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Upper Bounds on Return Predictability

Abstract

This paper investigates whether the degree of predictability can be explained by existing asset pricing models, and provides two theoretical upper bounds on the R-square of the regression of stock returns on predictors for given classes of models of interest. Empirically, we find that the predictive R-square is significantly larger than the upper bounds permitted by well known asset pricing models. Our findings suggest new asset pricing models are needed to have state variables highly correlated with stock returns.

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Keywords: Return predictability, asset pricing, stochastic discount factor, habit formation, long-run risks, rare disaster

1 Introduction

In the past four decades, financial economists and investors have found hundreds of economic variables that can predict stock returns. Examples include the short-term interest rate (Fama and Schwert, 1977; Breen, Glosten, and Jagannathan, 1989; Ang and Bekaert, 2007), the dividend yield (Fama and French, 1988; Campbell and Yogo, 2006; Ang and Bekaert, 2007), the earnings-price ratio (Campbell and Shiller, 1988), term spreads (Campbell, 1987; Fama and French, 1988), the book-to-market ratio (Kothari and Shanken, 1997), inflation (Campbell and Vuolteenaho, 2004), corporate issuing activity (Baker and Wurgler, 2000), the consumption-wealth ratio (Lettau and Ludvigson, 2001), stock volatility (French, Schwert, and Stambaugh, 1987; Guo, 2006). The evidence on return predictability has led to the development of new asset pricing models, such as the habit formation model (Campbell and Cochrane 1999), the long-run risks model (Bansal and Yaron, 2004), and the rare disaster model (Barro, 2006; Gabaix, 2012; Gourio, 2012; Wachter, 2013). While these models allow for time-varying expected returns, it is unclear whether they can explain the degree of predictability found in the data.

This paper provides two upper bounds on predictability given that a set of asset pricing models are true, of which the above three models are special cases. Empirically, we find that the bounds are violated, implying that the above three models plus asset pricing models of the same state variables cannot explain the degree of predictability found in the data.

Our bounds are related to a few studies. Kirby (1998) is the first who relates the stochastic discount factor (SDF) to the R^2 of predictive regressions. However, to test whether a given asset pricing model can explain the degree of predictability, he needs the full specification of the SDF. In contrast, our bounds are non-parametric. They depend on only the state variables of the model and the absence of arbitrage, the necessary condition for rational asset pricing. Therefore, the bounds hold for all asset pricing models of the same state variables and under the same no arbitrage conditions.

Ross (2005, 2014) is the pioneer of providing bounds on predictability. His bound is for *all* asset pricing models under no arbitrage conditions. For example, Ross's bound is about 5% for the monthly data we have. If a variable predicts the market with an R^2 of 6%, then the predictability cannot be explained by any rational asset pricing model according to Ross (2005, 2014). In practice, however, no predictor with R^2 greater than 5% has been uncovered yet. In fact, the best predictor to-date does not generate an R^2 exceeding 2% with monthly data (see, e.g., Rapach and Zhou (2013) for a recent survey of stock return predictability).

In this paper, we investigate Ross's bound by restricting it to a smaller set of asset pricing models, all of which are using the same state variables x, say the consumption growth. With this restriction, we can improve the bound substantially. In other words, for the smaller set of models, the bound can be much smaller than 5% for the monthly data, say it is 1%. Then, if we find empirically that one predictor has an R^2 of 2%, we can claim that all asset pricing models with the same state variables x cannot explain the predictability. Interestingly, the rejection of the models based on our bounds is constructive: it suggests that an asset pricing model that uses state variables $y \neq x$ may explain the predictability as long as y have greater correlation with the asset returns. This is because it is the correlation that drives the bounds. The greater the correlation, the greater the bounds, and so the easier to be satisfied by the data.

While the above bounds are developed in a frictionless market as typically done with standard asset pricing models and other bounds such as the variance bounds of Hansen and Jagannathan (1991) and Bakshi and Chabi-Yo (2012). Our paper also explores the role of market frictions on the bounds. Following Nagel (2013), we augment the SDF with a factor that captures different notions of transaction costs, such as the trading costs of Acharya and Pedersen (2005), the funding liquidity of Brunnermeier and Petersen (2009), or the leverage constraint of Adrian, Etula and Muir (2013). When the liquidity factor of Pátor and Stambaugh (2003) and the leverage factor of Adrian, Etula and Muir (2013) are used as proxies for transaction costs, the proposed bounds implied by some of the well known asset models become larger as they should, but they are still less than the predictive R^2s found in the data. Hence, accounting for transaction cost or market friction still cannot help the aforementioned three major models to explain return predictability.

The rest of the paper is organized as follows. Section 2 provides two upper bounds on the predictive R^2 based the maximum risk aversion or the market Sharpe ratio. Section 3 presents the data and econometric method. Section 4 reports the empirical results for common predictors and some of the well known asset pricing models. Section 5 concludes.

2 Bounds

In this section, we show that the stochastic discount factor (SDF) of a rational asset pricing model imposes a constraint on the predictive regression, suggesting that the predictive R^2 cannot be arbitrarily large. An asset pricing model can potentially explain return predictability if it can pass this necessary bound condition.

2.1 Return predictability

Predictive regression is widely used in the study of return predictability,

$$r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}, \tag{1}$$

where z_t is a predictive variable known at the end of period t. The degree of predictability is measured by the regression R^2,

$$R^2 = \frac{\mathrm{Var}(\alpha + \beta z_t)}{\mathrm{Var}(r_{t+1})}. \tag{2}$$

When $R^2 > 0$, r_{t+1} can be forecasted by z_t. Otherwise, z_t is not a predictor of r_{t+1}. Harvey, Liu and Zhu (2013) provide the references of hundreds of predictors.

2.2 Bound on R^2

An important question is what an asset pricing theory tells us about degree of predictability is possible. Intuitively, the degree of predictability cannot be close to 1. If so, the risky asset is too predictable and one can easily arbitrage between this asset and the riskfree asset. Indeed, the R^2 allowed by asset pricing models is much smaller than 1 for monthly data.

