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Abstract

This paper examines crop planning decision in sustainable agriculture—that is, how to allocate farmland among multiple crops in each growing season when the crops have rotation benefits across growing seasons. We consider a farmer who periodically allocates the farmland between two crops in the presence of revenue uncertainty where revenue is stochastically larger and farming cost is lower when a crop is grown on rotated farmland (where the other crop was grown in the previous season). We characterize the optimal dynamic farmland allocation policy and perform sensitivity analysis to investigate how revenue uncertainty of each crop affects the farmer's optimal allocation decision and profitability. Using a calibration based on a farmer growing corn and soybean in Iowa we show that growing only one crop over the entire planning horizon, as employed in industrial agriculture, leads to a considerable profit loss—that is, making crop planning based on principles of sustainable agriculture has substantial value. We propose a simple heuristic allocation policy which we characterize in closed form. Using our model calibration we show that (i) the proposed policy not only outperforms the commonly suggested heuristic policies in the literature, but also provides a near-optimal performance; (ii) compared to the optimal policy, the proposed policy has a higher allocation of crops to rotated farmland, and thus it is potentially more environmentally friendly.

Keywords: Farm Planning, Crop Rotation, Sustainability, Agriculture, Commodity, Uncertainty, Dynamic Programming, Corn, Soybean, Fallow

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1 Introduction

Sustainable agriculture aims at growing food in an ecologically and ethically responsible manner by using practices that enhance environmental quality and natural resource base (e.g., land, soil and water) while maintaining the economic viability of farm operations. The focus on sustainable agriculture is increasing owing to a variety of factors including surging demand for food—there will be 2 billion people more to feed by 2050—and heightened economic implications of agriculture—it is the main source of income for more than a third of world's population (European Commission 2012). In this paper we study a key decision in sustainable agriculture, crop planning—that is, how should a farmer allocate the available farmland among multiple crops in each growing season?

In sustainable agriculture crop planning is made based on crops that have rotation benefits across growing seasons (USDA 2015a). When two crops have rotation benefits growing a crop on rotated farmland (where the other crop was grown in the previous season) is more profitable than growing it on non-rotated farmland (where the same crop was grown in the previous season). As highlighted by Hennessy (2006), these rotation benefits can be attributed to increasing crop revenues owing to improved soil structure and broken reproductive cycles of pests, and to decreasing farming costs owing to reduced need for fertilizers (as a result of improved soil structure) and pesticides (as a result of lower pest populations). Consider, for example, corn and soybean, the two most planted crops in the U.S.[1] Because both crops are planted within the same time period—between late March and June—they compete for the allocation of farmland. Rotating these two crops is beneficial because, for instance, soybean improves the soil structure by fixing its nitrogen content, which is crucial for corn growth, and at the same time reduces the nitrogen (fertilizer) need for corn (Livingston et al. 2015). Rotating corn with soybean also breaks the reproductive cycle of corn rootworm—the most common corn insect—and reduces the need for pesticide.

Making the crop planning based on multiple crops with rotation benefits is a part of sustainable agriculture because it reduces the need for synthetic chemicals (e.g., fertilizers and pesticides), improves the soil structure, increases the biodiversity in the farm, and enhances the resilience of the farmer to adverse environmental conditions (e.g., unfavorable weather conditions, high infestation of pests and diseases) because crops, in general, are

[1] In the U.S., corn and soybean account for 55.5% of total acres harvested in 2014 (USDA 2015b) with an estimated total market value of $92 billion in the same year (USDA 2015c).

not affected in the same manner. It also improves the local community's diet because multiple crops are grown simultaneously. All these effects are in line with the objectives of sustainable agriculture as defined by the 1990 U.S. Farm Bill (USDA 2015a). In this paper our main focus is on the economic implications of crop planning in sustainable agriculture and we examine how it affects farmers' profitability.

An important feature of crop planning decision is that crop revenue is uncertain in each growing season. The revenue uncertainty of each crop is driven by the uncertainty in its harvest volume and the uncertainty in its sales price at the end of the growing season. The harvest volume is uncertain owing to uncertain weather conditions and potential infestation of pests and diseases during the growing season (Kazaz and Webster 2011). The sales price is uncertain because it is typically tied to the prevailing price at the regional exchange (spot) markets (Goel and Tanrısever 2017). In practice crop revenues show considerable variability, as illustrated for corn and soybean in Figure 1.

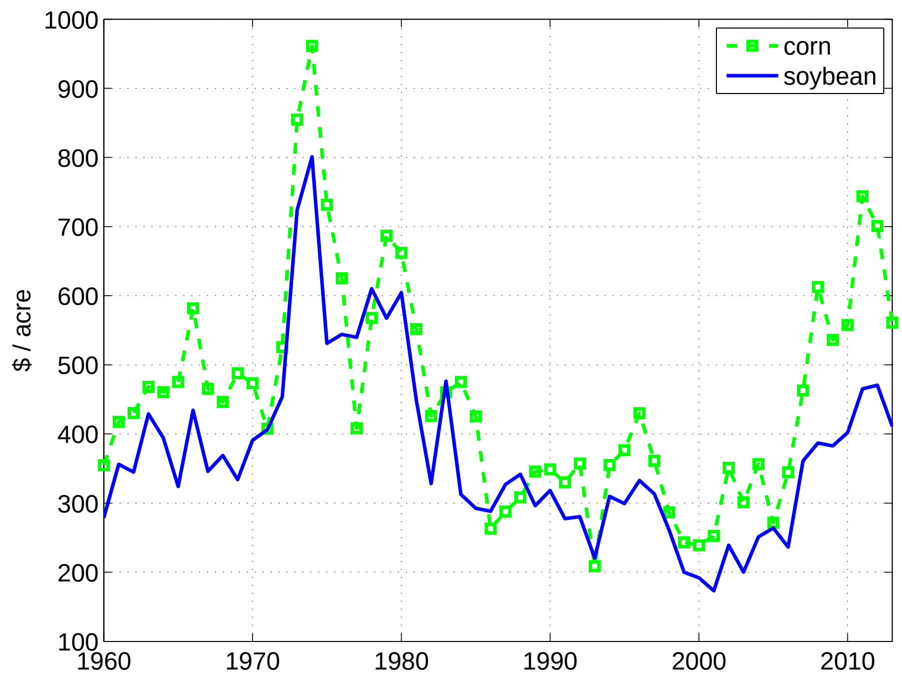

Figure 1: Annual corn and soybean revenues per acre (in $\$/acre$) in Iowa for the period 1960 to 2013 as calculated from the data reported by the U.S. Department of Agriculture.

As reviewed by Glen (1987) and Lowe and Preckel (2004), crop planning (or farmland allocation) problem has received considerable attention both in the operations management and agricultural economics literatures. The majority of papers in these literatures focuses on either single-period models where crop rotation benefits are irrelevant, or deterministic multi-period models where revenue uncertainty is absent. As highlighted by Livingston et al. (2015), the few papers which consider both revenue uncertainty and rotation benefits propose heuristic allocation policies and evaluate their performance using numerical experiments. In summary, there is no work that characterizes the optimal dynamic allocation policy under revenue uncertainty in the presence of crop rotation benefits. Therefore, there is also no work that examines the effect of key factors (e.g., revenue variability) on a farmer's optimal allocation policy and profitability. In this paper we attempt to fill this void.

Toward this end, we consider a multi-period optimization problem in which a farmer decides how to allocate the available farmland between two crops in each growing season to maximize the total expected profit over a finite planning horizon. In each season (period) the allocation decision is made with respect to revenue uncertainty of each crop while considering the crop rotation benefits across seasons. We characterize the optimal dynamic farmland allocation policy and answer the following research questions.

(1) How does revenue uncertainty affect the farmer's allocation decision and profitability?

(2) What is the additional value of making crop planning based on multiple crops with rotation benefits, as employed in sustainable agriculture, over continuously growing only one of the crops, as employed in industrial agriculture (USDA 2015a)?

(3) How do the performance of heuristic allocation policies commonly suggested in the extant literature compare to that of the optimal policy? And, is there a simple heuristic allocation policy that can be obtained from our analysis?

In answering these questions when analytical results are not attainable we conduct numerical experiments using realistic instances. To this end, we calibrate our model to represent a farmer growing corn and soybean in Iowa—the largest corn and soybean producing state in the U.S. based on the total acreage planted and harvested in 2014 (USDA 2015b). The model calibration is based on the publicly available data from United States Department of Agriculture (USDA) as complemented by the data obtained from the extant literature. Our main findings can be summarized as follows.

(1) We characterize the optimal dynamic farmland allocation policy and identify two strategies that emerge as a part of the optimal policy: *rotate*, where each crop is only grown

on rotated farmland; and *monoculture*, where only one of the crops is grown on the entire farmland. We provide specific conditions under which each strategy is optimal.

(2) We conduct sensitivity analysis, both analytically (in a special case of our model that limits the planning horizon to two periods) and numerically, to investigate the effects of revenue correlation between the two crops, and revenue volatility of each crop on the farmer's optimal allocation decision and profitability.

Effects on Allocation Decision. One of the general insights from our analytical analysis is that an increase in revenue correlation incents the farmer to increase the allocation of the crop with lower rotation benefits. Based on our model calibration we find that higher revenue correlation incents the farmer growing corn and soybean in Iowa to increase the corn allocation. Another general insight from our analytical analysis is that an increase in revenue volatility of each crop incents the farmer to decrease that crop's farmland allocation. Based on our model calibration we observe this finding to hold for a typical farmer in Iowa. These results provide insights on how to implement crop planning based on principles of sustainable agriculture as a response to changes in revenue uncertainty.

Effects on Profitability. The general insights from our analytical analysis are that the farmer always benefits from a lower revenue correlation but benefits from a lower revenue volatility only when this volatility is low; otherwise, a higher volatility is beneficial. Based on our model calibration we find that corn revenue volatility in Iowa in practice is high so a typical farmer in Iowa always benefits from a higher corn volatility. In contrast, soybean revenue volatility is not as high so the farmer may benefit from a lower soybean volatility. Because a farmer growing only one of the crops over the entire planning horizon, as employed in industrial agriculture, is not impacted by revenue correlation or revenue volatility of each crop, these results provide insights on how revenue uncertainty shapes the value of making crop planning based on principles of sustainable agriculture.

(3) Using our model calibration we find that a farmer growing only one of the crops over the entire planning horizon incurs a considerable profit loss—a minimum profit loss of 9.68% in the numerical instances considered—in comparison with a farmer using the optimal allocation policy. This result indicates that making crop planning based on principles of sustainable agriculture has substantial economic value.

