9-2007

Asymmetries in stock returns: Statistical tests and economic evaluation

Yongmiao HONG
Cornell University and Xiamen University

Jun TU
Singapore Management University, tujun@smu.edu.sg

Guofu ZHOU
Washington University in St. Louis

Citation

HONG, Yongmiao; TU, Jun; and ZHOU, Guofu. Asymmetries in stock returns: Statistical tests and economic evaluation. (2007). *Review of Financial Studies*. 20, (5), 1547-1581.

Available at: https://ink.library.smu.edu.sg/lkcsb_research/4574

Published in Review of Financial Studies, Volume 20, Issue 5, September 2007, Pages 1547–1581.
https://doi.org/10.1093/rfs/hhl037
Submitted Version

Asymmetries in Stock Returns: Statistical Tests and Economic Evaluation

Yongmiao Hong, Jun Tu and Guofu Zhou[1]

First draft: October, 2002
This version: October, 2003

[1]We are grateful for many helpful comments of Andrew Ang, Joseph Chen, Alex David, Heber Farnsworth, Michael Faulkender, Liu Hong, Tom Miller, Ľuboš Pástor, Bruno Solnik, and seminar participants at Washington University in St. Louis. Hong is from Department of Economics and Department of Statistical Science, Cornell University, who acknowledges financial support for this project from NSF Grant SES-0111769. Both Jun Tu and Guofu Zhou are from Olin School of Business, Washington University in St. Louis. Correspondence: Guofu Zhou, phone: (314) 935-6384 and e-mail: zhou@olin.wustl.edu.

Asymmetries in Stock Returns:

Statistical Tests and Economic Evaluation

ABSTRACT

In this paper, we provide a model-free test for asymmetric correlations which suggest stocks tend to have greater correlations with the market when the market goes down than when it goes up. We also provide such tests for asymmetric betas and covariances. In addition, we evaluate the economic significance of asymmetric correlations by answering the question that what is the utility gain for an investor who switches from a belief of symmetric stock returns into a belief of asymmetric returns. Applying our methodology to three portfolios grouped by size, Fama and French's size and book-to-market, and industry, we find that asymmetries show up in sample estimates for all the portfolios, but they are statistically significant primarily for small size portfolios. Nevertheless, asymmetries are of substantial economic importance for an investor who switches her symmetry belief into an asymmetric one, irrespective of the portfolios.

JEL classification: C11; C15; G11; G12

1 Introduction

Recently, there have been a number of studies on asymmetric characteristics of asset returns. Ball and Kothari (1989), Schwert (1989), Conrad, Gultekin and Kaul (1991), Cho and Engle (2000) and Bekaert and Wu (2000), among others, document asymmetries in the covariances, volatilities and betas of stock returns. Harvey and Siddique (2000) analyze asymmetry in higher moments. Of particular interest to this paper is asymmetric correlations of stock returns with the market indices. This line of research includes Karolyi and Stulz (1996), Longin and Solnik (2001), Ang and Bekaert (2000), and Bae, Karolyi and Stulz (2000). In particular, Login and Solnik (2001) find that international markets have greater correlations with the US market when it is going down than when it is going up. Ang and Chen (2002) study the correlations between stock portfolios and the US market and also find strong asymmetric correlations. The study of asymmetric correlations is important for several reasons. For instance, hedging relies crucially on the correlations between the assets hedged and the financial instruments used. The presence of asymmetric correlations can potentially cause problems to hedging effectiveness. More importantly, the standard mean-variance investment theory advises portfolio diversification, but the value of this advice might be questionable if all the stocks tend to fall as the market falls.

However, assessing asymmetric correlations is not an easy matter. Stambaugh (1995), Boyer, Gibson and Loretan (1999), and Forbs and Rigobon (2002) find that the correlation computed conditional on the variables being high or low is an biased estimation of the unconditional correlation by construction. Therefore, even if one obtains a conditional correlation from the real data that is much higher than the unconditional sample correlation, it is not sufficient to claim the existence of asymmetric correlations. A formal statistical test must be used to account for both sample variations and the bias induced by conditioning. Ang and Chen (2002) seem the first to propose such a formal test. Given a statistical model for the data, their test compares the sample conditional correlations with those implied by the model. If there is a large difference, then the data cannot be explained by the model. Despite the novelty of their test, it has two major weaknesses. First, if the model is symmetric and if the null of no difference is rejected, the test suggests asymmetry. But there may exist another symmetric model that fits the data perfectly well. Second, if an asymmetric model is used by the researcher and if the null of no difference is not rejected, the test also suggests asymmetry. But the possibility that the asymmetric model might degenerate into a symmetric one is not ruled out.

The first contribution of this paper is to propose a new test for symmetry in correlation. There are several appealing features of this test. First, it is model-free. Unlike Ang and Chen's (2002) test, ours is computed without having to specify a statistical model for the data. This is an advantage because a rejection of symmetry may be due to the rejection of the specified model rather than the symmetry itself. As a result, if symmetry is rejected by our test, then the data cannot be modelled by *any* symmetry distributions (under standard regularity conditions). Second, the test allows for GARCH and general distributional assumptions on the data. Third, the test statistic is easy to compute and its asymptotic distribution follows a standard chi-square distribution under the null hypothesis of symmetry. Therefore, the proposed test can be straightforwardly applied to a variety of areas to provide insights on assessing whether or not the asymmetric correlations are statistically significant.

While correlations seem obviously important from a risk management perspective of hedging exposures, betas are closely related to asset pricing theories, and useful in understanding the riskiness of the associated stocks. Ball and Kothari (1989), Conrad, Gultekin and Kaul (1991), Cho and Engle (2000) and Bekaert and Wu (2000), among others, document asymmetries in the betas of stock returns, but there are no formal statistical tests. The second contribution of this paper is to adapt the correlation symmetry test to obtain a model-free test of beta symmetry. In addition, we also develop such a test for asymmetric covariances. This is of interest because covariances are usually direct inputs of parameters for optimal portfolio choice while betas are primarily useful in understanding the systematic risks associated with factors.

However, the presence of statistically significant asymmetry may not necessarily be economically important. On the other hand, a statistically insignificant result can be of great economic importance. The third contribution of this paper is to provide an easy and yet informative method to assess the economic importance of asymmetry. For this purpose, we consider the portfolio choice problem of an expected utility maximizing investor who is uncertain about whether there exists asymmetry in the asset returns. The portfolio choice problem is chosen because it is one of the most asked questions in investment practice, and it is this problem to which asset pricing theory has the most assertions and suggestions. In the spirit of Kandel and Stambaugh (1996) and Pástor and Stambaugh (2000), we ask the question that what utility gains an investor enjoys if she switches from a belief of symmetric returns into a belief of asymmetric returns. If the investor, who invests in the universe of Fama and French's (1993) 25 portfolios, believes in symmetric returns, she would choose her optimal portfolios based on the Fama-French 3-factor model. On the other hand, if she believes in asymmetric return, she would choose her portfolio by utilizing the asymmetric characteristics. We provide two ways for doing so. The first is intuitive in which she simply adds an asymmetric factor into the Fama-French model. Ideally, this factor should be chosen to capture all the asymmetries, but doing so is clearly not feasible. Ang, Chen and Xing (2002) suggest a downside correlation factor shown be useful in capturing some aspects of correlation asymmetry. We will use this factor as the staring point. Because the construction of the asymmetry factor is not limited to sorting by correlations, we also form alternative factors based on betas and covariances. Then, any utility gain beyond that of the Fama-French 3-factor model may be interpreted as measures for the economic gain of an investor's switching from a belief of symmetric returns into a belief of asymmetric returns.

The second way of incorporating asymmetries is to alter the data-generating process of the Fama-French 3-factor model. Ang and Chen (2002) show that certain GARCH models can capture a fair portion of asymmetries. However, the GARCH models are difficult to apply in high dimensional problems and their multivariate extensions, if used, often impose very restrictive assumption on covariances which might cause biases for portfolio choices. An alternative model is the regime-switching model of Hamilton (1989). Ang and Bekaert (2002) uses it to analyze international asset allocation, while Ang and Chen (2002) show that this model can better capture correlation asymmetries than the GARCH models. Hence, we will use the regime-switching model rather than GARCH models as an alternative data-generating process of the Fama-French 3-factor model to capture some of asymmetries in the data. If an investor believes in symmetry, she would assume the normal data-generating process for the Fama-French 3-factor model. On the other hand, if she believes in asymmetry, she would regard the regime-switching model as the true data-generating process. The associated utility difference from using the normality model versus using the regime-switching model then measures the economic gain of incorporating asymmetries.

Empirically, we find that sample estimates show asymmetric correlations in size, book-to-market and industry portfolios, but the asymmetric correlations are only statistically significant for the smallest size portfolio (out of the usual 10 sizes) at monthly frequency, and for the 4 smallest sizes at daily frequency. While the results on asymmetric betas are similar, there are in general much more asymmetries in the covariances. For example, the hypothesis of correlations or betas symmetry cannot be rejected for both daily and weekly industry returns, but covariance symmetry can be rejected strongly. In terms of economic value, we find that a mean-variance utility maximizing investor can achieve substantial certainty-equivalence gain by switching from a dogmatic belief of symmetry into a dogmatic belief of asymmetry, where the investment universe is the four factors plus the size portfolios or the Fama-French 25 assets or the industry portfolios. In addition, while still in the three factor world, the use of the regime-switching data-generating process also achieves substantial economic gain foe a power utility investor. Therefore, although asymmetry is not statistically significant in some cases for the Fama-French and 20 industry portfolios, it is still economically important.

The remainder of the paper is organized as follows. Section 2 provides the statistical tests for symmetry. Section 3 discusses portfolio decisions incorporating asymmetry. Section 4 applies the proposed approach to the size, Fama-French and industry portfolios to assess asymmetry and its economic value. Section 5 concludes.

2 Symmetry tests

In this section, we motivate and provide three model-free tests. The first tests symmetry in correlations and the other two do so for betas and covariances.

2.1 Test for correlation symmetry

Let $\{R_{1t}, R_{2t}\}$ be the returns on two portfolios in period t. Following Login and Solnik (2000) and Ang and Chen (2002), we consider the exceedance correlation between the two series. A correlation at an exceedance level c is defined as the correlation between the two variables when both of them exceed c standard deviations away from their means,

$$\rho^+(c) = \text{corr}(R_{1t}, R_{2t} | R_{1t} > c, R_{2t} > c), \tag{1}$$

$$\rho^-(c) = \text{corr}(R_{1t}, R_{2t} | R_{1t} < -c, R_{2t} < -c), \tag{2}$$

where, following Ang and Chen (2002) and many others in the asymmetry literature, the returns are standardized to have zero mean and unit variance so that the mean and variance do not appear explicitly in the right hand side of the definition, making easy both the computation and statistical analysis. The null hypothesis of symmetric correlation is

$$H_0: \quad \rho^+(c) = \rho^-(c) \quad \text{for all } c > 0. \tag{3}$$

That is, we are interested in testing whether the correlation between positive large returns of the two portfolios is the same as that between negative large returns of the two portfolios. As pointed out in the introduction, this null hypothesis is of interest for its important implications on investment diversification and risk management. If the null hypothesis is rejected, there must exist asymmetric correlations. The alternative hypothesis is

$$H_A: \quad \rho^+(c) \neq \rho^-(c) \quad \text{for some } c > 0. \tag{4}$$

Login and Solnik (2000) use extreme value theory to test whether $\rho^+(c)$ or $\rho^-(c)$ is zero as c becomes extremely large. In contrast, Ang and Chen (2002) provide a more direct test of the symmetry hypothesis. For a random sample, $\{R_{1t}, R_{2t}\}_{t=1}^T$, of size T, the exceedance correlations can be estimated by their sample analogues,

$$\hat{\rho}^+(c) = \widehat{\text{corr}}(R_{1t}, R_{2t} | R_{1t} > c, R_{2t} > c), \tag{5}$$

$$\hat{\rho}^-(c) = \widehat{\text{corr}}(R_{1t}, R_{2t} | R_{1t} < -c, R_{2t} < -c), \tag{6}$$

that is, $\hat{\rho}^+(c)$ and $\hat{\rho}^-(c)$ are the standard sample correlations computed based on only those data that satisfy the tail restrictions. Based on these sample estimates, Ang and Chen (2002) propose a H statistic for testing symmetry,

$$H = \left[\sum_{i=1}^m w(c_i)(\rho(c_i, \phi) - \hat{\rho}(c_i))^2 \right]^{1/2}, \tag{7}$$

where $c_1, c_2, \ldots, c_m$ are m chosen exceedance levels, $w(c_i)$'s are the weights whose sum is one, $\hat{\rho}(c_i)$ can be either $\hat{\rho}^+(c_i)$ or $\hat{\rho}^-(c_i)$, and $\rho(c_i, \phi)$ is the population exceedance correlation computed from a given model with parameter ϕ. If H is large, this implies that the given model cannot explain the observed sample exceedance correlations. If in addition the given model is symmetric, this may be interpreted as evidence against symmetry. However, as pointed out in the introduction, this does not exclude the possibility of the existence of another symmetry model that can fit the data perfectly. The H statistics is also used to assess whether a given asymmetric model, like an asymmetry GARCH one, can explain the observed sample exceedance correlations. Suppose some model of the GARCH class passes the H test, it is also interpreted as evidence against symmetry. This may not be adequate for two reasons. First, there is still the possibility of the existence of another symmetry model that fits the data perfectly. Second, there is also a possibility that the asymmetric model degenerates so that it is symmetric for the purpose of modelling the data. Despite of these difficulties, the H statistic is clearly an informative and interesting measure, telling to what extent the exceedance correlations computed from a given model match those observed in the data. However, the asymptotic distribution of H is difficult to derive because the first order derivatives of H with respect to ϕ are singular in moment estimations.

Fortunately, a new test can be proposed whose asymptotic distribution is the standard chi-square one. Intuitively, if the null is true, the following $m \times 1$ difference vector

$$\hat{\rho}^+ - \hat{\rho}^- = \left[\hat{\rho}^+(c_1) - \hat{\rho}^-(c_1), ..., \hat{\rho}^+(c_m) - \hat{\rho}^-(c_m)\right]' \tag{8}$$

must be close to zero. It can be shown (see the Appendix) that, under the null of symmetry and some regularity conditions, this vector has an asymptotic normal distribution with mean zero and a positive definite variance-covariance matrix Ω. To construct a feasible test statistic, we need to estimate Ω. For this purpose, we need to introduce some notations. Let T_c^+ be the number of the observations for which both R_{1t} and R_{2t} are larger than c simultaneously. Then the sample means and variances of the two conditional series are easily computed

$$\begin{aligned}
\hat{\mu}_1^+(c) &= \frac{1}{T_c^+}\sum_{t=1}^{T} R_{1t}1(R_{1t} > c, R_{2t} > c), \\
\hat{\mu}_2^+(c) &= \frac{1}{T_c^+}\sum_{t=1}^{T} R_{2t}1(R_{2t} > c, R_{2t} > c), \\
\hat{\sigma}_1^+(c)^2 &= \frac{1}{T_c^+ - 1}\sum_{t=1}^{T} [R_{1t} - \hat{\mu}_1^+(c)]^2 1(R_{1t} > c, R_{2t} > c), \\
\hat{\sigma}_2^+(c)^2 &= \frac{1}{T_c^+ - 1}\sum_{t=1}^{T} [R_{2t} - \hat{\mu}_2^+(c)]^2 1(R_{1t} > c, R_{2t} > c),
\end{aligned}$$

where $1(\cdot)$ is the indicator function. As a result, we can express the conditional correlation as

$$\hat{\rho}^+(c) = \frac{1}{T_c^+ - 1}\sum_{t=1}^{T} X_{1t}^+(c)X_{2t}^+(c)1(R_{1t} > c, R_{2t} > c), \tag{9}$$

where

$$\begin{aligned}
X_{1t}^+(c) &= \frac{R_{1t} - \hat{\mu}_1^+(c)}{\hat{\sigma}_1^+(c)}, \\
X_{2t}^+(c) &= \frac{R_{2t} - \hat{\mu}_2^+(c)}{\hat{\sigma}_2^+(c)}.
\end{aligned}$$

Clearly, we can have a similar expression for $\hat{\rho}^-(c)$.

Then, under general conditions, a consistent estimator of Ω is the following matrix,

$$\hat{\Omega} = \sum_{l=1-T}^{T-1} k(l/p)\hat{\gamma}_l, \tag{10}$$

where $\hat{\gamma}_l$ is an $N \times N$ matrix with (i, j)-th element

$$\hat{\gamma}_l(c_i, c_j) = \frac{1}{T} \sum_{t=|l|+1}^{T} \xi_t(c_i)\xi_{t-l}(c_j) \tag{11}$$

and

$$\begin{aligned}\xi_t(c) = & \frac{T}{T_c^+}[X_{1t}^+(c)X_{2t}^+(c) - \hat{\rho}^+(c)]1(R_{1t} > c, R_{2t} > c) \\ & -\frac{T}{T_c^-}[X_{1t}^-(c)X_{2t}^-(c) - \hat{\rho}^-(c)]1(R_{1t} < -c, R_{2t} < -c). \end{aligned} \tag{12}$$

In addition, $k(\cdot)$ is a kernel function that assigns weights to each lag of order l, and p is the smoothing parameter or lag truncation order (when $k(\cdot)$ has bounded support). An example of $k(\cdot)$, as used in Newey and West (1994), is the Bartlett kernel,

$$k(z) = (1 - |z|)1(|z| < 1). \tag{13}$$

With these preparations, we are ready to define a test statistic for the null of symmetry,

$$J_\rho = T(\hat{\rho}^+ - \hat{\rho}^-)'\hat{\Omega}^{-1}(\hat{\rho}^+ - \hat{\rho}^-), \tag{14}$$

which clearly summarizes the deviations from the null.

