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Entry of copycats of luxury brands

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Abstract

We develop a game-theoretic model to examine the entry of copycats and its implications by incorporating two salient features, namely, two product attributes: physical resemblance and product quality, and two consumer utilities: consumption utility and status utility. Our equilibrium analysis suggests that copycats with a high physical resemblance but low product quality are more likely to enter the market successfully by defying the deterrence of the incumbent. Furthermore, we show that higher quality can hinder copycat to successfully enter the market. Finally, we show that the entry of copycats does not always improve consumer surplus and social welfare. In particular, when the quality of the copycat is sufficiently low, the loss in status utility from consumers of the incumbent product overshadows the small gain in consumption utility from buyers of the copycat, leading to an overall decrease in consumer surplus and social welfare.

Keywords: Conspicuous consumption, copycat, counterfeit, entry deterrence, entry strategies, pricing strategies.

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1 Introduction

"Fake goods aren't totally bad, at least it created jobs at some counterfeit factories...

We don't want to be a brand that nobody wants to copy."

Prada CEO Patrizio Bertelli (2012)

Copycats are generally quickly (and sometimes illegally) produced, low-priced and lower-quality replicas of products that enjoy substantial brand value (Lai and Zaichkowsky (1999)). As articulated by Katz (1960) and Wilcox et al. (2009), many consumers *knowingly* purchase non-deceptive copycats of luxury brands mainly due to the social status associated with the luxury brands. For this reason, the market for copycat goods is huge. Last year, American border officials nabbed copies that, had they been genuine, would have been worth $1.2 billion. Their European Union counterparts seized EURO 768m ($1 billion) of fakes in 2013. But these were surely a fraction of the counterfeits being peddled. Estimates for the total value of fakes sold worldwide each year go as high as $1.8 trillion (The Economist, August 1, 2015).

Efficient supply networks, inconsistent law enforcement[1], and large under-served markets have enabled many firms in China and other developing countries to produce and sell imitation products. A recent report issued by the United Nation suggests that 70% of all copycats of fashion and luxury goods is produced in China. In fact, a new term "Shanzhai" has been created as a reference to those imitations and copycats produced in China (Siu et al. (2010) and Tse et al. (2010)).[2]

In general, most copycat products tend to exhibit the following five characteristics:

1. **High Resemblance**. By definition, copycat products usually exhibit **high resemblance** of genuine "branded" products in terms of brand names or external designs.

2. **Low Selling Price**. Copycat products are usually sold at **low price** partly because they enjoy extremely low production cost, i.e., they incur no R&D costs, promotion and marketing costs, licensing fees, etc.[3]

[1]For example, fashion design has no copyright protection laws, luxury brands such as Balmain and Givenchy cannot file legal claims against Zara and NastyGal for copying their designs of boots and handbags, respectively (Lieber (2014)). This legal loophole has created incentives for copycats to enter the market.

[2]In traditional Chinese history, Shanzhai means "mountain stronghold" in reference to historical warlord holdouts which were outside of government control. This term is now being used to refer to products outside of government regulations that are widely reflected in the numerous copycats mobile phones, apparels, watches, computers, and even cars.

[3]For example, Shanzhai cell phone manufacturers ignored the requirement that they purchase a cell phone-

3. **Non-deceptive**. Many copycat products are **non-deceptive to the buyers** in the sense that the buyers are fully aware that the products are not genuine either from the price they paid, or the channel from which the buyers buy the product. (Cho et al. (2015)). [4] However, even though copycat products are non-deceptive to the buyers, their physical resemblance level of the incumbent products can deceive other consumers in the market that the copycat products are genuine. Therefore, when the resemblance level of the copycat product becomes higher, it can deceive more people to perceive that the copycat is authentic. Consequently, the "social utility" derived from a copycat product depends on the proportion of the market perceives the product to be genuine, which depends on the physical resemblance level.

4. **Low Quality**. Relative to the luxury brand that they are mimicking, copycat products are generally of **low quality**. For instance, the curator of the Fashion Institute of Technology Ariele Elia commented that "With (genuine) designer items, there is quality." (Lieber (2014)).

5. **Rapid Product Launch**. Partly due to its efficient supply chain (Siu et al. (2010)), most copycat products are launched shortly after the launch of genuine brands.[5]

In emerging markets, copycats provide access of imitation products to those who either cannot afford or are unwilling to pay the high selling price of these genuine luxury products. At the same time, these copycats can generate profits from piggybacking on the product development of the incumbent firms. To a certain extent, copycats are encroaching the market of the incumbent firm and it raises major concerns from the incumbent's perspective. These observations have motivated us to examine the following research questions:

1. In the presence of a potential copycat, under what condition is it possible for the incumbent

manufacturing license (a requirement which the Chinese government abandoned in 2007 largely because it had become unenforceable) (Sun et al. (2015)). As reported in the New York Times, a typical Shanzhai phone selling for $150 usually costs only $40 to produce in China (Barboza (2000)). For example, a copycat iPhone is sold at RMB600, while the genuine iPhone is selling for RMB5888 by China Unicom.

[4]There are copycat products that are deceptive and consumers are not being notified that the products are not genuine. In China, deceptive products such as fake drugs, milk powder and other food products have created major concerns about food safety in China due to product adulteration (Tang and Babich (2014)). To avoid the the legal issues associated with the loss of human lives due to the use of deceptive imitation foods, drugs, etc., we shall focus on non-deceptive imitation durable goods in this paper.

[5]In some cases, the copycat products can be available even before the genuine branded goods. For example, Tom Ford lamented that "I hate being copied by Zara.... (My items) will be (copied and sold) at Zara's stores before I can get them in the store, and I don't like that." (London (2013)).

to deter its entry?

2. In the presence of a potential copycat, what is the incumbent's pricing strategy?

3. Which type of copycat, in terms of physical resemblance and product quality, can gain successful entry?

4. Will the presence of copycats always improve consumer surplus? What is the impact of the presence of potential copycats on social welfare?

The first two questions are intended to examine the strategic dynamics between the incumbent and the copycat, and to identify conditions under which the incumbent can deter the entrance of copycats. The third question seeks to provide an explanation about why we tend to observe copycat products of high resemblance and low quality in practice as discussed earlier. The fourth question seeks to examine the conventional wisdom that copycats create additional consumer surplus and social welfare as articulated by Prada CEO in 2012.

To examine the above research questions, we present a two-period dynamic game model to capture the strategic interactions between an incumbent (I) and a copycat (C) over two time periods. Our model incorporates two salient features: (a) *two* types of copycat characteristics – *resemblance* and *quality* relative to the incumbent product; and (b) *two* types of consumer utilities associated with a product – *consumption utility* that depends on the product quality, and *status utility* that depends on the copycat's resemblance level with the incumbent product and the consumer's purchasing decision of the incumbent and the copycat products.

Our equilibrium analysis enables us to establish the following answers to the above research questions as follows:

1. *Conditions under which the incumbent can deter copycat's entry.* When the production cost of the incumbent is sufficiently close to that of the copycat's, the incumbent should sell its product at a lower price to capture the entire market (so as to deter the copycat's entry). In other words, the incumbent can afford to "flood the market" so as to deter the copycat's entry. This result is consistent with the way genuine musical CDs deter the entrance of copycats in China in the 90s.

2. *Implications of the potential entry of copycat on the incumbent's selling price.* Regardless of the actual entry of the copycat, the potential threat associated with the copycat's entry is sufficient to force the incumbent to lower its selling price.

3. *Characteristics of copycats that can enter the market successfully without being deterred by the incumbent.* Our equilibrium analysis reveals that, when it is profitable for a copycat to enter the market, its product tends to exhibit high resemblance and low quality (relative to the incumbent product).

4. *Implications of the potential entry of copycat on consumer surplus and social welfare.* In contrast to conventional wisdom, we find that the entry of a copycat does not always improve consumer surplus or social welfare.

The contribution of this paper is three-fold. First, contrary to extant literature on copycats, our paper focuses exclusively on non-deceptive copycats whereby consumers know for certain, at the point of purchase, that the copycat product is an imitation. Second, we present a model that captures two salient features associated with copycat products: resemblance and quality of the copycat product, as well as consumption and status utilities for the consumers. These features have not been examined in the literature and thus offer a novel and substantive dimension to the paper. Third, besides establishing pricing as a deterrence strategy that the incumbent should adopt in equilibrium, our results explain why most copycat products tend to be high on physical resemblance and yet low on quality. At the same time, we show that the presence of copycats is not always beneficial in terms of consumer surplus and social welfare, despite conventional wisdom.

2 Related Literature

The extant literature on counterfeits (or copycats) has focused on the demand of copycats in which price, attitude towards branded companies, and the need for status signaling have been cited as the main factors of driving copycat demand. Examples include Han et al. (2010), Wilcox et al. (2009), Bloch et al. (2003), Kwong et al. (2003), Tom et al. (1998), Cordell et al. (1996), Wee et al. (1995). Using a Cournot competition model, Grossman and Shapiro (1988) studied the case when the quality of the product is not observable. Much as the paper modeled products along both dimensions of status and quality, status utility is modeled to be dependent on the brand itself and *independent* of the number of buyers of the product. They concluded that policies that deter copycats may not improve social welfare. This finding is consistent with Cho et al. (2015). Qian (2014) examined the economic impact of copycats and the impact on the brand management strategy of firms. Qian et al. (2014) further examined how incumbents may seek to differentiate their products in response to entries by (deceptive) copycat entities. Qian et al. (2015), on the other hand, used a signaling model to examine how incumbents may seek to differentiate the products along the dimensions of searchable and experiential qualities in response to entries by copycat entities. Unlike the existing literature, we consider the case when the status utility of the genuine product and that of the copycat *depend* on the number of buyers of each product.

Unlike this stream of research, we incorporate the social motivations for the consumption of copycats of luxury brands into our framework to examine the strategic interactions between the incumbent and the copycat. To the best of our knowledge, ours is the first paper that examines the impact of non-deceptive copycats by explicitly incorporating both physical resemblance *and* quality of the copycat product as well as the status utility of the consumers into the framework. To this end, we provide a theoretical explanation as to why copycats observed in practice are often high on physical resemblance but low on product quality. Furthermore, the issue of whether copycats bring value to the consumers and society at large depends on the quality of the copycat. Our analysis extends the findings in Grossman and Shapiro (1988) and Cho et al. (2015) that copycats can bring value to the society. Specifically, our result reveals that quality is a key dimension that will determine if it is indeed the case.

Finally, our paper is also related to the literature on limit pricing. Bain (1949) formally established the notion of an incumbent's decision to cut its prices to decrease the potential of entry. Friedman (1979) on the other hand, argued that a credible threat to cut prices upon entry can be sufficient to deter entry and is a more effective and profitable way for the incumbent. Spence (1977, 1979) further expanded on the use of strategic commitments to deter entry. Hall (2009) offered a review of this literature. In this paper, limit pricing exists as an equilibrium under some conditions. We also elaborate on the commitment by the incumbent on its price going forward.

The rest of the paper is organized as follows. In §3 and §4, we present the model and the findings. In §5, we discuss the implications on consumer surplus and social welfare. In §6, we discuss some extensions and conclude in §7. All proofs are provided in the appendix while the complete backward induction analysis is given in the supplementary appendix.

3 Model

We describe the sequence of events before we formulate the market and consumer characteristics.

3.1 Sequence of Events

We consider a two-period game with observable actions between the incumbent I and the copycat firm C. At the beginning of Period 1, I launches a new product of intrinsic quality q_I (normalized to 1) and determines its selling price p_I. Upon observing I's product price p_I, C first decides whether to *enter* or to *stay out* of the market at the beginning of Period 2. If C enters, it enters with a copycat product and sets it selling price p_C.[6] The unit production cost for I is k_I, where $k_I \in (0, 1)$. We assume that the copycat's unit production cost $k_C < k_I$. The reason for C to enjoy a lower marginal cost of production is because C neither invests in promotion and regular advertisement nor fulfils any regulatory or licensing requirement that will increase the marginal cost of production. Upon observing p_I, p_C, α and q, each consumer makes her purchase decision that maximizes her total (consumption and status) utility. The sequence of events is depicted in Figure 1.

