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Abstract

In this chapter, we recommend the use of both the mean-variance (MV) rule and mean-variance-ratio (MVR) test to examine the performance of investment assets. We illustrate the approaches by investigating the performance of different Asian hedge funds over an entire sample period as well as over sub-periods that may be described as boom, crisis, and recovery in the recent past. The MV criterion suggests that the largest mean fund, the smallest standard deviation fund, the largest mean-variance-ratio fund, and the largest Sharpe-ratio funds outperforms the S&P 500 either from the viewpoints of risk averters or risk seekers. Our MVR test results support the inference obtained using the MV criterion. This finding helps investors make informed decision when investing in Asian hedge funds.

Keywords: mean variance ratio; Sharpe ratio; hypothesis testing; uniformly most powerful unbiased test; internet bubble; fund management

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1.1 Introduction

In 1990, the entire hedge fund industry was estimated at about US$20 billion. Globally, assets under management (AUM) totaled over US$2,050 billion at the end of 2012. While hedge funds are well established in the United States and Europe, they had only begun to grow aggressively in Asia since the turn of the century. The Bank of Bermuda estimated that of hedge funds operating in Asia (including those in Japan and Australia), 30 were established in year 2000 and 20 in 2001. Subsequently, Asian hedge fund experienced tremendous growth from 2000 to 2007, when the number of funds increased six-fold and total AUM grew by more than 900% to reach US$176 billion. This momentum was halted during the global financial crisis when the funds faced heavy redemptions and significant losses. Growth in the post-2008 period has been slow and has yet to match what was seen in the industry in the years before the crisis.

With an estimated AUM of more than US$127 billion in 2012, hedge fund investments in Asia remain an important slice of the hedge fund investment universe. Hence, investing in Asian hedge funds requires a better understanding of their performance and risk, specifically the impact when such funds are included in the investors' portfolios. Since the financial crisis, investors have become more aware that constructing an investment portfolio that provides "limited losses and more predictable returns" remain the holy grail of investing. In 2008, many investors thought they had constructed well-diversified portfolios, and yet the global financial crisis showed that, contrary to their expectations, all assets went down like they were tied together with a rope.

The core-satellite concept is an approach that is the new mainstream portfolio construction. Brinson et al. (1986) has concluded that a portfolio's asset allocation is the primary determinant of portfolio return variability and that investors need to adopt a strategy that provides good portfolio diversification. Singleton (2002) explains that the core-satellite approach put most competitively priced assets at the core, while satellite assets allow professional managers a better chance to pick up bargains. In this chapter, we focus on the role of Asian hedge funds as a 'satellite' investment. Specifically, we examine the requirement that a satellite investment provides upside capture and downside protection to the core investment that is usually a stock or stock and bond portfolio.

Typically, hedge fund managers adopt investment strategies to provide absolute returns under different market conditions compared with traditional fund managers who manage relative to benchmarks. This characteristic of hedge fund makes them ideal inclusion as satellite assets. In addition, it is commonly believed that hedge funds generate positive alphas and the returns are generally uncorrelated with traditional asset classes. Amenc et al. (2003) have argued that hedge funds, including those in Asia have low correlation with traditional asset classes like stocks and bonds and attempt to offer protection in falling and/or volatile markets. Lee et al. (2006) have proposed a practical approach to filter hedge funds using past returns, where investors are assumed to have sophisticated preferences - i.e., they like downside protection, whilst looking for yield enhancement. Wong et al. (2008) have used the stochastic dominance (SD) approach to rank Asian hedge fund performance under negative domain or bear markets and positive domain or bull markets.

Findings on the benefits of hedge funds in general and Asian hedge funds in particular support their inclusion as satellite assets in the core-satellite investment approach. According to Vanguard Investment, actively managed funds of which hedge funds are a part provide the opportunity for outperformance, while minimizing potential losses along with the added advantage in providing access to a wide range of specialist styles, markets, sectors and geographies, offering infinite choice for diversification. Hence, in the implementation of the core-satellite approach, investors would first determine the asset allocation, allocate core and satellite proportions and finally select the active funds.

In this chapter, we apply the mean-variance-ratio test of Bai et al. (2011c, 2012) to analyze the risk and performance of Asian hedge funds from the viewpoints of U.S. equities investors benchmarked to the S\&P 500. Using the power of the mean-variance-ratio test we would be able to examine the performance of Asian hedge funds in different market conditions relative to that of the S&P 500. The span of eight years from 2005-2012 allows us to examine the performance of Asian hedge funds during a market boom, a financial crisis and subsequent recovery in a low growth and low interest rate environment. Asian hedge fund performance during differing market conditions allow us to examine whether the inclusion helps to insulate the overall portfolio when the market is down while benefitting investors during market booms and recovery. Section 1.2 describes the data used. The empirical methodology is described in Section 1.3. The results of our analysis of Asian hedge funds are presented in Section 1.4 and Section 1.5 concludes.

1.2 Data

The study uses Asian hedge funds monthly returns obtained from the Eurekahedge database and the S&P 500 index. We pick out three outstanding hedge funds from nearly 300 hedge funds that provided complete data over the sample period January 2005 through December 2012. The three outstanding funds chosen were (1) maximum mean fund: Golden China Fund (GC Fund) - Non-restricted Class, (2) minimum standard deviation (also highest MVR) fund: PM CAPITAL Enhanced Yield Fund (PMCEY Fund) and (3) highest Sharpe ratio (SR) fund: Evenstar Sub-Fund I (ES Fund).

1.3 Methodology

Before we discuss the MVR test approach, in this chapter, we first recommend that academics and practitioners use the mean-variance (MV) approach. Markowitz (1952) has introduced the MV rule (for risk averters). The idea is that for any two returns *X* and *Y* with means μ_X and μ_Y and standard deviations σ_X and σ_Y, respectively, *X* is said to dominate *Y* by the MV rule for risk averters, denoted by *X* MV_{RA} *Y*, if $\mu_X \geq \mu_Y$ and $\sigma_X \geq \sigma_Y$ with at least one strictly inequality holding. Wong (2007) has introduced the MV rule for risk seekers such that *X* is said to dominate *Y* by the MV rule for risk seekers, denoted by *X* MV_{RS} *Y*, if $\mu_X \geq \mu_Y$ and $\sigma_X \leq \sigma_Y$ in which the inequality holds in at least one of the two. In addition, Wong (2007) has proved that if both X and Y belongs to the same location-scale family or the same linear combination of location-scale families, X MV_{RA} Y implies $E[u(X)] \geq E[u(Y)]$ for any risk-averse (risk-seeking) investor. On the other hand, Markowitz (2012) tests the ability of six functions of the arithmetic mean and variance to approximate the geometric mean return.