However, the value of p has to be provided to compute the test statistic. There are two ways for choosing p. The first is to take p as a nonstochastic known number, especially in the case where one wants to impose some lag structure on the data. Another choice of p is to allow it be determined by the data with either Andrews's (1991) or Newey and West's (1994) procedure. Let $\hat{J}_\rho$ be the J_ρ statistic with the nonstochastic bandwidth p replaced with a data-driven p, say $\hat{p}$.

The following proposition provides the useful asymptotic theory necessary for making statistical inference based on J_ρ and $\hat{J}_\rho$:

Theorem 1: *Under the null hypothesis H_0 and under certain regularity conditions,*

$$J_\rho \to^d \chi_m^2, \tag{15}$$

and

$$\hat{J}_\rho \to^d \chi_m^2, \tag{16}$$

as $T \to \infty$.

Theorem 1 says that our symmetry test has a simple asymptotic chi-square distribution with degrees of freedom m. So, the P-value of the test is straightforward to compute in practice, making it easily applied to a wide range of data series to assess their asymmetric correlations.

As can be seen from the regularity conditions in the appendix, our test is completely model-free, and is also robust to volatility clustering which is a well-known stylized fact for most financial time series. We have also explicitly justified the use of a data-driven bandwidth $\hat{p}$, which has no impact on the asymptotic distribution of the test statistic provided $\hat{p}$ converges to p at a sufficiently fast rate. As J_ρ and $\hat{J}_\rho$ have the same asymptotic distributions, we in what follows use only the notation J_ρ while stating explicitly how J_ρ is computed.

2.2 Test for beta and covariance symmetries

As pointed out in the introduction, betas are of interest for understanding the riskiness of the associated stocks. Analogous to the conditional correlations, we can define conditional betas at any exceedance level c,

$$\beta^+(c) = \frac{\operatorname{cov}(R_{1t}, R_{2t} | R_{1t} > c, R_{2t} > c)}{\operatorname{var}(R_{2t} | R_{1t} > c, R_{2t} > c)} = \frac{\sigma_1^+(c)}{\sigma_2^+(c)} \rho^+(c), \tag{17}$$

$$\beta^-(c) = \frac{\operatorname{cov}(R_{1t}, R_{2t} | R_{1t} < -c, R_{2t} < -c)}{\operatorname{var}(R_{2t} | R_{1t} < -c, R_{2t} < -c)} = \frac{\sigma_1^-(c)}{\sigma_2^-(c)} \rho^-(c), \tag{18}$$

where

$$\sigma_1^+(c)^2 = \operatorname{var}(R_{1t} | R_{1t} > c, R_{2t} > c), \tag{19}$$

$$\sigma_2^+(c)^2 = \operatorname{var}(R_{2t} | R_{1t} > c, R_{2t} > c), \tag{20}$$

and $\sigma_1^-(c)$ and $\sigma_2^-(c)$ are similarly defined. In particular, when $c = 0$, $\beta^+(c)$ and $\beta^-(c)$ are the upside and downside betas of Ang and Chen (2002). Here $\beta^-(c)$ can still be interpreted as the upside and downside betas except that they are examined at an exceedance level c. If we interpret R_{2t} as the return on the market, then $\sigma_1^+(c)/\sigma_2^+(c)$ is the ratio of upside asset standard deviation (risk) to the market standard deviation (risk), and the upside beta is a product of this ratio with the conditional correlation. Because the ratio can be different in upside and downside markets, the betas can be asymmetric even if there are no asymmetries in the correlations. So, our earlier test for symmetry in correlations cannot be used for testing symmetry in betas.

To test symmetry in betas, we, similar to the correlation case, evaluate the difference,

$$\sqrt{T}(\hat{\beta}^+ - \hat{\beta}^-) = \sqrt{T}\left[\hat{\beta}^+(c_1) - \hat{\beta}^-(c_1), ..., \hat{\beta}^+(c_N) - \hat{\beta}^-(c_m)\right]', \tag{21}$$

where $c_1, c_2, \ldots, c_m$ are a set of m chosen exceedance levels. Here, the symmetry hypothesis of interest is

$$H_0: \quad \beta^+(c) = \beta^-(c) \quad \text{for all } c > 0. \tag{22}$$

Under the null and some regularity conditions, like the earlier correlation case, $\sqrt{T}(\hat{\beta}^+ - \hat{\beta}^-)$ has an asymptotic normal distribution with mean zero and a positive definite variance-covariance matrix Ψ which can be consistently estimated by

$$\hat{\Psi} = \sum_{l=T-1}^{T-1} k(l/p)\hat{g}_l, \tag{23}$$

where $\hat{g}_l$ is an $m \times m$ matrix with (i, j)-th element

$$\hat{g}_l(c_i, c_j) = \frac{1}{T} \sum_{t=|l|+1}^{T} \hat{\eta}_t(c_i)\hat{\eta}_{t-l}(c_j), \tag{24}$$

where

$$\begin{aligned}\eta_t(c) = &\frac{T}{T_c^+}\left[\frac{\hat{\sigma}_1^+(c)}{\hat{\sigma}_2^+(c)}\hat{X}_{1t}^+(c)\hat{X}_{2t}^+(c) - \hat{\beta}^+(c)\right] 1(R_{1t} > c, R_{2t} > c)\\ &-\frac{T}{T_c^-}\left[\frac{\hat{\sigma}_1^-(c)}{\hat{\sigma}_2^-(c)}\hat{X}_{1t}^-(c)\hat{X}_{2t}^-(c) - \hat{\beta}^-(c)\right] 1(R_{1t} < -c)1(R_{2t} < -c).\end{aligned} \tag{25}$$

Then the beta symmetry test can be constructed,

$$\mathcal{J}_\beta = T(\hat{\beta}^+ - \hat{\beta}^-)'\hat{\Psi}^{-1}(\hat{\beta}^+ - \hat{\beta}^-), \tag{26}$$

where the bandwidth p is a fixed constant. Similar to the correlation case, we denote $\hat{\mathcal{J}}_\beta$ as the same statistic with p estimated by the data.

Because of its importance in portfolio selections, consider now the symmetry hypothesis for the covariance,

$$H_0: \quad \sigma_{12}^+(c) = \sigma_{12}^-(c) \quad \text{for all } c > 0. \tag{27}$$

where

$$\sigma_{12}^+(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c) = \sigma_1^+(c)\sigma_2^+(c)\rho^+(c), \tag{28}$$

$$\sigma_{12}^-(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c) = \sigma_1^-(c)\sigma_2^-(c)\rho^-(c). \tag{29}$$

Similar to the beta symmetry test, we can construct a test for covariance symmetry,

$$\mathcal{J}_{\sigma_{12}} = T(\hat{\sigma}_{12}^{+} - \hat{\sigma}_{12}^{-})'\hat{\Phi}^{-1}(\hat{\sigma}_{12}^{+} - \hat{\sigma}_{12}^{-}), \tag{30}$$

where

$$(\hat{\sigma}_{12}^{+} - \hat{\sigma}_{12}^{-}) = \left[\hat{\sigma}_{12}^{+}(c_1) - \hat{\sigma}_{12}^{-}(c_1), ..., \hat{\sigma}_{12}^{+}(c_m) - \hat{\sigma}_{12}^{-}(c_m)\right]', \tag{31}$$

$$\hat{\Phi} = \sum_{l=T-1}^{T-1} k(l/p)\hat{h}_l, \tag{32}$$

and $\hat{h}_l$ is an $m \times m$ matrix with (i,j)-th element

$$\hat{h}_l(c_i, c_j) = \frac{1}{T}\sum_{t=|l|+1}^{T} \hat{\phi}_t(c_i)\hat{\phi}_{t-l}(c_j), \tag{33}$$

where

$$\begin{aligned}\phi_t(c) = & \frac{T}{T_c^{+}}\left[\hat{\sigma}_1^{+}(c)\hat{\sigma}_2^{+}(c)\hat{X}_{1t}^{+}(c)\hat{X}_{2t}^{+}(c) - \hat{\sigma}_{12}^{+}(c)\right] 1(R_{1t} > c, R_{2t} > c) \\ & - \frac{T}{T_c^{-}}\left[\hat{\sigma}_1^{-}(c)\hat{\sigma}_2^{-}(c)\hat{X}_{1t}^{-}(c)\hat{X}_{2t}^{-}(c) - \hat{\sigma}_{12}^{-}(c)\right] 1(R_{1t} < -c)1(R_{2t} < -c).\end{aligned} \tag{34}$$

The bandwidth p has analogous meaning as before, and $\hat{\mathcal{J}}_{\beta}$ is defined in the same way.

For hypothesis testing based on the above tests, we have

Theorem 2: *Under the null hypotheses, equation (22) and (27), and under certain regularity conditions,*

$$J_{\beta} \to^{d} \chi_m^2, \tag{35}$$

and

$$\mathcal{J}_{\sigma_{12}} \to^{d} \chi_m^2, \tag{36}$$

respectively, as $T \to \infty$. *Moreover, both* $\hat{J}_{\beta}$ *and* $\hat{\mathcal{J}}_{\sigma_{12}}$ *have the same* χ_m^2 *asymptotic distributions.*

The proof of Theorem 2 is similar to that of Theorem 1 and is hence omitted here. Again, the tests are model-free. It is unnecessary to find a parametric model to fit the data in order to answer the question whether or not the upside and downside betas or covariances are symmetric. Once the null is rejected, the data cannot be modelled by any regular symmetric distributions and so we can legitimately claim that there is the presence of asymmetric betas or covariances.

3 Portfolio decisions

As shown later in the paper that we find asymmetric correlations only for one or a few size portfolios. This says that asymmetric correlations are present not for *all* of the returns, but only to a small number of them. Then the question arises that how important the asymmetric correlations, or other asymmetric characteristics, are from an investor's portfolio decision point of view. In this section, we provide two ways to assess the economic importance of asymmetries.

3.1 Factor Approach

Consider an investor who invests in the universe of Fama and French's (1993) 25 portfolios. If she believes in symmetric returns, it is reasonable to assume that she would choose her optimal portfolios based on the Fama-French 3-factor model. On the other hand, if she believes in asymmetric returns, she would choose her portfolio by utilizing the asymmetric characteristics. While the optimal portfolio under asymmetric belief may have quite different asset allocations than the optimal portfolio under symmetric belief, the performance of the two portfolios can be similar due to correlations among the assets. Therefore, a measure of the overall performance difference has to be developed to assess the economic value of knowing the presence of asymmetric correlations.

If investors believe symmetric stock returns, there are well developed frameworks for the optimal portfolio decisions. The most widely used one is the mean-variance framework where investors are assumed to have mean-variance utilities. While the portfolio optimization problem based on linear factor models is well understood, both parameter estimation risk and model mispricing risk are usually ignored in the classical statistical framework. Fortunately, Pástor and Stambaugh's (2000) Bayesian set-up incorporate both uncertainties into investors' decision making. Hence, in factor models, we will follow primarily Pástor and Stambaugh (2000) to assess investors' economic well-being.

To evaluate the expected utility under asymmetry, the factor approach simply suggests the use of a fourth factor to capture the asymmetry. Ang, Chen and Xing (2002) suggests a downside correlation factor constructed by sorting primarily on correlations. Clearly this factor is unlikely to capture all the asymmetries. Hence, we also consider alternative factors sorted based on covariances (see Section 4.4 for details). Therefore, if an investor believes that there is asymmetry in the stock returns, she would use the 4-factor model instead of Fama-French's 3-factor one. However, it should be pointed out that the asymmetric factors may not necessarily be independent of those Fama and French (1993) factors.[1] Nevertheless, they seem to add some asymmetric characteristics into the standard Fama and French 3-factor model, and useful for providing a lower bound on the investor's utility when believing in asymmetry. This is to say, once we find the utility is sufficiently higher than that under symmetry, the optimal factor that maximizes the use of asymmetric characteristics must be even greater.

In the context of using a factor model, it is reasonable to assume that the investor has a mean-variance utility. The mean-variance utility bears a cost of losing generality, but has the benefit of being able to solve the optimal portfolio problem easily. However, the limitation may not be too unrealistic for two reasons. First, in a factor model where the returns are assumed approximately normal, then it is the mean and variance that matter for portfolio decisions. Second, any smooth utility functions can have a good first order mean-variance approximation. Nevertheless, we do allow power utility in the next subsection where the data-generating process is assumed to be regime-switching rather than normal.

Now we illustrate how to evaluate the utility in a factor model of the stock returns. Following the well established mean-variance framework of Pástor and Stambaugh (2000), we consider an investment universe consisting of cash plus n risky assets. Let r_t denote an n-vector with i-th element $r_{i,t}$ representing the return of the i-th risky position at time t. If there is a riskiness asset with a rate of return $R_{f,t}$, then the excess return of this portfolio is

$$R_{p,t} - R_{f,t} = \sum_{i=1}^{n} w_i r_{i,t}. \tag{37}$$

The investor is assumed to choose w so as to maximize the mean-variance objective function

$$U = E\left(R_{p,t}\right) - \frac{1}{2} A \; Var\left(R_{p,t}\right), \tag{38}$$

where A is interpreted as the coefficient of relative risk aversion and W_0 is the wealth level. If we denote the mean vector and variance-covariance matrix of r_t as E and V, then the investor's optimal portfolio choice problem can be rewritten as the solution to

$$\max_{w} \left(w'E - \frac{1}{2} A w' V w \right). \tag{39}$$

To conduct the necessary Bayesian analysis, let $r_t = (y_t, x_t)$, where y_t contains the excess returns of $n - k$ assets and x_t contains the excess returns of k factors. Consider the following familiar

[1] Pástor and Stambaugh (2003) provide evidence for the liquidity being the fourth factor.

multivariate regression,

$$y_t = \alpha + Bx_t + u_t, \tag{40}$$

where u_t is an $(n-k) \times 1$ vector with zero means and a non-singular covariance matrix. It is clear that α, B and the earlier parameters E, V obey the following relationship:

$$\alpha = E_1 - BE_2, \qquad B = V_{12}V_{22}^{-1}, \tag{41}$$

where

$$E = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}, \; V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}. \tag{42}$$

Consider now the priors on α, B and Σ. If the factor model is true, the asset pricing restriction is $\alpha = 0$. To allow for the possibility of mispricing, we, following Pástor and Stambaugh (2000) and Pástor (2000), assume the prior distribution of α is a normal distribution conditional on Σ,

$$\alpha|\Sigma \sim N\left(0, \sigma_\alpha^2\left(\frac{1}{s^2}\Sigma\right)\right), \tag{43}$$

where s^2 is a suitable prior estimate for the average diagonal elements of Σ. The above alpha-Sigma link is also explored by MacKinlay and Pástor (2000) in the frequentist set-up. The numerical value of σ_α represents an investor's level of uncertainty about a given model's pricing abilities. When $\sigma_\alpha = 0$, the investor believes dogmatically in the model and there is no mispricing uncertainty. On the other hand, when $\sigma_\alpha = \infty$, the investor believes the pricing model is entirely useless. The remaining priors are straightforward. Then the expected utility can be easily evaluated (see Pástor and Stambaugh, 2000, for details).

With above preparations, it is now ready to compare the economic value of the portfolio differences when one switches her belief from a belief of symmetric correlations into a belief of asymmetric correlations. Under the belief of symmetric correlations, the investor makes the portfolio decision under the Fama-French model. Given a certain mispricing prior and a certain prior on the data-generating process, the expected utility is

$$EU_S = w_S^{'}E^* - \frac{1}{2}Aw_S^{'}V^*w_S, \tag{44}$$

where w_S is the optimal portfolio allocation under the Fama-French model. If the belief is dogmatic, the pricing error is zero, and then w_S is the holdings in the Fama-French three factors because the investor invests only in the factors. If the belief is not dogmatic so that the model has a nonzero pricing error, the investor invests in all of the assets based the predicative moments computed from the Fama-French 3-factor model. However, E^* and V^* of equation (44) are the predicative moments computed from the 4-factor model. This is because the 4-factor model is assumed the true model and the investor should evaluate her utility based on it. If she is forced to believe a 3-factor model or the symmetry hypothesis to have a portfolio choice w_S, EU_S should be the resulted utility.

Similarly, under the belief of asymmetric correlations, the expected utility is

$$EU_A = w_A^{'} E^* - \frac{1}{2} A w_A^{'} V^* w_A, \tag{45}$$

where w_A is the optimal portfolio allocation in the four factor model by incorporating the asymmetry. Following Pástor and Stambaugh (2000), the difference

$$CE = EU_A - EU_S \tag{46}$$

is thus the 'perceived' certainty-equivalent gain to a mean-variance investor who switches her belief from a belief of symmetric correlations into a belief of asymmetric correlations.[2] It should be noticed that E^* and V^* in (45) are identically to those in (44). So the certainty-equivalent gain is always nonzero or positive. The ideas of this approach can be traced back to Kandel and Stambaugh (1996). The issue is how big this value can be. Generally speaking, values over a couple of percentage points per year are deemed as economically significant.

3.2 Modelling Approach

As an alternative to using factor to capture asymmetries, one can propose a data-generating process or a statistical model that would describe the asymmetries of the data. That is, one may model the asymmetric characteristics by using a parametric statistical model rather than by using a factor. However, modelling asymmetry is not an easy matter and the candidate data-generating process is usually very complex. Ang and Chen (2002) show that certain bivariate GARCH models can capture a fair portion of asymmetries, but these models are difficult to extend to high dimensional problems. For example, it is very difficult to find and estimate a multivariate GARCH model that is of dimension 10 or over (Bollerslev, 2001). In addition, the multivariate extensions often impose very restrictive assumption on covariances which might cause biases for portfolio choices. Ang and Chen (2002) and Ang and Bekaert (2002) also show that the regime-switching model of Hamilton

[2] Fleming, Kirby and Ostdiek (2001) provide an alternative measure for assessing the economic value of a trading strategy or model.

