k_{ij,q}}, \quad \forall p \in P^k, \forall k \in K, \tag{13}$$

$$0 \le Pr_{kp} \le 1, \quad p \in P^k, \forall k \in K, \tag{14}$$

$$x_{kp} \ge 0, \quad \forall p \in P^k, \forall k \in K, \tag{15}$$

$$u_k \ge 0, \quad \forall k \in K, \tag{16}$$

$$y_r \in \{0,1\}, \quad \forall r \in R, \tag{17}$$

$$v_r \ge 0, \text{integer}, \quad \forall r \in R, \tag{18}$$

$$z_{kp} \in \{0,1\}, \quad \forall p \in P^k, \forall k \in K, \tag{19}$$

$$PS_{kp} \ge 0, \quad \forall p \in P^k, \forall k \in K. \tag{20}$$

The objective function (3) aims to minimize the sum of the penalty cost for unserved passengers and the cost of passenger travel time. Constraint (4) provides a quantitative description of the demand for each OD pair. Constraint (5) ensures that the operated frequencies of the selected bus bridging routes are limited within a certain range, while the frequencies of unselected routes are set to zero. Constraint (6) ensures that the passenger flow in each bus bridging route does not exceed its capacity. Constraint (7) states that the number of buses used cannot exceed the available bus fleet size. Constraint (8) ensures that the number of selected bus bridging routes equals the planned number. Constraints (9) and (10) detect the feasibility of the passenger path. Constraint (11) estimate the number of passengers choosing each path. Constraint (12) apply the PS-Logit model to estimate the probability of passenger path choice. Constraint (13) calculate the PS factor in the PS-Logit model. Constraints (14)~(20) define the domain of the variables.

4. Solution algorithm

Because of the large scale of the problem and nonlinear constraints, the mathematical model cannot be directly solved using commercial solvers such as CPLEX. Non-linear models are generally solved by designing heuristic algorithms (Zhen et al., 2019, 2020). In this section, we propose a heuristic algorithm based on VNS to solve the model, in which we can define multiple neighborhood structures and do multiple transformations of solutions to help avoid local optima. After generating candidate bus bridging set R using the method introduced in Section 3.2, we determine the set of selected bus bridging routes, which is a subset of R. Then, we generate all reasonable passenger paths based on the subset using the method described in Section 3.3. Next, we establish a bus allocation model to determine the bus deployment for each selected bridging route. The VNS framework is used to update the selection of bus bridging routes, and the previous steps are iteratively performed until the termination criteria are met. The algorithm's flow chart is shown in Fig. 4.

4.1. Initial solution

In the practical operations of urban rail transit, when a service disruption occurs and bus bridging service is to be implemented, the standard solution typically involves establishing a single bus bridging route that sequentially serves all stations within the disrupted segment. Starting from only the standard bridging route being selected, we set the initial feasible solution by sequentially inserting additional bus bridging routes, with one route added at a time, until the planned number of bridging routes is reached. A more detailed description is provided below.

Let $\bar{R}$ denote the set of currently selected bridging routes, and define $R_1^N(r)$ as the set of bridging routes obtained by removing, inserting, or replacing one station in the bridging route r, and $\forall r' \in R_1^N(r)$, s.t. $r' \in R$. Divide all the unselected bridging routes $r \in R\backslash\bar{R}$ into three subsets $\widetilde{R}_1, \widetilde{R}_2$ and $\widetilde{R}_3$, and the definitions are as follows:

- $\widetilde{R}_1$ consists of routes serving main stations that have not been served by the currently selected bridging routes, and if $r \in \widetilde{R}_1, \forall r' \in \bar{R}$, s.t. $r \notin R_1^N(r')$;
- $\widetilde{R}_2$ consists of routes not in $\widetilde{R}_1$ but serving a different set of main stations than the currently selected bridging routes, and if $r \in \widetilde{R}_2, \forall r' \in \bar{R}$, s.t. $r \notin R_1^N(r')$;
- $\widetilde{R}_3 = R\backslash(\bar{R} \cup \widetilde{R}_1 \cup \widetilde{R}_2)$.

