Household Heterogeneity and Optimal Inter-Temporal Pricing for a Durable-Good Monopoly

Winston T. H. KOH
Singapore Management University, winstonthkoh@gmail.com

DOI: https://doi.org/10.1142/s0217590806002494

Follow this and additional works at: https://ink.library.smu.edu.sg/soe_research

Part of the Econometrics Commons, and the Industrial Organization Commons

Citation
Available at: https://ink.library.smu.edu.sg/soe_research/227
HOUSEHOLD HETEROGENEITY AND OPTIMAL INTER-TEMPORAL PRICING FOR A DURABLE-GOOD MONOPOLY

WINSTON T.H. KOH
School of Economics and Social Sciences
Singapore Management University, 90 Stamford Road
Singapore 178903, Singapore
winstonkoh@smu.edu.sg

In this paper, I extend the analysis in Koh (2006) to examine the optimality of inter-temporal price discrimination for a durable-good monopoly in a model where infinitely-lived households consume both durable goods and a stream of non-durable goods subject to different inter-temporal budget constraints. I also consider the multi-dimensional setting where households differ in both inter-temporal budget constraints and the utilities they derive from the consumption of the durable good.

Keywords: Durable good; monopoly; inter-temporal price discrimination.

1. Introduction

Most models of inter-temporal price discrimination trace the source of such pricing opportunities to institutional features such as market segmentation, existence of asymmetric information as well as industry practices, but rarely to household heterogeneity. Koh (2006) investigates the optimality of inter-temporal price discrimination (IPD) in a setting where households consume a durable good and a stream of a non-durable good, subject to an inter-temporal budget constraint. The durable good is supplied by a monopoly that can credibly commit to its pricing policy. It is shown that when households differ in the utilities that they derive from the consumption of the durable good, an IPD sales strategy is optimal even when households possess the same inter-temporal discount rate as the monopoly. This result is in contrast to the findings in Stokey (1979) and Landsberger and Meilijson (1985), where consumers do not face an inter-temporal budget constraint or consume a non-durable good; as a result, an IPD sales strategy is dominated by a constant price policy.

In this paper, we extend the analysis presented in Koh (2006) to consider the situation where households derive the same utilities from the consumption of the durable good, but differ in their inter-temporal budgets. We demonstrate that the optimal sales strategy for the durable-good monopoly is an IPD strategy. The direct proof presented in this paper (in Proposition 1) is considerably more complicated than for the case considered in Koh (2006), where households face identical inter-temporal budget constraint.

The paper is organized as follows. Section 2 describes a model of household consumption subject to an inter-temporal budget constraint. In Section 3, we solve for the monopoly's
optimal sales strategy and prove that it does not entail a constant price. In Section 4, we extend the analysis to a multi-dimensional setting. Section 5 concludes the paper.

2. The Model

A monopoly supplies a costlessly-produced infinitely durable good to infinitely-lived households that also consume a stream of instantaneously perishable non-durable good subject to an inter-temporal budget constraint. The supply of the non-durable good is perfectly elastic at a unit price. The monopoly is able to credibly commit to a pricing policy \( P(t) \) and to sell the durable good for a period of length \( T \), after which sales end.

A household’s utility function at time \( t \) is given by \( (A + s_t X)^\alpha q_t^{(1-\alpha)} \), where \( s_t = 0 \) if the new durable good is not purchased at time \( t \), and \( s_t = 1 \) if it has been purchased at \( t \) or earlier; \( A \) is a constant; \( X \) is the identical per period utility that each household derives from the new durable good; \( q_t \) is the amount of non-durable good consumed at time \( t \); \( \alpha \in (0, 1) \) measures the relative importance of the consumption of the durable good in the utility function.

We consider first the situation where households differ only in their inter-temporal budget \( Y \), where \( Y \in [Y^-, Y^+] \), and \( f(Y) \) is the density function of \( Y \). Let \( z \) denote the date of purchase of the durable good, \( r \) denote the common rate of discount for the households and the monopoly, and \( \{q_{t0}\} \) and \( \{q_{t1}\} \) denote, respectively, the household’s consumption plan of the non-durable good before and after the purchase of the durable good. The present value of a household’s utilities is

\[
W(z, \{q_{t0}\}, \{q_{t1}\}) = \int_0^z A^\alpha q_{t0}^{(1-\alpha)} e^{-rt} dt + \int_z^\infty (A + X)^\alpha q_{t1}^{(1-\alpha)} e^{-rt} dt, \tag{1}
\]

while the inter-temporal budget constraint is

\[
P(z)e^{-rz} + \int_0^z e^{-rt} q_{t0} dt + \int_z^\infty e^{-rt} q_{t1} dt \leq Y. \tag{2}
\]