(1989) can capture a large portion of asymmetries, but this model is still difficult to implement in our high-dimensional applications. Fortunately, Tu (2003) develops a feasible Bayesian version of the regime-switching model which can be applied here.

In the two-regime model, the excess returns can follow one of two possible normal distributions,

$$r_t \sim N\left(E^{s_t},\, V^{s_t}\right), \tag{47}$$

where $s_t \in S = (1, 2)$ and the two regimes switch between each other with the following transition probabilities:

$$\Pi = \begin{bmatrix} P & 1-P \\ 1-Q & Q \end{bmatrix}, \tag{48}$$

where $P = Pr(s_t = 1|s_{t-1} = 1)$ and $Q = Pr(s_t = 2|s_{t-1} = 2)$. As shown by Ang and Chen (2002) and Ang and Bekaert (2002), among others, the regime-switching model can generate asymmetries in moments. So, to assess the economic importance of asymmetries, we provide below how to compute the associated expected utility gain.

First, the earlier factor model can be rewritten as

$$y_t = \alpha^{s_t} + B^{s_t} x_t + u_t, \tag{49}$$

where u_t is the residuals. Define similar the returns first two moments,

$$E^{s_t} = \begin{pmatrix} E_1^{s_t} \\ E_2^{s_t} \end{pmatrix},\ V^{s_t} = \begin{pmatrix} V_{11}^{s_t} & V_{12}^{s_t} \\ V_{21}^{s_t} & V_{22}^{s_t} \end{pmatrix}, \tag{50}$$

then α^{s_t}, B^{s_t} obey similar earlier relationship:

$$\alpha^{s_t} = E_1^{s_t} - B^{s_t} E_2^{s_t}, \qquad B^{s_t} = V_{12}^{s_t} (V_{22}^{s_t})^{-1}, \tag{51}$$

and

$$\Sigma^{s_t} = V_{11}^{s_t} - B^{s_t} V_{22}^{s_t} (B^{s_t})'. \tag{52}$$

Again, following Pástor and Stambaugh (2000) and Pástor (2000), we can use the following priors to reflect the degree of mispricing uncertainty,

$$\alpha^{s_t}|\Sigma^{s_t} \sim N\left(0, \sigma_\alpha^2 \left(\frac{1}{s^2(s_t)} \Sigma^{s_t}\right)\right). \tag{53}$$

The rest of the priors are standard.

In the above Bayesian framework where both model mispricing and parameter uncertainties are incorporated, a power utility investor solves

$$\max_{w} \int \frac{W_{T+1}^{(1-\gamma)}}{1-\gamma} p(r_{T+1}|R) dr_{T+1}, \tag{54}$$

where $p(r_{T+1}|R)$ is the predictive density of the returns, $W_{T+1} = 1 + R_f + \sum_{i=1}^{n} w_i r_{i_{T+1}}$ is the wealth at $T+1$ when time T wealth W_T is assumed as \$1, $w_i = X_i/W_{t-1}$ and X_i is the size in asset i, $i = 1, 2, \cdots, n$. The first order condition is

$$\int [W_{T+1}^{-\gamma} r_{T+1}] p(r_{T+1}|R) dr_{T+1} = 0. \tag{55}$$

Analytical solution does not seem feasible. To solve (55) numerically, we have to evaluate high dimensional integrals and Monte Carlo simulation is the only tractable approach. Then the key difficulty lies in how to draw returns from their predictive density $p(r_{T+1}|R)$. Fortunately, this problem is resolved by Tu (2003), so that (55) can be well approximated by

$$\sum_{q=1}^{M} \left\{ (W_{T+1}^{q})^{-\gamma} r_{T+1}^{q} \right\} = 0, \tag{56}$$

where $W_{T+1}^{q} = 1 + R_f + \sum_{i=1}^{n} w_i r_{i_{T+1}}^{q}$, r_{T+1}^{q} is the q-th draw and M is the total number of draws.

We denote the optimal portfolio weight for the two-regime model as w_{2R}. For the one-regime model, we can obtain similarly the optimal portfolio weight w_{1R} by solving (56) by using draws from the predictive distribution of the one-regime model. Therefore, expected utilities are

$$\hat{u}_{2R} = \sum_{q=1}^{M} \left\{ \left(1 + R_f + \sum_{i=1}^{n} w_{i2R} r_{i_{T+1}}^{q} \right)^{1-\gamma} / (1-\gamma) \right\}. \tag{57}$$

and

$$\hat{u}_{1R} = \sum_{q=1}^{M} \left\{ \left(1 + R_f + \sum_{i=1}^{n} w_{i1R} r_{i_{T+1}}^{q} \right)^{1-\gamma} / (1-\gamma) \right\}, \tag{58}$$

where $r_{i_{T+1}}^{q}$ is drawn from the predictive distribution of the two-regime model which is the assumed true data-generating process capturing the asymmetry of the data. Then the difference in the certainty-equivalent excess returns is

$$CE = [(1-\gamma)\hat{u}_{2R}]^{\frac{1}{1-\gamma}} - [(1-\gamma)\hat{u}_{1R}]^{\frac{1}{1-\gamma}}. \tag{59}$$

This is the 'perceived' certainty-equivalent gain to a power utility investor who switches from a symmetry belief (the one-regime model) into a belief of asymmetry (the two-regime model).

4 Empirical results

In this section, we first apply the symmetry tests to size portfolios, the well-known Fama and French's (1993) 25 portfolios and a set of 20 industry portfolios. Then, after analyzing the statistical significance of asymmetric correlations, betas and covariances, we further examine the economic gain to a mean-variance investor who invests in the three investment universes, respectively, when the investor makes use of the asymmetry of the data by incorporating an asymmetric factor into the CAPM and the Fama-French 3-factor model, respectively. We also examine the gain to a power utility investor maximizer who uses the regime-switching model to capture asymmetry rather than the asymmetry factor.

4.1 The data

The first data set is the 10 standard size portfolios of the Center for Research in Security Prices (CRSP). Both monthly and daily returns of the 10 size deciles portfolios as well as the value-weighted market portfolios based on stocks in NYSE/AMEX/NASDAQ are available directly from CRSP. To examine how the result of the symmetry test is affected by the sampling frequency, we also use weekly returns. The weekly returns are computed, following Ang and Chen (2002), as the holding period return from the end of Wednesday of the first week through the end of the next Wednesday by compounding daily returns in this holding period. As it is the returns in excess of the riskfree rate that are of interest, a proxy of the riskfree rate must be supplied. We use both the monthly and daily returns on the one-month Treasury bill from French's homepage.[3]

In recent empirical asset pricing studies, especially in linear factor models, Fama and French's (1993) 25 portfolios formed on size and book-to-market, are the standard test assets. As a result, we also apply our tests to them to provide some potentially very useful asymmetric information on this widely used data. The data is monthly returns from January 1965 through December 1999, available from French's website. So are available the Fama and French's (1993) three factors. As Fama-French 25 test assets are not available in daily form, but a set of 6 portfolios (grouped by 2 size and 3 book-to-market) is. We will also use this set in our study to provide asymmetry information on size and book-to-market portfolios at both weekly and daily frequencies.

[3]We are grateful to Ken French for making this data and many others used below available at his website: www.mba.tuck.dartmouth.edu/pages/faculty/ken.french.

Besides those sorted on size and book-to-market, we apply our test to 20 industry portfolios. The industry portfolios are constructed by sorting their 2-digit SIC codes following Moskowitz and Grinblatt (1999). King (1966) shows that industry groupings well proxy the investment opportunity set: they maximize intragroup and minimize intergroup correlations. In addition, this data set makes a nice comparison with the size portfolios since it is also available at monthly, weekly and daily frequencies.

4.2 Correlations

Following Ang and Chen (2002), we choose four exceedance levels, $c_1 = 0, c_2 = 0.5, c_3 = 1.0$ and $c_4 = 1.5$. In addition, we implement the symmetry test $\hat{J}_\rho$ by using Bartlett kernel and by letting the data to tell what the value of p is based on Newey and West's (1994) estimator for both weekly and daily data. However, for the monthly data, as the observations in the tail are sometimes as few as 10, there is simply not enough sample to estimate p. So we will use a fixed value of $p = 3$ for all monthly data which should capture a good amount of serial correlations, if any, in the data.

Table 1 provides the results of testing symmetry for monthly excess returns on the CRSP 10 size portfolios. The assets are in the first column. They range from the smallest size of 1 to the largest of size 10. The second column reports the symmetry test statistic, J_ρ, and the next column is the associated P-value in percentage points. It is seen that the P-values are all greater than 5% except for the first asset. Hence, the null hypothesis of symmetric correlations are not rejected statistically for the 9 assets at the usual 5% significance level. However, it is interesting to observe, from columns 4 through 7, that the sample differences of asymmetric conditional correlations, $\rho^+ - \rho^-$ are all less than zero at all values of c. This means that the downside correlations are indeed greater than the upside correlations based on the standard correlation estimates. For example, $\rho^-(0) - \rho^+(0)$ for the second decile portfolio is as large as 43.96%! However, this does not mean that there is necessarily a genuine difference in the population parameters as there are always differences in the estimated conditional correlations due to sample variations. Indeed, the earlier test results show only one rejection of symmetry after accounting for the sample variations.

There are in addition several interesting facts. First, the test statistic tends to get smaller as the firm size increases. For example, J_ρ is great than 5.6 for sizes 2 through 4, but less than 0.59 for sizes 7 through 10. Second, the test statistic appears positively related to the skewness. For example, size 1 has the largest J_ρ, 10.00 and a corresponding largest skewness of 0.87. In contrast, size 10 has a J_ρ value of 0.01 and a corresponding small skewness of -0.37. Finally, the mean returns, as well known, are almost strictly decreasing with firm size. The return on the size 1 portfolio is substantial greater than that on size 2, 1.373% versus 0.909% per month, and both are much greater than 0.505% per month of the largest firm size. To the extent that size 1 portfolios drop more when the market does, its substantial high return, 1.373 per month, seems to suggest that investors earn the high return by taking the substantial downside risk.

While our use of the returns here and later replies on the usual simple or percentage returns, Ang and Chen (2002) use continuously compounded returns in their study which may be well motivated by a continuous-time utility maximizing framework. A question then arises whether the test statistic changes drastically with respect to the use of continuously compounded returns versus to the use of simple returns. To answer this question, we repeat our test above for the same size portfolios with all of the excess returns being computed by continuous compounding. The results are reported in the second column panel of Table 1. The test statistic changes from 10.00 to 10.07, and from 1.69 to 2.25, for the size 1 and 5 portfolio, respectively. Clearly the differences are small and they have no impact on the rejection decision. Similarly, the changes on $\rho^-(0) - \rho^+(0)$ are also small. This suggests that the use of continuously compounded returns has little effect on symmetry test, at least for the current size portfolios. However, it might be of interest to note that the continuously compounded returns do make important differences in some other aspects. For example, the mean returns are smaller by construction. In addition, the skewness is shifted to the negative side. For instance, size 1 portfolio has a skewness of 0.87 in terms of simple returns, but has a skewness of 0.30 in terms of continuous returns. Similarly, the skewness of size 10 portfolio is shifted from -0.37 to -0.65.

Table 2 reports the results of testing symmetry for the Fama-French 25 excess returns. The 3 smallest P-values occur at size 1 portfolio with book-to-market grouping 2, 3 and 5. The S1B3 asset has the lowest P-value of 50.6% when using simple returns, or of 56.99% when using continuous returns. This ranking of P-values is consistent with the earlier results on size portfolios. However, there are no rejections at the usual 5% level. This seems to suggest that book-to-market is unrelated to size since grouping by book-to-market out of the now larger size 1 portfolio (all the stocks are grouped into 5 rather 10 sizes now) cannot single out the asymmetric property of the earlier CRSP smallest size portfolio. Because the differences between continuous and simple returns are small, we will focus on the results for the simple returns. Examining the individual differences in $\rho^-(c) - \rho^+(c)$, we see that $\rho^-(1.5) - \rho^+(1.5)$ of the first asset, S1B1, is equal to a value as large as 0.9126! On the other hand, the symmetry is not even rejected for this asset. The reason is that there are few samples at the exceedance level of $c = 1.5$, and relatively much greater samples at the exceedance level of $c = 0$. A further examination of $\rho^-(1.5) - \rho^+(1.5) = 0.9126$ for S1B1 reveals that $\rho^+(1.5) = -0.0379$, far away from the unconditional correlation of 0.8151. The sample size for this tail is 10, so the estimate does not seem to be very accurate. On the other hand, $\rho^- = 0.8747$, so their difference is big. Notice that the test utilizes the effective sample sizes for each tail and aggregate them accordingly. Because of this, the variance of the test statistic J_ρ is predominantly determined by $\rho^-(0) - \rho^+(0)$ rather than by $\rho^-(1.5) - \rho^+(1.5)$. The same reason applies to S1B3. Although it has a smaller deviation of $\rho^-(c) - \rho^+(c)$ at $c = 1.5$, its P-value is smaller than S1B1's because $\rho^-(0) - \rho^+(0) = 0.3432$ is greater than 0.3349. This is also consistent with asset S2B1 which has a P-value of 95.09% while $\rho^-(1.5) - \rho^+(1.5) = 0.7703$. The statistical behavior of the symmetry test is also confirmed with bootstrap simulations below. Another interesting fact about the Fama-French portfolios is that the skewness is small and negative for all but two. In contrast, the skewness of the CRSP size portfolios are in general larger in absolute value and have three positive ones.

Because the industry portfolios well proxy the investment opportunity set, it is also of interest to examine their asymmetry. Table 3 reports the results for both the simple and continuous returns. Like size and the Fama-French portfolios, the differences between using simple and continuous returns are very small so we will focus only on the results for the simple returns. Like the Fama-French portfolios, there are no rejections of symmetry for the industry portfolios. Moreover, the industry portfolios seem very diversified in the sense that $\rho^-(0) - \rho^+(0)$, which primarily determines the rejection, does not exceed 0.2886 for all the assets. In contrast, there are 5 Fama-French portfolios that have values over 0.3116, and 5 size portfolios that have values over 0.3369 (to a high of 0.5226). In terms of standard deviation, the traditional measure for risk, the industry portfolios also seem less risky. Their largest standard deviation is 6.98% per month, whereas the largest standard deviations are 7.75% and 7.70% for the CRSP size and Fama-French portfolios, respectively.

Now we examine how the test statistic varies at higher sampling frequencies. As the difference is tiny between the results of using simple and continuous returns, we in what follows apply our test only to simple returns. Table 4 reports the symmetry test results for the weekly and daily CRSP size portfolio returns. The most noticeable fact is that the test statistic increases in magnitude compared with the monthly data. For the weekly data, half of the magnitudes are in the 10s.

In contrast, the maximum test statistic is 10.00 for the monthly data. Because of the increasing magnitudes of the test statistics with both weekly and daily data, the symmetry hypothesis is rejected for four of the assets. However, the rejection is not perfectly consistent with the use of the same asset. While the first 3 decile portfolios are rejected by both the weekly and daily tests, the fifth decile is rejected by the weekly test, but not by the daily one. One interesting fact is that the skewness tends to decrease as the frequency increases. For example, the skewness value deceases from 0.87 to 0.32 and to -0.17 for the first decile as the data frequency goes up from monthly to weekly, and daily. It is still true that the small deciles tend to have larger skewnesses. Another noticeable fact is that the differences in conditional correlations, $[\rho^-(c) - \rho^+(c)]$'s, are in general smaller as the frequency increases. But why do we have rejections with the smaller differences? This is because the sample size is now quite large, $T = 1825$ and 8813, respectively, for the weekly and daily data. When the sample size increases, it would be more unlikely to still observe a sizable difference if the population parameter were equal.

Consider now a similar weekly and daily analysis for the Fama-French 25 portfolios. However, these portfolios are not available at the daily and weekly frequencies. But, as mentioned earlier, Fama-French 6 portfolios are, which formed by size and boo-to-market, 2×3 rather than 5×5. Hence, we have to apply our test to these 6 portfolios which should provide some useful information on the Fama-French 25 portfolios. Table 5 reports the results. Unlike the size portfolios which have more rejections at the daily and weekly frequencies than monthly, Fama-French 6 portfolios still do not have any rejections of the symmetry hypothesis. Indeed, the sample conditional correlation differences, $[\rho^-(c) - \rho^+(c)]$'s, actually shrink to much smaller numbers as sample sizes go large, suggesting equal up- and down-side correlations. For example, the difference for the S1B1 asset varies from 0.2053 to 0.7967, from 0.0174 to 0.1103, and from 0.0677 to 0.0943 for the monthly, weekly and daily data, respectively. Therefore, a failure of rejecting symmetry for the Fama-French 25 portfolios does not seem a problem with the sample size. Rather, it looks like that asymmetry simply disappears when one sorts portfolios by both size and book-to-market instead of by size alone. The new mix of the firms does not carry the asymmetry characteristics of the smallest decile of the CRSP size portfolios.

Similar results hold true for the 20 industry portfolios as well. As shown by Table 6, there are no rejections and the P-values are more or less the same at the daily and weekly frequencies. In addition, the sample conditional correlation differences also decrease as the frequency increases. This is expected as there are no rejections. However, the decreases in the differences are smaller than those in the Fama-French 6 portfolios. The reason is that such differences are relatively smaller to begin with at the monthly frequency. Overall, although the sample estimates of $[\rho^-(c) - \rho^+(c)]$'s indicate some asymmetry, but it is not statistically significant at all data frequencies.