An asset pricing model typically implies, as shown in Cochrane (2005), that the price of any asset is uniquely determined by a Euler equation, and hence its return must satisfy

$$\mathrm{E}[m(x_{t+1}) r_{t+1} | I_t] = 0, \tag{3}$$

where $m(x_{t+1})$ is the stochastic discount factor (SDF) with state variables x_{t+1}, r_{t+1} is the return on the asset in excess of the riskfree rate.

While Kirby (1998) is the first to link R^2 to a given SDF $m(x_{t+1})$, Ross (2005, 2014) is the first to provide an upper bound on R^2. Our result below improves Ross's bound substantially.

Proposition 1 *Let γ be the maximum risk aversion of the investors. If the K-dimensional state variables x_{t+1} satisfy certain distributional assumptions, such as normal distribution, then,*

$$R^2 \leq \bar{R}^2_{RA} = \phi^2_{x,rz}\gamma^2\sigma^2(r_{mkt}). \tag{4}$$

where r_{mkt} is the return of market portfolio, μ_z is the unconditional mean of z_t,

$$\phi^2_{x,rz} = \rho^2_{x,rz}\frac{\mathrm{Var}[r_{t+1}(z_t-\mu_z)]}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)}, \tag{5}$$

and

$$\rho^2_{x,rz} = \frac{\mathrm{Cov}[x_{t+1}, r_{t+1}(z_t-\mu_z)]'\mathrm{Var}^{-1}(x_{t+1})\mathrm{Cov}[x_{t+1}, r_{t+1}(z_t-\mu_z)]}{\mathrm{Var}[r_{t+1}(z_t-\mu_z)]}. \tag{6}$$

Proof. See Appendix A.1.

Proposition 1 provides a benchmark to evaluate whether an asset pricing model can explain the degree of predictability found in the data. If an asset pricing model generates an upper bound of 5%, larger than an $R^2 = 3\%$ from the data, then the model can potentially explain the degree of predictability. However, if the data yield an R^2 of 6%, it will be impossible for the model to explain the predictability. As the bound is free of the functional form of $m(\cdot)$, so all asset pricing models with the same state variables x cannot explain the predictability. A research needs to search new state variables to build a model to explain the time-varying expected returns of the asset.

There are three terms in the bound (4). The first term can be broken down further into two terms as (5). The first term is the key as the second term of (5) is a standardized variance. Since z_t is in the time t information set, $r_{t+1}(z_t - \mu_z)$ can be interpreted as a position of $z_t - \mu_z$ units of investment in r_{t+1}. Therefore, (6), the first term of (5), measures the correlation between the asset return and the state variables.[1] If the state variables have zero multiple correlation with the asset return, the SDF $m(x_{t+1})$ will be uncorrelated with the asset return, so it will not price the asset properly and cannot explain the predictability either.

The second term of the bound is the variance of the market portfolio which is easily estimated and computed in practice. The last term of the bound, γ, is known to be below 10, as argued by Mehra and Prescott (1985). Ross (2005) uses the insurance premium to explain that a value of 5 is large enough. Barro and Ursúa (2012) suggest that "a γ [risk aversion] of 6 seems implausibly high." Empirically, Guiso, Sapienza and Zingales (2011) find that the average risk aversion increases from 2.85 before the 2008 crisis to 3.27 after the collapse of the financial market. Paravisini, Rappoport and Ravina (2012) estimate the risk aversion from investors' financial decisions and find that the average risk aversion is 2.85 with a median of 1.62. We follow Ross (2005) in our empirical applications later by setting the maximum risk aversion to be 5.

It is worth emphasizing that (4) depends on only the state variables of the model and the absence of arbitrage. It is hence a non-parametric bound, depending neither on parameters of the model nor functional form of $m(\cdot)$. What matters only is the state variables. For example, although Basan and Yaron (2004) and Basal, Kiku and Yaron (2012) assume different persistence in the consumption volatility, Proposition 1 treats them as the same one since the two models share the same state variables. As a consequence of being non-parametric or independent of $m(\cdot)$, we no longer worry about how to estimate some complex parameters of a model to apply the bound test. For example, the SDF with the habit formation model is

$$m_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-A}, \tag{7}$$

where S_t is the surplus consumption ratio. Even if we do not know A, we can apply the bound test as long as we know the state variables $x = (\log(\frac{S_{t+1}}{S_t}), \log(\frac{C_{t+1}}{C_t}))'$. The functional form of (7) is unnecessary either.

[1] z_t may be replaced by any function $f(z_t)$. It is an open and technically complex question whether the function $f(z_t)$ that maximizes the predictability will also optimize the bound.

Our bound (4) is a substantial improvement over the bound of Ross (2005),

$$R^2 \leq \bar{R}^2_{Ross} = \gamma^2 \sigma^2(r_{mkt}). \tag{8}$$

This improvement is made possible because we have exploited the information of x_{t+1} in m_{t+1}. Comparing (4) with (8), we have improved the bound by introducing the term $\phi^2_{x,rz}$, which measures the squared correlation between x and the asset return. In applications, $\phi^2_{x,rz}$ is often less than 10%. This implies that we improve the bound 10 times or more.

Zhou (2010), based on Kan and Zhou (2007), provides the following upper bound

$$R^2 \leq \rho^2_{x,m_0} \gamma^2 \sigma^2(r_{mkt}), \tag{9}$$

where m_0 is the minimum variance SDF in Hansen and Jagannation (1991) and ρ_{x,m_0} is the multiple correlation between the state variable x and m_0. While there is no analytical relation between $\phi^2_{x,rz}$ and ρ^2_{x,m_0}, our empirical applications later reveal that $\phi^2_{x,rz}$ is almost always smaller than ρ^2_{x,m_0}, and often much smaller. Hence, our bound here is generally much tighter.

Instead of using maximum risk aversion, the predictive R^2 can alternatively be bounded above by the market Sharpe ratio. Ross (1976) shows that the market Sharpe ratio is closely related to the volatility of SDF, which implies that extremely high Sharpe ratios are unlikely to persist. With this insights, Cochrane and Saá-Requejo (2000) use the market Sharpe ratio to bound option prices when there are either market frictions or non-market risks. In short, if there is no arbitrage, the volatility of any SDF must satisfy the following constraint,

$$\mathrm{Std}(m_{t+1}) \leq h \cdot \mathrm{SR}(r_{mkt}), \tag{10}$$

where h is a parameter chosen by the marginal investor and $\mathrm{SR}(r_{mkt})$ is the market Sharpe ratio. Cochrane and Saá-Requejo (2000) suggest the choice of $h = 2$ to rule out "good deals" (arbitrage opportunities), which is also the choice in our applications later.