Each household’s maximization problem is given by the following Lagrangian (LP):

\[
L(z, \{q_{t0}\}, \{q_{t1}\}, \lambda) = W(z, \{q_{t0}\}, \{q_{t1}\}, Y) + \lambda \left\{ Y - P(z)e^{-rz} - \int_0^z e^{-rt} q_{t0} dt + \int_z^\infty e^{-rt} q_{t1} dt \right\}. \tag{3}
\]

Differentiating \( LP \) with respect to \( q_{t0} \) and \( q_{t1} \) yields the following optimality condition:

\[
\frac{A}{q_{t0}} = \frac{A + X}{q_{t1}} = \left[ \frac{\lambda}{1 - \alpha} \right]^\frac{1}{\alpha}. \tag{4}
\]

The condition in Equation (4) implies that \( q_{t0} \) and \( q_{t1} \) are constant for \( t = [0, z) \) and \( t = [z, \infty) \), respectively. We denote the optimal per period consumption of the non-durable good before and after the purchase of the durable good by \( q_0(Y) \) and \( q_1(Y) \), respectively.
Let \( z(Y) \) denote the optimal purchase date for the durable good. Differentiating \( LP \) in Equation (3) with respect to \( z \), the first-order condition for \( z(Y) \) when it is an interior solution, is given by

\[
\beta r X \left[ \frac{Y - P(z(Y)) e^{-rz(Y)}}{A + X e^{-rz(Y)}} \right] = r P(z(Y)) - \frac{d P(t)}{dt} \bigg|_{t=z(Y)},
\]

where \( \beta \equiv \alpha/(1 - \alpha) \). The necessary second-order condition for \( z(Y) \) is

\[
\frac{r d P(t)}{dt} \bigg|_{t=z(Y)} - \frac{d^2 P(t)}{dt^2} \bigg|_{t=z(Y)} < 0.
\]

Denote the household’s optimal consumption plan by \( \{z(Y), q_0(Y), q_1(Y)\} \). A household’s utility under the optimal consumption plan can be shown to be

\[
V(z(Y), Y) = \left[ \frac{A + X e^{-rz(Y)}}{r} \right]^{\alpha} \left[ Y - P(z(Y)) e^{-rz(Y)} \right]^{(1-\alpha)}.
\]

3. The Monopoly’s Optimal Sales Strategy

3.1. The feasible pricing policies

Using the first-order condition given in Equation (5) for \( z(Y) \), the implicit differentiation of \( z(Y) \) with respect to \( Y \) yields

\[
\frac{dz(Y)}{dY} = \beta r Y \left\{ \left[ r \frac{d P(t)}{dt} \bigg|_{t=z(Y)} - \frac{d^2 P(t)}{dt^2} \bigg|_{t=z(Y)} \right] \left[ A + X e^{-rz(Y)} \right] \right. \\
- \left. (\beta + 1) \left[ r P(z(Y)) - \frac{d P(t)}{dt} \bigg|_{t=z(Y)} \right] \right\}^{-1} < 0.
\]

Therefore, households with a larger budget would purchase the durable good earlier. Let \( Y_L \equiv \text{Min}\{Y \geq Y^-|z(Y) \text{ is an interior solution}\} \), \( Y_H \equiv \text{Max}\{Y \leq Y^+|z(Y) \text{ is an interior solution}\} \), and \( Y_M \in [Y^-, Y_L] \) such that a household with \( Y = Y_M \) is indifferent between buying the durable good at \( T \), or not at all. If the marginal household \( Y_M \) does without the durable good, its utility from consuming the non-durable good is \( (A/r)^\alpha Y_M^{(1-\alpha)} \). Using Equation (7), we can solve for \( P(T) \):

\[
P(T) = e^{rT} Y_M \left\{ 1 - \left[ \frac{A}{A + X e^{-rT}} \right]^\beta \right\}.
\]