(4) Based on our theoretical analysis we suggest a simple and practically implementable heuristic allocation policy where the periodic allocation decision is made based on a two-

period horizon. We characterize the optimal allocation decision of this policy in closed form. Using our model calibration we find that the proposed policy not only outperforms the commonly suggested heuristic allocation policies in the literature (e.g., rotation-based policy where each crop is planted only on rotated farmland and myopic policy where the allocation decision is made based on a single-period horizon) but also provides a near-optimal performance—a maximum profit loss of 0.13% in the numerical instances considered. These results have important managerial implications. In practice farmers may feel hesitant to make the allocation decision by considering a long-term planning horizon due to complexity of such a decision. Our results demonstrate that making that allocation decision by considering a short-term horizon (specifically, a two-period horizon) does not lead to a significant loss in profit, and our analysis provides a recipe for making that decision.

(5) Using our model calibration we find that, compared to the optimal allocation policy, the proposed heuristic policy has a larger allocation of crops to rotated farmland (for example, an additional 1.71% allocation in the baseline scenario). Because rotating crops improves the soil structure and reduces the need for synthetic chemicals, the proposed heuristic policy is potentially more environmentally friendly than the optimal policy.

The remainder of this paper is organized as follows. §2 surveys the related literature and discusses the contribution of our work. §3 describes the general model and the basis for our assumptions. By focusing on a special case of our model that limits the planning horizon to two periods, §4 derives the optimal allocation policy in closed form and analytically characterizes the effect of revenue uncertainty on the farmer's allocation decision and profitability. §5 extends the characterization of the optimal allocation policy to the general model and provides a practical application in the context of a farmer growing corn and soybean. In particular, using a model calibration that represents a farmer in Iowa we examine the effect of revenue uncertainty and the value of making crop planning based on principles of sustainable agriculture. We also compare the optimal allocation policy's performance with that of heuristic allocation policies. §6 discusses two extensions: i) examining the proportion of farmland allocated to rotated crops under the optimal and the relevant heuristic allocation policies, and ii) introducing fallow farmland. §7 concludes with a discussion of the limitations of our analysis and future research directions.

2 Literature Review

Crop planning problem has received considerable attention from the operations management and agricultural economics literatures. We refer the reader to Glen (1987), Lowe and Preckel (2004) and Ahamuda and Villalobos (2009) for a review of papers that study the crop planning problem under certainty and focus here on papers that incorporate uncertainty. The majority of these papers considers a single-period model where crop rotation benefits are irrelevant and examines the interplay between the crop planning decision and operational features including penalties associated with cash flow variability (Collender and Zilberman 1985), government price support for crops (Alizamir et al. 2017), other government interventions (Kazaz et al. 2016), rainfall uncertainty (Maatman et al. 2002) and management of that uncertainty through irrigation planning (Huh and Lall 2013). Only a few papers in the literature consider crop rotation benefits in crop planning and study the farmland allocation problem under uncertainty in a dynamic setting. The focus of these papers is to propose heuristic allocation policies and evaluate their performance using numerical experiments (see Livingston et al. (2015) for a review). Among these papers Taylor and Burt (1984) consider a farmer's decision of whether to grow wheat or leave the farmland lie fallow in a growing season. They develop a heuristic policy and numerically analyze the policy's performance using a calibration based on a typical wheat farmer in Montana. Considering the farmland allocation between corn and soybean, Cai et al. (2013) numerically compare the performance of different heuristic allocation policies such as growing each crop only on rotated farmland and growing only one crop over the entire planning horizon.

Closest to our work, Livingston et al. (2015) examine the farmland allocation decision between corn and soybean in a multi-period framework considering the crop rotation benefits. They formulate an infinite horizon stochastic dynamic programming model where in each period the farmer chooses which one of the two crops to grow and the amount of fertilizer to use for cultivation facing uncertainties in fertilizer cost and crop revenue. They do not provide a theoretical characterization of the optimal solution and instead numerically analyze the farmer's decisions. Their main conclusion is to suggest that the farmer should implement a rotation-based heuristic allocation policy—that is, grow one of the crops in one season and rotate to the other crop in the subsequent season. In contrast to their work, we do not consider fertilizer application decision or fertilizer cost uncertainty but we extend their model to consider the possibility of growing more than one crop in the same season—a future research direction suggested in their paper. We show that consideration of that possibility is important for a farmer that employs a rotation-based heuristic allocation policy. More importantly, we characterize the optimal dynamic allocation policy based on which we propose a simple heuristic policy. Using our model calibration we show that our proposed policy outperforms the rotation-based allocation policy and provides a near-optimal performance. In addition, we offer insights that are of practical importance to the farmers on how revenue uncertainty of each crop shapes their allocation decisions and profitability.

Our paper is also related to the growing operations management literature that examines operational decisions of supply chain agents in the agricultural sector. The majority of papers in this literature focuses on processors and studies their strategic (e.g., capacity investment) and operating (e.g., procurement and production planning) decisions. These papers consider idiosyncratic features of different agricultural industries including beef (Boyabatlı et al. 2011), citrus fruit (Kazaz and Webster 2011), cocoa (Boyabatlı 2015), corn (Goel and Tanrısever 2017), olive (Kazaz 2004), palm oil (Boyabatlı et al. 2017, Sunar and Plambeck 2016), seed (Jones et al. 2001, Burer et al. 2008), soybean (Devalkar et al. 2011) and wine (Noparumpa et al. 2015). There are also papers that focus on commoditized industries outside of the agricultural sector but their research questions are also relevant in the context of agricultural industries. For example, Chen et al. (2013) examine the processing decision of a semi-conductor manufacturer in a co-production environment where a single input gives rise to multiple outputs. That production environment is also relevant for several agricultural industries including grains and oilseeds. Another example is Plambeck and Taylor (2013) who examine process improvement investment decision of a clean-tech manufacturer that faces input and output price uncertainties. Similar investment decision is also relevant for an agri-processor (e.g., soybean crusher) who faces input (soybean) and output (soybean oil) spot price uncertainties. Dong et al. (2014) study the value of two types of operational flexibility in an oil refinery, range flexibility (the ability to process crude oil of diverse quality) and conversion flexibility (the ability to convert low-quality crude oil to high-quality crude oil). Those two types of operational flexibility are also relevant for a vegetable oil (e.g., palm oil) refinery. Paralleling the papers cited here we use methods and findings from financial engineering—for example, modeling of correlated bivariate uncertainty and its evolution over multiple periods, approximating that stochastic evolution using a lattice approach for numerical computation and insights related to how a financial option's value is affected by its volatility—and apply those in a specific problem domain—that is, crop planning in sustainable agriculture. Some of our analytical sensitivity results—i.e., how revenue uncertainty affects the farmer's profitability—are reminiscent of the sensitivity results in Plambeck and Taylor (2013), Dong et al. (2014) and Boyabatlı et al. (2017) who apply the insights from financial engineering in the context of a clean-tech manufacturer's process improvement decision, an oil refinery's technology investment decision and an oilseed processor's capacity investment decision, respectively. That being said, our sensitivity results are new to the literature on crop planning.

Our work also relates to the literature on sustainable operations. As highlighted by K-leindorfer et al. (2005), the main objective in this literature is to consider environmental (and natural resource) consequences of operational decisions and to help decision makers devise profitable operational practices to enhance environmental quality by, for instance, reducing greenhouse gas emissions (Plambeck 2012), converting waste into a saleable byproduct (Lee 2012) or energy (Ata et al. 2012). In the context of agricultural industries, there is a growing interest in the operations management literature that examines sustainability related issues; see Li et al. (2014) for a recent review. This literature mainly focuses on downstream supply chain (i.e., food retailers) and considers the impact of operating (e.g., procurement, processing, inventory) decisions on food waste—see, for instance, Lee and Tongarlak (2017). Our paper's focus is on the upstream supply chain (i.e., farmers) and we study sustainable way of making crop planning. As highlighted on page 211 of National Research Council (2010), there is vast amount of research in agricultural economics that examines the environmental impact of farming practices such as rotating crops—that is, environmental benefits of making crop planning based on crops with rotation benefits is well-established—yet research that examines the economic impact of those farming practices is limited. Based on this important lacuna in the literature we focus on the economic implications of crop planning decision and examine how it affects farmer's profitability. Using a model calibration we demonstrate that making crop planning based on the principles of sustainable agriculture has substantial economic value.

3 Model Description and Assumptions

The following mathematical representation is used throughout the text: A realization of the random variable $\tilde{y}$ is denoted by y. The expectation operator is denoted by $\mathbb{E}$. Bold face letters represent row vectors of the required size. We use $(u)^+ = \max(u, 0)$ and $(-j) = S \setminus j$ for $j \in S$. All the proofs are relegated to §A of the Online Appendix.

We consider a farmer who allocates his farmland between two crops in each growing season to maximize the total expected profit over a finite number of growing seasons. The farmland acreage is fixed throughout the planning horizon which we normalize to one acre without loss of generality. Though our model is generic, for the concreteness of the exposition we label the two crop choices available for the farmer as corn and soybean. We use superscript c (s) to denote the corn (soybean) related parameters.

In each growing season (time period) t, the farmer allocates $\alpha_t \in [0, 1]$ proportion of the farmland to corn with the remaining $1 - \alpha_t$ proportion allocated to soybean.[2] The allocation decision α_t is made with respect to revenue uncertainty of each crop. Let $\tilde{r}_t^c$ and $\tilde{r}_t^s$ denote the uncertain corn and soybean revenue per acre in period t, respectively. We assume that $\tilde{\boldsymbol{r}}_t = (\tilde{r}_t^c, \tilde{r}_t^s)$ follow correlated stochastic processes with Markovian property; that is, the current revenue realizations are sufficient to characterize the distribution of the future revenues. We make further assumptions about these stochastic processes later in §4.2 to study the effect of revenue uncertainty.