In terms of the market Sharpe ratio, $\mathrm{SR}^2(r_{mkt})$, we have

Proposition 2 *Under the same distributional assumption of Proposition 1 and (10), the* *predictive R^2 is bounded above,*

$$R^2 \leq \bar{R}^2_{SR} = \phi^2_{x,rz} \cdot h^2 \cdot \mathrm{SR}^2(r_{mkt}). \tag{11}$$

Proof. See Appendix A.2.

The bound (11) is very similar to the earlier one. It is also non-parametric and easy to compute. From an economic perspective, a given maximum risk aversion γ should have close relation to h that ensures the absence of arbitrage. As a result, the bounds with the choice of $\gamma = 5$ and $h = 2$ are numerically close in applications. One may apply one or both depending on one's preference on choosing γ or h or both.

It is worth noting that the bounds have an interesting implication on cross-sectional return predictability. In the finance literature, a large number of studies find that return predictability exists and varies over portfolios sorted by market capitalization (Ferson and Harvey, 1991; Kirby, 1998), book-to-market ratio (Ferson and Harvey, 1991), industry (Ferson and Harvey, 1991), and volatility (Han, Yang and Zhou, 2013). Propositions 1 and 2 suggest that the maximum predictability of the portfolios is likely determined by their correlations with the state variables in the SDF. An asset is allowed to be more predictable if it has a greater correlation with the state variables, regardless of the specification of the functional form of $m(\cdot)$. This suggests a direction of developing new models to identify suitable state variables in order to explain cross-section return predictability or anomaly.

In summary, our bounds, (4) and (11), provide a simple test of whether a class of asset pricing models can explain the degree of predictability, R^2, found in the data. They highlight the fact that the state variables in SDF are the key factor in explaining return predictability. Therefore, if an asset pricing model with state variables x fails to explain the predictability, new state variables $y \neq x$ may explain the predictability as long as y have greater correlation with the asset return. This insight may help explain why Savov (2011) finds garbage, as a measure of consumption, can explain well asset prices as it is more volatile and more correlated with stocks than standard consumption measures.

2.3 Bounds with market frictions

Our bounds are derived, like many other bounds in the literature such as Hansen and Jagannathan (1991) and Bakshi and Chabi-Yo (2012), under the assumption that the market is frictionless and investors can trade freely without constraints. In practice, however, there are various market frictions that can make some profitable opportunities hard to arbitrage, and hence lead to return predictability. This implies that the R^2 upper bound may have to be re-set higher if market frictions are incorporated.

Market frictions can be the transaction costs in Acharya and Pedersen (2005), the funding liquidity of Brunnermeier and Pedersen (2009), or the leverage constraint of Adrian, Etula, and Muir (2013). Nagel (2013) reviews these models and shows that the SDF in a frictionless market can be augmented with a factor Λ_t that captures the state of transaction costs,

$$m_{t+1}^F = m_{t+1} \frac{\Lambda_t}{\Lambda_{t+1}}. \tag{12}$$

Let $\triangle\omega_{t+1} = \log(\Lambda_{t+1}/\Lambda_t)$. Then, we can rewrite m_{t+1}^F as

$$m_{t+1}^F = m^F(x_{t+1}, \triangle\omega_{t+1}). \tag{13}$$

In this way, a higher $\triangle\omega_{t+1}$ means a higher transaction cost, and an asset paying well in the state of higher $\triangle\omega_{t+1}$ earns a low expected return. The bounds in (4) and (11) can be adjusted easily by including $\triangle\omega_{t+1}$ into the state variables. In Section 4, we will show that $\triangle\omega_{t+1}$ will raise the upper bounds as expected, but the raises are quantitatively small. This implies that accounting for market frictions in the three major asset pricing models still cannot explain the return predictability of the data.

3 Data and Econometric Estimation

In this section, we introduce the predictors and state variables used in this paper. We also provide the econometric framework for testing whether the predictive R^2 is less than the upper bounds.

3.1 Data

The data used in this paper are from Welch and Goyal (2008), the Ken French data library and Bureau of Economic Analysis (BEA), where the sources are described in detail. Due to their availability, the monthly data span only over 1959:01–2012:12 and the quarterly data are over 1947Q1–2012Q4. The excess return of the market portfolio is the gross return on the S&P 500 (including dividends) minus the gross return on a risk-free treasury bill. As discussed by Ferson and Korajczyk (1995), it is more appropriate to use the simple return instead of the continuously compounded returns in the context of this paper. This is because the pricing equation says that the expected returns are equal to the conditional covariances of returns with the marginal utility for wealth, which depends on the simple arithmetic return of the optimal portfolio. However, if continuously compounded returns are used, there results will have little changes and the conclusions are exactly the same.

Ten economic predictors are:

1. *Dividend-price ratio* (dp): log of a twelve-month moving sum of dividends paid on the S&P 500 index minus the log of stock prices (S&P 500 index;
2. *Treasury bill rate* (tbl): three-month Treasury bill rate (secondary market);
3. *Long-term yield* (lty): long-term government bond yield;
4. *Long-term return* (ltr): return on long-term government bonds;
5. *Term spread* (tms): difference between the long-term yield on government bonds and the Treasury bill rate;
6. *Default yield spread* (dfy): difference between Moody's BAA- and AAA-rated corporate bond yields;
7. *Default return spread* (dfr): long-term corporate bond return minus the long-term government bond return;
8. *Stock variance* (svar): monthly sum of squared daily returns on the S&P 500 index;
9. *Investment-capital ratio* (ik): ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy;
10. *Consumption-wealth ratio* (cay): cointegration residual between log consumption, broadly-defined financial wealth, and labor income.[2]

[2]Since ik and cay are available at a quarterly frequency, we convert them into monthly frequency when we explore monthly predictability by assigning the most recent quarterly value to each month. For example, the observations of ik and cay in the first quarter of 2014 are assigned to March, April and May 2014,

To calculate the R^2 upper bounds, we need the consumption growth rate which is one of the state variables in the consumption-based asset pricing models. Following common practice, we compute it as the percentage change in the seasonally adjusted, aggregate, real per capita consumption expenditures on nondurable goods and services. We use the annual and quarterly seasonally adjusted aggregate nominal consumption expenditures on nondurables and services from National Income and Product Accounts (NIPA) Table 2.3.5, and the monthly nominal consumption expenditures from NIPA Table 2.8.5. Population numbers from NIPA Tables 2.1 and 2.6 and price deflator series from NIPA Tables 2.3.4 and 2.8.4 are used to construct the time series of per capita real consumption figures. Finally, data on the cross-sectional portfolio returns sorted by size, book-to-market ratio, momentum, and industry are taken from Kenneth French's web site.

