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Is Regime Switching in Stock Returns Important in Portfolio Decisions?

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The stock market displays regime switching between upturns and downturns. This paper provides a Bayesian framework for making portfolio decisions that takes this regime switching into account, together with asset pricing model uncertainty and parameter uncertainty. The findings reveal that the economic value of accounting for regimes is substantially independent of whether or not model and parameter uncertainties are incorporated: the certainty-equivalent losses associated with ignoring regime switching are generally above 2% per year and can be as high as 10%. These results suggest that the more realistic regime switching model is fundamentally different from the commonly used single-state model, and hence should be employed instead in portfolio decisions irrespective of concerns about model or parameter uncertainty.

Key words: investments; regime switching; model uncertainty; parameter uncertainty; Bayesian analysis
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1. Introduction

The stock market goes through periods during which equity prices persistently rise or fall. Investors therefore tend to decompose market fluctuations into bull and bear markets. In particular, investors often use available realized returns at a given point in time to determine whether the market is in a bull or bear state. Turner et al. (1989) provide a rigorous econometric model for analyzing bull and bear markets and find that the S&P 500 index displays different means and variances across these markets. Schwert (1989) and Hamilton and Susmel (1994) also document regime-dependent market volatility, whereas Ang and Bekaert (2002, 2004) and Guidolin and Timmermann (2005, 2007, 2008a, b) provide important economic insights on how investments vary across different market regimes. Motivated by these insights, this paper proposes a Bayesian framework for analyzing portfolio decisions under a regime switching model.[1]

The approach in this paper is distinct from existing regime switching studies in three ways. First, the methodology incorporates asset-pricing-model uncertainty into portfolio decision making. Given that existing asset pricing models are unable to fully capture empirical patterns in the cross-section of stock returns, investors face model uncertainty in making their portfolio decisions. As a result, their portfolio holdings may deviate significantly from benchmark portfolios, as shown by recent studies that incorporate model uncertainty (e.g., Pástor 2000, Pástor and Stambaugh 2000, Avramov 2004, Tu and Zhou 2004).[2] However, existing regime switching studies do not consider asset-pricing-model uncertainty. This paper seeks to fill this gap in the literature by combining the more realistic regime switching data-generating process with asset-pricing-model uncertainty in portfolio decisions.

Second, the Bayesian approach taken here also incorporates parameter uncertainty into portfolio decisions. Without knowledge of the true parameter values, investors face parameter uncertainty when choosing their optimal portfolio because the accuracy of estimates based on a finite historical sample is likely to be imperfect.[3] Although existing regime switching studies often ignore parameter uncertainty, the Bayesian setting in this paper provides a natural way to incorporate parameter uncertainty into portfolio decisions.

Third, the approach is applicable in portfolio decisions with a large number of assets. For instance, this study examines portfolio decisions in the case of 28 risky assets in the context of the Fama and French three-factor model (1993; henceforth FF) under

[1] The methodology of this paper is based on the first chapter of Tu (2004).

[2] These studies rely on the standard single-state model. Additionally, Brandt et al. (2009) show that the optimal portfolio can deviate from the market portfolio based on firm characteristics, such as market capitalization, book-to-market ratio, and lagged return.

[3] Many studies show that parameter uncertainty is important for investment decisions (e.g., Zellner and Chetty 1965, Klein and Bawa 1976, Brown 1976, Jorion 1986, Barberis 2000, Avramov 2004).

a wide range of prior beliefs about the pricing ability of the model (ranging from total confidence to complete skepticism). The set of investable assets consists of cash, the value-weighted Center for Research in Security Prices (CRSP) market index portfolio, the size factor portfolio, the value factor portfolio, and 25 nonbenchmark portfolios sorted by size and book-to-market. In contrast, it would be difficult, if not impossible, to apply existing approaches to such a high-dimensional regime switching model because the likelihood function would be too complex to evaluate and optimize.

Applying the approach to the data, the findings reveal that the economic value of accounting for regimes is substantially independent of whether or not model and parameter uncertainties are incorporated. In particular, the certainty-equivalent losses associated with ignoring regime switching are generally above 2% per year, and can be as high as 10%.[4] The results support the qualitative conclusions of the earlier regime studies by Ang and Bekaert (2002, 2004) and Guidolin and Timmermann (2005, 2007, 2008a, b), despite their classical framework, which does not incorporate model or parameter uncertainty. This is because the impact of these types of uncertainty could be less important than the impact of regime changes.

In addition, the findings show that in line with prior regime switching studies, asset returns generally have higher means and lower standard deviations in the bull regime than in the bear regime, and the bull regime appears to be more typical in that it accounts for approximately two-thirds of the entire sample period. Moreover, the results on cross-regime differences in correlations, betas, and mispricing alphas show that the correlations and betas between the 25 nonbenchmark portfolios and the three factor portfolios are regime-dependent, with sizable cross-regime differences. Furthermore, there is evidence of mispricing in both bull and bear regimes, with the cross-regime differences in mispricing alphas nontrivial in many cases.

The remainder of this paper is organized as follows. Section 2 develops a Bayesian framework for making portfolio decisions that includes regime switching together with model and parameter uncertainty. In §3, the methodology is applied to the data and the results are discussed. Section 4 concludes.

2. Investing with Regime Switching

To investigate whether or not regime switching in stock returns is important for portfolio decisions when model and parameter uncertainties are also taken into account, this section first presents a regime switching model in the Bayesian framework that incorporates model and parameter uncertainties. Economic measures are then constructed to evaluate the differences between the optimal portfolios implied by the regime-switching model and those implied by a single-state model.

2.1. Regime Switching Under Asset-Pricing-Model Uncertainty

Assume that the investment opportunity set consists of n risky assets and one riskless asset. Under a single-state model (SSM), the excess returns of the risky assets over the riskless asset are typically assumed to follow a multivariate normal distribution. Under regime switching between multiple states, the returns are assumed to follow an l-regime Markov regime switching model (RSM) with l multivariate normal distributions associated with the l regimes:

$$r_t \backsim MVN(E^{s_t}, V^{s_t}), \tag{1}$$

where MVN represents a multivariate normal distribution, E^{s_t} is an $n \times 1$ vector, V^{s_t} is an $n \times n$ matrix, and E^{s_t} and V^{s_t} are both associated with the prevailing state at time t, namely, $s_t \in S = \{1, 2, \ldots, l\}$.[5] The transition probabilities are determined by an $l \times l$ matrix Π, whose generic elements π_{ij} are defined as

$$\Pr(s_t = i \mid s_{t-1} = j) = \pi_{ij}, \quad i, j = 1, \ldots, l.$$

In this paper, the number of states, l, is set to two (as discussed in the empirical results). When $l = 2$, the transition probability matrix Π becomes a 2×2 matrix:

$$\Pi = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}, \tag{2}$$

where $P = \Pr(s_t = 1 \mid s_{t-1} = 1)$ and $Q = \Pr(s_t = 2 \mid s_{t-1} = 2)$. Therefore, in any given period t, r_t follows the normal distribution associated with the state in S that is prevailing at t. In period $t+1$, r_{t+1} may stay in the same regime and hence follow the same normal distribution at the given transition probability, P or Q, or it can switch to the other regime and therefore follow the normal distribution associated with

[4] Tu and Zhou (2004) find that the certainty-equivalent losses associated with ignoring fat tails are typically less than 1% for mean-variance investors facing model and parameter uncertainty. However, the findings of this study reveal that ignoring regime switching can lead to sizable economic costs.

[5] Ang and Bekaert (2004) and Guidolin and Timmermann (2007, 2008b) examine regime switching models in which the dividend yield and/or short-term interest rates are used to predict excess returns. Guidolin and Timmermann (2006) study forecasting accuracy under regime switching. In addition, Avramov and Chordia (2006b) provide important insights on the economic value of predictability at the stock level. For tractability, the approach here is limited by not using any variables to predict means and covariances.

the other regime at the given transition probability, $1-P$ or $1-Q$.

Note that although the aforementioned model can be used to capture potential regime switching in the data, it is silent about asset-pricing-model uncertainty, which can fundamentally change the way investors make portfolio decisions (e.g., Pástor and Stambaugh 2000, Avramov 2004). To incorporate model uncertainty, the problem is cast into a regression setting. Let $r_t = (y_t, x_t)$, where y_t contains the returns of m nonbenchmark positions and x_t contains the returns of k $(= n - m)$ benchmark positions. In the regime switching framework, consider the multivariate regression:

$$y_t = \alpha^{s_t} + B^{s_t} x_t + u_t^{s_t}, \qquad (3)$$

where $u_t^{s_t}$ is an $m \times 1$ vector with zero mean and nonsingular covariance matrix Σ^{s_t}, and $s_t \in S = (1, 2)$. To relate α^{s_t}, B^{s_t}, and Σ^{s_t} to the earlier parameters E^{s_t} and V^{s_t}, consider the corresponding partition:

$$E^{s_t} = \begin{pmatrix} E_1^{s_t} \\ E_2^{s_t} \end{pmatrix}, \quad V^{s_t} = \begin{pmatrix} V_{11}^{s_t} & V_{12}^{s_t} \\ V_{21}^{s_t} & V_{22}^{s_t} \end{pmatrix}. \qquad (4)$$

Under the usual multivariate normal distribution for each regime, it is clear that the distribution of y_t conditional on x_t and s_t is also normal, and that the conditional mean is a linear function of x_t. Hence,

$$E(y_t \mid x_t, s_t) = E_1^{s_t} + V_{12}^{s_t}(V_{22}^{s_t})^{-1}(x_t - E_2^{s_t}), \qquad (5)$$

$$\mathrm{Var}(y_t \mid x_t, s_t) = V_{11}^{s_t} - V_{12}^{s_t}(V_{22}^{s_t})^{-1}V_{21}^{s_t}. \qquad (6)$$

Therefore, the parameters α^{s_t}, B^{s_t}, and Σ^{s_t} and the earlier parameters E^{s_t} and V^{s_t} obey the relationships:

$$\alpha^{s_t} = E_1^{s_t} - B^{s_t}E_2^{s_t}, \quad B^{s_t} = V_{12}^{s_t}(V_{22}^{s_t})^{-1}, \qquad (7)$$

$$\Sigma^{s_t} = V_{11}^{s_t} - B^{s_t}V_{22}^{s_t}(B^{s_t})', \qquad (8)$$

and the returns of the benchmark positions are normally distributed in each state,

$$x_t \backsim N(E_2^{s_t}, V_{22}^{s_t}).$$

An asset pricing model such as FF restricts α^{s_t} to be zero. However, any given model is likely to be imperfect, in which case investors are uncertain about the pricing ability of the given model, that is, they face asset pricing model uncertainty or mispricing uncertainty. To incorporate mispricing uncertainty, following Pástor and Stambaugh (2000) and Pástor (2000), the model specifies, in a Bayesian framework, the prior distribution of α^{s_t} as a normal distribution conditional on Σ^{s_t}:

$$\alpha^{s_t} \mid \Sigma^{s_t} \sim N\left(0, \sigma_\alpha^2\left(\frac{1}{(s^{s_t})^2}\Sigma^{s_t}\right)\right), \qquad (9)$$

where $(s^{s_t})^2$ is a suitable prior estimate for the average diagonal elements of Σ^{s_t}.[6] The aforementioned alpha-Sigma link is also explored by MacKinlay and Pástor (2000) in a frequentist set up. The value of σ_α represents an investor's level of uncertainty about a given model's pricing ability.[7] When $\sigma_\alpha = 0$, the investor believes dogmatically in the model, and there is no mispricing uncertainty. In contrast, when $\sigma_\alpha = \infty$, the investor believes that the pricing model is completely useless. Turning to P and Q, in recent regime switching studies that also use a two-regime model (see Ang and Bekaert 2002, 2004), the values of P and Q are around 90% and 80%, respectively.[8] In this paper, prior beliefs on P and Q center around 91.67% and 83.33%, respectively. These values correspond to expected average durations of 12 months and 6 months for the bull and bear regimes, respectively. Robustness checks indicate that the results are generally qualitatively the same under different specifications of the priors on P and Q. Finally, in addition to the aforementioned priors on α, P, and Q, the remaining priors are fairly standard (see the online appendix, which is provided in the e-companion, for details).[9]

2.2. Performance Measures

Without knowledge of the true parameter values, investors use historical data to assess investment opportunities. However, investors encounter parameter uncertainty when choosing their optimal portfolio because the accuracy of estimates based on a finite sample is likely to be imperfect. Bayesian predictive distribution provides a natural way to express investment opportunities in the presence of parameter uncertainty by integrating over the posterior distribution that summarizes such uncertainty. The predictive

[6] As detailed in Pástor and Stambaugh (2000), following an "empirical Bayes" approach, the value of $(s^{s_t})^2$ is set equal to the average of the diagonal elements of the sample estimate of Σ^{s_t}.