A key feature of the farmland allocation decision is the crop rotation benefits across growing seasons. In particular, the profit from growing a crop on rotated farmland, where the other crop was grown in the previous season, is (stochastically) larger than the profit from growing it on non-rotated farmland, where the same crop was grown in the previous season. As discussed in §1, the crop rotation benefits are attributed to increasing crop revenues and decreasing farming costs. To capture the revenue-enhancing crop rotation benefits, we assume that the uncertain revenue per acre of crop $j \in \{c, s\}$ grown on rotated farmland in period t is $(1 + b^j)\tilde{r}_t^j$ ($b^j \geq 0$), where $\tilde{r}_t^j$, as mentioned above, is the uncertain revenue per acre of the same crop grown on non-rotated farmland. To capture the cost-reducing crop rotation benefits, we assume that the unit farming cost of crop j is ω^j if it is grown on non-rotated farmland, and $(1-\gamma^j)\omega^j$ for $\gamma^j \geq 0$ if it is grown on rotated farmland. When $b^j = 0$ and $\gamma^j = 0$, there is no rotation benefit for crop $j \in \{c, s\}$. Our model can be extended to include period-dependent farming cost ω^j and crop rotation parameters b^j and

[2]In §6.2, we relax the assumption that farmland is fully allocated to corn and soybean in each growing season, and incorporate the farmer's decision to let some portion of the farmland lay fallow to rejuvenate the soil and increase the revenue for the crop grown on this farmland in the subsequent seasons.

γ^j. Because it does not affect the structural analysis, for brevity, we assume that they are fixed throughout the planning horizon. We also assume that rotation benefits carry through only one growing season and the allocation decision α_t in period t is impacted by only the allocation in period $t-1$ and not the earlier periods. One-season crop rotation history assumption is reasonable for corn-soybean rotation as documented in Hennessy (2006).[3]

We formulate the farmer's problem as a finite horizon stochastic dynamic program. In each period $t \in [1, T]$, the sequence of events is as follows: (i) at the beginning of period t, the farmer observes the corn allocation α_{t-1}, and corn and soybean revenues $\boldsymbol{r}_{t-1} = (r^c_{t-1}, r^s_{t-1})$ from period $t-1$ and chooses the corn allocation α_t; (ii) at the end of period t, the corn and soybean revenues $\tilde{\boldsymbol{r}}_t = (\tilde{r}^c_t, \tilde{r}^s_t)$ are realized and the farmer collects the revenues from the crop sales. The farmer's immediate payoff in period $t \in [1, T]$ is given by

$$\begin{aligned} L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r}_{t-1}) \doteq & - \quad (\alpha_t - \gamma^c \min(\alpha_t, 1-\alpha_{t-1}))\,\omega^c - (1-\alpha_t - \gamma^s \min(1-\alpha_t, \alpha_{t-1}))\,\omega^s \\ & + \quad \mathbb{E}_t\left[(\alpha_t + b^c \min(\alpha_t, 1-\alpha_{t-1}))\,\tilde{r}^c_t + (1-\alpha_t + b^s \min(1-\alpha_t, \alpha_{t-1}))\,\tilde{r}^s_t\right], \end{aligned} \tag{1}$$

where $\mathbb{E}_t[.]$ denotes the expectation operator conditional on the available information at time t, i.e., $\mathbb{E}_t[.] = \mathbb{E}[.|\boldsymbol{r}_{t-1}]$. In (1), the first line corresponds to the total farming cost and the second line corresponds to the expected revenues from crop sales in period t. When the farmer decides to allocate α_t proportion of the farmland to corn, to leverage crop rotation benefits, the farmer starts planting corn from the rotated farmland (where soybean was grown in the previous period) which is $1-\alpha_{t-1}$ proportion of the farmland. Therefore, rotation benefits for corn plantation are only relevant for $\min(\alpha_t, 1-\alpha_{t-1})$ proportion of the farmland which yields the revenue $(1+b^c)\tilde{r}^c_t$ with the farming cost $(1-\gamma^c)\omega^c$. Similarly, rotation benefits for soybean plantation are only relevant for $\min(1-\alpha_t, \alpha_{t-1})$ proportion of the farmland which yields the revenue $(1+b^s)\tilde{r}^s_t$ with the farming cost $(1-\gamma^s)\omega^s$.

Let $V_t(\alpha_{t-1}, \boldsymbol{r}_{t-1})$ for $t \in [1, T]$ denote the optimal value function from period t onwards given α_{t-1} and $\boldsymbol{r}_{t-1}$, which satisfies

$$V_t(\alpha_{t-1}, \boldsymbol{r}_{t-1}) = \max_{0 \le \alpha_t \le 1} \Big\{ L(\alpha_t \mid \alpha_{t-1}, \boldsymbol{r}_{t-1}) + \mathbb{E}_t[V_{t+1}(\alpha_t, \tilde{\boldsymbol{r}}_t)] \Big\}, \tag{2}$$

with a boundary condition $V_{T+1}(\cdot) = 0$. The farmer's optimal total expected profit over the entire planning horizon is given by $V_1(\alpha_0, \boldsymbol{r}_0)$, where α_0 and $\boldsymbol{r}_0$ denote the observed corn allocation and crop revenues at the beginning of the planning horizon, respectively.

[3] We relax the one-season rotation history assumption in §C of the Online Appendix, and show that it is not a critical assumption for the characterization of the optimal allocation policy—that is, although the optimal policy is more complex than the optimal policy presented in this paper, it follows the same structure.

4 Two-Period Model

In this section, we limit the planning horizon to two periods, i.e., $T = 2$, and characterize the optimal allocation policy (§4.1) and conduct sensitivity analysis to examine the effects of revenue uncertainty on the farmer's optimal allocation decision and profitability (§4.2). Focusing on a two-period model enables us to analytically characterize the sensitivity results and to provide a closed-form solution for the optimal allocation decision which we use as a heuristic policy for the general T-period problem in §5. We examine how the optimal policy characterization and the sensitivity results extend to the T-period problem in §5.

4.1 Optimal Allocation Policy

We solve the farmer's problem using backward induction. In period 1, the farmer allocates α_1 proportion of the farmland to corn and observes crop revenues $\boldsymbol{r}_1 = (r_1^c, r_1^s)$. In period 2, the farmer chooses the corn allocation α_2. The farmer's optimization problem in this period follows from (2) by substituting $t = 2$ and using the boundary condition $V_3(\cdot) = 0$.

Proposition 1 *In period 2, the optimal corn allocation, α_2^*, is given by*

$$\alpha_2^* = \begin{cases} 0 & \textit{if } -(1-\gamma^c)\omega^c + \mathbb{E}_2\left[(1+b^c)\tilde{r}_2^c\right] \leq -\omega^s + \mathbb{E}_2\left[\tilde{r}_2^s\right], \\ 1 & \textit{if } -\omega^c + \mathbb{E}_2\left[\tilde{r}_2^c\right] \geq -(1-\gamma^s)\omega^s + \mathbb{E}_2\left[(1+b^s)\tilde{r}_2^s\right], \\ 1-\alpha_1 & \textit{otherwise.} \end{cases} \tag{3}$$

The optimal expected profit in this period is $V_2(\alpha_1, \boldsymbol{r}_1) = \alpha_1 K_2^c(\boldsymbol{r}_1) + (1-\alpha_1) K_2^s(\boldsymbol{r}_1)$ where

$$\begin{aligned} K_2^c(\boldsymbol{r}_1) &\doteq \max\left\{-\omega^c + \mathbb{E}_2\left[\tilde{r}_2^c\right], -(1-\gamma^s)\omega^s + \mathbb{E}_2\left[(1+b^s)\tilde{r}_2^s\right]\right\}, \\ K_2^s(\boldsymbol{r}_1) &\doteq \max\left\{-(1-\gamma^c)\omega^c + \mathbb{E}_2\left[(1+b^c)\tilde{r}_2^c\right], -\omega^s + \mathbb{E}_2\left[\tilde{r}_2^s\right]\right\}. \end{aligned} \tag{4}$$

To reap the crop rotation benefits, the farmer has an incentive to grow each crop on rotated farmland. However, it can be profitable to break the rotation and grow *additional* crop $j \in \{c, s\}$ on non-rotated farmland if this crop's profit is expected to be *sufficiently* larger than the other crop's profit. The optimal allocation decision in (3) follows this intuition. In particular, if the expected marginal profit of growing corn on non-rotated farmland is larger than the expected marginal profit of growing soybean on rotated farmland then the farmer only grows corn, i.e., $\alpha_2^* = 1$. Similarly, if the expected marginal profit of growing soybean on non-rotated farmland is larger than the expected marginal profit of growing corn on rotated farmland then the farmer only grows soybean, i.e., $\alpha_2^* = 0$. Otherwise, the farmer optimally grows each crop on rotated farmland, i.e., $\alpha_2^* = 1 - \alpha_1$. The optimal expected profit in period 2 also follows an intuitive structure: it is characterized by the product of the proportion of farmland allocated to crop $j \in \{c, s\}$ in period 1 (α_1 for corn and $1 - \alpha_1$ for soybean) and its expected marginal profit in period 2 ($K_2^j(\boldsymbol{r}_1)$ in (4)). Here, $K_2^j(\boldsymbol{r}_1)$ is given by the maximum profit from two options available to the farmer: growing crop j, which does not involve rotation benefits as it is grown on non-rotated farmland, and growing the other crop, which involves rotation benefits as it is grown on rotated farmland.

We now proceed to characterize the optimal allocation decision in period 1. The farmer's optimization problem in this period follows from (2) by substituting $t = 1$ and using the characterization of $V_2(\alpha_1, \boldsymbol{r}_1)$ as given by Proposition 1.

Proposition 2 *In period 1, the optimal corn allocation, α_1^*, is given by*

$$\alpha_1^* \;=\; \begin{cases} 0 & \textit{if } \; -(1-\gamma^c)\omega^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right] \le -\omega^s + \mathbb{E}_1\left[\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right], \\ 1 & \textit{if } -\omega^c + \mathbb{E}_1\left[\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right] \ge -(1-\gamma^s)\omega^s + \mathbb{E}_1\left[(1+b^s)\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_2)\right], \\ 1-\alpha_0 & \textit{otherwise,} \end{cases} \tag{5}$$

where $K_2^j(\boldsymbol{r}_1)$ for $j \in \{c, s\}$ is as given in (4). The optimal total expected profit over the planning horizon is $V_1(\alpha_0, \boldsymbol{r}_0) = \alpha_0 K_1^c(\boldsymbol{r}_0) + (1-\alpha_0) K_1^s(\boldsymbol{r}_0)$ where

$$\begin{aligned} K_1^c(\boldsymbol{r}_0) \;&\doteq\; \max\left\{-\omega^c + \mathbb{E}_1\left[\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right], -(1-\gamma^s)\omega^s + \mathbb{E}_1\left[(1+b^s)\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right]\right\}, \\ K_1^s(\boldsymbol{r}_0) \;&\doteq\; \max\left\{-(1-\gamma^c)\omega^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c + K_2^c(\tilde{\boldsymbol{r}}_1)\right], -\omega^s + \mathbb{E}_1\left[\tilde{r}_1^s + K_2^s(\tilde{\boldsymbol{r}}_1)\right]\right\}. \end{aligned} \tag{6}$$

The optimal allocation decision in (5) follows the same structure as in Proposition 1: if the expected marginal profit of growing corn (soybean) on non-rotated farmland is larger than the expected marginal profit of growing soybean (corn) on rotated farmland then the farmer only grows corn (soybean), i.e., $\alpha_1^* = 1$ ($\alpha_1^* = 0$); otherwise, the farmer optimally grows each crop on rotated farmland, i.e., $\alpha_1^* = 1 - \alpha_0$. The only difference from Proposition 1 is that the expected marginal profit expressions not only include the profit in period 1 but also the profit in period 2 which is captured by $K_2^j(\boldsymbol{r}_1)$, $j \in \{c, s\}$. For example, the expected marginal profit of growing corn on rotated farmland is given by the sum of the expected corn profit in period 1, i.e., $-(1-\gamma^c)\omega^c + \mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c\right]$, and the expected marginal profit in period 2 obtained from the farmland where corn was grown in period 1, i.e., $\mathbb{E}_1\left[K_2^c(\tilde{\boldsymbol{r}}_1)\right]$.