Let X_i and Y_i (i = 1, 2, … , n) be independent excess returns drawn from the corresponding normal distributions $N(\mu,\sigma^2)$ and $N(\eta,\tau^2)$ with joint density p(x,y) such that

$$p(x,y) = k \times \exp\left(\frac{\mu}{\sigma^2}\sum x_i - \frac{1}{2\sigma^2}\sum x_i^2 + \frac{\eta}{\tau^2}\sum y_i - \frac{1}{2\tau^2}\sum y_i^2\right) \quad (1.1)$$

where k = $\left(2\pi\sigma^2\right)^{-n/2}\left(2\pi\tau^2\right)^{-n/2} \exp\left(-\frac{n\mu^2}{2\sigma^2}\right)\exp\left(-\frac{n\eta^2}{2\tau^2}\right)$

To evaluate the performance of the prospects X and Y, financial professionals are interested in testing the hypotheses

$$H_0^* : \frac{\mu}{\sigma} \leq \frac{\eta}{\tau} \quad \text{versus} \quad H_1^* : \frac{\mu}{\sigma} > \frac{\eta}{\tau} \quad (1.2)$$

to compare the performance of their corresponding SRs, $\frac{\mu}{\sigma}$ and $\frac{\eta}{\tau}$, the ratios of the excess expected returns to their standard deviations.

Rejecting H_0^* implies *X* to be the better investment prospect with larger SR because *X* has either larger excess mean return or smaller standard deviation or both. Jobson and Korkie (1981) and Memmel (2003) have developed test statistics to test the hypotheses in Equation (1.2) for large samples but their tests are not appropriate for testing small samples as the distribution of their test statistics is only valid asymptotically, but is not valid for small samples. However, it is especially relevant in investment decisions to test the hypothesis in Equation (1.2) for small samples to provide useful investment information to investors. Furthermore, as it is impossible to obtain any UMPU test statistic to test the inequality of the SRs in Equation (1.2) for small samples. Bai, et al. (2011c, 2012) have proposed to use the following hypothesis to test the inequality of the MVRs:

$$H_{01} : \frac{\mu}{\sigma^2} \le \frac{\eta}{\tau^2} \quad \text{versus} \quad H_{11} : \frac{\mu}{\sigma^2} > \frac{\eta}{\tau^2} \tag{1.3}$$

In addition, they have developed the UMPU test statistic to test the above hypotheses. Rejecting H_0 suggests *X* will have smaller variance or larger excess return or both leading to the conclusion that *X* is the better investment. As investors may be interested in conducting the two-sided test to compare the MVRs, the following hypotheses are included in our study:

$$H_{02} : \frac{\mu}{\sigma^2} = \frac{\eta}{\tau^2} \quad \text{versus} \quad H_{12} : \frac{\mu}{\sigma^2} \ne \frac{\eta}{\tau^2} \tag{1.4}$$

One may argue that the SR test is better because it is scale invariant whereas the MV ratio test is not. To support the MVR test as an acceptable alternative test statistic, Bai, et al. (2011c, 2012) show that in some financial processes, the mean change in a short period of time is proportional to the variance change. Thus, when the time period is small, the MVR will be advantageous over the SR.

To further support the use of the MVR test, Bai, et al. (2011c, 2012) have documented the MVR in the context of Markowitz MV optimization theory. An advantage of using the MVR test over the SR test is that it not only allows investors to compare the performance of different assets, but it also provides investors with information of the asset weights. The MVR test enables investors to compute the corresponding allocation for the assets. On the other hand, as the SR is not proportional to the weight of the corresponding asset, an asset with the highest SR would not infer that one should put highest weight on this asset as compared with our MVR. In this sense, the test proposed by Bai, et al. (2011c, 2012) is superior to the SR test.

Bai, et al. (2011c, 2012) have developed both one-sided UMPU test and two-sided UMPU test equality of the MVRs in comparing the performances of different prospects with hypotheses stated in Equations (1.3) and (1.4) respectively. We first state the one-sided UMPU test for the MVRs as follows:

Theorem 1.1 *Let X_i and Y_i $(i = 1, 2, \dots, n)$ be independent random variables with joint distribution function defined in Equation (1.1). For the hypotheses setup in Equation (1.3), there exists a UMPU level-α test with the critical function $\phi(u,t)$ such that*

$$\phi(u,t) = \begin{cases} 1, & \text{when } u \geq C_0(t) \\ 0, & \text{when } u < C_0(t) \end{cases} \tag{1.5}$$

where C_0 is determined by

$$\int_{C_0}^{\infty} f_{n,t}^{*}(u)du = K_1; \tag{1.6}$$

with

$$f_{n,t}^{*}(u) = \left(t_2 - \frac{u^2}{n}\right)^{\frac{n-1}{2}-1} \left(t_3 - \frac{(t_1 - u)^2}{n}\right)^{\frac{n-1}{2}-1},$$

$$K_1 = \alpha \int_{\Omega} f_{n,t}^{*}(u)du;$$

in which

$$U = \sum_{t=1}^{n} X_t \quad T_1 = \sum_{t=1}^{n} X_t + \sum_{t=1}^{n} Y_t, \quad T_2 = \sum_{t=1}^{n} X_t^2, \quad T_3 = \sum_{t=1}^{n} Y_t^2, \quad T = (T_1, T_2, T_3);$$

with $\Omega = \left\{u \,\middle|\, \max\left(-\sqrt{nt_2}, t_1 - \sqrt{nt_3}\right) \leq u \leq \min\left(\sqrt{nt_2}, t_1 + \sqrt{nt_3}\right)\right\}$ *to be the support of the joint density function of (U,T).*

We call the statistic U in Theorem 1.1 the one-sided MVR test statistic or simply the MVR test statistic for the hypotheses setup in Equation (1.3) if no confusion arises. In addition, Bai, et al. (2011c, 2012) have introduced the two-sided UMPU test statistic as stated in the following theorem to test for the equality of the MVRs listed in Equation (1.4):

Theorem 1.2 *Let X_i and Y_i (i = 1, 2, ... , n) be independent random variables with joint distribution function defined in Equation (1.1). Then, for the hypotheses setup in Equation (1.4), there exists a UMPU level-α test with critical function:*

$$\phi(u,t) = \begin{cases} 1, & \text{when } u \le C_1(t) \text{ or } \ge C_2(t) \\ 0, & \text{when } C_1(t) < u < C_2(t) \end{cases} \tag{1.7}$$

in which C_1 and C_2 satisfy

$$\begin{cases} \int_{c_1}^{c_2} f_{n,t}^{*}(u)du = K_2 \\ \int_{c_1}^{c_2} uf_{n,t}^{*}(u)du = K_3 \end{cases} \tag{1.8}$$

where

$$K_2 = (1-\alpha)\int_{\Omega} f_{n,t}^{*}(u)du,$$

$$K_3 = (1-\alpha)\int_{\Omega} uf_{n,t}^{*}(u)du.$$

The terms $f_{n,t}^{*}(u)du$, T_i (i=1,2,3) and T are defined in Theorem 1.1.

We call the statistic U in Theorem 1.2 the two-sided MVR test statistic or simply the MVR test statistic for the hypotheses setup in Equation (1.4) if no confusion occurs. To obtain the critical values, C_1 and C_2 for the test, readers may refer to Bai, et al. (2011c, 2012).