[7] Before looking at the data, investors may not be clear as to whether the regimes are significantly different from each other and whether asset pricing models perform very differently across regimes. Therefore, it is assumed here that investors have the same prior belief (a "belief" before looking at the data) on the pricing ability of an asset pricing model across regimes. Although not examined here, a potentially richer model could be employed that allows different prior beliefs on model performance across regimes.

[8] The values of the transition probabilities for Regime 3 and Regime 1 in Guidolin and Timmermann (2008b) (as shown in Table 1 of their paper) are also around 90% and 80%. As is shown later, the bull and bear regimes of this paper seem to correspond to Regime 3 and Regime 1 of Guidolin and Timmermann (2008b).

[9] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

distribution approach has been used by many studies, such as Barberis (2000), Pástor (2000), Pástor and Stambaugh (2000), and Avramov (2002, 2004).

With the previously discussed priors, the posterior distribution on the model parameters, $p(\theta \mid R)$, is obtained by updating the priors in light of the data $\{R\colon r_t, t = 1, \ldots, T\}$. Denoting the first and second moments of the predictive distribution of r_{T+1} by $E(r_{T+1} \mid R)$ and $\mathrm{Var}(r_{T+1} \mid R)$, respectively (see the online appendix for details on the derivation of the two moments), the optimal portfolio solves

$$\max_{\omega}\left(\omega' E(r_{T+1} \mid R) - \frac{\gamma}{2}\omega' \mathrm{Var}(r_{T+1} \mid R)\omega\right), \tag{10}$$

where γ is the coefficient of risk aversion.[10] In addition, because Regulation T requires the use of margins for risky investments, and following Pástor and Stambaugh (2000), a constraint is imposed on the portfolio weights, namely, $\sum_{i\in\Lambda} 2|w_i| + \sum_{i\notin\Lambda} |w_i| \leqslant c$, where $w_i = X_i/W_T$, X_i is the dollar amount invested in asset i, W_T is the investor's initial wealth at time T, Λ denotes the set of FF size (SMB) and value (HML) factor portfolios, and $c \geq 0$ is used to characterize the margin requirements.[11] For example, if $c = 5$, the setup implies 20% margin requirements, whereas if $c = \infty$, there are no margin requirements, and the resulting optimal portfolio is explicitly given by $\omega = 1/\gamma[\mathrm{Var}(r_{T+1} \mid R)]^{-1} E(r_{T+1} \mid R)$.[12]

We now construct two measures for gauging the economic importance of incorporating regime switching into portfolio decisions. Assume that portfolio ω_{RSM} is optimal under RSM (for a given prior belief on an asset pricing model), and that portfolio ω_{SSM} is optimal under SSM (for the same prior belief on the asset pricing model as used under RSM).[13] The certainty-equivalent returns (*CERs*) and Sharpe ratios (*SRs*) associated with portfolios ω_{RSM} and ω_{SSM} are defined as

$$CER_{\mathrm{RSM}} = \omega'_{\mathrm{RSM}} E^*(r_{T+1} \mid R) - \frac{\gamma}{2}\omega'_{\mathrm{RSM}} \mathrm{Var}^*(r_{T+1} \mid R)\omega_{\mathrm{RSM}}, \tag{11}$$

$$CER_{\mathrm{SSM}} = \omega'_{\mathrm{SSM}} E^*(r_{T+1} \mid R) - \frac{\gamma}{2}\omega'_{\mathrm{SSM}} \mathrm{Var}^*(r_{T+1} \mid R)\omega_{\mathrm{SSM}}, \tag{12}$$

$$SR_{\mathrm{RSM}} = \frac{\omega'_{\mathrm{RSM}} E^*(r_{T+1} \mid R)}{\sqrt{\omega'_{\mathrm{RSM}} \mathrm{Var}^*(r_{T+1} \mid R)\omega_{\mathrm{RSM}}}}, \tag{13}$$

$$SR_{\mathrm{SSM}} = \frac{\omega'_{\mathrm{SSM}} E^*(r_{T+1} \mid R)}{\sqrt{\omega'_{\mathrm{SSM}} \mathrm{Var}^*(r_{T+1} \mid R)\omega_{\mathrm{SSM}}}}, \tag{14}$$

where $E^*(r_{T+1} \mid R)$ and $\mathrm{Var}^*(r_{T+1} \mid R)$ are the first two predictive moments under RSM. The two proposed measures are the difference $CER_{\mathrm{RSM}} - CER_{\mathrm{SSM}}$ and the difference $SR_{\mathrm{RSM}} - SR_{\mathrm{SSM}}$, which reflect the "perceived" certainty-equivalent return gain and Sharpe ratio gain, respectively, associated with incorporating regime switching, or certainty-equivalent return loss and Sharpe ratio loss to a hypothetical investor who believes RSM but is forced to hold the portfolio that is optimal to an investor who believes SSM. In the Bayesian setting, following many recent studies (e.g., Avramov 2004), we use a single predictive distribution to compute the *CERs* and *SRs* of more than one portfolio.[14]

Note that Guidolin and Timmermann (2008a) provide valuable insights for making portfolio decisions with higher-order moments, such as skewness and kurtosis, under regime switching. In contrast, our paper uses the mean-variance framework. There are two reasons for this design choice. First, we can illustrate (as shown in the empirical results) that even under this framework, differences in mean and variance estimates with or without regime switching can cause sizable performance differences.[15] Second, because the mean-variance framework is the major tool used in quantitative equity management (due to its tractability), it may be of interest to see what

[10] In some studies, such as Pástor and Stambaugh (2000), γ is actually referred to as the "coefficient of relative risk aversion," in the sense that the mean-variance preference is considered a second-order Taylor expansion of a power utility function.

[11] Both the SMB and HML portfolios involve two risky assets. Therefore, the weight on each portfolio is multiplied by two in the constraint, as in Pástor and Stambaugh (2000). In other words, if investing $|X|$ dollars in SMB or HML, $(2/c)|X|$ dollars of capital are required and the rest can be borrowed. As for the market portfolio and the 25 nonbenchmark portfolios, each involves only one risky asset. Hence, if investing $|Y|$ dollars in one of these portfolios, only $(1/c)|Y|$ dollars of capital are required.

[12] When there are margin requirements, the resulting optimal portfolios are not explicitly given. The MATLAB function "fmincon" in the optimization toolbox is employed to find the solutions numerically.

[13] The procedure used to obtain the predictive distribution of returns under SSM is largely the same as that used under RSM with some modifications, such as dropping the regime switching feature.

[14] Notice that a bootstrap analysis in a frequentist framework, like the one considered in Guidolin and Timmermann (2008b), can potentially be used to test the economic value of regime switching. Such analysis would not need to assume RSM to be correctly specified. Unfortunately, we are not able to run bootstrap analysis because of computational constraints. Nevertheless, later in the paper, we run further analysis that uses a realized recursive (pseudo) out-of-sample performance measure that does not need to assume RSM to be correctly specified.

[15] The differences in performance measures between the single-state model and the regime switching model could become larger if higher-order moments are taken into consideration as well. Perez-Quiros and Timmermann (2001) provide an excellent study on higher-order moments under regime switching.

insights regime switching can offer for applications using this framework when model and parameter uncertainties are also incorporated.[16]

3. Empirical Results

In this section, the methodology proposed in §2 is applied to the data to investigate the economic value of regime switching under model and parameter uncertainties. First, the data are briefly described. Next, the extent of regime switching in the data is investigated. Third, the impact of regime switching on portfolio decisions and the associated economic gains under various mispricing uncertainty scenarios are examined. Finally, additional analyses are conducted, including out-of-sample analysis. The empirical results show that the economic value of regime switching is independent of whether or not model and parameter uncertainties are incorporated.

3.1. Data

Although the methodology can be applied to any data set, we focus here on the investment universe consisting of cash (which earns the risk-free interest rate), the value-weighted CRSP market index portfolio (MKT), the size factor portfolio (SMB), the value factor portfolio (HML), and the FF 25 portfolios sorted on size and book-to-market. Similar data sets are used by many studies, such as Pástor and Stambaugh (2000), Perez-Quiros and Timmermann (2000), and Avramov (2004).[17] The data comprise monthly observations over a period of 512 months, from July 1963 to February 2006.[18]

3.2. Regime Switching in the Data

First of all, the findings reveal that there is evidence of regime switching in the data. Because the posterior distributions of the three factor portfolios' expected returns are not affected by the prior beliefs on the validity of FF, we focus on the case of dogmatic beliefs ($\sigma_\alpha = 0$) in analyzing the factor portfolios. As the first row of Table 1 shows, the annualized expected excess return of the market portfolio (MKT) has a sizable positive posterior mean of 12.9% in the bull regime and a negative posterior mean of −11.0% in the bear regime. With fewer observations in the bear regime, the posterior standard deviation (pstd) in the bear regime is much larger than that in the bull regime.[19] Under SSM, the annualized expected excess return of MKT has a positive posterior mean of 5.7%, which is between the posterior means of 12.9% in the bull regime and −11.0% in the bear regime under RSM. Moreover, whereas the expected return of HML has positive posterior means in both the bull and bear regimes, the expected return of SMB has a positive posterior mean in the bull regime but a negative posterior mean in the bear regime. In contrast, under SSM, the expected returns of both SMB and HML have positive posterior means between those of the bull and bear regimes under RSM.

Unlike the three factors, the posterior distributions of the expected excess returns of the nonbenchmark assets, that is, the FF 25 size and book-to-market portfolios, are affected by the prior beliefs on the validity of FF. However, as Table 1 shows, regardless of the priors on the mispricing errors, the expected excess returns of the 25 nonbenchmark portfolios generally have positive posterior means and relatively small pstds in the bull regime, but negative posterior means and relatively large pstds in the bear regime. In addition, the cross-regime differences tend to be larger for small size or low book-to-market portfolios. Under SSM, the expected excess returns of the 25 nonbenchmark portfolios have positive posterior means, which again are between the posterior means of the bull and bear regimes.

Turning to volatilities, Table 2 shows volatilities are generally much larger in the bear regime than in the bull regime across various beliefs on FF. In addition, the cross-regime differences again tend to be larger for small size or low book-to-market portfolios. Under SSM, as in the case of expected returns, the volatilities of all 28 portfolios have posterior means that are between the posterior means of the bull and bear regimes.[20]

[16] For practical applications of the mean-variance framework, see Grinold and Kahn (1999) and Meucci (2005).

[17] Portfolios sorted by industry, beta, profitability, or other criteria can also be used. Later in this paper, we provide results when the set of 25 portfolios sorted on size and book-to-market is replaced by an alternative set of 17 industry portfolios.

[18] All data are from Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). We are grateful to Ken French for making the data available. In addition, because SMB and HML are zero-investment portfolios, their net returns as opposed to returns in excess of the risk-free interest rate are considered. Regarding MKT and the FF 25 portfolios sorted on size and book-to-market, their excess returns over the risk-free interest rate are used.

[19] If investors believed that the excess return of the market index portfolio is characterized by the regime switching model, and if they were able to observe the current state, it would be hard to understand why they would hold the index portfolio in the bear regime. However, in this paper, similar to Turner et al. (1989), investors are assumed to be unable to observe the current regime. Figure EC.1 in the e-companion displays the posterior distributions of the expected returns of the FF three factors.