The characterization of the optimal total expected profit over the entire planning horizon also follows the same structure as Proposition 1. In particular, this profit is given by the product of the proportion of farmland allocated to crop $j \in \{c, s\}$ at the beginning of the planning horizon (α_0 for corn and $1-\alpha_0$ for soybean) and its expected marginal profit over the entire planning horizon ($K_1^j(\boldsymbol{r}_0)$ in (6)). Here, $K_1^j(\boldsymbol{r}_0)$ is given by the maximum profit from two options available to the farmer: (i) growing crop j in period 1 and optimally using the farmland in period 2 (which yields the expected marginal profit $\mathbb{E}_1\left[K_2^j(\tilde{\boldsymbol{r}}_1)\right]$) and (ii) growing the other crop ($-j$) in period 1 and optimally using the farmland in period 2 (which yields the expected marginal profit $\mathbb{E}_1\left[K_2^{(-j)}(\tilde{\boldsymbol{r}}_1)\right]$).

Proposition 2 identifies two strategies that emerge as a part of the optimal allocation policy: *rotate*, where each crop is only grown on rotated farmland, and *monoculture*, where only one of the crops is grown on the entire farmland. This proposition provides specific conditions under which each strategy is optimal. We will later show in §5 that this optimal policy structure extends to a more general T-period model.

4.2 Effects of Revenue Uncertainty

In this section, we examine the effects of revenue uncertainty on the farmer's optimal allocation decision and optimal total expected profit over the planning horizon. To this end, we impose additional structure on our model of the revenue processes. In particular, paralleling Boyabatlı et al. (2017), we use a single-factor, bivariate mean-reverting process to describe the evolution of the corn and soybean revenues. Specifically, corn and soybean revenues at time τ, $\boldsymbol{r}_\tau = (r_\tau^c, r_\tau^s)$, are modeled as

$$\begin{aligned} dr_\tau^c &= \kappa^c(\xi^c - r_\tau^c)d\tau + \sigma^c d\tilde{W}_\tau^c, \\ dr_\tau^s &= \kappa^s(\xi^s - r_\tau^s)d\tau + \sigma^s d\tilde{W}_\tau^s, \end{aligned} \tag{7}$$

where $\kappa^j > 0$ is the mean-reversion parameter, ξ^j is the long-term revenue level and σ^j is the volatility for $j \in \{c, s\}$, whereas $(d\tilde{W}_\tau^c, d\tilde{W}_\tau^s)$ denotes the increment of a standard bivariate Brownian motion with correlation ρ. We assume $\rho > 0$. This is a reasonable assumption for corn and soybean as we empirically demonstrate in §5.2.1. Because the allocation decision is made at discrete time periods $t \in [1, T]$, although the revenue process in (7) evolves on a continuous time τ, we only need to focus on the revenue evolution at these discrete time periods. We assume that τ and t are in the same time units (which we consider to be a year for our model calibration in §5.2.1). This revenue model implies that at period $\hat{t}$ with realized revenues $\boldsymbol{r}_{\hat{t}} = (r_{\hat{t}}^c, r_{\hat{t}}^s)$, the revenues $\tilde{\boldsymbol{r}}_t = (\tilde{r}_t^c, \tilde{r}_t^s)$ at a future period $t > \hat{t}$ follow a bivariate Normal distribution with

$$\begin{aligned}
\mathbb{E}[\tilde{r}_t^j|\boldsymbol{r}_{\hat{t}}] &= e^{-\kappa^j(t-\hat{t})}r_{\hat{t}}^j + \left(1-e^{-\kappa^j(t-\hat{t})}\right)\xi^j, \\
\mathrm{VAR}[\tilde{r}_t^j|\boldsymbol{r}_{\hat{t}}] &= \left(\frac{1-e^{-2\kappa^j(t-\hat{t})}}{2\kappa^j}\right)(\sigma^j)^2, \\
\mathrm{COV}[\tilde{r}_t^c,\tilde{r}_t^s|\boldsymbol{r}_{\hat{t}}] &= \left(\frac{1-e^{-(\kappa^c+\kappa^s)(t-\hat{t})}}{\kappa^c+\kappa^s}\right)\rho\sigma^c\sigma^s,
\end{aligned} \tag{8}$$

where VAR and COV denote variance and covariance, respectively.

We conduct sensitivity analyses to study the effects of revenue correlation ρ and revenue volatility σ^j of crop $j \in \{c,s\}$. In particular, we examine the effects on $V_1(\alpha_0,\boldsymbol{r}_0)$, the optimal total expected profit over the planning horizon, and on α_1^*, the first period's optimal allocation decision.[4] To this end, we first provide the following result which enables us to provide a closed-form characterization of α_1^* and $V_1(\alpha_0,\boldsymbol{r}_0)$.

Proposition 3 *Under the bivariate Normal distribution specified in (8),* $V_1(\alpha_0,\boldsymbol{r}_0)$ *and* α_1^* *in Proposition 2 can be characterized in closed form by using* $K_2^j(\boldsymbol{r}_1) = \max\{\underline{u}r_1^j + \underline{v}, \overline{u}r_1^{(-j)} + \overline{v}\}$ *for* $j \in \{c,s\}$ *where* $\underline{u} = e^{-\kappa^j}$, $\underline{v} = \left(1-e^{-\kappa^j}\right)\xi^j - \omega^j$, $\overline{u} = (1+b^{(-j)})e^{-\kappa^{(-j)}}$, $\overline{v} = (1+b^{(-j)})\left(1-e^{-\kappa^{(-j)}}\right)\xi^{(-j)} - (1-\gamma^{(-j)})\omega^{(-j)}$ *and the following identity*

$$\begin{aligned}
\mathbb{E}_1[\max\{\underline{u}\tilde{r}_1^j + \underline{v}, \overline{u}\tilde{r}_1^{(-j)} + \overline{v}\}] &= (\underline{u}\mu_1^j + \underline{v})\Phi\left(\frac{\underline{u}\mu_1^j + \underline{v} - \overline{u}\mu_1^{(-j)} - \overline{v}}{\lambda}\right) \\
&+ (\overline{u}\mu_1^{(-j)} + \overline{v})\Phi\left(\frac{\overline{u}\mu_1^{(-j)} + \overline{v} - \underline{u}\mu_1^j - \underline{v}}{\lambda}\right) + \lambda\phi\left(\frac{\overline{u}\mu_1^{(-j)} + \overline{v} - \underline{u}\mu_1^j - \underline{v}}{\lambda}\right),
\end{aligned} \tag{9}$$

where $\Phi(.)$ *and* $\phi(.)$ *denote the cumulative distribution function and the probability density function of the standard Normal distribution respectively,* $\lambda \doteq \sqrt{\underline{u}^2\, VAR_1^j + \overline{u}^2\, VAR_1^{(-j)} - 2\underline{u}\overline{u}COV_1}$, *and* $\mu_1^j \doteq \mathbb{E}[\tilde{r}_1^j|\boldsymbol{r}_0]$, $VAR_1^j \doteq VAR[\tilde{r}_1^j|\boldsymbol{r}_0]$, *and* $COV_1 \doteq COV[\tilde{r}_1^c,\tilde{r}_1^s|\boldsymbol{r}_0]$ *follow from (8) with* $t=1$ *and* $\hat{t}=0$.

The key observation from Propositions 2 and 3 is that revenue correlation and revenue volatility of each crop affect both the farmer's profitability and the first period's optimal allocation decision through their impacts on the expected marginal profit from optimally using the farmland in period 2, i.e., $\mathbb{E}_1\left[K_2^j(\tilde{\boldsymbol{r}}_1)\right]$ for $j \in \{c,s\}$. As discussed in §4.1, this expected marginal profit is given by the maximum profit from two options available to the

[4] As seen from Proposition 1, the second period's optimal allocation decision α_2^* depends on $\mathbb{E}[\tilde{r}_2^j|\boldsymbol{r}_1] = e^{-\kappa^j}r_1^j + \left(1-e^{-\kappa^j}\right)\xi^j$ for $j \in \{c,s\}$, and thus, it is not impacted by ρ or σ^j.

farmer, growing corn and growing soybean. We will use this observation to explain our sensitivity results as discussed next.

Effects on the Farmer's Profitability.[5] We first start with the effect of revenue correlation. As proven in Proposition 4, the farmer benefits from a lower revenue correlation:

Proposition 4 (Revenue correlation ρ) $\frac{\partial V_1(\alpha_0, \boldsymbol{r}_0)}{\partial \rho} < 0$.

With a higher ρ, there will be a higher likelihood that when the profit from growing one of the crops is low, the profit from growing the other crop will be low. Therefore, as ρ increases the maximum profit from these two options, and thus the optimal expected profit over the planning horizon decreases.

