# Cooperative approaches to managing social responsibility in a market with externalities

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# Cooperative Approaches to Managing Social Responsibility in a Market with Externalities

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**Problem Definition:** This paper studies two cooperative approaches of firms in managing social responsibility violations of their supplier: auditing a common supplier jointly ("joint auditing") and sharing independent audit results with other firms ("audit sharing"). We study this problem in a market with externalities and a large number of firms.

**Academic/Practical Relevance:** With numerous firms procuring their materials and parts worldwide, there are many cases in which overseas suppliers violate safety, labor or environmental standards. Those violations have externalities in the sense that one firm's violation affects other firms in the same market. It is not clear how such externalities affect competing firms' incentives to cooperate and the effectiveness of such cooperation.

**Methodology:** We develop a model based on a cooperative game in partition function form, which enables us to analyze the competitive and cooperative interactions of a large number of firms in a market.

**Results:** Although there has been concern about cooperation for fear of compromising a competitive advantage, firms have incentives to cooperate in managing their suppliers when one firm can be hurt by others' violations, i.e., the negative externality is high. However, neither cooperative approach necessarily improves social responsibility, especially when one firm can benefit from others' violations, i.e., the positive externality is high. Finally, even if agreement is not reached for cooperation before conducting individual audits, social responsibility can still be improved by incentivizing firms to share their private audit results with others under a properly designed mechanism.

**Managerial Implications:** The careful assessment of the externalities associated with social responsibility violations is a key to the success of joint auditing and audit sharing. Although firms cooperate voluntarily in some cases, a government agency or an industry association should intervene in other cases to motivate cooperation if it is beneficial. In addition, caution must be taken to monitor manufacturers' audit efforts, especially when cooperative approaches are implemented in the market where competition is fierce and consumers switch brands easily.

Key words: Game Theory, Global Operations Management, Supply Chain Management

## 1 Introduction

A building that housed several garment factories in Bangladesh collapsed into a deadly heap on April 24, 2013, only five months after a horrific fire at a similar facility (Manik and Yardley 2013). The factories were producing garments for major American and European brands such as Mango, Benetton, Primark, and Walmart. Substandard construction and inadequate safety protocols were the major causes of the accident, and in the aftermath of the disaster, the media focused its attention on the firms who knowingly sourced products from the factories that maintained such poor safety conditions (Al-Mahmood et al. 2013).

With numerous firms procuring their materials and parts worldwide and outsourcing their manufacturing functions to overseas suppliers, there are many similar cases in which overseas suppliers violate safety, labor or environmental standards. For example, Nike and Adidas were criticized by the public for their suppliers' labor practices, including low wages, excessive working hours, and child labor (Nisen 2013, Burke 2000). Apple, HP, and Sony were accused of sourcing from suppliers with environmental violations (Mozur and Dou 2013).

When a social responsibility violation of a firm's supplier is revealed, it damages the firm's brand, and it also affects other firms in the same market. On the one hand, a violation that involves one firm may have a *negative externality* on other firms. This may happen when consumers lose confidence in the whole market and associate the violation with other firms as well (Roehm and Tybout 2006, Yu and Lester 2008). For example, after Bangladesh fires, twenty three global apparel brands were reported to source from one of the factories in the Rana Plaza building (Clean Clothes Campaign 2014). This incident created sustained attention from the public to a wider set of labor problems in the whole apparel industry, such as the poor working conditions in other factories in Bangladesh and other developing countries (Mestrich 2014). On the other hand, more intuitively, a social responsibility violation may have a *positive externality* if consumers switch from the firm involved in the violation to its competitors (Tsang 2000, Guo et al. 2015, Plambeck and Taylor 2016). For example, after several social responsibility scandals, many British shoppers switched to Fairtrade products that are ethically sourced (Lucas 2013).

To manage these supplier risks, firms have started to cooperate with each other in auditing their common suppliers. Many believe that audits are the best deterrence to supplier risks (Grocery Manufacturers Association (GMA) 2010). There are two approaches of cooperation that have been implemented. First, under the approach referred to as "joint auditing," multiple firms pool their resources to conduct an audit jointly on their common supplier instead of each conducting an audit individually. For example, after tragic Bangladesh fires, global apparel brands formed the Accord on Fire and Building Safety in Bangladesh and the Alliance of Bangladesh Worker Safety to jointly establish a safety program and conduct factory audits in Bangladesh (Thomasson 2014). In the pharmaceutical industry, major firms across the world, including Johnson & Johnson, GlaxoSmithKline and Bayer, have formed the Pharmaceutical Supply Chain Initiative (PSCI) to conduct joint audits on their suppliers (PSCI 2014). Since audit costs are shared among firms, the cost incurred by an individual firm is reduced under joint auditing. Second, under the approach referred to as "audit sharing," firms share results from their individual audits. For example, Rx-360 provides a platform for its members to share audit reports with each other (RX-360 2014). Similarly, in the chemical industry, Together for Sustainability (TfS) provides a website where audit reports are uploaded and accessed by all of the TfS members (TfS 2019). With the results from multiple audits, firms may obtain better knowledge about suppliers' practices.

Without taking into account the externalities of social responsibility violations, the extant model in the literature shows that firms *always* have incentives to cooperate and such cooperation *always* improves social responsibility (Caro et al. 2018). This result seems too ideal, considering that industrywide cooperation is still limited to only some industries such as the examples mentioned above, and firms in many industries still work independently. There have been concerns about cooperation for fear of compromising such competitive advantage (GMA 2010). Thus, it is unclear whether firms always have incentives to collaborate with their direct competitors and whether such cooperation will improve social responsibility. Furthermore, the extant model in the literature is limited to *two monopolistic firms* that potentially cooperate, although almost all industries have more than two competing firms in reality, and some industries even have multiple coalitions. For example, in the apparel industry, the Alliance for Bangladesh Worker Safety has 29 firms (Alliance 2017) and the Accord on Fire and Building Safety in Bangladesh involves more than 220 apparel brands (Smith 2016). Therefore, it is important to establish results about firms' cooperation in a realistic setting where multiple firms compete and they can potentially form multiple coalitions.

The objective of this paper is to investigate firms' incentives to cooperate and effectiveness of such cooperation in a market with externalities and a large number of firms. More specifically, we address the following research questions: (Q1) Do firms always have incentives to conduct joint audits or share their audit results with their competitors? If not, how can we design an incentive mechanism to motivate cooperation among competing firms? How does the degree of externalities in a market affect firms' incentives? (Q2) Does joint auditing or audit sharing guarantee improvement in social responsibility? If not, under what condition does either cooperative approach lead to improvement? How does the number of firms in a market affect the level of improvement?

To answer these questions, we develop an analytical model in which multiple manufacturers audit a common supplier to decide whether to source parts from this supplier or switch to a backup supplier who is reliable but more expensive.[1] Our model is based on *a cooperative game in partition function form*. This form of a game enables us to analyze the competitive and cooperative interactions of a large number of manufacturers in a market with externalities. Furthermore, this game allows manufacturers to potentially form multiple coalitions (as in the Alliance and the Accord). Thus, our model and analysis greatly enrich the extant literature that considers only one coalition between two monopolistic firms.

Our analysis shows that when the negative externality dominates the positive externality, manufacturers have incentives to voluntarily cooperate through joint auditing or audit sharing despite the concern of compromising a competitive advantage. This may explain the formation of the Accord, the Alliance, and the initiatives in the pharmaceutical and chemical industries. In these examples, a fatal accident can potentially have significant ripple effects beyond a single manufacturer. We also find that cooperation serves as a win-win solution in this case for both manufacturers and society by increasing profits and social responsibility simultaneously.

[1] For ease of exposition, a downstream firm who sources parts from an upstream supplier is referred to as a manufacturer, but it can be a retailer as well. Similarly, the parts can be materials or finished goods.

Different from the extant result that firms always have incentives to cooperate, we find that when the positive externality dominates the negative externality, manufacturers may not cooperate. They may audit the supplier jointly only when the audit cost is high, and they are unlikely to agree upon sharing audit results even with the substantial risk of social responsibility violations. This is the case when consumers are less sensitive to responsibility violations; for example, no cooperative initiative has been undertaken to address the environmental violations mentioned earlier which involved Apple, HP, and Sony. For this case, our result suggests that a government agency or a third-party organization such as an industry association should intervene, and offer an incentive-compatible mechanism to facilitate industry cooperation. We propose such a mechanism that reallocates profits among manufacturers and specifies the minimum amount of subsidy that is needed from the outside.

Our analysis further reveals that, contrary to the extant result in the literature, neither cooperative approach necessarily improves social responsibility. This is because cooperation reduces the intensity of competition among manufacturers, and manufacturers reduce their audit efforts significantly under joint auditing or audit sharing when the negative externality is very low or the positive externality is very high. Moreover, with a larger number of manufacturers, it is more likely for joint auditing to be effective: this is intuitive because as more manufacturers share audit costs under joint auditing, they can choose higher audit effort. In contrast, the cooperation among a larger number of manufacturers can hurt the effectiveness of audit sharing. This is because as more manufacturers share audit results, they have incentives to free-ride on others' audit efforts. The formation of multiple coalitions such as the Accord and the Alliance makes it more likely for joint auditing to be effective than a single industrywide coalition, but this is not always the case with audit sharing. Lastly, when comparing the effectiveness of joint auditing with that of audit sharing, we find that when the risk of social responsibility violations is high, audit sharing is more effective than joint auditing (and vice versa). The reason is that when anticipating higher risk, manufacturers choose higher audit efforts for their individual audits. This makes audits more informative so that audit sharing, which leverages the value of the information from individual audits, becomes more effective.

We consider several extensions, including a hybrid scenario under which joint auditing and audit sharing coexist, heterogeneity of manufacturers in their contributions to the supplier's profit, payment decisions of manufacturers to the supplier, different ways in which the supplier's violation is handled by manufacturers, and the audit fatigue of the supplier. Notably, we find that the program (such as the Accord), which helps suppliers to correct their problems, can be more effective in inducing higher compliance than the program (such as the Alliance) under which manufacturers can walk away when suppliers do not fix their problems.

## 2 Related Literature

This paper is related to the nascent literature on socially-responsible operations and to a stream of research that applies cooperative game theory to firms' operational decisions.

The literature on socially-responsible operations studies various challenges firms face in managing social responsibility in their operations. Our work is particularly relevant to a stream of research that studies the management of suppliers in decentralized supply chains to improve social responsibility. Several papers in this stream examine various means such as contract penalty, inspection, and certification to induce a supplier to exert high effort for quality improvement (e.g., Baiman et al. (2000), Hwang et al. (2006), Chao et al. (2009), Chen and Deng (2013), Lewis et al. (2015)). Babich and Tang (2012) investigate a mechanism with deferred payments to improve product safety, and Rui and Lai (2015) extend it to broader settings. Chen and Lee (2017) examine screening mechanisms to distinguish suppliers with different ethical levels, and Guo et al. (2015) analyze the sourcing decision of a buyer choosing between a responsible supplier and a risky supplier. Agrawal and Lee (2018) compare a sustainable preferred policy with a sustainable required sourcing policy. Chen et al. (2015), Huang et al. (2015), Cho et al. (2016) and Plambeck and Taylor (2016) study the effectiveness of supplier audits conducted by firms and NGOs to mitigate suppliers' violation of social responsibility in various settings. Whereas a typical setup in these prior papers involves only one manufacturer, our paper studies cooperation among multiple manufacturers to induce a supplier to behave in a socially responsible manner. Following our paper, Orsdemir et al. (2018) investigate the impacts of vertical integration and horizontal sourcing on social responsibility; Levi et al. (2018) examine farms' strategic adulteration behavior and how quality uncertainty, supply chain dispersion, traceability, and testing sensitivity jointly impact the equilibrium adulteration behavior; and Chen et al. (2019) investigate whether buyers should prioritize the auditing of their common supplier in an assembly network consisting of two buyers and three suppliers.

Manufacturers in our model compete in their social responsibility levels, and they are influenced by such externalities that a violation incident of one manufacturer not only affects her own profit, but also has an impact on other manufacturers' profits. Although Caro et al. (2018) study the same topic of joint auditing and audit sharing as this paper, they consider two monopolistic manufacturers with no externalities. Using cooperative game theory, our paper analyzes cooperation among more than two manufacturers who compete in social responsibility levels under externalities – in other words, at a high level, the model of Caro et al. (2018) is a special case (with two manufacturers and no externalities) of our general model, although there are some differences between the two models. Although Caro et al. (2018) is written independently of us, we are the first to study joint auditing and audit sharing among multiple firms (Fang 2014). Our analysis highlights a crucial role of externalities: In the absence of externalities, all manufacturers always have incentives to cooperate and such cooperation always improves social responsibility. When taking into account externalities, we find that this is true only when the negative externality dominates the positive externality. Finally, even if agreement is not reached for cooperation (ex-ante) before conducting individual audits, social responsibility can still be improved by incentivizing firms to share their private audit results with others (i.e., ex-post audit sharing) under the incentive-compatible mechanism we propose.

In order to examine manufacturers' incentives to cooperate, we apply cooperative game theory. Cooperative game theory has been applied to various operational problems, including inventory transshipment (e.g., Anupindi et al. 2001, Granot and Sošić 2003, Fang and Cho 2014), decentralized assembly systems (e.g., Granot and Yin 2008, Nagarajan and Sošić. 2009, Yin 2010), group buying (e.g., Chen and Yin 2010, Nagarajan et al. 2010), capacity allocation and scheduling (e.g., Hall and Liu 2010), risk sharing alliances (Huang et al. 2016), and product recycling (Gui et al. 2016). Most papers in this literature use a cooperative game in *characteristic* function form, under which the profit generated by a coalition depends only on the actions chosen by members of the coalition. Instead, we employ a cooperative game in *partition* function form to model the externalities of violation incidents which are affected by the audit efforts of both members and non-members of each coalition. Our work explores an important new application area of cooperative game theory.

# 3 Model

We consider a supply chain in which a set of manufacturers ("she") source parts from a supplier ("he"). Let $N = \{1, 2, ..., n\}$ denote the set of manufacturers, where $n$ is the total number of manufacturers. In §3.1, we present the model of individual auditing under which manufacturers do not cooperate in managing the supplier. In §3.2, we consider two forms of cooperation: joint auditing or audit sharing. Our notation is summarized in Table 1.

Table 1. Summary of Key Notation

| Symbol | Definition |
|---|---|
| $N$ | Set of manufacturers; $N = \{1, 2, ..., n\}$, where $n$ is the number of manufacturers |
| $i$ | Index for a manufacturer; $i \in N$ |
| $a$ | Indicator of whether the supplier produces irresponsibly $(= 1)$ or responsibly $(= 0)$ |
| $\theta$ | Supplier's decision: probability of social responsibility violation; $\theta \in [0, 1]$ |
| $r$ | Supplier's profit from selling parts produced responsibly to a manufacturer; $r > 0$ |
| $g$ | Supplier's cost saving by cutting corners; $g > 0$ |
| $s_i$ | Indicator of whether the supplier fails manufacturer $i$'s audit $(= 1)$ or not $(= 0)$ |
| $e_i$ | Manufacturer $i$'s decision: audit effort; $e_i \in [0, 1]$ |
| $c$, $x$ | Audit cost parameters; $c$, $x > 0$ |
| $l$ | Additional cost when a manufacturer uses the backup supplier; $l \geq 0$ |
| $q_i$ | Indicator of manufacturer $i$'s involvement with social responsibility violation; $q_i = a - s_i$ |
| $\lambda$ | Probability of the revelation of social responsibility violation; $\lambda \in [0, 1]$ |
| $z_i$ | Indicator of whether manufacturer $i$'s involvement is revealed $(= 1)$ or not $(= 0)$ |
| $\alpha_i$ | Gross profit of manufacturer $i$ with no social responsibility violation; $\alpha_i > 0$ |
| $\beta$ | Degree of the negative externality; $\beta > 0$ |
| $\gamma$ | Degree of the positive externality; $\gamma > 0$ |

### 3.1 Individual Auditing

We first provide an overview of the model. The supplier makes a decision regarding whether to produce parts responsibly or not. Each manufacturer then conducts an individual audit on the supplier. When the supplier produces parts irresponsibly, he may fail the audits of some manufacturers, who then switch to a backup supplier that is reliable but more expensive. For the other manufacturers who fail to detect the supplier's violation, there is a potential risk that the violation will be revealed to consumers – in this case, those manufacturers incur losses because some consumers switch to the manufacturers who use the reliable backup supplier (i.e., the positive externality exists). However, even the manufacturers who use the reliable backup supplier may also incur losses when consumers lose confidence in the whole market and reduce their consumption (i.e., the negative externality exists). Next, we present details of our model.