The timing of the purchase is

\[
Y \in [Y_H, Y^+], \quad z(Y) = 0, \\
Y \in (Y_L, Y_H), \quad \frac{dz(Y)}{dY} < 0, \\
Y \in [Y_M, Y_L], \quad z(Y) = T, \\
Y \in [Y^-, Y_M], \quad z(Y) = \infty.
\]
Since $z(Y)$ is monotonic, we can invert $z(Y)$ to define a purchase schedule $Y(t)$, implicitly chosen by the monopoly for a feasible IPD pricing policy $P(t)$, such that

\[ t = 0, \quad Y(t) \in [Y_H, Y^+], \]
\[ t \in (0, T), \quad Y(t) = z^{-1}(t) \in (Y_L, Y_H), \]
\[ t = T, \quad Y(t) \in [Y_M, Y_L], \]
\[ t = \infty, \quad Y(t) \in [Y^-, Y_M), \]

where $Y(t)$ satisfies, when it is an interior solution, the following conditions:

\[
\frac{dP(t)}{dt} - \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] rP(t) = -\frac{\beta rX}{A + X e^{-rt}} Y(t), \tag{10}
\]

\[
\frac{dY(t)}{dt} = -\frac{1}{\beta rX} \left[ r \frac{dP(t)}{dt} - \frac{d^2 P(t)}{dt^2} \right] [A(t) + X e^{-rt}] - \left\{ \frac{r^{-\beta}(1 + 1)}{\beta} \right\} [rP(t) - \frac{dP(t)}{dt}] < 0. \tag{11}
\]

To derive a feasible $P(t)$, first multiply both sides of Equation (10) with the integrating factor $\exp \left\{ r \int_t^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz \right\}$ to obtain

\[
\left\{ \frac{dP(t)}{dt} - \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] rP(t) \right\} e^{\int_t^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz} = -\frac{\beta rXY(t)}{A + X e^{-rt}} e^{\int_t^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz}. \tag{12}
\]

Integrating both sides of Equation (12), this leads to

\[
P(t) = e^{-r \int_t^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz} \times \left\{ \int_t^T \frac{\beta rXY(w)}{A + X e^{-rw}} e^{\int_w^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz} \, dw \right\} + e^{\int_0^T Y_M \left\{ 1 - \left[ \frac{A}{A + X e^{-rt}} \right]^{\beta} \right\} \, dz}. \tag{13}
\]

Next, we note that

\[
\exp \left\{ -r \int_t^T \left[ 1 + \frac{\beta X e^{-rt}}{A + X e^{-rt}} \right] \, dz \right\} = \exp \left\{ -r(T - t) - \int_t^T \frac{\beta rX e^{-rt}}{A + X e^{-rt}} \, dz \right\} = e^{-r(T - t)} \left[ \frac{A + X e^{-rT}}{A + X e^{-rt}} \right]^{\beta}, \tag{14}
\]

so that substituting Equation (14) into (13), and using the boundary condition $P(T)$, we obtain

\[
P(t) = \int_t^T \left\{ \frac{\beta rXY(z)}{A + X e^{-rz}} e^{(T-t) \left[ \frac{A + X e^{-rz}}{A + X e^{-rt}} \right]^{\beta}} \right\} \, dz + e^{rT} Y_M \left\{ 1 - \left[ \frac{A}{A + X e^{-rT}} \right]^{\beta} \right\} \left[ \frac{A + X e^{-rT}}{A + X e^{-rt}} \right]^{\beta}. \tag{15}
\]
3.2. The monopoly’s revenue function

The monopoly’s revenue function, denoted $\Pi(Y(t), Y_M, T)$, is

$$\int_{Y^{-}}^{Y^{+}} e^{-r_z(Y)} P(z(Y)) f(Y) dY = \int_{Y_H}^{Y^{+}} P(0) f(Y) dY + \int_{Y_M}^{Y_L} e^{-r_T} P(T) f(Y) dY$$

$$+ \int_{Y_L}^{Y_H} e^{-r_z(Y)} P(z(Y)) f(Y) dY.$$ Integrating by parts,

$$\Pi(Y(t), Y_M, T) = [1 - F(Y_M)] e^{-r_T} P(T)$$

$$+ \int_{Y_L}^{Y_H} e^{-r_z(Y)} \left[ rP(z(Y)) - \frac{dP(t)}{dt} \bigg|_{t=z(Y)} \right] \frac{dz(Y)}{dY} [1 - F(Y)] dY.$$ Next, using the first-order condition for $z(Y)$, and using a change in variable, $z(Y) = t$, we obtain

$$\Pi(Y(t), Y_M, T) = [1 - F(Y_M)] e^{-r_T} P(T)$$

$$+ \beta \int_{0}^{T} \left\{ \frac{rXe^{-r_T} [1 - F(Y(t))]}{A + Xe^{-r_T}} \left[ Y(t) - P(t)e^{-r_T} \right] \right\} dt. \quad (16)$$ Substituting $P(t)$ and $P(T)$ into Equation (16), and simplifying, we obtain