[20] Interestingly, the volatilities under nondogmatic beliefs, such as $\sigma_\alpha = \infty$, are almost the same as those under $\sigma_\alpha = 0$. Intuitively, this is because the priors on pricing errors are primarily about the mean returns and not about the volatilities. Similar results under other nondogmatic beliefs, such as $\sigma_\alpha = 1\%$, are omitted from Tables 1 and 2 but are available upon request.

Table 1 Expected Return Estimates

	$\sigma_\alpha = 0$				$\sigma_\alpha = \infty$			
		RSM				RSM		
Portfolio	SSM	Bear	Bull	BMB	SSM	Bear	Bull	BMB
MKT	5.7 (2.3)	−11.0 (7.9)	12.9 (2.7)	−23.9 (8.8)	5.7 (2.4)	−11.0 (7.9)	12.9 (2.7)	−23.9 (8.8)
SMB	3.2 (1.7)	−3.5 (4.7)	6.1 (2.1)	−9.6 (5.5)	3.2 (1.7)	−3.5 (4.7)	6.1 (2.1)	−9.6 (5.5)
HML	5.3 (1.6)	9.6 (4.5)	3.5 (1.9)	6.1 (5.3)	5.3 (1.5)	9.6 (4.5)	3.5 (1.9)	6.1 (5.3)
S1B1	8.4 (4.2)	−20.4 (13.9)	20.3 (4.8)	−40.7 (15.7)	3.1 (4.3)	−25.0 (14.3)	15.3 (5.2)	−40.3 (16.3)
S1B2	9.9 (3.6)	−14.3 (11.6)	20.7 (4.3)	−35.0 (13.2)	10.1 (3.7)	−13.5 (11.9)	20.3 (4.4)	−33.8 (13.6)
S1B3	10.2 (3.1)	−10.5 (9.9)	19.7 (3.8)	−30.3 (11.3)	10.5 (3.2)	−9.3 (10.2)	19.2 (3.8)	−28.5 (11.6)
S1B4	10.8 (2.9)	−8.8 (9.3)	19.8 (3.5)	−28.5 (10.6)	13.0 (3.0)	−5.6 (9.5)	21.1 (3.5)	−26.7 (10.8)
S1B5	12.6 (3.0)	−8.0 (9.8)	21.9 (3.7)	−29.9 (11.1)	14.3 (3.1)	−6.9 (10.0)	23.5 (3.8)	−30.4 (11.3)
S2B1	7.2 (3.8)	−19.2 (12.6)	18.9 (4.5)	−38.1 (14.3)	5.0 (3.9)	−20.3 (12.8)	16.0 (4.7)	−36.3 (14.7)
S2B2	9.4 (3.1)	−12.0 (10.2)	18.7 (3.7)	−30.7 (11.5)	8.1 (3.2)	−12.2 (10.3)	16.9 (3.8)	−29.1 (11.7)
S2B3	10.2 (2.7)	−8.8 (9.0)	18.6 (3.3)	−27.4 (10.2)	11.3 (2.8)	−8.9 (9.2)	20.1 (3.3)	−29.0 (10.3)
S2B4	10.9 (2.6)	−7.2 (8.6)	19.0 (3.2)	−26.1 (9.7)	11.9 (2.7)	−5.7 (8.7)	19.6 (3.2)	−25.3 (9.8)
S2B5	12.9 (2.9)	−7.0 (9.4)	22.0 (3.6)	−29.0 (10.7)	12.8 (3.0)	−6.7 (9.5)	21.3 (3.6)	−28.0 (10.7)
S3B1	5.9 (3.5)	−18.6 (11.7)	16.8 (4.1)	−35.4 (13.3)	5.1 (3.6)	−18.3 (11.8)	15.3 (4.2)	−33.6 (13.4)
S3B2	8.8 (2.7)	−10.6 (9.2)	17.1 (3.2)	−27.7 (10.4)	9.0 (2.9)	−10.1 (9.5)	17.3 (3.4)	−27.5 (10.8)
S3B3	9.7 (2.5)	−7.4 (8.2)	16.9 (2.9)	−24.3 (9.2)	9.1 (2.6)	−8.8 (8.5)	16.9 (3.0)	−25.7 (9.5)
S3B4	10.4 (2.4)	−5.4 (7.7)	17.5 (2.8)	−23.0 (8.7)	10.3 (2.5)	−5.8 (8.0)	17.3 (3.0)	−23.1 (9.1)
S3B5	12.4 (2.7)	−5.7 (8.8)	20.0 (3.2)	−25.8 (9.9)	12.5 (2.9)	−4.2 (9.0)	19.8 (3.3)	−23.9 (10.2)
S4B1	4.9 (3.1)	−16.8 (10.3)	14.6 (3.6)	−31.3 (11.7)	6.5 (3.2)	−13.1 (10.6)	15.0 (3.7)	−28.1 (12.1)
S4B2	8.1 (2.5)	−9.9 (8.7)	15.4 (2.9)	−25.2 (9.7)	6.2 (2.7)	−10.3 (8.9)	13.5 (3.2)	−23.8 (10.1)
S4B3	9.3 (2.4)	−6.9 (7.9)	16.2 (2.8)	−23.0 (8.9)	9.1 (2.6)	−6.4 (8.2)	15.9 (3.0)	−22.2 (9.3)
S4B4	9.9 (2.3)	−5.3 (7.4)	17.0 (2.8)	−22.2 (8.3)	10.5 (2.5)	−4.0 (7.7)	16.9 (3.0)	−20.9 (8.8)
S4B5	11.7 (2.6)	−5.2 (8.4)	19.0 (3.0)	−24.2 (9.4)	10.8 (2.8)	−5.5 (8.6)	17.9 (3.3)	−23.4 (9.8)
S5B1	2.7 (2.5)	−12.9 (7.9)	9.9 (3.0)	−22.7 (9.1)	5.0 (2.5)	−10.9 (8.2)	11.8 (2.9)	−22.6 (9.3)
S5B2	5.9 (2.3)	−8.8 (7.3)	12.2 (2.7)	−21.0 (8.2)	5.8 (2.4)	−8.1 (7.5)	11.9 (2.8)	−19.9 (8.6)
S5B3	6.4 (2.1)	−7.0 (6.8)	11.8 (2.4)	−18.8 (7.6)	5.9 (2.3)	−6.8 (7.0)	11.5 (2.6)	−18.3 (7.9)
S5B4	8.1 (2.1)	−3.8 (6.4)	13.3 (2.4)	−17.1 (7.2)	7.0 (2.2)	−4.3 (6.6)	12.0 (2.6)	−16.3 (7.6)
S5B5	9.8 (2.3)	−3.7 (6.8)	16.2 (2.7)	−20.0 (7.7)	7.1 (2.6)	−7.6 (7.6)	13.6 (3.1)	−21.2 (8.6)

Notes. This table reports in percentage points the posterior means and standard variations (in parentheses) of the annualized expected returns of SMB and HML, and the annualized expected excess returns of MKT and the 25 nonbenchmark portfolios (S1B1, S1B2, …, S5B5). The columns under the subtitle "SSM" provide the results corresponding to the single-state model, whereas the columns under the subtitle "RSM" provide the results corresponding to the regime switching model, for the two regimes (bear and bull) and for the difference between the two regimes (BMB, bear minus bull). The mispricing priors imposed on FF are $\sigma_\alpha = 0$ and ∞, respectively.

The results on regime-dependent means and variances discussed so far are largely consistent with the existing regime switching literature, in particular, Guidolin and Timmermann (2008b). Recently, however, a number of studies (e.g., Hong et al. 2007) on asymmetric comovements between asset returns and market indices suggest that stocks are more likely to move with the market when the market goes down than when it goes up. Regime-dependent comovements between nonbenchmark and benchmark portfolios may therefore be interesting though they are largely ignored by the existing regime switching literature. In particular, regime-dependent comovements could be important for examining the economic value of regime switching for portfolio decisions under model uncertainty, because although standard investment theory advises portfolio diversification under pricing model uncertainty, the value of this advice might be questionable if all stocks tend to fall a lot as the market falls in a bear regime. Thus, next we examine whether the comovements between the nonbenchmark assets and the three factors are different across bull and bear regimes.

Consider first the market correlations and betas.[21] As the first column of Figure 1 shows, there appear to be some cross-regime differences in the correlations with MKT. For example, the cross-regime difference in the correlation between MKT and S1B1 has a posterior mean of 14%. When size and book-to-market increase, however, the cross-regime differences in the correlations tend to decrease. For instance, the cross-regime difference in the correlation between MKT and S5B5 has a posterior mean of 5%, which is much less than the 14% for S1B1. Hence, large size and high book-to-market portfolios tend to have less cross-regime

[21] Because the priors on pricing errors are primarily about the mean returns and not about the correlations and betas, the results are almost the same under different prior beliefs on FF. Therefore, we report the results when the mispricing prior imposed on the FF three-factor model is diffuse ($\sigma_\alpha = \infty$) and omit the results under the other prior beliefs on FF.

Table 2 Standard Deviation Estimates

	$\sigma_\alpha = 0$				$\sigma_\alpha = \infty$			
		RSM				RSM		
Portfolio	SSM	Bear	Bull	BMB	SSM	Bear	Bull	BMB
MKT	15.4 (0.5)	21.1 (1.6)	11.4 (0.7)	9.7 (1.6)	15.4 (0.5)	21.1 (1.6)	11.4 (0.7)	9.7 (1.6)
SMB	11.2 (0.4)	13.3 (1.4)	9.8 (0.8)	3.5 (2.1)	11.2 (0.4)	13.3 (1.4)	9.8 (0.8)	3.5 (2.1)
HML	10.1 (0.3)	13.0 (1.1)	8.6 (0.6)	4.4 (1.4)	10.1 (0.3)	13.0 (1.1)	8.6 (0.6)	4.4 (1.4)
S1B1	28.4 (0.9)	37.6 (2.9)	22.1 (1.6)	15.5 (3.5)	28.5 (0.9)	37.4 (2.9)	22.5 (1.6)	14.9 (3.6)
S1B2	24.4 (0.8)	31.7 (2.6)	19.4 (1.4)	12.3 (3.3)	24.4 (0.8)	31.7 (2.6)	19.4 (1.4)	12.3 (3.3)
S1B3	20.9 (0.7)	27.0 (2.0)	16.8 (1.0)	10.3 (2.3)	20.8 (0.7)	27.1 (2.0)	16.8 (1.0)	10.2 (2.3)
S1B4	19.6 (0.6)	25.4 (1.9)	15.8 (0.9)	9.6 (2.1)	19.5 (0.6)	25.5 (1.9)	15.6 (0.9)	9.9 (2.1)
S1B5	20.6 (0.6)	26.8 (2.0)	16.6 (0.9)	10.1 (2.2)	20.5 (0.6)	26.8 (2.0)	16.4 (0.9)	10.3 (2.3)
S2B1	25.7 (0.8)	33.7 (2.4)	20.4 (1.2)	13.4 (2.7)	25.8 (0.8)	33.7 (2.4)	20.6 (1.2)	13.1 (2.7)
S2B2	20.9 (0.7)	27.6 (2.0)	16.4 (0.9)	11.3 (2.1)	20.9 (0.7)	27.6 (2.0)	16.5 (0.9)	11.1 (2.1)
S2B3	18.7 (0.6)	24.8 (1.8)	14.7 (0.8)	10.1 (1.9)	18.6 (0.6)	24.8 (1.8)	14.5 (0.8)	10.3 (1.9)
S2B4	17.9 (0.6)	23.7 (1.8)	14.2 (0.7)	9.5 (1.9)	17.8 (0.6)	23.7 (1.8)	14.1 (0.7)	9.6 (1.9)
S2B5	19.9 (0.6)	26.1 (2.0)	15.9 (0.9)	10.2 (2.1)	19.9 (0.6)	26.1 (2.0)	15.9 (0.9)	10.1 (2.1)
S3B1	23.6 (0.7)	31.3 (2.3)	18.4 (1.1)	12.8 (2.5)	23.6 (0.7)	31.3 (2.3)	18.6 (1.1)	12.7 (2.5)
S3B2	18.9 (0.6)	25.5 (1.9)	14.6 (0.8)	10.9 (1.9)	18.9 (0.6)	25.5 (1.9)	14.6 (0.8)	10.9 (1.9)
S3B3	17.0 (0.5)	23.0 (1.7)	13.1 (0.7)	9.9 (1.7)	17.0 (0.5)	23.0 (1.7)	13.1 (0.7)	9.9 (1.7)
S3B4	16.4 (0.5)	21.9 (1.6)	13.0 (0.7)	8.9 (1.7)	16.4 (0.5)	21.9 (1.6)	13.0 (0.7)	8.9 (1.7)
S3B5	18.7 (0.6)	25.1 (1.9)	14.6 (0.8)	10.5 (2.0)	18.7 (0.6)	25.1 (1.9)	14.6 (0.8)	10.5 (2.0)
S4B1	21.0 (0.7)	28.1 (2.1)	16.2 (1.0)	12.0 (2.3)	20.9 (0.7)	28.3 (2.1)	16.1 (1.0)	12.1 (2.3)
S4B2	17.8 (0.6)	24.3 (1.8)	13.4 (0.8)	10.9 (1.8)	17.8 (0.6)	24.3 (1.8)	13.6 (0.8)	10.7 (1.8)
S4B3	16.8 (0.5)	22.8 (1.7)	12.9 (0.7)	9.9 (1.7)	16.8 (0.5)	22.8 (1.7)	13.0 (0.7)	9.9 (1.7)
S4B4	16.2 (0.5)	21.5 (1.6)	12.9 (0.7)	8.6 (1.7)	16.2 (0.5)	21.5 (1.6)	12.9 (0.7)	8.6 (1.7)
S4B5	18.4 (0.6)	24.5 (1.8)	14.6 (0.8)	9.9 (2.0)	18.4 (0.6)	24.5 (1.8)	14.7 (0.8)	9.8 (2.0)
S5B1	16.6 (0.5)	22.1 (1.6)	13.1 (0.7)	9.0 (1.7)	16.6 (0.5)	22.2 (1.6)	13.0 (0.7)	9.2 (1.7)
S5B2	15.7 (0.5)	21.0 (1.5)	12.4 (0.7)	8.6 (1.6)	15.7 (0.5)	21.0 (1.5)	12.4 (0.7)	8.6 (1.6)
S5B3	14.9 (0.5)	20.0 (1.5)	11.6 (0.7)	8.3 (1.6)	14.9 (0.5)	20.0 (1.5)	11.7 (0.7)	8.3 (1.6)
S5B4	14.6 (0.5)	19.2 (1.4)	11.7 (0.7)	7.6 (1.6)	14.6 (0.5)	19.2 (1.4)	11.8 (0.7)	7.4 (1.6)
S5B5	16.5 (0.5)	21.2 (1.5)	13.7 (0.7)	7.6 (1.7)	16.6 (0.5)	21.1 (1.5)	13.9 (0.7)	7.2 (1.7)