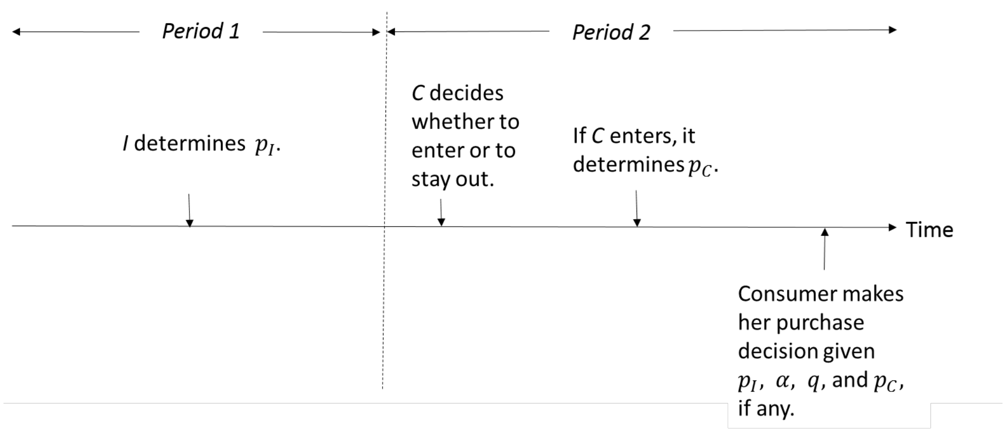


Figure 1: Sequence of events

In our model, the copycat product is different from the incumbent product along two attributes: (a) the physical resemblance α, where $\alpha \in (0, 1)$; and (b) the product quality q, where $q \in (0, 1)$ (i.e., the copycat product is of lower quality than that of the incumbent). As an initial attempt to examine the implications of resemblance α and quality q of copycat products in a game theoretic game, we shall assume both resemblance and quality of the copycat product are given exogenously. This assumption enables us to explore which type of copycat tend to gain entry successfully in equilibrium.

[6] The framework in which the incumbent sets the price before the copycat is also adopted in Qian (2014) and Qian, Gong and Chen (2015).

First, we use physical resemblance α to refer to the extent in which the products are physically alike and the "likelihood" that the copycat product will be identified by the market as the incumbent product. For instance, by copying prominent insignias used in luxury leather goods, a copycat product can increase its physical resemblance α. In general, it is relatively less costly for the copycat to increase its physical resemblance than to increase product quality.

We assume that the quality of the copycat product q is always strictly less than that of the incumbent's (i.e., $q < 1$). This assumption is reasonable and necessary for the following reason. Suppose that the copycat product is of exactly the same quality as that of the incumbent's. Then the copycat can easily have a product that is an exact physical replica of the incumbent's product. In this case, owing to a lower marginal production cost, the copycat product is actually more competitive than that of the incumbent product and the copycat can foreclose the incumbent's market, in which case, the copycat does not have anything in the market to 'copy'. Imagine a leather bag with an insignia that does not have a well-known brand attached to it. What value is then of the copycat brand? Thus, for the framework to be relevant to the research question in this paper, we assume that the quality of the copycat product is always lower than that of the incumbent. Nonetheless, it can be sufficiently close (q close to 1).

3.2 Market Conditions

Our base model is based on the following market conditions. First, we assume that the time interval between the two periods is so short that consumers are confronted with both products of I and C at almost the same time if C chooses to enter the market. This assumption implies that there is virtually no sales of the incumbent product in Period 1. As described earlier, this assumption is observed in practice due to the speed at which C can enter the market. As stated earlier, Tom Ford lamented that "(My items) will be (copied and sold) at Zara's stores before I can get them in the store." (London (2013)). (To examine the robustness of our results obtained in the base model, we shall relax this assumption in Section 6.1 so that the time interval between the two periods is long enough so that the sales of the incumbent can take place in Period 1 before the entry of the copycat in Period 2. By using the same approach to analyze this extension, we find that the key results continue to hold.)

We incorporate the notion of irreversibility of the incumbent's price p_I in our model. As highlighted in Spence (1977), irreversibility is a way for the firm to commit itself in advance and a way to issue a credible threat to potential entry. Specifically, to avoid diluting the brand image, most luxury brands do not lower selling price or launch new products to compete with a copycat. For this reason and for tractability, we shall assume that the incumbent will not change its selling price and will not launch *another* product to compete with the copycat product upon its entry (Figure 1).

3.3 Two Types of Consumer Utility

In our model, there are N (normalized to 1) infinitesimal consumers in the market: each consumer i has wealth v_i, where $v_i \sim U[0, 1]$. Instead of considering the case when social status is an increasing function of wealth, we assume that Consumer i's wealth v_i corresponds *perfectly* to her social status for ease of exposition. For each consumer, we first define the *intrinsic consumption utility* as well as the *status utility* for buyers and non buyers derived from a "generic" product with a functional quality q. (In Section 3.5., we shall describe how the status utility depends on the resemblance factor α and the number of buyers who purchased Product I versus C.)

Consumption Utility for Buyers and Non-buyers. We model a consumer willingness to pay (WTP) for a product of quality q as directly proportional to her wealth level v_i so that the lifetime utility from consumption is $v_i q$. In other words, if a consumer with wealth v_i purchases a product of quality q at price p, she obtains a net *consumption utility* of $(v_i q - p)$. For non-buyers, the consumption utility is equal to 0.

Status Utility for Buyers. Consider the case when there are a continuum $[\underline{v}, \overline{v}]$ of buyers and a continuum $[0, \underline{v}]$ of non-buyers. By adapting the status utility model developed by Rao and Schaefer (2013), the entire group of buyers with $v_i \in [\underline{v}, \overline{v}]$ will share the same status utility, which can be written as:[7]

$$\lambda \frac{\int_{\underline{v}}^{\overline{v}} v_i dv_i}{\int_{\underline{v}}^{\overline{v}} dv_i} = \lambda \frac{\overline{v} + \underline{v}}{2}, \tag{1}$$

where λ ($\lambda \in (0, 1)$) represents the consumer's sensitivity towards status utility. Essentially, the status utility is equal to the sensitivity λ times the expected wealth level of the buyers in the group.

Status Utility for Non-Buyers. To incorporate the notion of "social comparison", the status utility of the non-buyers should be lower than that of the buyers and it should depend on the number of buyers and non-buyers. Therefore, if we set the status utility for non-buyers to 0, the

[7]Furthermore, if a consumer $v^j < \underline{v}$ ($v_j > \overline{v}$) purchases the product, then the status utility of the existing group of buyers in $[\underline{v}, \overline{v}]$ will decrease (increase) because an additional consumer with lower (higher) wealth level is "joining" the existing group of buyers.

status utility for non-buyers will be independent of the wealth level of the non-buyers with wealth $[0, \underline{v}]$ as well as the number of buyers and non-buyers. For this reason, we set the status utility for the group of non-buyers with $v_i \in [0, \underline{v}]$ as

$$\lambda \frac{\int_0^{\underline{v}} v_i dv_i}{\int_0^{\underline{v}} dv_i} = \lambda \frac{\underline{v}}{2}. \tag{2}$$

Observe from (2) that the status utility for non-buyers is increasing in $\underline{v}$. Hence, the status utility for non-buyers can increase (decrease) when there are fewer (more) buyers for the product.

3.4 Consumer's Threshold Purchasing Policy

Before we analyze the pricing strategies of I and C (with potential entry), we now examine the consumer's rational purchasing behavior in equilibrium. As we shall show later, regardless of the entry of C, all rational consumers will follow a "threshold purchasing policy" in equilibrium. Instead of proving similar threshold policies for different settings (I operates as a monopoly, I and C operate as a duopoly with both incumbent and copycat products, etc.), we now present a *unified model* to analyze the consumer's purchasing policy in equilibrium so as to avoid repetition. Without loss of generality, we shall consider the most general case in which both products from I and C are available in the market. As we shall show later, all consumers will follow a threshold purchasing policy $[\tau_C, \tau_I]$ that can be described as follows: (a) consumers with wealth $v_i \in [0, \tau_C]$ will buy nothing, (b) consumers with wealth $v_i \in [\tau_C, \tau_I]$ will buy C, and (c) consumers with wealth $v_i \in [\tau_I, 1]$ will buy I. Notice that the thresholds τ_C and τ_I depend on p_I, p_C, α, q and λ. Also, it can be shown that this form of threshold purchasing policy is an equilibrium policy, i.e., no consumer can improve her utility by deviating from this policy unilaterally.

3.5 Consumption Utility and Status Utility associated with a Threshold Purchasing Policy

For any threshold purchasing policy $[\tau_C, \tau_I]$, we now determine the consumption utility and status utility of three different groups of consumers: (1) buyers of I with wealth $v_i \in [\tau_I, 1]$; (2) buyers of C with wealth $v_i \in [\tau_C, \tau_I]$; and (3) non-buyers with wealth $v_i \in [0, \tau_C]$.

Consumption Utility. By using the consumption utility as defined in Section 3.2 along with the fact that the quality of I is equal to 1 and the quality of C is equal to q, it is easy to check that the consumption utility for each buyer of I is equal to $(v_i \cdot 1 - p_I)$, for each buyer of C is equal to $(v_i \cdot q - p_C)$, and for each non-buyer is equal to 0.

Status Utility. We can use the same approach as presented in Section 3.2 to determine the status utility of each of the three groups of consumers. However, in the presence of copycat C, the status utility of each group depends on whether the market can identify the copycat product C as "fake" or not. For this reason, we consider two separate cases by incorporating the notion of social comparison as explained in Section 3.3 :

Case 1: The market cannot identify C as fake with probability α. When the market cannot distinguish between the incumbent product I and the copycat product C, the market will recognize the buyers of C and the buyers of I as having the same social status. Hence, both groups of buyers will be treated as a "combined" group of buyers with wealth level $v_i \in [\tau_C, 1]$. By applying (1), the buyers of I and the buyers of C will enjoy the same status utility, which is equal to $\lambda\frac{1+\tau_C}{2}$. By applying (2), the non-buyers will obtain a status utility of $\lambda\frac{\tau_C}{2}$.

Case 2: The market can identify C as fake with probability $(1-\alpha)$. When the market can clearly distinguish between the incumbent product I and the copycat product C, the market will recognize the buyers of I are consumers with wealth level $v_i \in [\tau_I, 1]$. By applying (1), the buyers of I will obtain a status utility $\lambda\frac{1+\tau_I}{2}$. At the same time, the market recognizes that the buyers of C are different from the buyers of I and they have a lower social status than the buyers of I. In the base model, we assume that the market will treat the buyers of C as non-buyers (i.e., as if they did not buy anything). Consequently, the buyers of C and those non-buyers will be treated as a "combined" group of "non-buyers" with wealth level $v_i \in [0, \tau_I]$. By applying (2), the buyers of C and the non-buyers will enjoy the same status utility, which is equal to $\lambda\frac{\tau_I}{2}$, which is lower than that of the buyers of I. (In Section 6.2., we shall examine an alternative scenario under which the buyers of C and non-buyers obtain different status utility.)

By considering the consumption utility and the status utility of these three groups of consumers along with the probability associated with Case 1 (i.e., α) and Case 2 (i.e., $(1-\alpha)$), we can compute the total expected utility $U(v_i)$ for each consumer with wealth v_i as follows:

$$U(v_i) = \begin{cases} (v_i \cdot 1 - p_I) + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}) & \text{if } v_i \in [\tau_I, 1], \\ (v_i \cdot q - p_C) + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}) & \text{if } v_i \in [\tau_C, \tau_I], \\ 0 + \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}) & \text{if } v_i \in [0, \tau_C]. \end{cases} \tag{3}$$

In summary, we have identified a threshold purchasing policy (τ_C, τ_I) that consumers will adopt in equilibrium so that the demand for product I is equal to $(1-\tau_I)$ and the demand for product C is equal to $(\tau_I - \tau_C)$. Also, we have determined the total expected utility of each consumer as given in (3). In the next section, we shall use the demand functions of I and C along with the total expected utility of each consumer given in (3) to determine the pricing strategy for I and the entrance and pricing strategy for C in equilibrium so that we can answer those four research questions.