The supplier decides whether to produce parts responsibly or irresponsibly to save costs (e.g., employing child labor, using cheap facilities involving safety or environmental risks). We denote by $a = 0$ the case when the supplier produces parts responsibly, and denote by $a = 1$ the case when the supplier produces them irresponsibly. Define the supplier's probability of a social responsibility violation as $\theta = P(a = 1)$. The supplier may adopt a pure strategy of either producing parts responsibly (i.e., $\theta = 0$) or irresponsibly (i.e., $\theta = 1$), or adopt a mixed strategy of choosing $\theta \in (0, 1)$.[2]

Under individual auditing, each manufacturer conducts an audit on the supplier. Let $s_i = 1$ ($s_i = 0$) represent the case when the supplier fails (passes) manufacturer $i$'s audit. When the supplier produces parts responsibly, he passes the audits of all manufacturers; i.e., $P(s_i = 1|a = 0) = 0$ for all $i \in N$. When the supplier produces parts irresponsibly, he fails manufacturer $i$'s audit with probability $e_i = P(s_i = 1|a = 1) \in [0, 1]$, where $e_i = 0$ may represent the case in which manufacturer $i$ conducts no audit. Each manufacturer $i$ decides on the probability $e_i$, called "audit effort," without the knowledge of $\theta$ chosen by the supplier, and incurs audit cost $C(e_i)$. We assume an exponential cost function, $C(e_i) = c(1 - e_i)^{-x}$ where $c, x > 0$, which is increasing and convex. Parameter $x$ captures the degree of convexity in the cost function. This cost function suggests that an audit does not generate perfect information (i.e., $e_i < 1$). A similar function has been used in the literature (e.g., Huang et al. 2015). Our qualitative insights continue to hold with other cost functions such as a power cost function $C(e_i) = ce_i^x$ where $x \geq 1$.

If the supplier passes manufacturer $i$'s audit, the supplier can sell parts to manufacturer $i$. Let $r$ ($> 0$) denote the supplier's profit from selling the parts produced responsibly to one manufacturer, and let $g$ ($> 0$) denote the supplier's cost saving from producing parts irresponsibly. Although the supplier can lower his cost by cutting corners, if he fails manufacturer $i$'s audit, then manufacturer

[2] A mixed strategy can be interpreted as the belief held by manufacturers concerning the supplier's action. This is the standard interpretation in Game theory – see, e.g., Rubinstein (1991). Alternatively, $(1 - \theta)$ can be interpreted as a continuous social responsibility effort, e.g., hiring a portion of workforce with child labor.

$i$ does not source parts from the supplier; instead, manufacturer $i$ switches to the backup supplier with additional cost $l$ ($\geq 0$), who always produces his product in a responsible manner.[3] If only some manufacturers, not all of them, detect the supplier's violation, manufacturers in the market may end up sourcing from different suppliers. The expected profit of the supplier, $E\pi_0$, can then be expressed as a function of his decision variable $\theta$:

$$E\pi_0(\theta) = nr - \theta \left( r \sum_{i \in N} e_i - g \right), \tag{1}$$

where the second term represents the expected cost of a social responsibility violation to the supplier.[4]

The supplier's violation of social responsibility can result in a loss of profit to manufacturers. Existing models in the literature reviewed in §2 deal with a single manufacturer who may lose a part of her profit (in particular, due to the loss of socially conscious customers) when the supplier's violation is disclosed to the public. A main departure of our model from those existing models is that a social responsibility violation of one manufacturer not only affects her own profit, but also has an impact on other manufacturers' profits.

To model such externalities, we define $q_i \equiv a - s_i$ which indicates whether or not manufacturer $i$ is involved with a social responsibility violation. If the supplier produces parts responsibly or the violation is detected during manufacturer $i$'s audit (i.e., $a = s_i = 0$ or $a = s_i = 1$), then manufacturer $i$ is not involved with any violation (i.e., $q_i = 0$). In contrast, if the supplier produces parts irresponsibly but he passes manufacturer $i$'s audit (i.e., $a = 1$ and $s_i = 0$), then manufacturer $i$ is involved with the violation (i.e., $q_i = 1$). Given that manufacturer $i$ is involved with the social responsibility violation, the involvement is revealed to consumers with probability $\lambda$. Using a random variable $z_i$, we denote this scenario by $z_i = 1$, while denoting by $z_i = 0$ the scenario that manufacturer $i$ is not involved with any violation or the involvement is not revealed to consumers.[5] Note that we do not require the revelation of social responsibility violations of different

[3] Our assumption that a manufacturer does not source parts from the supplier who fails the audit is consistent with Plambeck and Taylor (2016), who do not consider a manufacturer's switching cost by assuming $l = 0$. Guo et al. (2015) study a manufacturer's decision to choose between a reliable supplier and a risky supplier, in which a reliable supplier has zero responsibility risk. Likewise, we assume for simplicity that a back-up supplier involves no risk of a social responsibility violation. In §5.4, we further consider a case in which a manufacturer helps the supplier to correct the violation if the supplier fails her audit.

[4] Alternatively, if the supplier saves cost $g$ for each manufacturer who sources from the supplier, then $(r \sum_{i \in N} e_i - g)$ in (1) can be replaced with $\{r \sum_{i \in N} e_i - g \sum_{i \in N}(1 - e_i)\} = (r + g) \sum_{i \in N} e_i - ng$. Or if the supplier incurs an additional cost $r_b$ from audit failure (e.g., opportunity cost of lost future profit or material cost from production), then this term can be replaced with $\{(r + r_b) \sum_{i \in N} e_i - g\}$. These modifications do not affect our qualitative insights.

[5] When a violation is revealed to the public, it is plausible the supplier may also incur monetary penalty cost $r_c$. This can be modelled by revising $r \sum_{i \in N} e_i$ in (1) to $r \sum_{i \in N} e_i + \lambda r_c \sum_{i \in N}(1 - e_i) = (r - \lambda r_c) \sum_{i \in N} e_i + \lambda r_c n$. As long as manufacturers' audit efforts reduce the supplier's propensity to violate (i.e., $r > \lambda r_c$), our qualitative insights continue to hold.

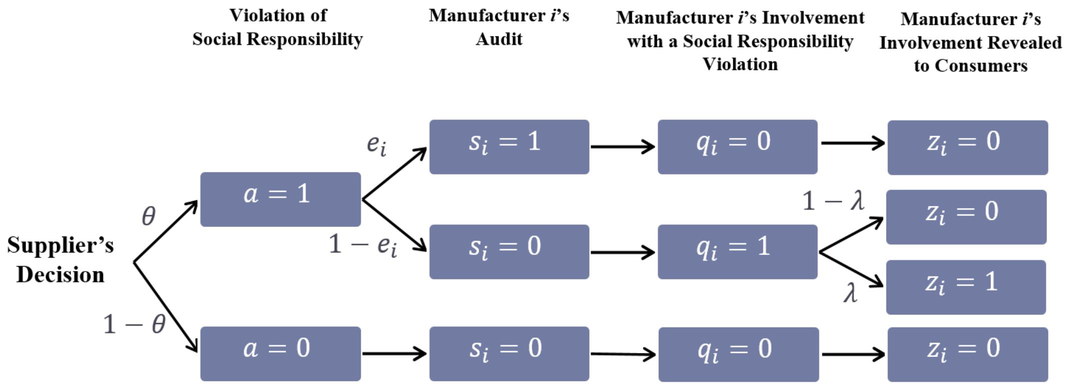


Figure 1: Manufacturer $i$'s involvement with a social responsibility violation revealed to consumers ($z_i$) that depends on the violation of the supplier ($a$) and the audit result from manufacturer $i$ ($s_i$).

manufacturers to be independent. Figure 1 illustrates all different scenarios.

As discussed in §1, one manufacturer's social responsibility violation has two distinct impacts on other manufacturers. On the one hand, it may reduce the demands of other products in the market through the *negative externality* because consumers may lose confidence in the whole market. On the other hand, consumers may switch from the product involved in a social responsibility violation to other products, creating the *positive externality.* These externalities are captured in manufacturer $i$'s profit $\pi_i$ as follows. Let $\alpha_i$ ($> 0$) represent manufacturer $i$'s gross profit in the market with no social responsibility violation. Using the indicator variable $z_i$, manufacturer $i$'s profit $\pi_i$ can be expressed as a function of her decision variable $e_i$:

$$\pi_i(e_i) = (1-\theta)\alpha_i + \theta \left\{ \alpha_i - s_i l - \beta \frac{\sum_{j \in N} z_j}{n} + \gamma \left( \frac{\sum_{j \in N} z_j}{n} - z_i \right) \right\} - C(e_i), \tag{2}$$

where the first term is manufacturer $i$'s profit when the supplier produces parts responsibly with probability $(1-\theta)$, the second term is her profit when the supplier produces parts irresponsibly with probability $\theta$, and the last term $C(e_i)$ is the audit cost. In the second term, $l$ is the additional cost when sourcing parts from the backup supplier, and $\beta > 0$ (resp., $\gamma > 0$) captures the degree of the negative (resp., positive) externality. We explain these two externality terms intuitively as follows (while providing a consumer choice model in Appendix, where socially conscious consumers do not purchase products of manufacturers who are involved in violations or switch to other manufacturers' products). First, the term $\beta \sum_{j \in N} z_j/n$ increases with the number of manufacturers involved in social responsibility violations in the market. The more products from such manufacturers in the market, the larger loss in the profit for manufacturer $i$. Thus, this term captures the negative externality of one manufacturer's social responsibility violation on other manufacturers in the market. Second, the term $\gamma(\sum_{j \in N} z_j/n - z_i)$, which captures the positive externality, increases with the difference between the average level of social responsibility risks among all manufacturers in the market and manufacturer $i$'s own level. This term will be positive when manufacturer $i$'s level is lower than the average of competitors' levels; in other words, there will be an increase in the profit due to consumers who will switch from other manufacturers involved in social responsibility violations to manufacturer $i$. To illustrate, suppose, for example, that the supplier had produced his parts irresponsibly, and that only manufacturer 1 among four manufacturers in the market detected the violation and switched to the backup supplier. If the social responsibility violations of manufacturers 3 and 4 (not that of manufacturer 2) are revealed to consumers (i.e., $z_1 = z_2 = 0$ and $z_3 = z_4 = 1$), then manufacturers' profits are: $\pi_1 = \alpha_1 - l - \frac{1}{2}\beta + \frac{1}{2}\gamma - C(e_1)$, $\pi_2 = \alpha_2 - \frac{1}{2}\beta + \frac{1}{2}\gamma - C(e_2)$ and $\pi_i = \alpha_i - \frac{1}{2}\beta - \frac{1}{2}\gamma - C(e_i)$ for $i = 3$ and 4.

Finally, noting that the expected value of $z_i$ conditional on the supplier's violation (i.e., $a = 1$) is $\lambda(1 - e_i)$, we obtain the following $E\pi_i$ by taking the expectation of $\pi_i$ in (2) with respect to $z_j$ and $s_j$ for $j \in N$:[6]

$$E\pi_i(e_i) = (1-\theta)\alpha_i + \theta\left\{\alpha_i - e_i l - \beta\lambda\left(1 - \frac{\sum_{j\in N} e_j}{n}\right) + \gamma\lambda\left(e_i - \frac{\sum_{j\in N} e_j}{n}\right)\right\} - C(e_i). \quad (3)$$

As in the literature (e.g., Babich and Tang 2012, Plambeck and Taylor 2016), we make a set of assumptions on parameters to rule out some unrealistic or uninteresting cases: (A1) $g < nr$, (A2) $1 - \left\{\frac{\beta\lambda + (n-1)\gamma\lambda - nl}{cxn}\right\}^{-\frac{1}{x+1}} > \frac{g}{nr}$, and (A3) $\beta\lambda > l$ and $\gamma\lambda > l$. (A1) rules out the case where the supplier always produces parts irresponsibly regardless of manufacturers' audit efforts. In (A2), the left-hand side of the inequality is a manufacturer's optimal audit effort when anticipating that the supplier will produce parts irresponsibly with probability one. The right-hand side of (A2) is the audit effort above which the supplier prefers to produce parts responsibly. This condition rules out the case where a manufacturer always exerts no audit effort regardless of the supplier's propensity of violation. (A3) means that when a manufacturer detects the supplier's violation, the benefits from the externalities are large enough to cover the extra cost of the backup supplier so that the manufacturer does not source parts from the supplier who fails the audit.

### 3.2 Two Cooperative Approaches

Suppose manufacturers cooperate in managing their common supplier. For generality, we do not require that each manufacturer cooperates with all other manufacturers, but instead we assume that every manufacturer belongs to one coalition of which the size can be between 1 and $n$. Let $m$ ($\leq n$) denote the number of coalitions in the market, and let $B_k$ (where $k = 1, 2, ..., m$) denote a coalition. Define a *coalition structure* $B = \{B_1, B_2, ..., B_m\}$ as a set of coalitions. We write $i \in B_k$ if manufacturer $i$ belongs to coalition $B_k$. We next describe how manufacturers can collaborate through joint auditing or audit sharing, respectively. See Figure 2 for illustration.

[6] One can see that the revelation of social responsibility violations of different manufacturers need not be independent because $E\pi_i$ is based on the expectation of the sum of random variables $z_i$ for $i = 1, 2, ..., n$ (which is the same when $z_i$ and $z_j$ are correlated for $i \neq j$).

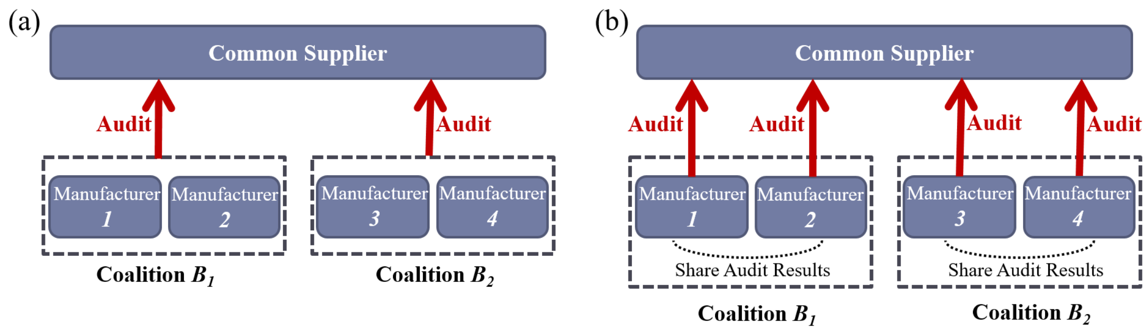


Figure 2: Two coalitions in the market with four manufacturers: (a) joint auditing, and (b) audit sharing.

Under joint auditing, manufacturers in coalition $B_k$ jointly decide audit effort $e_{B_k}$ and obtain result $s_{B_k}$ from the joint audit. The cost of this joint audit is $C(e_{B_k})$. There are three possible scenarios. First, if the supplier chooses to produce parts responsibly, then he will pass the audit and therefore all manufacturers $i$ in $B_k$ will face no social responsibility risk (i.e., $z_i = 0 \ \forall i \in B_k$). Second, if the supplier chooses to produce parts irresponsibly but fails the joint audit of $B_k$ (i.e., $a = 1$ and $s_{B_k} = 1$), then all manufacturers in $B_k$ will not source parts from the supplier (i.e., $z_i = 0 \ \forall i \in B_k$). Lastly, if the supplier chooses to produce parts irresponsibly and passes the joint audit of $B_k$ (i.e., $a = 1$ and $s_{B_k} = 0$), then all manufacturers in $B_k$ will face the risk of social responsibility violations being revealed (i.e., $z_i = 0$ with probability $1 - \lambda$ or 1 with probability $\lambda$ $\forall i \in B_k$). Since the audit effort is determined by a coalition instead of an individual manufacturer, we consider the total expected profit of manufacturers in $B_k$, which is obtained by aggregating the profits in (3) over all manufacturers in $B_k$ as follows:

$$E\pi_{B_k}(e_{B_k}) = (1-\theta)\sum_{i\in B_k}\alpha_i + \theta\left\{\sum_{i\in B_k}\alpha_i - n_k\beta\lambda\left(1 - \frac{\sum_{h=1}^{m} n_h e_{B_h}}{n}\right) + n_k\gamma\lambda\left(e_{B_k} - \frac{\sum_{h=1}^{m} n_h e_{B_h}}{n}\right) - n_k e_{B_k} l\right\} - C(e_{B_k}), \tag{4}$$

where $n_k$ is the number of manufacturers in $B_k$. There are $m$ coalitions conducting joint auditing, so the average audit effort across all coalitions is computed as $\sum_{h=1}^{m} n_h e_{B_h}/n$ using the ratio of the number of manufacturers in each coalition over the total number of manufacturers ($n_h/n$) as a weight. Similar to (3) under individual auditing, $n_k\beta\lambda\,(1 - \sum_{h=1}^{m} n_h e_{B_h}/n)$ represents the profit loss of coalition $B_k$ due to the negative externality, $n_k\gamma\lambda\,(e_{B_k} - \sum_{h=1}^{m} n_h e_{B_h}/n)$ represents the profit gain due to the positive externality, and $C(e_{B_k})$ is the audit cost of coalition $B_k$.