3.2 State variables in SDF

Since Mehra and Prescott (1985), there are various consumption-based models that have been developed to explain the equity risk premium puzzle and other features of the data. Among them, the habit formation model, the long-run risks model, and the rare disaster model are three especially noteworthy. Also, all these three models can generate time-varying expected returns and therefore can explain predictability. For this reason, we focus on these three models and investigate whether they can allow for the degree of predictability of the data.

3.2.1 Habit formation

The habit formation model assumes that the risk aversion is time-varying over business cycles. The risk aversion is high in economic recessions when investors require a high premium for taking risk, and the risk aversion is low in economic expansions when investors require a low premium. The countercyclical risk aversion suggests that the risk premium is countercyclical, and hence the stock returns are predictable.

The SDF of the habit formation model is

$$m_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-A}. \tag{14}$$

respectively.

Campbell and Cochrane (1999) assume that the ratios in m_{t+1} are conditionally lognormal, suggesting that we can take

$$x_{t+1} = (\triangle c_{t+1}, \triangle s_{t+1})$$

as the two state variables of the model, where $\triangle c_{t+1} = \log(C_{t+1}/C_t)$ and $\triangle s_{t+1} = \log(S_{t+1}/S_t)$. However, the surplus consumption ratio $S_t = (C_t - X_t)/C_t$ is unobservable since the habit level X_t is latent. Campbell and Cochrane (1999) assume the log surplus consumption ratio S_{t+1} follows

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\triangle c_{t+1} - \mu_c),$$

where ϕ, μ_c and $\bar{s}$ are parameters. The sensitivity function $\lambda(s_t)$ is given by

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1, & s_t < \bar{s} + \frac{1}{2}(1-\bar{S}^2), \\ 0, & s_t \geq \bar{s} + \frac{1}{2}(1-\bar{S}^2), \end{cases}$$

where $\bar{S} = \sigma_c\sqrt{\gamma/(1-\phi)}$ is the steady-state surplus consumption ratio, $\bar{s} = \log(\bar{S})$, and μ_c and σ_c are the mean and standard deviation of the log consumption growth and hence can be easily estimated from the data. We follow Campbell and Cochrane (1999) by extracting s_{t+1} from the model and calculate the multiple correlation between the state variables $x_{t+1} = (\triangle c_{t+1}, \triangle s_{t+1})$ and the excess return with $z_t - \mu_z$ units of investment in the market portfolio.

3.2.2 Long-run risks

The long-run risks model makes use of the low-frequency time series properties of the dividends and aggregate consumption, and thus it can explain simultaneously the equity risk premium puzzle, the risk-free rate puzzle, and the high level of market volatility. The key assumptions of the model are that the consumption growth rate and the dividend growth rate follow the following joint dynamics:

$$\begin{aligned} \triangle c_{t+1} &= \mu_c + \mu_{c,t} + \sigma_t \epsilon_{c,t+1}, \\ \mu_{c,t+1} &= \rho_\mu \mu_{c,t} + \psi_c \sigma_t \epsilon_{\mu,t+1}, \\ \sigma_{t+1}^2 &= (1-\nu)\bar{\sigma}^2 + \nu\sigma_t^2 + \sigma_w \epsilon_{\sigma,t+1}, \\ \triangle d_{t+1} &= \mu_d + \phi\mu_{c,t} + \phi\sigma_t \epsilon_{d,t+1}, \end{aligned}$$

where c_{t+1} is the log aggregate consumption and d_{t+1} is the log dividends. The shocks $\epsilon_{c,t+1}$, $\epsilon_{\mu,t+1}$, $\epsilon_{\sigma,t+1}$, and $\epsilon_{d,t+1}$ are assumed to be i.i.d. normally distributed.

With log-affine approximation, the SDF is

$$\log m_{t+1} = A_0 + A_1\mu_{c,t} + A_2\sigma_t^2 + A_3\triangle c_{t+1} + A_4\mu_{c,t+1} + A_5\sigma_{t+1}^2, \tag{15}$$

where $A_0, \cdots, A_5$ are parameters to be estimated. There are two latent state variables in the SDF, the conditional mean of the consumption growth rate y_t and the conditional variance of its innovation σ_t^2, which are unobserved latent data. Based on Dai and Singleton (2000), Constantinides and Ghosh (2011) find that these two latent variables can be projected on the log risk-free rate $r_{f,t}$ and the log dividend-price ratio dp_t:

$$\begin{aligned} \mu_{c,t} &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 dp_t, \\ \sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 dp_t. \end{aligned}$$

In this way, the log SDF is an affine function of the log risk-free rate, the log dividend-price ratio, and the consumption growth rate:

$$\log m_{t+1} = B_0 + B_1 r_{f,t} + B_2 dp_t + B_3 r_{f,t+1} + B_4 dp_{t+1} + B_5\triangle c_{t+1}.$$

As a result, the state variables in SDF for the long-run risks model are

$$x_{t+1} = (\triangle c_{t+1}, r_{f,t+1}, dp_{t+1})'. \tag{16}$$

3.2.3 Rare disaster

The rare disaster model revived by Barro (2006) is intended to solve the equity risk premium puzzle and does not accommodate time-varying expected returns. Gourio (2008), Gabaix (2012), and Wachter (2013) allow for time-varying probability of disasters, thereby generating return predictability.