In contrast to the correlation effect, the farmer benefits from a lower revenue volatility of each crop only when this volatility is low; otherwise, a higher volatility is beneficial:

Proposition 5 (Revenue volatility σ^j) *For $j \in \{c, s\}$, there exists a unique $\widehat{\sigma}^j$ such that $\frac{\partial V_1(\alpha_0, \boldsymbol{r}_0)}{\partial \sigma^j} \leq 0$ if $\sigma^j \leq \widehat{\sigma}^j$; and $\frac{\partial V_1(\alpha_0, \boldsymbol{r}_0)}{\partial \sigma^j} \geq 0$ if $\sigma^j \geq \widehat{\sigma}^j$.*

The intuition behind Proposition 5 follows from how the expected maximum profit from two options, growing corn or soybean, changes in σ^j. Although the actual bivariate corn and soybean profit distribution is more complex (as follows from (8)), for illustrative purposes, we will focus on the special case where profits have the same mean. Because $\rho > 0$, high (low) corn profit realizations are more likely to occur with high (low) soybean profit realizations. Changes in σ^j alter the standard deviation of crop j's profit distribution. In particular, with a higher σ^j, there will be a higher likelihood of observing low and high profit from crop j. Consider first the case where σ^j is small. In this case, the other crop's profit distribution has a larger spread than crop j's profit distribution around the same mean. When σ^j increases, while a higher likelihood of observing low profit from crop j decreases the maximum profit from two options, a higher likelihood of observing high profit from crop j does not increase the maximum profit from two options. The latter argument follows because the other crop's profit, owing to the larger spread of its distribution, exceeds the profit from crop j and determines the maximum profit from two options. Therefore, when σ^j is small, a higher σ^j decreases the optimal expected profit over the planning horizon. This result is reversed

[5] Our sensitivity results in this section are reminiscent of the sensitivity results in the financial engineering literature that are related to the value of compound exchange options. We refer the reader to Carr (1988) for a description of compound exchange options and related sensitivity results.

when σ^j is large. In that case, the other crop's profit distribution has a smaller spread than crop j's profit distribution. As σ^j increases while a higher likelihood of observing high profit from crop j increases the maximum profit from two options, a higher likelihood of observing low profit from crop j is less consequential. The latter argument follows because the other crop's profit, owing to the smaller spread of its distribution, exceeds the profit from crop j and determines the maximum profit from two options.

Effects on the Farmer's Allocation Decision. It follows from (5) in Proposition 2 that the first period's optimal corn allocation is given by

$$\alpha_1^* \quad = \quad \begin{cases} 0 & \text{if } \Gamma \leq \underline{\Gamma} \doteq \mathbb{E}_1\left[\tilde{r}_1^s\right] - \omega^s - (\mathbb{E}_1\left[(1+b^c)\tilde{r}_1^c\right] - (1-\gamma^c)\omega^c), \\ 1-\alpha_0 & \text{if } \underline{\Gamma} < \Gamma < \overline{\Gamma}, \\ 1 & \text{if } \Gamma \geq \overline{\Gamma} \doteq \mathbb{E}_1\left[(1+b^s)\tilde{r}_1^s\right] - (1-\gamma^s)\omega^s - (\mathbb{E}_1\left[\tilde{r}_1^c\right] - \omega^c), \end{cases} \tag{10}$$

where $\Gamma \doteq \mathbb{E}_1\left[K_2^c(\tilde{\boldsymbol{r}}_1)\right] - \mathbb{E}_1\left[K_2^s(\tilde{\boldsymbol{r}}_1)\right]$. In (10), revenue correlation or volatility of each crop affects α_1^* through their effects on Γ ($\underline{\Gamma}$ and $\overline{\Gamma}$ are not affected). An increase (a decrease) in Γ incents the farmer to increase (decrease) the corn allocation, and thus decrease (increase) the soybean allocation.

For the effect of ρ on Γ, because an increase in ρ decreases both $\mathbb{E}_1\left[K_2^c(\tilde{\boldsymbol{r}}_1)\right]$ and $\mathbb{E}_1\left[K_2^s(\tilde{\boldsymbol{r}}_1)\right]$, characterizing this effect, in general, is not tractable. However, Proposition 6 characterizes this effect in the special case where crops have symmetric parameters except for their cost-reducing rotation benefits.

Proposition 6 (Revenue correlation ρ) *Assume symmetric distribution for $\tilde{\boldsymbol{r}}_1 = (\tilde{r}_1^c, \tilde{r}_1^s)$, i.e., $\kappa^c = \kappa^s$, $\xi^c = \xi^s$, $\sigma^c = \sigma^s$ and $r_0^c = r_0^s$, and let $b^c = b^s$ and $\omega^c = \omega^s$. In this case, $\frac{\partial \Gamma}{\partial \rho} > 0$ if $\gamma^c < \gamma^s$ and $\frac{\partial \Gamma}{\partial \rho} < 0$ if $\gamma^c > \gamma^s$.*

The general insight from Proposition 6 is that an increase in revenue correlation incents the farmer to increase the allocation of the crop with lower rotation benefits.

For the effect of σ^j for $j \in \{c, s\}$ on Γ, because an increase in σ^j decreases (increases) both $\mathbb{E}_1\left[K_2^c(\tilde{\boldsymbol{r}}_1)\right]$ and $\mathbb{E}_1\left[K_2^s(\tilde{\boldsymbol{r}}_1)\right]$ for low (high) volatility levels, this effect can only be characterized for moderate volatility levels:

Proposition 7 (Revenue volatility σ^j) *Let $\underline{\sigma}^j \doteq \frac{U^j}{(1+b^j)}$ and $\overline{\sigma}^j \doteq (1+b^j)U^j$ for $j \in \{c, s\}$ where $U^j \doteq \frac{e^{-\kappa^{(-j)}}}{e^{-\kappa^j}} \left(\frac{1-e^{-(\kappa^c+\kappa^s)}}{1-e^{-2\kappa^j}}\right) \left(\frac{2\kappa^j}{\kappa^c+\kappa^s}\right) \rho\sigma^{(-j)}$. For $\sigma^c \in [\underline{\sigma}^c, \overline{\sigma}^c]$, $\frac{\partial \Gamma}{\partial \sigma^c} < 0$ and for $\sigma^s \in [\underline{\sigma}^s, \overline{\sigma}^s]$, $\frac{\partial \Gamma}{\partial \sigma^s} > 0$.*

The general insight from Proposition 7 is that an increase in revenue volatility of each crop incents the farmer to decrease that crop's farmland allocation.

5 T-Period Model and Its Application in Crop Planning with Corn and Soybean

In this section, we extend our analysis in §4 that focuses on a two-period problem to a general T-period problem. In §5.1, we demonstrate that the optimal allocation policy as characterized in §4.1 continues to hold for the T-period problem. In §5.2, we conduct numerical experiments by calibrating our model parameters to represent a typical farmer growing corn and soybean.

5.1 Optimal Allocation Policy

We now solve for the farmer's optimization problem stated in (2) and characterize the optimal allocation decision and the optimal value function in period $t \in [1, T]$.

Proposition 8 *In period $t \in [1, T]$, the optimal corn allocation, α_t^*, is given by*

$$\alpha_t^* = \begin{cases} 0 & \text{if } -(1-\gamma^c)\omega^c + \mathbb{E}_t[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)] \leq -\omega^s + \mathbb{E}_t[\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)], \\ 1 & \text{if } -\omega^c + \mathbb{E}_t[\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)] \geq -(1-\gamma^s)\omega^s + \mathbb{E}_t[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)], \\ 1 - \alpha_{t-1} & \text{otherwise,} \end{cases} \tag{11}$$

where

$$\begin{aligned} K_t^c(\boldsymbol{r}_{t-1}) &\doteq \max\left\{-\omega^c + \mathbb{E}_t\left[\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], -(1-\gamma^s)\omega^s + \mathbb{E}_t\left[(1+b^s)\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right]\right\}, \quad (12) \\ K_t^s(\boldsymbol{r}_{t-1}) &\doteq \max\left\{-(1-\gamma^c)\omega^c + \mathbb{E}_t\left[(1+b^c)\tilde{r}_t^c + K_{t+1}^c(\tilde{\boldsymbol{r}}_t)\right], -\omega^s + \mathbb{E}_t\left[\tilde{r}_t^s + K_{t+1}^s(\tilde{\boldsymbol{r}}_t)\right]\right\}, \end{aligned}$$

with $K_{T+1}^j(\boldsymbol{r}_T) = 0$ for $j \in \{c, s\}$. The optimal value function from period t onwards is

$$V_t(\alpha_{t-1}, \boldsymbol{r}_{t-1}) = \alpha_{t-1}K_t^c(\boldsymbol{r}_{t-1}) + (1-\alpha_{t-1})K_t^s(\boldsymbol{r}_{t-1}). \tag{13}$$

The characterizations of the optimal allocation decision and the optimal value function follow the same structure as the characterizations in Proposition 2 except that the recursive operators $K_1^j(\boldsymbol{r}_0)$ for $j \in \{c, s\}$ are generalized to suit the T-period model. In particular, $K_t^j(\boldsymbol{r}_{t-1})$ denotes the expected marginal profit of farmland in the remaining planning horizon (from period t onwards) where crop j was grown in period $t-1$. It is easy to establish that Proposition 8 is identical to Proposition 2 when $t = T-1$ and it is identical to Proposition 1 when $t = T$ if the planning horizon is limited to two periods, i.e., $T = 2$.

Unlike the two-period problem, the optimal policy and the optimal value function for a general T-period problem cannot be characterized in closed form under the specific revenue processes defined in (7) because $K_t^j(\boldsymbol{r}_{t-1})$ in (12) cannot be characterized in closed form. Therefore, we resort to numerical experiments for further analysis.

5.2 Application in Crop Planning with Corn and Soybean

In this section, we provide a practical application in the context of a farmer growing corn and soybean and calibrate our model parameters to represent a typical farmer in Iowa. We describe the data and calibration used for our numerical experiments in §5.2.1. Using these experiments we examine the effect of revenue uncertainty on the farmer's profitability and the optimal allocation decision (§5.2.2), the value of making crop planning based on principles of sustainable agriculture (§5.2.3) and the performance of a variety of heuristic allocation policies in comparison with the optimal policy (§5.2.4).

5.2.1 Data, Model Calibration and Computation for Numerical Experiments

Our numerical experiments use publicly available data from USDA complemented by data reported in other academic studies.