Under audit sharing, each manufacturer $i$ decides her own audit effort $e_i$ as in individual auditing, but manufacturers in coalition $B_k$ share the results from their individual audits with other manufacturers in $B_k$. As in joint auditing, if the supplier chooses to produce parts responsibly, then every manufacturer $i$ in $B_k$ will face no social responsibility risk (i.e., $z_i = 0 \ \forall i \in B_k$). Now suppose the supplier has chosen to produce parts irresponsibly. In this case, if at least one manufacturer $j \in B_k$ detects the supplier's violation (i.e., $s_j = 1$), then any manufacturer $i$ in the same coalition $B_k$ will learn this audit result and will not source parts from the supplier (i.e.,

$q_i = a - \max_{j\in B_k} s_j = 0\ \forall i \in B_k$); otherwise, all manufacturers in $B_k$ will face the risk of violations being revealed.[7] Since each manufacturer decides her audit effort independently to maximize her own profit instead of the total profit of the coalition, we consider the expected profit of manufacturer $i$ in $B_k$ given as

$$\begin{aligned} E\pi_{i,B_k}(e_i) \quad = \quad & (1-\theta)\alpha_i - C(e_i) + \theta \left[ \alpha_i - \beta\lambda \left\{ 1 - \sum_{h=1}^{m} \frac{\left(1 - \prod_{j\in B_h}(1-e_j)\right) n_h}{n} \right\} \right. \qquad (5) \\ & \left. + \gamma\lambda \left\{ \left(1 - \prod_{j\in B_k}(1-e_j)\right) - \sum_{h=1}^{m} \frac{\left(1 - \prod_{j\in B_h}(1-e_j)\right) n_h}{n} \right\} - \left(1 - \prod_{j\in B_k}(1-e_j)\right) l \right]. \end{aligned}$$

The average audit effort across all coalitions is given by $\sum_{h=1}^{m}\left(1-\prod_{j\in B_h}(1-e_j)\right) n_h/n$, where $1-\prod_{j\in B_h}(1-e_j)$ represents the probability that the supplier who produces parts irresponsibly fails at least one audit from members in coalition $B_h$, and $n_h/n$ is the ratio of the number of manufacturers in coalition $B_h$ over the total number of manufacturers. The profit loss due to the negative externality and the profit gain due to the positive externality are similar to the respective terms in (3) under individual auditing and (4) under joint auditing. $C(e_i)$ is the audit cost of manufacturer $i$ in coalition $B_k$.

Before we proceed to our analysis, we discuss several model extensions presented later. In §5.1 we consider a hybrid scenario under which some coalitions may conduct joint audits, while others may share the audit results from individual audits. In §5.2 we consider that manufacturers may purchase different amounts of parts from the supplier and contribute differently to the profit of the supplier. In §5.3 we extend our model by analyzing a setting where each manufacturer can determine her payment to the supplier. In §5.4 we consider a case in which a manufacturer helps the supplier to correct the violation. In §5.5 we model the audit fatigue of the supplier by considering the cost of audits to the supplier. Lastly, in Appendix, we analyze the case in which the compliance cost of the supplier is convex, and compare the supplier's profits with and without manufacturers' cooperation.

# 4 Analysis

As a benchmark, we first analyze individual auditing without cooperation in §4.1. We then compare its results with those under joint auditing or audit sharing in §4.2. We use superscript (0) to indicate equilibrium for individual auditing, (1) for joint auditing, and (2) for audit sharing.

## 4.1 Analysis of Individual Auditing

[7] In practice, audit results are verifiable as manufacturers need to provide original audit reports before sharing (RX-360 2014). As such, if manufacturers choose to share their audit results, our model assumes they will share them truthfully.

Under individual auditing, the supplier decides on the probability of social responsibility violation, and each manufacturer decides on her audit effort independently. Since manufacturers cannot observe the supplier's decision, a game between the supplier and manufacturers is a simultaneous game in which every firm makes a decision, anticipating the best response of other firms to his/her decision.[8] In the following, we first consider the best response of manufacturers and then that of the supplier.

From manufacturer $i$'s expected profit given in (3), if the supplier chooses to produce parts responsibly (i.e., $\theta = 0$), then the optimal audit efforts of all manufacturers are equal to zero. Now suppose the supplier produces parts irresponsibly with probability $\theta > 0$. Since manufacturer $i$'s expected profit $E\pi_i$ in (3) is concave in her audit effort $e_i$, i.e., $\frac{\partial^2 E\pi_i}{\partial e_i^2} = -cx(x+1)(1-e_i)^{-x-2} < 0$, we can obtain the following optimal audit effort of manufacturer $i$ by solving the first-order condition of (3):

$$e_i^{(0)}(\theta) = 1 - \left[\frac{\theta[\beta\lambda + (n-1)\gamma\lambda - nl]}{cxn}\right]^{-\frac{1}{x+1}}. \tag{6}$$

From (6), we observe that for any given violation probability $\theta$, the manufacturer would increase her audit effort when a higher level of the negative or positive externality exists (i.e., higher $\beta$ or $\gamma$). The former indicates that a manufacturer has an incentive to dedicate more resources in auditing the supplier to keep the confidence of consumers in the market, and the latter indicates that a manufacturer has an incentive to improve her social responsibility when competition becomes more intense. As expected, a manufacturer would reduce her audit effort as the audit becomes more costly (i.e., higher $c$ and $x$) or the reliable backup supplier becomes more expensive (i.e., higher $l$).

From the supplier's profit given in (1), it is easy to see that the optimal probability of violation $\theta^{(0)}(e_i)$ should satisfy one of the following three cases: $\theta^{(0)}(e_i) = 0$ if the expected loss from violation is larger than the cost saving from violation (i.e., $r\sum_{i\in N} e_i > g$); $\theta^{(0)}(e_i) = 1$ if the expected loss is smaller than the saving; and $\theta^{(0)}(e_i) \in [0,1]$ if they are the same. By analyzing these cases, we obtain the supplier's probability of violation $\theta^{(0)}$ in equilibrium in the following lemma. All proofs are provided in Appendix.

**Lemma 1** *The supplier's probability of violation, $\theta^{(0)}$, under individual auditing is given by*

$$\theta^{(0)} = \frac{cxn}{\beta\lambda + (n-1)\gamma\lambda - nl}\left(1 - \frac{g}{nr}\right)^{-(x+1)}. \tag{7}$$

One can verify that $\theta^{(0)}$ is increasing in the audit cost parameters $c$ and $x$, and the additional backup cost $l$, while it is decreasing in the externality parameters $\beta$ and $\gamma$. This is exactly opposite to their respective impact on the manufacturer's audit effort $e_i^{(0)}(\theta)$ in (6). Also, it is intuitive that the supplier is more likely to produce his parts irresponsibly when the cost saving from violation ($g$) is higher or the loss of profit from the audit failure ($r$) is lower.

[8] We have also analyzed a sequential-move game in which manufacturers first decide on their audit efforts, and then the supplier makes his compliance decision. Our main results continue to hold in the sequential-move game.

## 4.2 Analysis of the Two Cooperative Approaches

We analyze manufacturers' incentives to cooperate by investigating the stable coalitions formed by manufacturers. In §4.2.1, we introduce the basic definitions and stability concepts used in a cooperative game in partition function form. In §4.2.2 and §4.2.3, we present the results under joint auditing and audit sharing, respectively.

### 4.2.1 Preliminary: A Cooperative Game in Partition Function Form

To form a coalition for joint auditing or audit sharing, manufacturers need to negotiate how to allocate the cost and benefit of cooperation. Without a proper allocation scheme, manufacturers do not necessarily conduct a joint audit or share audit results with other manufacturers. It is well-known that a cooperative game can be used to analyze multi-lateral negotiations in such a setting (e.g., Brandenburger and Stuart 2007). The common approach that has been used to analyze the formation of coalitions in the existing work reviewed in §2 is the use of a cooperative game in *characteristic* function form. It is represented by the pair $(N, w)$, in which $N$ is a set of players and $w : 2^N \rightarrow R$ is a characteristic function of the game. A subset $S$ of $N$ is called a coalition, and $N$ itself is the grand coalition. The characteristic function $w(S)$ captures the profit generated by a coalition $S$. In our model, however, the profit generated by coalition $S$ not only depends on the audit efforts of manufacturers in $S$, but also depends on the audit efforts of manufacturers outside of $S$ due to the negative and positive externalities. As a result, a cooperative game in characteristic function form cannot be used in analyzing our model. Therefore, we employ a different approach based on a cooperative game in *partition* function form that was first introduced by Thrall and Lucas (1963).

A cooperative game in partition function form is defined by $(N, \Pi, \{v_B\}_{B\in\Pi})$, where $\Pi$ is a set of all coalition structures, $B = \{B_1, B_2, ..., B_m\} \in \Pi$ is a coalition structure that is a partition of $N$, $m$ is the number of coalitions in $B$, and $v_B$ is a partition function that associates each coalition $B_k \in B$ with the profit it can generate, $v_B(B_k)$, where $k = 1, 2, ..., m$. The value of $v_B(B_k)$ depends on how manufacturers outside $B_k$ form coalitions; i.e., $v_B(B_k)$ and $v_{B'}(B_k)$ may be different if $B \neq B'$. Given a coalition structure $B = \{B_1, B_2, ..., B_m\}$, an *allocation* is a payoff vector $\varphi = \{\varphi_1, \varphi_2, ..., \varphi_n\}$, which specifies how much of the profit generated by one coalition is attributed to each of its members. An allocation is feasible under $B$ if it satisfies $\sum_{i\in B_k} \varphi_i \leq v_B(B_k)$ for $k = 1, 2, ..., m$. Let $\phi^B$ denote a set of all feasible allocations under $B$, and define $\phi \equiv \bigcup_{B\in\Pi} \phi^B$.

In order to define the stability concept for our analysis, we introduce a domination relation for two allocations. Consider two allocations $\varphi$ and $\varphi'$ in $\phi$ and a coalition $S$ in $N$. We say that $\varphi$ *dominates* $\varphi'$ *via* $S$ and denote $\varphi$ $dom_S$ $\varphi'$ if the following two conditions hold: (i) $\sum_{i\in S} \varphi_i \leq v_B(S)$ for all $B$ for which $S \in B$, and (ii) $\varphi_i > \varphi_i'$ for all $i \in S$. When $\varphi$ $dom_S$ $\varphi'$, each member of $S$ receives a larger payoff under the feasible allocation $\varphi$ than under the present allocation $\varphi'$, and this property holds true for any coalition formation among outsiders (i.e., for all $B$ for which $S \in B$).

In addition, we say that $\varphi$ *dominates* $\varphi'$ and denote $\varphi$ *dom* $\varphi'$ if there exists $S \subseteq N$ such that $\varphi$ $dom_S$ $\varphi'$.

We use the notion of a *core* to analyze manufacturers' incentives to cooperate through joint auditing or audit sharing. The core is a set of feasible allocations that are not dominated by any other allocations, i.e., $\{\varphi \in \phi \mid \nexists \varphi' \in \phi \ s.t. \ \varphi' \ dom \ \varphi\}$. In a cooperative game in characteristic function form, a core allocation leads to a stable outcome in the sense that no subset of players has an incentive to secede from the grand coalition. This interpretation of the core can be extended to a cooperative game in partition function form as follows. Suppose $v_{B^N}(N) \geq \sum_{k=1}^{m} v_B(B_k)$ for any $B \in \Pi$, where $B^N = \{N\}$ is the coalition structure that contains only the grand coalition. This condition means that the total profit from all manufacturers is the largest when they form the grand coalition. Then, any allocation $\varphi$ that satisfies $\sum_{i\in N} \varphi_i > v_{B^N}(N)$ is not feasible, and any allocation $\varphi$ that satisfies $\sum_{i\in N} \varphi_i < v_{B^N}(N)$ is dominated via $N$ by some feasible allocation. As a result, any allocation $\varphi$ in the core satisfies $\sum_{i\in N} \varphi_i = v_{B^N}(N)$, which means that a core allocation attributes the profit generated by the grand coalition to each of its members. A core allocation is not dominated by any other allocation, so any members of the grand coalition have no incentives to secede from the grand coalition under a core allocation.

### 4.2.2 Incentives and Effectiveness of Joint Auditing

In this section, we first derive the conditions under which stable coalition structures are formed for joint auditing, and then examine the effectiveness of joint auditing by comparing the supplier's violation probability $\theta$ with that under individual auditing.

Manufacturers consider forming coalitions to audit the supplier jointly. For coalition $B_k$, where $k = 1, 2, ..., m$, the partition function $v_B^{(1)}(B_k)$ is defined as the total expected profit generated by members of $B_k$. In order to obtain $v_B^{(1)}(B_k)$, we first compute the optimal joint audit effort of coalition $B_k$. Noting that $E\pi_{B_k}$ in (4) is concave in $e_{B_k}$, we obtain from the first-order condition:

$$e_{B_k}^{(1)}(\theta) = 1 - \left[\frac{\theta n_k\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+1}}. \qquad (8)$$

Since manufacturers in the same coalition share the cost of the joint audit, one may expect that the optimal audit effort should increase with the number of manufacturers in the coalition, $n_k$. However, observe from (8) that the optimal audit effort $e_{B_k}^{(1)}(\theta)$ may not necessarily increase with $n_k$. This is due to the existence of the positive externality: With more manufacturers in a coalition, there are fewer manufacturers outside the coalition, and hence manufacturers in the coalition derive the lower benefit of ensuring high social responsibility from the positive externality. As a result, having more manufacturers in a coalition does not guarantee a higher audit effort of the coalition.

By substituting the optimal audit effort $e_{B_k}^{(1)}(\theta)$ into $E\pi_{B_k}$ in (4), we obtain the following expected profit of coalition $B_k$:

$$v_B^{(1)}(B_k) = \sum_{i \in B_k} \alpha_i - \theta n_k \left\{ \beta\lambda \left(1 - \frac{\sum_{h=1}^m n_h e_{B_h}^{(1)}}{n}\right) + \gamma\lambda \left(\frac{\sum_{h=1}^m n_h e_{B_h}^{(1)}}{n} - e_{B_k}^{(1)}\right) + e_{B_h}^{(1)} l \right\} - C(e_{B_k}^{(1)}). \tag{9}$$

In (9), the first term is the sum of all manufacturers' gross profits in $B_k$ without social responsibility violations, the second term captures the effect of externalities on profits, and the third term is the audit cost.

The following proposition presents the conditions under which the core is non-empty under joint auditing.

**Proposition 1** *Suppose the audit cost is sufficiently high (i.e., $\exists t_{cost}$ such that $c \geq t_{cost}$) or the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$). Then the core of cooperative game $(N, \Pi, \{v_B^{(1)}\}_{B \in \Pi})$ under joint auditing is non-empty, and furthermore it contains the egalitarian allocation $\varphi_i^{Eg} = \alpha_i + \frac{v_{B^N}(N) - \sum_{j \in N} \alpha_j}{n}$ for all $i \in N$.*

The first condition in Proposition 1 shows that when the audit cost is sufficiently high, manufacturers have incentives to cooperate through joint auditing. This is intuitive because manufacturers in a coalition share the joint audit cost.

The second condition shows that when the negative externality dominates the positive externality, manufacturers also have incentives to cooperate through joint auditing. To understand this result, we discuss two effects of joint auditing on the profits of manufacturers. On the one hand, joint auditing reduces the audit cost by pooling resources. The reduced audit cost enables manufacturers to choose a higher audit effort, which increases the supplier's social responsibility level. This lowers a potential loss from the negative externality. On the other hand, since manufacturers in one coalition face the same risk of social responsibility violations being revealed, they do not benefit from the positive externality of social responsibility violations from other manufacturers in the same coalition. The former effect creates incentives for manufacturers to conduct audits jointly, whereas the latter effect reduces such incentives. Therefore, when the former effect outweighs the latter effect with high $\beta/\gamma$, manufacturers would cooperate through joint auditing.

Proposition 1 further shows that joint auditing can be implemented by using the egalitarian allocation $\varphi^{Eg}$, which allocates to manufacturers the audit cost as well as the loss or profit due to the externalities from potential social responsibility violations. Note that the Shapley value, another commonly used allocation in cooperative game theory, coincides with the egalitarian allocation in our setting. This is because all manufacturers in a coalition make the same marginal contributions to the coalition's audit cost and loss/profit from the externalities.