1.4 Analysis of Asian Hedge Funds

In this section, we examine the performance of Asian hedge funds over a sample period from January 2005 to December 2012 and its sub-periods. The objectives of our study includes (1) to compare the performance of the funds being chosen, (2) to compare the performance of the funds with the S&P 500, and (3) to examine the robustness of the funds' performance in different market environments. The time series plot of the S&P 500 stock index from January 2005 to December 2012 is shown in Figure 1. From the figure, we note that the stock index peaked in September 2007; before collapsing to a trough in February 2009. Subsequent, the index underwent a period of gradual recovery. In order to analyze the funds in different market conditions, we divide the sample period into three sub-periods: January 2005 to September 2007, October 2007 to February 2009 and March 2009 to December 2012 that we describe as boom, crisis, and recovery periods, respectively. Since most investors prefer to invest in funds with higher expected returns and smaller risk, we selected the funds with the largest sample mean, smallest standard deviation, highest Sharpe ratio, and highest mean-variance ratio. Nonetheless, the result in Table 1 shows that the fund with the smallest standard deviation also has the highest mean-variance ratio. Hence, in our analysis, it only suffices to use the monthly returns of three hedge funds and the S&P 500 index.

Insert Figure 1

Insert Table 1

We let X_1, X_2, X_3, and Y be the monthly returns of Golden China Fund - Non Restricted Class, PM CAPITAL Enhanced Yield Fund, Evenstar Sub-Fund I, and the S&P 500 respectively, of which X_1 has the largest mean, X_2 has the smallest standard deviation and the largest mean-variance ratio, and X_3 has the largest Sharpe ratio in the entire period. The plot of the returns of the S&P 500 and the three hedge funds are presented in Figure 2.

Insert Figure 2

To compare the performance of the chosen three funds: X_1, X_2, X_3, we (a) compare the performance among the funds and (b) compare the performance of the fund with the S&P 500 index, Y, for the entire sample period and for each of the sub-periods - boom, crisis, and recovery. We apply the mean-variance criterion for both (a) and (b) but, for simplicity, we use the MVR test to conduct (b) only. In addition, we check whether the performance of a fund is robust. Here, ``robustness'' means that the performance of a fund be the same or does not change too much in different conditions.

We first discuss the results of applying the mean-variance criterion to compare the performance among the three funds chosen and between each of these funds with the S&P. To do so, for the returns of a pair of funds, *X* and *Y* with means μ_X and μ_Y and standard deviations σ_X and σ_Y, respectively, we will test whether $\mu_X \geq \mu_Y$ and whether $\sigma_X \leq \sigma_Y$ or $\sigma_X \geq \sigma_Y$ with at least one strictly inequality holding. If $\mu_X \geq \mu_Y$ and $\sigma_X \leq \sigma_Y$, *X* is said to dominate *Y* by the MV rule for risk averters, denoted by *X* MV_{RA} *Y* and risk averters prefer *X* to *Y*. On the other hand, if $\mu_X \geq \mu_Y$ and $\sigma_X \geq \sigma_Y$, *X* is said to dominate *Y* by the MV rule for risk seekers, denoted by *X* MV_{RS} *Y* and risk seekers will prefer *X* to *Y*.

We first apply the MV criterion to compare the performance of X_i with the S&P 500, *Y* for i = 1, 2, 3 for the entire sample period as well as each of the sub-periods. To do so, we first apply the t-test to test whether $\mu_X \geq \mu_Y$ and thereafter apply the F-test to test whether $\sigma_X \leq \sigma_Y$ or $\sigma_X \geq \sigma_Y$. The results are shown in Panel A of Table 2.

Insert Table 2

From the results of the t-test in the Table 2, we conclude that $\mu_{X_1} \geq \mu_Y$ for the entire sample period as well as for the boom period while we do not reject $\mu_{X_1} \geq \mu_Y$ in both the crisis and the recovery periods. On the other hand, from the results of the F-test, we conclude that $\sigma_{X_1} > \sigma_Y$ for the entire period as well as all the sub-periods viz. the boom, crisis and the recovery periods. Thus, we conclude that X_1 MV_{RS} Y and risk seekers will prefer X_1 to Y for the entire period and all the sub-periods. This, in turn, implies that (a1) in the viewpoints of risk seekers, the Golden China Fund - Non Restricted Class outperforms the S&P 500 for the entire period and during the boom and the recovery sub-periods but do not underperform the S&P 500 during the crisis, and (a2) the performance of Golden China Fund - Non Restricted Class is robust for the entire sample period and for all of the sub-periods (boom, crisis, and the recovery) when we compare its performance with that of the S&P 500.

On the other hand, from the results of the t-test in Table 2, we conclude that $\mu_{X_i} \geq \mu_Y$ in the crisis sub-period and do not reject that $\mu_{X_i} \geq \mu_Y$ for i=2, 3 for the entire sample period and for the sub-periods including the boom and the recovery periods. The results of the F-test show that $\sigma_{X_i} < \sigma_Y$ for i=2, 3 for the entire period and in any of the sub-periods. Thus, we conclude that X_i MV_{RA} Y for i=2, 3 and risk averters will prefer X_i to Y for i=2, 3 for the entire sample period and in any of the sub-periods. This finding in turn, implies that (b1) in the viewpoint of risk averters, PM CAPITAL Enhanced Yield Fund outperforms the S&P 500 for the entire sample period and in the crisis sub-period while (c1) Evenstar Sub-Fund I outperforms the S&P 500 for the entire sample period and in the boom and crisis sub-periods while these two funds do not underperform the S&P 500 in all other sub-periods, and (b2 and c2) the performance of both PM CAPITAL Enhanced Yield Fund and Evenstar Sub-Fund I is robust for the entire sample period and in any of the sub-periods when we compare its performance with that of the S&P 500.

We next apply the MV criterion to compare the performance among the funds X_i for i=1, 2, 3. The results of the t-test in Table 2 lead us to conclude that $\mu_{X_1} \geq \mu_{X_2}$ for the entire sample period as well as in the boom and recovery sub-periods while we do not reject $\mu_{X_1} \geq \mu_{X_2}$ for the crisis sub-period. In addition, the results of the F-test in Table 2 exhibits that $\sigma_{X_1} < \sigma_{X_2}$ for the entire sample period and all the sub-periods, Thus, we conclude that X_1 MV_{RS} X_2 and risk seekers will prefer X_1 to X_2 for the entire sample period and any of the sub-periods. This, in turn, implies that (d1) in the viewpoint of risk seekers Golden China Fund - Non Restricted Class outperforms PM CAPITAL Enhanced Yield Fund for the whole sample period and in both boom and recovery sub-periods and it does not underperform PM CAPITAL Enhanced Yield Fund during the crisis, and (d2) the performance of Golden China Fund - Non Restricted Class is robust when compared with the PM CAPITAL Enhanced Yield Fund.

Nonetheless, when we compare the performance between Golden China Fund - Non Restricted Class, X_1, and Evenstar Sub-Fund I, X_3, and between PM CAPITAL Enhanced Yield Fund, X_2, and Evenstar Sub-Fund I, X_3, the results are not robust. This finding can be explained as follows: the results of the t-test in Table 2 show that $\mu_{X_1} \geq \mu_{X_3}$ during the entire sample period and the boom and recovery sub-periods. However, the same test concludes that $\mu_{X_3} \geq \mu_{X_1}$ during the crisis sub-periods. On the other hand, the results of the F-test show that $\sigma_{X_1} > \sigma_{X_3}$ for the entire sample period and all the sub-periods. Thus, we conclude that X_1 MV_{RS} X_3 for the entire sample period as well as the boom and recovery periods and risk seekers will prefer X_1 to X_3 for the entire sample period and the boom and recovery sub-periods. However, the result concludes that X_3 MV_{RA} X_1 for the crisis period and risk averters will prefer X_3 to X_1 during the crisis sub-period. This result implies that (e1) in the viewpoint of risk seekers, Golden China Fund - Non Restricted Class outperforms Evenstar Sub-Fund I for the entire sample period and during both the boom and crisis but in the viewpoint of risk averters the preference order reverses in the crisis period. (e2) The performance between Golden China Fund - Non Restricted Class and Evenstar Sub-Fund I is not robust.