Calibration for Revenue Process Parameters. The revenue per acre of each crop (\$/acre) is determined by the product of its yield (bushel/acre) and sales price (\$/bushel). Because a time period in our model corresponds to a year and because corn and soybean are mostly harvested in October in Iowa, we use the annual yield data and October price data for each crop between 1960 and 2013 as reported by USDA for farmlands in Iowa. To obtain the real values for prices, we adjust the (nominal) data based on the consumer price index (available at United States Department of Labor) with a base year of 2000. For each year, we multiply the price and yield to obtain the revenue per acre of each crop, as plotted in Figure 1. The obtained revenue data are weighted average of the revenue from rotated and non-rotated farmlands. In particular, the observed revenue corresponds to $r_t^j[(1+b^j)\times\varphi_t^j+(1-\varphi_t^j)]$ where φ_t^j is the fraction of rotated farmland in year t of crop $j\in\{c,s\}$. To calibrate the revenue process parameters, we need to adjust the data for crop rotation benefits and obtain the (raw) revenue observation r_t^j. To do so, we use the values of φ_t^c and φ_t^s reported in Livingston et al. (2015) for Iowa in the years 1981-1982, 1986-1987, 1991-1992 and 1996-2007.[6] For missing years, we use the average of the data available and obtain $\varphi_t^c=0.77$ and $\varphi_t^s=0.93$. To obtain the values of b^c and b^s we refer to Cai et al. (2013) who report the yield of corn and soybean on non-rotated farmland as 92.2% and 85.5% of the yield on rotated farmland, respectively. Based on these values we obtain $\hat{b}^c=1/0.922-1\cong 0.08$ and $\hat{b}^s=1/0.855-1\cong 0.17$. Using φ_t^j and $\hat{b}^j$, we calculate

[6] The available data for φ_t^c and φ_t^s range from 0.59 to 0.88 and from 0.76 to 0.98, respectively.

the revenue observations r_t^j from the data plotted in Figure 1.

To describe the evolution of corn and soybean revenues, we continue to use the single-factor bivarite mean-reverting process as specified in (7). According to this specification, as also highlighted in Boyabatlı et al. (2017), the yearly revenues evolve as

$$\begin{aligned} \tilde{r}_t^c &= e^{-\kappa^c} r_{t-1}^c + (1-e^{-\kappa^c})\xi^c + \sigma^c \sqrt{\frac{1-e^{-2\kappa^c}}{2\kappa^c}}\tilde{z}^c, \\ \tilde{r}_t^s &= e^{-\kappa^s} r_{t-1}^s + (1-e^{-\kappa^s})\xi^s + \sigma^s \sqrt{\frac{1-e^{-2\kappa^s}}{2\kappa^s}}\tilde{z}^s, \end{aligned} \tag{14}$$

where $(\tilde{z}^c, \tilde{z}^s)$ follows a standard bivariate Normal distribution with correlation ρ. The equations in (14) are a system of simultaneous equations of $(\tilde{r}_t^c, \tilde{r}_t^s)$ on (r_{t-1}^c, r_{t-1}^s) with the form $\tilde{r}_t^j = \vartheta^j r_{t-1}^j + \eta^j + \tilde{\epsilon}^j$ for $j \in \{c, s\}$. Because the error terms $(\tilde{\epsilon}^c, \tilde{\epsilon}^s)$ are correlated, we use the Seemingly Unrelated Regression (henceforth SUR, see Zellner 1962) to estimate ϑ^j, η^j and the covariance matrix of $(\tilde{\epsilon}^c, \tilde{\epsilon}^s)$. Based on these estimates, using (14), we obtain $\hat{\kappa}^c = 0.33$, $\hat{\xi}^c = 439.07$, $\hat{\sigma}^c = 108.22$, $\hat{\kappa}^s = 0.35$, $\hat{\xi}^s = 328.64$, $\hat{\sigma}^s = 79.69$ and $\hat{\rho} = 0.73$. The root mean-squared errors are 92.49 for corn and 67.64 for soybean, the mean absolute percentage errors are 17% for corn and 16.45% for soybean, the adjusted R^2 of the individual regression equations are 60.89% for corn and 62.66% for soybean, and the (system-wide) R^2 of the SUR is 50.66%.

Calibration for other operational parameters. We calibrate the (variable) farming cost ω^j and the cost-reducing crop rotation benefit γ^j for crop $j \in \{c, s\}$ using the data presented in Iowa State University extension and outreach report[7]. Similar to this report, we assume that the variable farming cost is characterized by the cost of seeds, chemicals (fertilizers and herbicides), and the variable cost of machinery. The report presents each of these cost components between 1994 and 2013 for a typical farmer in Iowa that grows corn following corn, corn following soybean and soybean following corn. Because there is no cost-reducing rotation benefit of soybean reported in the literature, we assume the variable cost for the farmer growing soybean after soybean (which is not reported in our data source) is the same as the variable cost for the farmer growing soybean following corn, i.e., $\hat{\gamma}^s = 0$. To obtain the real values for the variable cost, we adjust the (nominal) data based on the consumer price index with a base year of 2000. To estimate the farming cost ω^c and ω^s, we take the average of the cost data for corn following corn and soybean following soybean, and obtain $\hat{\omega}^c$ = \$251.61/acre and $\hat{\omega}^s$ = \$122.15/acre. To estimate

[7]http://www.extension.iastate.edu/agdm/cdcostsreturns.html

the cost-reducing crop rotation benefit for corn, we compare the average cost for corn following corn (\$251.61/acre) and corn following soybean (\$225.68/acre), and obtain $\hat{\gamma}^c = 1 - 225.68/251.61 = 0.103 \cong 0.10$. For the initial corn allocation α_0, we use the total farmland in Iowa where corn (13,600,000 acres) and soybean (9,950,000 acres) were grown in 2014 (USDA 2015b). We estimate α_0 by using the fraction of the total farmland allocated to corn and obtain $\hat{\alpha}_0 = 0.58$. The planning horizon is set to be 10 years, i.e., $\hat{T} = 10$.

Numerical Computation. For numerical computation, we follow the standard procedure in the literature and discretize the continuous revenue process in (7) to a lattice. In particular, we represent the stochastic evolution of corn and soybean revenues as a two-dimensional trinomial recombinant lattice based on the method presented in Tseng and Lin (2007; see §3.2). In this lattice each period t is discretized into δ time steps, and we set $\delta = 12$, i.e., each time step corresponds to a month in practice. Each lattice node (in a given time step) can transit to 3×3 nodes (in the subsequent time step) each of which is defined by the two-dimensional values (that correspond to corn and soybean revenues) and a particular transition probability. The two-dimensional values and the transition probabilities are computed based on the formulas given in Tseng and Lin (2007). The continuous distribution of corn and soybean revenues at period t is then approximated by a discrete distribution represented by a set of lattice nodes at period t with a value and a probability for each node. As discussed in Tseng and Lin (2007), this discrete distribution asymptotically converges to the exact continuous distribution when $\delta \to \infty$. To evaluate the optimal policy and the optimal value function in period $t \in [1, T]$, as given by Proposition 8, we compute recursively $K_t^j(\mathbf{r}_{t-1})$ for $j \in \{c, s\}$ in (12) for any given $\mathbf{r}_{t-1}$ on the lattice.[8]

Baseline Scenario. We obtain the optimal total expected profit over the planning horizon for the baseline scenario as \$2543.0/acre, i.e., an annual value of \$254.30/acre (with a base year of 2000), which is \$352.41/acre in 2015 dollars. This is comparable with the values reported by Economic Research Services (2015): an average farmer in the U.S. growing corn earns \$364.91/acre and \$246.26/acre in 2013 and 2014, respectively, which is \$373.81/acre and \$248.24/acre, respectively, in 2015 dollars; an average farmer in the U.S.

[8] Another way of conducting the numerical experiments is to use the simulation approach—that is, simulating multiple sample paths, computing the performance measure along each sample path and averaging across all the sample paths—following the work of Devalkar et al. (2011). We replicated our numerical analysis in this section using the simulation approach and verified that our main insights do not change. We refer the reader to §D of the Online Appendix for the details of this analysis.

planting soybean earns \$359.92/acre and \$317.66/acre in 2013 and 2014, respectively, which is \$368.70/acre and \$320.21/acre, respectively, in 2015 dollars.

5.2.2 Effects of Revenue Uncertainty

In this section, we numerically examine the effects of revenue correlation between the two crops and revenue volatility of each crop on the farmer's optimal total expected profit over the planning horizon and the optimal first-period allocation decision. We compare our numerical results with the analytical results obtained from the two-period model in §4.2.

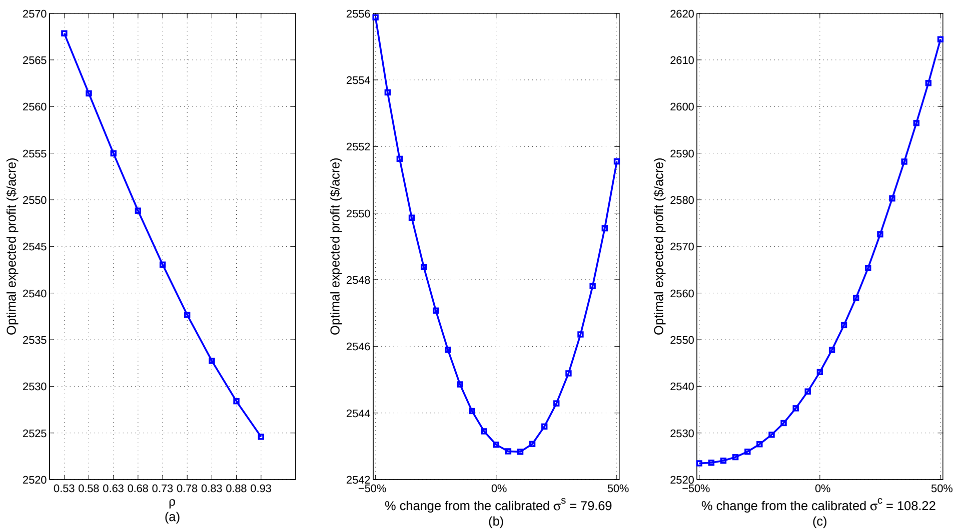

Figure 2: Effects of revenue correlation ρ (Panel a), soybean revenue volatility σ^s (Panel b) and corn revenue volatility σ^c (Panel c) on the farmer's optimal total expected profit over the planning horizon in the baseline scenario. In Panel a, $\rho \in [0.53, 0.93]$ evenly-spaced around the baseline value $\hat{\rho} = 0.73$ with a step size of 0.05 whereas in Panel b (Panel c) $\sigma^s(\sigma^c) \in [-50\%, 50\%]$ of the baseline value $\hat{\sigma}^s = 79.69$ ($\hat{\sigma}^c = 108.22$) with 5% increments.