In summary, although sample estimates show asymmetric correlations in the three sets of portfolios, these asymmetric correlations are statistically important only for the size portfolios. This also makes intuitive economic sense. As the market goes down, say when the economy is in recession, small firms usually get a disproportional impact than the large firms in terms of sales and financing. As is well-known, small firms have higher risks in terms of standard deviation than large firms, but the higher standard deviation measures only symmetric risk to suggest that small firms are more volatile than large firms whether the market is up or down. In contrast, what we found here is that the small firms exert asymmetric correlations. So, after controlling for the market, the small size portfolios must be asymmetric. The failure of the CAPM to explain the well-known size anomaly, e.g., Banz (1981), may be due, in part, to the use of a symmetric model to explain the asymmetric movements of the returns with the market.

Statistically, an unanswered issue is that whether the symmetry test is reliable in the sample size we use. To address this question, consider the first asset of the Fama-French 25 portfolios. If there are any rejections, this would be one of the most likely asset. We use bootstrap to analyze the distribution of the symmetry test by drawing 10,000 samples from the data with replacements. Table 7 reports the results. With the true data, $J_\rho = 2.02$ and the asymptotic P-value is 73.29%. With the bootstrap simulations, the mean estimate of J_ρ is 3.82 with standard error 1.98, statistically indistinguishable from 2.02. The empirical P-value, the percentage of simulated J_ρ being greater than 2.02, is 85.25%, very close to 73.29%. The small and inconsequential differences are due to the joint effect of small sample deviations and the independent and identical distribution assumption imposed by the bootstrap. Similar results also hold for the first asset of the Fama-French 6 portfolios at all the data frequencies. So, in our applications, there is no evidence on small sample problems of using the symmetry asymptotic test.

4.3 Betas and covariances

Now let us examine asymmetry in betas. Table 8 provides the results for the monthly size portfolios. We have no in fact no rejections of symmetry in the betas. In contrast to the correlation symmetry test, the smallest P-value occurs at the 4th decide rather than the smallest one. This is true regardless of using simple or continuously compounded returns. However, Table 9 shows that there are one or two more rejections in beta symmetry than correlation symmetry at both the daily and weekly frequencies. In addition, when one asset is rejected by correlation symmetry test, it is also rejected by beta symmetry test. For other assets, the beta symmetry test yields very similar results and hence they are omitted.

Finally, consider asymmetry in covariances. Tables 10 and 11 report the results on size portfolios. For the monthly data, it is seen that the smallest P-value also occurs at at the 4th decide, but is close to the value at the smallest decile. For the week and daily data, symmetry is strongly rejected for all deciles except for the largest one! So the data is much more asymmetry in covariances than in correlations or betas. While strong asymmetry does not show up at monthly frequency for Fama and French's (1993) 25 assets or the 20 industries, Table 12 and 13 demonstrate that covariance symmetry hypothesis is rejected for many of the assets at both weekly and daily frequencies. This is surprising as there are no rejections by using the correlation symmetry test. Therefore, we find that asymmetry in covariances is pervasive in the data.

4.4 Utility gains

Whether or not there are statistical rejections of symmetric characteristics, it is of interest to assess the economic value of knowing the presence of such asymmetries. To implement the utility gain measures discussed earlier in Section 3, we need to specify some initial values for the parameters. Following Pástor and Stambaugh (2000), we consider a mean-variance-optimizing investor with relative risk aversion equal to 3. For the power utility, we allow it to be 3, 6 and 9. Given these specifications, we can compute the utility gain measures based on 10,000 draws of the parameters based on the predictive distributions.

Consider first the case where a factor approach is used to capture the asymmetry of the data. Ang, Chen and Xing (2002) construct such a factor, CMC, based on downside correlation. As alternatives, we also construct four similar factors, Dcov10, Dcov, CC3 and Dcorr10. The Dcov10 factor is obtained as the difference between the largest and smallest portfolio after sorting all stocks into ten groups by downside covariance with the market. Dcov is obtained by sorting first the stocks into two groups according to market beta, (L1, H1), and then according to downside covariance, (L2, M2, H2). This gives rise to Dcov $= (L1H2 + H1H2)/2 - (L1L2 + H1L2)/2$. CC3 is obtained similarly as the difference portfolio when the stocks are first sorted into 3 groups by downside covariance, and then into 3 groups by downside correlation. Finally, Dcorr10 is the difference portfolio after sorting stocks into ten groups by downside correlation alone.

In light of the well-known Fama-French model, a mean-variance utility maximizing investor would make her investment decisions based on this model and the 10 test size portfolios if she has a belief of symmetric correlations, and if the investment universe consists of the CRSP 10 size portfolios, the Fama-French three factors and one of the asymmetry factors. If the investor's belief is dogmatic, she would take the Fama-French model as the exact asset pricing model, so that the pricing error $\alpha = 0$. Now if she switches into a dogmatic belief of asymmetry, she would make her investment decisions based on the 4-factor model (the Fama-French 3 factors plus one of the asymmetry factors) with $\alpha = 0$. The first panel of Table 14 presents the results. The annualized certainty-equivalent gain of switching the beliefs, can be as low as 0.02% for the Dcov10 factor, and as high as 9.64% for the CMC factor. It is seen that this is quite sensitive to the asymmetric factor used. The economic gain is quite substantial for the CMC factor, but insignificant for the Dcov10 factor. To the extent that only the largest gain is of concern, the results clearly show asymmetry is of great economic importance.

A dogmatic belief assumes no pricing errors, which is obviously an unrealistic assumption in practice. So we also compute the gains when the model pricing errors is 0.5%, 1% to 6% away from the standard error. When the error is infinity, it means that the pricing model is useless in pricing the assets. At the 1% level, the largest gain is still as high as 3.10%. However, as the pricing error grows, it is seen that the gain gets smaller and smaller, and eventually becomes insignificant, suggesting that the gain is sensitive to model pricing errors. Intuitively, this should be the case. If one does not believe the 4-factor model that much, the gain should be small when using it.

Now when we assume that the investment universe consists of the Fama-French 25 portfolios, their three factors as well as one of the asymmetry factors, the gains are in general greater. For example, as reported in the second panel of Table 14, the largest gain under a dogmatic belief is 13.54%, much greater than the earlier gain of 9.64%. The intuition is that the Fama-French 25 test assets is more difficult to model in the factor model regression than the 10 size portfolios because the 25 asset represent more cross-sectional difference in expected returns. Hence, once the asymmetry factor is introduced, there is more gain for the Fama-French 25 test assets than the the 10 size portfolios. In general, consistent with earlier results, we find substantial economic value of utilizing asymmetric characteristics in investment decisions. The results are also similar for the 20 industry portfolios as reported in the third panel of table 14.

For interest of comparison, we also compute the economic gains if the benchmark model is the standard CAPM, rather than the Fama-French model. In this case, the gain is still substantial, though smaller. For example, as reported in table 15, the largest gain under a dogmatic belief is 2.73%, 2.5%, and 1.7% for the three alternative investment universes, respectively. Although these values are smaller than the earlier ones, they are clearly of significant economic importance.

Now consider the case where the regime-switching model is used to capture the asymmetry of the data. The results are provided in table 16. Under a dogmatic belief, the largest gain is 0.47%, 3.38%, and 2.79% for the three alternative investment universes, respectively. These gains are smaller than those by using the factor approach. Notice that, under dogmatic beliefs, the investor invest only in the factors and the regime-switching model just provides different weights than the normality case. Empirically as we just found out, this performs less well than adding a fourth asymmetry factor. However, when the model mispricing error is large, the utility gain from using the regime-switching model is actually much greater than using the factor approach. The reason is that the pricing ability is more concerned with the non-factor assets. When the mispricing error is large, investors invest in all assets rather than just three factors. When the joint dynamics of all of the assets is better modelled by the regime-switching data-generating process, the net impact becomes greater. As a result, the gain is still substantial even when there is a large model mispricing uncertainty. In contrast, the gain in the factor approach shrinks as the model mispricing uncertainty increases. Similar results also follow, as reported in table 17, if the factors are replaced by the market factor alone, i.e., if the CAPM is used to gauge the economic gain rather than the Fama-French 3-factor model.

5 Conclusion

There are a great number of studies on asymmetric characteristics of asset returns in both domestic and international markets. Of particular interest are the asymmetric correlations where returns tend to have higher correlations with the market when it goes down than when it goes up. Ang and Chen (2002) seem the first to provide a novel test for the null hypothesis of symmetric correlations, but the test, unfortunately, has a few important weaknesses. For example, their test depends on a pre-specified model for the data and the test is a joint test of the null and the specified model. A rejection of the symmetry hypothesis does not preclude the possibility that there is another symmetric model that fits the data perfectly. To overcome this problem, we propose a new symmetry test that is completely model-free. A rejection of the symmetry hypothesis by our test tells us exactly that *any* symmetric model (subject to of course some standard regularity conditions) cannot model the data. In addition, our test is easier to justify rigorously on the econometric ground and has a simple asymptotic chi-square distribution. Moreover, we also provide model-free tests for beta and covariance symmetries.

Complementing existing studies in the asymmetry literature, our paper seems the first to provide a way formally assessing the economic value of asymmetries. We find that, while sample estimates indicate asymmetric correlations in the US stock market, asymmetric correlations are primarily in existence in small stocks, significant only for the CRSP smallest size portfolio at monthly frequency, and for the 4 smallest sizes at daily frequency. The results on asymmetric betas are similar. However, asymmetric covariances are pervasive, statistically significant for size, Fama and French's (1993) and industry portfolios at both daily and weekly frequency. In spite of insignificant correlation and beta asymmetry, we find that the economic value of utilizing the asymmetric characteristics is substantial. Our method seems to apply not only in asymmetric correlations, betas and covariances, but also in many types of other asymmetries and other markets, leaving ample room for future research.

References

Andrews, D., 1991, Heteroskedasticity and autocorrelation consistent covariance estimation, *Econometrica* 59, 817–858.

Ang, A., Bekaert, G., 2000, International asset allocation with time-varying correlations, *Review of Financial studies* 15, 1137–1187.

Ang, A., Chen, J., 2002, Asymmetric correlations of equity portfolios, *Journal of Financial Economics* 63, 443–494.

Ang, A., Chen, J., Xing, Y., 2002, Downside correlation and expected stock returns, working paper, Columbia Business School.

Bae, K.H., Karolyi, G.A., Stulz, R.M., 2001, A new approach to measuring financial contagion, Unpublished working paper, National Bureau of Economic Research, Cambridge, MA.

Ball, R., Kothari, S.P., 1989, Nonstationary expected returns: implications for tests of market efficiency and serial correlation in returns, *Journal of Financial Economics* 25, 51–74.

Banz, R.W., Breen, J.W., 1986, 'Sample-dependent results using accounting and market data: Some evidence, *Journal of Finance 41*, 779–793.

Bekaert, G., Wu, G., 2000, Asymmetric volatility and risk in equity markets, *Review of Financial Studies* 13, 1–42.

Bollerslev, T., 2001, Financial econometrics: past developments and future challenges, *Journal of Econometrics* 100, 41–51.

Boyer, B.H., Gibson, M.S., Loretan, M., 1999, Pitfalls in tests for changes in correlations. International Finance Discussion Paper 597, Board of Governors of the Federal Reserve System, Washington, DC.

Cho, Y.H., Engle, R.F., 2000, Time-varying betas and asymmetric effects of news: empirical analysis of blue chip stocks, working paper, National Bureau of Economic Research, Cambridge, MA.

Conrad, J., Gultekin, M., Kaul, G., 1991, Asymmetric predictability of conditional variances, *Review of Financial Studies* 4, 597–622.

Fama E.F., French, K.R., 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fleming, J., Kirby, C., Ostdiek, B., 2001, The economic value of volatility timing, *Journal of Finance* 56, 329–352.

Forbes, K., Rigobon, R., 2002, No contagion, only interdependence: measuring stock market co-movements, *Journal of Finance* 57, 2223–2261.

Gibbons, M.R., Ross, S.A., Shanken J., 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Hamilton, J. D., 1989, A new approach to the economic analysis of nonstationary time series and the business cycle, Econometrica 57, 357–384.

Hannan, E., 1970, Multiple Time Series, John Wiley: New York.

Hansen, L.P., 1982, Large sample properties of the generalized method of moments estimators, *Econometrica* 50, 1029–1054.

Harvey, C.R., Zhou, G., 1990, Bayesian inference in asset pricing tests, *Journal of Financial Economics* 26,221–254.

Karolyi, A., Stulz, R., 1996, Why do markets move together? an investigation of US-Japan stock return comovements, *Journal of Finance* 51, 951–986.

Kandel, S., Stambaugh, R.F., 1996, On the predictability of stock returns: An asset-allocation perspective, *Journal of Finance* 51, 385–424.

King, B., 1966, Market and industry factors in stock price behavior, *Journal of Business* 39, 139–190.

Longin, F., Solnik, B., 2001, Extreme correlation of international equity markets. *Journal of Finance* 56, 649-676.

MacKinlay, A.C., Pástor, Ľ., 2000, Asset pricing models: implications for expected returns and portfolio selection, *Review of Financial Studies* 13, 883–916.

Moskowitz, T., Grinblatt, M., 1999, Do industries explain momentum? *Journal of Finance 54*, 1249–1290.

Newey, W., West, K., 1994, Automatic lag selection in covariance matrix estimation, *Review of Economic Studies* 61, 631–653.

Pástor, Ľ., 2000, Portfolio selection and asset pricing models, *Journal of Finance* 55, 179–223.

Pástor, Ľ., Stambaugh, R.F., 1999, Costs of equity capital and model mispricing, *Journal of Finance* 54, 67–121.

Pástor, Ľ., Stambaugh, R.F., 2000, Comparing asset pricing models: an investment perspective, *Journal of Financial Economics* 56, 335–381.

Pástor, Ľ., Stambaugh, R.F., 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Priestley, M., 1981, Time Series and Spectral Analysis, Academic Press: London.

Schwert, G.W., 1989, Why does stock market volatility change over time? *Journal of Finance* 44, 1115–1153.

Stambaugh, R., 1995, Unpublished discussion of Karolyi and Stulz (1996). National Bureau of Economic Research, Universities Research Conference on Risk Management, May 1995.

Tu, J., 2003, Are bull and bear markets economically important? working paper, Washington University in St. Louis.

White, H., 1984, *Asymptotic Theory for Econometricians*, Academic Press: San Diego.

A Appendix

A.1 Proof of Theorem 1

To derive the asymptotic distribution of our test statistic J_ρ, we first spell out clearly what the regularity conditions are. Throughout the appendix, we use C to denote a generic bounded constant that may differ from place to place.

Assumption A.1: (i) The return series of the two portfolio returns, $\{R_{1t}, R_{2t}\}$, is a bivariate fourth order stationary process with $E(|R_{1t}|^{4\nu} + E|R_{2t}|^{4\nu}) \leq C$ for some $\nu > 1$; (ii) $\{R_{1t}, R_{2t}\}$ is a α-mixing process with α-mixing coefficient satisfying $\sum_{j=-\infty}^{\infty} j^2 \alpha(j)^{\frac{\nu}{\nu-1}} < \infty$.

Assumption A.2: For $j = 1, 2$, put $X_{jt}^{+}(c) = [R_{1t} - E(R_{1t}|R_{1t} > c)]/[\text{var}(R_{1t}|R_{1t} > c)]^{1/2}$ and $X_{jt}^{-}(c) = [R_{1t} - E(R_{1t}|R_{1t} < -c)]/[\text{var}(R_{1t}|R_{1t} < -c)]^{1/2}$. Define

$$\xi_t(c) = \frac{[X_{1t}^{+}(c)X_{2t}^{+}(c) - \rho^{+}(c)]}{\Pr(R_{1t} > c, R_{2t} > c)} - \frac{[X_{1t}^{-}(c)X_{2t}^{-}(c) - \rho^{-}(c)]}{\Pr(R_{1t} < -c, R_{2t} < -c)}.$$

Let Ω be an $m \times m$ matrix with (i,j)-th element $\Omega_{ij} \equiv \sum_{l=-\infty}^{\infty} \text{cov}[\xi_t(c_i), \xi_{t-l}(c_j)]$. Then for the prespecified vector $c = (c_1, c_2, ..., c_m)' \in \mathbb{R}^m$, the variance-covariance matrix Ω is finite and nonsingular.

Assumption A.3: The kernel function $k : \mathbb{R} \to [-1, 1]$ is symmetric about zero and is continuous at all points expect a finite number of them on $\mathbb{R}$, with $k(0) = 1$ and $\int_{-\infty}^{\infty} |k(z)| dz < \infty$.

Assumption A.4: The bandwidth $p = p(T) \to \infty,\ p/T \to 0$ as the sample size $T \to \infty$.

Assumption A.5: (i) For some $b > 1$, $|k(z)| \leq C|z|^{-b}$ as $z \to \infty$; (ii) $|k(z_1) - k(z_2)| \leq C|z_1 - z_2|$ for any z_1, z_2 in $\mathbb{R}$, where C is a bounded constant.

Assumption A.6: $\hat{p}$ is a data-dependent bandwidth such that $\hat{p}/p = 1 + O_P(p^{1+b}/T^{\kappa(1+b)})$ for any $0 < \kappa < \frac{1}{2}$ and some nonstochastic bandwidth p satisfying $p = p(T) \to \infty, p/T^{\kappa} \to 0$.