Notes. This table reports in percentage points the posterior means and standard variations (in parentheses) of the annualized standard deviations of the three factors (MKT, SMB, and HML) and the 25 nonbenchmark portfolios (S1B1, S1B2, ..., S5B5). The columns under the subtitle "SSM" provide the results corresponding to the single-state model, whereas the columns under the subtitle "RSM" provide the results corresponding to the regime switching model, for the two regimes (bear and bull) and for the difference between the two regimes (BMB, bear minus bull). The mispricing priors imposed on FF are $\sigma_\alpha = 0$ and ∞, respectively.

differences in their market correlations than small size and low book-to-market portfolios. Turning to the cross-regime differences in market betas, the first column of Figure 2 shows that the relatively significant cross-regime differences in market correlations for small size and low book-to-market portfolios are reduced in the market beta case. For instance, whereas the cross-regime difference in the correlation between MKT and S1B1 has a posterior mean of 14%, the cross-regime difference in the market beta is smaller and has a posterior mean of 10%. Moreover, such reduction can be much larger for some portfolios, such as S1B5. The posterior mean of the cross-regime difference in the beta, 2%, is much smaller than 11%, the posterior mean of the cross-regime difference in the correlation for S1B5. Because betas are closely related to asset pricing theories, it is of interest to determine why S1B5 shows this large reduction in the cross-regime difference in market beta, especially given the fact that if the bull and bear variances were equal for both S1B5 and MKT, the market correlation difference and the market beta difference should be equal to each other. Here, the bull and bear variances are different, and we have (roughly):[22]

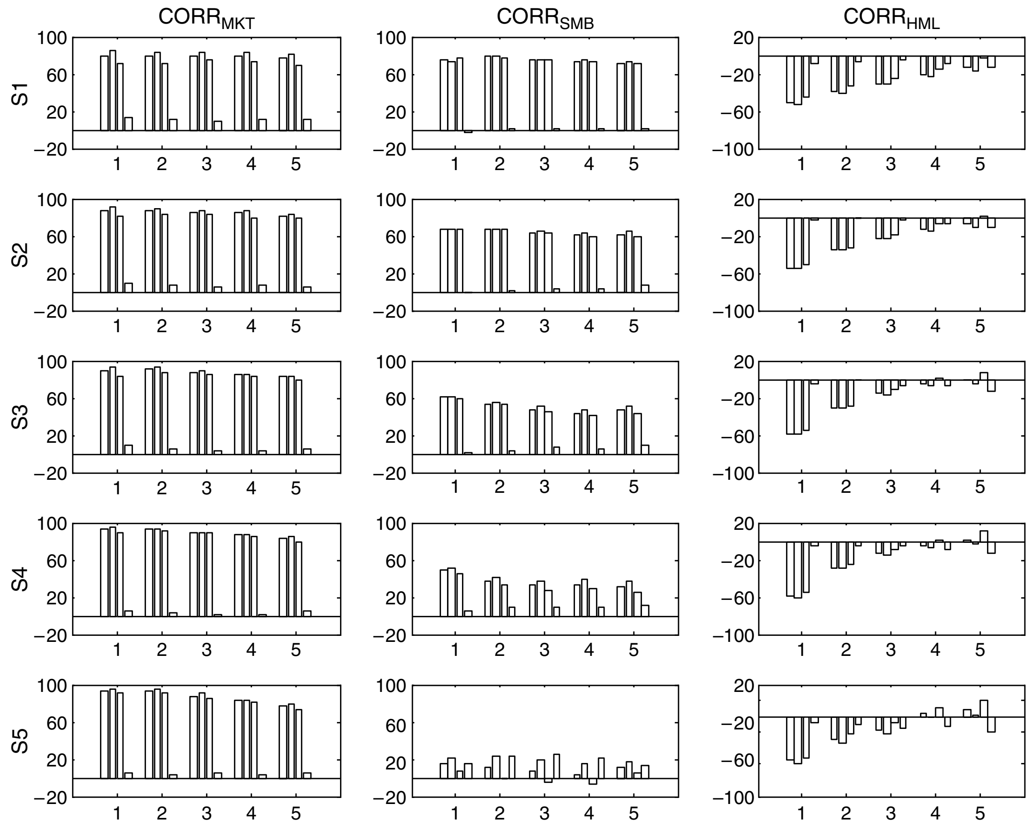
$$\beta^{+}_{\mathrm{S1B5,\,MKT}} = \frac{\sigma^{+}_{\mathrm{S1B5}}}{\sigma^{+}_{\mathrm{MKT}}} \times \rho^{+}_{\mathrm{S1B5,\,MKT}} = \frac{0.0479}{0.0329} \times \rho^{+}_{\mathrm{S1B5,\,MKT}} = 1.4559\rho^{+}_{\mathrm{S1B5,\,MKT}},$$

$$\beta^{-}_{\mathrm{S1B5,\,MKT}} = \frac{\sigma^{-}_{\mathrm{S1B5}}}{\sigma^{-}_{\mathrm{MKT}}} \times \rho^{-}_{\mathrm{S1B5,\,MKT}} = \frac{0.0774}{0.0609} \times \rho^{-}_{\mathrm{S1B5,\,MKT}} = 1.2709\rho^{-}_{\mathrm{S1B5,\,MKT}}.$$

Because $\rho^{+}_{\mathrm{S1B5,\,MKT}} < \rho^{-}_{\mathrm{S1B5,\,MKT}}$, the larger standard deviation ratio in the bull regime, $\sigma^{+}_{\mathrm{S1B5}}/\sigma^{+}_{\mathrm{MKT}}$, helps inflate $\beta^{+}_{\mathrm{S1B5,\,MKT}}$ substantially to narrow its difference

[22] Here, "+" denotes the bull regime and "−" denotes the bear regime. The numerical values of $\sigma^{+}_{\mathrm{S1B5}}$, $\sigma^{-}_{\mathrm{S1B5}}$, $\sigma^{+}_{\mathrm{MKT}}$, and $\sigma^{-}_{\mathrm{MKT}}$ in the two equations are the posterior means of these conditional moments.

Figure 1 **Correlations**

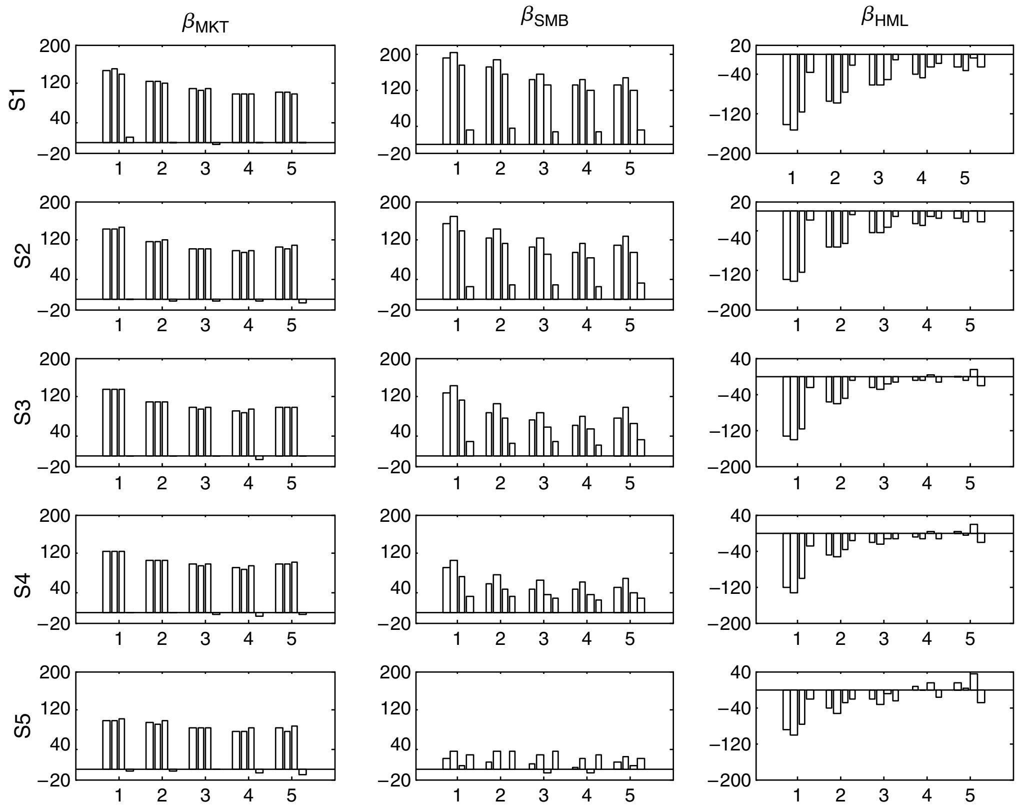


Notes. This figure displays in percentage points the correlations between the 25 nonbenchmark portfolios and the three factor portfolios: MKT (first column), SMB (second column), and HML (third column). The first row corresponds to the first five assets, that is, S1B1, S1B2, …, S1B5. Similarly, the second, third, fourth, and fifth rows correspond to the second, third, fourth, and fifth five assets, respectively. There are four bars for each asset, among which the first three bars depict the posterior means of the correlations under the SSM (Bar 1), the bear regime (Bar 2), and the bull regime (Bar 3), and the fourth bar depicts the posterior means of the differences in correlations between the bear regime and the bull regime (bear minus bull; Bar 4). The mispricing prior imposed on the FF three-factor model is diffuse ($\sigma_\alpha = \infty$).

with $\beta^{-}_{\mathrm{S1B5,\,MKT}}$. Hence, for S1B5, the posterior mean of the cross-regime difference in the beta is much smaller than that in the correlation. Finally, under SSM, as in the case of expected returns and volatilities, the correlations and betas with MKT of all 25 nonbenchmark portfolios have posterior means in between those of the bull and bear regimes.