The basic assumption for the rare disaster model is that the consumption growth rate follows the stochastic process:

$$\triangle c_{t+1} = \begin{cases} \mu_c + \sigma\epsilon_{t+1}, & \text{with probability } 1 - p_t; \\ \mu_c + \sigma\epsilon_{t+1} + \log(1-b), & \text{with probability } p_t. \end{cases} \tag{17}$$

where ϵ_{t+1} is i.i.d. $N(0,1)$, and $0 < b < 1$ is the size of the disaster. The crucial question is to find a variable to proxy the unobservable probability of disasters. Wachter (2013) considers the rare disaster model in a continuous-time setting, and finds that the dividend-price ratio is a strictly increasing function of the disaster probability, which implies that one can invert this function to find the disaster probability given the observations of the dividend-price ratio. Hence, in addition to the consumption growth rate, the dividend-price ratio can be used as an observable state variable for the rare disaster model. That is,

$$x_{t+1} = (\triangle c_{t+1}, dp_{t+1})'$$

are the state variables we need.

3.3 Wald test

The parameters needed to calculate the predictive R^2 and its upper bounds involve only the mean and covariance of $y_{t+1} = (r_{t+1}, z_t, r_{t+1}z_t, x'_{t+1})'$, where x_{t+1} can be multi-dimensional. The moment conditions are

$$h(y_{t+1}, \theta) = \begin{pmatrix} y_{t+1} - \mu_y \\ y_{t+1}y'_{t+1} - (\Sigma_y + \mu_y\mu'_y) \end{pmatrix}, \tag{18}$$

where $\mu_y = E(y_{t+1})$ and $\Sigma_y = Cov(y_{t+1})$. Since the econometric specification in (18) is exactly identified, the GMM estimator of $\theta = (\mu'_y, \Sigma_y)$ is the value that sets $1/T\sum_{t=1}^{T} h(y_{t+1}, \theta)$ equal to zero.

The distribution of $\hat{\theta}$ takes the form

$$\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{d} N(0, S), \tag{19}$$

where $S = \sum_{j=-\infty}^{\infty} E[h(y_{t+1}, \theta)h(y_{t+1-j}, \theta)']$.

We use a Wald test to evaluate whether $R^2 \leq \bar{R}^2_{RA}$ or $\bar{R}^2_{SR}$, which is equivalent to a one-sided test for $g(\theta_{RA}) = 0$ or $g(\theta_{SR}) = 0$, where θ_{RA} and θ_{Sh} are the moment parameters used in $g(\theta_{RA}) = R^2 - \bar{R}^2_{RA}$ and $g(\theta_{SR}) = R^2 - \bar{R}^2_{SR}$. Let Σ_{RA} and Σ_{SR} be the corresponding covariances of θ_{RA} and θ_{SR}. The Wald statistic is

$$W_{RA} = Tg(\hat{\theta}_{RA})\left[\frac{dg}{d\theta_{RA}}\hat{\Sigma}_{RA}\frac{dg}{d\theta_{RA}}\right]^{-1} g(\hat{\theta}_{RA}) \xrightarrow{d} \chi^2(1) \tag{20}$$

for the bound with the maximum risk aversion, and

$$W_{SR} = Tg(\hat{\theta}_{SR})\left[\frac{dg}{d\theta_{SR}}\hat{\Sigma}_{SR}\frac{dg}{d\theta_{SR}}\right]^{-1} g(\hat{\theta}_{SR}) \xrightarrow{d} \chi^2(1) \tag{21}$$

for the bound with the market Sharpe ratio.

4 Empirical Results

In this section, we compute the bounds for the common predictors, and examine whether or not the three major asset pricing models can explain the degree of predictability found in the data. We investigate both the market predictability and cross-sectional portfolio predictability.

4.1 Market predictability

Consider the predictive market regression,

$$r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1},$$

where r_{t+1} is the excess return on the market portfolio, and z_t is a predictor of interest. Table 1 reports the predictive R^2s, the upper bound of Ross (2005), $\bar{R}^2_{Ross}$, $\phi_{x,rz}$, the coefficient that determines the improvement of our bounds over Ross's, and our two bounds. All the values are presented in percentage points, and statistical significance is assessed by the Wald statistic for testing the hypothesis that the predictive R^2 is less than the upper bound.

The first column of 1 indicates the predictors used. The associated predictive R^2s are reported in the second column, which range from 0.02% for the long-term yield (lty) to

1.62% for consumption-wealth ratio (cay). Positive R^2s suggest that the excess return of the market portfolio is predictable and the degree of predictability varies across the predictors.

The upper bound of Ross (2005), $\bar{R}^2_{Ross}$, is reported in the third column, which has a constant value of 5.09% regardless of what the predictor is used and what asset pricing model is of consideration. Since the maximum R^2 is only 1.62%, the bound is satisfied for all predictors and all models. To the best of our knowledge, there is no single predictor that can generate an R^2 as large as 5.09% or close to it at the monthly frequency. Therefore, $\bar{R}^2_{Ross}$ is unable to reject any of the models for explaining the R^2s.

Column 4 reports the coefficient $\phi_{x,rz}$, which captures the multiple correlation between the state variables and the stock return. Compared with Ross's bound, the proposed bounds improve it by a factor of $1/\phi^2_{x,rz}$. The results shows that, with several exceptions, $\phi_{x,rz}$ is less than 10%. This implies that the bounds improve Ross's (2005) over 100 times in almost all cases. Among the three sets of state variables, the values of $\phi_{x,rz}$ are all small and similar. In other words, all the state variables used by the three asset models have low correlations of about the same magnitude. As a result, our new bounds should be much smaller than Ross's (2005) bound, and are in the same range across the state variables.

Columns 5 and 6 report the numerical values of the two bounds, $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$. As expected, the low value $\phi_{x,rz}$ drives the R^2 upper bound close to zero for all the three sets of state variables of the habit formation model, the long-run risks model and the rare disaster model. Of the 10 predictors, nine display larger R^2s than the two bounds. The only exception is the long-term yield (lty) with a predictive R^2 of 0.02. This value implies very small predictability and so it satisfies the bounds. In other words, from the perspective of the bounds, it is possible for models based on the three sets of state variables to explain the small predictability. Overall, except lty, we can conclude that asset pricing models with the same state variables of the habit formation, of the long-run risks or of the rare disaster models cannot explain the magnitude of return predictability.