Assumption A.1 allows for the existence of volatility clustering, which is a well-known stylized fact for most financial time series. The mixing condition is commonly used for nonlinear time series analysis, as is the case with our test because we only consider the cross-correlation in the tail distributions of the returns $\{R_{1t}, R_{2t}\}$. This condition characterizes temporal dependence in return series, and rules out long memory processes. However, it is well-known that returns of portfolios have weak serial correlation. Therefore, the mixing condition is quite reasonable in the present context.

Assumption A.2 is assumed to prevent degeneracy of our test statistic. A necessary but not sufficient condition is that threshold levels c_i's should be distinct. Also, for all c, $\Pr(R_{1t} > c, R_{2t} > c)$ and $\Pr(R_{1t} < -c, R_{2t} < -c)$ are bounded away from below from zero. This condition is easily satisfied in practice, given the fact that financial returns usually have heavy tails.

Assumptions A.3 and A.4 are standard conditions on the kernel function $k(\cdot)$ and bandwidth p. These conditions are sufficient when we use nonstochastic bandwidths. Assumptions A.5 imposes some extra conditions on the kernel function, which is needed when we use data-dependent bandwidth $\hat{p}$. Many commonly used kernels, such as the Bartlett, Parzen, and quadratic-spectral kernels are included. However, Assumption A.5 rules out the truncated and Daniell kernels. For various kernels, see, e.g., Priestely (1981, p.442). Assumption A.6 imposes a rate condition on the data-driven bandwidth $\hat{p}$, which ensures that using $\hat{p}$ rather than p has no impact on the limit distribution of our test statistic. Commonly used data-driven bandwidths are Andrews's (1991) parametric plug-in method or Newey and West's (1994) nonparametric plug-in method. Note that the condition on p in Assumption A.7 is more restrictive than Assumption A.4, but it still allows for optimal bandwidths for most commonly used kernels. All of these ensures that our test is completely model-free. Right prior to the proof, we re-state Theorem 1 in the following technically more clear way,

Theorem 1: Suppose Assumptions A.1–A.4 hold. Then, under H_0, we have (i)

$$\mathcal{J}_\rho = (\hat{\rho}^+ - \hat{\rho}^-)'\hat{\Omega}^{-1}(\hat{\rho}^+ - \hat{\rho}^-) \to^d \chi^2_m$$

as $T \to \infty$; and (ii), if in addition Assumptions A.5 and A.6 hold, $\hat{\mathcal{J}}_\rho - \mathcal{J}_\rho \to^p 0$, and

$$\hat{\mathcal{J}}_\rho \to^d \chi^2_m.$$

Proof of Theorem 1: (i) We first use the Cramer-Wold device to show $\sqrt{T}(\hat{\rho}^+ - \hat{\rho}^-) \to^d N(0, \Omega)$. Put $\hat{\xi}_t = \Sigma_{j=1}^m \lambda_j \hat{\xi}_t(c_j)$ and $\xi_t = \Sigma_{j=1}^m \lambda_j \xi_t(c_j)$, where $\hat{\xi}_t(c)$ and $\xi_t(c)$ are defined in (12) and Assumption A.2 respectively, and $\lambda = (\lambda_1, ..., \lambda_m)'$ is a $m \times 1$ vector such that $\lambda'\lambda = 1$. We then have $\lambda'(\hat{\rho}^+ - \hat{\rho}^-) = \Sigma_{j=1}^m \lambda_j[\hat{\rho}^+(c_j) - \hat{\rho}^-(c_j)] = T^{-1}\Sigma_{t=1}^T \hat{\xi}_t$, and by tedious but straightforward algebra, $\lambda'(\hat{\rho}^+ - \hat{\rho}^-) = T^{-1}\Sigma_{t=1}^T \xi_t + o_P(T^{-1/2})$. In other words, the replacement of the sample means, sample variances, and sample proportions with their population counterparts have no impact on the asymptotic distribution of $\sqrt{T}\lambda'(\hat{\rho}^+ - \hat{\rho}^-)$.

Given Assumption A.1, $\{R_{1t}, R_{2t}\}$ is an α-mixing process, so is ξ_t, which is an instantaneous function of (R_{1t}, R_{2t}). Under $H_0 : \rho^+(c) = \rho^-(c)$ for all c, we have $E(\xi_t) = 0$ because $E[\xi_t(c_j)] = 0$. In addition, given Assumptions A.1 and A.2, we have

$$\begin{aligned} V &= \lim_{T\to\infty} \text{var}\left[T^{-1/2}\sum_{t=1}^{T}\xi_t\right] = \sum_{j=-\infty}^{\infty} \text{cov}(\xi_t, \xi_{t-j}) \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i\lambda_j \sum_{l=-\infty}^{\infty} \text{cov}[\xi_t(c_i), \xi_{t-l}(c_j)] \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i\lambda_j\Omega_{ij} \\ &= \lambda'\Omega\lambda. \end{aligned}$$

Note that $0 < V < \infty$ for all λ such that $\lambda'\lambda = 1$, because Ω is positive definite. Thus, using the central limit theorem for mixing processes (e.g., White 1984, Theorem 5.19), we have

$$\sqrt{T}(\hat{\rho}^+ - \hat{\rho}^-)/\sqrt{V} \to^d N(0, 1),$$

It follows from Cramer-Wold device that $\sqrt{T}(\hat{\rho}^+ - \hat{\rho}^-) \to^d N(0, \Omega)$. It follows that

$$T(\hat{\rho}^+ - \hat{\rho}^-)'\Omega^{-1}(\hat{\rho}^+ - \hat{\rho}^-) \to^d \chi^2_m.$$

Next, we show $\hat{\Omega} \to^p \Omega$. Write $\hat{\Omega} - \Omega = [\hat{\Omega} - E\hat{\Omega}] + [E\hat{\Omega} - \Omega]$. By Andrews (1991, Lemma 1), Assumption A.1 implies that Assumption A of Andrews (1991) hold. It follows from Andrews (1991, Proposition 1(a)) that $\text{var}(\hat{\Omega}) = E[(\hat{\Omega} - E\hat{\Omega})(\hat{\Omega} - E\hat{\Omega})'] = O(p/T)$. Therefore we have $\hat{\Omega} - \Omega = O_P(p^{1/2}/T^{1/2})$ by Chebyshev's inequality. In addition, because Assumption A.1(ii) implies $\Sigma_{j=-\infty}^{\infty}\Omega(j) \le C$, we have

$$E\hat{\Omega} - \Omega = \Sigma_{j=1-T}^{T-1}[(1 - |j|/T)k(j/p) - 1]\Omega(j) + \Sigma_{|j|>T}\Omega(j) \to 0$$

as $T \to \infty$ by Assumption A.4, $p \to \infty$, and dominated convergence. Consequently, we have $\hat{\Omega} \to^p \Omega$. By Slutsky theorem and (A2), we then obtain

$$J = T(\hat{\rho}^+ - \hat{\rho}^-)'\hat{\Omega}^{-1}(\hat{\rho}^+ - \hat{\rho}^-) \to^d \chi^2_m.$$

(ii) Let $\hat{\Omega}^*$ and $\hat{\Omega}$ be the kernel estimators for Ω using the bandwidth $\hat{p}$ and p respectively. It suffices to show $\hat{\Omega}^* - \hat{\Omega} \to^p 0$ and then apply Slutsky theorem. By the definition of $\hat{\Omega}$, we have for the (i,j)-th element,

$$\begin{aligned}
\hat{\Omega}^*_{ij} - \hat{\Omega}_{ij} &= \sum_{l=1-T}^{T-1} [k(l/\hat{p}) - k(l/p)]\,\hat{\gamma}_l(c_i, c_j) \\
&= \sum_{|l|\le q} [k(l/\hat{p}) - k(l/p)]\,\hat{\gamma}_l(c_i, c_j) + \sum_{q<|l|<T} [k(l/\hat{p}) - k(l/p)]\,\hat{\gamma}_l(c_i, c_j) \\
&= \hat{A}_1(i,j) + \hat{A}_2(i,j), \text{ say,} \qquad \text{(A1)}
\end{aligned}$$

where $q = T^{\kappa}$ for κ as in Assumption A.6.

We now consider the first term $\hat{A}_1$. Using Assumption A.5(ii) and the triangle inequality, we have

$$\begin{aligned}
|\hat{A}_1(i,j)| &\le \sum_{|l|\le q} C|(l/\hat{p}) - (l/p)| \cdot |\hat{\gamma}_l(c_i, c_j)| \\
&\le C|\hat{p}^{-1} - p^{-1}|q \sum_{|l|\le q} |\hat{\gamma}_l(c_i, c_j) - \gamma_l(c_i, c_j)| + C|\hat{p}^{-1} - p^{-1}|q \sum_{|l|\le q} |\gamma_l(c_i, c_j)| \\
&= |\hat{p}^{-1} - p^{-1}| O_P(q/T^{1/2} + q) \\
&= O(q|\hat{p}^{-1} - p^{-1}|), \qquad \text{(A2)}
\end{aligned}$$

where we have made use of the facts that $\Sigma_{l=-\infty}^{\infty} |\gamma_l(c_i, c_j)| \le C$ and $\sup_{0<l<T} E[\hat{\gamma}_l(c_i, c_j) - \gamma_l(c_i, c_j)]^2 = O(T^{-1})$, which follows by Hanan (1970, equation (3.3), p. 209) and Assumption A.1 (recall that Assumption A.1 ensures that the fourth order cumulant condition holds).

For the second term $\hat{A}_2(i,j)$, using Assumption A.5(i), we have

$$\begin{aligned}
|\hat{A}_2(i,j)| &\le \sum_{q<|l|<T} C(|l/\hat{p}|^{-b} + |l/p|^{-b})|\hat{\gamma}_l(c_i, c_j)| \\
&\le C(\hat{p}^b + p^b) q^{1-b} q^{-1} \sum_{q<|l|<T} (l/q)^{-b} |\hat{\gamma}_l(c_i, c_j) - \gamma_l(c_i, c_j)| \\
&\quad + C(\hat{p}^b + p^b) q^{-b} \sum_{q<|l|<T} |\gamma_l(c_i, c_j)| \\
&= C(\hat{p}^b + p^b) q^{-b} [O_P(q/T^{1/2}) + o_P(1)], \qquad \text{(A3)}
\end{aligned}$$

where again we have used the facts that $\Sigma_{l=-\infty}^{\infty} |\gamma_l(c_i, c_j)| \le C$ and $\sup_{0<l<T} E[\hat{\gamma}_l(c_i, c_j) - \gamma_l(c_i, c_j)]^2 = O(T^{-1})$.

Combining (A1)–(A3), $q = o(T^{1/2})$ and $\hat{p}/p = 1 + O_P(p^{1+b}/q^{1+b})$ as implied by Assumption A.6, we have $\hat{\Omega}^* - \hat{\Omega} = o_P(1)$. This completes the proof. Q.E.D.

Table 1: Symmetry test for size portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the CRSP 10 size portfolios with monthly data from Jan, 1965 through Dec, 1999. J_ρ is the test statistic for the symmetry hypothesis $H_0: \quad \rho^+(c) = \rho^-(c)$ for all $c > 0$, where $\rho^+(c) = \text{corr}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\rho^-(c) = \text{corr}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional correlations, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. J_ρ has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is chosen as $p = 3$ (for the monthly data).

	simple excess return										continuous excess return									
portfolio	J_ρ	$P(\%)$	$\hat\rho^+(c_i) - \hat\rho^-(c_i)$				summary statistics				J_ρ	$P(\%)$	$\hat\rho^+(c_i) - \hat\rho^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	10.00	4.05	-0.5226	-0.3873	-0.2984	-0.4263	0.87	7.12	1.373	7.75	10.07	3.93	-0.5633	-0.3997	-0.3162	-0.4286	0.30	6.22	1.078	7.48
Size2	5.67	22.56	-0.4396	-0.3485	-0.3267	-0.1468	0.25	5.89	0.909	6.97	3.53	47.33	-0.4678	-0.4844	-0.3700	-0.1514	-0.26	6.27	0.666	6.89
Size3	5.89	20.74	-0.4246	-0.4022	-0.2233	-0.4242	0.01	5.99	0.791	6.65	5.40	24.89	-0.4550	-0.4111	-0.2415	-0.2948	-0.51	6.69	0.568	6.64
Size4	7.14	12.84	-0.4091	-0.3346	-0.1820	-0.3892	-0.03	6.60	0.692	6.34	7.50	11.16	-0.4304	-0.3405	-0.1904	-0.4078	-0.57	7.19	0.490	6.34
Size5	1.69	79.24	-0.3369	-0.3725	-0.4453	-0.4199	-0.31	6.43	0.680	6.18	2.25	69.06	-0.3689	-0.3647	-0.4555	-0.4356	-0.82	7.55	0.486	6.23
Size6	1.00	91.00	-0.2795	-0.3645	-0.3732	-0.3678	-0.35	6.18	0.647	5.98	0.89	92.62	-0.2826	-0.3665	-0.3710	-0.3904	-0.83	7.30	0.464	6.03
Size7	0.54	96.94	-0.2092	-0.2852	-0.3115	-0.2578	-0.53	6.42	0.648	5.76	0.53	97.08	-0.2086	-0.2965	-0.2992	-0.2721	-1.01	7.90	0.477	5.83
Size8	0.57	96.64	-0.1450	-0.1984	-0.3327	-0.7114	-0.58	6.13	0.669	5.50	0.53	97.08	-0.1395	-0.2027	-0.3358	-0.7263	-1.01	7.69	0.512	5.57
Size9	0.59	96.36	-0.0828	-0.1578	-0.1995	-0.4912	-0.59	6.28	0.627	5.14	0.49	97.42	-0.0824	-0.1583	-0.2016	-0.4903	-1.01	8.04	0.490	5.20
Size10	0.01	100.00	-0.0063	-0.0141	-0.0182	-0.0508	-0.37	5.22	0.508	4.33	0.01	100.00	-0.0067	-0.0151	-0.0211	-0.0612	-0.65	6.04	0.412	4.34

Table 2: Symmetry test for Fama-French 25 portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the Fama-French 25 portfolios with monthly data from Jan, 1965 through Dec, 1999.

	simple excess return											continuous excess return										
portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics				J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
S1B1	2.02	73.29	-0.3349	-0.3971	-0.4748	-0.9126	-0.33	4.90	0.302	7.70	1.97	74.19	-0.3576	-0.4019	-0.4159	-0.9246	-0.82	6.28	0.002	7.84
S1B2	2.06	72.42	-0.3321	-0.3627	-0.3804	-0.5462	-0.29	5.62	0.779	6.73	2.28	68.37	-0.3548	-0.3795	-0.3495	-0.5796	-0.78	6.76	0.548	6.79
S1B3	3.31	50.76	-0.3432	-0.4238	-0.2090	-0.4385	-0.32	5.89	0.781	6.07	2.93	56.99	-0.3535	-0.4245	-0.1984	-0.4522	-0.78	7.01	0.593	6.11
S1B4	1.62	80.47	-0.3184	-0.3597	-0.4362	-0.5452	-0.25	6.66	0.981	5.71	1.34	85.39	-0.3321	-0.3762	-0.4179	-0.2847	-0.76	7.85	0.811	5.72
S1B5	2.59	62.78	-0.3116	-0.2932	-0.4029	-0.1344	-0.10	7.21	1.086	6.01	2.36	66.92	-0.3307	-0.3171	-0.4067	-0.1568	-0.66	8.04	0.899	6.00
S2B1	0.70	95.09	-0.1868	-0.2699	-0.3946	-0.7703	-0.46	4.65	0.498	7.28	0.95	91.76	-0.1903	-0.2850	-0.2697	-0.7798	-0.89	6.08	0.228	7.40
S2B2	1.20	87.89	-0.1846	-0.3440	-0.3965	-0.3762	-0.55	5.91	0.657	6.13	0.97	91.48	-0.1864	-0.3325	-0.3683	-0.3849	-1.03	7.75	0.463	6.22
S2B3	0.78	94.07	-0.2269	-0.3162	-0.3844	-0.6272	-0.50	6.67	0.861	5.50	1.71	78.96	-0.2225	-0.3230	-0.2017	-0.5957	-0.97	7.93	0.703	5.54
S2B4	1.07	89.96	-0.2032	-0.2323	-0.3495	-0.3618	-0.38	6.79	0.921	5.24	1.12	89.13	-0.2112	-0.2365	-0.3718	-0.4679	-0.84	7.80	0.777	5.26
S2B5	1.87	75.90	-0.2495	-0.2598	-0.2850	-0.4532	-0.21	7.27	0.959	5.73	1.00	90.95	-0.2499	-0.2694	-0.3098	-0.2480	-0.76	8.33	0.789	5.74
S3B1	0.63	95.97	-0.1559	-0.2168	-0.3697	-0.4124	-0.41	4.54	0.546	6.65	0.47	97.63	-0.1651	-0.2283	-0.2426	-0.4022	-0.78	5.71	0.321	6.73
S3B2	1.04	90.37	-0.1280	-0.1548	-0.2374	-0.6015	-0.68	6.19	0.712	5.57	0.73	94.80	-0.1285	-0.1632	-0.2313	-0.6059	-1.13	7.99	0.550	5.65
S3B3	0.67	95.45	-0.1405	-0.1865	-0.3506	-0.2293	-0.65	5.91	0.666	5.06	0.50	97.32	-0.1408	-0.2052	-0.3691	-0.2783	-1.01	7.10	0.533	5.12
S3B4	0.20	99.53	-0.1206	-0.1563	-0.2141	-0.2942	-0.36	5.89	0.814	4.76	0.16	99.69	-0.1186	-0.1608	-0.2209	-0.3030	-0.70	6.57	0.696	4.77
S3B5	0.45	97.80	-0.1797	-0.2294	-0.2877	-0.4617	-0.35	7.07	0.927	5.35	0.85	93.16	-0.2098	-0.2350	-0.3251	-0.4843	-0.85	8.29	0.777	5.37
S4B1	0.18	99.64	-0.0766	-0.1290	-0.1978	-0.4190	-0.29	4.49	0.615	5.90	0.42	98.06	-0.0802	-0.1388	-0.1351	-0.4746	-0.62	5.32	0.438	5.92
S4B2	0.63	95.91	-0.0648	-0.1255	-0.1763	-0.5070	-0.58	6.14	0.423	5.29	0.62	96.11	-0.0656	-0.1308	-0.1762	-0.5087	-1.02	8.13	0.280	5.36
S4B3	0.73	94.76	-0.1067	-0.1451	-0.2128	-0.5566	-0.50	6.33	0.648	4.90	0.85	93.23	-0.1225	-0.1524	-0.2187	-0.5532	-0.90	7.66	0.524	4.93
S4B4	0.75	94.47	-0.0942	-0.1659	-0.1905	-0.5911	0.06	5.41	0.756	4.63	0.81	93.72	-0.0945	-0.1657	-0.1861	-0.6021	-0.23	5.24	0.645	4.59
S4B5	0.15	99.74	-0.1069	-0.1347	-0.1553	-0.1795	-0.14	5.78	0.902	5.37	0.14	99.76	-0.1109	-0.1443	-0.1674	-0.2027	-0.52	6.20	0.753	5.35
S5B1	0.14	99.76	0.0071	-0.0432	-0.0997	-0.1982	-0.16	4.82	0.532	4.83	0.11	99.86	-0.0020	-0.0499	-0.1136	-0.2194	-0.44	5.28	0.414	4.82
S5B2	0.51	97.28	-0.0804	-0.0733	-0.1198	-0.3505	-0.38	4.94	0.504	4.64	0.63	95.91	-0.0832	-0.0834	-0.1327	-0.4411	-0.67	5.93	0.394	4.65
S5B3	0.53	97.09	-0.0784	-0.1275	-0.1424	-0.7388	-0.25	5.74	0.512	4.36	0.65	95.78	-0.1130	-0.1441	-0.1370	-0.7321	-0.58	6.76	0.415	4.36
S5B4	0.29	99.04	-0.0660	-0.1026	-0.1171	-0.3193	0.03	4.62	0.574	4.24	0.76	94.43	-0.0681	-0.1134	-0.1228	-0.4956	-0.20	4.62	0.483	4.20
S5B5	1.63	80.27	-0.1619	-0.1333	-0.0302	-0.3522	-0.08	4.00	0.619	4.66	1.89	75.51	-0.1885	-0.1597	-0.0398	-0.3821	-0.29	4.28	0.509	4.63