Having characterized the incentives of manufacturers to conduct joint audits, we now investigate when joint auditing is effective in improving social responsibility. To be general, in the following, we do not require all manufacturers to cooperate through joint auditing, i.e., to form the grand coalition. Instead, we consider the Nash equilibrium violation probability of the supplier $\theta^{(1)}$ for any given coalition structure, and compare it with $\theta^{(0)}$ under individual auditing. Using $e^{(1)}_{B_k}(\theta)$ in (8), we can evaluate the supplier's violation probability $\theta^{(1)}$ in equilibrium similarly to that under individual auditing. Although the closed-form expression for $\theta^{(1)}$ does not always exist, we can still compare $\theta^{(1)}$ with $\theta^{(0)}$ in (7) as shown in the following proposition.

**Proposition 2** *There exists a threshold $\xi^{(1)} \in [0,1]$ such that joint auditing is more effective in improving social responsibility than individual auditing (i.e., $\theta^{(1)} \leq \theta^{(0)}$) if and only if $\beta/\gamma \geq \xi^{(1)}$. Furthermore, when the grand coalition is formed (i.e., $m = 1$), $\xi^{(1)} (> 0)$ is decreasing in $n$ and it is larger than that when $m > 1$.*

One might expect that cooperation among firms would help improve social responsibility. Proposition 2 cautions that this may not be always true. Specifically, when the grand coalition is formed in the market (i.e., $m = 1$) and the ratio of the negative externality to the positive externality ($\beta/\gamma$) is sufficiently small, joint auditing is less effective in improving social responsibility than individual auditing. The reason is that joint auditing lessens the intensity of competition among manufacturers within a coalition by equalizing their levels of social responsibility. This reduced competitive intensity, which is significant when $\beta/\gamma$ is sufficiently small, drives down the joint audit effort, making joint auditing less effective. Therefore, in a market that features high positive externality or low negative externality, governments and non-governmental organizations (NGOs) should closely monitor the audit effort of a consortium. For example, apparel industry may exhibit high positive externality due to intense competition among firms as well as high negative externality due to consumers' increased awareness about social responsibility after the Bangladesh fires (e.g., Gough 2012). If the positive externality dominates (i.e., a large number of consumers switch brands after social responsibility violations), then the coalitions such as the Accord and the Alliance may not improve firms' social responsibility as compared to individual approaches.

Proposition 2 also suggests that it is more likely for joint auditing to be effective if there are more manufacturers engaging in joint auditing (i.e., larger $n$) or multiple coalitions are formed (i.e., $m > 1$). The former is intuitive because, as more manufacturers share audit costs under joint auditing, they can choose higher audit efforts. For the latter, the competition among multiple coalitions induces manufacturers to maintain high audit efforts as compared to universal cooperation under the grand coalition (i.e., $m = 1$) which eliminates competition among manufacturers. With multiple coalitions, it is even possible that joint auditing is always more effective than individual auditing (i.e., $\xi^{(1)} = 0$). This result indicates that the formation of two coalitions such as the Accord and the Alliance may be more effective in improving social responsibility, ceteris paribus, than the industrywide joint audit sought by Florida tomato industry (Boyd 2012).

This proposition has the following implications in relation to the two conditions given in Proposition 1. First, when manufacturers cooperate simply to save high audit cost (see $c \geq t_{cost}$ in

Proposition 1), Proposition 2 suggests that such cooperation does not necessarily improve social responsibility. The mere fact that an industry consortium is formed to jointly audit their suppliers does not imply that it will improve social responsibility. Second, recall from Proposition 1 that when $\beta/\gamma \geq 1$, manufacturers have incentives to conduct joint auditing. Since $\xi^{(1)} \leq 1$ as shown in Proposition 2, when $\beta/\gamma \geq 1$, joint auditing not only improves social responsibility, but also benefits manufacturers by increasing their expected profits. Suppose social welfare is the summation of all manufacturers' expected profits, the supplier's expected profit and consumer surplus minus the expected societal cost of social responsibility violations. Since the supplier's expected profit and consumer surplus are unaffected by the implementation of joint auditing in our base model, the impact of joint auditing on social welfare relative to individual auditing can be defined as $\Delta^{(1)} = v^{(1)}_{B^N}(N) - \sum_{i \in N} E\pi_i^{(0)} - \delta(\theta^{(1)} - \theta^{(0)})$, where $\delta$ captures the magnitude of societal cost of violations. Propositions 1 and 2 show that the impact of joint auditing on social welfare $\Delta^{(1)}$ is positive when $\beta/\gamma \geq 1$. In this case, joint auditing is a win-win solution for both manufacturers and society.

Lastly, note from (2) that, when $\beta/\gamma = 1$, the negative externality offsets the positive externality, so it is equivalent to a setting without externalities. In particular, Caro et al. (2018) consider the setting where $\beta/\gamma = 1$ and $n = 2$. In this special case, Propositions 1 and 2 imply that all manufacturers always have incentives to conduct joint auditing and that joint auditing is always more effective than individual auditing. Yet we do not observe such industrywide cooperation in many markets. Our results indicate that externalities are crucial to the understanding of firms' incentives and effectiveness of joint auditing.

#### 4.2.3 Incentives and Effectiveness of Audit Sharing

Suppose manufacturers consider sharing the audit results that they obtain from their individual audits. Manufacturers can form coalitions at two different points in time: They can make these decisions *ex-ante* before conducting their individual audits, or they can decide whether or not to share the audit results *ex-post* after they observe their private signals about the supplier. We first analyze the ex-ante audit sharing and then analyze the ex-post audit sharing.

**Ex-ante Audit Sharing** Suppose manufacturers in coalition $B_k$ (where $k = 1, 2, ..., m$) agreed to share their audit results with each other ex-ante. We can define the partition function $v^{(2)}_B(B_k)$ similar to joint auditing based on the expected profit of manufacturers in coalition $B_k$. To obtain the expected profit, we consider manufacturer $i$'s effort $e_{i,B_k}(\theta, e_j)$ in coalition $B_k$, given the other manufacturers' effort in the same coalition, $e_j$ for $j \in B_k \backslash \{i\}$. From the first-order condition of

$E\pi_{i,B_k}$ in (5), we obtain the following optimal effort $e^{(2)}_{i,B_k}(\theta, e_j)$:

$$e^{(2)}_{i,B_k}(\theta, e_j) = 1 - \left[\frac{\theta \prod_{j\in B_k\setminus i}(1-e_j)\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+1}}. \tag{10}$$

From (10), one can see that the optimal audit effort of manufacturer $i$ is decreasing in the other manufacturers' effort in the same coalition. This free-riding effect of ex-ante audit sharing implies that a manufacturer devotes less resources in auditing the supplier when she expects her partners in the same coalition to conduct more comprehensive audits. In the symmetric equilibrium in which the audit efforts of manufacturers within the same coalition are the same,[9] we can simplify (10) into the following by substituting $e_j = e^{(2)}_{i,B_k}$:

$$e^{(2)}_{i,B_k}(\theta) = 1 - \left[\frac{\theta\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+n_k}}. \tag{11}$$

The expected profit of $B_k$, $v^{(2)}_B(B_k)$, can be obtained by substituting the optimal audit effort $e^{(2)}_{i,B_k}(\theta)$ into $E\pi_{i,B_k}$ given in (5). The following corollary presents the condition under which the core is non-empty under ex-ante audit sharing.

**Corollary 1** *If the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), then the core of cooperative game $(N, \Pi, \{v^{(2)}_B\}_{B\in\Pi})$ under ex-ante audit sharing is non-empty, and it contains the egalitarian allocation $\varphi^{Eg}$ specified in Proposition 1.*

Corollary 1 shows that when the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), it is more important for manufacturers to improve social responsibility of the entire market than to gain a competitive advantage over others, so they have incentives to cooperate through ex-ante audit sharing. Recall from Proposition 1 that the same condition applies to joint auditing. However, different from joint auditing, Corollary 1 implies that even if the audit cost is sufficiently high, manufacturers do not necessarily have incentives to share audit results ex-ante. This is expected because manufacturers do not share their audit costs while sharing their audit results.

Next, we investigate when ex-ante audit sharing improves social responsibility. As in joint auditing, we consider the Nash equilibrium violation probability of the supplier $\theta^{(2)}$ for any given coalition structure. Using $e^{(2)}_{i,B_k}(\theta)$ given in (11), we can evaluate the supplier's violation probability $\theta^{(2)}$ in equilibrium. Although a closed-form expression for $\theta^{(2)}$ does not always exist, we can still evaluate the effectiveness of ex-ante audit sharing as compared to that of individual auditing in the following proposition.

[9] We focus on the symmetric equilibrium mainly for trackability. In the asymmetric equilibrium, it is possible that only a few manufacturers in a coalition choose positive audit efforts, while others conduct no audits. Such asymmetric equilibrium is not realistic in practice because manufacturers can later verify each other's audit report.

**Proposition 3** *There exists a threshold $\xi^{(2)} \in [0,1]$ such that ex-ante audit sharing is more effective in improving social responsibility than individual auditing (i.e., $\theta^{(2)} \leq \theta^{(0)}$) if and only if $\beta/\gamma \geq \xi^{(2)}$. Furthermore, when the grand coalition is formed (i.e., $m = 1$), $\xi^{(2)}$ is increasing in $n$, and when $g/r$ is sufficiently small, $\xi^{(2)} > 0$.*

Proposition 3 suggests that audit sharing may not always be effective in improving social responsibility. Under ex-ante audit sharing, manufacturers have incentives to free-ride on others' audit efforts, so they may reduce their own audit efforts. Yet, it is not obvious whether this free-ride effect can outweigh the benefit of shared audit results, and if so, when. In the absence of externalities (i.e., $\beta/\gamma = 1$), Proposition 3 suggests that the (negative) free-ride effect never dominates the (positive) benefit of shared audit results, so that ex-ante audit sharing is always more effective. In this case, ex-ante audit sharing is always incentive-compatible (Corollary 1) and always more effective. Unfortunately, this seems too ideal to be true, considering that audit sharing is not common in many markets. Thus, we need to examine how externalities affect the incentives and effectiveness of ex-ante audit sharing.

Under ex-ante audit sharing, although manufacturers do not benefit from the positive externality of social responsibility violations from their partners (due to the same social responsibility level in a coalition), they may still lose profits due to the negative externality when no manufacturers in a coalition identify the supplier's violation. Thus, when the negative externality $\beta$ is low relative to the positive externality $\gamma$, social responsibility has a less significant impact on manufacturers' profits under ex-ante audit sharing than individual auditing without cooperation. In this case, manufacturers reduce their individual audit efforts significantly, and consequently ex-ante audit sharing becomes less effective despite its advantage of having multiple audit results.

Proposition 3 further reveals that it is less likely for ex-ante audit sharing to be effective when there are more manufacturers (i.e., larger $n$). This is different from the case of joint auditing, in which more manufacturers make joint auditing more effective. Unlike the case of joint auditing, manufacturers do not share the audit costs under ex-ante audit sharing, and thus they do not have incentives to increase their audit efforts with more manufacturers. On the contrary, they have incentives to reduce their audit efforts due to the stronger free-ride effect with larger $n$. Furthermore, whereas Proposition 2 shows that the formation of multiple coalitions (i.e., $m > 1$) makes joint auditing more effective, it is not always the case with ex-ante audit sharing. In fact, as long as the supplier's cost saving from irresponsible production is small or his profit from selling parts without violation is high (i.e., $g/r$ is sufficiently small), a sufficiently small ratio of $\beta/\gamma$ can make ex-ante audit sharing less effective than individual auditing (i.e., $\xi^{(2)} > 0$) no matter how many coalitions are formed. This is because when $g/r$ is small, the audit efforts of manufacturers are low, and thus the value created by audit sharing is limited. This difference implies that competition among audit-sharing coalitions may not be sufficient to induce high audit efforts from individual manufacturers.

When the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), manufacturers have incentives to share audit results ex-ante to increase their own expected profits (Corollary 1), and such sharing also improves social responsibility (Proposition 3). Define the impact of ex-ante audit sharing on social welfare relative to individual auditing as $\Delta^{(2)} = v^{(2)}_{B^N}(N) - \sum_{i \in N} E\pi^{(0)}_i - \delta(\theta^{(2)} - \theta^{(0)})$. When $\beta/\gamma \geq 1$, the impact of ex-ante audit sharing on social welfare $\Delta^{(2)}$ is positive so that it is a win-win solution for both manufacturers and society. This is consistent with joint auditing.

Lastly, we compare the effectiveness of joint auditing and ex-ante audit sharing as well as their impacts on social welfare.

**Corollary 2** *Suppose the grand coalition is formed under joint auditing and ex-ante audit sharing. Ex-ante audit sharing is more effective in improving social responsibility and has a larger impact on social welfare than joint auditing (i.e., $\theta^{(1)} \geq \theta^{(2)}$ and $\Delta^{(1)} \leq \Delta^{(2)}$) if and only if $g/r \geq n\left\{1 - \left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$.*

Corollary 2 shows that when the supplier's cost saving from violation ($g$) is sufficiently high or his profit from selling parts without violation ($r$) is sufficiently low, ex-ante audit sharing is more effective and has a larger impact on social welfare than joint auditing. In such a case, there is a stronger incentive for the supplier to produce parts irresponsibly. Anticipating this, each manufacturer chooses higher audit effort. Therefore, by pooling multiple audit results generated from higher efforts, audit sharing becomes more effective, which leads to a lower societal cost.

**Ex-post Audit Sharing** Suppose manufacturers conducted audits individually without agreeing to cooperate through joint auditing or ex-ante audit sharing. After observing their audit results, manufacturers still have opportunities to share their results with others. If coalition $B_k$ is formed ex-post, the partition function $v'^{(2)}_B(B_k)$ can be defined as the sum of ex-post profits given by

$$v'^{(2)}_B(B_k) = \sum_{i \in B_k} \alpha_i - n_k \left\{ \beta\lambda \left(1 - \frac{\sum_{h=1}^m n_h I(B_h)}{n}\right) + \gamma\lambda \left(\frac{\sum_{h=1}^m n_h I(B_h)}{n} - I(B_k)\right) + I(B_k)l \right\}, \tag{12}$$

where $I(B_h) = 1$ if there exists $j \in B_h$ such that $s_j = 1$, and otherwise $I(B_h) = 0$. Note that the audit cost is sunk after manufacturers obtain their results, and hence it is irrelevant to their ex-post decisions about whether or not to cooperate.

When the supplier passed or failed every manufacturer's audit, it is easy to see that the only allocation in the core is the egalitarian allocation. A more interesting case is when the supplier produced parts irresponsibly, passed at least one manufacturer's audit, and failed at least one manufacturer's audit (i.e., $a = 1$, $\min_{i \in N} s_i = 0$ and $\max_{i \in N} s_i = 1$). In this case, some manufacturers may have better knowledge about the supplier than others. In this case, ex-post audit sharing clearly improves social responsibility. Thus, in what follows, we focus on the incentives of manufacturers to share their audit results ex-post by characterizing the core of this game.

**Proposition 4** *Suppose* $a = 1$, $\min_{i \in N} s_i = 0$ *and* $\max_{i \in N} s_i = 1$. *Then;*
*(i) If* $\frac{\beta}{\gamma} \geq 1$, *then the egalitarian allocation* $\varphi^{Eg}$ *is in the core of cooperative game* $(N, \Pi, \{v_B'^{(2)}\}_{B \in \Pi})$. *If* $\frac{\beta}{\gamma} < 1$, *then* $\varphi^{Eg}$ *is not in the core of cooperative game* $(N, \Pi, \{v_B'^{(2)}\}_{B \in \Pi})$.
*(ii) If* $\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1$ *and only manufacturer i found the supplier's violation (i.e.,* $s_i = 1$*), then the following allocation is in the core:* $\varphi_i^{Un} = \alpha_i + \frac{n-1}{n}\{\beta\lambda(n-1) + \gamma\lambda\}$ *and* $\varphi_{i'}^{Un} = \alpha_{i'} - \frac{n-1}{n}\left\{\beta\lambda + \frac{\gamma\lambda}{n-1}\right\}$ *for* $i' \in N \backslash \{i\}$.
*(iii) If* $\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1$ *and more than one manufacturer found the supplier's violation or if* $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$, *then the core is empty.*

Proposition 4 suggests that when ex-ante agreement is not reached for joint auditing or audit sharing, ex-post audit sharing among manufacturers is still possible in some circumstances. When the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), Proposition 4(i) suggests that even if some manufacturers have better information about the supplier's social responsibility level, they would share the audit results ex-post with others. Interestingly, they would do so under the egalitarian allocation without any side payment from other manufacturers. Such cooperation is possible when a manufacturer's concern about social responsibility in the whole market outweighs her concern about compromising a competitive advantage.