4 Analysis

We use backward induction to analyze the sequential game between I and C as depicted in Figure 1. First, we determine the threshold purchasing policy in Period 2 for any *given* p_I, p_C. Then we derive the optimal p_C as a function of p_I and the entry decision of C. Finally, we determine the optimal p_I for I. Once we determine the optimal p_I, we can retrieve the equilibrium outcomes through substitutions as well as the entry strategy of C and the deterrence strategy of I, if any.

Let us recall the threshold purchasing policy under which a consumer with wealth v_i will buy I if $v_i \in [\tau_I, 1]$, buy C if $v_i \in [\tau_C, \tau_I]$, and buy nothing if $v_i \in [0, \tau_C]$. Hence, by considering the total expected utility as given in (3), we know that consumer with $v_i = \tau_I$ is indifferent between buying I or C, and consumer with $v_i = \tau_C$ is indifferent between buying C or buying nothing. By using these observations and (3), thresholds τ_I and τ_C satisfy the following equations simultaneously:

$$\begin{aligned}
\tau_I \cdot 1 - p_I + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}) &= \tau_I \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}), \\
\tau_C \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}) &= \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}).
\end{aligned}$$

By solving these two equations, we get:

$$\tau_I = \frac{p_I - p_C - (1-\alpha)\frac{\lambda}{2}}{1-q}, \text{ and } \tau_C = \frac{p_C - \alpha\frac{\lambda}{2}}{q}. \tag{4}$$

4.1 Pricing Strategies of C and I

For any given threshold policy (τ_C, τ_I), the demand for product I is equal to $(1 - \tau_I)$ and the demand for product C is equal to $(\tau_I - max(0, \tau_C))$. (Note that $\tau_C < 0$ when p_C is too low.) Also, it is easy to check from (4) that Product I's demand is decreasing in p_I, while Product C's demand is decreasing in p_C (due to smaller τ_I and bigger p_C). We now use these demand characteristics to determine the pricing strategies of C and I in equilibrium.

Given p_I, we first determine the copycat C's best response $p_C^*(p_I)$. By noting that the demand for Product C is equal to $(\tau_I - max(0, \tau_C))$, copycat C will determine its best response $p_C^*(p_I)$ by solving the following problem:

$$\begin{aligned} \max_{p_C} \quad & \pi_C(p_I, p_C) = (p_C - k_C)(\tau_I - max(0, \tau_C)) \\ s.t. \quad & \tau_I \in [max(0, \tau_C), 1], \end{aligned}$$

where τ_I, τ_C are given in (4).

By considering the cases where τ_C is positive and negative and various boundary conditions, we summarize the best response of C with respect to p_I as follows:

1. When $k_C \leq \frac{\lambda\alpha}{2}$,

$$p_C^*(p_I) = \begin{cases} p_I - \frac{\lambda}{2}(1-\alpha) & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)], \\ \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2} & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C], \\ \frac{\lambda\alpha}{2} & \text{if } p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)], \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q-\alpha)}{2} & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)]. \end{cases} \tag{5}$$

2. When $k_C > \frac{\lambda\alpha}{2}$,

$$p_C^*(p_I) = \begin{cases} k_C & \text{if } p_I < \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q)) \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q-\alpha)}{2}, & \text{if } p_I \geq \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q)) \end{cases} \tag{6}$$

Anticipating Copycat C's best response $p_C^*(p_I)$ as given in (5) and (6) for different scenarios and applying (4) along with the boundary constraints, Incumbent I can determine its optimal pricing strategy p_I^* by solving the following problem:

$$\begin{aligned} \max_{p_I} \quad & \pi_I(p_I) = (p_I - k_I)(1 - \tau_I) \\ s.t. \quad & \tau_I = \frac{p_I - p_C^*(p_I) - (1-\alpha)\frac{\lambda}{2}}{1-q} \\ & \tau_I \in [0, 1]. \end{aligned}$$

By considering $p_C^*(p_I)$ as given in (5) and (6) for different ranges of values of p_I, we obtain the optimal p_I^* for each of these ranges by solving the above problem. Then, we can retrieve the equilibrium outcomes of all possible ranges. Due to the boundary constraints associated with the incumbent's problem as stated above and due to 6 different scenarios that affect C's best response $p_C^*(p_I)$ as given in (5) and (6), the detailed expressions for the optimal p_I^* for I and the corresponding best response $p_C^*(p_I^*)$ for C and the equilibrium profits for I and C are tedious. Given the focus of our paper is to examine those 4 aforementioned research questions, we shall omit the detailed expressions of p_I^* and $p_C^*(p_I^*)$ for different scenarios in the main text. (The detailed expressions are embedded in the proof as shown in Supplementary Appendix A.) Instead, we shall compare the resulting profit of both firms associated with different scenarios to map out the entry and deterrence strategies of C and I. This is the focus of the next subsection.

4.2 Entry and Deterrence Strategies of C and I

By considering the boundary conditions for p_I^* along with those 6 different scenarios that affect C's best response $p_C^*(p_I^*)$ as given in (5) and (6), we can compare the resulting profit of both firms to map out the deterrence and entry strategies that I and C will adopt in equilibrium in terms of k_I and k_C (Figure 2). Observe from Figure 2 that the lower right-hand corner corresponds to the case when $k_I \leq k_C$, which is inadmissible (because we assume that the production cost of I is higher than that of C (i.e., $k_I > k_C$). Also, the upper left-hand corner of Figure 2 corresponds to the case when the incumbent I does not enter the market because k_I is too high. Without an incumbent firm, there is no product to "copy" and so the problem becomes trite. Therefore, it suffices to consider those shaded cases as depicted in Figure 2.

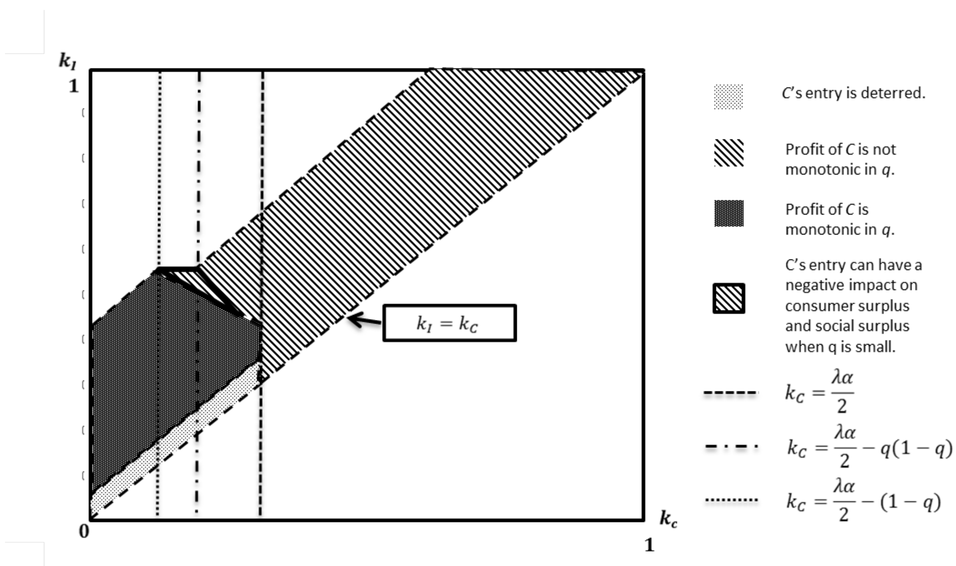


Figure 2: Equilibrium strategies

To begin, let us examine the entry strategy of the copycat as well as the deterrence strategy of the incumbent in equilibrium. Our main question is: Under what conditions can limit pricing enable I to deter the entrance of C? Proposition 1 provides an answer.

Proposition 1. (Entry Deterrence.) *Copycat C will not enter the market in equilibrium if and only if $k_I - k_C \leq \frac{\lambda(1-\alpha)}{2} - 2(1-q)$ and $k_C \leq \frac{\lambda\alpha}{2}$.*

Proposition 1 has the following implications. First, when $k_C \leq \frac{\lambda\alpha}{2}$, the incumbent I is able to deter the entry of C when the cost differential between I and C (i.e., $(k_I - k_C)$) satisfies: $(k_I - k_C) \leq \frac{\lambda(1-\alpha)}{2} - 2(1-q)$. Second, besides cost differential, the condition for deterrence is likely to hold when the consumer's sensitivity towards status utility (λ) is high, when the copycat's quality (q) is high, or when the copycat's physical resemblance (α) is low. While Incumbent I cannot control copycat's quality q, Incumbent I can deter C's entry by increasing λ. To do so, I can enhance the status image through advertising as well as celebrity endorsements. Also, I can deter C's entry by designing a product that is difficult or costly for the copycat to replicate so that α is kept low.

On the flip side, Proposition 1 reveals the condition when the incumbent I cannot deter the entrance of C, say, for instance, $k_I - k_C > \frac{\lambda(1-\alpha)}{2} - 2(1-q)$. Upon close examination of this condition, we can conclude that the copycat can enter the market successfully with a product that is high in physical resemblance α, and low in quality q. This result is consistent with the characteristics of most copycat products that are commonly observed in practice. Furthermore, in the event when the two product attributes (quality q and physical resemblance α) are correlated, it is easy to check from those two conditions as stated in Proposition 1 that only copycat with low quality q and high resemblance α can enter the market successfully.

While it is desirable for C to enter the market by offering products with high resemblance so that buyers can enjoy a higher expected status utility, it is less obvious why C would prefer to enter the market with low quality products. The rationale for low quality copycat can be explained as follows. First, suppose C attempts to enter the market with a product that has a high quality (q is close to 1) as well as high resemblance (α is close to 1). Then all consumers, regardless of their wealth level, will either prefer the product of I or that of C. This preference is determined solely by the prices p_I, p_C. Hence, this entry strategy of copycat C will trigger a price war between I and C, and the chances for obtaining market share for C is limited. Clearly, it is not optimal for C, as a copycat firm that leverages on the status utility that the incumbent provides, to 'kick' the incumbent out of the market. If that happens, the value of the product of C immediately diminishes and C will end up with no demand as well - imagine a Prada look-a-like bag when Prada is no longer in the market.[8] Second, suppose C attempts to enter the market with a product with high

[8]In our model, we do not consider the possibility that C builds up its own brand while being a copycat.

resemblance level (α is close to 1) and low quality (q is close to 0). Then, it becomes clear that the deterrence condition stated in Proposition 1 does not hold. In this case, to gain some market share, C has to set p_C sufficiently low so that the incumbent I cannot afford to undercut C's price and stay profitable. Consequently, the incumbent I cannot afford to deter C's entry, and the market is segmented between I and C.

Next, let us consider the case when C can gain access to the market (i.e., without being deterred by I). Given successful market entry, would C benefit from offering a product with higher quality? The following proposition provides an answer.

Proposition 2. (Copycat's Profit is Non-Monotonic in Quality.) *When* $k_C \in [\frac{\lambda\alpha}{2} - (1-q), k_I]$, $k_I \in [\min(-2k_C + \frac{\lambda}{2}(1+2\alpha) - (1-q), \frac{\lambda\alpha}{2}), \max(\frac{\lambda}{2} + (1-q), \frac{1}{2-q}(k_C - \frac{\lambda\alpha}{2}) + \frac{\lambda\alpha}{2} + \frac{2(1-q)}{2-q})]$, *copycat C will enter the market successfully in equilibrium. However, the profit of C is non-monotonic in its quality level q.*

Proposition 2 further affirms that higher product quality does not always generate higher profit for the copycat, even when the conditions for successful entry have been satisfied and the cost of producing a higher-quality product is ignored. This is because a higher-quality copycat product inevitably triggers a more intense pricing competition between I and C. However, under the conditions above when both k_C, k_I are relatively large, intense pricing competition can lead to the copycat dropping out of the market. As such, the copycat is better off by entering the market with a product of a lower quality.