Priority declines in order $\widetilde{R}_1, \widetilde{R}_2$ and $\widetilde{R}_3$. Bridging routes are randomly selected from the corresponding subset according to their priority and inserted into $\bar{R}$, which ensures the feasibility of the solution. If no routes satisfy the conditions in the current subset, a bridging route is randomly selected from the next priority-level subset and inserted until the planned number of bus bridging routes is achieved.

This approach is based on the intuition that bus bridging routes should prioritize serving stations with high passenger volumes. In addition, the selected bus bridging routes should have a certain level of differentiation from each other to meet the demands of diverse OD pairs.

4.2. Bus allocation

Once the set of selected bus bridging routes $\bar{R}$ is determined, it is possible to calculate sets of reasonable passenger paths $\bar{P}^k$ under current $\bar{R}$. The values of y_r in the PCBBD model can also be determined, with $y_r = 1, \forall r \in \bar{R}$ and $y_r = 0, \forall r \in R\backslash\bar{R}$. Subsequently, we can calculate the values of Pr_{kp} according to Eqs. (12) and (13), and then establish the following bus allocation (BA) model.

$$\text{[BA] minimize} \quad c_t \sum_{k\in K} \sum_{p\in\bar{P}^k} t_{kp} x_{kp} + \sum_{k\in K} c_k u_k, \tag{21}$$

subject to

$$\sum_{p\in\bar{P}^k} x_{kp} + u_k = Q_k, \quad \forall k \in K, \tag{22}$$

$$f_{min} \le \frac{v_r}{t_r} \le f_{max}, \quad \forall r \in \bar{R}, \tag{23}$$

$$\sum_{k\in K} \sum_{p\in\bar{P}^k} \beta_{ij,r,p}^k x_{kp} \le C_B \frac{v_r}{t_r}, \quad \forall r \in \bar{R}, \forall (i,j) \in A_B, \tag{24}$$

$$\sum_{r\in\bar{R}} v_r \le N_B, \tag{25}$$

$$x_{kp} \le Q_k Pr_{kp} + e, \quad \forall p \in \bar{P}^k, \forall k \in K, \tag{26}$$

$$x_{kp} \ge 0, \quad \forall p \in \bar{P}^k, \forall k \in K, \tag{27}$$

$$u_k \ge 0, \quad \forall k \in K, \tag{28}$$

$$v_r \ge 0, \text{integer}, \quad \forall r \in \bar{R}. \tag{29}$$

4.3. VNS for the selection of bus bridging routes

The VNS algorithm was first proposed by Mladenović and Hansen (1997). In our work, a VNS algorithm framework is used to update the selection of bus bridging routes. VNS defines multiple neighborhood structures that allow for moving to the next neighborhood to search for better solutions after obtaining a locally optimal solution under the current neighborhood structure, and thereby reduces the probability of local optima. VNS consists of two main procedures: the variable neighborhood descent (VND) procedure, which iteratively updates the solution by searching for new solutions in multiple neighborhoods according to a certain order and rules, and the shaking procedure, which mutates the current solution to another solution in a specific neighborhood structure to explore the diversity of solutions and help avoid getting stuck in local optima. Algorithm 1 presents the pseudo-code of the VNS algorithm.

Algorithm 1: VNS

Input: neighbor function for shaking, $M_s(s = 1, \dots, s_{max})$;
neighbor function for VND, $N_l(l = 1, \dots, l_{max})$;
initial solution, $\bar{\boldsymbol{y}}$; objective function, $F(*)$

Output: best solution, $\boldsymbol{y}^*$

$y^* \leftarrow \bar{y}$
repeat
  $s \leftarrow 1$
  while $s \leq s_{max}$ **do**
    select a random solution $\bar{\boldsymbol{y}}$ of $\boldsymbol{y}^*$ in $M_s(y^*)$
    $l \leftarrow 1$
    while $l \leq l_{max}$ **do**
      find the best feasible neighbor $\boldsymbol{y}$ of $\bar{\boldsymbol{y}}$ in $N_l(\bar{\boldsymbol{y}})$
      if $F(\boldsymbol{y}) < F(\bar{\boldsymbol{y}})$ **then**
        $\bar{\boldsymbol{y}} \leftarrow \boldsymbol{y}$
        $l \leftarrow 0$
      end
      $l \leftarrow l + 1$
    end
    if $F(\bar{\boldsymbol{y}}) < F(\boldsymbol{y}^*)$ **then**
      $\boldsymbol{y}^* \leftarrow \bar{\boldsymbol{y}}$
      $s \leftarrow 0$
    end
    $s \leftarrow s + 1$
  end
until stopping criteria;