Effects on the Farmer's Profitability. As illustrated in Panel a of Figure 2, paralleling Proposition 4, the optimal total expected profit decreases in ρ, i.e., a typical farmer growing corn and soybean in Iowa benefits from a lower correlation between corn and soybean revenues. Panel b illustrates that, paralleling Proposition 5, the optimal total expected profit decreases (increases) in soybean volatility when this volatility is low (high). In contrast, we observe in Panel c that the optimal total expected profit always increases in corn volatility σ^c. When lower σ^c values—beyond -50% of the baseline value—are considered (not reported here), paralleling Proposition 5, we observe that the total expected profit first decreases then increases in σ^c. However, for realistic values of σ^c only increasing behavior is relevant. In summary, while the farmer in Iowa always benefits from a higher corn volatility, a higher soybean revenue volatility is beneficial only when this volatility is sufficiently high; otherwise a lower soybean volatility is beneficial.

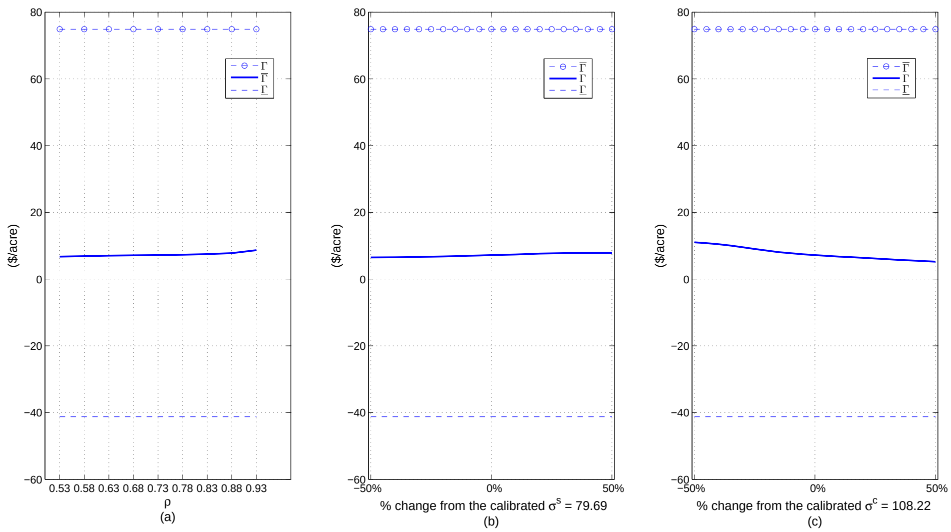

Figure 3: Effects of revenue correlation ρ (Panel a), soybean revenue volatility σ^s (Panel b) and corn revenue volatility σ^c (Panel c) on the first period's optimal corn allocation α_1^* in the baseline scenario. Here, $\alpha_1^* = 0$ if $\Gamma \leq \underline{\Gamma}$, $\alpha_1^* = 1 - \alpha_0$ if $\underline{\Gamma} < \Gamma < \overline{\Gamma}$, and $\alpha_1^* = 1$ if $\Gamma \geq \overline{\Gamma}$. The expressions for Γ, $\underline{\Gamma}$ and $\overline{\Gamma}$ are the same as those in (10) defined for the two-period model except for $K_2^j(\tilde{\boldsymbol{r}}_1)$ for $j \in \{c, s\}$ in Γ is modified to suit the T-period model.

Effects on the Farmer's Allocation Decision. To be able to make comparisons with our analytical results in §4.2, we examine the effects on the first period's optimal corn allocation α_1^*. Recall from (10) that ρ or σ^j for $j \in \{c, s\}$ affects α_1^* through their effects on $\Gamma = \mathbb{E}_1 [K_2^c(\tilde{\boldsymbol{r}}_1)] - \mathbb{E}_1 [K_2^s(\tilde{\boldsymbol{r}}_1)]$; an increase (a decrease) in Γ incents the farmer to increase (decrease) the corn allocation. Figure 3 illustrates these effects around the baseline scenario. We observe that although changes in ρ and σ^j for $j \in \{c, s\}$ affect Γ, the effects are not sufficient to alter α_1^*; in all reported instances across three panels we have $\underline{\Gamma} < \Gamma < \overline{\Gamma}$, and thus the farmer optimally rotates in the first period, i.e., $\alpha_1^* = 1 - \alpha_0$.

For the ρ effect, our general insight from Proposition 6 was that an increase in revenue correlation incents the farmer to increase the allocation of the crop with lower rotation benefits. In our baseline scenario, corn has higher cost-reducing rotation benefits ($\hat{\gamma}^c > \hat{\gamma}^s$)

but lower revenue-enhancing rotation benefits ($\hat{b}^c < \hat{b}^s$). Therefore, it is not clear which crop has higher rotation benefits. For a typical farmer in Iowa, Panel a of Figure 3 illustrates that higher ρ incents the farmer to increase corn allocation. For the σ^j effect, as observed from Panels b and c, higher σ^s increases Γ whereas higher σ^c decreases it. These observations are consistent with our general insight from Proposition 7; that is, an increase in revenue volatility of each crop incents the farmer to decrease that crop's farmland allocation.

5.2.3 Value of Sustainable Crop Planning

We now examine the value of making crop planning based on multiple crops with rotation benefits, as employed in sustainable agriculture, in comparison with continuously growing only one of the crops, as employed in industrial agriculture. To this end, we consider the benchmark case in which the farmer grows the same crop over the entire planning horizon. We define the profit loss due to continuously growing the same crop as

$$\Delta \doteq \left[\frac{V_1(\alpha_0, \boldsymbol{r}_0) - V_1^B(\alpha_0, \boldsymbol{r}_0)}{V_1(\alpha_0, \boldsymbol{r}_0)}\right],$$

where $V_1(\alpha_0, \boldsymbol{r}_0)$ is the optimal total expected profit over the planning horizon and $V_1^B(\alpha_0, \boldsymbol{r}_0)$ denotes the expected profit in the benchmark case. In the benchmark case the farmer has two options, growing corn or growing soybean over the entire planning horizon. Here, $V_1^B(\alpha_0, \boldsymbol{r}_0)$ denotes the maximum expected profit from these two options.

We numerically compute the percentage profit loss $\Delta * 100$. To this end, we extend our numerical instances around the baseline scenario to consider sensitivity of our result-s based on several key parameters. In particular, we consider revenue correlation $\rho = \{0.53, 0.63, 0.73, 0.83, 0.93\}$, evenly-spaced around the baseline value 0.73; we consider corn (soybean) volatility σ^c (σ^s) that are $\{-50\%, -25\%, 0\%, 25\%, 50\%\}$ of their baseline values. We also consider yield-enhancing rotation benefit b^j for crop $j \in \{c, s\}$ and cost-reducing rotation benefit γ^c for corn that are $\{-50\%, -25\%, 0\%, 25\%, 50\%\}$ of their baseline values (we continue to assume $\gamma^s = 0$). We set initial corn allocation $\alpha_0 \in \{0.38, 0.48, 0.58, 0.68, 0.78\}$, evenly-spaced around the baseline value 0.58. We also use different planning horizons, specifically $T \in \{5, 10, 15, 20\}$. In summary, we consider 312,500 numerical instances.

We find that the average profit loss in the numerical instances considered is 18.67% with a minimum and a maximum loss of 9.68% and 27.12%, respectively. This result indicates that *making crop planning based on principles of sustainable agriculture has substantial economic value.* We note here that our analysis (implicitly) normalizes the additional costs associated with making crop planning based on multiple crops instead of a single crop to zero. In practice, these additional costs may exist owing to, for example, different set of management skills, labor, or harvesting machinery required for each crop. Nevertheless, our analysis provides the benefit of growing multiple crops instead of a single crop which can later be compared with these additional costs.

5.2.4 Analysis of Heuristic Allocation Policies

In this section, we study the performance of a variety of heuristic allocation policies in comparison with the optimal policy. To this end, we numerically compute the percentage profit loss $\Delta^H * 100$ due to employing heuristic policy (H). Here, Δ^H is defined as

$$\Delta^H \doteq \left[\frac{V_1(\alpha_0, \boldsymbol{r}_0) - V_1^H(\alpha_0, \boldsymbol{r}_0)}{V_1(\alpha_0, \boldsymbol{r}_0)}\right],$$

where $V_1(\alpha_0, \boldsymbol{r}_0)$ is the optimal total expected profit over the planning horizon and $V_1^H(\alpha_0, \boldsymbol{r}_0)$ denotes the expected profit under the heuristic allocation policy. We use the same 312,500 numerical instances as in §5.2.3.

We restrict our attention to heuristic policies in which the periodic allocation decision, as denoted by α_t^H for $t \in \{1, T\}$, can be characterized in closed form, and thus are easily implementable in practice. In particular, we consider the following heuristic policies:

Always Rotate. Under this policy the farmer grows each crop only on rotated farmland in each period, i.e., $\alpha_t^H = 1 - \alpha_{t-1} \in \{\alpha_0, 1 - \alpha_0\}$ for $t \in [1, T]$.

Always Rotate (Monoculture). Under this policy the farmer grows only one of the crops in each period and rotates to the other crop in the subsequent period. This is a commonly suggested heuristic policy in the literature (see, for example, Livingston et al. 2015). We consider two heuristics based on the crop choice in the first period: The farmer first grows corn, i.e., $\alpha_1^H = 1$, or the farmer first grows soybean, i.e., $\alpha_1^H = 0$. In both cases the allocation in the rest of the planning horizon ($t \in [2, T]$) is given by $\alpha_t^H = 1 - \alpha_{t-1} \in \{0, 1\}$. In each numerical instance, we only report the better performing heuristic—that is, $V_1^H(\alpha_0, \boldsymbol{r}_0)$ denotes the higher expected profit of the two heuristics.

Myopic. Under this policy the farmer chooses the allocation in each period ignoring the cash flows from future periods. The optimal allocation in period $t \in [1, T]$ under the myopic policy can be obtained from (3) of Proposition 1 by substituting $t = 2$ with an arbitrary t, and using the identity $\mathbb{E}_t[\tilde{r}_t^j] = e^{-\kappa^j} r_{t-1}^j + \left(1 - e^{-\kappa^j}\right) \xi^j$ for $j \in \{c, s\}$.

	Always Rotate	Always Rotate (Monoculture)	Myopic	One-period Lookahead
Average	1.13%	1.85%	0.80%	0.03%
Min	0.23%	0.60%	0.17%	0.00%
Max	3.83%	4.09%	2.20%	0.13%

Table 1: Performance of heuristic allocation policies. For each heuristic (H), "Average" denotes the average percentage profit loss ($\Delta^H * 100$), whereas "Min" and "Max" denote the minimum and the maximum percentage profit loss observed in all numerical instances.