5 Consumer Surplus and Social Welfare

It is clear that successful entry of C will enable consumers with lower wealth level $v_i \in [\tau_C, \tau_I]$ to gain access to the copycat product at a lower price (instead of the incumbent product that they may not be able to afford (or willing) to buy). In addition, buying the copycat product can enable the buyers to obtain consumption utility and perhaps even status utility (especially when the market cannot identify the copycat product as fake). Therefore, conventional wisdom has it that the presence of C would improve both consumer surplus and social welfare (in terms of consumer surplus and the total profit of both I and C). Is this conventional wisdom always true?

While this belief is correct in many instances, we find that that are instances under which the presence of copycat C will actually reduce consumer surplus and/or social welfare. To construct a specific counter example to show that the presence of copycat C can reduce consumer surplus and/or social welfare, we shall compare the consumer surplus and social welfare between 2 cases: (a) the benchmark case when I operates as a monopoly without any potential entry threat from copycat C; and (b) when I and C co-exist and capture the entire market together when

$$\begin{aligned} k_C &\in [\frac{\lambda\alpha}{2} - (1-q), \frac{\lambda\alpha}{2}], \\ k_I &\in [min(\frac{\lambda\alpha}{2} + (1-q), \frac{\lambda}{2} - \frac{2(1-q)}{2-q} + \frac{4-q}{q(2-q)}(-k_C + \frac{\lambda\alpha}{2}))]. \end{aligned} \tag{7}$$

In preparation, let us compute the consumer surplus and social welfare associated with cases (a) and (b) in the following subsections.

5.1 Incumbent I Operates as a Monopoly

Consider the benchmark (Case (a)) when I is the only firm in the market without any potential entry threat of the copycat. When I operates as a monopoly, the corresponding game becomes a single-person decision problem. Specifically, upon observing p_I, the corresponding threshold purchasing policy is: buy I if $v_i \in [\tau_I^B, 1]$; and buy nothing, otherwise. Hence, by applying (3) to this special case and by using the approach as presented in Sections 4 and 4.1, we can show that $p_I^B = \frac{1+k_I}{2} + \frac{\lambda}{4}$, $\tau_I^B = \frac{1+k_I}{2} - \frac{\lambda}{4}$, and $\pi_I^B = (\frac{1-k_I}{2} + \frac{\lambda}{4})^2$, where the superscript/subscript 'B' is used to denote the benchmark case.

Before we compute the consumer surplus and social surplus for the case when I operates as a monopoly, we notice that the optimal monopoly price p_I^B is higher than the optimal price p_I^* associated with the scenario when C is present but deterred from entry. Specifically, we have:

Corollary 1. *The potential threat associated with the copycat's entry can pressure Incumbent I to lower its selling price: $p_I^B > p_I^*$.*

Corollary 1 reveals that, in the presence of a copycat, its potential entry is sufficient to force the incumbent to lower it selling price in equilibrium. This result suggests the presence of copycat can increase consumer surplus due to a lower selling price of the incumbent product.

By using p_I^B and τ_I^B and π_I^B as stated above, we can compute the consumer surplus as follows:[9]

[9] Because the copycat C does not exist in the Benchmark case, the copycat's resemblance level α and quality q are irrelevant to the consumer surplus CS_B and social welfare SS_B.

$$\begin{aligned} CS_B &= \int_{\tau_I^B}^1 [v_i - p_I^B + \lambda \frac{1+\tau_I^B}{2}] dv_i + \int_0^{\tau_I^B} \lambda \frac{\tau_I^B}{2} dv_i \\ &= \frac{(\tau_I^B)^2}{2} + (\frac{\lambda}{2} - 1)\tau_I^B + \frac{1}{2} \\ &= \frac{1}{8}((1-k_I)^2 - \frac{\lambda^2}{4} + 2\lambda). \end{aligned}$$

Also, the social surplus is simply the sum of the consumer surplus and the profit of I so that:

$$\begin{aligned} SS_B &= CS_B + \pi_I^B \\ &= \frac{(\tau_I^B)^2}{2} + (\frac{\lambda}{2} - 1)\tau_I^B + \frac{1}{2} + (\frac{1-k_I}{2} + \frac{\lambda}{4})^2 \\ &= \frac{1}{8}((1-k_I)^2 - \frac{\lambda^2}{4} + 2\lambda) + (\frac{1-k_I}{2} + \frac{\lambda}{4})^2. \end{aligned}$$

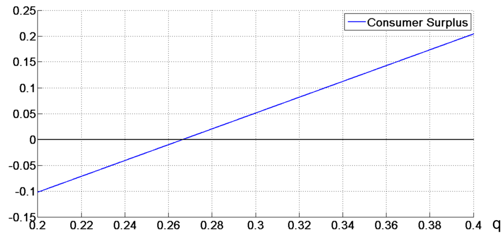
5.2 Both Incumbent I and Copycat I Co-exist in the Market

Consider Case (b) when both I and C co-exist in the market under the conditions stated in (7). The calculation of the consumer surplus for a consumer depends on the aforementioned threshold purchasing policy and the entry and pricing strategies of both I and C, which in turn depends on those 6 scenarios as depicted in (5) and (6). For ease of exposition, we shall state the consumer surplus and the social surplus below and refer the reader to Appendix B for details.

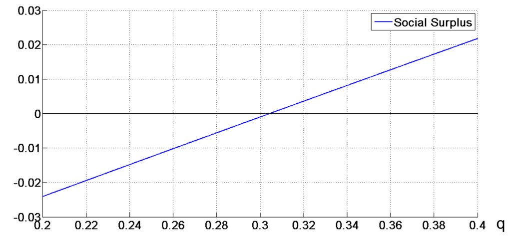
$$\begin{aligned} CS &= \frac{1-q}{2}(\tau_I)^2 + (\tau_I)(\frac{\lambda}{2}(1-\alpha) - (1-q)) + \frac{1}{2}, \\ SS &= CS + \pi_I + \pi_C \\ &= \frac{1-q}{2}(\tau_I)^2 + (\tau_I)(\frac{\lambda}{2}(1-\alpha) - (1-q)) + \frac{1}{2} + \frac{1}{1-q}(\frac{-k_I + \frac{\lambda}{2} + (1-q)}{2})^2 \\ &+ \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I - \frac{\lambda}{2} + (1-q)}{2}), \end{aligned}$$

and $\tau_I = \frac{1}{2(1-q)}(1 - q + k_I - \frac{\lambda}{2})$ and $\pi_I = \frac{1}{4(1-q)}(-k_I + \frac{\lambda}{2} + (1-q))^2$ and $\pi_C = \frac{1}{2(1-q)}(\frac{\lambda\alpha}{2} - k_C)(k_I - \frac{\lambda}{2} + (1-q))$ corresponds to the profit of I and C under the specified conditions as stated in Case (b).

Since CS and SS are complex function of q, to illustrate, we examine the difference between CS and CS_B and the difference between SS and SS_B numerically by considering the case when $k_C = 0.3, k_I = 0.4, \lambda = 0.9$, $\alpha = 0.95$, and $q \in [0.2, 0.4]$, which satisfy (7). It is easy to observe from Figures 3a and 3b that $CS < CS_B$ and $SS < SS_B$ when q is sufficiently low. This is to say, the entry of copycat C can lower the consumer surplus and social welfare when q is below a certain threshold.



(a) $CS - CS_B$



(b) $SS - SS_B$

Figure 3: A case when CS and SS are smaller than the benchmark case

This observation that the entry of copycat C can lower the consumer surplus and social welfare when q is below a certain threshold has motivated us to establish the following proposition:

Proposition 3. (Consumer Surplus and Social Surplus.) *Relative to the Benchmark case in which Copycat C is absent from the market, the presence of copycat can reduce consumer surplus and social welfare when copycat's quality q is sufficiently low. In other words, $CS_B > CS$ and $SS_B > SS$ when q is sufficiently small. Specifically, as q converges to 0, $CS_B - CS$ converges to $\frac{\lambda\alpha}{2}(\frac{1+k_I}{2} - \frac{\lambda}{4}) > 0$. Also, $SS_B - SS$ converges to $k_C(\frac{1+k_I}{2} - \frac{\lambda}{4}) > 0$.*

Proposition 3 provides a counter example to illustrate that the presence of copycat can indeed reduce consumer surplus and social welfare when the copycat's quality q is sufficiently low. This result can be explained as follows. Consider the case when copycat C enters the market with quality q closed to 0. By noting from above that τ_I converges to τ_I^B when (7) hold and that the corresponding τ_C is equals to 0, we can conclude that the presence of copycat in this case will entice the same group of consumers with wealth $v_i \in [\tau_I^B, 1]$ to buy the incumbent product as in the Benchmark case. However, the presence of copycat will entice the consumers with wealth $v_i \in [0, \tau_I^B]$ to buy the copycat product so that the entire market is captured by both firms I and C. In this case, Copycat C's entry will increase consumption utility (due to those buyers of the copycat product C). However, this gain is overshadowed by the loss in the status utility of those buyers of the incumbent product I (due to the resemblance level of the copycat product). Consequently, the presence of copycat can reduce consumer surplus and social welfare when copycat's quality q is sufficiently low. In summary, much as copycat products seem to bring value to consumers who would not purchase the incumbent product otherwise, we find that there are instances under which the presence of the copycat product can reduce consumer surplus and social welfare.

6 Extensions

To examine the robustness of our results associated our base model as stated in Propositions 1, 2, and 3 as well as Corollary 1, we now examine two different extensions.

6.1 Extension 1: Sequential Sales of I and C

In the base model as depicted in Figure 1, we have assumed that Period 1 is very short so that there is no sales of the incumbent product in Period 1. Hence, the sales of both I and C take place in Period 2. We now extend our analysis to the case when time interval between the introduction of the incumbent's product and the copycat product is sufficiently large so that the sales of product I can take place in Period 1. More formally, we consider a sequential game that is associated with the following sequence of events.

At the beginning of Period 1, I launches a new product of intrinsic quality 1 and determines its selling price p_I. Upon observing I's price (p_I), consumers with wealth level $v_i \in [0, 1]$ decide whether to buy I or not. Here, we assume that consumers in Period 1 are strategic: they make their purchase decisions by taking into account the potential entry of a copycat product C and it selling price p_C in Period 2. At the beginning of Period 2, C first decides whether to *enter* or to *stay out* of the market. If C enters, it enters with a product with quality q and physical resemblance α as before. At the same time, C decides on prices p_C. As before, we assume that the incumbent will continue its commitment (in the context of irreversibility) by not adjusting it price in Period 2. Due to the fact that sales can take place in Periods 1 and 2, we introduction the discount factor δ ($\delta \in [0, 1]$) to capture the time value of money.

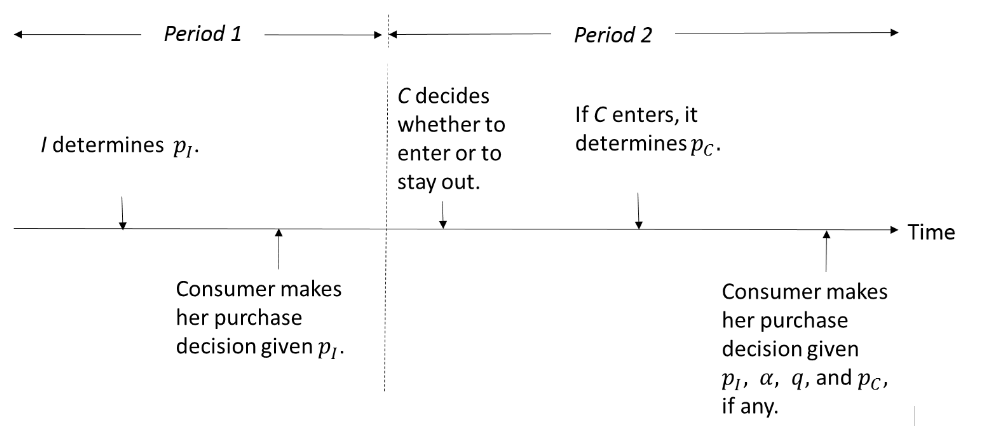


Figure 4: Sequence of events for Extension 1

By using the same approach as presented in Section 4 (the details are provided in Supplementary Appendix B), we can show that the main results obtained in the base model (i.e., Propositions 1, 2, and 3 and Corollary 1) continue to hold as follows:

Proposition 4. (Entry Deterrence.) *Suppose $k_I > k_C$. Copycat C does not enter at the equilibrium if and only if $k_I - \delta k_C \leq \frac{\lambda(1-\alpha)}{2} - 2(1-\delta q)$ and $k_C \leq \frac{\lambda\alpha}{2}$.*

Observe that Proposition 4 is identical to Proposition 1 when the discount factor $\delta = 1$. To explain this result, observe that I can deter C's entry only when the incumbent I can capture the entire market, which can occur only when the thresholds $\tau_I = \tau_C = 0$. This deterrence condition is essentially the same under the base model and the extension except the discount factor δ. Consequently, the structure of Proposition 1 is preserved except that the boundary conditions are adjusted to capture the discount factor δ. By considering the flip side, we can also conclude that C can gain entry by offering a product that is high on resemblance and low on quality.