Table 3: Symmetry test for industry portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the 20 industry portfolios with monthly data from Jan, 1965 through Dec, 1999.

	simple excess return											continuous excess return										
portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics				J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Misc.	0.29	99.05	-0.0958	-0.1753	-0.2135	-0.4332	-0.40	4.89	0.752	4.81	0.21	99.48	-0.1034	-0.1781	-0.2403	-0.4389	-0.70	5.83	0.632	4.82
Mining	1.87	76.04	-0.1886	-0.1623	-0.4112	-0.1615	-0.20	5.29	0.418	6.49	2.43	65.77	-0.2133	-0.1662	-0.4352	-0.0867	-0.63	6.04	0.206	6.53
Food	0.74	94.64	-0.1508	-0.1877	-0.2913	0.0101	-0.09	5.31	0.697	4.74	0.54	96.92	-0.1585	-0.1927	-0.2497	-0.0084	-0.39	5.48	0.581	4.71
Apparel	1.07	89.85	-0.2151	-0.3053	-0.2404	-0.0910	-0.28	5.52	0.374	6.39	0.85	93.14	-0.2201	-0.3178	-0.2723	-0.1067	-0.76	7.02	0.167	6.46
Paper	4.08	39.56	-0.1231	-0.2367	-0.0642	-0.3529	0.05	5.38	0.539	5.42	3.85	42.65	-0.1583	-0.2556	-0.0640	-0.3637	-0.33	6.10	0.391	5.39
Chemical	2.15	70.84	-0.0044	-0.1376	-0.0635	0.0159	0.05	5.53	0.598	4.75	2.36	66.90	-0.0118	-0.1580	-0.0730	-0.0021	-0.26	5.62	0.483	4.72
Petroleum	0.48	97.52	-0.1340	-0.1768	-0.2004	-0.3862	0.18	4.59	0.598	5.19	0.58	96.54	-0.1523	-0.1831	-0.1897	-0.3952	-0.08	4.47	0.463	5.13
Construction	1.51	82.45	-0.1362	-0.3154	-0.3684	-0.3562	-0.27	5.71	0.556	5.87	0.98	91.33	-0.1751	-0.3346	-0.3918	-0.2316	-0.73	7.44	0.381	5.91
Prim. Metals	3.38	49.70	-0.2655	-0.4939	-0.4183	-0.4012	0.05	5.39	0.284	6.42	2.92	57.16	-0.2836	-0.5119	-0.4198	-0.3851	-0.42	6.53	0.080	6.42
Fab. Metals	0.62	96.08	-0.1050	-0.0856	-0.1569	-0.3158	-0.52	5.72	0.577	5.35	0.37	98.49	-0.1054	-0.1032	-0.1610	-0.3120	-0.93	7.36	0.430	5.40
Machinery	0.32	98.85	-0.1388	-0.1708	-0.2379	-0.2831	-0.03	3.94	0.614	5.72	0.35	98.65	-0.1728	-0.2225	-0.2736	-0.2989	-0.30	4.62	0.450	5.69
Electrical Eq.	0.24	99.34	-0.0503	-0.1067	-0.2178	-0.5652	-0.12	4.20	0.772	6.04	0.29	99.07	-0.0580	-0.1114	-0.2166	-0.5551	-0.44	5.05	0.587	6.02
Transport Eq.	1.42	84.03	-0.1938	-0.2834	-0.1762	-0.2699	-0.28	5.78	0.398	5.60	1.13	88.88	-0.2166	-0.2967	-0.2058	-0.2849	-0.73	7.48	0.240	5.64
Manufacturing	1.43	83.94	-0.1358	-0.1171	-0.2793	-0.4247	-0.31	4.13	0.513	5.57	1.70	79.06	-0.1434	-0.1326	-0.3062	-0.5387	-0.59	4.92	0.356	5.59
Railroads	1.42	83.99	-0.0995	-0.0185	0.0132	-0.2211	-0.00	4.15	0.794	6.53	0.99	91.13	-0.1139	-0.0749	-0.0125	-0.2299	-0.32	4.51	0.579	6.49
Other Transport.	4.22	37.76	-0.2886	-0.4511	-0.2695	-0.2393	-0.23	4.01	0.326	6.98	3.82	43.10	-0.2813	-0.4359	-0.2587	-0.2453	-0.56	4.66	0.082	7.03
Utilities	0.15	99.74	-0.0016	-0.0251	0.0480	0.2745	0.32	4.31	0.236	4.01	0.40	98.26	0.0083	-0.0253	0.1017	0.2351	0.14	4.06	0.156	3.96
Dept. Stores	2.71	60.74	-0.1144	-0.3218	-0.2962	0.0885	-0.06	4.36	0.669	6.28	2.74	60.23	-0.1246	-0.3401	-0.3115	0.0649	-0.41	5.39	0.470	6.26
Retail	0.54	96.93	-0.1273	-0.2153	-0.2684	-0.4110	-0.43	5.27	0.672	5.75	1.38	84.82	-0.1280	-0.2177	-0.2007	-0.4033	-0.83	6.70	0.502	5.80
Financial	0.11	99.86	-0.0641	-0.0811	-0.0508	0.0430	-0.19	4.31	0.606	5.44	0.34	98.71	-0.0817	-0.0961	-0.0194	0.0122	-0.46	4.59	0.455	5.44

Table 4: Symmetry test for weekly and daily size portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the CRSP 10 size portfolios with weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data. J_ρ is the test statistic for the symmetry hypothesis $H_0: \quad \rho^+(c) = \rho^-(c)$ for all $c > 0$, where $\rho^+(c) = \text{corr}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\rho^-(c) = \text{corr}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional correlations, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. J_ρ has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is automatically selected based on the data by following Newey and West's (1994) procedure (for the weekly and daily data).

	daily excess return										weekly excess return									
portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics				J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	29.12	0.00	-0.3431	-0.2805	-0.2064	-0.2047	-0.17	11.49	0.053	0.75	19.48	0.06	-0.3547	-0.2420	-0.3792	-0.2841	0.32	8.64	0.266	2.38
Size2	15.38	0.40	-0.2677	-0.2469	-0.2187	-0.1037	-0.86	14.53	0.035	0.74	17.44	0.16	-0.3146	-0.2110	-0.3232	-0.2556	-0.19	8.56	0.177	2.28
Size3	18.20	0.11	-0.2864	-0.2447	-0.2242	-0.1087	-0.99	17.96	0.030	0.77	23.02	0.01	-0.2898	-0.1693	-0.3376	-0.4070	-0.43	8.81	0.151	2.28
Size4	14.42	0.61	-0.2120	-0.1877	-0.1813	-0.1201	-1.37	19.44	0.027	0.75	7.41	11.57	-0.2361	-0.1862	-0.2542	-0.2180	-0.54	9.65	0.137	2.23
Size5	4.88	29.99	-0.1556	-0.1485	-0.1444	-0.0822	-1.28	19.84	0.026	0.77	10.04	3.98	-0.1987	-0.1218	-0.2162	-0.1703	-0.70	9.46	0.134	2.24
Size6	2.90	57.41	-0.1222	-0.1180	-0.1076	-0.1105	-1.25	18.49	0.025	0.78	8.41	7.78	-0.1651	-0.1004	-0.2176	-0.1245	-0.75	9.04	0.132	2.24
Size7	1.06	90.12	-0.0932	-0.1020	-0.0833	-0.0672	-1.18	18.79	0.026	0.79	2.00	73.56	-0.1132	-0.0870	-0.1075	-0.0748	-0.81	9.28	0.136	2.24
Size8	0.75	94.44	-0.0741	-0.0797	-0.0750	-0.0698	-1.11	18.43	0.028	0.78	1.15	88.63	-0.0927	-0.0821	-0.1111	-0.0674	-0.75	8.87	0.141	2.20
Size9	0.44	97.89	-0.0494	-0.0621	-0.0622	-0.0799	-1.07	20.04	0.027	0.79	0.22	99.42	-0.0667	-0.0747	-0.0920	-0.0652	-0.73	8.96	0.137	2.16
Size10	0.03	99.99	-0.0017	-0.0038	-0.0021	-0.0080	-1.09	29.79	0.024	0.88	0.00	100.00	-0.0004	-0.0025	-0.0064	-0.0126	-0.38	5.87	0.116	2.04

Table 5: Symmetry test for Fama-French 6 portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the Fama-French 6 portfolios formed on 2 size × 3 book-to-market with monthly (Jan 1965 through Dec 1999; sample size 420), weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data.

portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Fama-French 6 portfolios: 2 size × 3 book-to-market monthly excess return										
S1B1	0.84	93.35	-0.2053	-0.2891	-0.3718	-0.7967	-0.52	5.07	0.545	6.83
S1B2	0.80	93.90	-0.1940	-0.2781	-0.3975	-0.3200	-0.55	6.63	0.831	5.45
S1B3	1.68	79.41	-0.2555	-0.2783	-0.4133	-0.1507	-0.29	7.56	0.980	5.47
S2B1	0.07	99.94	-0.0229	-0.0470	-0.0824	-0.2207	-0.29	5.08	0.537	4.80
S2B2	0.24	99.31	-0.0760	-0.0831	-0.1313	-0.3061	-0.29	5.40	0.511	4.31
S2B3	0.53	97.05	-0.1442	-0.1559	-0.2382	-0.3055	-0.10	5.25	0.708	4.39
Fama-French 6 portfolios: 2 size × 3 book-to-market weekly excess return										
S1B1	1.47	83.28	-0.1103	-0.0887	-0.0730	-0.0174	-0.64	7.77	0.101	2.69
S1B2	2.70	60.85	-0.1222	-0.0944	-0.1271	-0.0414	-0.81	9.36	0.168	2.05
S1B3	7.13	12.94	-0.1715	-0.1193	-0.2058	-0.2928	-0.70	9.35	0.200	2.01
S2B1	0.05	99.97	-0.0152	-0.0223	-0.0218	-0.0504	-0.32	5.43	0.121	2.28
S2B2	0.47	97.60	-0.0532	-0.0540	-0.0956	-0.0674	-0.38	6.62	0.111	1.93
S2B3	1.49	82.76	-0.1027	-0.1069	-0.2001	-0.2583	-0.24	5.23	0.157	1.94
Fama-French 6 portfolios: 2 size × 3 book-to-market daily excess return										
S1B1	2.18	70.30	-0.0815	-0.0943	-0.0677	-0.0932	-0.97	15.20	0.019	0.96
S1B2	1.09	89.53	-0.1060	-0.1197	-0.1250	-0.1278	-1.23	18.47	0.033	0.70
S1B3	1.76	78.07	-0.1409	-0.1644	-0.1649	-0.1718	-1.14	19.65	0.040	0.69
S2B1	0.03	99.99	-0.0080	-0.0128	-0.0246	-0.0371	-0.74	21.57	0.024	0.97
S2B2	0.12	99.83	-0.0316	-0.0516	-0.0615	-0.0728	-1.50	40.20	0.023	0.81
S2B3	0.56	96.79	-0.0693	-0.0981	-0.1037	-0.1220	-1.21	32.23	0.032	0.81

Table 6: Symmetry test for weekly and daily industry portfolios

The table reports symmetric correlation test between the market excess return and the excess return on one of the 20 industry portfolios with weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data.

	daily excess return										weekly excess return									
portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics				J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Misc.	0.13	99.80	-0.0527	-0.0724	-0.0923	-0.0896	-1.46	35.03	0.029	0.88	0.30	99.00	-0.0818	-0.1069	-0.1180	-0.1063	-0.47	6.36	0.160	2.13
Mining	0.93	92.07	-0.1401	-0.2010	-0.2078	-0.1855	-0.59	15.99	0.017	1.10	3.57	46.76	-0.1194	-0.0914	-0.0936	-0.1299	-0.00	6.57	0.119	2.75
Food	1.71	78.97	-0.1034	-0.1633	-0.1522	-0.1988	-1.05	29.87	0.029	0.88	3.29	51.11	-0.1486	-0.1589	-0.1196	-0.2666	-0.25	4.90	0.163	2.05
Apparel	2.03	72.95	-0.1548	-0.1968	-0.1685	-0.1895	-1.55	50.57	0.013	0.95	5.56	23.41	-0.1357	-0.0856	-0.0452	-0.1356	-0.39	8.27	0.093	2.53
Paper	1.38	84.85	-0.1174	-0.1447	-0.1668	-0.1636	-1.61	44.09	0.021	1.00	1.20	87.86	-0.0927	-0.1349	-0.1410	-0.2712	-0.19	6.35	0.131	2.45
Chemical	0.19	99.58	-0.0453	-0.0763	-0.0963	-0.1074	-1.22	33.27	0.026	0.98	0.65	95.70	-0.0684	-0.0836	-0.1676	-0.2116	-0.31	5.63	0.152	2.27
Petroleum	1.53	82.15	-0.1411	-0.2387	-0.2731	-0.3535	-0.70	24.00	0.026	1.12	4.04	40.05	-0.1359	-0.1622	-0.0669	-0.0463	0.12	4.44	0.156	2.47
Construction	3.16	53.14	-0.1391	-0.1741	-0.2302	-0.1711	-1.64	50.08	0.022	0.98	2.13	71.15	-0.1288	-0.1456	-0.1550	-0.2674	-0.30	9.56	0.137	2.54
Prim. Metals	2.15	70.77	-0.1596	-0.1890	-0.2137	-0.2070	-1.82	56.90	0.008	1.08	6.09	19.27	-0.1759	-0.1789	-0.1075	-0.2730	-0.30	7.62	0.073	2.73
Fab. Metals	0.22	99.44	-0.0690	-0.0996	-0.1338	-0.1479	-1.30	29.88	0.024	0.92	2.39	66.40	-0.0617	-0.0103	-0.0408	-0.0618	-0.49	8.44	0.145	2.35
Machinery	0.55	96.83	-0.0539	-0.0969	-0.1203	-0.1407	-0.97	24.20	0.022	1.18	0.42	98.11	-0.0520	-0.0862	-0.1586	-0.1532	-0.15	5.35	0.145	2.74
Electrical Eq.	0.18	99.64	-0.0509	-0.0792	-0.0993	-0.1121	-0.65	16.53	0.032	1.15	0.23	99.39	-0.0790	-0.1287	-0.2066	-0.3264	-0.17	4.96	0.190	2.73
Transport Eq.	3.56	46.88	-0.0894	-0.1653	-0.1856	-0.2796	-0.74	19.04	0.017	1.09	0.83	93.46	-0.1243	-0.1887	-0.2233	-0.2140	-0.12	5.10	0.113	2.58
Manufacturing	0.89	92.54	-0.0688	-0.0883	-0.1312	-0.1814	-0.95	23.27	0.023	1.15	0.33	98.81	-0.0750	-0.1066	-0.1502	-0.2428	-0.32	4.92	0.144	2.64
Railroads	1.54	81.95	-0.1421	-0.1953	-0.2569	-0.2146	-0.83	25.09	0.023	1.11	4.44	34.92	-0.1782	-0.1866	-0.2583	-0.5054	-0.30	7.63	0.148	2.68
Other Transport.	4.39	35.58	-0.1573	-0.1937	-0.2277	-0.2307	-0.27	10.96	0.012	1.29	4.54	33.80	-0.1672	-0.1432	-0.2076	-0.2508	-0.06	5.07	0.109	3.25
Utilities	1.54	81.99	-0.1329	-0.1857	-0.2056	-0.2648	-1.05	38.35	0.011	0.62	3.06	54.76	-0.1639	-0.1569	-0.2381	-0.3118	0.18	5.49	0.067	1.64
Dept. Stores	0.70	95.16	-0.0641	-0.1148	-0.1681	-0.1790	-0.53	17.39	0.026	1.20	0.64	95.86	-0.1087	-0.1395	-0.1986	-0.3828	-0.03	5.56	0.167	2.91
Retail	0.69	95.25	-0.0542	-0.0583	-0.0630	-0.0749	-1.00	21.79	0.029	0.92	0.82	93.64	-0.0730	-0.0868	-0.0591	-0.0846	-0.41	6.32	0.168	2.42
Financial	0.27	99.16	-0.0527	-0.0696	-0.0899	-0.1113	-0.93	22.78	0.027	0.87	2.02	73.23	-0.0777	-0.0549	-0.0637	-0.1249	-0.24	5.01	0.156	2.25