In contrast, when the negative externality does not dominate the positive externality (i.e., $\beta/\gamma < 1$), manufacturers do not have sufficient incentives to share the audit results ex-post under the egalitarian allocation. However, we find the allocation $\varphi^{Un}$ presented in Proposition 4(ii) (where the superscript represents "unequal" allocation) is in the core if: (i) the supplier's violation is found by only one manufacturer, and (ii) the ratio of the negative externality to the positive externality is not very low (i.e., $\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1$). This allocation requires the manufacturer with better information to be compensated by other manufacturers. The amount of compensation is increasing with the negative externality ($\beta$) because the audit results to be shared becomes more valuable as the negative externality increases, and it is also increasing with the positive externality ($\gamma$) because the manufacturer with better information becomes more reluctant to share it as the positive externality increases.

Lastly, Proposition 4(iii) presents two cases when the core is empty. First, when $\beta/\gamma < 1$ and more than one manufacturers discovered the supplier's violation, manufacturers may have incentives to form subcoalitions such that each subcoalition has exactly one manufacturer who discovered the supplier's violation. In this case, the empty core may not be an issue from a practical perspective because all manufacturers get to know the supplier's violation. Moreover, the allocation given in Proposition 4(ii) can still be applied to each subcoalition by replacing $n$ with the number of manufacturers in the subcoalition. Second, if the negative externality is very low or the positive externality is very high (i.e., $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$), then audit sharing cannot be achieved ex-post without the intervention of a third party. This represents the case in which the compensation required by a manufacturer with better information is so high that other manufacturers cannot afford it. For example, as mentioned in §1, there has been no cooperation so far in addressing the environmental violations which involved Apple, HP, and Sony. Therefore, the intervention of a government agency or a third-party organization such as an industry association is necessary to facilitate ex-post audit sharing in this case. For this case, the following corollary presents the amount of subsidies needed for audit sharing.

**Corollary 3** *Suppose* $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$ *and there exists only one manufacturer* $i \in N$ *such that* $s_i = 1$*. If* $\frac{n-1}{n}\{(n-2)\gamma\lambda - (2n-2)\beta\lambda\}$ *(*$> 0$*) is paid to the grand coalition (i.e.,* $v''^{(2)}_{B^N}(N) = \sum_{i \in N} \alpha_i + \frac{n-1}{n}\{(n-2)\gamma\lambda - (2n-2)\beta\lambda\}$ *and* $v''^{(2)}_B(B_k) = v^{(2)}_B(B_k)$ *for* $B \neq B^N$*), then the core of cooperative game* $(N, \Pi, \{v''^{(2)}_B\}_{B \in \Pi})$ *is always non-empty.*

When there are more than one manufacturers who discovered the supplier's violation, a subsidy scheme similar to the one in Corollary 3 can be applied to each subcoalition that has only one such manufacturer.

Lastly, note that manufacturers do not fully internalize the consequence of low social responsibility level. This is evident from the result that no allocation can induce ex-post audit sharing in some cases (i.e., the second case of Proposition 4(iii) discussed above). This lack of internalization creates inefficiency in manufacturers' cooperative decisions, and our result demonstrates that the subsidy can correct such inefficiency.

# 5 Extensions

## 5.1 Hybrid Scenario

In the base model, manufacturers engage in either joint auditing or audit sharing. In this subsection, we consider a hybrid scenario under which some coalitions may conduct joint audits while others may share the audit results from individual audits. Specifically, suppose that manufacturers $1, 2, ..., \widehat{n}$ $(1 \leq \widehat{n} \leq n)$ can potentially form coalitions $B_1, B_2, ..., B_{\widehat{m}}$ $(1 \leq \widehat{m} \leq m)$ for joint auditing and manufacturers $\widehat{n}+1, \widehat{n}+2, ..., n$ can potentially form coalitions $B_{\widehat{m}+1}, B_{\widehat{m}+2}, ..., B_m$ for ex-ante audit sharing. We use superscript (3) to indicate equilibrium for this model, and show that our insights from the base model continue to hold under this hybrid scenario.

Under this scenario, the total expected profit of manufacturers in coalition $B_k$ for joint auditing (where $k = 1, 2, ..., \widehat{m}$) is as follows:

$$E\pi_{B_k}(e_{B_k}) = (1-\theta)\sum_{i\in B_k}\alpha_i + \theta\left[\sum_{i\in B_k}\alpha_i - n_k\beta\lambda\left\{1 - \frac{\sum_{h=1}^{\widehat{m}} n_h e_{B_h} + \sum_{h=\widehat{m}+1}^{m}\left(1-\prod_{j\in B_h}(1-e_j)\right)n_h}{n}\right\}\right.$$
$$\left. + n_k\gamma\lambda\left\{e_{B_k} - \frac{\sum_{h=1}^{\widehat{m}} n_h e_{B_h} + \sum_{h=\widehat{m}+1}^{m}\left(1-\prod_{j\in B_h}(1-e_j)\right)n_h}{n}\right\} - n_k e_{B_k} l\right] - C(e_{B_k}).$$

The average audit effort across all manufacturers (cf., $\sum_{h=1}^{m} n_h e_{B_h}/n$ in (4)) depends on the number of coalitions that conduct joint auditing, $\widehat{m}$, because manufacturers choose different audit efforts under joint auditing and audit sharing. Similarly, the expected profit $E\pi_{i,B_k}(e_i)$ of manufacturer $i$ in coalition $B_k$ for ex-ante audit sharing (where $k = \widehat{m}+1, \widehat{m}+2, ..., m$) can also be obtained by modifying the average audit effort in (5).

To define the partition function under the hybrid scenario, we assume that if partition $B$ includes coalition $S$ in which some manufacturers engage in joint auditing and others engage in ex-ante audit sharing, the total expected profit of coalition $S$ is the sum of the expected profits of coalitions $B_k$ and $B'_k$, where $B_k = S \cap \{1, 2, ..., \widehat{n}\}$ and $B'_k = S \cap \{\widehat{n}+1, \widehat{n}+2, ..., n\}$. Then we can define the partition function as $v_B^{(3)}(S) = E\pi_{B_k}(e_{B_k}^{(3)}) + \sum_{i\in B'_k} E\pi_{i,B'_k}(e_i^{(3)})$, where $e_{B_k}^{(3)}$ is the optimal audit effort of coalition $B_k$ that conducts a joint audit and $e_i^{(3)}$ is the optimal audit effort of manufacturer $i$ in coalition $B'_k$ that agreed ex-ante audit sharing. Following the analysis similar to that of the base model, we obtain the following corollary:

**Corollary 4** *(i) If the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), then the core of cooperative game $(N, \Pi, \{v_B^{(3)}\}_{B\in\Pi})$ under the hybrid scenario is non-empty, and it contains the egalitarian allocation $\varphi^{Eg}$ specified in Proposition 1.*
*(ii) There exists a threshold $\xi^{(3)} \in [0, 1]$ such that collaboration under the hybrid scenario is more effective in improving social responsibility than individual auditing (i.e., $\theta^{(3)} \leq \theta^{(0)}$) if and only if $\beta/\gamma \geq \xi^{(3)}$.*

Since collaboration under the hybrid scenario combines joint auditing and audit sharing, it is not surprising that manufacturers have incentives to collaborate in the hybrid scenario when they have incentives for both joint auditing and ex-ante audit sharing (i.e., $\beta/\gamma \geq 1$). The effectiveness of this collaboration also depends on the externalities. Furthermore, it can be shown that its effectiveness is always between that of joint auditing and ex-ante audit sharing: For example, when joint auditing is more effective than ex-ante audit sharing, hybrid auditing is more effective than ex-ante audit sharing but less effective than joint auditing.

## 5.2 Heterogeneous Manufacturers

In the base model, manufacturers contribute the same amount ($r$) to the supplier's profit. Our base model abstracts away from a manufacturer's quantity decision to focus on the impact of audits on firms' socially responsible behavior. This is common in the related literature (e.g., Babich and Tang 2012, Plambeck and Taylor 2016, Chen and Lee 2017, Caro et al. 2018). In this subsection, we consider that manufacturers may purchase different amounts of parts from the supplier and contribute differently to his profit.

Suppose that there are two types of manufacturers. Manufacturers $1, 2, ..., \widetilde{n}$ ($1 \leq \widetilde{n} \leq n$) are type $H$ and manufacturers $\widetilde{n}+1, \widetilde{n}+2, ..., n$ are type $L$. By selling parts produced responsibly to a type $H$ (resp., type $L$) manufacturer, the supplier generates a profit of $r_H$ (resp., $r_L$). We assume that $r_H > r_L$. Under this scenario, the expected profit of the supplier $E\pi_0$ is as follows:

$$E\pi_0(\theta) = \widetilde{n}r_H + (n - \widetilde{n})r_L - \theta \left( \sum_{i=1}^{\widetilde{n}} r_H e_i + \sum_{i=\widetilde{n}}^{n} r_L e_i - g \right).$$

The following corollary shows the preference of manufacturers between joint auditing and ex-ante audit sharing as well as their effectiveness.

**Corollary 5** *Suppose the grand coalition is formed under joint auditing and ex-ante audit sharing. As the number of type $H$ manufacturers ($\widetilde{n}$) decreases, we have the following results:*
*(i) It is more likely for manufacturers to prefer ex-ante audit sharing to joint auditing (i.e., $v^{(2)}_{B^N}(N) \geq v^{(1)}_{B^N}(N)$ is more likely to hold).*
*(ii) It is more likely for ex-ante audit sharing to be more effective in improving social responsibility than individual auditing or joint auditing (i.e., $\theta^{(2)} \leq \theta^{(0)}$ and $\theta^{(2)} \leq \theta^{(1)}$ are more likely to hold).*

Corollary 5(i) shows that, as the number of manufacturers who contribute more to the supplier's profit decreases, manufacturers are less likely to engage in joint auditing. This is because there is a stronger incentive for the supplier to produce parts irresponsibly as his profit from selling those parts produced responsibly decreases. Anticipating this, manufacturers need to choose higher audit efforts under either joint auditing or audit sharing. However, this has a smaller impact on audit sharing, since audit sharing requires lower efforts from individual manufacturers than one comprehensive effort for joint auditing. Furthermore, Corollary 5(ii) shows that audit sharing is more effective in this case than individual auditing and joint auditing. The intuition for this result is similar to that of Corollary 2.

## 5.3 Contracting Stage

In addition to audits, manufacturers can influence the supplier's social responsibility practice through the contracts that they offer. Specifically, if manufacturers' contracts bring considerable profits to the supplier, the supplier is less likely to violate safety, labor or environmental standards because his opportunity costs of losing manufacturers' business are very high when he gets caught.

In this subsection, we consider a contracting stage that occurs before manufacturers' audits and the supplier's violation decision. Each manufacturer $i$ offers a contract to the supplier, which guarantees a gross profit $r_i$ for the parts she sources from the supplier. Then the supplier decides whether or not to accept the contract. If the supplier rejects the contract from manufacturer $i$, the supplier earns zero profit from manufacturer $i$, and manufacturer $i$ also earns zero profit. Under joint auditing, if the supplier passes a joint audit of coalition $B_k$, he can generate profits from all manufacturers in $B_k$, i.e., $\sum_{i \in B_k} r_i$. Under ex-ante audit sharing, the supplier can generate $\sum_{i \in B_k} r_i$ from coalition $B_k$ if he passes all the audits by manufacturers in the coalition. Under ex-post audit sharing, manufacturers consider sharing audit results only after they have conducted their individual audits, so their decisions in the contracting stage are the same as those under individual auditing. The interactions between manufacturers and the supplier in the subsequent stages of the game remain the same. For tractability, we focus our analysis on a symmetric coalition structure under which the number of manufacturers in every coalition is the same.

The following corollary compares the optimal payments of manufacturers under individual auditing, joint auditing, and ex-ante audit sharing.

**Corollary 6** *Suppose the number of manufacturers in every coalition is the same.*
*(i) If* $\beta/\gamma \geq 1$, *the optimal payment under individual auditing is higher than that under joint auditing or ex-ante audit sharing (i.e.,* $r_i^{(0)} \geq r_i^{(1)}$ *and* $r_i^{(0)} \geq r_i^{(2)}$*).*
*(ii) The optimal payment under joint auditing is lower than that under ex-ante audit sharing (i.e.,* $r_i^{(1)} \leq r_i^{(2)}$*).*

In relation to the conditions given in Propositions 2 and 3, Corollary 6(i) shows that when joint auditing and ex-ante audit sharing are more effective in improving social responsibility than individual auditing, the optimal payments under joint auditing and under ex-ante audit sharing are also lower. This happens because contract payments and audits work as strategic substitutes in deterring the supplier's violation (i.e., audit effort $e_i$ is decreasing in $r_i$). As audits become more effective, manufacturers reply more on audits and reduce their payments to the supplier. Further, Corollary 6(ii) shows that the optimal payment under joint auditing is always lower than that under ex-ante audit sharing. Under ex-ante audit sharing, manufacturers invest less resources in auditing due to the free-ride effect discussed earlier, and instead specify higher payments to the supplier.

## 5.4 Violation Correction

In the base model, we assume that if the supplier fails a manufacturer's audit, then the manufacturer does not source parts from the supplier and switches to the backup supplier with additional cost $l$. As discussed in §3, this is a common assumption in prior literature. However, in some cases, the manufacturer may help the supplier to correct his problem that caused the violation detected during the audit. For example, after Bangladesh fires, the Alliance plan holds suppliers responsible for making safety improvements and manufacturers terminate their contracts with suppliers in case of violations, whereas with the Accord plan, manufacturers help suppliers to improve their compliance

(Greenhouse and Clifford 2013). In this subsection, we investigate manufacturers' incentives to help the supplier to correct his problem, and examine how such corrective actions affect the supplier's compliance under individual auditing, joint auditing, and ex-ante audit sharing.

Suppose that when the supplier fails manufacturer $i$'s audit, manufacturer $i$ can choose to help the supplier to correct his problem with investment $l'$. Apparently, if the investment $l'$ is sufficiently larger than the additional cost $l$ of using a reliable backup supplier, manufacturers would not have incentives to help the supplier to correct his problem. Thus, in order to focus on the effects of cooperation and competition among manufacturers, we assume $l = l' = 0$ in this subsection. We further assume that if manufacturer $i$ helps the supplier to correct his problem, all manufacturers who source from the supplier will face no social responsibility risk (i.e., $z_i = 0\ \forall i \in N$). Then manufacturer $i$'s expected profit under individual auditing is given by

$$E\pi_i(e_i) = (1-\theta)\alpha_i + \theta \left\{ \alpha_i - \beta\lambda \prod_{j\in N}(1-e_j) \right\} - C(e_i). \tag{13}$$

One can see from (13) that the positive externality (determined by $\gamma$) does not affect the profit, since manufacturers have the same social responsibility level after the supplier's problem is corrected. Similarly, we can express a manufacturer's expected profit under joint auditing or ex-ante audit sharing; see Appendix. As in §5.3, we consider a symmetric coalition structure.

**Corollary 7** *Suppose the number of manufacturers in every coalition is the same. If $\beta/\gamma \geq 1$, the manufacturer's expected profit is higher and the supplier's violation probability is lower when manufacturers help the supplier to correct his problem rather than switching to the backup supplier under individual auditing, joint auditing or ex-ante audit sharing.*

When the negative externality dominates the positive externality (i.e., $\beta/\gamma \geq 1$), a manufacturer has the incentive to help the supplier correct his problem rather than to switch to the backup supplier. This is because the manufacturer can reduce a potential loss from the negative externality by eliminating other manufacturers' risks. However, this causes the free-ride effect as in audit sharing: By anticipating that other manufacturers may detect the supplier's problem and help him correct the problem, manufacturers may reduce their audit efforts. Corollary 7 indicates that when the negative externality is high, the former (positive) effect dominates the latter (negative) free-ride effect, so that corrective actions are more effective than immediate contract termination. For example, if the negative externality created by Bangladesh fires dominates the positive externality due to the competition in the apparel industry, the Accord plan, which helps suppliers to correct their problems, may be more effective in inducing higher compliance than the Alliance plan under which manufacturers can walk away when suppliers do not fix their problems.

### 5.5 Supplier's Audit Fatigue

When manufacturers audit their supplier, they incur audit costs. In some cases, these audits can be costly to the supplier as well. Especially for a small supplier who serves many manufacturers, preparing for and hosting multiple on-site audits can take a lot of time and resources, leaving the supplier less amount of resources to actually execute environmental and social performance improvements (McKinnon 2012). We can model the audit fatigue of the supplier by assuming that, if a manufacturer conducts an audit, the supplier's profit from the manufacturer is $r - f$, where $f$ ($\geq 0$) represents the supplier's cost to host the manufacturer's audit. When manufacturers within a coalition conduct a joint audit, the supplier incurs such cost for the coalition instead of every individual manufacturer that the supplier's profit from the coalition having $n_k$ manufacturers is $n_k r - f$. In the case of audit sharing, the supplier's cost due to audit fatigue is the same as that under individual auditing as all manufacturers conduct their own audits. If some manufacturers choose to reply entirely on the shared audit results, then the effect of the supplier's audit fatigue is similar to that under joint auditing. With this extension, we show in Corollary 8 that our results about the incentives and effectiveness of joint auditing and audit sharing continue to hold.