Proposition 5. (Non-Monotonicity in Quality.) *When $\frac{\lambda\alpha}{2} - (1-\delta q) \leq k_C \leq k_I$, $\min(-2\delta k_C + \frac{\lambda}{2}(1+\delta+2\delta\alpha) - (1-\delta q), \frac{\lambda\alpha}{2}) \leq k_I \leq \max(\frac{\lambda}{2}(1+\delta) + (1-\delta q), \frac{1}{2-\delta q}(2(1-\delta q) - \frac{\delta\lambda\alpha}{2} + \delta k_C) + \frac{\lambda}{2}(1+\delta))$, C always enters at the equilibrium but its profit is non-monotonic in q.*

Next, when the copycat successfully enters the market and competes with the incumbent in Period 2, a high quality copycat product can attract intensive pricing competition from the incumbent (by setting a lower price in Period 1). Hence, the profit of the copycat is non-monotonic in the quality. This explains why Proposition 5 is akin to Proposition 2.

Regardless of whether the sales of Products I and C take place in Period 2 only (as in the base case) or in Periods 1 and 2 respectively, the underlying threat imposed by the potential entry of C in Period 2 can pressure I to lower its price in Period 1 (compared to the case when I operates as a monopoly). This intuition continues to hold in this extension. Hence, Corollary 1 continues to hold in this extension as stated in Corollary 2.

Corollary 2. *In the sequential game as depicted in Figure 4, the potential threat associated with the copycat's entry can also pressure Incumbent I to lower its selling price: $p_I^B > p_I^*$.*

Finally, consider the following conditions that are akin to (7):

$$k_I \in [-2\delta k_C + \frac{\lambda}{2}(1+\delta+2\delta\alpha) - (1-\delta q), \min(1-\delta q + \frac{\lambda}{2}(1+\delta), \frac{\lambda}{2}(1+\delta) + \frac{1}{2-\delta q}(\frac{4-\delta q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1-\delta q)))],$$

$$k_C \in [\frac{\lambda\alpha}{2} - (1-\delta q), \frac{\lambda\alpha}{2}].$$

By using the fact that these conditions ensure both I and C co-exist and capture the entire market, we get:

Proposition 6. (Consumer Surplus and Social Surplus.) *Relative to the Benchmark case in which copycat C is absent from the market, the presence of copycat can reduce consumer surplus and social welfare when copycat's quality q is sufficiently low. In other words, $CS_B > CS$ and $SS_B > SS$ when q is sufficiently small. Specifically, as q converges to 0, $(CS_B - CS)$ converges to $\frac{\delta\alpha\lambda}{2}(\frac{1+k_I}{2} - \frac{\lambda(1+\delta)}{4}) > 0$. Also, $(SS_B - SS)$ converges to $\delta k_C(\frac{1+k_I}{2} - \frac{\lambda(1+\delta)}{2}) > 0$.*

In the same vein, when the negative externality generated by the copycat product is sufficient large, it overshadows any gain from the consumption utility of copycat buyers. This explains why Proposition 6 is akin to Proposition 3.

6.2 Extension 2: An Alternative Formulation of Status Utility

Recall from Section 3.5 that we have formulated the consumption utility and the status utility for three different groups of consumers associated with a threshold purchasing policy (τ_C, τ_I) so that the total expected utility $U(v_i)$ for each consumer with wealth v_i is given in (3). This formulation is based on one key assumption as stated in Case 2 in Section 3.5. Specifically, when the market can identify C as fake with probability $(1-\alpha)$, we have assumed that the market will treat the buyers of C as non-buyers (i.e., as if they did not buy anything) so that the buyers of C and those non-buyers will be treated as a "combined" group of "non-buyers" with wealth level $v_i \in [0, \tau_I]$. Based on this assumption, the buyers of C and the non-buyers have the same status utility $\lambda\frac{\tau_I}{2}$.

One may argue that, when the market can identify C as fake, the buyers of C and non-buyers of both I and C should not share the same status utility.[10] There are certainly many plausible alternatives especially when there is no formal analysis of status utility comparison between buyers of C and non-buyers in the literature. In the absence of a grounded theory, we shall consider one alternative scenario for illustrative purposes. We consider the following scenario. When the market can identify C as fake, the market will recognize the buyers of I are consumers with wealth level $v_i \in [\tau_I, 1]$. By applying (1), the buyers of I will obtain a status utility $\lambda\frac{1+\tau_I}{2}$. Also, relative to the buyers of I, the market will recognize the buyers of C as a different group of consumers who purchased a product with lower social status. Specifically, the market will treat the buyers as consumers with wealth level $v_i \in [\tau_C, \tau_I]$ so that the buyers of C will obtain a status utility

[10]We thank an anonymous reviewer for his/her suggestion.

$\lambda\frac{\tau_C+\tau_I}{2}$. Finally, relative to the buyers of I, the market will treat the non-buyers as consumers with wealth level $v_i \in [0, \tau_I]$ (as non-buyers of I) so that these non-buyers will obtain a status utility $\lambda\frac{\tau_I}{2}$. Unlike the base model, the buyers of C have a higher status utility than the non-buyers in this scenario. This is certainly a plausible scenario where a copycat product grants its owner a higher social utility than not owning any product at all, particularly so in developing societies. As we shall see, the key results obtained in the base model continue to hold in this setting.

By considering the consumption utility stated in Section 3.5 and the status utility associated with the case when the market cannot identify C as fake (i.e., Case 1 in Section 3.5) and the case when the market can identify C as fake as stated above, we can compute the total expected utility $U(v_i)$ as before for each consumer with wealth v_i as follows:

$$U(v_i) = \begin{cases} (v_i \cdot 1 - p_I) + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}) & \text{if } v_i \in [\tau_I, 1], \\ (v_i \cdot q - p_C) + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I+\tau_C}{2}) & \text{if } v_i \in [\tau_C, \tau_I], \\ 0 + \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}) & \text{if } v_i \in [0, \tau_C]. \end{cases} \tag{8}$$

As expected, the total expected utility $U(v_i)$ given in (8) under the alternative scenario resembles (3) as in the base case except the status utility of the buyers of C. By using the same approach as presented in Section 4 (the details are provided in Supplementary Appendix C), we can show that the main results obtained in the base model (i.e., Propositions 1, 2, and 3 and Corollary 1) continue to hold. To avoid repetition, we omit the details here. Essentially, the underlying intuition regarding why our results continue to hold in this scenario is the same as explained in Section 6.1. For instance, I can deter C's entry only when the incumbent I can capture the entire market, which can occur only when the thresholds $\tau_I = \tau_C = 0$. In this case, there are no buyers of C and there are no non-buyers. Hence, considering a different status utility for the buyers of C in our scenario has no impact on the deterrence condition as stated in Proposition 1. Hence, Proposition 1 continues to hold in this scenario. By using the same approach as presented in Section 4, we can show that the structure of all other results as stated in Propositions 2 and 3 and Corollary 1 continue to hold.

7 Conclusion

Copycats of luxury brands are prevalent in the market place. This paper seeks to better understand their entry strategy, the deterrent strategy for the luxury brands, if any, as well as the implications for consumers and the society at large. We have developed a model to capture two salient features: (a) consumption utility and status utility; and (b) resemblance level and product quality of the copycat product (relative to the incumbent luxury brand product). By solving a dynamic game between the incumbent and the copycat, we have identified the conditions under which the incumbent can deter the entry of the copycats.

Our analysis reveals that a copycat can gain entry to the market successfully (without being blocked by the incumbent) by launching a product that exhibits high resemblance and low quality. This result provides an explanation regarding why most copycat products available in the market tend to have high resemblance and yet low quality.

We have shown that the conventional wisdom that the presence of copycat product can always increase consumer surplus and social welfare is not true. Specifically, when the copycat's product quality is sufficiently low, we have identified specific instances under which the presence of the copycat can actually reduce consumer surplus and social welfare. This finding suggests that the quote by Patrizio Bertelli (2012) can be modified as follows: copycats are totally bad *except* when the quality of these copycats are not too low.

There are several limitations to our modeling framework. Firstly, we have assumed in the base model that copycats may enter the market speedily, presenting its products to consumers at the same time as the incumbent. To address this issue, we have showed in Extension 1 that, even when this assumption is relaxed and the incumbent's product is launched way before the copycat, the key results obtained in the base model continue to hold. Secondly, we have also checked for robustness of our formulation of the status utility of consumers. An alternative formulation of the status utility of copycat buyers is considered in Extension 2. We showed that even when copycat buyers can obtain a higher status utility than that of the non-buyers of either products (i.e., I or C), our key results continue to hold, albeit with different boundary conditions.

Next, we have assumed throughout that the incumbent does not develop a lower-quality product to compete directly with the copycat. In a way, our findings actually lend support to this assumption because it was found that successful copycat entrants are likely to be high in physical resemblance to the incumbent's product but low in quality. This segmentation strategy of the copycat is to avoid intensive pricing competition from the incumbent in order to ensure its own successful entry. Thus, the copycat will not choose a quality that is sufficiently high that will enable the incumbent to develop another product to compete directly with it. As a future research, it is of interest to explore the conditions under which the incumbent would launch a lower quality product to compete.

Finally, in our base model and Extension 2, we have assumed that the buyers of C will always obtain a higher status utility than the non-buyers (who bought nothing). However, when the market can identify C as fake, it is possible that the market may view buyers of C to have a lower status utility than the non-buyers. There is existing research suggesting that the status of the buyers of C can be even lower than the non-buyers. This result is due to 'loss of face' when the buyers of C are known to be buying fake products in certain product categories or in certain sub-culture that owners of copycats have a lower social standing (Gentry et al., 2006; Wilcox et al., 2009; and Grubb and Grathwohl, 1967). The analysis of this scenario is complex because there exist instances under which a different threshold purchasing policy will occur: consumers with wealth $v_i \in [\tau_I, 1]$ will buy I; consumers with wealth $v_i \in [\tau_C, \tau_I]$ do not buy anything because they are not wealthy enough to buy I and they are afraid of lower status utility if they were exposed; and consumers with wealth $v_i \in [0, \tau_C]$ would buy C. Because of the non-contiguous purchasing policy under this scenario, the corresponding analysis is highly complex and our key results in the base model no longer hold. We shall defer this scenario as future research.

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Appendix

A Proof of Propositions 1-6 and Corollary 1-2

Proof of Proposition 1

Proof. **Outline of the proof:**

Applying backward induction, C chooses its entry decision optimally in the second stage given p_I. We will show that under the conditions presented in Proposition 1 , I is optimal to select a p_I in the first stage such that C cannot select any p_C to have a positive profit if entering. Specifically, we divide the proof in two cases, i.e., $k_C \leq \frac{\lambda\alpha}{2}$ and $k_C > \frac{\lambda\alpha}{2}$. In the first case, when $k_C \leq \frac{\lambda\alpha}{2}$, we show that under the conditions in Proposition 1, C's best response in the second stage leads to $\tau_I = 0$. Therefore, C cannot enter the market. In the second case, when $k_C > \frac{\lambda\alpha}{2}$, we will show that there is no feasible conditions under which I is optimal to choose a price such that C cannot have a positive profit. Therefore, C always enters the market when $k_C > \frac{\lambda\alpha}{2}$. Combining these two cases, we have Proposition 1. In the following, we present the details of the proof.