Proposed Policy: One-period Lookahead. Under this policy the farmer chooses the allocation in period t based on a two-period horizon—that is, by considering the future cash flows only from the subsequent period $t+1$. The optimal allocation in period $t \in [1, T]$ under this policy can be obtained from (5) of Proposition 2 by substituting $t = 1$ ($t-1 = 0$, $t+1 = 2$) with an arbitrary t ($t-1$, $t+1$), and using the identities given in Proposition 3. We note that the closed-form characterization of the optimal periodic allocation decision under this policy is only made possible by our theoretical analysis in §4.

Table 1 summarizes the average, minimum and maximum percentage profit losses $\Delta^H * 100$ observed under each heuristic policy in all numerical instances. We make the following important observations:

1) We observe in all numerical instances that the profit loss is the smallest with the One-period Lookahead policy and the maximum percentage loss with this policy is only 0.13% as reported in Table 1. In other words, *the One-period Lookahead policy not only outperforms the commonly suggested heuristic policies in the literature but also provides a near-optimal performance.* The One-period Lookahead policy outperforms the other heuristic policies because in making the allocation decision in each period, unlike other policies, this policy uses the information about revenue uncertainty—i.e., revenue volatility of each crop and revenue correlation between crops—which is a critical feature of growing corn and soybean as discussed in §1. The performance of the One-period Lookahead policy is very close to the performance of the optimal policy because (i) by assumption (one-period carrying through of crop rotation benefits) the allocation decision in each period only impacts the allocation decision in the subsequent period, and (ii) this impact is captured by the One-

period Lookahead policy owing to the two-period horizon considered.

2) In all numerical instances the profit loss with the Always Rotate (Monoculture) policy is larger than the Always Rotate policy. In the literature, papers studying the farmland allocation decision in a multi-period setting (for example, Livingston et al. 2015) suggest farmers to use rotation-based allocation policy. Because these papers assume growing of a single crop every season their proposed policy corresponds to Always Rotate (Monoculture) in our framework. Our results underline the value of considering the possibility of growing more than one crop in the same season in employing a rotation-based allocation policy (as in the case of Always Rotate policy).

6 Extensions

In this section, we provide two extensions to our analysis. We only provide a summary of our results; the details of the analyses are relegated to §B of the Online Appendix.

6.1 Examining The Proportion of Farmland Allocated to Rotated Crops

In §5.2.4, we proposed One-period Lookahead policy as a heuristic allocation policy. In this section, we examine the prevalence of crop rotations under this heuristic policy and make a comparison with the prevalence under the optimal allocation policy. Because rotating crops improves the soil structure and reduces the need for synthetic chemicals, this comparison is one way to assess the relative environmental impact of these two policies. To this end, we numerically compute the expected proportion of farmland allocated to rotated crops per growing season under each policy. We calculate this metric using the same 312,500 numerical instances as in §5.2.3. We find that this metric under the One-period Lookahead policy in the baseline scenario is 90.57% with an average, a minimum, and a maximum level (in the numerical instances considered) of 85.39%, 41.43% and 100%, respectively. We also find that this metric under the optimal policy in the baseline scenario is 88.86% with an average, a minimum, and a maximum level (in the numerical instances considered) of 84.45%, 41.43% and 100%, respectively. Because this metric is higher in the baseline scenario and also has a higher average in the numerical instances considered under the One-period Lookahead policy, we conclude that *the proposed heuristic policy is potentially more environmentally friendly than the optimal policy.*

6.2 Introducing Fallow Farmland

Throughout the paper, we assume that farmland is always fully allocated to the two crops in each growing season. In practice, farmers may also let the farmland lay fallow—that is, deliberately not use the farmland to grow any crop—to rejuvenate the soil and increase the revenue for the crop grown on this farmland in the subsequent seasons. In this section, we generalize our model to incorporate the farmer's decision to let some portion of the farmland lay fallow in each growing season (which we capture through a "fallow crop").

Different from the base model in §3, the decision variables are $\alpha_t^j \in [0, 1]$ and $\beta_t^j \in [0, 1]$ for $j \in \{c, s, f\}$ (f corresponding to fallow crop), which denote the proportion of farmland allocated to corn and fallow crop in time period t on which crop j was grown in period $t-1$, respectively. Again different from the base model, we add for each cash crop $j \in \{c, s\}$ the revenue-enhancing crop rotation benefit as well as the cost-reducing crop rotation benefit if this crop is grown on rotated farmland where fallow crop was grown in the previous period (where these two benefits are no smaller than their respective benefits of this crop if it is grown on rotated farmland where the other cash crop was grown in the previous period). We assume no farming cost for the farmland that is laid fallow.

In this new model, we characterize the optimal allocation decisions by defining a new set of recursive operators which are generalized versions of the recursive operators in Proposition 8, as defined in (12), to suit the consideration of fallow crop. In particular, there is a new (third) recursive operator $K_t^f(\boldsymbol{r}_{t-1})$ which denotes the expected marginal profit of farmland in the remaining planning horizon where fallow crop was grown in period $t-1$. Moreover, each operator is given by the maximum profit from *three* (not two) options available to the farmer: growing corn, soybean or fallow crop in period t (and optimally using the farmland in the remaining periods). The optimal allocation decisions in each period are characterized in thirteen different cases. Besides *rotate* and *monoculture*, as identified in Proposition 8, a third strategy—in which one of the cash crops is grown on rotated farmland where fallow crop was grown in the previous period, and the other cash crop is grown both on rotated farmland where the other cash crop was grown in the previous period and on non-rotated farmland—-emerges as a part of the optimal allocation policy. Although the optimal policy is more complex (due to a larger set of decision variables) it follows the same structure as the optimal allocation policy presented in Proposition 8 for the following reasons: (i) The optimal allocation decisions in each period are characterized based on which of the three options is the most profitable on the farmland where crop $j \in \{c, s, f\}$ was grown in the previous period, as captured by the recursive operators; (ii) the optimal total expected profit from period t onwards is given by the product of the proportion of farmland allocated to crop $j \in \{c, s, f\}$ in the previous period and its corresponding expected marginal profit (as captured by the recursive operators).

We also conduct a comparable set of computational experiments as those in §5.2.3 and §5.2.4 in the presence of fallow crop and find that our key insights continue to hold. In particular, the average percentage profit loss due to continuously growing the same crop in the numerical instances considered is 18.16% with a minimum and a maximum loss of 12.94% and 23.03%, respectively—that is, *making crop planning based on principles of sustainable agriculture has a substantial economic value.* Moreover, in all numerical instances the profit loss due to employing heuristic policy is the smallest with the One-period Lookahead policy (a maximum percentage loss of 0.10% in the numerical instances considered)—that is, *the One-period Lookahead policy not only outperforms the commonly suggested heuristic policies in the literature but also provides a near-optimal performance.*

7 Conclusion

This paper examines crop planning decision in sustainable agriculture—that is, how to allocate farmland among multiple crops in each growing season when the crops have rotation benefits across growing seasons. Because farmers in practice face significant uncertainty for their crop revenues, we study the crop planning decision under revenue uncertainty. This is the first paper that characterizes the optimal dynamic farmland allocation policy under uncertainty in the presence of crop rotation benefits. As summarized in the Introduction, we provide insights on how revenue uncertainty of each crop shapes the value of making crop planning based on principles of sustainable agriculture and the way it is practised, and insights on the optimality gap when using heuristic policies suggested in the literature. Based on our optimal policy characterization we propose a simple heuristic allocation policy and show that (i) the proposed policy not only outperforms the other suggested heuristic policies but also provides a near-optimal performance; (ii) compared to the optimal policy, the proposed policy has a higher allocation of crops to rotated farmland, and thus it is potentially more environmentally friendly.

In our computational study throughout §5.2, we calibrated our model to represent a typical farmer growing corn and soybean in Iowa. We expect our insights to continue to hold for a farmer growing corn and soybean in another location (e.g., Illinois in the U.S., Brazil) or growing other crops with rotation benefits (e.g., cotton, wheat and rice). For example, we expect the farmer to benefit from a lower (higher) revenue volatility when this volatility is low (high); but the actual range in which the volatility in practice falls depends on the specific crop considered. We also expect our proposed heuristic policy to outperform the other suggested heuristic policies in the literature. This is because, unlike those other policies, our suggested policy uses the information about revenue volatility of each crop and revenue correlation between crops which are critical features of crops with rotation benefits listed above. The performance of the proposed heuristic policy in comparison to the optimal policy again depends on the specific crops considered. Future research may re-calibrate our model to represent different farmers by using the methodology presented in this paper.

Relaxing the assumptions made about the crop features gives rise to a number of interesting areas for future research. First, we assume that crop related uncertainty is captured through uncertainty in its revenue. In practice, the revenue uncertainty of each crop is driven by the uncertainty in its yield and the uncertainty in its sales price at the end of the growing season. Farmers may be interested to know how each of these uncertainties individually affects their profitability and farmland allocation decisions. Second, we (implicitly) normalize the additional costs associated with making crop planning based on multiple crops instead of a single crop to zero. In practice, these additional costs may exist owing to, for example, different set of labor or harvesting machinery required for each crop. Third, we only consider the inter-temporal benefits of growing multiple crops (which are captured through rotation benefits across growing seasons). In practice, growing multiple crops may also have spatial benefits—that is, growing multiple crops in the same season can increase plant diversity in order to avoid pest infestation, and provide shade, nitrogen fixation, or other benefits to the crops being grown within that season (Lithourgidis et al. 2011).

Our model (implicitly) assumes that the farmer's crop planning decision has no effect on crop revenues, an assumption commonly made in the literatures of operations management and agricultural economics. This is a reasonable assumption for commodity crops—including corn and soybean as considered in this paper—where production volume of an individual farmer is insignificant in comparison to the aggregate production volume traded in the exchange (spot) markets. For other crops the farmer's crop planning decision may affect the crop revenue by altering its availability in the market. Examining the crop planning decision in this setting requires an equilibrium model that captures the interplay between crop availability and crop revenue, following the work of Mendelson and Tunca (2007) who provide an equilibrium model in the context of commodity procurement.

Finally, we restrict our attention to crop planning decision. Crop production, however, involves subsequent operational decisions during cultivation (e.g., fertilizer and pesticides application, irrigation planning) and harvesting (e.g., harvest timing). Those operational decisions have an impact on crop revenues which we assume uncertain but exogenous in our model. Combining the crop planning decision with those other operational decisions in crop production should prove to be an interesting avenue for future research.

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