**Corollary 8** *When the supplier incurs the audit hosting cost $f$, Propositions 1, 2, 3, and 4 continue to hold.*

Moreover, since the supplier's cost due to audit fatigue is lower under joint auditing, his opportunity cost of losing business when a violation is caught becomes larger, and the supplier is more likely to produce parts responsibly. Therefore, as one may expect, when the supplier's audit cost is significant, joint auditing is more effective than audit sharing as shown in Corollary 9 below.

**Corollary 9** *As audit hosting cost $f$ becomes larger, it is more likely for joint auditing to be more effective in improving social responsibility than individual auditing (i.e., $\xi^{(1)}$ given in Proposition 2 is decreasing in $f$).*

## 6 Conclusion

The incidents of social and environmental violations in recent years have compelled many firms to rethink their approaches to managing suppliers. This paper investigates two cooperative approaches that are used in practice: auditing a common supplier jointly ("joint auditing") and sharing independent audit results with other firms ("audit sharing") either ex-ante or ex-post. We investigate the incentives of competing firms to cooperate by analyzing a cooperative game in partition function form, and examine when cooperation improves social responsibility in supply chains.

Our analysis shows that the incentives for cooperation as well as the effectiveness of cooperation depend crucially on the externalities of social responsibility violations. On the one hand, the

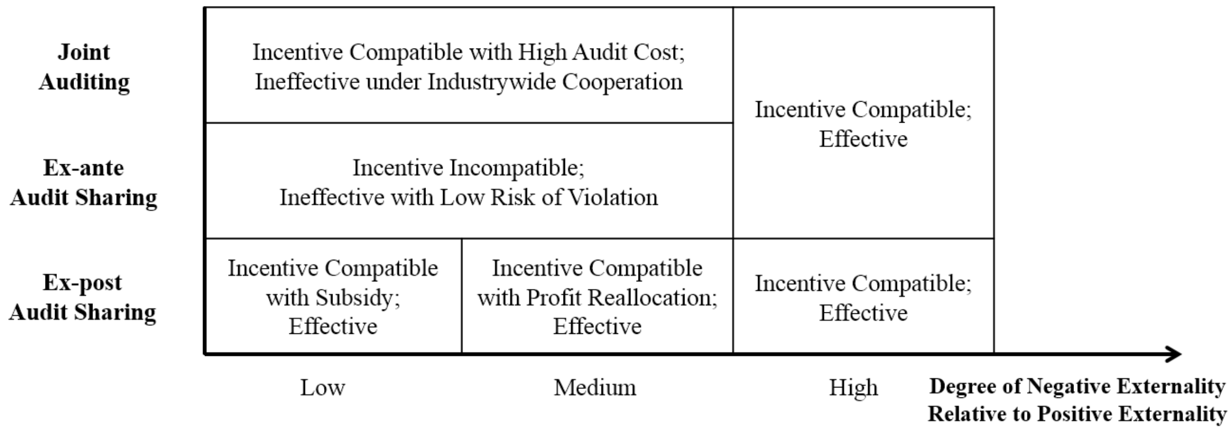


Figure 3: Incentive compatibility and effectiveness of joint auditing, ex-ante audit sharing, and ex-post audit sharing (Note: A cooperative approach is incentive-compatible if it increases manufacturers' profits, and effective if it reduces the supplier's probability of social responsibility violation).

violation of one firm may have a negative externality on the profit of other firms because consumers may lose confidence in all products in the market as a result of the violation. On the other hand, the externality of such a violation can be positive if consumers switch from the manufacturer involved in the violation to others. The degree of these externalities has the following implications (see Figure 3 for the summary):

(1) When the negative externality is high and/or the positive externality is low, manufacturers have incentives to audit their common supplier jointly and to share their audit results with each other. This is true even when some manufacturers have better knowledge ex-post about the supplier than others, and they would voluntarily share the audit results without any side payment from other manufacturers because improving the social responsibility of the entire market is more important than gaining competitive advantages over others. In this case, both joint auditing and audit sharing have positive impacts on social welfare relative to individual auditing. Therefore, industry cooperation should be encouraged in this case to achieve a win-win outcome for both manufacturers and society. When the risk of the supplier's violation is high, the positive impact of audit sharing on social welfare is more significant than that of joint auditing.

(2) When the negative externality is close to the positive externality, manufacturers have incentives to conduct joint auditing if the audit cost is high, but they may not agree to share their private audit results ex-ante. In this case, joint auditing can still be effective in improving social responsibility when multiple coalitions are formed, and ex-ante audit sharing can be effective when the risk of the supplier's violation is high. Even if ex-ante agreement is not reached for joint auditing or audit sharing, it is still possible to improve social responsibility by incentivizing manufacturers to share audit results ex-post through a side payment to those with better audit results. To facilitate such cooperation, the side payment should be set larger when the degree of either positive or negative

externality is higher.
(3) When the negative externality is low and/or the positive externality is high, manufacturers may still audit the supplier jointly if the audit cost is high, but they are unlikely to agree upon sharing audit results ex-ante. They have incentives to share audit results ex-post only with the subsidy provided by a third-party organization. Although ex-post audit sharing is effective in improving social responsibility, both joint auditing and ex-ante audit sharing may not improve social responsibility. This is because both approaches reduce the intensity of competition, lowering manufacturers' incentives to exert audit efforts. Therefore, in this case, governments and NGOs should pay close attention to industry cooperation.

Although manufacturers cooperated voluntarily in the case of the Bangladesh fires and in the pharmaceutical industry, a government agency or an industry association should intervene in other cases to motivate cooperation, for example, in addressing the environmental violations which involved Apple, HP, and Sony. Because industry cooperation does not necessarily improve social responsibility, caution must be taken to monitor manufacturers' audit efforts, especially when cooperative approaches are implemented in the market where competition is fierce and consumers switch easily. The careful assessment of the risk and externalities associated with social responsibility violations is a key to the success of joint auditing and audit sharing.

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Online Appendix for "Cooperative Approaches to Managing Social Responsibility in a Market with Externalities"

# A1 Proofs of Analytical Results

**Proof of Lemma 1:** We show in the following that $\theta \in (0,1)$ and $e_i > 0$ in the equilibrium. To see that $\theta = 0$ cannot be an equilibrium, one can observe from (3) that $e_i^{(0)}(\theta) = 0$ when $\theta = 0$, whereas $\theta^{(0)}(0) = 1$ from (1). Similarly, $\theta = 1$ cannot be an equilibrium because $e_i^{(0)}(\theta) = 1 - \left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}} \equiv \bar{e}$ from (6) when $\theta = 1$, whereas $\theta^{(0)}(\bar{e}) = 0$ due to (A2). Therefore, $\theta = 0$ or 1 can never be an equilibrium. When $\theta \in (0,1)$, $e_i = \frac{g}{nr} > 0$. By substituting (6) into $r\sum_{i\in N} e_i^{(0)} = g$ and solving for $\theta$, we obtain the equilibrium supplier's probability of violation $\theta^{(0)}$ in the lemma. □

**Proof of Proposition 1:** First, we verify $v_{B^N}(N) \geq \sum\limits_{k=1}^{m} v_B(B_k)$ for any $B \in \Pi$. From (9), $v_{B^N}(N) = \sum_{i\in N}\alpha_i - \theta n\beta\lambda\left(1 - e_N^{(1)}\right) - \theta nle_N^{(1)} - c(1-e_N^{(1)})^{-x} \geq \sum_{i\in N}\alpha_i - \theta n\beta\lambda\left(1 - e_{B_{\max}}^{(1)}\right) - \theta nle_{B_{\max}}^{(1)} - c(1-e_{B_{\max}}^{(1)})^{-x} \geq \sum_{i\in N}\alpha_i - \sum\limits_{k=1}^{m}\left\{\theta n_k\beta\lambda\left(1 - \frac{\sum_{h=1}^{m} n_h e_{B_h}^{(1)}}{n}\right) + \theta n_k le_{B_k}^{(1)} + c(1-e_{B_k}^{(1)})^{-x}\right\} = \sum\limits_{k=1}^{m} v_B(B_k)$, where $e_N^{(1)}$ is the optimal audit effort of the grand coalition $N$ under $B^N$ and $e_{B_{\max}}^{(1)} = \max\limits_h e_{B_h}^{(1)}$. The first inequality is due to the optimality of $e_N^{(1)}$ given $B^N$ and the second inequality follows from the definition of $e_{B_{\max}}^{(1)}$ and $\beta\lambda > l$.

Next, we prove that if (i) $c \geq t_{cost}$ or (ii) $\beta/\gamma \geq 1$, then manufacturers in $S \subset N$ have no incentives to secede from the grand coalition $N$, by comparing the allocation to $S$ under $B$ with that under $B^N$. Suppose coalition structure $B$ satisfies $B = \arg\min\limits_{B'\ni S} v_{B'}(S)$. We consider allocation $\varphi$ with $\sum_{i\in S}\varphi_i = v_B(S)$ such that it is the largest allocation that satisfies $\sum_{i\in S}\varphi_i \leq v_{B'}(S)$ for all $B' \ni S$. From (9), the allocation to $S$ under $B^N$ satisfies $\sum_{i\in S}\varphi_i^{Eg} = \sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1 - e_N^{(1)}\right) - \theta n_s le_N^{(1)} - \frac{n_s}{n}c(1-e_N^{(1)})^{-x} \geq \sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1 - e_S^{(1)}\right) - \theta n_s le_S^{(1)} - \frac{n_s}{n}c(1-e_S^{(1)})^{-x}$, where $n_s$ is the number of manufacturers in $S$, $e_S^{(1)}$ is the optimal audit effort of coalition $S$ under $B$, and the inequality is due to the optimality of $e_N^{(1)}$ given $B^N$. Similarly, the allocation to $S$ under $B$ satisfies $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - \theta n_s\left\{\beta\lambda\left(1 - \frac{\sum_{k=1}^{m} n_k e_{B_k}^{(1)}}{n}\right) + \gamma\lambda\left(\frac{\sum_{k=1}^{m} n_k e_{B_k}^{(1)}}{n} - e_S^{(1)}\right) + le_S^{(1)}\right\} - c(1-e_S^{(1)})^{-x}$. Then, by solving $\sum_{i\in S}\alpha_i - \theta n_s\beta\lambda\left(1 - e_S^{(1)}\right) - \theta n_s le_S^{(1)} - \frac{n_s}{n}c(1-e_S^{(1)})^{-x} \geq \sum_{i\in S}\varphi_i$, we obtain $c \geq \theta n_s\lambda(1-\frac{n_s}{n})^{-1}(1-e_S^{(1)})^x(\beta-\gamma)\left(\frac{\sum_{k=1}^{m} n_k e_{B_k}^{(1)}}{n} - e_S^{(1)}\right) \equiv t_{cost}$. Therefore, if $c \geq t_{cost}$, then $\sum_{i\in S}\varphi_i^{Eg} \geq \sum_{i\in S}\varphi_i$ so that $S$ has no incentives to secede from the grand coalition.

Similarly, for condition (ii), we consider coalition structure $B$ which minimizes $v_B(S)$ and allocation $\varphi$ with $\sum_{i\in S}\varphi_i = v_B(S)$. Note from (8) that when $\frac{\beta}{\gamma} \geq 1$, $v_B(S)$ is increasing in $e_{B_k}^{(1)}$ and $e_{B_k}^{(1)}$ is increasing in $n_k$. Thus, except $S$, every coalition $B_k$ in $B$ includes only one manufacturer (i.e., $n_k = 1$). Then, we obtain $\sum_{i\in S}\varphi_i^{Eg} - \sum_{i\in S}\varphi_i \geq \theta n_s\lambda(\beta-\gamma)\left(e_S^{(1)} - \frac{\sum_{k=1}^{m} n_k e_{B_k}^{(1)}}{n}\right) \geq 0$. □

**Proof of Proposition 2:** By substituting $e_{B_k}^{(1)}(\theta)$ in (8) into $r\sum_{k=1}^{m} n_k e_{B_k}^{(1)}(\theta) = g$, we obtain the following equation that $\theta^{(1)}$ satisfies:

$$r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(1)} n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right]=g. \tag{14}$$

Since the left-hand side of (14) is increasing in $\theta^{(1)}$, when $r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(0)} n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right]$ $\geq g$, $\theta^{(1)}\leq\theta^{(0)}$. By substituting $\theta^{(1)}$ in the left-hand side of (14) with $\theta^{(0)}=\frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$, we obtain $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]$, where $X_k=\frac{n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$. When $X_k\geq 1$ for $k=1,2,...,m$, $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})\right]=r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. By solving $X_k\geq 1$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma}\geq 1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)$.

When $m=1$, define $\xi^{(1)}=1+\frac{n}{n+1}(\frac{l}{\gamma\lambda}-1)$. One can see that $\xi^{(1)}\in(0,1]$ and $\xi^{(1)}$ is decreasing in $n$ because $\gamma\lambda>l$. If $\frac{\beta}{\gamma}\geq\xi^{(1)}$, then $X_k\geq 1$ and $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. Similarly, if $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$, then $X_k\geq 1$ so $\frac{\beta}{\gamma}\geq\xi^{(1)}$. Therefore, $\theta^{(1)}\leq\theta^{(0)}$ if and only if $\frac{\beta}{\gamma}\geq\xi^{(1)}$.

When $m>1$, define $\overline{\xi}^{(1)}=\max_k\{1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)\}$ and $\underline{\xi}^{(1)}=\min_k\{1+\frac{n}{n_k+1}(\frac{l}{\gamma\lambda}-1)\}$. One can see that $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$ if $\frac{\beta}{\gamma}\geq\overline{\xi}^{(1)}$, and $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\leq g$ if $\frac{\beta}{\gamma}\leq\underline{\xi}^{(1)}$. Since we assume $1-\left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}}>\frac{g}{nr}$, $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]$ is increasing in $X_k$. Further, $X_k$ is increasing in $\frac{\beta}{\gamma}$. Thus, there exists $\xi'^{(1)}\in[\underline{\xi}^{(1)},\overline{\xi}^{(1)}]$ such that $r\sum_{k=1}^{m} n_k\left[1-(1-\frac{g}{nr})X_k^{-\frac{1}{x+1}}\right]\geq g$ and $\theta^{(1)}\leq\theta^{(0)}$ if and only if $\frac{\beta}{\gamma}\geq\xi'^{(1)}$. Since $\overline{\xi}^{(1)}\leq\xi^{(1)}$ and $\frac{\beta}{\gamma}>0$, we have $\xi'^{(1)}\in[0,\xi^{(1)}]$. □

**Proof of Corollary 1:** The proof is similar to that of Proposition 1 and is omitted.

**Proof of Proposition 3:** By substituting $e_{i,B_k}^{(2)}(\theta)$ in (11) into $r\sum_{k=1}^{m} n_k e_{i,B_k}^{(2)}(\theta)=g$, we obtain the following equation that $\theta^{(2)}$ satisfies:

$$r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(2)}\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{n_k}{x+n_k}}\right]=g. \tag{15}$$

Since the left-hand side of (15) is increasing in $\theta^{(2)}$, when $r\sum_{k=1}^{m} n_k \left[1-\left[\frac{\theta^{(0)}\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{n_k}{x+n_k}}\right]$ $\geq g$, $\theta^{(2)}\leq\theta^{(0)}$. By substituting $\theta^{(2)}$ in the left-hand side of (15) with $\theta^{(0)}=\frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$, we obtain $r\sum_{k=1}^{m} n_k Y_k$, where $Y_k=1-\left[\frac{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl}{\beta\lambda+(n-1)\gamma\lambda-nl}\right]^{-\frac{n_k}{x+n_k}}(1-\frac{g}{nr})^{\frac{n_k(x+1)}{x+n_k}}$. When $Y_k\geq\frac{g}{nr}$ for $k=1,2,...,m$, $r\sum_{k=1}^{m} n_k Y_k\geq r\sum_{k=1}^{m}\frac{n_k g}{nr}=g$. By solving $Y_k\geq\frac{g}{nr}$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma}\geq 1+n\left[1-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]\left[n_k-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]^{-1}(\frac{l}{\gamma\lambda}-1)\equiv A_k$. Define $\overline{\xi}^{(2)}=\max_k A_k$ and $\underline{\xi}^{(2)}=\min_k A_k$. One can see that $r\sum_{k=1}^{m} n_k Y_k\geq g$ if $\frac{\beta}{\gamma}\geq\overline{\xi}^{(2)}$, and $r\sum_{k=1}^{m} n_k Y_k\leq g$ if

$\frac{\beta}{\gamma} \leq \underline{\xi}^{(2)}$. Since $r\sum_{k=1}^{m} n_k Y_k$ is increasing in $Y_k$ and $Y_k$ is increasing in $\frac{\beta}{\gamma}$ due to our assumption $1-\left\{\frac{\beta\lambda+(n-1)\gamma\lambda-nl}{cxn}\right\}^{-\frac{1}{x+1}} > \frac{g}{nr}$, there exists $\xi^{(2)} \in [\underline{\xi}^{(2)}, \overline{\xi}^{(2)}]$ such that $r\sum_{k=1}^{m} n_k Y_k \geq g$ and $\theta^{(2)} \leq \theta^{(0)}$ if and only if $\frac{\beta}{\gamma} \geq \xi^{(2)}$. Since $\gamma\lambda > l$, we have $\xi^{(2)} \leq \overline{\xi}^{(2)} \leq 1$, and further $\frac{\beta}{\gamma} > 0$ so $\xi^{(2)} \in [0,1]$.