In comparing the performance between PM CAPITAL Enhanced Yield Fund X_2 and Evenstar Sub-Fund I X_3, the results of the t-test in Table 2 show that $\mu_{X_3} > \mu_{X_2}$ for the entire sample period and during the boom and crisis sub-periods. Based on the results, we may conclude that $\mu_{X_3} \geq \mu_{X_2}$ in the recovery sub-periods. Thus, Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund in sample mean. However, the results of the F-test show that $\sigma_{X_3} > \sigma_{X_2}$ for the entire sample period and for both the boom and recovery sub-periods but $\sigma_{X_2} > \sigma_{X_3}$ during the crisis sub-period. Thus, the MV analysis concludes that X_3 MV_{RS} X_2 for the entire sample period and for the boom and recovery sub-periods while X_3 MV_{RA} X_2 for the crisis sub-period. Thus, we conclude that (1) Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund and the results are robust in terms of the mean for the entire sample period and for any sub-periods and the results are robust, (2) from the viewpoint of risk seekers, Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund for the entire period and the boom and recovery sub-periods, and (3) for the viewpoints of risk averters, Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund during the crisis sub-period.

After practitioners have obtained the results using the MV criterion, we recommend that they use the MVR test to confirm the results. The advantage of using the MVR test is that we can use very few past observations to conduct the test and the test value can be used for prediction of the future performance of the funds. For simplicity, we have only applied the MVR test to compare the performance of X_i with the S&P 500, Y for i=1, 2, 3 for the entire sample period and in each of the sub-periods. For simplicity, we will only demonstrate the two-sided UMPU test.[1] To do so, we let X (presenting each of X_i) with mean μ_X and variance σ_X^2 be the monthly return on a hedge fund while Y with mean μ_Y and variance σ_Y^2 be the monthly return on the S&P 500 index. We test the following hypotheses:

$$H_0 : \frac{\mu_X}{\sigma_X^2} = \frac{\mu_Y}{\sigma_Y^2} \quad versus \quad H_1 : \frac{\mu_X}{\sigma_X^2} \neq \frac{\mu_Y}{\sigma_Y^2} \tag{1.9}$$

To test the hypotheses in Equation (1.9), we first compute the values of the test function U for the MVR statistic shown in Equation (1.7) and thereafter compute the critical values C_1 and C_2 under the test level of 5% for each pair of indices. The results are shown in Tables 3 to 5.

For comparison, we also compute the corresponding SR statistic developed by Jobson and Korkie (1981) and Memmel (2003) such that

[1] The results of the one-sided test which draw a similar conclusion are available on request.

$$z = \frac{\hat{\sigma}_2 \hat{\mu}_1 - \hat{\sigma}_1 \hat{\mu}_2}{\sqrt{\hat{\theta}}} \tag{1.10}$$

which follows standard normal distribution asymptotically with

$$\theta = \frac{1}{T}\left[2\sigma_X^2\sigma_Y^2 - 2\sigma_X\sigma_Y\sigma_{X,Y} + \frac{1}{2}\mu_X^2\sigma_Y^2 + \frac{1}{2}\mu_Y^2\sigma_X^2 - \frac{\mu_X\mu_Y}{\sigma_X\sigma_Y}\sigma_{X,Y}^2\right]$$

to test for the equality of the SRs for the funds by setting the following hypotheses such that

$$H_o^* : \frac{\mu_X}{\sigma_X^2} = \frac{\mu_Y}{\sigma_Y^2} \quad \textit{versus} \quad H_1^* : \frac{\mu_X}{\sigma_X^2} \neq \frac{\mu_Y}{\sigma_Y^2} \tag{1.11}$$

Instead of using six monthly returns to compute the values of our proposed statistic, we use all seventeen samples to compute the SR statistic. The results are also reported in Tables 3 to 5.

Insert Tables 3 to 5

Now, we use the MVR test to complement the findings from the MV criterion. Our MV criterion concludes that in the viewpoints of risk seekers, Golden China Fund - Non Restricted Class outperforms the S&P 500 in the entire period as well as each of the sub-periods. Our MVR test results do not reject this claim but does not strongly support this claim because the results of the MVR test in Tables 3(a) to 3(c) shows that the MVR test is not significant in any of the sub-periods (boom, crisis and recovery).

Nonetheless, our MVR test strongly supports the claim base on the MV criterion of the outperformance of PM CAPITAL Enhanced Yield Fund over the S&P 500. The MVR test results in Tables 4(a) to 4(c) show that (a) the difference of the MVR of PM CAPITAL Enhanced Yield Fund over the S&P 500 is positive for all sub-periods, (b) the averages of the differences of the MVR of PM CAPITAL Enhanced Yield Fund over the S&P 500 are positive for all sub-periods, and (c) the U statistic in most of the time periods in both boom and recovery sub-periods is significant. Thus, our MVR test strongly supports the claim from our MV criterion that (a) PM CAPITAL Enhanced Yield Fund outperforms the S&P 500 in all the sub-periods, and (b) the performance of PM CAPITAL Enhanced Yield Fund over the S&P 500 is robust.

On the other hand, our MVR test does support (but not strongly) the claim from our MV criterion of the outperformance of Evenstar Sub-Fund I over the S&P 500 for the entire sample period as well as all the sub-periods because (a) the difference of the MVR of Evenstar Sub-Fund I over the S&P 500 is positive for all sub-periods except two in the boom sub-periods, (b) the averages of the differences of the MVR of Evenstar Sub-Fund I over the S&P 500 are positive for all sub-periods. However, there is only one value of the U statistic is significant in the boom time period. Thus, the MVR test does support the claim from our MV criterion that Evenstar Sub-Fund I performed better than the S&P 500 in the entire sample period as well as in all the sub-periods but not strongly.

Overall, the results of MVR test show that Golden China Fund - Non Restricted Class which has highest sample mean during the whole period has the lowest robustness while PM CAPITAL Enhanced Yield Fund with smallest standard deviation, which also has highest mean-variance ratio possesses the highest robustness. As we can see Golden China Fund - Non Restricted Class's MVR is smaller than that of the S&P 500 five times in the boom market, four times in the crisis market and twelve times when market recovers, although the differences are not significant. On the other hand, we find from Tables 4 (a) to 4(c) that the MVR of PM CAPITAL Enhanced Yield Fund is greater than that of the S&P 500 in all three different market environments. In addition, PM CAPITAL Enhanced Yield Fund outperforms the S&P 500 significantly 10 times during the boom market and 8 times during the recover periods. The results in Table 5 (a) to 5(c) show that Evenstar Sub-Fund I with highest Sharpe ratio in the whole period also perform with robustness. Except for two sub-periods during the boom, all mean-variance ratios of Evenstar Sub-Fund I are larger than those of the S&P 500. We note that the above inference is in the eyes of risk averters as the MVR test cares of both larger mean and smaller variance. However, Golden China Fund - Non Restricted Class has the highest sample mean and also has larger variance. Thus, Golden China Fund - Non Restricted Class had outperformed the S&P 500 significantly and robustly for the entire sample period and during sub-periods in the eyes of risk seekers but not in the eyes of risk averters.