Next, we consider whether the comovements of the 25 nonbenchmark portfolios with SMB and HML are different across bull and bear regimes. As the second column of Figure 1 shows, the cross-regime differences in the correlations with SMB are generally smaller for small size portfolios and become larger as size increases. As for the cross-regime differences in the betas with SMB, the second column of Figure 2 shows that they are also generally large for large size portfolios but they are not particularly small for small size portfolios. The reason for the finding of more cross-regime differences in the betas with SMB than in the correlations with SMB is the same as that for the finding of more cross-regime differences in the correlations with MKT than in the betas with MKT, though the effect is in the opposite direction: we now have a larger standard deviation ratio in the bear regime, which helps increase the differences in the betas with SMB relative to the differences in the correlations with SMB. Turning to HML, as the third column of Figure 1 shows, the cross-regime differences in the correlations with HML are generally small for low book-to-market portfolios and become larger when book-to-market increases. As for the cross-regime differences in the betas with HML, the third column of Figure 2 shows that they are also generally large for high book-to-market portfolios but are not particularly small for low book-to-market ones. The reason for the finding of more cross-regime differences in the betas with HML than in the correlations with HML is the same as that for the finding of more cross-regime differences in the betas with SMB than in the correlations with SMB. Finally, under SSM, as in the case of the correlations and betas with MKT, the correlations and betas with SMB and HML generally have posterior means in between those of the bull and bear regimes.

Figure 2 **Betas**



Notes. This figure displays in percentage points the betas between the 25 nonbenchmark portfolios and the three factor portfolios: MKT (first column), SMB (second column), and HML (third column). The first row corresponds to the first five assets, that is, S1B1, S1B2, …, S1B5. Similarly, the second, third, fourth, and fifth rows correspond to the second, third, fourth, and fifth five assets, respectively. There are four bars for each asset, among which the first three bars depict the posterior means of the betas under the SSM (Bar 1), the bear regime (Bar 2), and the bull regime (Bar 3), and the fourth bar depicts the posterior means of the differences in betas between the bear regime and the bull regime (bear minus bull; Bar 4). The mispricing prior imposed on the FF three-factor model is diffuse ($\sigma_\alpha = \infty$).

In summary, correlations and betas are regime-dependent, with some cross-regime differences. Whereas small size and low book-to-market portfolios tend to have more cross-regime differences in their correlations with MKT, large size (high book-to-market) assets tend to have more cross-regime differences in their correlations with SMB (HML). In addition, although the cross-regime differences in the betas with MKT are small, the cross-regime differences in the betas with SMB and HML tend to be large.

Finally, we consider the posterior distributions of mispricing αs. Our empirical objective is to investigate the economic value of regime switching for portfolio decisions when pricing model uncertainty is taken into account. Model uncertainty is reflected by prior distributions of mispricing α, which impact portfolio decisions to a large degree by affecting posterior distributions of α. Table 3 shows that generally there is some mispricing under both SSM and RSM, with the cross-regime difference not negligible in many cases. For example, under a diffuse prior on FF ($\sigma_\alpha = \infty$), the mispricing α for S1B1 has a posterior mean of −5.6% under SSM, and of −4.9% in the bear regime and −6.0% in the bull regime under RSM, with a cross-regime difference of 1.1%. Overall, of the 25 portfolios, we observe a 1% or larger absolute mispricing α for 13 portfolios under SSM and for 12 portfolios in both the bull and bear regimes under RSM, with a cross-regime difference of 1% or larger for 17 portfolios. The aggregate mispricing, $\alpha'\Sigma^{-1}\alpha$, is also not negligible under both SSM and RSM, with a posterior mean of 23.6% under SSM, and of 56.2% in the bear regime and 36.4% in the bull regime under RSM. The degree of the aggregate mispricing is larger in the bear regime, with a cross-regime difference of 19.8%. Under an informative prior ($\sigma_\alpha = 1\%$), the aggregate mispricing has a posterior mean of 8.4% under SSM, and of 5.6% in the bear regime and 10.3% in the bull regime under RSM. The aggregate mispricing is still sizable, although smaller than that under the diffuse prior ($\sigma_\alpha = \infty$). Intuitively, under the informative prior ($\sigma_\alpha = 1\%$), FF should be considered more useful than under the diffuse prior ($\sigma_\alpha = \infty$). Thus, after updating using the same amount of data, under the informative prior ($\sigma_\alpha = 1\%$), FF should be

Table 3 α Estimates

	$\sigma_\alpha = 1\%$				$\sigma_\alpha = \infty$			
		RSM				RSM		
Portfolio	SSM	Bear	Bull	BMB	SSM	Bear	Bull	BMB
S1B1	−3.2 (1.0)	−1.2 (1.4)	−3.0 (1.1)	1.8 (1.9)	−5.6 (1.3)	−4.9 (3.4)	−6.0 (1.7)	1.1 (4.1)
S1B2	0.1 (0.7)	0.2 (0.9)	−0.2 (0.8)	0.4 (1.3)	0.1 (0.9)	0.8 (2.4)	−0.5 (1.2)	1.3 (2.9)
S1B3	0.2 (0.6)	0.3 (0.8)	−0.3 (0.7)	0.6 (1.0)	0.3 (0.7)	1.3 (1.8)	−0.6 (1.0)	1.9 (2.2)
S1B4	1.3 (0.6)	0.8 (0.8)	0.8 (0.7)	−0.0 (1.1)	2.3 (0.7)	3.4 (1.8)	1.7 (1.0)	1.8 (2.2)
S1B5	1.0 (0.6)	0.3 (0.8)	1.0 (0.7)	−0.7 (1.1)	1.7 (0.8)	1.2 (1.9)	1.9 (1.0)	−0.7 (2.3)
S2B1	−1.3 (0.7)	−0.3 (0.9)	−1.7 (0.8)	1.4 (1.3)	−2.4 (0.9)	−1.2 (2.2)	−3.5 (1.2)	2.3 (2.7)
S2B2	−0.8 (0.6)	−0.1 (0.9)	−1.1 (0.7)	1.0 (1.1)	−1.4 (0.8)	−0.2 (2.1)	−2.1 (1.0)	1.9 (2.5)
S2B3	0.6 (0.6)	−0.0 (0.8)	0.9 (0.7)	−0.9 (1.1)	1.1 (0.8)	−0.1 (1.9)	1.7 (1.0)	−1.9 (2.3)
S2B4	0.6 (0.5)	0.3 (0.7)	0.4 (0.6)	−0.0 (1.0)	1.1 (0.7)	1.5 (1.8)	0.8 (1.0)	0.8 (2.3)
S2B5	−0.1 (0.6)	0.1 (0.8)	−0.4 (0.7)	0.5 (1.0)	−0.1 (0.7)	0.3 (1.8)	−0.8 (1.0)	1.1 (2.2)
S3B1	−0.5 (0.6)	0.1 (0.8)	−0.9 (0.8)	1.0 (1.2)	−0.9 (0.8)	0.3 (2.0)	−1.8 (1.2)	2.1 (2.4)
S3B2	0.1 (0.7)	0.1 (1.0)	0.1 (0.8)	−0.0 (1.3)	0.2 (0.9)	0.5 (2.4)	0.3 (1.2)	0.2 (2.9)
S3B3	−0.4 (0.7)	−0.4 (0.9)	−0.0 (0.7)	−0.3 (1.2)	−0.7 (0.9)	−1.5 (2.2)	−0.1 (1.1)	−1.5 (2.7)
S3B4	−0.1 (0.6)	−0.1 (0.9)	−0.1 (0.7)	0.0 (1.2)	−0.2 (0.8)	−0.4 (2.1)	−0.3 (1.1)	−0.1 (2.6)
S3B5	0.1 (0.7)	0.4 (1.0)	−0.2 (0.8)	0.6 (1.3)	0.1 (1.0)	1.7 (2.4)	−0.3 (1.2)	2.0 (2.9)
S4B1	1.0 (0.6)	0.9 (0.9)	0.2 (0.8)	0.7 (1.2)	1.7 (0.8)	3.9 (2.2)	0.5 (1.2)	3.4 (2.8)
S4B2	−1.1 (0.7)	−0.1 (1.0)	−1.1 (0.8)	1.0 (1.4)	−1.9 (1.0)	−0.4 (2.5)	−2.3 (1.3)	1.8 (3.0)
S4B3	−0.1 (0.7)	0.1 (1.0)	−0.2 (0.8)	0.3 (1.3)	−0.2 (0.9)	0.6 (2.4)	−0.3 (1.2)	0.9 (2.8)
S4B4	0.4 (0.7)	0.3 (0.9)	−0.1 (0.8)	0.4 (1.2)	0.7 (0.9)	1.3 (2.3)	−0.1 (1.1)	1.4 (2.7)
S4B5	−0.6 (0.8)	−0.1 (1.1)	−0.7 (1.0)	0.6 (1.5)	−1.0 (1.1)	−0.3 (2.5)	−1.3 (1.4)	1.0 (3.1)
S5B1	1.4 (0.5)	0.5 (0.7)	1.1 (0.6)	−0.6 (1.0)	2.4 (0.7)	2.1 (1.7)	2.3 (0.9)	−0.1 (2.1)
S5B2	−0.0 (0.6)	0.2 (0.8)	−0.2 (0.7)	0.4 (1.1)	−0.1 (0.8)	0.7 (2.0)	−0.4 (1.0)	1.2 (2.4)
S5B3	−0.3 (0.7)	0.0 (1.0)	−0.2 (0.8)	0.2 (1.3)	−0.5 (0.9)	0.2 (2.4)	−0.4 (1.2)	0.6 (2.9)
S5B4	−0.7 (0.6)	−0.1 (0.9)	−0.8 (0.7)	0.7 (1.1)	−1.2 (0.8)	−0.6 (2.0)	−1.6 (1.1)	1.1 (2.4)
S5B5	−1.6 (0.9)	−1.0 (1.2)	−1.6 (1.0)	0.6 (1.6)	−2.8 (1.2)	−4.1 (3.0)	−3.2 (1.6)	−0.9 (3.6)
$\alpha'\Sigma^{-1}\alpha$	8.4 (1.8)	5.6 (1.6)	10.3 (2.7)	−4.7 (3.2)	23.6 (4.3)	56.2 (20.1)	36.4 (9.0)	19.8 (24.4)

Notes. This table reports in percentage points the posterior means and standard variations (in parentheses) of the annualized αs of the 25 nonbenchmark portfolios (S1B1, S1B2, …, S5B5), and the aggregate mispricing, $\alpha'\Sigma^{-1}\alpha$. The columns under the subtitle "SSM" provide the results corresponding to the single-state model, whereas the columns under the subtitle "RSM" provide the results corresponding to the regime switching model, for the two regimes (bear and bull) and for the difference between the two regimes (BMB, bear minus bull). The mispricing priors imposed on FF are $\sigma_\alpha = 1\%$ and ∞, respectively.

expected to be more useful posteriorly as well and to have smaller posterior aggregate mispricing than under the diffuse prior ($\sigma_\alpha = \infty$).[23]