$$\Pi(Y(t), Y_M, T)$$

$$= Y_M[1 - F(Y_M)] \left\{ 1 - \left[ \frac{A}{A + Xe^{-r_T}} \right]^\beta \right\}$$

$$+ \int_{0}^{T} \left\{ \frac{\beta rXe^{-r_T} [1 - F(Y(t))]}{A + Xe^{-r_T}} \left[ Y(t) - \int_{t}^{T} \left\{ \frac{\beta rXe^{-r_T} Y(z)}{A + Xe^{-r_T}} \left[ \frac{A + Xe^{-r_T}}{A + Xe^{-r_T}} \right]^\beta \right\} dz \right. \right.$$  

$$\left. - Y_M \left\{ 1 - \left[ \frac{A}{A + Xe^{-r_T}} \right]^\beta \right\} \left[ \frac{A + Xe^{-r_T}}{A + Xe^{-r_T}} \right]^\beta \right\} dt. \quad (17)$$

3.3. The optimal sales strategy

Let $\{Y^*(t), Y^*_M, T^*\}$ denote the optimal sales strategy of the durable-good monopoly. The first-order conditions that characterize $\{Y^*(t), Y^*_M, T^*\}$ are

$$Y^*(t) : \quad Y^*(t) - \int_{t}^{T} \left\{ \frac{\beta rXe^{-r_T} Y^*(z)}{A + Xe^{-r_T}} \left[ \frac{A + Xe^{-r_T}}{A + Xe^{-r_T}} \right]^\beta \right\} dz$$

$$- Y_M \left\{ 1 - \left[ \frac{A}{A + Xe^{-r_T}} \right]^\beta \right\} \left[ \frac{A + Xe^{-r_T}}{A + Xe^{-r_T}} \right]^\beta$$

$$= \left[ 1 - \frac{\beta rXe^{-r_T}}{A + Xe^{-r_T}} \right] \left[ \frac{1 - F(Y^*(t))}{f(Y^*(t))} \right], \quad t \in [0, T]. \quad (18)$$
\[
T^* : \int_0^{T^*} \left\{ \frac{\beta r X e^{-rt}}{A + X e^{-rt}} \left[ \frac{A + X e^{-rT^*}}{A + X e^{-rt}} \right]^\beta \left[ 1 - F(Y(t)) \right] \right\} dt
= [1 - F(Y_L)] - Y_M \left[ A \right] \left[ A + X e^{-rT^*} \right]^\beta \left[ F(Y_L) - F(Y_M) \right] Y_L - Y_M. \tag{19}
\]

\[
Y_M^* : \int_0^{T^*} \left\{ \frac{\beta r X e^{-rt}}{A + X e^{-rt}} \left[ \frac{A + X e^{-rT^*}}{A + X e^{-rt}} \right]^\beta \left[ 1 - F(Y(t)) \right] \right\} dt
= [1 - F(Y_M^*)] - Y_M^* f(Y_M^*). \tag{20}
\]

Substituting the above first-order conditions into Equation (17), the monopoly's revenue function under the optimal sales strategy can be written as

\[
\Pi(Y^*(t), Y_M^*, T^*) = Y_M^* \left[ 1 - F(Y_M^*) \right] \left\{ 1 - \left[ \frac{A}{A + X e^{-rT^*}} \right]^\beta \right\}
+ \beta \int_0^{T^*} \left\{ r e^{-rt} X \left[ 1 - \frac{\beta r X e^{-rt}}{A + X e^{-rt}} \right] \frac{[1 - F(Y^*(t))]^2}{f(Y^*(t))} \right\} dt. \tag{21}
\]

**Proposition 1.** The monopoly practices inter-temporal price discrimination under the optimal sales strategy \( \{Y^*(t), Y_M^*, T^*\} \).