1.5 Concluding Remarks and Discussions

In summary, in this chapter, we recommend the use of both mean-variance (MV) rule and mean-variance-ratio (MVR) test to examine the performance of financial assets. We illustrate the approaches by investigating the performance of different Asian hedge funds over a sample period from January 2005 to December 2012 and over sub-periods. In this study, we examined three funds, viz. the funds with the largest mean (Golden China Fund - Non Restricted Class), the smallest standard deviation (PM CAPITAL Enhanced Yield Fund), the largest mean-variance ratio (PM CAPITAL Enhanced Yield Fund), and the largest Sharpe ratio (Evenstar Sub-Fund I) and the S&P 500. Since PM CAPITAL Enhanced Yield Fund has the smallest standard deviation and the largest mean-variance ratio, The objectives of our paper includes (1) to compare the performance of the funds being chosen, (2) to compare the performance of the funds with the S&P 500, and (3) to examine the robustness of the funds' performance in different market environments: boom, crisis, and recovery periods.

The MV criterion shows that (a) in the viewpoints of risk seekers, Golden China Fund - Non Restricted Class outperforms the S&P 500, (b) in the viewpoint of risk averters, PM CAPITAL Enhanced Yield Fund outperforms the S&P 500, (c) in the viewpoints of risk seekers, Evenstar Sub-Fund I outperforms the S&P 500, and (d) in the viewpoint of risk seekers Golden China Fund - Non Restricted Class outperforms PM CAPITAL Enhanced Yield Fund in the entire sample period and for all the sub-periods. The above results are robust.

However, our MV criterion documents that (d) from the viewpoint of risk seekers, Golden China Fund - Non Restricted Class outperforms Evenstar Sub-Fund I in the entire period as well as in both the boom and crisis but in the viewpoint of risk averters the preference order reverses in the crisis period, (e) from the viewpoint of risk seekers, Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund for the entire sample period and for the boom and recovery sub-periods, and (f) from the viewpoints of risk averters, Evenstar Sub-Fund I outperforms PM CAPITAL Enhanced Yield Fund in the crisis sub-periods. These results are not robust.

We next conducted the MVR tests to complement the findings using the MV criterion. Basically, the results of the MVR test support (but not strongly) the results using the MV criterion that risk seekers, Golden China Fund - Non Restricted Class outperforms the S&P 500 and Evenstar Sub-Fund I outperforms the S&P 500. On the other hand, the MVR test strongly supports the finding using the MV criterion that PM CAPITAL Enhanced Yield Fund outperforms the S&P 500 and these results are robust.

Overall, the results of MVR test show that Golden China Fund - Non Restricted Class which has highest sample mean during the whole period has the lowest robustness while PM CAPITAL Enhanced Yield Fund with smallest standard deviation, which also has highest mean-variance ratio possesses the highest robustness. We note that the above inference is in the eyes of risk averters because MVR test concerns both larger mean and smaller variance. However, Golden China Fund - Non Restricted Class has the highest sample mean and but also the larger variance. Thus, Golden China Fund - Non Restricted

Class could outperform the S&P 500 significantly and robustly in the entire sample period and during the sub-periods in the eyes of risk seekers but not in the eyes of risk averters.

We note that Sharpe ratios of, say, PM CAPITAL Enhanced Yield Fund and Evenstar Sub-Fund I are all significantly larger than those of the S&P 500 in all three different market environments. But we cannot tell which if this finding is robust. In relation to our objective of examining performance over different market conditions, the Sharpe ratio cannot detect the vibration of the performance of, say, Golden China Fund - Non Restricted Class in different market environments because the Sharpe ratios of Golden China Fund - Non Restricted Class are all 'slightly' larger than those of the S&P 500. This is because Sharpe ratio applies in large-samples. So during significant market changes and with only a small sample, we can make wrong decisions using Sharpe ratio as the inference based on Sharpe ratio test may not be reliable.

Lastly, we note that the findings from our the MV criterion and the MVR test are useful for investors because, for example, different robustness of the three funds found from our analysis can assist the fund managers to manage the Asian hedge funds managers more effectively, especially in managing their risk - managing their downside while allowing for upside capture. For investors who want higher returns like Golden China Fund - Non Restricted Class, they should understand that the price to pay may be increased risk and lower robustness.

There are two basic approaches to the problem of portfolio selection under uncertainty. One approach is based on the concept of utility theory (Gasbarro, et al., 2007, 2012; Wong et al., 2006, 2008). Several stochastic dominance (SD) test statistics have been developed, see, for example, Bai, et al. (2011a) and the references therein for more information. This approach offers a mathematically rigorous treatment for portfolio selection but it is not popular among investors since investors would have to specify their utility functions and choose a distributional assumption for the returns before making their investment decisions.

The other approach is the mean-risk (MR) analysis that has been discussed in this chapter. In this approach, the portfolio choice is made with respect to two measures -- the expected portfolio mean return and portfolio risk. A portfolio is preferred if it has higher expected return and smaller risk. These are convenient computational recipes and they provide geometric interpretations for the trade-off between the two measures. A disadvantage of the latter approach is that it is derived by assuming the Von Neumann-Morgenstern quadratic utility function and that returns are normally distributed (Hanoch and Levy, 1969). Thus, it cannot capture the richness of the former approach. Among the MR analyses, the most popular measure is the SR introduced by Sharpe (1966). As the SR requires strong assumptions that the returns of assets being analyzed have to be iid, various measures for MR analysis have been developed to improve the SR, including the Sortino ratio (Sortino and van der Meer, 1991), the conditional SR (Agarwal and Naik, 2004), the modified SR (Gregoriou and Gueyie, 2003), Value-at-Risk (Ma and Wong, 2010), Expected Shortfall (Chen, 2008), mixed Sharpe ratio (Wong, et al., 2012) and others. However, most of the empirical studies, see, for example, Eling and Schuhmacher (2007), find that the conclusions drawn by using these ratios are basically the same as that drawn by the SR. Nonetheless, Leung and Wong (2008) have developed a multiple SR statistic and find that the results drawn from the multiple Sharpe ratio statistic can be different from its counterpart pair-wise SR statistic comparison, indicating that there are some relationships among the assets that have not being revealed using the pair-wise SR statistics. The MVR test could be the right candidate to reveal these relationships.