While our paper focuses on the most frequent used monthly frequency of the data, it is of interest to see how the results of Table 1 will change if the predictability is examined quarterly. Table 2 reports the results with the quarterly data over 1952Q1–2012Q4. In comparison with Table 1, the predictive R^2s increase significantly, and seven of them are larger than 1%. Again, cay stands out as the most pronounced predictor with an R^2 of

4.77%. This is consistent with the predictability literature that it is generally true that the longer the horizon, the greater the degree of predictability. Theoretically, this appears true too as the Ross's (2005) bound increases to 16.7%, which is much larger than any of the R^2s of the data. Our proposed bounds are greater than before as well. Note that coefficient $\phi_{x,rz}$ is now generally larger, implying smaller improvement over the Ross's bound than the earlier monthly frequency. However, $\phi_{x,rz}$ is still less than 0.3 for almost all the predictors. This implies that our new bounds can improve Ross's bound 11 times or more. For the state variables of the rare disaster model, the associated bounds are all below the R^2, and so we reject asset pricing models based on these state variables for explaining the predictability. For the state variables of the habit model, we reject the model for nine of the predictors. Finally, for the state variables of the long-run risk model, we see models based on them have difficulties for eight of the predictors. Overall, models based on each of the three sets of state variables cannot explain the predictability of the data.

Now we examine the effects of market frictions on the upper bounds. Consider first the liquidity factor constructed by Pástor and Stambaugh (2003) as the proxy of transaction costs. The monthly data span from August 1962 to December 2012. Table 3 reports the results. Ross's (2005) bound increases from the earlier 5.09% of the frictionless case to 5.19%. The change is small and it makes no differences in the inference. However, the percentage changes for our new bounds are relatively large. For example, for the state variables of the habit formation model, the bounds increase about three time from 0.02% percent to 0.06% and 0.05%, respectively. However, the bounds are still small compared with the R^2 values. Indeed, like Table 1, the bounds are binding in almost all cases. Hence, the conclusions are the almost identical to the earlier ones.

Consider next the leverage constraint of Adrian, Etula and Muir (2013) as the proxy of market frictions. In this case, following Nagel (2013), their broker-dealer leverage is a proxy state variable for the friction. The rationale is that de-leveraging indicates deteriorating funding conditions. The data are quarterly and over 1968Q1–2009Q4.[3] Table 4 reports the results. In contrast with the Pástor and Stambaugh liquidity factor, the bounds are generally greater, and the number of non-rejections increases slightly. However, most of the R^2s still violate the bounds. Summarizing Tables 3 and 4, market friction may be a factor to weaken the upper bounds, but the bounds are still binding in most cases. This indicates that, even

[3]We are grateful to Tyler Muir for making the data available on his web page.

after accounting for market frictions, asset pricing models based on one of the three sets of state variables still have difficulties in explaining the magnitude of predictability in the data.

4.2 Portfolio predictability

In this subsection, we examine whether the proposed bounds are also binding for cross-sectional portfolio predictability. Theoretically, our proposed bounds, (4) and (11), should have different values for different portfolios since they have different correlations with the state variables. Hence, it is an empirical question how the bounds vary at the portfolio level.

Tables 5, 6, 7 and 8 report the R^2s and their upper bounds on portfolios sorted by size, value (book-to-market ratio) and momentum. There are a few interesting observations. First, the macroeconomic predictors not only predict the market as shown in Table 1, but also predict all of the cross sectional portfolios with positive R^2s. However, the predictability is generally smaller than that of the market. Second, the upper bounds are smaller too in almost all cases. Third, as a result, it is not surprising that, despite of lower R^2, the bounds are still violated in most cases.

Table 8 reports further results on portfolios sorted by industry. For brevity, we consider only three of the most promising predictors, the dividend-price ratio (dp), the term spread (tms) and the consumption-wealth ratio (cay). Consistent with Ferson and Harvey (1991) and Ferson and Korajczky (1995), the industry portfolios are significantly predictable. However, the predictability varies substantially across industries. The most predictable industry has an R^2 of 1.68%, greater than the market, and the least predictable ones have R^2s virtually zeros across the predictors. The bounds are still of the same magnitude as for other portfolio sorts.

Overall, results on the cross-section portfolios are similar to those on the market predictability, and imply that the three sets of state variables have difficulties in explaining the magnitude of predictability in the portfolio returns.

5 Conclusion

This paper investigates whether or not a given degree of return predictability found in the data is consistent with asset pricing models. To answer this question, we develop two upper bounds on the predictive R^2. Our bounds improve substantially over the non-binding bound of Ross (2005, 2014), and provide likely reasons as to why a given asset pricing model cannot explain the predictability. In forecasting the market return or returns sorted by size, value, momentum and industry, we find that the high predictive R^2s almost always exceed the proposed upper bounds, implying that return predictability cannot be fully explained by asset pricing models based on three sets of well known state variables. The reason is that the correlations between the return(s) and the state variables are low. This conclusion is unaltered even if market frictions are accounted for.

While our study is focused on the stock market, it seems useful to study other asset classes, such as options, bonds and foreign exchanges, to examine whether predictability of the data is consistent with rational models. Technically, it appears a challenging problem to extend our bounds to allow for parameter instability and structural breaks. While these issues are of interest, we leave them for future research.

Appendix

In this appendix, we provide detailed proofs of Propositions 1 and 2.