4.3.1. VND

In the VND procedure, we iteratively update the solution by searching for better decisions on the selection of bus bridging routes under three neighborhood structures. We first define the following sets:

$R^N_{1M}(r)$: set of bus bridging routes and entails changing one major station from the original route r. Routes can be classified into three categories: (a) removing one major station; (b) inserting one major station; and (c) replacing one station s in r with another station s', where at least one of s and s' is a major station. $R^N_{1M}(r)$ is a subset of R—i.e., $R^N_{1M}(r) \subset R$—which means that all routes in $R^N_{1M}(r)$ must be in the set of candidate bus bridging routes R generated in Section 3.2.

$R^N_{1m}(r)$: set of bus bridging routes that entails changing one minor station and keeping major stations unchanged from the original route r. Routes can be classified into three categories: (a) removing one minor station; (b) inserting one minor station; and (c) replacing one minor station in r with another minor station. $R^N_{1m}(r) \subset R$.

$R^N_{nm}(r)$: set of bus bridging routes that entails changing multiple minor stations and keeping major stations unchanged from the original route r—i.e., for $r' \in R^N_{nm}(r)$, r' has the same major stations as route r but different minor stations, and $r' \notin R^N_{1m}(r)$. $R^N_{nm}(r) \subset R$.

Fig. 5 provides several examples of these sets. Note that when performing a replace operator for a bridging route, stations moving in and out do not need to be the same position in the station sequence; for instance, the third change in $R^N_{1m}(r)$ in Fig. 5.

Based on the above definitions, our three VND neighborhood structures are as follows:

(1) Replace a route r in the set of currently selected bus bridging routes $\bar{R}$ with r' where $r' \in R^N_{1M}(r)$.
(2) Replace a route r in $\bar{R}$ with r', where $r' \in R^N_{nm}(r)$.
(3) Replace a route r in $\bar{R}$ with r', where $r' \in R^N_{1m}(r)$.

4.3.2. Shaking

VNS for the selection of bus bridging routes performs a random move using one shaking neighborhood structure, in which two route replacement operations are performed. First, we identify a route r_1 with the minimum operated frequency among the set of currently selected bus bridging routes $\bar{R}$ and randomly select a route r_1' from $R^N_{1M}(r_1)$ in place of r_1. Second, we randomly select another route r_2 from the remaining routes in $\bar{R}$ and replace it with a randomly chosen route r_2' from $R^N_{nm}(r_2)$.

5. Computational study

In this section, we conduct a real-world case study based on the Shanghai rail transit network to demonstrate the applicability of the proposed model and VNS algorithm. All computations are performed on a personal computer with a 3.00 GHz Intel Core i7 processor and 16 GB RAM. The BA model was solved using the CPLEX 12.8.0 solver.

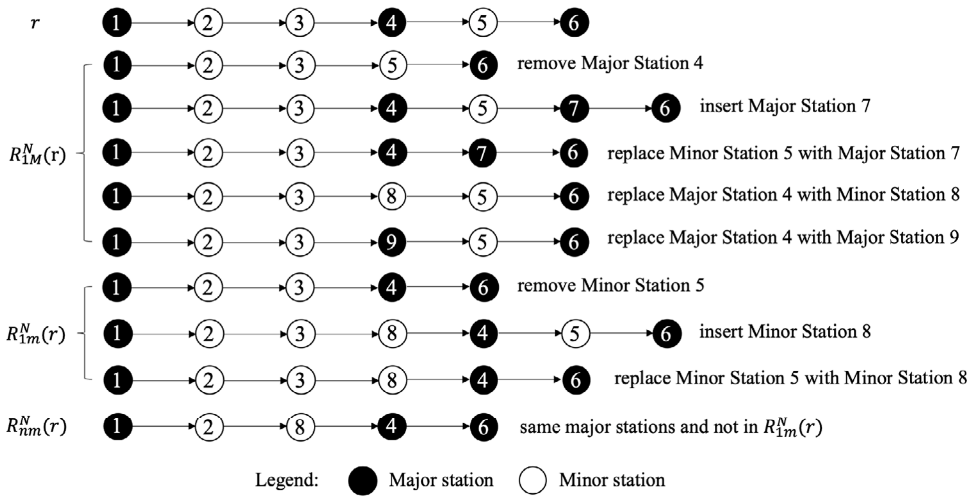

Fig. 5. Illustration of neighboring sets.