One may claim that the limitation of the MVR test statistic is that it can only draw conclusion for investors with quadratic utility functions and for normal-distributed assets. Wong (2006), Wong and Ma (2008), and others have shown that the conclusion drawn from the MVR comparison is equivalent to the comparison of expected utility maximization for any risk-averse investor, not necessarily with only quadratic utility function, and for assets with any distribution, not necessarily normal distribution, if the assets being examined belong to the same location-scale family. In addition, one can apply the results of Li and Wong (1999) and Egozcue and Wong (2010) to generalize the result so that it will be valid for any risk-averse investor and for portfolios with any distribution if the portfolios being examined belong to the same convex combinations of (same or different) location-scale families. The location-scale family can be very large, containing normal distributions as well as t-distributions, gamma distributions, etc. The stock returns could be expressed as convex combinations of normal distributions, t-distributions and other location-scale families, see, for example, Wong and Bian (2000)

and the references therein for more information. Thus, the conclusions drawn from the MVR test statistics are valid for most of the stationary data including most,
if not all, of the returns of different portfolios.

Lastly, we note the MVR test can be used to evaluate financial assets performance and the effectiveness of investment techniques, approaches and models, for example, fundamental analysis (Wong and Chan, 2004), technical analysis (Wong, et al., 2001, 2003), behavioral finance (Matsumura, et al., 1990), prospect theory (Broll, et al., 2010; Egozcue, et al., 2011), and advanced econometrics (Wong and Miller, 1990; Bai, et al. 2010, 2011b) allowing investors to be better informed about asset performance and investment management approaches.

Figure 1: S&P 500 index (January 2005 to December 2012)

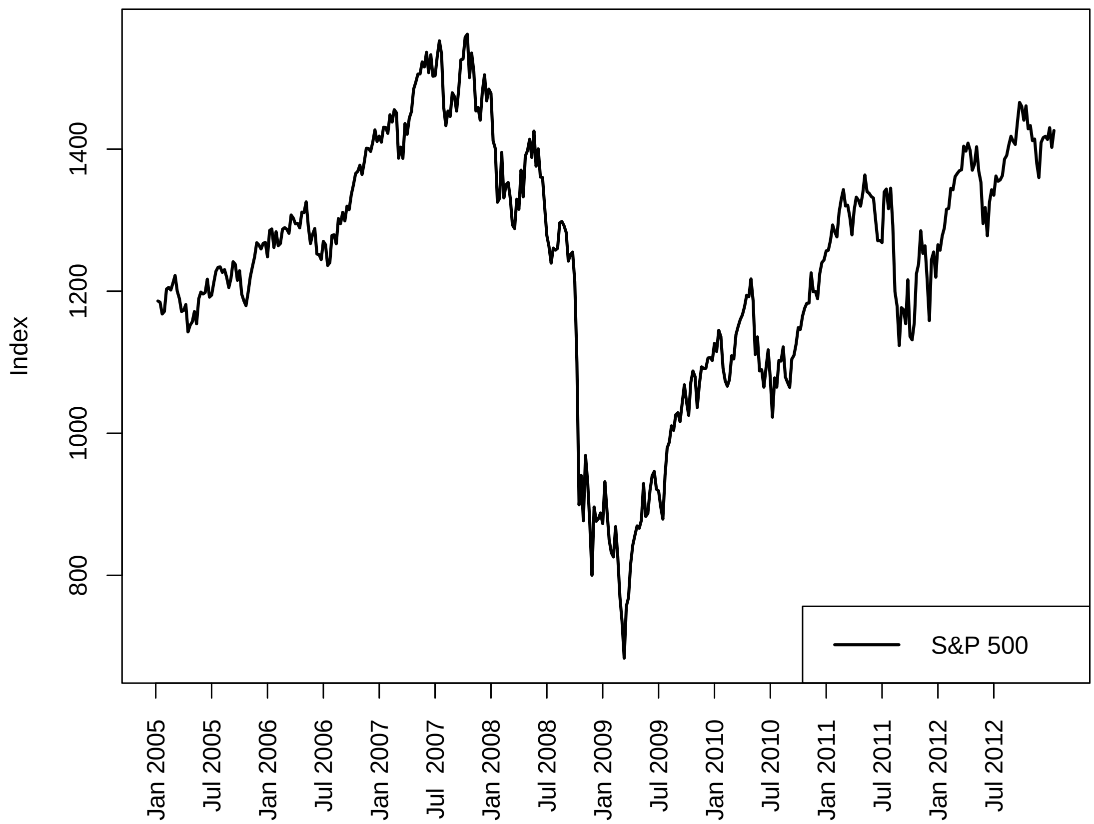

Figure 2: Monthly returns of hedge funds and S&P 500 index
(January 2005 to December 2012)

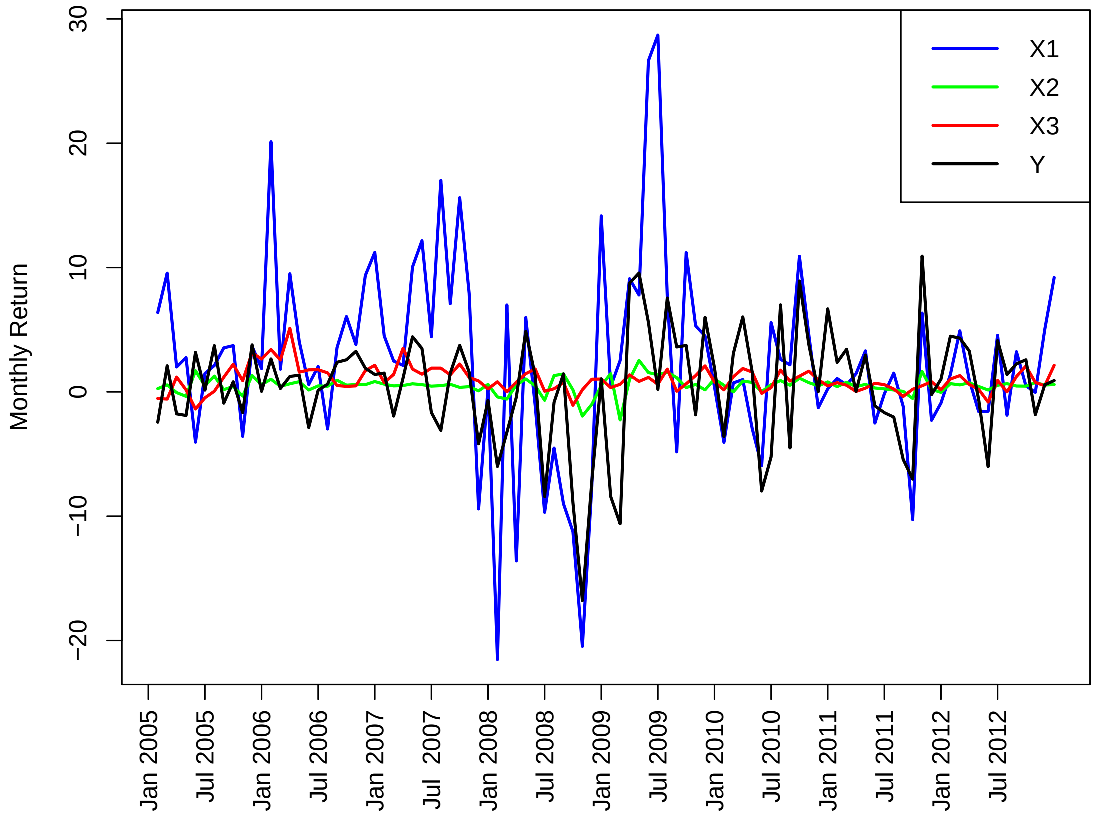

Table 1: Summary statistics of monthly returns of hedge funds and S&P 500 index (January 2005 to December 2012)

	Mean	SD	SR	MVR	Skewness	Kurtosis
Y	0.4493	4.5426	0.0989	0.0217	-0.7146	1.4654
X_1	**2.4563**	7.7833	0.3155	0.0405	0.1519	2.3251
X_2	0.5008	**0.6578**	0.7736	**1.1763**	-1.0193	4.4750
X_3	0.9932	1.0038	**0.9894**	0.9957	08276	2.2122

Y is the monthly return of the S&P 500; X_1 is the monthly return (with largest mean) of Golden China Fund - Non Restricted Class; X_2 is the monthly return (with smallest standard deviation) of PM CAPITAL Enhanced Yield Fund; X_3 is the monthly return (with largest Sharpe ratio) of Evenstar Sub-Fund I. SD is standard deviation, SR is Sharpe ratio, MVR is mean-variance ratio. We note that though the numbers are different from zero and three for skewness and kurtosis, respectively, normality is not rejected for the four variables.