Table 7: Bootstrap of the symmetry test

The table reports the bootstrap results of the symmetry test on the first asset of the Fama-French 25 portfolios 25, the first asset of Fama-French 6 portfolios to examine the small sample properties of the proposed symmetry test. The bootstrap makes an identical and independent distribution assumption on the data. The results are based on 10,000 repetitions with replacement. The first row reports the sample value computed from the raw data. The rest of each panels report the empirical P-value, mean, standard deviation of the empirical distribution of 10,000 repetitions and the 10%, 5% and 1% percentiles.

portfolio	J_ρ	$P(\%)$	$\hat{\rho}^+(c_i) - \hat{\rho}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
FF 25 portfolios: 5 size × 5 book-to-market monthly excess return										
sample	2.02	73.29	-0.3349	-0.3971	-0.4748	-0.9126	-0.33	4.90	0.302	7.70
empirical		85.25								
mean	3.82		-0.3393	-0.4067	-0.4406	-0.8717	-0.32	4.82	0.301	7.67
std	1.98		0.0797	0.1250	0.1885	0.4071	0.25	0.62	0.375	0.37
10	6.34		-0.4424	-0.5711	-0.6875	-1.3917	-0.00	5.62	0.780	8.15
5	7.56		-0.4701	-0.6180	-0.7581	-1.5030	0.08	5.87	0.920	8.28
1	10.34		-0.5260	-0.7121	-0.8943	-1.7262	0.25	6.33	1.189	8.55
Fama-French 6 portfolios: 2 size × 3 book-to-market monthly excess return										
Sample	0.84	93.35	-0.2053	-0.2891	-0.3718	-0.7967	-0.52	5.07	0.545	6.83
empirical		90.52								
mean	1.98		-0.2031	-0.2918	-0.3394	-0.8058	-0.50	4.95	0.550	6.81
std	1.13		0.0586	0.0964	0.1504	0.3634	0.26	0.85	0.330	0.33
10	3.36		-0.2779	-0.4176	-0.5373	-1.3213	-0.16	6.07	0.973	7.25
5	4.10		-0.2987	-0.4565	-0.6023	-1.4400	-0.07	6.40	1.092	7.38
1	5.97		-0.3407	-0.5338	-0.7344	-1.6450	0.08	7.04	1.319	7.62
Fama-French 6 portfolios: 2 size × 3 book-to-market weekly excess return										
Sample	1.47	83.28	-0.1103	-0.0887	-0.0730	-0.0174	-0.64	7.77	0.101	2.69
empirical		83.27								
mean	2.88		-0.1117	-0.0854	-0.0688	-0.0037	-0.63	7.60	0.100	2.69
std	1.54		0.0355	0.0598	0.1024	0.1587	0.30	1.79	0.062	0.08
10	4.94		-0.1575	-0.1615	-0.1960	-0.1735	-0.25	9.98	0.180	2.79
5	5.79		-0.1707	-0.1832	-0.2319	-0.2288	-0.16	10.62	0.202	2.83
1	7.72		-0.1941	-0.2220	-0.3060	-0.3470	-0.01	11.87	0.242	2.89
Fama-French 6 portfolios: 2 size × 3 book-to-market daily excess return										
sample	2.18	70.30	-0.0815	-0.0943	-0.0677	-0.0932	-0.97	15.20	0.019	0.96
empirical		87.64								
mean	4.33		-0.0814	-0.0944	-0.0682	-0.0923	-0.96	15.05	0.019	0.96
std	1.97		0.0219	0.0386	0.0634	0.1046	0.34	3.55	0.010	0.02
10	6.93		-0.1091	-0.1435	-0.1499	-0.2269	-0.54	19.78	0.032	0.99
5	7.95		-0.1169	-0.1581	-0.1726	-0.2673	-0.43	21.29	0.035	0.99
1	10.07		-0.1311	-0.1823	-0.2144	-0.3437	-0.25	23.85	0.042	1.01

Table 8: Beta symmetry test for size portfolios

The table reports symmetric beta test between the market excess return and the excess return on one of the CRSP 10 size portfolios with monthly data from Jan, 1965 through Dec, 1999. J_β is the test statistic for the symmetry hypothesis $H_0: \quad \beta^+(c) = \beta^-(c)$ for all $c > 0$, where $\beta^+(c) = \text{beta}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\beta^-(c) = \text{beta}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional betas, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. J_β has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is chosen as $p = 3$ (for the monthly data).

	simple excess return										continuous excess return									
portfolio	J_β	$P(\%)$	$\hat\beta^+(c_i) - \hat\beta^-(c_i)$				summary statistics				J_β	$P(\%)$	$\hat\beta^+(c_i) - \hat\beta^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	5.99	19.95	-0.2815	0.0322	0.4456	0.7489	0.87	7.12	1.373	7.75	6.51	16.44	-0.3683	-0.0568	0.2553	0.4487	0.30	6.22	1.078	7.48
Size2	3.61	46.19	-0.2868	-0.0926	0.1237	0.9193	0.25	5.89	0.909	6.97	2.89	57.70	-0.3526	-0.3819	-0.0503	0.2698	-0.26	6.27	0.666	6.89
Size3	6.40	17.09	-0.3439	-0.2644	0.2400	0.1877	0.01	5.99	0.791	6.65	5.23	26.45	-0.4202	-0.3248	0.1175	0.3520	-0.51	6.69	0.568	6.64
Size4	8.17	8.55	-0.3149	-0.1760	0.3320	0.4189	-0.03	6.60	0.692	6.34	8.49	7.51	-0.3927	-0.2536	0.2119	0.1947	-0.57	7.19	0.490	6.34
Size5	1.32	85.73	-0.2924	-0.3296	-0.3801	0.1796	-0.31	6.43	0.680	6.18	1.98	73.94	-0.3737	-0.3835	-0.4598	0.0053	-0.82	7.55	0.486	6.23
Size6	0.98	91.25	-0.2575	-0.3449	-0.2817	0.0608	-0.35	6.18	0.647	5.98	1.09	89.60	-0.3122	-0.3984	-0.3434	-0.0902	-0.83	7.30	0.464	6.03
Size7	0.91	92.33	-0.2377	-0.3343	-0.2882	0.2156	-0.53	6.42	0.648	5.76	1.07	89.90	-0.2845	-0.3935	-0.3427	0.0630	-1.01	7.90	0.477	5.83
Size8	0.69	95.31	-0.1782	-0.2301	-0.3492	-0.7254	-0.58	6.13	0.669	5.50	0.78	94.15	-0.2111	-0.2738	-0.4169	-0.7910	-1.01	7.69	0.512	5.57
Size9	0.38	98.43	-0.1226	-0.1905	-0.2615	-0.5166	-0.59	6.28	0.627	5.14	0.42	98.05	-0.1512	-0.2293	-0.2978	-0.5630	-1.01	8.04	0.490	5.20
Size10	0.11	99.86	0.0632	0.0651	0.0764	0.1466	-0.37	5.22	0.508	4.33	0.08	99.92	0.0639	0.0678	0.0801	0.1141	-0.65	6.04	0.412	4.34

Table 9: Beta symmetry test for weekly and daily size portfolios

The table reports symmetric beta test between the market excess return and the excess return on one of the CRSP 10 size portfolios with weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data. J_β is the test statistic for the symmetry hypothesis $H_0: \quad \beta^+(c) = \beta^-(c)$ for all $c > 0$, where $\beta^+(c) = \text{beta}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\beta^-(c) = \text{beta}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional betas, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. J_β has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is automatically selected based on the data by following Newey and West's (1994) procedure (for the weekly and daily data).

	daily excess return										weekly excess return									
portfolio	J_β	$P(\%)$	$\hat\beta^+(c_i) - \hat\beta^-(c_i)$				summary statistics				J_β	$P(\%)$	$\hat\beta^+(c_i) - \hat\beta^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	25.34	0.00	-0.3054	-0.1970	-0.0600	-0.0309	-0.17	11.49	0.053	0.75	10.32	3.54	-0.2562	-0.1562	-0.2648	-0.0468	0.32	8.64	0.266	2.38
Size2	15.30	0.41	-0.2916	-0.2561	-0.1693	0.0206	-0.86	14.53	0.035	0.74	12.80	1.23	-0.2805	-0.1842	-0.2896	-0.1918	-0.19	8.56	0.177	2.28
Size3	21.37	0.03	-0.3361	-0.2776	-0.2175	0.0092	-0.99	17.96	0.030	0.77	22.96	0.01	-0.2940	-0.1514	-0.2896	-0.4390	-0.43	8.81	0.151	2.28
Size4	22.37	0.02	-0.3134	-0.2592	-0.1979	-0.0669	-1.37	19.44	0.027	0.75	8.78	6.69	-0.2625	-0.1900	-0.2020	-0.2733	-0.54	9.65	0.137	2.23
Size5	14.44	0.60	-0.2586	-0.2257	-0.1326	0.0192	-1.28	19.84	0.026	0.77	10.96	2.70	-0.2529	-0.1685	-0.2240	-0.2409	-0.70	9.46	0.134	2.24
Size6	12.20	1.59	-0.2380	-0.1930	-0.1012	-0.0238	-1.25	18.49	0.025	0.78	9.85	4.30	-0.2361	-0.1617	-0.2587	-0.2079	-0.75	9.04	0.132	2.24
Size7	6.50	16.49	-0.2022	-0.1779	-0.0788	0.0358	-1.18	18.79	0.026	0.79	4.43	35.08	-0.2010	-0.1752	-0.1555	-0.1640	-0.81	9.28	0.136	2.24
Size8	5.90	20.65	-0.1714	-0.1424	-0.0637	0.0242	-1.11	18.43	0.028	0.78	2.41	66.11	-0.1610	-0.1501	-0.1583	-0.1660	-0.75	8.87	0.141	2.20
Size9	3.29	51.13	-0.1235	-0.1106	-0.0573	-0.0135	-1.07	20.04	0.027	0.79	0.98	91.24	-0.1216	-0.1295	-0.1440	-0.1522	-0.73	8.96	0.137	2.16
Size10	1.37	84.89	0.0431	0.0337	0.0088	-0.0182	-1.09	29.79	0.024	0.88	0.84	93.28	0.0499	0.0377	0.0082	0.0074	-0.38	5.87	0.116	2.04

Table 10: Covariance symmetry test for size portfolios

The table reports symmetric covariance test between the market excess return and the excess return on one of the CRSP 10 size portfolios with monthly data from Jan, 1965 through Dec, 1999. $J_{\sigma_{12}}$ is the test statistic for the symmetry hypothesis $H_0: \quad \sigma_{12}^+(c) = \sigma_{12}^-(c)$ for all $c > 0$, where $\sigma_{12}^+(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\sigma_{12}^-(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional covariances, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. $J_{\sigma_{12}}$ has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is chosen as $p = 3$ (for the monthly data).

	simple excess return										continuous excess return									
portfolio	$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^+(c_i) - \hat{\sigma}_{12}^-(c_i)$				summary statistics				$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^+(c_i) - \hat{\sigma}_{12}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	9.00	6.11	-0.2414	-0.1919	-0.1400	-0.3380	0.87	7.12	1.373	7.75	11.11	2.54	-0.3687	-0.3731	-0.4444	-0.7570	0.30	6.22	1.078	7.48
Size2	7.15	12.81	-0.2711	-0.2705	-0.3197	-0.3672	0.25	5.89	0.909	6.97	7.46	11.36	-0.4025	-0.4766	-0.6213	-0.8874	-0.26	6.27	0.666	6.89
Size3	7.97	9.26	-0.2846	-0.3188	-0.3124	-0.5657	0.01	5.99	0.791	6.65	9.65	4.68	-0.4131	-0.5037	-0.6208	-0.9235	-0.51	6.69	0.568	6.64
Size4	9.53	4.90	-0.2798	-0.2833	-0.2537	-0.5902	-0.03	6.60	0.692	6.34	10.50	3.28	-0.4050	-0.4728	-0.5709	-1.0619	-0.57	7.19	0.490	6.34
Size5	5.30	25.75	-0.2807	-0.3340	-0.4061	-0.6957	-0.31	6.43	0.680	6.18	7.58	10.84	-0.4170	-0.5163	-0.6802	-1.1700	-0.82	7.55	0.486	6.23
Size6	5.13	27.43	-0.2599	-0.3412	-0.3686	-0.6108	-0.35	6.18	0.647	5.98	6.46	16.76	-0.3788	-0.5155	-0.6411	-1.0168	-0.83	7.30	0.464	6.03
Size7	5.38	25.05	-0.2662	-0.3238	-0.3844	-0.6063	-0.53	6.42	0.648	5.76	6.78	14.80	-0.3845	-0.4999	-0.6363	-1.0412	-1.01	7.90	0.477	5.83
Size8	3.84	42.87	-0.2467	-0.3011	-0.4544	-0.8364	-0.58	6.13	0.669	5.50	4.91	29.66	-0.3589	-0.4706	-0.7044	-1.2148	-1.01	7.69	0.512	5.57
Size9	3.35	50.06	-0.2204	-0.2955	-0.4054	-0.7542	-0.59	6.28	0.627	5.14	4.78	31.07	-0.3353	-0.4507	-0.6666	-1.1542	-1.01	8.04	0.490	5.20
Size10	2.23	69.27	-0.1540	-0.2109	-0.2531	-0.4320	-0.37	5.22	0.508	4.33	3.44	48.67	-0.2556	-0.3447	-0.4500	-0.7389	-0.65	6.04	0.412	4.34

Table 11: Covariance symmetry test for weekly and daily size portfolios

The table reports symmetric covariance test between the market excess return and the excess return on one of the CRSP 10 size portfolios with weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data. $J_{\sigma_{12}}$ is the test statistic for the symmetry hypothesis $H_0: \quad \sigma_{12}^+(c) = \sigma_{12}^-(c)$ for all $c > 0$, where $\sigma_{12}^+(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} > c, R_{2t} > c)$, and $\sigma_{12}^-(c) = \text{cov}(R_{1t}, R_{2t}|R_{1t} < -c, R_{2t} < -c)$ are the conditional covariances, R_{1t} is the return on the CRSP value-weighted market portfolio and R_{2t} is the return on one of the 10 size portfolios. P is the P-value of the test in percentage points. $J_{\sigma_{12}}$ has an asymptotic chi-square distribution with degrees of freedom $m = 4$. The statistic is computed by using Bartlett kernel. The exceedance levels are $c_1 = 0$, $c_2 = 0.5$, $c_3 = 1.0$ and $c_4 = 1.5$, and the lag of the test is automatically selected based on the data by following Newey and West's (1994) procedure (for the weekly and daily data).

	daily excess return											weekly excess return										
portfolio	$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^+(c_i) - \hat{\sigma}_{12}^-(c_i)$				summary statistics				$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^+(c_i) - \hat{\sigma}_{12}^-(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Size1	34.22	0.00	-0.2792	-0.3281	-0.3488	-0.4607	-0.17	11.49	0.053	0.75	24.67	0.01	-0.2243	-0.1716	-0.2722	-0.4904	0.32	8.64	0.266	2.38
Size2	37.69	0.00	-0.2985	-0.3720	-0.4930	-0.6112	-0.86	14.53	0.035	0.74	28.33	0.00	-0.2420	-0.1967	-0.2921	-0.4533	-0.19	8.56	0.177	2.28
Size3	37.59	0.00	-0.3258	-0.4050	-0.4835	-0.5777	-0.99	17.96	0.030	0.77	40.00	0.00	-0.2554	-0.1934	-0.3020	-0.5825	-0.43	8.81	0.151	2.28
Size4	55.08	0.00	-0.3235	-0.4060	-0.4994	-0.6929	-1.37	19.44	0.027	0.75	25.32	0.00	-0.2484	-0.2187	-0.2734	-0.4428	-0.54	9.65	0.137	2.23
Size5	30.73	0.00	-0.2971	-0.3687	-0.4729	-0.5929	-1.28	19.84	0.026	0.77	30.07	0.00	-0.2505	-0.2051	-0.2792	-0.4125	-0.70	9.46	0.134	2.24
Size6	35.75	0.00	-0.2852	-0.3593	-0.3975	-0.6136	-1.25	18.49	0.025	0.78	31.45	0.00	-0.2483	-0.2114	-0.2970	-0.4463	-0.75	9.04	0.132	2.24
Size7	34.55	0.00	-0.2691	-0.3388	-0.3947	-0.6299	-1.18	18.79	0.026	0.79	24.03	0.01	-0.2365	-0.2247	-0.2636	-0.4097	-0.81	9.28	0.136	2.24
Size8	31.77	0.00	-0.2519	-0.3205	-0.4039	-0.6264	-1.11	18.43	0.028	0.78	21.93	0.02	-0.2223	-0.2091	-0.2555	-0.3679	-0.75	8.87	0.141	2.20
Size9	44.78	0.00	-0.2271	-0.2977	-0.4246	-0.7331	-1.07	20.04	0.027	0.79	16.98	0.19	-0.2122	-0.2182	-0.2727	-0.4104	-0.73	8.96	0.137	2.16
Size10	5.85	21.08	-0.1330	-0.2160	-0.3960	-0.8599	-1.09	29.79	0.024	0.88	8.89	6.40	-0.1450	-0.1531	-0.1996	-0.3392	-0.38	5.87	0.116	2.04