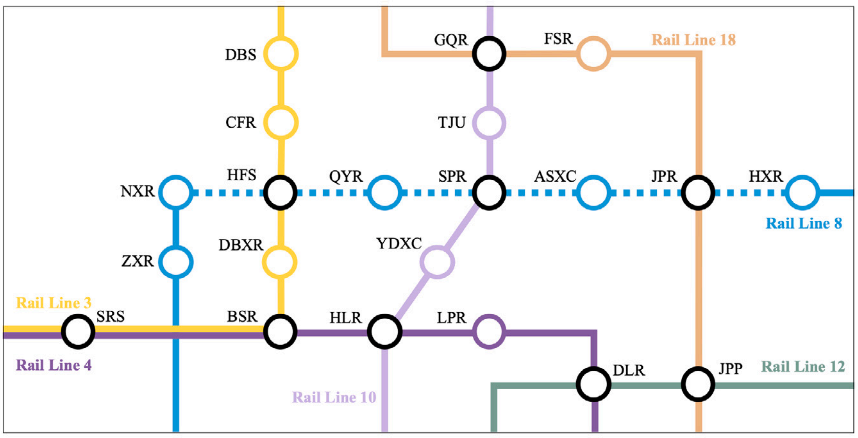

Fig. 6. Disruption area.

5.1. Disruption case description

We consider a disruption case study of the Shanghai Metro Line 8 segment between stations NXR and HXR, which occurred from 13:00 to 15:00. As shown in Fig. 6, there are 7 stations within the disrupted segment and 14 nearby stations, which are part of 6 metro lines. There are 42 affected OD pairs within the disruption segment with 7 stations. In addition to this, we use *Amap* to search for paths from each of the disrupted stations to stations outside the disrupted segment, and if the original recommended best path is infeasible during the disruption, the corresponding OD pair is defined as affected. With this calculation method, the disruption in this case impacts a total of 188 OD pairs, and according to the AFC data, it will affect over 11,000 passengers per hour.

Parameter settings for the bus bridging services are as follows:

(1) The upper bound of travel time for a complete bus bridging route (including both upstream and downstream) is 75 min.
(2) The maximum number of served stations is 5 for network bridging routes.
(3) The capacity of buses is 80.
(4) The operated frequencies for bus bridging routes range from 6 to 60 vehicles per hour.
(5) The unit cost of passenger travel time is 1.
(6) A penalty of 150 is assigned to 1 unserved passenger—i.e., unsatisfied transportation demand.

In addition, considering passenger path choice behaviors, we used the PS-Logit model to estimate the probabilities of passenger path choices. Parameter values for the PS-Logit model are as follows: $\theta_1 = -0.24, \theta_2 = -0.24, \theta_3 = -0.967, \theta_4 = -3.699, \theta = 0.138$ (Tan et al., 2015).

5.2. Computational results

For urban rail service disruptions, the bus bridging service solution most commonly adopted by transportation authorities is to establish a single bridging route that sequentially serves all stations within the disrupted segment, which we refer to as the standard solution. However, this design lacks flexibility and precision in specific scenarios. As a result, many studies have attempted to design alternative bridging routes that deviate from conventional bus routes to achieve better performance by integrating demand patterns. In this case study, we generate a total of 142 bus bridging routes, with 74 line bridging routes and 68 network bridging routes. These bridging routes can provide a total of 166,887,793 potential paths for all affected passengers to continue their journey. Our proposed PCBBD model considers passengers' autonomous path choices using the PS-Logit model. To validate the effectiveness, we conduct a test in which we compare our model with the standard solution using a model that does not consider passenger path choice behaviors (non-PC model). The non-PC model is established as follows:

$$\text{[non-PC] minimize} \quad c_t \sum_{k\in K} \sum_{(i,j)\in A} t_{ij} x_{ij}^k + \sum_{k\in K} c_k u_k, \tag{30}$$