<u>**Consumer purchasing decisions.**</u> τ_1 is solved by $\tau_I \cdot 1 - p_I + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}) = \tau_I \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2})$. We get $\tau_I = \frac{p_I - p_C - (1-\alpha)\frac{\lambda}{2}}{1-q}$. τ_C can be solved similarly by $\tau_C \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}) = \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2})$. We have $\tau_C = \frac{p_C - \alpha\frac{\lambda}{2}}{q}$.

<u>**Firm pricing decision**</u>

Case 1. <u>$k_C \leq \frac{\lambda\alpha}{2}$</u>

Applying backward induction, we first consider Period 2.

<u>Period 2</u>

C does not enter the market if $\tau_I \leq 0$, which is equivalent to $p_C \geq p_I - \frac{\lambda}{2}(1-\alpha)$. Since $\tau_C \leq \tau_I$, τ_C has to be smaller than 0 if C cannot enter the market. So we will only consider the case when $\tau_C \leq 0$, which is equivalent to $p_C \leq \frac{\lambda\alpha}{2}$. Now suppose C can ensure to price not larger than $p_I - \frac{\lambda}{2}(1-\alpha)$. Then in Period 2, C maximizes its profit with respect to price p_C under those constraints on p_C.

$$\begin{aligned} \max \quad & \pi_C(p_I) = (p_C - k_C)\tau_I \\ \text{s. t.} \quad & p_C \leq \frac{\lambda\alpha}{2} \\ & p_C \leq p_I - \frac{\lambda}{2}(1-\alpha) \end{aligned}$$

We have the optimal price,

$$p_C^*(p_I) = \begin{cases} p_I - \frac{\lambda}{2}(1-\alpha), & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2}, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C] \\ \frac{\lambda\alpha}{2}, & \text{if } p_I > \frac{\lambda}{2}(1+\alpha) - k_C \end{cases}$$

Hence, there are three scenarios depending on I's price in Period 1. In the first scenario, $\tau_I = 0$; in the second scenario, $\tau_I \geq 0$ and if $p_I^* = k_C + \frac{\lambda}{2}(1-\alpha)$ is the solution in the first period, $\tau_I = 0$; while in the last scenario, $\tau_I > 0$. So we are only interested in the first two scenarios where C might be out of the market.

Period 1

In Period 1, I maximizes its profit with respect to price p_I. I should also ensure $\tau_I \leq 1$ to have nonnegative demand.

Scenario 1.1: $p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)]$. In this scenario, $\tau_I = 0$ which satisfies the condition $\tau_I \leq 1$ automatically. Since I covers the whole market, C will not enter the market.

Scenario 1.2: $p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C]$. In this scenario, $\tau_C < 0$ and $\tau_I \geq 0$. $p_C^*(p_I) = \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2}$. $\tau_I \leq 1$ implies $p_I \leq 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha)$. I maximizes its own profit function under those listed constraints on p_I.

$$\begin{aligned} \max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\ \text{s. t.} \quad & p_I \leq 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha) \\ & p_I \leq \frac{\lambda}{2}(1+\alpha) - k_C \\ & p_I \geq k_C + \frac{\lambda}{2}(1-\alpha) \end{aligned}$$

Only when $\frac{\lambda(1-\alpha)}{2} - 2(1-q) \geq k_I - k_C$, $k_C + \frac{\lambda}{2}(1-\alpha)$ is the optimal price, which implies $\tau_I = 0$. In this case, C cannot enter the market. Combining these two scenarios, we have when $\frac{\lambda\alpha}{2} \geq k_C$ and $\frac{\lambda(1-\alpha)}{2} - 2(1-q) \geq k_I - k_C$, C does not enter the market.

Case 2. $k_C > \frac{\lambda\alpha}{2}$

Period 2

Since $k_C > \frac{\lambda\alpha}{2}$, we have $p_C \leq \frac{\lambda\alpha}{2} < k_C$ when $\tau_C \leq 0$. Hence, C cannot make positive profit margin if entering. Therefore, C will always make sure $\tau_C > 0$. Suppose $\tau_C > 0$, C can enter the market if $p_C \geq k_C$ and $\tau_I > \tau_C$. C maximizes its profit with respect to price p_C

$$\begin{aligned} \max \quad & \pi_C(p_I) = (p_C - k_C))(\tau_I - \tau_C) \\ \text{s. t.} \quad & p_C \geq k_C \\ & p_C \leq q p_I - \frac{\lambda}{2}(q-\alpha) \end{aligned}$$

We have the optimal price

$$p_C^*(p_I) = \begin{cases} k_C, & \text{if } p_I \leq \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))] \\ \frac{k_C + q p_I - \frac{\lambda}{2}(q-\alpha)}{2}, & \text{if } p_I \geq \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q)) \end{cases}$$

Hence, there are two scenarios depending on I's price in Period 1. In the first scenario, C cannot make positive profit; in the second scenario, if $p_I^* = \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))$, C also cannot have a positive margin.

Period 1

Scenario 2.1**:** $p_I \in [0, \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))]$. In this scenario, C cannot get positive profit margin. C will not enter the market.

Scenario 2.2**:** $p_I \geq \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))$ In this scenario, $\tau_C > 0$ and $\tau_I > \tau_C$. $p_C^*(p_I) = \frac{k_C + qp_I - \frac{\lambda}{2}(\alpha - q)}{2}$. $\tau_I \leq 1$ implies $p_I \leq \frac{1}{2-q}(2(1-q) + k_C - \frac{\lambda}{2}(\alpha + q) + \lambda)$. I maximizes its own profit function given the constraints on p_I.

$$\begin{aligned} \max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\ \text{s. t.} \quad & p_I \leq \frac{1}{2-q}(2(1-q) + k_C - \frac{\lambda}{2}(\alpha + q) + \lambda) \\ & p_I \geq \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q)) \end{aligned}$$

In both scenarios, we know C cannot have positive margin only when $p_I^* = \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))$, which is equivalent to $k_I \leq \frac{\lambda}{2} + \frac{1}{2-q}(\frac{4-3q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1-q))$ and $k_C \geq \frac{\lambda\alpha}{2} + q$. It can be shown that these two conditions cannot both hold given $k_I \geq k_C$. Therefore, C can always enter the market. Hence, we have Proposition 1.

□

Supplementary Appendix A gives the detailed backward induction analysis of this game. Before we show the proof of Proposition 2 and Corollary 1, we first highlight the main results in the Supplementary Appendix A. Define

$$\begin{aligned}
& p_0 := k_C + \tfrac{\lambda}{2}(1-\alpha) && \pi_0 := k_C + \tfrac{\lambda}{2}(1-\alpha) - k_I \\
& p_1 := \tfrac{k_I + 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha)}{2} && \pi_1 := \tfrac{1}{2(1-q)}(\tfrac{2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha) - k_I}{2})^2 \\
& p_3 := \tfrac{\lambda}{2}(1+\alpha) - k_C && \pi_3 := \tfrac{1}{1-q}(1 - q - \tfrac{\lambda\alpha}{2} + k_C)(\tfrac{\lambda}{2}(1+\alpha) - k_C - k_I) \\
& p_4 := \tfrac{k_I + (1-q) + \frac{\lambda}{2}}{2} && \pi_4 := \tfrac{1}{1-q}(\tfrac{1-q+\frac{\lambda}{2} - k_I}{2})^2 \\
& p_7 := \tfrac{k_I + \frac{2(1-q) - \frac{\lambda\alpha}{2} + k_C}{2-q} + \frac{\lambda}{2}}{2} && \pi_7 := \tfrac{2-q}{2(1-q)}(\tfrac{\frac{2(1-q) - \frac{\lambda\alpha}{2} + k_C}{2-q} + \frac{\lambda}{2} - k_I}{2})^2 \\
& l_1 := k_C + \tfrac{\lambda}{2}(1-\alpha) - 2(1-q) && l_2 := 2(1-q) + k_C + \tfrac{\lambda}{2}(1-\alpha) \\
& l_3 := -3k_C + \tfrac{3\lambda\alpha}{2} + \tfrac{\lambda}{2} - 2(1-q) && l_5 := \tfrac{\lambda}{2} + \lambda\alpha - 2k_C - (1-q) \\
& l_6 := 1 - q + \tfrac{\lambda}{2} && l_8 := \tfrac{\lambda\alpha}{2} - q(1-q) \\
& l_9 := \tfrac{\lambda}{2} + \tfrac{1}{2-q}(\tfrac{4-q}{q}(\tfrac{\lambda\alpha}{2} - k_C) - 2(1-q)) && l_{10} := \tfrac{1}{2-q}(2(1-q) - \tfrac{\lambda\alpha}{2} + k_C) + \tfrac{\lambda}{2}
\end{aligned}$$

(Equilibrium Case 1) When $k_C \leq \frac{\lambda\alpha}{2}$ and $k_I \in [k_C, l_1]$, the copycat C will not enter the market. As incumbent I operates as a monopoly, it will set its price $p_{I,1} = p_0$ and earns a profit $\pi_{I,1} = \pi_0$ in equilibrium. Also, $\tau_{I,1} = 0$ so that the incumbent will capture the entire market.

(Equilibrium Case 2) When $k_C \leq \frac{\lambda\alpha}{2}$ and $k_I \in [l_1, \min(l_2, l_3)]$, copycat C will enter the market with $p_{C,2} = \frac{1}{4}(k_I + 3k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q))$, $\tau_{C,2} = \frac{1}{4q}(k_I + 3k_C - \frac{\lambda}{2}(1+3\alpha) + 2(1-q)) < 0$, $\pi_{C,2} = \frac{1}{16(1-q)}(k_I - k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q))^2$. Also, incumbent I will set $p_{I,2} = p_1$ and $\tau_{I,2} = \frac{1}{4(1-q)}(2(1-q) + k_I - k_C - \frac{\lambda}{2}(1-\alpha))$ so that $\pi_{I,2} = \pi_1$.

(Equilibrium Case 3) When $k_I \in [l_3, l_5]$, C enters the market with $p_{C,3} = \frac{\lambda\alpha}{2}$, $\tau_{C,3} = 0$, and $\pi_{C,3} =$

$\frac{1}{1-q}(\frac{\lambda\alpha}{2}-k_C)^2$. Also, the incumbent will set $p_{I,3}=p_3$ and $\tau_{I,3}=\frac{1}{1-q}(\frac{\lambda\alpha}{2}-k_C)$ so that $\pi_{I,3}=\pi_3$.

(Equilibrium Case 4) When $k_I\in[l_5,\min(l_6,l_9)]$, C enters the market with $p_{C,4}=\frac{\lambda\alpha}{2}$, $\tau_{C,4}=0$, and $\pi_{C,4}=\frac{1}{2(1-q)}(\frac{\lambda\alpha}{2}-k_C)(k_I-\frac{\lambda}{2}+(1-q))$. Also, the incumbent will set $p_{I,4}=p_4$ and $\tau_{I,4}=\frac{1}{2(1-q)}(1-q+k_I-\frac{\lambda}{2})$ so that $\pi_{I,4}=\pi_4$.

(Equilibrium Case 5) When $k_I\in[\min\{\max(l_5,l_9),\frac{\lambda\alpha}{2}\},l_{10}]$, $k_C\in[l_8,1]$, C enters the market and sets $p_{C,5}=\frac{q}{4}k_I-\frac{\lambda q}{8}+\frac{2q(1-q)+(4-q)k_C+(4-3q)\frac{\lambda\alpha}{2}}{4(2-q)}$ and $\tau_{C,5}=\tau_I^5-\frac{1}{4q(1-q)}(qk_I+\frac{2q(1-q)}{2-q}+\frac{\lambda q}{2})$ so that $\pi_{C,5}=\frac{q}{16(1-q)}(k_I-\frac{\lambda}{2}+\frac{2(1-q)}{2-q}+\frac{4-3q}{q(2-q)}(\frac{\lambda\alpha}{2}-k_C))^2$. Also, the incumbent sets $p_{I,5}=p_7$ and $\tau_{I,5}=\frac{2-q}{4(1-q)}(\frac{2(1-q)+\frac{\lambda\alpha}{2}-k_C}{2-q}-\frac{\lambda}{2}+k_I)$ so that $\pi_{I,5}=\pi_7$.