**Proof.** The proof is as follows. Suppose, to the contrary, the optimal sales strategy involves an optimal constant price \( P^* \) over the period \( T^* \). Let \( Y^*(t) \) be the associated optimal purchase schedule. Using Equations (5) and (14),

\[
P^* = e^{T^*} Y_M^* \left\{ 1 - \left[ \frac{A}{A + X e^{-rT^*}} \right]^\beta \right\}; \tag{22a}
\]

\[
Y^*(t) = \left[ \frac{A + (\beta + 1) X e^{-rt}}{\beta X} \right] P^*. \tag{22b}
\]

Substituting into Equation (17), this in turn implies that the monopoly’s revenue function is

\[
\Pi(Y^*(t), Y_M^*, T^*) = e^{T^*} Y_M^* \left\{ 1 - \left[ \frac{A}{A + X e^{-rT^*}} \right]^\beta \right\} \left\{ e^{-rT^*} [1 - F(Y_M^*)] \right\}
+ \int_0^{T^*} r e^{-rt} [1 - F(Y^*(t))] dt. \tag{23}
\]
Since the revenue function given in Equation (23) must be equivalent to the one given in Equation (21), we obtain the following equality:

\[
e^{\int_t^{T^*} Y^*_M \left(1 - \left[\frac{A}{A + Xe^{-rt}}\right]^\beta\right)} \int_0^{T^*} re^{-rt} \left[1 - F(Y^*(t))\right] dt
\]

\[
= \int_0^{T^*} \left[\frac{\beta r Xe^{-rt}}{A + Xe^{-rt}}\right] \left\{1 - \frac{\beta r Xe^{-rt}}{A + Xe^{-rt}}\right\} \frac{[1 - F(Y^*(t))]^2}{f(Y^*(t))} dt,
\]

which simplifies to

\[
\int_0^{T^*} \left[\frac{\beta r Xe^{-rt}}{A + Xe^{-rt}}\right] \left\{1 - \frac{\beta r Xe^{-rt}}{A + Xe^{-rt}}\right\} \frac{[1 - F(Y^*(t))]^2}{f(Y^*(t))} dt = 0.
\]

Using the first-order condition in Equation (18), we can show that

\[
\left[\frac{A + Xe^{-rt}}{A + (\beta + 1)Xe^{-rt}}\right] Y^*(t) - \left\{1 - \frac{\beta r Xe^{-rt}}{A + Xe^{-rt}}\right\} \frac{[1 - F(Y^*(t))]^2}{f(Y^*(t))}
\]

\[
= - \frac{\beta Xe^{-rt} Y^*(t)}{A + (\beta + 1)Xe^{-rt}} + \int_0^{T^*} \left[\frac{\beta r Xe^{-rt} Y^*(z)}{A + Xe^{-rt}}\left[\frac{A + Xe^{-rt}}{A + (\beta + 1)Xe^{-rt}}\right]\right]^\beta dz
\]

\[
+ Y^*_M \left\{1 - \left[\frac{A}{A + Xe^{-rt}}\right]^\beta\right\} \left[\frac{A + Xe^{-rt}}{A + Xe^{-rt}}\right]^\beta.
\]

Since \(\beta > 0\), we have

\[
- \frac{\beta Xe^{-rt} Y^*(t)}{A + (\beta + 1)Xe^{-rt}} > - \frac{\beta Xe^{-rt} Y^*(t)}{A + Xe^{-rt}}.
\]

Next,

\[
\int_0^{T^*} \left[\frac{\beta r Xe^{-rt} Y^*(z)}{A + Xe^{-rt}}\left[\frac{A + Xe^{-rt}}{A + Xe^{-rt}}\right]\right]^\beta dz > \frac{\beta r Xe^{-rt} Y^*(t)}{A + Xe^{-rt}}.
\]

These two conditions in Equations (27) and (28) imply that the right-hand side of Equation (26) is always greater than zero. This in turn implies that the left-hand side of Equation (25) is always positive, which leads to a contradiction. Therefore, a constant price is not a feature of the optimal sales strategy of the durable-good monopoly.

The intuition that an IPD strategy is optimal in our model is as follows. Each household must make a decision, given its inter-temporal budget constraint and the monopoly’s announced pricing policy \(P(t)\), whether to purchase the new durable good, and when to do so. The timing decision affects the budget share that will be allocated to the new durable good, as well as the marginal utility [given by \(\lambda\) in Equation (4)] of consuming the stream.
of non-durable good. Suppose, for the moment, that the monopoly were to decide to sell the new durable good at a uniform price. As shown in Equation (22b), even with uniform pricing, households with a smaller inter-temporal budget constraint will optimally choose to purchase it later, in order to increase the consumption of the non-durable good. Suppose now the monopoly were to raise the price slightly at \( t = 0 \), and leave the price unchanged for \( t \in (0, T) \). Some households with an inter-temporal budget constraint larger than \( Y_H \) would be willing to continue to buy it at \( t = 0 \), while others may choose to delay the purchase, including those with an inter-temporal budget of \( Y_H \). By choosing a suitable small increase in price at \( t = 0 \), the monopoly would be able to increase its profits.