When $m = 1$, $\xi^{(2)} = 1 + n\left[1-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\right]\left[n-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\right]^{-1}(\frac{l}{\gamma\lambda}-1)$. We obtain $\frac{\partial\xi^{(2)}}{\partial n} = (\frac{l}{\gamma\lambda}-1)(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}[n(g-nr)\{1-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\}-g(n-1)\{n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr})\}]/[n\{n-(1-\frac{g}{nr})^{\frac{x(n-1)}{n}}\}^2(nr-g)]$. Since $g < nr$ and $\gamma\lambda > l$, $\frac{\partial\xi^{(2)}}{\partial n} > 0$ if $n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) > 0$. It is easy to see that $(1-\frac{nr}{g})\ln(1-\frac{g}{nr})$ is decreasing in $\frac{g}{nr}$ and $\lim_{\frac{g}{nr}\to 0}(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) = 1$. Thus, $n-1-(1-\frac{nr}{g})\ln(1-\frac{g}{nr}) > n-2 \geq 0$, so $\xi^{(2)}$ is increasing in $n$.

When $g/r$ is sufficiently small, $\left[1-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]\left[n_k-(1-\frac{g}{nr})^{\frac{x(n_k-1)}{n_k}}\right]^{-1}$ is sufficiently small so $A_k > 0$ for $k = 1, 2, ..., m$. Thus, $\underline{\xi}^{(2)} > 0$ and $\xi^{(2)} \geq \underline{\xi}^{(2)} > 0$. □

**Proof of Corollary 2:** Under the grand coalition, we obtain $\theta^{(1)} = \frac{cx}{\beta\lambda n-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)}$ by solving $e_N^{(1)}(\theta) = \frac{g}{nr}$ similar to the base model. We obtain $\theta^{(2)} = \frac{cx}{\beta\lambda-l}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ by solving $1-(1-e_{i,N}^{(2)}(\theta))^n = \frac{g}{nr}$. We simplify the inequality $\frac{cx}{\beta\lambda n-nl}\left(1-\frac{g}{nr}\right)^{-(x+1)} \geq \frac{cx}{\beta\lambda-l}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ and obtain $g/r \geq n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$.

By substituting $e_N^{(1)} = \frac{g}{nr}$ and $e_{i,N}^{(2)} = 1-(1-\frac{g}{nr})^{1/n}$ into $\Delta^{(1)}$ and $\Delta^{(2)}$, respectively, we obtain $\Delta^{(1)}-\Delta^{(2)} = -c\left(1-\frac{g}{nr}\right)^{-x} + nc\left(1-\frac{g}{nr}\right)^{-x/n} - \delta(\theta^{(1)}-\theta^{(2)})$. Since $-c\left(1-\frac{g}{nr}\right)^{-x} + nc\left(1-\frac{g}{nr}\right)^{-x/n} \leq 0$ if and only if $g/r \geq n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$ and $\theta^{(1)}-\theta^{(2)} \geq 0$ under the same condition, $\Delta^{(1)}-\Delta^{(2)} \leq 0$ if and only if $g/r \geq n\left\{1-\left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$. □

**Proof of Proposition 4:** From (12), we can see that $v'_{B^N}(N) \geq \sum_{k=1}^{m} v'_B(B_k)$ for any $B \in \Pi$. In what follows, we prove in (i) and (ii) that manufacturers in $S \subset N$ have no incentives to secede from the grand coalition $N$ if $\frac{\beta}{\gamma} \geq 1$ and $\varphi^{Eg}$ is used, or if $\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1$ and $\varphi^{Un}$ is used. Lastly, we prove in (iii) that manufacturers always have incentives to secede if $\frac{n-2}{2n-2} \leq \frac{\beta}{\gamma} < 1$ and more than one manufacturers found the supplier's violation, or if $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$.

(i) From (12), $\sum_{i\in S}\varphi_i^{Eg} = \sum_{i\in S}\alpha_i$. We first prove that any coalition $S$, whose members fail to detect social responsibility risk (i.e., $s_i = 0$ for all $i \in S$), has no incentive to secede from the grand coalition $N$. For such a coalition $S$, since $\frac{\beta}{\gamma} \geq 1$, $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - n_s\left\{\beta\lambda\left(1-\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right)\right.$ $\left.+\gamma\lambda\left(\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right)\right\} \leq \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$. Next, we show that coalition $S$ with at least one manufacturer who has detected social responsibility risk has no incentive to secede from the grand coalition $N$ as well. For such a coalition $S$, $\sum_{i\in S}\varphi_i = \sum_{i\in S}\alpha_i - n_s\lambda(\beta-\gamma)\left(1-\frac{\sum_{k=1}^{m} n_k I(B_k)}{n}\right) \leq \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$, where the inequality holds because $\frac{\beta}{\gamma} \geq 1$. On the contrary, when $\frac{\beta}{\gamma} < 1$, $\sum_{i\in S}\varphi_i > \sum_{i\in S}\alpha_i = \sum_{i\in S}\varphi_i^{Eg}$ for coalition $S$ with at least one manufacturer who has detected social responsibility risk, so $\varphi^{Eg}$ is not in the core.

(ii) We first consider coalition $S$ such that $i \notin S$. Under $\varphi^{Un}$, $\sum_{j\in S}\varphi_j^{Un} = \sum_{j\in S}\alpha_j - \frac{n_s}{n}\{\beta\lambda(n-1)+\gamma\lambda\}$. Similar to the proof of Proposition 1, we consider coalition structure $B$ which minimizes $v'_B(S)$ and allocation $\varphi$ with $\sum_{j\in S}\varphi_j = v'_B(S)$. Since $\frac{\beta}{\gamma} < 1$, the coalition structure $B$

that minimizes $v'_B(S)$ should have the highest social responsibility level. This can be achieved by letting all manufacturers that are not in $S$ form one coalition (i.e., $m = 2$). Thus, $\sum_{j\in S}\varphi_j = \sum_{j\in S}\alpha_j - \frac{n_s}{n}\{\beta\lambda n_s + \gamma\lambda(n-n_s)\} < \sum_{j\in S}\varphi_j^{Un}$ because $\frac{\beta}{\gamma} < 1$. Next, for coalition $S$ such that $i \in S$, $\sum_{j\in S}\varphi_j^{Un} = \sum_{j\in S}\alpha_j + \lambda\left(1-\frac{n_s}{n}\right)\{n\beta+\gamma-\beta\}$. Yet, $\sum_{j\in S}\varphi_j = \sum_{j\in S}\alpha_j + \lambda(\gamma-\beta)\left(1-\frac{n_s}{n}\right) = \sum_{j\in S}\varphi_j^{Un} - \lambda\left(1-\frac{n_s}{n}\right)n\beta < \sum_{j\in S}\varphi_j^{Un}$.
(iii) We first prove that when $\frac{\beta}{\gamma} < 1$, the core is empty when there exists another manufacturer $j$ ($\neq i$) such that $s_j = 1$. On the one hand, the allocation $\varphi$ that is not dominated via $\{i,j\}$ satisfies $\varphi_i + \varphi_j \geq \alpha_i + \alpha_j + 2\lambda\left(1-\frac{2}{n}\right)(\gamma-\beta) > \alpha_i + \alpha_j$, where the first inequality is because $\varphi_i + \varphi_j \geq v_B(\{i,j\})$ for all $B$ for which $\{i,j\} \in B$ from the definition of domination and the second inequality holds because $\frac{\beta}{\gamma} < 1$. On the other hand, the allocation $\varphi$ that is not dominated via $N\backslash i$ satisfies $\sum_{z\in N\backslash i}\varphi_z \geq \sum_{z\in N\backslash i}\alpha_z$ because $\sum_{z\in N\backslash i}\varphi_z \geq v_B(N\backslash i)$ for all $B$ for which $N\backslash i \in B$ from the definition of domination. Then, we obtain $\sum_{z\in N}\varphi_z = \sum_{z\in N\backslash i}\varphi_z + \varphi_i > \sum_{z\in N}\alpha_z$, which means that $\varphi$ is not feasible. Therefore, the core is empty.

Next, we prove that when $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$, the core is empty even when there exists only one manufacturer $i$ such that $s_i = 1$. Similar to the case above, according to the definition of domination, the allocation $\varphi$ that is not dominated via $N\backslash i$ satisfies $\sum_{z\in N\backslash i}\varphi_z \geq \sum_{z\in N\backslash i}\alpha_z - (n-1)\left\{\beta\lambda\left(1-\frac{1}{n}\right) + \frac{\gamma\lambda}{n}\right\}$. Suppose $j \in N\backslash i$ so that $s_j = 0$. The allocation $\varphi$ that is not dominated via $N\backslash j$ satisfies $\sum_{z\in N\backslash j}\varphi_z \geq \sum_{z\in N\backslash j}\alpha_z + \frac{n-1}{n}\lambda(\gamma-\beta)$ according to the definition of domination. Then, we obtain $\sum_{z\in N}\varphi_z = \alpha_j + \frac{1}{n-1}\left(\sum_{z\in N\backslash i}\varphi_z - \sum_{z\in N\backslash i}\alpha_z\right) + \sum_{z\in N\backslash j}\varphi_z \geq \alpha_j - \beta\lambda\left(1-\frac{1}{n}\right) - \frac{\gamma\lambda}{n} + \sum_{z\in N\backslash j}\alpha_z + \frac{n-1}{n}\lambda(\gamma-\beta) = \sum_{z\in N}\alpha_z - 2\beta\lambda\left(1-\frac{1}{n}\right) + \gamma\lambda\frac{n-2}{n} > \sum_{z\in N}\alpha_z$, where the last inequality holds because $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. Thus, $\varphi$ is not feasible, so the core is empty. □

**Proof of Corollary 3:** We prove that when $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$, the following allocation is in the core:

$$\begin{aligned}\varphi_i^{Sub} &= \alpha_i + \frac{n-1}{n}\{\beta\lambda(n-1)+\gamma\lambda\} + \frac{n-1}{n}\{(n-2)\gamma\lambda - (2n-2)\beta\lambda\} \text{ and} \\ \varphi_{i'}^{Sub} &= \alpha_i - \frac{n-1}{n}\{\beta\lambda + \frac{\gamma\lambda}{n-1}\} \text{ for } i' \in N\backslash i.\end{aligned}$$

First, consider coalition $S$ such that $i \notin S$. For any allocation $\varphi$ to dominate $\varphi^{Sub}$ via $S$, $\sum_{z\in S}\varphi_z \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{n-n_s}{n}\right) + \gamma\lambda\left(\frac{n-n_s}{n}\right)\right\} \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{1}{n}\right) + \gamma\lambda\left(\frac{1}{n}\right)\right\} = \sum_{z\in S}\varphi_z^{Sub}$, where the first inequality is due to the definition of domination and the second inequality is due to $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. For coalition $S$ with $i \in S$, if any allocation $\varphi$ dominates $\varphi^{Sub}$ via $S$, then $\sum_{z\in S}\varphi_z - \sum_{z\in S}\varphi_z^{Sub} \leq \sum_{z\in S}\alpha_z - n_s\left\{\beta\lambda\left(1-\frac{n_s}{n}\right) + \gamma\lambda\left(\frac{n_s}{n}-1\right)\right\} - \sum_{z\in S}\varphi_z^{Sub} = (n-n_s)\lambda\left\{\frac{n_s-1}{n}\gamma - \frac{n_s-1+n}{n}\beta\right\} - \frac{n-1}{n}\lambda\{(n-2)\gamma - (2n-2)\beta\} \leq 0$, where the first inequality is due to the definition of domination and the second inequality is due to $1 \leq n_s \leq n-1$ and $\frac{\beta}{\gamma} < \frac{n-2}{2n-2}$. Therefore, $S$ has no incentives to secede from the grand coalition and $\varphi^{Sub}$ is in the core. □

**Proof of Corollary 4:** (i) Similar to the proof of Proposition 1, we consider coalition structure $B$ which minimizes $v_B^{(3)}(S)$ and allocation $\varphi$ with $\sum_{i\in S}\varphi_i = v_B^{(3)}(S)$. When $\frac{\beta}{\gamma} \geq 1$, $v_B^{(3)}(S)$ is increasing in $e_{B_k}^{(3)}$ and $e_i^{(3)}$. Further, $e_{B_k}^{(3)}$ and $e_i^{(3)}$ are increasing in $n_k$. Thus, except $S$, every coalition $B_k$ in $B$ includes only one manufacturer (i.e., $n_k = 1$). Then, we obtain $\sum_{i\in S}\varphi_i^{Eg} - \sum_{i\in S}\varphi_i \geq \theta\widehat{n}_s\lambda(\beta-\gamma)\left(e_S^{(3)} - \frac{\sum_{k=1}^{\widehat{m}} n_k e_{B_k}^{(3)}}{n}\right) + \theta(n_s-\widehat{n}_s)\lambda(\beta-\gamma)\left\{\left(1-\prod_{i\in S\backslash\widehat{S}}(1-e_i^{(3)})\right) - \sum_{k=\widehat{m}+1}^{m}\frac{\left(1-\prod_{j\in B_k}(1-e_j^{(3)})\right)n_k}{n}\right\} \geq 0$, where $\widehat{S} = S \cap \{1,2,...,\widehat{n}\}$ and $\widehat{n}_s$ is the number of manufacturers in $\widehat{S}$.

(ii) It can be easily shown that either $\theta^{(1)} \leq \theta^{(3)} \leq \theta^{(2)}$ or $\theta^{(2)} \leq \theta^{(3)} \leq \theta^{(1)}$. Define $\underline{\xi}^{(3)} = \min\{\xi^{(1)}, \xi^{(2)}\}$ and $\overline{\xi}^{(3)} = \max\{\xi^{(1)}, \xi^{(2)}\}$. According to Propositions 2 and 3, $\theta^{(3)} \leq \theta^{(0)}$ if $\beta/r \leq \underline{\xi}^{(3)}$ and $\theta^{(3)} \geq \theta^{(0)}$ if $\beta/r \geq \overline{\xi}^{(3)}$. Due to the monotonicity and continuity of $\theta^{(3)} - \theta^{(0)}$ with respect to $\beta/r$, we obtain that there exists $\xi^{(3)}$ such that $\theta^{(3)} \leq \theta^{(0)}$ if and only if $\beta/\gamma \geq \xi^{(3)}$. $\square$

**Proof of Corollary 5:** (i) Under the grand coalition, by substituting $e_N^{(1)} = \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}$ and $e_{i,N}^{(2)} = 1 - (1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L})^{1/n}$ into $v_{B^N}^{(1)}(N)$ and $v_{B^N}^{(2)}(N)$, respectively, we obtain $v_{B^N}^{(2)}(N) - v_{B^N}^{(1)}(N) = -nc\left(1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-x/n} + c\left(1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-x}$. Then we can obtain $v_{B^N}^{(2)}(N) - v_{B^N}^{(1)}(N) \geq 0$ if and only if $\frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L} \geq n\left\{1 - \left(\frac{1}{n}\right)^{\frac{n}{x(n-1)}}\right\}$, which is more likely to hold as $\widetilde{n}$ decreases because $r_H > r_L$.