subject to

$$\sum_{(i,j)\in A|i=o_k} x_{ij}^k + u_k = Q_k, \quad \forall k \in K, \tag{31}$$

$$\sum_{(i,j)\in A|j=o_k} x_{ij}^k = 0, \quad \forall k \in K, \tag{32}$$

$$\sum_{(i,j)\in A} x_{ij}^k - \sum_{(j,i)\in A} x_{ji}^k = 0, \quad \forall k \in K, \forall i \in V\backslash\{o_k, d_k\}, \tag{33}$$

$$\sum_{(i,j)\in A|j=d_k} x_{ij}^k + u_k = Q_k, \quad \forall k \in K, \tag{34}$$

$$\sum_{(i,j)\in A|i=d_k} x_{ij}^k = 0, \quad \forall k \in K, \tag{35}$$

$$y_r f_{min} \le \frac{v_r}{t_r} \le y_r f_{max}, \quad \forall r \in R, \tag{36}$$

$$\sum_{k\in K} x_{ij}^k \le \sum_{r\in R} \xi_{ij,r} C_B \frac{v_r}{t_r}, \quad \forall (i,j) \in A_B, \tag{37}$$

$$\sum_{r\in R} v_r \le N_B, \tag{38}$$

$$\sum_{r\in R} y_r = N_R, \tag{39}$$

$$x_{ij}^k \ge 0, \quad \forall (i,j) \in A, \forall k \in K, \tag{40}$$

$$u_k \ge 0, \quad \forall k \in K, \tag{41}$$

$$y_r \in \{0,1\}, \quad \forall r \in R, \tag{42}$$

$$v_r \ge 0, \text{integer}, \quad \forall r \in R. \tag{43}$$

In the non-PC model, the decision variable x_{ij}^k represents the number of passengers in OD group k using arc (i,j). The dummy parameter $\xi_{ij,r} = 1$ if bridging route r covers arc (i,j) and $\xi_{ij,r} = 0$ otherwise. o_k and d_k represent the origin and destination, respectively, of OD group k. The non-PC model is directly solved by CPLEX. After calculating the results of the non-PC model, we consider passenger route choice behaviors and calculate the objective function value under their chosen paths.

Assuming there are 30 available buses, Table 3 presents results of the standard solution and of optimization models for different numbers of planned bridging routes, $N_R = 2, 3, 4$ and 5. The result of the standard solution is obtained through BA model, with the set of selected bus bridging routes only containing the standard one. It is evident that considering passenger path choice behaviors significantly reduces the number of unserved passengers. Moreover, even with a substantial increase in the percentage of served passengers, the average travel time for served passengers only slightly increases. The main advantage of the PCBBD method is a significant reduction in the number of unserved passengers, because the routes generated by PCBBD consider passenger preferences and the allocation of bus resources aligns better with actual demand. Since the standard solution without any optimization algorithm is the most commonly adopted bus bridging service solution in the real world, we treat it as a baseline: any optimization algorithm is expected to achieve better performance than the baseline.

The results indicate that PCBBD increases the number of served passengers by over 10% for all scenarios, with the highest improvement exceeding 15% ($N_R = 5$), and the objective function value decreases by 20∼31%. In contrast, if passenger path choice behaviors are not considered during the modeling process, the solutions generated may be considerably less effective in practical applications. Compared with the baseline, the number of served passengers in the non-PC model may even decrease, and the objective function may be even higher ($N_R = 2$). The maximum percentage decrease in the objective function value ($N_R = 4$) in the non-PC model is only approximately 9.08% compared with the baseline.

In addition, as N_R increases the objective function value decreases, the number of unserved passengers decreases, and the average travel time for served passengers exhibits a decreasing trend. However, the degree of improvement becomes smaller as N_R increases.

Table 3
Comparison among PCBBD, non-PC model, and standard solution ($N_B = 30$).