4. The Multi-Dimensional Setting

In a general setting, households may differ in more than one dimension; in addition to differences in the inter-temporal budget constraint \( (Y) \), households may also differ in the utilities they derive from the consumption of the durable good \( (X) \), and/or the utilities they derive from the existing stock of durable good \( (A) \). In such a setting, IPD remains an optimal sales strategy. To show that this is the case, let \( I \) denote \( (A, X, Y) \), and \( z(I) \) denote the optimal timing of purchase. Utilizing the earlier analysis in Section 2, the household’s utility under the optimal consumption plan is given by

\[
V(z(I), I) = \left[ \frac{A + Xe^{-rz(I)}}{r} \right]^{\gamma^a} \left[ Y - P(z(I))e^{-rz(I)} \right]^{(1-a)}. \tag{29}
\]

Suppose we fix \( A \) and \( X \) at \( \bar{A} \) and \( \bar{X} \), respectively, and then vary \( Y \) by an amount \( \Delta Y \) such that \( V(z(I), I) = V(z(I), \hat{I}) \), where \( \hat{Y} \equiv Y + \Delta Y \) and \( \hat{I} \equiv (\bar{A}, \bar{X}, \hat{Y}) \). This leads to the following condition:

\[
\left[ \frac{A + Xe^{-rz(I)}}{A + Xe^{-rz(I)}} \right]^{\gamma^a} = \left[ \frac{\hat{Y} - P(z(I))e^{-rz(I)}}{Y - P(z(I))e^{-rz(I)}} \right]^{(1-a)}. \tag{30}
\]

Next, in order that the optimal timing of purchase under \( I \) and \( \hat{I} \) is the same, i.e., \( z(I) = z(\hat{I}) \), the first-order condition in Equation (6) must be satisfied as follows:

\[
\beta r X \left[ \frac{Y - P(z(I))e^{-rz(I)}}{A + Xe^{-rz(I)}} \right] = r P(z) - \frac{dP(t)}{dt} \bigg|_{t=z} = \beta r \bar{X} \left[ \frac{\hat{Y} - P(z(\hat{I}))e^{-rz(I)}}{\bar{A} + \bar{X}e^{-rz(I)}} \right]. \tag{31}
\]

This yields the following condition:

\[
\bar{A} + \bar{X}e^{-rz(I)} \equiv \bar{X} \left[ \frac{\hat{Y} - P(z(\hat{I}))e^{-rz(I)}}{Y - P(z(I))e^{-rz(I)}} \right] \tag{32}
\]

Suppose \( X \) is fixed at \( \bar{X} \). Then, the conditions in Equations (30) and (32) imply that in order that \( V(z(I), I) = V(z(\hat{I}), \hat{I}) \) and \( z(I) = z(\hat{I}) \) for all \( I, \bar{A} \) and \( \hat{Y} \) must be chosen as follows:

\[
\bar{A} = \left[ \frac{\bar{X}}{\bar{X}} \right]^{(1-a)} \left[ A + Xe^{-rz(I)} \right] - \bar{X}e^{-rz(I)}, \tag{33}
\]
\[
\hat{Y} = \left[ \frac{X}{X} \right]^\alpha \left[ Y - P(z(I))e^{-rz(I)} \right] + P(z(I))e^{-rz(I)}.
\] (34)

Therefore, while households may differ in several dimensions, their decision on the optimal timing of purchase of the durable good can be made to correspond to the problem of a particular household type where the difference is along only one dimension — the inter-temporal budget constraint \( \hat{Y} \). It follows from an application of Proposition 1 that an IPD pricing policy must be part of the optimal sales strategy for the multi-dimensional setting.

5. Concluding Remarks

In this paper, we relate the optimality of inter-temporal price discrimination to various sources of household heterogeneity, in a model where households consume both durable goods and a stream of non-durable goods. As discussed in Koh (2006), the optimal monopoly prices may be increasing initially under certain circumstances. In these situations, households may optimally choose to delay the purchase of the durable good and pay a higher price, since they are also left with a larger budget to purchase and consume the non-durable good. The optimal purchase time is chosen such that the marginal gain in utility from the consumption of more of the non-durable good is equal to the marginal loss in utility from delaying the consumption of the new durable good.

Acknowledgments

I would like to thank two anonymous referees for their comments and suggestions. All remaining errors are mine. Research support from Singapore Management University is gratefully acknowledged.

References