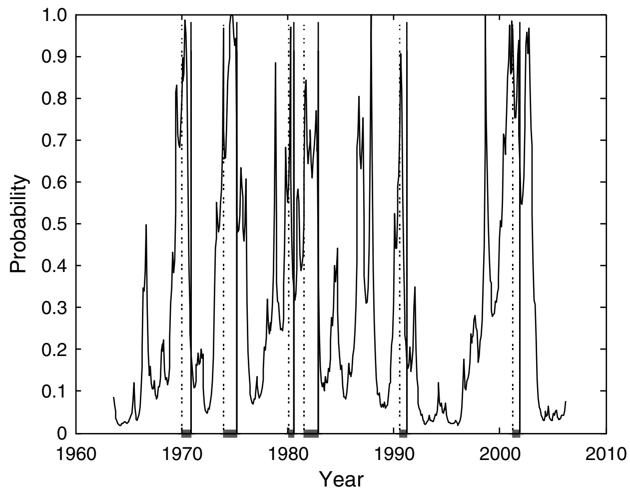
Although the regimes are not observable, we can compute the empirical probability of being in the bear regime by dividing the number of draws associated with the bear regime by the total number of sample draws (set to 10,000). Figure 3 plots this empirical probability. One interesting pattern in the figure is that almost all of the recessionary periods (periods between a National Bureau of Economic Research (NBER) peak and the following NBER trough) are associated with high bear regime probabilities. Regime switching therefore appears to be related to the business cycle to some extent. However, some periods associated with high bear regime probabilities are not classified as NBER recessions.[24] Overall, the correlation between the empirical probability of being in a bear regime and the NBER recession dummy variable is 30.6%. In addition, the posterior means of the transition probabilities for staying in the bull and bear regimes are 92.4% and 83.7%, respectively, and the implied average durations for the bull and bear regimes are 13.1 and 6.1 months, respectively.[25] The bull regime is also more typical than the bear regime, as the implied steady state probabilities for the bull

[23] Furthermore, the reduction in the degree of aggregate mispricing from 56.2% to 5.6% for the bear regime is more than the reduction from 36.4% to 10.3% for the bull regime. The larger reduction for the bear regime is partially due to the relatively larger uncertainty associated with the degree of aggregate mispricing in the bear regime under the diffuse prior, which has a pstd of 20.1% compared to the pstd of 9.0% in the bull regime. Also in an extreme case, if an investor has a dogmatic prior belief on FF, he will believe posteriorly that mispricing α's are all zeros as well.

[24] One reason may be that stock markets also react to sectoral or shorter-lived contractions in the economy that are not designated as recessions by NBER.

[25] The posterior mean, 83.7%, happens to be close to the prior mean, 83.3%. Indeed, the posterior mean of the transition probability for staying in the bear regime is not sensitive to the prior specification. For instance, even when assuming a diffuse prior, the posterior mean of the transition probability for staying in the bear regime takes a similar value of 82.6%.

Figure 3 **Probability in the Bear Regime**



Notes. This figure plots the empirical probability of being in the bear regime from July 1963 through February 2006. The vertical dotted and solid lines represent the NBER peaks and troughs, respectively.

and bear regimes are approximately 68% and 32%, respectively.[26]

Moreover, in our experimental analysis, we find that when the two-regime model is extended to a three-regime model, one regime has very few observations.[27] Given the large number of assets (28 portfolios) in our study, in the three-regime model the estimation errors associated with the estimates of the mean (a 28×1 vector) and the covariance matrix (a 28×28 matrix) of the regime with very few observations are too large to obtain robust optimal portfolios. When more regimes are added, for instance, when we extend the analysis to a four-regime model, similar results obtain: the regimes other than the two major regimes have very few observations and the implied optimal portfolios are not robust because of large estimation errors. Furthermore, because of the large number of assets examined in this paper, the computing time needed increases significantly as the number of regimes increases. Because of these difficulties, we choose to use a two-regime model and leave investigation of models with three or more regimes for future work.[28] However, when we compare the bull and bear regimes in our two-regime model with Regime 1 and Regime 3 in Guidolin and Timmermann (2008b), we find that the bear (bull) regime of this paper appears to correspond to their Regime 1 (Regime 3), because both have low (high) returns and high (low) risk. Moreover, the correlation between HML and MKT in our bull regime is negative, which is consistent with their Regime 3.

3.3. Investments Under Regime Switching

In this subsection, we investigate the impact of regime switching on portfolio decisions under various prior beliefs on FF. By combining the regime switching data-generating process with asset pricing model uncertainty in portfolio decisions, our results shed new light on the economic importance of regime switching under model uncertainty.

Given the large cross-regime differences in the various moments of asset returns, it may not be surprising that the optimal portfolio weights change significantly when regime switching is taken into account, as shown in Table 4.[29] In general, regardless of the degree of model mispricing uncertainty, investors are more aggressive (defensive) in taking risks conditional on a bull (bear) regime under RSM than under SSM.[30] The resulting differences in the position-by-position allocations are substantial. For instance, panel A of Table 4 shows that under SSM, for each \$100 the optimal weight allocated to the market portfolio MKT is \$40.0 ($\sigma_\alpha = 0$), \$−21.7 ($\sigma_\alpha = 1\%$), and \$−74.8 ($\sigma_\alpha = \infty$), whereas under RSM, conditional on a bull regime, the optimal weight allocated to MKT increases by \$58.0 ($\sigma_\alpha = 0$), \$113.9 ($\sigma_\alpha = 1\%$), and \$204.7 ($\sigma_\alpha = \infty$).[31] The reason is that the risky assets become relatively more attractive (with larger means and smaller variances) conditional on a bull regime under RSM than under SSM. Furthermore, the differences in portfolio weights are still generally sizable under 20% margin

[26] The regime switching model here seems able to avoid classifying most of the periods as "uncertain" between the bull and bear regimes: the probability of being in the bear or bull regime is between 40% to 60% only 10.6% of the time.

[27] In a thorough analysis, Guidolin and Timmermann (2008b) identify four regimes in the joint process of the returns of MKT, SMB, and HML. In particular, their Regime 2 (p. 8) "captures long periods with growing stock prices during the 1940s, the 1950s, …" In contrast, this paper's sample period starts in July 1963, with the 1940s and the 1950s not covered in our data. Therefore, it is possible that their Regime 2 will be missing in our analysis. In addition, the regime with very few observations may correspond to Guidolin and Timmermann's (2008b) Regime 4, which has a steady state probability as small as 1%.

[28] In a different application, Guidolin and Timmermann (2007) explicitly discuss whether two or more regimes should be considered.

[29] When the prior belief on FF is dogmatic ($\sigma_\alpha = 0$), the weights on the nonbenchmark assets other than the three factor portfolios (MKT, SMB, and HML) are all zeros. However, some degree of skepticism with respect to FF (such as $\sigma_\alpha = 1\%$) leads to substantial deviations from FF-implied weights, let alone the completely skeptical view ($\sigma_\alpha = \infty\%$).

[30] Following Avramov (2004), we choose $\gamma = 10$. A number larger than 10 seems implausible according to Mehra and Prescott (1985).

[31] We report the regime-specific (bull and bear) portfolio allocations because doing so helps a Bayesian investor who believes RSM to decide how he should invest next period if he is convinced with unit probability of a bull or bear regime. We thank an anonymous referee for this and numerous other thoughtful observations and suggestions.

Table 4 Portfolio Weights

	$c=\infty$						$c=5$					
	$\sigma_\alpha=0$		$\sigma_\alpha=1\%$		$\sigma_\alpha=\infty$		$\sigma_\alpha=0$		$\sigma_\alpha=1\%$		$\sigma_\alpha=\infty$	
Portfolio	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS
						Panel A: Bull regime						
MKT	40.0	58.0	−21.7	113.9	−74.8	204.7	40.0	58.0	−9.0	48.5	−0.0	0.0
SMB	29.0	21.0	24.1	87.3	21.2	215.0	29.0	21.0	−0.1	0.1	0.0	−0.0
HML	85.0	36.0	48.7	25.6	21.9	32.7	85.0	36.0	25.8	−20.3	0.0	−0.0
S1B1	0	0	−74.2	−0.4	−140.9	−38.7	0	0	−69.1	−2.0	−114.8	−3.0
S1B2	0	0	32.0	−4.2	60.8	9.6	0	0	22.0	5.6	25.5	−8.3
S1B3	0	0	−9.0	−11.4	−17.1	−45.7	0	0	1.5	−1.5	0.0	−0.0
S1B4	0	0	69.3	−36.5	131.6	−48.0	0	0	65.9	−29.7	116.9	−55.8
S1B5	0	0	24.7	19.9	47.0	59.2	0	0	23.6	35.2	39.3	56.3
S2B1	0	0	−10.4	−7.7	−19.7	−34.3	0	0	−7.3	−7.1	−7.5	−5.2
S2B2	0	0	−20.4	−8.6	−38.8	−36.6	0	0	−11.9	−9.6	−17.0	5.1
S2B3	0	0	38.9	13.4	73.8	61.3	0	0	29.7	22.3	12.6	58.3
S2B4	0	0	9.8	−15.3	18.5	−38.0	0	0	4.7	4.7	0.0	−0.0
S2B5	0	0	−12.3	1.4	−23.3	−26.7	0	0	−1.1	8.9	0.0	−0.0
S3B1	0	0	−21.6	1.0	−41.1	−7.8	0	0	−16.6	−2.3	−10.2	10.2
S3B2	0	0	5.4	−0.5	10.2	10.0	0	0	−1.4	1.5	−0.0	0.0
S3B3	0	0	−22.1	10.4	−42.0	13.2	0	0	−8.5	8.5	−0.0	0.0
S3B4	0	0	−27.0	3.5	−51.2	−12.9	0	0	−10.2	6.3	−0.0	0.0
S3B5	0	0	14.5	−23.7	27.6	−43.6	0	0	8.2	−8.2	0.0	−0.0
S4B1	0	0	67.6	−28.8	128.4	−27.7	0	0	51.5	−21.1	65.9	−35.9
S4B2	0	0	−52.4	6.7	−99.6	−8.1	0	0	−41.6	−3.9	−62.7	16.0
S4B3	0	0	5.1	2.7	9.7	13.8	0	0	−2.0	2.1	−0.0	0.0
S4B4	0	0	25.0	3.5	47.4	9.6	0	0	16.4	11.0	12.1	−6.9
S4B5	0	0	−12.2	3.3	−23.1	6.0	0	0	−4.4	−1.4	−0.7	0.7
S5B1	0	0	32.2	−0.3	61.1	23.5	0	0	15.5	−15.5	11.5	17.6
S5B2	0	0	2.3	−1.2	4.3	1.8	0	0	−7.4	−0.5	−0.0	0.0
S5B3	0	0	26.0	−7.6	49.4	−4.1	0	0	14.0	−6.4	0.0	0.0
S5B4	0	0	−5.0	12.1	−9.4	24.6	0	0	−0.3	0.9	−0.0	0.0
S5B5	0	0	−3.5	6.6	−6.7	0.1	0	0	−4.1	6.2	−3.2	1.5
CER	7.0	2.0	9.4	2.6	18.4	4.5	7.0	2.0	9.3	2.6	15.9	3.2
SR	131.0	7.0	141.2	13.8	197.1	16.9	131.0	7.0	144.5	9.8	195.3	2.7
						Panel B: Bear regime						
MKT	40.0	−43.0	−21.7	−29.7	−74.8	−123.7	40.0	−43.0	−9.0	−42.4	−0.0	−0.0
SMB	29.0	−25.0	24.1	−68.3	21.2	−197.2	29.0	−25.0	−0.1	−44.1	0.0	−0.0
HML	85.0	−31.0	48.7	−17.5	21.9	−5.3	85.0	−31.0	25.8	5.4	0.0	−0.0
S1B1	0	0	−74.2	39.4	−140.9	21.8	0	0	−69.1	34.3	−114.8	8.7
S1B2	0	0	32.0	−14.1	60.8	8.5	0	0	22.0	−4.1	25.5	−5.3
S1B3	0	0	−9.0	16.9	−17.1	35.7	0	0	1.5	6.3	0.0	0.0
S1B4	0	0	69.3	−26.0	131.6	38.3	0	0	65.9	−22.6	116.9	8.1
S1B5	0	0	24.7	−23.2	47.0	−56.0	0	0	23.6	−22.1	39.3	−39.3
S2B1	0	0	−10.4	13.1	−19.7	29.0	0	0	−7.3	10.0	−7.5	6.1
S2B2	0	0	−20.4	13.7	−38.8	21.0	0	0	−11.9	5.2	−17.0	8.5
S2B3	0	0	38.9	−31.9	73.8	−58.0	0	0	29.7	−22.7	12.6	−12.6
S2B4	0	0	9.8	5.6	18.5	49.0	0	0	4.7	10.6	0.0	−0.0
S2B5	0	0	−12.3	12.7	−23.3	11.0	0	0	−1.1	1.6	0.0	−0.0
S3B1	0	0	−21.6	17.8	−41.1	29.0	0	0	−16.6	12.8	−10.2	−7.8
S3B2	0	0	5.4	−9.1	10.2	−14.4	0	0	−1.4	−2.4	−0.0	0.0
S3B3	0	0	−22.1	9.9	−42.0	−11.6	0	0	−8.5	−3.7	−0.0	−41.8
S3B4	0	0	−27.0	14.4	−51.2	−6.5	0	0	−10.2	−2.3	−0.0	−17.7
S3B5	0	0	14.5	2.6	27.6	53.1	0	0	8.2	8.9	0.0	33.6
S4B1	0	0	67.6	−28.2	128.4	30.1	0	0	51.5	−12.1	65.9	8.9
S4B2	0	0	−52.4	24.9	−99.6	4.8	0	0	−41.6	14.1	−62.7	24.6
S4B3	0	0	5.1	−2.5	9.7	8.9	0	0	−2.0	4.6	−0.0	−0.0
S4B4	0	0	25.0	−15.9	47.4	−32.8	0	0	16.4	−7.3	12.1	−9.4
S4B5	0	0	−12.2	5.0	−23.1	−2.2	0	0	−4.4	−2.8	−0.7	0.7