Proof of Proposition 2

Proof. Given the profits of C in different equilibrium cases, we first show how they are affected by q. Then those cases where π_C is not monotonic in q are identified.

Equilibrium Case 1: C does not enter the market.

Equilibrium Case 2: $\pi_{C,2}=\frac{1}{1-q}(\frac{k_I-k_C-\frac{\lambda}{2}(1-\alpha)+2(1-q)}{4})^2$. $\frac{\partial\pi_{C,2}}{\partial q}=\frac{1}{16(1-q)^2}(k_I-k_C-\frac{\lambda}{2}(1-\alpha)+2(1-q))(k_I-k_C-\frac{\lambda}{2}(1-\alpha)-2(1-q))$. Due to the boundary condition that $k_I\in(k_C+\frac{\lambda}{2}(1-\alpha)-2(1-q),min(k_C+\frac{\lambda}{2}(1-\alpha)+2(1-q),-3k_C+\frac{\lambda}{2}(1+3\alpha)-2(1-q)))$, we have $\frac{\partial\pi_{C,2}}{\partial q}\le 0$. Therefore, profit of C in Case 2 is **decreasing in** q.

Equilibrium Case 3: $\pi_{C,3}=\frac{1}{1-q}(\frac{\lambda\alpha}{2}-k_C)^2$. The profit of C in Case 3 is easily to be seen as **increasing in** q.

Equilibrium Case 4: $\pi_{C,4}=\frac{1}{1-q}(\frac{\lambda\alpha}{2}-k_C)(\frac{k_I-\frac{\lambda}{2}+(1-q)}{2})$. $\frac{\partial\pi_{C,4}}{\partial q}=\frac{1}{2(1-q)^2}(k_I-\frac{\lambda}{2})(\frac{\lambda\alpha}{2}-k_C)$. Due to the boundary condition that $k_C\in[\frac{\lambda\alpha}{2}-(1-q),\frac{\lambda\alpha}{2}]$, $k_I\in[-2k_C+\frac{\lambda}{2}(1+2\alpha)-(1-q),\min(\frac{\lambda}{2}+(1-q),\frac{\lambda}{2}-\frac{2(1-q)}{2-q}+\frac{4-q}{q(2-q)}(-k_C+\frac{\lambda\alpha}{2})]$, $\frac{\partial\pi_{C,4}(q)}{\partial q}\ge 0$ when $k_I\ge\frac{\lambda}{2}$, and $\frac{\partial\pi_{C,4}(q)}{\partial q}\le 0$ when $k_I\le\frac{\lambda}{2}$. Therefore, profit of C in Case 4 is **non-monotone in** q.

Equilibrium Case 5: $\pi_{C,5}=\frac{q}{1-q}(\frac{k_I-\frac{\lambda}{2}+\frac{2(1-q)}{2-q}+\frac{4-3q}{q(2-q)}(\frac{\lambda\alpha}{2}-k_C)}{4})^2$, which is a complicated fractional function of q. It is **non-monotone in** q.

The union of feasible regions of Case 4 and 5 gives the condition in Proposition 2. Hence, we have Proposition 2. □

Proof of Corollary 1

Proof. Given I's optimal price the first equilibrium case (i.e. when C is deterred from entry), $p_I^*=p_{I,1}=k_C+\frac{\lambda}{2}(1-\alpha)$, we compare it with $p_I^B=\frac{1}{2}(k_I+1+\frac{\lambda}{2})>\frac{1}{2}$. Since in this case, $\frac{\lambda\alpha}{2}\ge k_C$, we have $p_{I,1}\le\frac{\lambda}{2}<\frac{1}{2}$. Because $p_I^B>\frac{1}{2}$, we have $p_{I,1}<p_I^B$

□

Proof of Proposition 3

Proof. **Outline of the proof:**

Appendix section B gives the detailed consumer surplus analysis. Here, we look at Equilibrium case 4 (when $k_C \in [\frac{\lambda\alpha}{2} - (1-q), \frac{\lambda\alpha}{2}])$ and show how the consumer surplus and social surplus change when q tends to 0 compared with the benchmark case. In the benchmark case, we have $CS_B = \frac{(\tau_I^B)^2}{2} + (\frac{\lambda}{2} - 1)\tau_I^B + \frac{1}{2}$, where $\tau_I^B = \frac{1+k_I}{2} - \frac{\lambda}{4}$. $SS_B = CS_B + \pi_I^B$, where $\pi_I^B = (\frac{1-k_I}{2} + \frac{\lambda}{4})^2$. When $k_C \in [\frac{\lambda\alpha}{2} - (1-q), \frac{\lambda\alpha}{2}]$, we have $CS = \frac{1-q}{2}(\tau_I)^2 + (\tau_I)(\frac{\lambda}{2}(1-\alpha) - (1-q)) + \frac{1}{2}$, where $\tau_I = \frac{1}{2(1-q)}(1 - q + k_I - \frac{\lambda}{2})$. $SS = CS + \pi_I + \pi_C$, where $\pi_I = \frac{1}{4(1-q)}(-k_I + \frac{\lambda}{2} + (1-q))^2$ and $\pi_C = \frac{1}{2(1-q)}(\frac{\lambda\alpha}{2} - k_C)(k_I - \frac{\lambda}{2} + (1-q))$. When q tends to 0, τ_I tends to τ_B. Hence, CS tends to $CS_B - \tau_I^B\frac{\lambda\alpha}{2} = CS_B - \frac{\lambda\alpha}{2}(\frac{1+k_I}{2} - \frac{\lambda}{4})$. Since $k_I \geq 0$, $\lambda, \alpha \in (0,1)$, we have the term $\frac{\lambda\alpha}{2}(\frac{1+k_I}{2} - \frac{\lambda}{4}) > 0$. Similarly, when q tends to 0, π_I tends to π_I^B. π_C tends to $\tau_I^B(\frac{\lambda\alpha}{2} - k_C)$. Hence, SS tends to $SS_B - \tau_I^B k_C = SS_B - k_C(\frac{1+k_I}{2} - \frac{\lambda}{4})$. The term $k_C(\frac{1+k_I}{2} - \frac{\lambda}{4})$ is also strictly greater than 0. Therefore, we have when q is sufficiently small, $CS_B > CS$ and $SS_B > SS$. This completes the proof. □

Proof of Proposition 4 and Proposition 5.

Proof. **Outline of the proof:**

Proposition 4 and Proposition 5 can be proved similarly to the way we prove Proposition 1 and Proposition 2. We here directly refer the reader to Supplementary Appendix B which gives the full backward induction analysis of Extension 1. Proposition 4 and Proposition 5 are natural results from the analysis.

We here firstly conclude the main results of backward induction analysis in Supplementary Appendix B.

1. When $k_I \in [k_C, \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) - 2(1-\delta q)]$, $k_C \leq \frac{\lambda\alpha}{2}$. C will not enter the market.

2. When $k_I \in [\delta k_C + \frac{\lambda\delta}{2}(1-\alpha) - 2(1-\delta q), \min(2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}, -3\delta k_C + \frac{3\delta\lambda\alpha}{2} + \frac{\lambda}{2}(1 + \delta) - 2(1-\delta q))]$, C enters the market. $p_C = \frac{k_I + 2(1-\delta q) + 3\delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha)}{4\delta}$ $\pi_C = \frac{1}{16\delta(1-\delta q)}(k_I + 2(1 - \delta q) - \delta k_C - \frac{\lambda}{2}(1 + \delta - \delta\alpha))^2$.

3. When $k_I \in [-3\delta k_C + \frac{3\delta\lambda\alpha}{2} + \frac{\lambda}{2}(1+\delta) - 2(1-\delta q), \frac{\lambda}{2}(1+\delta) + \delta\lambda\alpha - 2\delta k_C - (1-\delta q)]$, $k_C \in [\frac{\lambda\alpha}{2} - (1-\delta q), \frac{\lambda\alpha}{2}]$, C enters the market. $p_C = \frac{\lambda\alpha}{2}$. $\pi_C = \frac{\delta}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)^2$.

4. When $k_I \in [\frac{\lambda}{2}(1+\delta) + \delta\lambda\alpha - 2\delta k_C - (1-\delta q), \min(1 - \delta q + \frac{\lambda}{2}(1+\delta), \frac{\lambda}{2}(1+\delta) + \frac{1}{2-\delta q}(\frac{4-\delta q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1 - \delta q)))]$, $k_C \in [\frac{\lambda\alpha}{2} - (1-\delta q), \frac{\lambda\alpha}{2}]$, C enters the market.$p_C = \frac{\lambda\alpha}{2}$. $\pi_C = \frac{1}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I + 1 - \delta q - \frac{\lambda(1+\delta)}{2}}{2})$.

5. When $k_I \in [\min\{\max(\frac{\lambda}{2}(1+\delta) + \delta\lambda\alpha - 2\delta k_C - (1-\delta q), \frac{\lambda}{2}(1+\delta) + \frac{1}{2-\delta q}(\frac{4-\delta q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1 - \delta q))), \frac{\lambda\alpha}{2}\}, \frac{1}{2-\delta q}(2(1-\delta q) - \frac{\delta\lambda\alpha}{2} + \delta k_C) + \frac{\lambda}{2}(1+\delta)]$, $k_C \in [\frac{\lambda\alpha}{2} - q(1-\delta q), 1]$, C enters the market. $p_C = \frac{2q(1-\delta q) + (4-\delta q)k_C + (4-3\delta q)\frac{\lambda\alpha}{2}}{4(2-\delta q)} - \frac{\lambda q(1+\delta)}{8} + \frac{qk_I}{4}$. $\pi_C = \frac{q}{16(1-\delta q)}(k_I - \frac{\lambda(1+\delta)}{2} + \frac{2(1-\delta q)}{2-\delta q} + \frac{4-3\delta q}{q(2-\delta q)}(\frac{\lambda\alpha}{2} - k_C))^2$.

The only case when C does enter the market is when $k_I \in [k_C, \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) - 2(1-\delta q)]$, $k_C \leq \frac{\lambda\alpha}{2}$, i.e. $k_I - \delta k_C \leq \frac{\lambda(1-\alpha)}{2} - 2(1-\delta q)$, $k_C \leq \frac{\lambda\alpha}{2}$. We have Proposition 4.

Equilibrium Case 1: C does not enter the market.

Equilibrium Case 2: $\pi_{C,2} = \frac{1}{16\delta(1-\delta q)}(k_I + 2(1-\delta q) - \delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha))^2$. $\frac{\partial \pi_{C,2}}{\partial q} = \frac{1}{16(1-\delta q)^2}(k_I - \delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha) + 2(1-\delta q))(k_I - \delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha) - 2(1-\delta q))$. Due to the boundary conation that

$k_I \in [\delta k_C + \frac{\lambda\delta}{2}(1-\alpha) - 2(1-\delta q), \min(2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}, -3\delta k_C + \frac{3\delta\lambda\alpha}{2} + \frac{\lambda}{2}(1+\delta) - 2(1-\delta q))]$, $\frac{\partial \pi_{C,2}}{\partial q} \leq 0$. Therefore, profit of C in Case 2 is **decreasing in** q.

Equilibrium Case 3: $\pi_{C,3} = \frac{\delta}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)^2$. The profit of C in Case 3 is easily to be seen as **increasing in** q.