Model	N_R	Obj.	#Unserved passengers	Avg travel time of the served passengers (min)
PCBBD	2	458,425	1,564	22.82
	3	432,389	1,434	21.85
	4	410,758	1,263	21.88
	5	396,304	1,149	21.89
Non-PC	2	609,543	2,872	21.01
	3	569,168	2,543	21.24
	4	521,041	2,192	20.93
	5	538,380	2,350	20.59
Standard	1	573,078	2,552	21.57

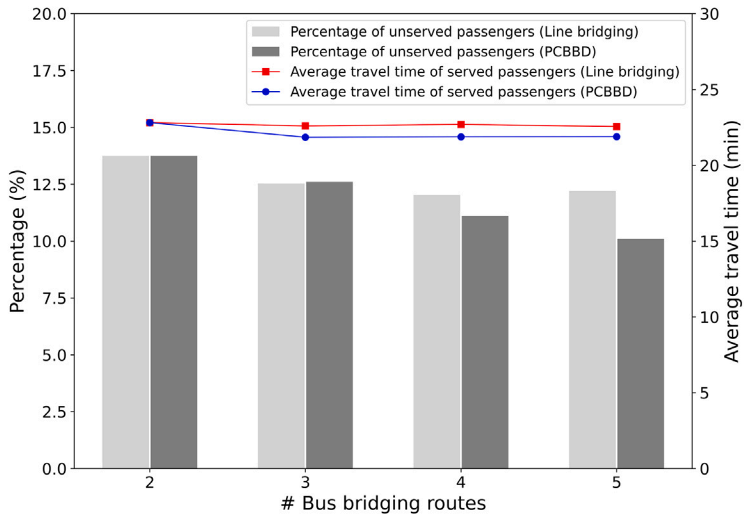

Fig. 7. Comparison between PCBBD and line bridging only.

The rise of N_R increases the number of links between stations, which enables more journeys to resume and reduces detours for passengers. At the same time, due to the minimal frequency constraint and limited bus resources, the number of buses allocated to each route may decrease and the evacuation of passengers in the most overcrowded stations may be affected, which reduces the increase in effectiveness.

5.3. Comparison with line bridging only

In our study, we consider both line and network bridging. To evaluate the effectiveness of our method, we compared it with a scenario that only uses line bridging. We find that for $N_R = 2, 3, 4$, and 5, employing only line bridging would result in an increase in the objective function value by approximately 0%, 1.42%, 5.27%, and 9.38%, respectively. As N_R increases, the disparity in the objective function value also becomes more pronounced. As depicted in Fig. 7, employing both line and network bridging offers potential to serve a larger number of passengers affected by service disruptions, and also reduces the average travel time for served passengers. This can be attributed to the use of network bridging, which enables long-distance travelers to access bus bridging services and reduces their travel time. Moreover, with an increasing N_R—i.e., enhanced flexibility in bus bridging– this advantage becomes more prominent.

5.4. Computational speed and robustness

In our proposed method, the computational time for generating the set of bus bridging routes is approximately 0.013 s, which can be negligible. The efficiency of the bus bridging design primarily depends on the runtime of the VNS algorithm. Due to the presence of mutation operators in VNS, the computational results of the model do not always converge to the same value; also, the convergence speed may vary. To test the efficiency and stability of the algorithm, we conduct 20 experimental instances for each N_R value (Table 4). The average solution time of the algorithm rises with the increase in N_R but remains within 8 min on average, which demonstrates a fast computation speed that satisfies the requirements of quick response during disruptions. For the same N_R, the difference between the mean and minimum values of the objective function is small (<3%). In addition, in the 20 experimental instances, although there are fluctuations in the results, the coefficient of variation (CV) is generally maintained at 1∼2%. Therefore, the computational results of the algorithm are relatively stable.

Table 4
Results of 20 experimental instances.