Table 4 **(Continued)**

	$c=\infty$						$c=5$					
	$\sigma_\alpha=0$		$\sigma_\alpha=1\%$		$\sigma_\alpha=\infty$		$\sigma_\alpha=0$		$\sigma_\alpha=1\%$		$\sigma_\alpha=\infty$	
Portfolio	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS	SSM	RMS
						Panel B: Bear regime						
S5B1	0	0	32.2	−30.1	61.1	−48.3	0	0	15.5	−13.4	11.5	−11.5
S5B2	0	0	2.3	−3.3	4.3	4.0	0	0	−7.4	6.4	−0.0	0.0
S5B3	0	0	26.0	−17.0	49.4	−14.1	0	0	14.0	−5.0	0.0	3.8
S5B4	0	0	−5.0	1.4	−9.4	−2.4	0	0	−0.3	−3.3	−0.0	0.0
S5B5	0	0	−3.5	0.5	−6.7	−24.9	0	0	−4.1	1.0	−3.2	−4.9
CER	−1.0	4.0	−2.1	5.3	6.7	7.0	−1.0	4.0	−1.5	4.7	5.7	5.2
SR	35.0	31.0	54.8	24.7	132.2	33.3	35.0	31.0	53.2	26.3	114.0	37.0

Notes. For a mean-variance investor with risk aversion $\gamma=10$ and with varying mispricing priors, this table reports the differences (under the subtitle "RMS," RSM − SSM) between the optimal portfolio weights (for each \$100), CERs and SRs under RSM, and those under SSM (under the subtitle "SSM"). In panel A (B), a unit probability of a bull (bear) regime is assigned under RSM. There are two scenarios: one with no margin requirements ($c=\infty$) and one with 20% margin requirements ($c=5$). In each scenario, the prior beliefs on FF range from first a dogmatic view ($\sigma_\alpha=0$), to a somewhat skeptical view ($\sigma_\alpha=1\%$), to a completely skeptical view ($\sigma_\alpha=\infty$).

requirements ($c=5$) though become smaller in general than those under no margin requirements ($c=\infty$). On the other hand, conditional on a bear regime under RSM, the risky assets become relatively less attractive (with smaller means and larger variances) than under SSM. Therefore, investors should decrease their overall holding of risky assets under RSM compared to under SSM, as shown by panel B of Table 4, conditional on a bear regime.

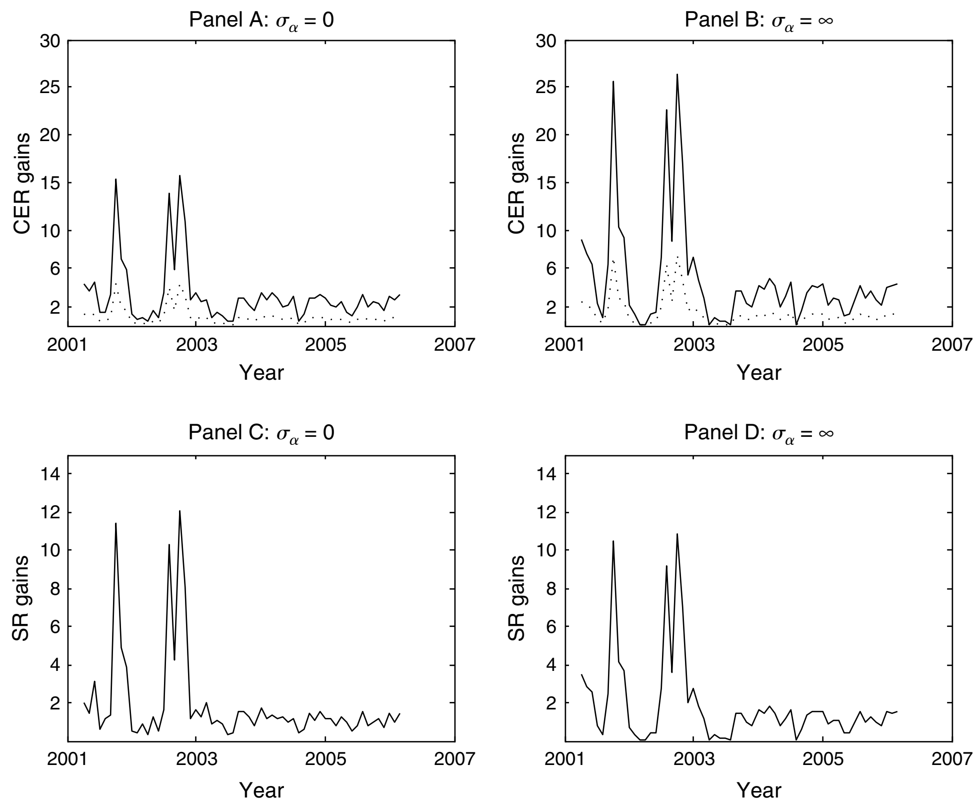
Although the differences in the optimal portfolio weights implied by RSM and SSM are large, overall portfolio performance may still be similar because of correlations among the payoffs of risky positions. To address this concern, we now report the CER gains and SR gains associated with incorporating regime switching below the portfolio weights in Table 4.[32] As reported in panel A of Table 4, under a dogmatic belief on FF ($\sigma_\alpha=0$) and no margin requirements ($c=\infty$), the CER gain is 2.0% per year in a bull regime. The gains become even larger when mispricing uncertainty is taken into account. For instance, under a completely skeptical belief on FF ($\sigma_\alpha=\infty$), the CER gain of 2.0% under $\sigma_\alpha=0$ rises to 4.5%. As reported in panel B of Table 4, the CER gains are generally larger in a bear regime. The results on the SR gains are similar, as shown in Table 4.

To assess the economic value of incorporating regime switching under model uncertainty more generally, in Figure 4 we report the CER gains and SR gains for the five-year period from March 2001 to February 2006 under two priors on mispricing uncertainty, that is, $\sigma_\alpha=0$ and $\sigma_\alpha=\infty$.[33] In addition, to facilitate comparison with Tu and Zhou (2004), in which risk aversion γ is set to 2.83, we also report the CER gains for the case of $\gamma=2.83$. The findings reveal that, regardless of the degree of mispricing uncertainty, the CER gains and SR gains generally appear to be economically meaningful, and they appear to be particularly large during market downturns. For instance, when $\gamma=2.83$, the CER gains are 2.8% ($\sigma_\alpha=0$) and 4.3% ($\sigma_\alpha=\infty$) in February 2006, when the probability of being in a bull regime in the next period is 85.2%, and they are 15.5% ($\sigma_\alpha=0$) and 26.3% ($\sigma_\alpha=\infty$) in September 2002, when the probability of being in a bear regime in the next period is 81.8%. Because of heavy computational burden, in Figure 4 we do not report the gains under 20% margin requirements ($c=5$). Nevertheless, the results under 20% margin requirements appear to still be significant based on test cases. For instance, under 20% margin requirements ($c=5$), the aforementioned CER gains of 2.8% and 4.3% in February 2006 and 15.5% and 26.3% in September 2002 become 2.3% and 2.2% in February 2006 and 13.0% and 12.7% in September 2002. Recall that Tu and Zhou (2004) find that although the optimal portfolio weights that account for fat tails can be substantially different from those obtained under the usual normality assumption, the resulting utility loss is actually small in terms of the certainty-equivalent return. In particular, under 20% margin requirements, the *maximum* loss across various priors on asset pricing models is only 0.7% per year for a mean-variance

[32] Under a frequentist framework, Guidolin and Timmermann (2007, 2008a, b) provide various novel ways to examine the economic value of incorporating regime switching into asset allocations.

[33] To avoid look-ahead bias, all of the results for a given month t are obtained for a sample that ends in month $t-1$, one month before the given month t. In addition, because of heavy computational burden, in Figure 4 we calculate the gains for the most recent five years. Moreover, in contrast to Table 4, here the probability of being in a bull or bear regime is not set to 100%.

Figure 4 **Certainty-Equivalent Return Gains and Sharpe Ratio Gains**



Notes. This figure displays in percentage points the annualized certainty-equivalent return (CER) gains in panels A and B and the monthly Sharpe ratio (SR) gains in panels C and D associated with incorporating regime switching. In panels A and B, the risk aversion coefficient is equal to 10 (dotted line) and 2.83 (solid line), respectively. The mispricing priors are $\sigma_\alpha = 0$ (panels A and C) and $\sigma_\alpha = \infty$ (panels B and D). We report the results for the five-year period from March 2001 to February 2006, under no margin constraints ($c = \infty$).

investor with a risk aversion coefficient of 2.83. In contrast, if regime switching is not taken into account, with the same level of risk aversion and under the same margin requirements, even the *minimum* loss across various priors on asset pricing models is larger than 2%, and the larger losses can exceed 10%. These findings suggest that the economic value of regime switching is largely independent of whether model uncertainty with respect to the underlying asset pricing models is taken into account or not.[34] Therefore, it is important to take regime switching into account in making portfolio decisions irrespective of any concerns about model uncertainty.

3.4. Further Analyses

In this subsection, additional analyses are conducted to address several issues. First, although we document significant economic value of incorporating regime switching into portfolio decisions from an ex ante perspective, the ex ante in-sample gains may not be evident out-of-sample. To address this concern, we run ex post out-of-sample analysis. Second, we examine the robustness of RSM using simulated data. Third, we investigate in-sample and out-of-sample forecasting performances. Fourth, we analyze whether RSM can capture the time variation in size and value premia, and whether RSM can capture industry rotations. Finally, given the recent turmoil in financial markets, we examine the performance of the proposed regime switching approach for the 2006–2008 period.

First, we analyze the ex post out-of-sample performance.[35] Specifically, we implement a recursive scheme. For a testing period with a length of T^* months, the optimal portfolios under RSM and SSM are computed using the data from July 1963 to each of the months from $T^* - 1$ months before January 2006 to January 2006. For example, if $T^* = 120$, then the optimal portfolios under RSM and SSM are first computed using the data from July 1963 to March 1996, are then computed using the data from July 1963 to April 1996, etc., and finally are computed using the data from July 1963 to January 2006. This procedure produces a

[34] Interestingly, under a special case of RSM where only the means are allowed to vary across regimes but the covariances are assumed to be constant, the losses tend to be greatly reduced though remain economically significant in market downturns.

[35] In a different context, Guidolin and Timmermann (2008b) systematically analyze out-of-sample performance of regime switching models versus alternative models.