Equilibrium Case 4: $\pi_{C,4} = \frac{1}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I - \frac{\lambda}{2}(1+\delta) + (1-\delta q)}{2})$. $\frac{\partial \pi_{C,4}}{\partial q} = \frac{1}{2(1-\delta q)^2}(k_I - \frac{\lambda}{2}(1+\delta))(\frac{\lambda\alpha}{2} - k_C)$. Due to the boundary conation that $k_I \in [l_5^d, \min(l_6^d, l_9^d)]$, $k_C \in [l_4^d, \frac{\lambda\alpha}{2}]$, $\frac{\partial \pi_{C,4}(q)}{\partial q} \geq 0$ when $k_I \geq \frac{\lambda}{2}(1+\delta)$, and $\frac{\partial \pi_{C,4}(q)}{\partial q} \leq 0$ when $k_I \leq \frac{\lambda}{2}(1+\delta)$. Therefore, profit of C in Case 4 is **non-monotone in** q.

Equilibrium Case 5: $\pi_{C,5} = \frac{q}{16(1-\delta q)}(k_I - \frac{\lambda(1+\delta)}{2} + \frac{2(1-\delta q)}{2-\delta q} + \frac{4-3\delta q}{q(2-\delta q)}(\frac{\lambda\alpha}{2} - k_C))^2$, which is a complicated fractional function of q. It is **non-monotone in** q.

We then have Proposition 5. □

Proof of Proposition 6

Proof. 1. Benchmark case:

Consumer purchasing decisions

τ_I^B is solved by $\tau_I^B \cdot 1 - p_I + \lambda\frac{1+\tau_I^B}{2} + \delta\lambda\frac{1+\tau_I^B}{2} = \lambda\frac{\tau_I^B}{2} + \delta\lambda\frac{\tau_I^B}{2}$. We get $\tau_I^B = p_I - \frac{\lambda(1+\delta)}{2}$.

Firm pricing decision

I maximizes its profit

$$\pi_I^B = (p_I - k_I)(1 - \tau_I^B)$$

We have $p_I^B = \frac{k_I+1}{2} + \frac{\lambda(1+\delta)}{4}$. Hence $\pi_I^B = (\frac{1-k_I}{2} + \frac{\lambda(1+\delta)}{4})^2$; and $\tau_I^B = \frac{k_I+1}{2} - \frac{\lambda(1+\delta)}{4}$.

Consumer Surplus and Social Surplus

$$CS_B \equiv \int_{\tau_I}^1 v_i \cdot 1 - p_I + \lambda\frac{1+\tau_I^B}{2} + \delta\lambda\frac{1+\tau_I^B}{2}dv_i + \delta\int_0^{\tau_C} \lambda\frac{\tau_I^B}{2} + \delta\lambda\frac{\tau_I^B}{2}dv_i = \frac{1}{2} + \frac{(\tau_I^B)^2}{2} + (\frac{\lambda}{2}(1+\delta) - 1)\tau_I^B.$$

$$SS_B = CS_B + \pi_I^B$$

2. Equilibrium case 4:

We have obtained $p_I = \frac{k_I + (1-\delta q) + \frac{\lambda}{2}(1+\delta)}{2}$, $\pi_I = \frac{1}{1-\delta q}(\frac{1-\delta q + \frac{\lambda}{2}(1+\delta) - k_I}{2})^2$. $p_C = \frac{\lambda\alpha}{2}$. $\pi_C = \frac{1}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I + 1 - \delta q - \frac{\lambda(1+\delta)}{2}}{2})$; and $\tau_I = \frac{1}{1-\delta q}(p_I - \frac{\lambda(1+\delta)}{2})$.

Consumer Surplus and Social Surplus

$$CS \equiv \int_{\tau_I}^1 v_i \cdot 1 - p_I + \lambda(\frac{(1-\alpha)(1+\tau_I)}{2} + \frac{\alpha}{2}) + \delta\lambda(\frac{(1-\alpha)(1+\tau_I)}{2} + \frac{\alpha}{2})dv_i + \delta\int_0^{\tau_C} v \cdot q - p_C + \lambda((1-\alpha)\frac{\tau_I}{2} + \frac{\alpha}{2})dv_i$$

$$= \frac{1}{2} + \frac{(1-\delta q)(\tau_I^B)^2}{2} + (\frac{\lambda}{2}(1+\delta-\delta\alpha) - 1)\tau_I^B.$$

$$SS = CS + \pi_I + \delta\pi_C$$

When q tends to 0, τ_I tends to τ_I^B. Hence, CS tends $CS_B - \frac{\delta\lambda\alpha}{2} < CS_B$. When q tends to 0, π_I tends to π_I^B. π_C tends to $(\frac{\lambda\alpha}{2} - k_C)\tau_I^B$. Hence, SS tends to $SS_B - \delta k_C \tau_I^B < SS_B$. This completes the proof.

□

Proof of Corollary 2

Proof. We have already obtained $p_I^B = \frac{k_I+1}{2} + \frac{\lambda(1+\delta)}{4}$. When C is deterred from entry, we know I's optimal price is $p_I^* = \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)$ from Supplementary Appendix B.$p_I^B - p_I^* = \frac{1}{2}(k_I + 1 - \frac{\lambda}{2}(1+\delta)$. Since $k_I + 1 > 1$, and $\frac{\lambda}{2}(1+\delta) \leq \lambda < 1$, we have $p_I^B > p_I^*$. □

B Analysis of Consumer Surplus when C is a Potential Entrant

Recall that the total utility $U(v_i)$ is given by

$$U(v_i) = \begin{cases} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}), & \text{if } v_i \in [\tau_I, 1], \\ v_i \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [\tau_C, \tau_I] \\ \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [0, \tau_C]. \end{cases}$$

Equilibrium Case 1: $k_C \in [0, \frac{\lambda\alpha}{2}]$ and $k_I \in [k_C, l_1]$ In this case, only I is in the market. Consumer v_i's surplus is, thus, $v_i \cdot 1 - p_I + \lambda \cdot 1$ where, $p_I = k_C + \frac{\lambda}{2}(1-\alpha)$. Total consumer surplus is, thus, $CS_1 \equiv \int_0^1 v_i \cdot 1 - p_I + \lambda \cdot 1 dv_i = \frac{1}{2} - k_C + \frac{\lambda}{2}(1+\alpha)$

Equilibrium Case 2: $k_C \in [0, \frac{\lambda\alpha}{2}]$ and $k_I \in [l_1, \min(l_2, l_3)]$ In this case, both I and C are in the market. $\tau_C < 0$ and the market is fully covered. Consumer v_i's surplus is, thus,

$$U(v_i) = \begin{cases} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2}), & \text{if } v_i \in [\tau_I, 1], \\ v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [0, \tau_I] \end{cases}$$

where, $\tau_I = \frac{1}{4(1-q)}(2(1-q) + k_I - k_C - \frac{\lambda}{2}(1-\alpha))$, $p_I = 2(1-q)\tau_I) + k_C + \frac{\lambda}{2}(1-\alpha)$, and $p_C = \frac{k_I + 2(1-q) + 3k_C - \frac{\lambda}{2}(1-\alpha)}{4} = \tau_I(1-q) + k_C$.

Total consumer surplus is, $CS_2 \equiv \int_{\tau_I}^1 v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2})dv_i + \int_0^{\tau_I} v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2})dv_i = \frac{1-q}{2}\tau_I^2 + 2(\frac{\lambda}{2}(1-\alpha) - (1-q))\tau_I + \frac{1}{2} + \frac{\lambda\alpha}{2} - k_C$ where, $\tau_I = \frac{1}{4(1-q)}(2(1-q) + k_I - k_C - \frac{\lambda}{2}(1-\alpha))$.

Equilibrium Case 3: $k_C \in [l_4, \frac{\lambda\alpha}{2}]$ and $k_I \in [l_3, l_5]$ In this case, both I and C are in the market. $\tau_C = 0$ and the market is fully covered. Consumer v_i's surplus is, thus,

$$U(v_i) = \begin{cases} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2}), & \text{if } v_i \in [\tau_I, 1], \\ v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [0, \tau_I] \end{cases}$$

where, $\tau_I = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)$, $p_I = (1-q)\tau_I + \frac{\lambda}{2}$, and $p_C = \frac{\lambda\alpha}{2}$.

Total consumer surplus is, $CS_3 \equiv \int_{\tau_I}^1 v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2})dv_i + \int_0^{\tau_I} v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2})dv_i = \frac{1-q}{2}\tau_I^2 + (\frac{\lambda}{2}(1-\alpha) - (1-q))\tau_I + \frac{1}{2}$ where, $\tau_I = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)$.

Equilibrium Case 4: $k_C \in [l_4, \frac{\lambda\alpha}{2}]$ and $k_I \in [l_5, \min(l_6, l_9)]$ In this case, both I and C are in the market. $\tau_C = 0$ and the market is fully covered. Consumer v_i's surplus is, thus,

$$U(v_i) = \begin{cases} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2}), & \text{if } v_i \in [\tau_I, 1], \\ v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [0, \tau_I] \end{cases}$$

where, $\tau_I = \frac{1}{2(1-q)}((1-q)+k_I-\frac{\lambda}{2})$, $p_I = (1-q)\tau_I + \frac{\lambda}{2}$, and $p_C = \frac{\lambda\alpha}{2}$.

Total consumer surplus is, $CS_4 \equiv \int_{\tau_I}^{1} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{1+\tau_I}{2})dv_i + \int_0^{\tau_I} v_i \cdot q - p_C + \lambda(\alpha\frac{1}{2} + (1-\alpha)\frac{\tau_I}{2})dv_i = \frac{1-q}{2}\tau_I^2 + (\frac{\lambda}{2}(1-\alpha) - (1-q))\tau_I + \frac{1}{2}$ where, $\tau_I = \frac{1}{2(1-q)}((1-q)+k_I-\frac{\lambda}{2})$.

Equilibrium Case 5: $k_C \in [l_8, 1]$ and $k_I \in [\min\{\max(l_5, l_9), \frac{\lambda\alpha}{2}\}, l_{10}]$ In this case, both I and C are in the market. $\tau_C > 0$ and the market is not fully covered. Consumer v_i's surplus is, thus,

$$U(v_i) = \begin{cases} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}), & \text{if } v_i \in [\tau_I, 1], \\ v_i \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [\tau_C, \tau_I] \\ \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2}), & \text{if } v_i \in [0, \tau_C]. \end{cases}$$

where, $\tau_I = \frac{2-q}{4(1-q)}(\frac{2(1-q)+\frac{\lambda\alpha}{2}-k_C}{2-q} - \frac{\lambda}{2} + k_I)$, $\tau_C = \tau_I - \frac{1}{4q(1-q)}(qk_I + \frac{2q(1-q)}{2-q} + \frac{4-3q}{2-q}(\frac{\lambda\alpha}{2} - k_C) - \frac{\lambda q}{2})$, $p_I = \frac{1}{2-q}((2(1-q)\tau_I + k_C - \frac{\lambda\alpha}{2}) + \frac{\lambda}{2}$, $p_C = \frac{2q(1-q)+(4-q)k_C+(4-3q)\frac{\lambda\alpha}{2}}{4(2-q)} - \frac{\lambda q}{8} + \frac{qk_I}{4} = q(1-q)(\tau_I - \tau_C) + k_C$. Total consumer surplus is, $CS_5 \equiv \int_{\tau_I}^{1} v_i \cdot 1 - p_I + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2})dv_i + \int_{\tau_C}^{\tau_I} v_i \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2})dv_i + \int_0^{\tau_C} \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2})dv_i = (1-q)(\frac{2}{2-q} - q - \frac{1}{2})\tau_I^2 - q(\frac{3}{2} - q)\tau_C^2 + 2q(1-q)\tau_I\tau_C + \{\frac{\lambda}{2} - \frac{1}{2-q}[\frac{\lambda\alpha}{2} + (1-q)k_C + 2(1-q)]\}\tau_I + k_C\tau_C + \frac{1}{2} - \frac{1}{2-q}(k_C - \frac{\lambda\alpha}{2})$ where, $\tau_I = \frac{2-q}{4(1-q)}(\frac{2(1-q)+\frac{\lambda\alpha}{2}-k_C}{2-q} - \frac{\lambda}{2} + k_I)$, $\tau_C = \tau_I - \frac{1}{4q(1-q)}(qk_I + \frac{2q(1-q)}{2-q} + \frac{4-3q}{2-q}(\frac{\lambda\alpha}{2} - k_C) - \frac{\lambda q}{2})$.