(ii) Under the grand coalition, we obtain $\theta^{(0)} = \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x+1)}$, $\theta^{(1)} = \frac{cx}{\beta\lambda n-nl}\left(1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x+1)}$ and $\theta^{(2)} = \frac{cx}{\beta\lambda-l}\left(1 - \frac{g}{\widetilde{n}r_H+(n-\widetilde{n})r_L}\right)^{-(x/n+1)}$. Similar to the proof of part (i) above, we obtain $\theta^{(2)} \leq \theta^{(0)}$ and $\theta^{(2)} \leq \theta^{(1)}$ are more likely to hold as $\widetilde{n}$ decreases. $\square$

**Proof of Corollary 6:** Under individual auditing, the expected profit of manufacturer $i$ is given by

$$E\pi_i = (1-\theta)\alpha_i + \theta\left\{\alpha_i - \beta\lambda\left(1 - \frac{\sum_{j\in N} e_j}{n}\right) + \gamma\lambda\left(e_i - \frac{\sum_{j\in N} e_j}{n}\right) - e_i l\right\} - (r_i+v) - C(e_i), \quad (16)$$

where $(r_i + v)$ denotes manufacturer $i$'s payment to the supplier given the supplier's production cost $v$. By substituting $e_i = \frac{g}{nr_i}$ and $\theta$ in (7) into (16), we obtain

$$E\pi_i = \alpha_i - \frac{[cxn(\beta\lambda - l) + c]\left(1 - \frac{g}{nr_i}\right)^{-x} + cxnl\left(1 - \frac{g}{nr_i}\right)^{-x-1}}{\beta\lambda + (n-1)\gamma\lambda - nl} - (r_i + v).$$

The first-order condition implies that $r_i^{(0)}$ satisfies $[cx^2n(\beta\lambda - l) + cx]\left(1 - \frac{g}{nr_i^{(0)}}\right)^{-x-1}\frac{g}{n\{r_i^{(0)}\}^2} + cxnl(x+1)\left(1 - \frac{g}{nr_i^{(0)}}\right)^{-x-2}\frac{g}{n\{r_i^{(0)}\}^2} - [\beta\lambda + (n-1)\gamma\lambda - nl] = 0$. Similarly, under joint auditing, $r_i^{(1)}$ satisfies $[cx^2n(\beta\lambda - l) + cx]\left(1 - \frac{g}{nr_i^{(1)}}\right)^{-x-1}\frac{g}{n\{r_i^{(1)}\}^2} + cxnl(x+1)\left(1 - \frac{g}{nr_i^{(1)}}\right)^{-x-2}\frac{g}{n\{r_i^{(1)}\}^2} - n_k[\beta\lambda n_k + (n-n_k)\gamma\lambda - nl] = 0$. Under ex-ante audit sharing, $r_i^{(2)}$ satisfies $[cx^2n(\beta\lambda - l) + cx]\left(1 - \frac{g}{nr_i^{(2)}}\right)^{-x/n_k-1}\frac{g}{n\{r_i^{(2)}\}^2} + cxnl(x+n_k)\left(1 - \frac{g}{nr_i^{(2)}}\right)^{-x/n_k-2}\frac{g}{n\{r_i^{(2)}\}^2} - n_k[\beta\lambda n_k + (n-n_k)\gamma\lambda - nl] = 0$.

When $\beta/\gamma \geq 1$, $\beta\lambda + (n-1)\gamma\lambda - nl \leq \beta\lambda n_k + (n-n_k)\gamma\lambda - nl$. Since the left-hand sides of the three equations above are all decreasing in $r_i$, we obtain that $r_i^{(0)} \geq r_i^{(1)}$ and $r_i^{(0)} \geq r_i^{(2)}$. Since $\left(1 - \frac{g}{nr_i}\right)^{-x/n_k} \leq \left(1 - \frac{g}{nr_i}\right)^{-x}$, we obtain that $r_i^{(2)} \geq r_i^{(1)}$. $\square$

**Proof of Corollary 7:** When the number of manufacturers in every coalition is the same, the expected profit of manufacturer $i$ with violation correction under ex-ante audit sharing is the same as that under individual auditing in (13). The expected profit of coalition $B_k$ under joint auditing is given by

$$E\pi_{B_k}(e_{B_j}) = (1-\theta)\alpha_i + \theta\left\{\alpha_i - \beta\lambda\prod_{B_j\in B}(1-e_{B_j})\right\} - C(e_{B_j}).$$

By comparing this expression with (13), one can see that by taking one coalition under joint auditing as one manufacturer under individual auditing, we can analyze joint auditing similar to individual auditing (except that we have $m$ instead of $n$ manufacturers). Therefore, in the following, we show the proof for individual auditing only.

By solving the first-order condition of (13), we obtain manufacturer $i$'s optimal audit effort $e^*(\theta) = 1-\left(\frac{\theta\beta\lambda}{cx}\right)^{-\frac{1}{x+n}}$ with violation correction. Then we obtain $\theta^* = \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x/n+1)}$ by solving $1-(1-e^*(\theta))^n = \frac{g}{nr}$. When $\beta \geq \gamma$, we obtain $\theta^* = \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x/n+1)} \leq \frac{cx}{\beta\lambda}\left(1-\frac{g}{nr}\right)^{-(x+1)} \leq \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda}\left(1-\frac{g}{nr}\right)^{-(x+1)} = \theta^{(0)}$; i.e., the violation probability of the supplier is lower when manufacturers help the supplier to correct the violation.

By substituting $\theta^*$ into $e^*(\theta)$, we obtain $e^* = 1-\left(1-\frac{g}{nr}\right)^{\frac{1}{n}}$. According to (13), we have the equilibrium profit $E\pi_i(e^*,\theta^*) = (1-\theta^*)\alpha_i + \theta^*\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e^*) \geq (1-\theta^{(0)})\alpha_i + \theta^{(0)}\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e^*) \geq (1-\theta^{(0)})\alpha_i + \theta^{(0)}\left\{\alpha_i - \beta\lambda(1-\frac{g}{nr})\right\} - C(e_i^{(0)})$, where the last expression is the profit of manufacturer $i$ in the base model. Therefore, the expected profits of manufacturers are higher. □

**Proof of Corollary 8:** Under individual auditing and audit sharing, the expected profit of the supplier is $E\pi_0 = n(r-f) - \theta\left\{(r-f)\sum_{i\in N} e_i - g\right\}$. Then the equilibrium decisions of the manufacturers and supplier are the same as those in the original model except that $r$ is replaced by $r-f$. Therefore, Propositions 3 and 4 continue to hold.

Under joint auditing, the expected profit of the supplier is $E\pi_0 = nr - mf - \theta\{\sum_{k=1}^m(n_kr - f)e_{B_k} - g\}$. Proposition 1 continues to hold as $\theta$ is fixed. In the following, we show that Proposition 2 also holds. By substituting $e_{B_k}^{(1)}(\theta)$ in (8) into $\sum_{k=1}^m(n_kr-f)e_{B_k}^{(1)} = g$, we obtain the following equation that $\theta^{(1)}$ satisfies:

$$\sum_{k=1}^m (n_kr-f)\left[1-\left[\frac{\theta^{(1)}n_k\{\beta\lambda n_k + (n-n_k)\gamma\lambda - nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right] = g. \tag{17}$$

Since the left-hand side of (17) is increasing in $\theta^{(1)}$, when $\sum_{k=1}^m(n_kr-f)\left[1-\left[\frac{\theta^{(0)}n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{cxn}\right]^{-\frac{1}{x+1}}\right] \geq g$, $\theta^{(1)} \leq \theta^{(0)}$. By substituting $\theta^{(1)}$ in the left-hand side of (17) with $\theta^{(0)} = \frac{cxn}{\beta\lambda+(n-1)\gamma\lambda-nl}\left(1-\frac{g}{nr-nf}\right)^{-(x+1)}$, we obtain $\sum_{k=1}^m(n_kr-f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right]$, where $X_k = \frac{n_k\{\beta\lambda n_k+(n-n_k)\gamma\lambda-nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$ is increasing in $\beta/\gamma$. When $m=1$, $X_k = \frac{n\{\beta\lambda n-nl\}}{\beta\lambda+(n-1)\gamma\lambda-nl}$ and $\theta^{(1)} \leq \theta^{(0)}$ if and only if $X_k \geq [\{1-g/(nr-f)\}/\{1-g/(nr-nf)\}]^{-(x+1)}$. One can see that in this case, $X_k$ is increasing in $n$ and the right-hand side of the inequality is decreasing in $n$, so the threshold for $\beta/\gamma$, $\xi^{(1)}$, is decreasing in $n$. Furthermore, if $\beta/\gamma = 1$, $X_k \geq 1$ so $\sum_{k=1}^m(n_kr-f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right] \geq \sum_{k=1}^m(n_kr-f)\frac{g}{nr-nf} \geq g$, and thus $\xi^{(1)} \leq 1$. The proof for the case when $m > 1$ is similar to that of Proposition 2. □

**Proof of Corollary 9:** In the proof of Corollary 8, one can see $\sum_{k=1}^{m}(n_k r - f)\frac{g}{nr-nf}$ is increasing in $f$. Since $\sum_{k=1}^{m}(n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right]$ is increasing in $X_k$, there exists $x(f)$, which is decreasing in $f$, such that when $X_k \geq x(f)$ for $k = 1, 2, ..., m$, $\sum_{k=1}^{m}(n_k r - f)\left[1-(1-\frac{g}{nr-nf})X_k^{-\frac{1}{x+1}}\right] \geq g$ and $\theta^{(1)} \leq \theta^{(0)}$. By solving $X_k \geq x(f)$ for $\frac{\beta}{\gamma}$, we obtain that $\frac{\beta}{\gamma} \geq 1 + \frac{n}{n_k + x(f)}(\frac{l}{\gamma\lambda} - 1)$, which is decreasing in $f$ because $\gamma\lambda > l$ and $x(f)$ is decreasing in $f$. □

# A2 Additional Results

## A2.1 Consumer Choice Model

Suppose there are two groups of socially conscious consumers for manufacturer $i$. The first group of $\gamma'$ consumers is informed of manufacturer $i$'s social responsibility level $z_i$ and their utilities from purchasing manufacturer $i$'s product are given by $v - z_i$, where $v \sim U[0,1]$ represents the brand loyalty to manufacturer $i$. A consumer in this group switches to other manufacturers if $v - z_i < 0$. The second group of $\beta'$ consumers is uninformed of manufacturer $i$'s social responsibility level $z_i$ and their utilities from purchasing manufacturer $i$'s product are given by $u - \widetilde{z}_i$, where $u \sim U[0,1]$ and $\widetilde{z}_i$ is the belief about manufacturer $i$'s social responsibility level. Although these consumers are uninformed of one particular manufacturer's social responsibility level, they can learn the overall social responsibility level of the market from the media. We assume that they take the average level of social responsibility among all manufacturers as their belief for $z_i$; i.e., $\widetilde{z}_i = \sum_{j\in N} z_j/n$. A consumer in this group does not purchase any product in the market if $u - \widetilde{z}_i < 0$. The consumer does not switch to other manufacturers because given the consumer is uninformed, other manufacturers appear to have the same social responsibility level as manufacturer $i$ to the consumer. With this setup, the number of consumers that switch to manufacturer $i$ from other manufacturers is given by $\gamma' \sum_{j\in N\setminus i} z_j/(n-1)$. The number of consumers that switch from manufacturer $i$ to others or choose not to purchase is given by $\gamma' z_i + \beta' \sum_{j\in N} z_j/n$. Therefore, manufacturer $i$'s demand from socially conscious consumers is given by $\gamma' + \beta' + \gamma' \sum_{j\in N\setminus i} z_j/(n-1) - \gamma' z_i - \beta' \sum_{j\in N} z_j/n$. Let $\alpha_i = \gamma' + \beta'$, $\beta = \beta'$ and $\gamma = \gamma' n/(n-1)$. Then the demand can be rewritten as $\alpha_i - \beta \sum_{j\in N} z_j/n + \gamma\left(\sum_{j\in N} z_j/n - z_i\right)$. This supports the functional form of $\pi_i$ in (2).

## A2.2 Extension to Convex Cost and Supplier's Profit

In the base model, we assume that the supplier may adopt a pure strategy of either producing parts responsibly (i.e., $\theta = 0$) or irresponsibly (i.e., $\theta = 1$), or adopt a mixed strategy of choosing $\theta \in (0,1)$. In this case, the expected cost saving of the supplier from producing parts irresponsibly is $g\theta$, which is linear in $\theta$. It is plausible in some other cases that the compliance cost of the supplier is convex in his compliance effort. In the following, we first show that our insights from the base model continue to hold under the convex compliance cost and then analyze supplier's profit in this case.

Let $\varepsilon \in [0,1]$ denote the compliance effort of the supplier and we assume that the compliance cost takes a quadratic form as $g\varepsilon^2$. Then the expected profit of the supplier, $E\pi_0$, under individual auditing (cf. (1) in the base model) can be expressed as $E\pi_0(\varepsilon) = r\left\{n - (1-\varepsilon)\sum_{i\in N} e_i\right\} - g\varepsilon^2$, where the first term represents the expected revenue of the supplier from selling parts with compliance effort $\varepsilon$. By solving the first-order condition, we can obtain the optimal compliance effort of the supplier under individual auditing $\varepsilon^{(0)}(e_i) = r/(2g)\sum_{i\in N} e_i$.

For trackability, we assume that the audit costs of manufacturers are also quadratic in their audit efforts; i.e., $C(e_i) = ce_i^2$. Then we can obtain the following optimal audit effort of manufacturer $i$ under individual auditing (cf. (6) in the base model) by solving the first-order condition of (3):

$$e_i^{(0)}(\varepsilon) = \frac{(1-\varepsilon)\{\beta\lambda + (n-1)\gamma\lambda - nl\}}{2cn}.$$

The optimal audit efforts under joint auditing and ex-ante audit sharing can be obtained similarly. Following the analysis similar to that of the base model, we obtain the following corollary:

**Corollary 10** *With the quadratic compliance cost and audit cost (i.e., $g\varepsilon^2$ and $ce_i^2$), Propositions 1, 2, 3, and 4 continue to hold except that joint auditing (resp., ex-ante audit sharing) being more effective than individual auditing means $\varepsilon^{(1)} \geq \varepsilon^{(0)}$ (resp., $\varepsilon^{(2)} \geq \varepsilon^{(0)}$).*

**Proof:** The quadratic costs do not affect our analysis of the partition functions, and thus Propositions 1 and 4 continue to hold. For Proposition 2, by substituting $e_i^{(0)}(\varepsilon)$ into $\varepsilon = r/(2g)\sum_{i\in N} e_i$ and solving the equation for $\varepsilon$, we obtain the equilibrium compliance effort of the supplier under individual auditing $\varepsilon^{(0)} = r\{\beta\lambda + (n-1)\gamma\lambda - nl\}/[4gc + r\{\beta\lambda + (n-1)\gamma\lambda - nl\}]$. On the other hand, the equilibrium compliance effort under joint auditing $\varepsilon^{(1)}$ satisfies $\varepsilon^{(1)} = r/(2g)\sum_{k=1}^m n_k e_{B_k}^{(1)}$, where $e_{B_k}^{(1)} = n_k(1-\varepsilon^{(1)})\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(2cn)$. By replacing $\varepsilon^{(1)}$ in $\varepsilon^{(1)}/(1-\varepsilon^{(1)})$ with $\varepsilon^{(0)}$ and comparing it with $r\sum_{k=1}^m n_k^2\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(4cng)$, following a similar proof to that of Proposition 2, one can show that the proposition continues to hold. Proposition 3 can be proved similarly. □

Lastly, the following corollary compares the supplier's profit with and without the cooperation in auditing.

**Corollary 11** *The expected profit of the supplier under joint auditing or ex-ante audit sharing is lower than that under individual auditing if $\beta/\gamma$ is sufficiently large.*

**Proof:** By comparing the optimal audit effort under individual auditing $e_i^{(0)}(\varepsilon) = (1-\varepsilon)\{\beta\lambda + (n-1)\gamma\lambda - nl\}/(2cn)$ and that under joint auditing $e_{B_k}^{(1)}(\varepsilon) = n_k(1-\varepsilon^{(1)})\{n_k\beta\lambda + (n-n_k)\gamma\lambda - nl\}/(2cn)$, it is easy to see that $e_i^{(0)}(\varepsilon) < e_{B_k}^{(1)}(\varepsilon)$ if $\beta/\gamma$ is sufficiently large. Thus, the expected profit of the supplier under individual auditing satisfies $E\pi_0^{(0)}(\varepsilon^{(0)}) = r\left\{n - (1-\varepsilon^{(0)})\sum_{i\in N} e_i^{(0)}(\varepsilon^{(0)})\right\} - g\varepsilon^{(0)2} \geq r\left\{n - (1-\varepsilon^{(1)})\sum_{i\in N} e_i^{(0)}(\varepsilon^{(1)})\right\} - g\varepsilon^{(1)2} > r\left\{n - (1-\varepsilon^{(1)})\sum_{k=1}^m n_k e_{B_k}^{(1)}(\varepsilon^{(1)})\right\} - g\varepsilon^{(1)2} = E\pi_0^{(1)}(\varepsilon^{(1)})$ if $\beta/\gamma$ is sufficiently large, where the first inequality is due to the optimality of $\varepsilon^{(0)}$ under individual auditing. The result under ex-ante audit sharing can be proved similarly. □

When the negative externality is high or the positive externality is low, the manufacturers conduct more comprehensive audits when they cooperate than when they do not. As a result, the supplier has to choose a higher compliance effort under joint auditing or ex-ante audit sharing, which leads to a higher compliance cost and a lower profit.