Table 5 Ex Post Out-of-Sample Performance

	$\sigma_\alpha = 0$	$\sigma_\alpha = 1\%$	$\sigma_\alpha = 2\%$	$\sigma_\alpha = \infty$
		Panel A: $c = \infty$		
$T^* = 60$				
RSM	37.9	56.4	55.6	54.8
SSM	34.8	54.7	54.4	54.3
$T^* = 120$				
RSM	25.8	45.3	45.1	44.8
SSM	25.1	43.7	43.7	43.7
		Panel B: $c = 5$		
$T^* = 60$				
RSM	37.1	61.2	57.3	62.1
SSM	34.0	52.7	53.3	58.1
$T^* = 120$				
RSM	23.0	39.0	40.0	43.7
SSM	24.7	39.5	34.8	37.5

Note. This table reports ex post out-of-sample Sharpe ratios generated by various optimal portfolios corresponding to RSM and SSM for priors on FF of $\sigma_\alpha = 0$, $\sigma_\alpha = 1\%$, $\sigma_\alpha = 2\%$, and $\sigma_\alpha = \infty$, for the unconstrained case ($c = \infty$) and the constrained case with 20% margin requirements ($c = 5$), and for two choices of T^*, namely, $T^* = 60$ and $T^* = 120$.

time series of 120 "ex post" excess returns for the optimal portfolios under RSM and SSM. To illustrate, let $\omega_{\mathrm{RSM},t}$ and $\omega_{\mathrm{SSM},t}$ denote the optimal portfolios in a given month t under RSM and SSM, respectively, and let r_{t+1} denote the excess return realized in the next month, $t+1$. The realized excess returns of $\omega_{\mathrm{RSM},t}$ and $\omega_{\mathrm{SSM},t}$ are $r_{\mathrm{RSM},t+1} = \omega'_{\mathrm{RSM},t} r_{t+1}$ and $r_{\mathrm{SSM},t+1} = \omega'_{\mathrm{SSM},t} r_{t+1}$, respectively. Next, following Avramov (2004), we compute the ex post out-of-sample Sharpe ratio by dividing the average value of the T^* realized returns by the standard deviation. Table 5 reports the results on ex post out-of-sample performance. In particular, the table displays out-of-sample Sharpe ratios corresponding to RSM and SSM for priors on FF of $\sigma_\alpha = 0$, $\sigma_\alpha = 1\%$, $\sigma_\alpha = 2\%$, and $\sigma_\alpha = \infty$, for the unconstrained case ($c = \infty$) and the constrained case with 20% margin requirements ($c = 5$), and for two choices of T^*, namely, $T^* = 60$ and $T^* = 120$. In 14 of the 16 cases, the optimal portfolios under RSM generate higher out-of-sample Sharpe ratios. Therefore, consistent with the ex ante results, RSM generally outperforms SSM ex post as well.

Second, although it seems naive to believe that there exists only one state, some investors may believe that the true data-generating process (DGP) of stock returns follows SSM. In such a case, using RSM to model the DGP may lead to significant underperformance because RSM would be misspecified by definition. To address this concern, we run an analysis using simulated data. The sample means and covariance matrix of the full sample from July 1963 to February 2006 are treated as the true parameters in the simulation. Then, 1,000 data sets with sample size $T = 512$ are simulated from the normal distribution with the calibrated parameters. For each simulated data set, we compute the CERs and SRs in the ending month as in (11)–(14). However, the predictive means and covariance matrix are the first two predictive moments under SSM, and not under RSM as before, because SSM is now the true DGP. Although RSM should do worse than SSM by design, the underperformance turns out to be very small. For instance, across various mispricing priors, the largest difference in the average of the monthly SR of the 1,000 simulated data sets is 0.4% (=33.7% (SSM) − 33.3% (RSM)), and the largest difference in the average CER is 0.3% per year (=20.1% (SSM) − 19.8% (RSM)) when $\gamma = 10$. Intuitively, RSM contains SSM as a special case. Therefore, even when the true DGP follows SSM, the results under RSM can still be close to those under SSM.

Third, we examine whether the expected returns implied by RSM are closer to the realized returns than the expected returns implied by SSM. In doing so, we study the forecasting errors from both in-sample and out-of-sample perspectives. In the out-of-sample case, all the estimations for a given month t are done using the data up to month $t-1$, one month before the decision-making month. In the in-sample case, the estimations are conducted using all the data. Over the five-year period from March 2001 to February 2006, regardless of the degree of mispricing uncertainty, for 24 of the 28 risky portfolios (25 size and book-to-market portfolios plus MKT, SMB, and HML), the average of the absolute value of the difference between the implied expected returns and the realized returns is smaller under RSM compared to SSM. In the out-of-sample case, RSM has smaller average absolute differences for only 7 of the 28 portfolios. Therefore, RSM appears to provide superior forecasts of mean returns in-sample but not out-of-sample. Nevertheless, the out-of-sample underperformance of RSM is not significant. In the worst case, the average absolute difference of RSM is only 1.47% larger than that of SSM.[36]

Fourth, there is some evidence that the size and value premia disappear in the 1990s and reappear in more recent years.[37] This raises the question as to whether the RSM of this study can produce a similar pattern of time variation over the last 20 years or so. Under RSM, over the 20 years from March 1986 to February 2006, the implied in-sample average return

[36] Note that the performance of RSM in terms of mean-return forecasting is different from that in terms of CERs and SRs because the latter depend not only on the estimations of means but also on the estimations of variances and covariances.

[37] Among others, Fama and French (1992) document a weaker size premium since the 1980s. In a recent paper, Avramov and Chordia (2006a) show that when one allows stock-level beta to vary with firm-level size and book-to-market as well as with macroeconomic variables, the size and value anomalies can be explained.

(standard error) of the size factor is 2.97% (2.99%) per year for the subperiod from March 1986 to February 2003, and 5.44% (0.51%) per year for the subperiod from March 2003 to February 2006. Furthermore, the implied in-sample average return (standard error) of the value factor is 5.35% (1.88%) per year for the first subperiod (March 1986 to February 2003) and 3.86% (0.29%) per year for the second subperiod (March 2003 to February 2006). Therefore, RSM appears to deliver the disappearance and reappearance pattern of the size premium. However, a match in the time variation of the value premium appears to be more difficult to obtain.

Fifth, Avramov and Wermers (2006) examine optimal portfolios of mutual funds formed on macroeconomic information and find large variation in industry tilt over the business cycle. Therefore, it may be of interest to examine whether the RSM of this study can identify industries that outperform during different economic climates. To address this issue, we replace the set of FF 25 size and book-to-market portfolios by the set of FF 17 industry portfolios. We find that under RSM, some industries tend to perform relatively well in expansions whereas others tend to perform relatively well during recessions. For instance, for Financials and Utilities, the average excess returns implied by RSM are 9.61% (Financials) and 6.45% (Utilities) per year across NBER expansions, whereas they are −1.35% (Financials) and 1.44% (Utilities) per year across NBER recessions.[38] Hence, under the RSM of this study, Utilities outperform Financials during economic downturns whereas Financials outperform Utilities during economic upturns. In addition, when the set of FF 25 size and book-to-market portfolios is replaced by the set of FF 17 industry portfolios, the impact of incorporating regime switching into portfolio decisions remains significant, regardless of the degree of asset pricing model uncertainty.[39]

Finally, given the recent turmoil in financial markets, it is of interest to examine the performance of the proposed Bayesian portfolio model with regime switching during the 2006 to 2008 period.[40] The regime switching model is applied on the extended sample including the 34 observations from March 2006 to December 2008 on the 25 size and book-to-market portfolios and the three-factor portfolios. A few interesting results emerge. First, our model suggests that the stock market switches from "bull" to "bear" around December 2007 to January 2008. From March 2006 to December 2007, the probability of being in the bull regime is always above 50%. However, in January 2008, the probability of being in the bull regime falls to 44.2% whereas the probability of being in the bear regime rises to 55.8%, exceeding the 50% level. The probability of being in the bear regime stays above 50% for most of 2008, except for March 2008 when it drops to 48.2%, slightly below 50%.[41]

Second, the weights on the optimal portfolios change significantly when regime switching is incorporated, regardless of the degree of mispricing uncertainty. Here, we examine the portfolio impact of November 2006 and October 2008, given that in the extended period from March 2006 to December 2008, November 2006 is associated with the highest probability of being in the bull regime (98.3%), and October 2008 is associated with the highest probability of being in the bear regime (99.8%). In November 2006, the optimal weight allocated to the market portfolio MKT under SSM is \$42.0 ($\sigma_\alpha = 0$), \$−13.8 ($\sigma_\alpha = 1\%$), and \$−59.9 ($\sigma_\alpha = \infty$), and under RSM this weight increases by \$37.0 ($\sigma_\alpha = 0$), \$87.1 ($\sigma_\alpha = 1\%$), and \$136.5 ($\sigma_\alpha = \infty$), conditional on a bull regime prevailing. On the other hand, in October 2008, the optimal weight allocated to the market portfolio MKT under SSM is \$33.0 ($\sigma_\alpha = 0$), \$−14.1 ($\sigma_\alpha = 1\%$), and \$−51.6 ($\sigma_\alpha = \infty$), and under RSM it decreases by \$46.0 ($\sigma_\alpha = 0$), \$40.4 ($\sigma_\alpha = 1\%$), and \$136.5 ($\sigma_\alpha = \infty$), conditional on a bear regime prevailing.[42] In addition, the portfolio weights tend to change more under RSM than under SSM. For instance, between November 2006 and October 2008, under $\sigma_\alpha = 0$, for each \$100 the optimal weight allocated to the market portfolio MKT decreases by \$92.0 under RSM when the market switches from a upturn to a downturn whereas it decreases by only \$9.0 under SSM.

Moreover, we compare the out-of-sample performances of the optimal portfolios implied by RSM and SSM. Similar to Table 5, over the period from March 2006 to December 2008, the out-of-sample SRs corresponding to RSM are generally larger than those corresponding to SSM for various priors on FF, for both the unconstrained case ($c = \infty$) and the constrained case with 20% margin requirements ($c = 5$). For instance, for priors on FF of $\sigma_\alpha = 0$, $\sigma_\alpha = 1\%$, $\sigma_\alpha = 2\%$, and $\sigma_\alpha = \infty$ and for the unconstrained case ($c = \infty$), the out-of-sample Sharpe ratios are −1.2%,

[38] The results correspond to the diffuse prior on mispricing ($\sigma_\alpha = \infty$). Under the other priors on mispricing, the results are similar.

[39] Given space constraints, the details are omitted but available upon request.

[40] We thank David A. Hsieh (the department editor) for suggesting that we investigate the 2006–2008 period.

[41] Although the regimes are not observable, as in Figure 3, the empirical probability can be computed. Figure EC.2, which is available in the e-companion, plots this probability from July 1963 through December 2008.

[42] The portfolio weights on the other assets under the unconstrained case ($c = \infty$) and the portfolio weights under the constrained case with 20% margin requirements ($c = 5$) are omitted here but available in Tables EC.1 and EC.2 in the e-companion.

14.7%, 21.9%, and 26.5% for the optimal portfolios under RSM and −13.4%, 2.9%, 8.2%, and 10.3% for the optimal portfolios under SSM. Therefore, the regime switching model appears promising and performs better than the single-state model in the recent period of financial turmoil.

4. Conclusions

This paper provides a Bayesian framework for making portfolio decisions that accounts for regime switching together with pricing model uncertainty and parameter uncertainty. The findings reveal that when regime switching is taken into account, the optimal portfolio weights deviate substantially from those that obtain under the single-state model under various prior beliefs on the underlying asset pricing models. In terms of the certainty-equivalent return measure, the loss to an investor who is forced to hold the portfolio that is optimal under the single-state model is economically meaningful regardless of the degree of pricing model uncertainty. Furthermore, these results are generally robust out-of-sample. These findings suggest that the more realistic regime switching model is fundamentally different from the commonly used single-state model, and should be employed instead in portfolio decisions irrespective of any concerns about model or parameter uncertainty. Finally, the framework may have other applications. For example, one can potentially apply it to investigate whether hedge fund performance is regime dependent and whether hedge funds can indeed offer favorable returns during bear markets. These topics are beyond the scope of this paper, but they are interesting questions for future research.

5. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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