A.1 Bound with Maximum Risk Aversion

Proposition 1 *Let γ be the maximum risk aversion of the investors. If the state variables x_{t+1} satisfy certain distributional assumptions (detailed below), such as normal distribution, then,*

$$R^2 \leq \bar{R}^2_{RA} = \phi^2_{x,rz}\gamma^2\sigma^2(r_{mkt}). \tag{22}$$

where r_{mkt} is the return of market portfolio, μ_z is the unconditional mean of z_t,

$$\phi^2_{x,rz} = \rho^2_{x,rz}\frac{\text{Var}[r_{t+1}(z_t-\mu_z)]}{\text{Var}(r_{t+1})\text{Var}(z_t)}, \tag{23}$$

and

$$\rho^2_{x,rz} = \frac{\text{Cov}[x_{t+1}, r_{t+1}(z_t-\mu_z)]'\text{Var}^{-1}(x_{t+1})\text{Cov}[x_{t+1}, r_{t+1}(z_t-\mu_z)]}{\text{Var}[r_{t+1}(z_t-\mu_z)]}. \tag{24}$$

Proof. We prove this proposition in two steps. In the first step, we show that, with mild assumptions, the R^2 from the pedictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$ is bounded above as $R^2 \leq \phi^2_{x,rz}\text{Var}(m(x_{t+1}))$, where $m(x_{t+1})$ is a specific SDF. In the second step, we show that the variance of any SDF can be bounded above by the variance of a constant relative risk aversion (CRRA) utility's SDF with risk aversion γ.

Step 1 For ease of exposition, we follow Balduzzi and Kallai (1997) and normalize the SDF as

$$\tilde{m}_{t+1} = \frac{m_{t+1}}{\text{E}(m_{t+1})} \tag{25}$$

so that $\text{E}(\tilde{m}_{t+1}) = 1$. With this normalized SDF, the Euler equation (3) still holds as

$$\text{E}[\tilde{m}(x_{t+1})r_{t+1}|I_t] = 0. \tag{26}$$

Let μ_z denote the mean of predictor z. Since $z_t - \mu_z$ is in the information set I_t, we multiply (26) by $z_t - \mu_z$ in both sides and apply the law of iterated expectations to obtain

$$\mathrm{E}[\tilde{m}_{t+1} r_{t+1}(z_t - \mu_z)] = 0, \tag{27}$$

which can be rewritten as

$$\mathrm{Cov}(r_{t+1}, z_t) = -\mathrm{Cov}[\tilde{m}_{t+1}, r_{t+1}(z_t - \mu_z)]. \tag{28}$$

Since $\mathrm{Cov}(r_{t+1}, z_t) = \mathrm{Cov}(r_{t+1}, z_t - \mu_z) = \mathrm{E}[r_{t+1}(z_t - \mu_z)]$, equality (28) says that the expected excess return with $z_t - \mu_z$ units of investment in the asset r_{t+1} is equal to the negative covariance between the normalized SDF and the realized excess return of the investment, which implies that any dynamic trading strategy that exploits the predictability of r_{t+1} must be priced by the normalized SDF.

In the predictive regression (1), $\beta = \frac{\mathrm{Cov}(r_{t+1}, z_t)}{\mathrm{Var}(z_t)}$. Combining (2) and (28) gives

$$R^2 = \frac{\mathrm{Var}(\alpha + \beta z_t)}{\mathrm{Var}(r_{t+1})} = \frac{\beta^2 \mathrm{Var}(z_t)}{\mathrm{Var}(r_{t+1})} = \frac{\mathrm{Cov}^2(r_{t+1}, z_t)}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} = \frac{\mathrm{Cov}^2[\tilde{m}_{t+1}, r_{t+1}(z_t - \mu_z)]}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)}. \tag{29}$$

This equality is derived first by Kirby (1998) whose test depends only on specific functional form of $m(\cdot)$, but we derive non-parametric bounds here which is independent of $m(\cdot)$.

Consider the first case when x_{t+1} and $r_{t+1}(z_t - \mu_z)$ are jointly normally distributed conditional on time t. From (29), we have

$$\begin{aligned}
R^2 &= \frac{\mathrm{Cov}^2(\tilde{m}_{t+1}, r_{t+1}(z_t - \mu_z))}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \\
&= \frac{\left[\mathrm{Cov}(x_{t+1}, r_{t+1}(z_t - \mu_z))' \mathrm{Var}^{-1}(x_{t+1}) \mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})\right]^2}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \qquad (30) \\
&\leq \left[\mathrm{Cov}(x_{t+1}, r_{t+1}(z_t - \mu_z))' \mathrm{Var}^{-1}(x_{t+1}) \mathrm{Cov}(x_{t+1}, r_{t+1}(z_t - \mu_z))\right] \\
&\quad \times \frac{\left(\mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})' \mathrm{Var}^{-1}(x_{t+1}) \mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})\right)}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \qquad (31) \\
&= \frac{\rho^2_{x,rz} \mathrm{Var}(r_{t+1}(z_t - \mu_z)) \mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})' \mathrm{Var}^{-1}(x) \mathrm{Cov}(\tilde{m}_{t+1}, x_{t+1})}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \qquad (32) \\
&\leq \rho^2_{x,rz} \frac{\mathrm{Var}(r_{t+1}(z_t - \mu_z))}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \mathrm{Var}(\tilde{m}_{t+1}) = \phi^2_{x,rz} \mathrm{Var}(\tilde{m}_{t+1}), \qquad (33)
\end{aligned}$$

where (30) uses Stein's Lemma, which separates the underlying stochastic structure between r_{t+1} and x_{t+1} from the distortion of $\tilde{m}(\cdot)$ (Furman and Zitikis, 2008). Inequalities (31) and

(33) use the Cauchy-Schwarz inequality.

Consider the case when r_{t+1} and $x_{t+1}(z_t - \mu_z)$ follow a general distribution, but with the additional assumption that $\mathrm{E}_t(\varepsilon_{t+1}|x_{t+1}) = 0$, where ε_{t+1} is the residual in the orthogonal decomposition $r_{t+1}(z_t - \mu_z) = a + bx_{t+1} + \varepsilon_{t+1}$. A similar assumption is also used by Kan and Zhou (2007). As discussed there, a sufficient condition for this assumption is that the state variables are elliptically distributed (normal is a special case), which seems to fit state variables well. In fact, though technically very complex, one may expand the density function into Taylor series and plug in them into the bounds. The contributions of higher moments are likely smaller than the first two moments. Since doubling the bounds will not affect much our empirical results, we conjecture that our bounds can be extended by relaxing the assumption. However, we make that assumption here.