Table 2: Pairwise comparison among funds by the mean-variance criterion

Panel A						
Time period	t-test			F-test		
	$X_1 \leftrightarrow Y$	$X_2 \leftrightarrow Y$	$X_3 \leftrightarrow Y$	$X_1 \leftrightarrow Y$	$X_2 \leftrightarrow Y$	$X_3 \leftrightarrow Y$
Boom	4.22***	-0.85	1.29	6.65***	0.03***	0.38***
Crisis	-0.09	2.80**	3.22***	3.13**	0.03***	0.01***
Recovery	0.93	-1.63	-1.37	2.38***	0.01***	0.02***
Whole	2.18**	0.12	1.14	2.93***	0.02***	0.04***

Panel B						
Time period	t-test			F-test		
	$X_1 \leftrightarrow X_2$	$X_1 \leftrightarrow X_3$	$X_2 \leftrightarrow X_3$	$X_1 \leftrightarrow X_2$	$X_1 \leftrightarrow X_3$	$X_2 \leftrightarrow X_3$
Boom	4.85***	3.83***	-3.69***	182.42***	17.47***	0.09***
Crisis	-1.71	-1.93*	-1.73*	83.72***	22573***	2.69*
Recovery	2.17**	2.00*	-1.33	174.51***	103.97***	0.59*
Whole	2.44**	1.82*	-3.95***	140.05***	60.12***	0.42***

X_1 is the monthly return of Golden China Fund - Non Restricted Class; X_2 is monthly return on PM CAPITAL Enhanced Yield Fund; X_3 Evenstar Sub-Fund I. t-test and F-test are adopted to test the equality of mean and variance respectively for each pair funds.*, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 3(a): Test Results for the max-mean-return Fund and S&P500 during a boom

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
05/06-10/06	13.07	5.4033	21.5448	0.0225
06/06-11/06	21.85	20.0015	31.4463	-1.0329
07/06-12/06	31.03	30.0012	41.4636	-2.1261
08/06-01-07	38.52	38.2692	41.6047	-3.7053
09/06-02-07	37.44	33.9364	40.6144	0.0965
10/06-03-07	33.51	28.8669	39.4313	-0.0197
11/06-04-07	39.75	34.1889	45.5384	0.0647
12/06-05-07	42.54	36.8361	49.3372	-0.0004
01/07-06/07	35.75	26.8348	42.3861	0.1739
02/07-07/07	48.26	33.4038	58.4048	0.1828
03/07-08/07	52.87	41.7369	60.6286	0.1858
04/07-09/07	66.36	55.7219	71.5018	0.3241
Average				-0.4861

Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
05/06-09/07	1.9277	-1.96	+1.96	0.7078

The max-mean-return fund is Golden China Fund - Non Restricted Class. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$.The boom period is from January 2005 to September 2007.

Table 3(b): Test Results for the max-mean-return Fund and S&P500 during a crisis

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
10/07-03/08	-29.34	-62.3834	-22.2165	0.2406
11/07-04/08	-31.29	-64.0020	-17.9379	0.0712
12/07-05/08	-22.97	-48.0454	-6.2945	0.0206
01/08-06/08	-32.95	-70.4937	-15.5831	0.0408
02/08-07/08	-15.94	-48.0697	2.7171	0.0175
03/08-08/08	-31.94	-50.0461	-9.6014	-0.0900
04/08-09/08	-29.58	-46.3992	-7.4102	-0.0607
05/08-10/08	-56.03	-66.7266	-37.0533	-0.115
06/08-11/08	-62.50	-69.2027	-49.1593	-0.1930
07/08-12/08	-38.65	-73.6795	-19.8653	0.0531
08/08-01/09	-33.71	-72.8502	-18.4573	0.0965
09/08-02/09	-22.18	-69.7006	-13.1682	0.2294
Average				0.0262
Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
10/07-02/09	1.0250	-1.93	+1.96	0.2704

The max-mean-return fund is Golden China Fund - Non Restricted Class. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$.The boom period is from October 2007 to February 2009.

Table 3(c): Test Results for the max-mean-return Fund and S&P500 during a recovery

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2}-\frac{\mu_Y}{\sigma_Y^2}$
03/09-08/09	75.04	70.1076	102.7201	-0.3974
04/09-09/09	77.14	72.1695	103.9577	-0.3853
05/09-10/09	74.67	66.8176	103.0190	-0.1893
06/09-11/09	52.52	44.5486	80.4720	-0.1891
07/09-12/09	24.22	17.5753	39.1735	-0.1970
08/09-01/10	12.52	0.0367	35.8224	-0.0669
09/09-02/10	18.07	5.5401	33.8583	-0.0090
10/09-03/10	7.86	-5.2619	19.9906	-0.0054
11/09-04/10	-0.42	-9.9993	16.8042	-0.2053
12/09-05/10	-10.84	-19.2979	18.0083	-0.2195
01/10-06/10	-5.67	-23.6135	18.4617	-0.0191
02/10-07/10	1.01	-22.3581	22.3581	-0.0100
Average				-0.1578

Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X}-\frac{\mu_Y}{\sigma_Y}$
03/09-07/10	0.0789	-1.96	+1.96	0.0252

The max-mean-return fund is Golden China Fund - Non Restricted Class. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$.The boom period is from March 2009 to December 2012.

Table 4(a): Test Results for the min-s.d.-return Fund and S&P500 during a boom

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
05/06-10/06	3.15*	-2.1879	2.7564	7.8779
06/06-11/06	3.58*	1.9787	3.4765	17.8122
07/06-12/06	3.93*	3.0348	3.9176	16.1767
08/06-01-07	4.1	3.6004	4.1044	21.6748
09/06-02-07	3.64*	-0.1675	3.2109	41.9602
10/06-03-07	3.61*	-0.3389	3.1669	39.0286
11/06-04-07	3.67*	-0.9630	3.1566	38.9302
12/06-05-07	3.67*	-1.0823	3.1420	38.9270
01/07-06/07	3.3*	-2.2627	2.6287	90.2897
02/07-07/07	3.18	-3.2040	3.2040	103.4754
03/07-08/07	3.31*	-2.3838	2.6005	103.2585
04/07-09/07	3.16*	-2.2823	2.5234	45.3106
Average				47.0602
Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
05/06-09/07	4.5931*	-1.96	+1.96	2.6945

The min-s.d.-return fund is the minimum-standard-deviation-return fund which is PM CAPITAL Enhanced Yield Fund. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$. The boom period is from January 2005 to September 2007.