N_R	2		3		4		5	
Instance	Obj.	CPU time (s)	Obj.	CPU time (s)	Obj.	CPU time (s)	Obj.	CPU time (s)
1	469,112	56	436,639	186	410,416	290	403,837	353
2	475,902	45	444,574	179	424,631	232	404,681	408
3	464,372	117	444,237	226	410,416	244	396,304	436
4	481,926	35	440,270	195	422,931	149	413,691	904
5	464,372	209	441,870	61	410,416	266	398,352	576
6	464,372	200	440,490	146	412,560	336	398,352	374
7	481,926	52	432,389	158	416,891	308	416,277	446
8	469,112	56	441,870	53	417,631	219	407,517	283
9	464,372	179	432,389	192	416,891	229	409,752	494
10	464,372	320	432,389	211	416,115	326	398,352	404
11	458,425	195	439,726	116	410,416	252	396,304	421
12	481,926	43	451,320	92	417,222	457	409,566	586
13	469,112	60	441,870	158	418,685	184	398,352	757
14	464,372	165	438,833	108	410,758	172	398,352	841
15	464,372	124	436,639	146	412,560	134	404,681	274
16	464,372	168	441,870	63	410,879	240	404,040	371
17	464,372	113	432,389	182	412,560	382	416,277	419
18	464,372	124	441,870	149	410,758	343	405,489	391
19	464,372	112	448,037	145	410,879	194	406,617	201
20	458,425	142	444,237	161	412,560	187	414,057	383
Std. dev.	7160.8		5243.3		4468.3		6645.9	
Avg	467,697.9	126	440,195.45	146	414,358.75	257	405,042.5	466
CV (%)	1.53		1.19		1.08		1.64	

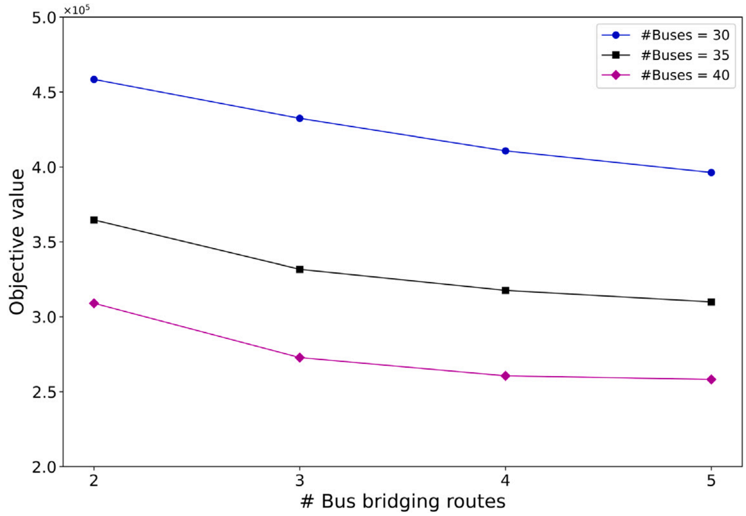

Fig. 8. Objective values under different bus fleet sizes.

5.5. Sensitivity analysis of bus fleet size

To investigate the impact of bus fleet size on our method's performance, we further conduct experiments with bus fleet sizes N_B = 35 and 40. Fig. 8 compares the objective value of the PCBBD model under different bus fleet sizes. As can be seen, as N_B increases, objective function values for the same N_R steadily decrease. Fig. 9 shows that for each N_R, an increase of 5 buses leads to a significant reduction in the number of unserved passengers and a slight rise (<1 min) in the travel time for served passengers, which indicates that the decrease in the objective function value is primarily driven by the reduction in the number of passengers unable to receive bus bridging services. Moreover, the increase in average travel time for served passengers can be attributed to more passengers with longer expected travel time being served by the bus bridging services.

Furthermore, from Tables 5 and 6, it can be seen that our proposed PCBBD model consistently outperforms the standard solution and non-PC model. Compared with the standard solution, the PCBBD model achieves a reduction in the objective function value by 22∼37%, which demonstrates its effectiveness. Moreover, when N_B = 35, PCBBD yields an increase in the number of served passengers by 10.55%, 12.84%, 13.68%, and 14.52% for increasing values of N_R. Similarly, for $N_B = 40$, the corresponding values are 7.59%, 9.60%, 10.18%, and 10.19%. In addition, as N_B increases, the gap in the average travel time of served passengers