Table 12: Covariance symmetry test for Fama-French 6 portfolios

The table reports symmetric covariance test between the market excess return and the excess return on one of the Fama-French 6 portfolios formed on 2 size × 3 book-to-market with monthly (Jan 1965 through Dec 1999; sample size 420), weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data.

portfolio	$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}^+_{12}(c_i) - \hat{\sigma}^-_{12}(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Fama-French 6 portfolios: 2 size × 3 book-to-market monthly excess return										
S1B1	3.88	42.25	-0.2482	-0.3167	-0.4395	-0.8281	-0.52	5.07	0.545	6.83
S1B2	3.58	46.55	-0.2612	-0.3356	-0.4534	-0.5924	-0.55	6.63	0.831	5.45
S1B3	4.45	34.82	-0.2569	-0.2910	-0.3871	-0.4006	-0.29	7.56	0.980	5.47
S2B1	1.84	76.57	-0.1472	-0.2149	-0.2685	-0.4070	-0.29	5.08	0.537	4.80
S2B2	1.73	78.53	-0.1547	-0.1911	-0.2854	-0.5257	-0.29	5.40	0.511	4.31
S2B3	3.49	47.86	-0.1625	-0.1674	-0.2320	-0.4617	-0.10	5.25	0.708	4.39
Fama-French 6 portfolios: 2 size × 3 book-to-market weekly excess return										
S1B1	19.46	0.06	-0.2182	-0.2023	-0.2040	-0.2734	-0.64	7.77	0.101	2.69
S1B2	21.29	0.03	-0.2447	-0.2201	-0.2557	-0.3211	-0.81	9.36	0.168	2.05
S1B3	36.66	0.00	-0.2411	-0.2083	-0.2625	-0.5252	-0.70	9.35	0.200	2.01
S2B1	7.55	10.94	-0.1375	-0.1469	-0.1950	-0.3592	-0.32	5.43	0.121	2.28
S2B2	12.92	1.17	-0.1550	-0.1547	-0.2008	-0.3279	-0.38	6.62	0.111	1.93
S2B3	14.61	0.56	-0.1505	-0.1391	-0.2051	-0.4098	-0.24	5.23	0.157	1.94
Fama-French 6 portfolios: 2 size × 3 book-to-market daily excess return										
S1B1	36.34	0.00	-0.2510	-0.3178	-0.3529	-0.5875	-0.97	15.20	0.019	0.96
S1B2	30.06	0.00	-0.2777	-0.3565	-0.4577	-0.7230	-1.23	18.47	0.033	0.70
S1B3	21.76	0.02	-0.2726	-0.3525	-0.4624	-0.7326	-1.14	19.65	0.040	0.69
S2B1	5.95	20.26	-0.1194	-0.1877	-0.3619	-0.7726	-0.74	21.57	0.024	0.97
S2B2	6.63	15.71	-0.1667	-0.2735	-0.5291	-1.1190	-1.50	40.20	0.023	0.81
S2B3	9.38	5.22	-0.1699	-0.2759	-0.4947	-1.1003	-1.21	32.23	0.032	0.81

Table 13: Covariance symmetry test for weekly and daily industry portfolios

The table reports symmetric covariance test between the market excess return and the excess return on one of the 20 industry portfolios with weekly (Jan 7th, 1965 through Dec 29th, 1999; sample size 1825) and daily (Jan 4th, 1965 through Dec 31th, 1999; sample size 8813) data.

	daily excess return										weekly excess return									
portfolio	$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^{+}(c_i) - \hat{\sigma}_{12}^{-}(c_i)$				summary statistics				$J_{\sigma_{12}}$	$P(\%)$	$\hat{\sigma}_{12}^{+}(c_i) - \hat{\sigma}_{12}^{-}(c_i)$				summary statistics			
			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std			$c_1 = 0$	$c_2 = 0.5$	$c_3 = 1.0$	$c_4 = 1.5$	skew	kurt	mean	std
Misc.	11.08	2.57	-0.1969	-0.3208	-0.5758	-1.1745	-1.46	35.03	0.029	0.88	10.92	2.75	-0.1817	-0.1999	-0.2652	-0.4316	-0.47	6.36	0.160	2.13
Mining	13.81	0.79	-0.2018	-0.3162	-0.5624	-1.0938	-0.59	15.99	0.017	1.10	13.34	0.97	-0.1231	-0.1162	-0.0829	-0.2176	-0.00	6.57	0.119	2.75
Food	7.32	11.98	-0.1842	-0.3207	-0.5805	-1.3484	-1.05	29.87	0.029	0.88	11.11	2.54	-0.1645	-0.1736	-0.2133	-0.3977	-0.25	4.90	0.163	2.05
Apparel	27.92	0.00	-0.2599	-0.4111	-0.6052	-1.1476	-1.55	50.57	0.013	0.95	24.55	0.01	-0.1783	-0.1452	-0.1070	-0.4682	-0.39	8.27	0.093	2.53
Paper	6.27	18.02	-0.1892	-0.3383	-0.6784	-1.5599	-1.61	44.09	0.021	1.00	3.30	50.84	-0.1171	-0.1485	-0.2307	-0.5319	-0.19	6.35	0.131	2.45
Chemical	4.97	29.06	-0.1615	-0.2779	-0.5267	-1.1331	-1.22	33.27	0.026	0.98	8.46	7.60	-0.1459	-0.1593	-0.2496	-0.4734	-0.31	5.63	0.152	2.27
Petroleum	1.68	79.44	-0.1479	-0.3103	-0.7196	-1.7368	-0.70	24.00	0.026	1.12	6.56	16.10	-0.0963	-0.0984	-0.0566	-0.0209	0.12	4.44	0.156	2.47
Construction	11.78	1.91	-0.2174	-0.3791	-0.7386	-1.3662	-1.64	50.08	0.022	0.98	11.90	1.81	-0.1548	-0.1736	-0.1949	-0.4561	-0.30	9.56	0.137	2.54
Prim. Metals	8.01	9.13	-0.2033	-0.3616	-0.7840	-1.7938	-1.82	56.90	0.008	1.08	8.65	7.06	-0.1549	-0.1769	-0.2284	-0.6210	-0.30	7.62	0.073	2.73
Fab. Metals	15.87	0.32	-0.2074	-0.3312	-0.6014	-1.2269	-1.30	29.88	0.024	0.92	24.84	0.01	-0.1630	-0.1343	-0.1732	-0.3636	-0.49	8.44	0.145	2.35
Machinery	3.61	46.18	-0.1393	-0.2418	-0.5020	-1.1836	-0.97	24.20	0.022	1.18	2.20	69.94	-0.1000	-0.1296	-0.2516	-0.4738	-0.15	5.35	0.145	2.74
Electrical Eq.	4.73	31.62	-0.1345	-0.2189	-0.4169	-0.8845	-0.65	16.53	0.032	1.15	3.98	40.90	-0.1163	-0.1334	-0.2157	-0.5219	-0.17	4.96	0.190	2.73
Transport Eq.	3.86	42.59	-0.1467	-0.2702	-0.5345	-1.2087	-0.74	19.04	0.017	1.09	5.80	21.47	-0.1282	-0.1400	-0.2077	-0.3987	-0.12	5.10	0.113	2.58
Manufacturing	11.78	1.91	-0.1591	-0.2408	-0.5029	-1.0358	-0.95	23.27	0.023	1.15	6.39	17.17	-0.1449	-0.1634	-0.2407	-0.3846	-0.32	4.92	0.144	2.64
Railroads	8.24	8.33	-0.1646	-0.2818	-0.6203	-1.4497	-0.83	25.09	0.023	1.11	9.45	5.08	-0.1589	-0.1746	-0.2915	-0.7042	-0.30	7.63	0.148	2.68
Other Transport.	8.39	7.82	-0.1395	-0.2131	-0.4055	-0.8347	-0.27	10.96	0.012	1.29	12.30	1.53	-0.1356	-0.1142	-0.1790	-0.4062	-0.06	5.07	0.109	3.25
Utilities	5.20	26.72	-0.1706	-0.3184	-0.6370	-1.4366	-1.05	38.35	0.011	0.62	5.63	22.87	-0.1102	-0.1066	-0.1632	-0.2900	0.18	5.49	0.067	1.64
Dept. Stores	1.35	85.37	-0.1050	-0.2178	-0.5217	-1.2588	-0.53	17.39	0.026	1.20	3.30	50.87	-0.1037	-0.1291	-0.2548	-0.6152	-0.03	5.56	0.167	2.91
Retail	25.27	0.00	-0.1997	-0.2841	-0.4436	-0.8777	-1.00	21.79	0.029	0.92	15.34	0.40	-0.1744	-0.1760	-0.1781	-0.2651	-0.41	6.32	0.168	2.42
Financial	14.20	0.67	-0.1696	-0.2493	-0.4408	-0.9296	-0.93	22.78	0.027	0.87	16.09	0.29	-0.1514	-0.1257	-0.1511	-0.2678	-0.24	5.01	0.156	2.25

Table 14: Utility gain in the Fama-French model

The table reports the annualized certainty-equivalence gain of expected utility of a mean-variance-optimizing investor with relative risk aversion equal to 3 who switches from believing the Fama-French 3-factor model to believing a factor model with an additional asymmetric factor, for varying degrees of pricing errors σ_α. The investment universe is the Fama-French three factors and the asymmetry factor plus 10 CRSP size portfolios (in the first panel), or plus the Fama-French 25 portfolios (in the second panel), or plus 20 industrial portfolios (in the third panel).

Asy Factor	$\sigma_\alpha = 0$	$\sigma_\alpha = 0.5\%$	$\sigma_\alpha = 1\%$	$\sigma_\alpha = 2\%$	$\sigma_\alpha = 3\%$	$\sigma_\alpha = 6\%$	$\sigma_\alpha = \infty$
			10 CRSP size portfolios				
Dcov10	0.02	0.05	0.11	0.07	0.03	0.00	0.01
Dcov	0.10	0.07	0.04	0.02	0.02	0.04	0.02
CC3	2.54	1.86	0.94	0.21	0.06	0.01	0.01
Dcorr10	3.45	2.54	1.24	0.22	0.05	0.02	0.02
CMC	9.64	6.91	3.10	0.49	0.12	0.03	0.03
			Fama-French 25 portfolios				
Dcov10	0.01	0.07	0.25	0.14	0.06	0.02	0.02
Dcov	0.07	0.05	0.05	0.03	0.04	0.06	0.05
CC3	3.04	2.11	0.83	0.16	0.07	0.03	0.04
Dcorr10	4.44	2.91	1.16	0.19	0.05	0.03	0.04
CMC	13.54	8.32	3.24	0.55	0.12	0.06	0.05
			20 industrial portfolios				
Dcov10	0.04	0.03	0.03	0.02	0.02	0.02	0.03
Dcov	0.05	0.05	0.03	0.03	0.02	0.03	0.03
CC3	2.30	2.02	1.51	0.60	0.23	0.06	0.03
Dcorr10	4.08	3.57	2.51	1.01	0.38	0.07	0.03
CMC	11.68	10.43	7.70	3.25	1.33	0.16	0.04

Table 15: Utility gain in the CAPM

The table reports the annualized certainty-equivalence gain of expected utility of a mean-variance-optimizing investor with relative risk aversion equal to 3 who switches from believing in the CAPM to believing in a model of CAPM plus an asymmetric factor, for varying degrees of pricing errors σ_α. The investment universe is the market factor and the asymmetry factor plus 10 CRSP size portfolios (in the first panel), or plus the Fama-French 25 portfolios (in the second panel), or plus 20 industrial portfolios (in the third panel).

Asy Factor	$\sigma_\alpha = 0$	$\sigma_\alpha = 0.5\%$	$\sigma_\alpha = 1\%$	$\sigma_\alpha = 2\%$	$\sigma_\alpha = 3\%$	$\sigma_\alpha = 6\%$	$\sigma_\alpha = \infty$
			10 CRSP size portfolios				
Dcov10	0.49	0.44	0.35	0.13	0.09	0.08	0.12
Dcov	0.15	0.12	0.10	0.05	0.09	0.14	0.17
CC3	0.19	0.15	0.11	0.05	0.03	0.02	0.06
Dcorr10	0.39	0.33	0.27	0.15	0.07	0.01	0.01
CMC	2.73	2.49	1.80	0.82	0.42	0.09	0.04
			Fama-French 25 portfolios				
Dcov10	1.27	1.01	0.68	0.25	0.10	0.03	0.07
Dcov	0.23	0.19	0.12	0.04	0.03	0.05	0.09
CC3	0.05	0.05	0.08	0.11	0.04	0.04	0.07
Dcorr10	0.21	0.21	0.22	0.26	0.12	0.02	0.03
CMC	2.50	2.13	1.63	0.97	0.51	0.07	0.03
			20 industrial portfolios				
Dcov10	1.46	1.34	0.96	0.41	0.15	0.02	0.02
Dcov	0.16	0.11	0.09	0.07	0.03	0.02	0.03
CC3	0.00	0.01	0.01	0.01	0.01	0.01	0.02
Dcorr10	0.09	0.07	0.05	0.02	0.02	0.02	0.02
CMC	1.70	1.53	1.07	0.49	0.18	0.04	0.03

Table 16: Utility gain in a regime-switching model

The table reports the annualized certainty-equivalence gain of expected utility of a power utility investor with relative risk aversion equal to 3, 6 and 9 who switches from believing in the one-regime Fama-French 3-factor model to believing in a two-regime Fama-French model capturing asymmetry, for varying degrees of pricing errors σ_α. The investment universe is the Fama-French three factors plus 10 CRSP size portfolios (in the first panel), or plus the Fama-French 25 portfolios (in the second panel), or plus 20 industrial portfolios (in the third panel).

	$\sigma_\alpha = 0$	$\sigma_\alpha = 0.5\%$	$\sigma_\alpha = 1\%$	$\sigma_\alpha = 2\%$	$\sigma_\alpha = 3\%$	$\sigma_\alpha = 6\%$	$\sigma_\alpha = \infty$
			10 CRSP size portfolios				
$\gamma = 3$	0.47	1.18	1.63	0.48	1.55	0.91	1.43
$\gamma = 6$	0.22	0.48	0.68	0.72	1.37	0.40	0.42
$\gamma = 9$	0.15	0.31	0.45	0.48	0.85	0.26	0.29
			Fama-French 25 portfolios				
$\gamma = 3$	3.38	1.92	3.58	3.87	4.75	2.90	4.64
$\gamma = 6$	1.67	0.88	1.70	1.71	1.78	1.21	1.79
$\gamma = 9$	1.11	0.58	1.12	1.10	1.21	0.77	1.04
			20 industrial portfolios				
$\gamma = 3$	2.79	1.24	0.90	2.67	2.54	3.07	1.54
$\gamma = 6$	1.41	0.65	0.62	0.63	1.10	0.91	1.74
$\gamma = 9$	0.94	0.42	0.40	0.42	0.71	0.54	0.49

Table 17: Utility gain in a regime-switching model

The table reports the annualized certainty-equivalence gain of expected utility of a power utility investor with relative risk aversion equal to 3, 6 and 9 who switches from believing in the one-regime CAPM to believing in a two-regime CAPM capturing asymmetry, for varying degrees of pricing errors σ_α. The investment universe is the market portfolio plus 10 CRSP size portfolios (in the first panel), or plus the Fama-French 25 portfolios (in the second panel), or plus 20 industrial portfolios (in the third panel).

	$\sigma_\alpha = 0$	$\sigma_\alpha = 0.5\%$	$\sigma_\alpha = 1\%$	$\sigma_\alpha = 2\%$	$\sigma_\alpha = 3\%$	$\sigma_\alpha = 6\%$	$\sigma_\alpha = \infty$
				10 CRSP size portfolios			
$\gamma = 3$	0.93	0.60	0.53	0.37	1.01	0.59	1.49
$\gamma = 6$	0.46	0.30	0.26	0.18	0.81	1.07	0.93
$\gamma = 9$	0.31	0.20	0.17	0.12	0.53	0.70	0.57
				Fama-French 25 portfolios			
$\gamma = 3$	1.46	1.55	0.94	1.98	2.39	2.60	3.77
$\gamma = 6$	0.73	0.77	0.47	0.93	1.07	1.36	1.22
$\gamma = 9$	0.49	0.51	0.31	0.61	0.69	0.86	0.75
				20 industrial portfolios			
$\gamma = 3$	1.15	2.00	1.03	0.83	2.91	1.43	1.02
$\gamma = 6$	0.58	0.99	0.52	0.41	3.06	1.70	0.30
$\gamma = 9$	0.39	0.65	0.35	0.27	1.24	0.60	0.19