Table 4(b): Test Results for the min-s.d.-return Fund and S&P500 during a crisis

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
10/07-03/08	0.66	-2.8458	2.8458	0.7059
11/07-04/08	1.28	-2.8323	2.7219	0.6490
12/07-05/08	1.69	-3.8642	3.8642	0.7499
01/08-06/08	0.41	-3.0360	2.9177	0.2166
02/08-07/08	2.13	-3.8123	3.7126	0.5810
03/08-08/08	4.15	-5.9378	5.9378	1.1680
04/08-09/08	3.76	-5.7962	5.7962	1.0211
05/08-10/08	0.77	-7.0655	7.0655	0.1798
06/08-11/08	-0.72	-7.3811	7.3811	0.0942
07/08-12/08	0.46	-7.2942	7.2942	0.1457
08/08-01/09	0.61	-7.4632	7.4632	0.1921
09/08-02/09	-3.09	-8.5967	5.9217	0.0147
Average				0.4765

Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
10/07-02/09	3.4099*	-1.96	+1.96	0.7437

The min-s.d.-return fund is the minimum-standard-deviation-return fund which is PM CAPITAL Enhanced Yield Fund. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference'' is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$. The boom period is from October 2007 to February 2009.

Table 4(c): Test Results for the min-s.d.-return Fund and S&P500 during a recovery

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
03/09-08/09	9.19*	4.9322	9.0205	4.7735
04/09-09/09	8.53	4.2865	8.6924	2.3512
05/09-10/09	6.6*	-0.0285	6.2802	3.8100
06/09-11/09	5.21*	-1.3188	5.2056	2.2802
07/09-12/09	4.86	-0.6452	4.9022	2.4621
08/09-01/10	3.8*	-3.0396	3.4087	4.1804
09/09-02/10	2.66*	-2.4108	2.5771	3.2204
10/09-03/10	3.19*	-2.7604	3.0114	3.2505
11/09-04/10	3.35*	-2.6573	3.2009	3.1595
12/09-05/10	3.2*	-3.0255	3.0455	2.7430
01/10-06/10	2.75	-3.4221	3.4221	3.3489
02/10-07/10	3.13	-3.8722	3.8722	2.9907
Average				3.2142
Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
03/09-07/10	2.5648*	-1.96	+1.96	0.8254

The min-s.d.-return fund is the minimum-standard-deviation-return fund which is PM CAPITAL Enhanced Yield Fund. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$. The boom period is from March 2009 to December 2012.

Table 5(a): Test Results for the max-Sharpe-ratio-return Fund and S&P500 during a boom

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
05/06-10/06	6.51*	0.0525	6.5088	2.1970
06/06-11/06	6.45	4.9369	7.0539	1.2148
07/06-12/06	6.81	6.0249	7.6718	-0.3040
08/06-01-07	6.02	5.6568	7.0122	-2.4791
09/06-02-07	6.91	3.1321	7.2689	1.8802
10/06-03-07	9.98	6.0769	10.8476	0.9873
11/06-04-07	11.33	6.5495	11.5824	1.8653
12/06-05-07	11.04	6.1587	11.3518	1.7222
01/07-06/07	10.81	1.8948	11.9602	1.8929
02/07-07/07	11.98	-2.8761	12.7141	3.2638
03/07-08/07	11.95	0.8169	12.6945	3.1420
04/07-09/07	10.68	0.0419	10.8321	16.1684
Average				2.6292

Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
05/06-09/07	3.0442*	-1.96	1.96	1.5859

The max-Sharpe-ratio-return fund is Evenstar Sub-Fund I. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference'' is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$.The boom period is from January 2005 to September 2007.

Table 5(b): Test Results for the max-Sharpe-ratio-return Fund and S&P500 during a crisis

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
10/07-03/08	3.88	-4.5445	4.5445	3.7403
11/07-04/08	4.18	-5.0367	5.0367	2.7574
12/07-05/08	5.14	-6.4059	6.4059	1.8084
01/08-06/08	4.95	-6.3801	6.3801	1.6107
02/08-07/08	4.39	-6.0946	6.0946	1.2845
03/08-08/08	5.13	-6.3654	6.3654	1.8236
04/08-09/08	3.27	-6.6231	6.6231	0.5478
05/08-10/08	1.98	-5.5902	5.5902	0.4639
06/08-11/08	1.16	-3.2329	3.0049	0.5273
07/08-12/08	2.14	-3.7593	3.5652	0.6669
08/08-01/09	2.23	-4.8834	4.8834	0.7276
09/08-02/09	2.09	-3.7643	3.3355	0.8246
Average				1.3986

Time period mm/yy-mm/yy	SR test			Difference
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
10/07-02/09	4.9987*	-1.93	+1.96	1.5917

The max-Sharpe-ratio-return fund is Evenstar Sub-Fund I. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference" is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$.The boom period is from October 2007 to February 2009.

Table 5(c): Test Results for the max-Sharpe-ratio-return Fund and S&P500 during a recovery

Time Period mm/yy-mm/yy	MVR Test			Difference
	U	C_1	C_2	$\frac{\mu_X}{\sigma_X^2} - \frac{\mu_Y}{\sigma_Y^2}$
03/09-08/09	5.86	1.7024	6.1177	2.0586
04/09-09/09	5.14	0.9846	5.4456	1.9229
05/09-10/09	5.61	-0.8975	5.6502	2.1189
06/09-11/09	6.54	-0.0878	6.8385	1.4973
07/09-12/09	6.73	1.1648	7.0729	1.5768
08/09-01/10	5.06	-6.5611	6.5611	1.3277
09/09-02/10	6.23	-6.2998	7.2086	2.2493
10/09-03/10	7.46	-5.6619	8.4044	2.3675
11/09-04/10	7.74	-1.8393	8.6895	2.2828
12/09-05/10	5.53	-7.0274	7.0274	1.4635
01/10-06/10	5.07	-6.7999	6.7999	1.2693
02/10-07/10	6.66	-8.0287	8.0287	1.6361
Average				1.8142
Time period mm/yy-mm/yy	**SR test**			**Difference**
	Z	$-z_{0.025}$	$+z_{0.025}$	$\frac{\mu_X}{\sigma_X} - \frac{\mu_Y}{\sigma_Y}$
03/09-07/10	3.1230*	-1.96	+1.96	0.9685

The max-Sharpe-ratio-return fund is Evenstar Sub-Fund I. The mean-variance-ratio (MVR) test statistic U is defined in Equation (1.7) and its critical values C_1 and C_2 are defined in Equation (1.8). The Sharpe ratio (SR) test statistic Z is defined in Equation (1.10), and ``Difference'' is the difference of the MVR estimates or SR estimates. The level is $\alpha = 0.05$. Here, the sample size of the MVR test is 6, while the sample size of the SR test is 17. Recall that $\pm z_{0.025} \approx \pm 1.96$ .The boom period is from March 2009 to December 2012.

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