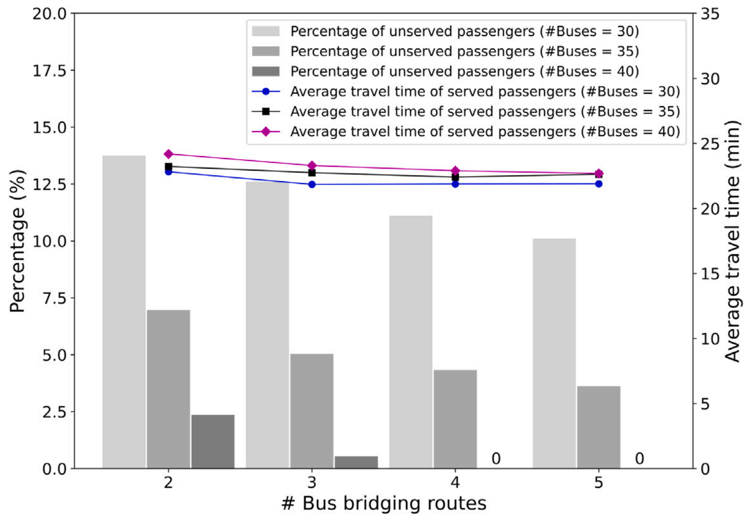

Fig. 9. Performance measures under different bus fleet sizes.

Table 5
Comparison among PCBBD, non-PC model, and standard solution ($N_B = 35$).

Model	N_R	Obj.	#Unserved passengers	Avg travel time of the served passengers (min)
PCBBD	2	364,605	792	23.23
	3	331,696	573	22.75
	4	317,628	492	22.41
	5	309,925	412	22.63
Non-PC	2	573,183	2,572	21.27
	3	563,536	2,501	21.23
	4	479,996	1,858	21.15
	5	415,222	1,344	21.29
Standard	1	485,395	1,802	22.47

Table 6
Comparison among PCBBD, non-PC model, and standard solution ($N_B = 40$).

Model	N_R	Obj.	#Unserved passengers	Avg travel time of the served passengers (min)
PCBBD	2	309,019	269	24.19
	3	272,746	61	23.30
	4	260,529	0	22.90
	5	258,142	0	22.69
Non-PC	2	501,591	1,972	21.87
	3	483,291	1,835	21.81
	4	457,799	1,677	21.27
	5	416,319	1,365	21.13
Standard	1	399,620	1,052	23.43

compared with the standard solution gradually diminishes. PCBBD can surpass the standard solution for $N_B = 35$ and $N_R = 4$, as well as $N_B = 40$ and $N_R = 3, 4$, and 5 in terms of both the two dimensions in the objective function—i.e., the number of unserved passengers and the average travel time of served passengers. Therefore, PCBBD consistently demonstrates a significant advantage by reducing the number of stranded passengers, and its effectiveness in achieving fewer passenger delays than the standard solution gradually becomes more prominent as N_B increases. In contrast, the performance of the non-PC model does not exhibit obvious improvement with an increase in the bus fleet size. The improvement is even less noticeable compared with the standard solution, and in many cases the results are inferior to the standard solution. This further confirms the approach proposed in this paper: Passengers' path choice behaviors cannot be neglected.

6. Conclusion

In this paper, we initially develop a mixed-integer nonlinear programming model that incorporates passenger path choice behaviors to address the problem of bus bridging design under urban rail transit service disruptions. The model simultaneously determines bus bridging route selection and bus allocation. To solve the model, we further design a customized VNS framework. A case study based on the Shanghai metro network is conducted to validate the effectiveness and efficiency of the proposed approach. The results demonstrate that considering passenger path choice behaviors in the design of bus bridging can significantly reduce the number of unserved passengers, because it is more likely to generate bridging routes that cater to passenger preferences and enables bus allocation to match with actual demand. Also, our method consistently provides effective bridging solutions within a reasonable timeframe—and despite the presence of mutation procedures, the computational results of the algorithm exhibit stability.

The main limitation of this work is that we model bus bridging service design in a static and deterministic environment, without considering real-time passenger flows and uncertainty. Therefore, a promising direction for future research is to develop dynamic time-space networks, stochastic programming, and distributionally robust optimization models that better align with real-world scenarios (Shehadeh et al., 2021). Also, the main purpose of this paper is to demonstrate the necessity of considering passenger path choice behaviors, and extensive development of the algorithm to ensure convergence to the optimal solution is not guaranteed. Furthermore, latest research has employed AI techniques to address behavioral choice problems (Liu et al., 2021, 2022, 2023), and they may provide a more precise estimation on passenger path choice behaviors than the PS-Logit model.

CRediT authorship contribution statement

Yiyang Zhu: Writing – original draft, Validation, Software, Methodology, Conceptualization. **Jian Gang Jin:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Hai Wang:** Writing – review & editing, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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