Under the assumption $\mathrm{E}_t(\varepsilon_{t+1}|x_{t+1}) = 0$, we have

$$\mathrm{Cov}(\varepsilon_{t+1}, \tilde{m}(x_{t+1}) = \mathrm{E}[\mathrm{E}(\varepsilon_{t+1}|x_{t+1})\tilde{m}(x_{t+1})] = 0.$$

In this case,

$$\mathrm{Cov}(r_{t+1}(z_t - \mu_z), \tilde{m}(x_{t+1})) = \mathrm{Cov}[b'x_{t+1}, \tilde{m}(x_{t+1})] = b'\Sigma_{x\tilde{m}}. \tag{34}$$

The Cauchy-Schwarz inequality generates

$$[\mathrm{Cov}(r_{t+1}(z_t - \mu_z), \tilde{m}(x_{t+1}))]^2 = (b'\Sigma_{xx}^{1/2}\Sigma_{xx}^{-1/2}\Sigma_{x\tilde{m}})^2 \leq (b'\Sigma_{xx}b)(\Sigma_{x\tilde{m}}'\Sigma_{xx}^{-1}\Sigma_{x\tilde{m}}). \tag{35}$$

With (35), (29) can be bounded as

$$\begin{aligned}
R^2 &= \frac{\mathrm{Cov}^2(\tilde{m}_{t+1}, r_{t+1}(z_t - \mu_z))}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} \\
&\leq \frac{b'\Sigma_{xx}b}{\mathrm{Var}(r_{t+1}(z_t - \mu_z))}\frac{\mathrm{Var}(r_{t+1}(z_t - \mu))(\Sigma_{x\tilde{m}}'\Sigma_{xx}^{-1}\Sigma_{x\tilde{m}})}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)} && (36) \\
&\leq \rho_{x,rz}^2\frac{\mathrm{Var}(r_{t+1}(z_t - \mu_z))}{\mathrm{Var}(r_{t+1})\mathrm{Var}(z_t)}\mathrm{Var}(\tilde{m}_{t+1}) && (37) \\
&= \phi_{x,rz}^2\mathrm{Var}(\tilde{m}_{t+1}). && (38)
\end{aligned}$$

From (33) and (38), we can conclude that, given that an asset pricing model can explain predictability, the predictive R^2 cannot be arbitrarily large, but is bounded above by the variance of the SDF that is derived from the asset pricing model.

Step 2 We show that the variance of SDF $\text{Var}(\tilde{m}_{t+1})$ in (33) and (38) can be bounded further, so that the final R^2 bound will not depend on the full sepecification of SDF.

Ross (2005) show that if a utility function, $U(w)$, is bounded above in the relative risk aversion by a utility function $V(w)$, i.e., the risk aversion of $U(w)$ is less than that of $V(w)$, then

$$\text{Var}(\tilde{m}_U) \leq \text{Var}(\tilde{m}_V),$$

where $\tilde{m}_U$ and $\tilde{m}_V$ are the corresponding SDFs. Moreover, if $V(w)$ is a constant relative risk aversion utility function with risk aversion γ ($\gamma \neq 1$), the optimal wealth is the market portfolio and lognormally distributed such as $\log w \sim N[\mu(r_{mkt}), \sigma^2(r_{mkt})]$, then

$$\text{Var}(\tilde{m}_U) \leq \gamma^2 \sigma^2(r_{mkt}). \tag{39}$$

This inequality says that the variance of any SDF can be bounded above by a maximum risk aversion.

Combining (33), (38) and (39), if investors are bounded above by the maximum risk aversion γ, we have the R^2 bound as

$$R^2 \leq \bar{R}^2_{RA} = \phi^2_{x,rz} \gamma^2 \sigma^2(r_{mkt}).$$

This completes Proposition 1.

A.2 Bound with Market Sharpe Ratio

Proposition 2 *Under the same distributional assumption of Proposition 1 and (10), the predictive R^2 is bounded above,*

$$R^2 \leq \bar{R}^2_{SR} = \phi^2_{x,rz} \cdot h^2 \cdot \text{SR}^2(r_{mkt}). \tag{40}$$

Proof. The proof of this proposition consists of two steps too. The first step is the same as that in the proof of Proposition 1, which shows that $R^2 \leq \rho^2_{x,rz}\text{Var}(\tilde{m}_{t+1})$. In the second step, to make the absence of arbitrage true, we assume the constraint (10), i.e.,

$$\text{Std}(m_{t+1}) \leq h \cdot \text{SR}(r_{mkt}).$$

Since $\tilde{m}_{t+1} = m_{t+1}/\text{E}(m_{t+1})$, we have

$$\text{Var}(\tilde{m}_{t+1}) = \frac{\text{Var}(m_{t+1})}{[\text{E}(m_{t+1})]^2} \leq \frac{h}{[\text{E}(m_{t+1})]^2}\text{SR}^2(r_{mkt}).$$

According to (3), in the presence of riskfree asset, $\frac{1}{[\mathrm{E}(m_{t+1})]^2}$ is equal to the riskfree rate. With monthly horizon, the riskfree rate is approximately 1, so we have

$$R^2 \leq \bar{R}^2_{SR} = \phi^2_{x,rz} \cdot h^2 \cdot \mathrm{SR}^2(r_{mkt}).$$

Now if the risk free rate is not equal to 1, we can re-define $\frac{h}{[\mathrm{E}(m_{t+1})]^2}$ as an alternative parameter $\tilde{h}$. The proof is complete.

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Table 1 Bounds on Market Predictability

This table reports the R^2 from the market predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from January 1959 to December 2012. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. $\phi_{x,rz}$ is the key coefficient that determines the improvement of our bounds over Ross's. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

z	R^2(%)	$\bar{R}^2_{Ross}$(%)	$\phi_{x,rz}$	$\bar{R}^2_{RA}$(%)	$\bar{R}^2_{SR}$(%)
			Panel A: Habit formation		
dp	0.23	5.09	0.02	0.00**	0.00**
tbl	0.23	5.09	0.06	0.02**	0.01**
lty	0.02	5.09	0.07	0.02	0.02
ltr	0.90	5.09	0.06	0.02**	0.02**
tms	0.50	5.09	0.08	0.03**	0.03**
dfy	0.26	5.09	0.06	0.02**	0.02**
dfr	0.36	5.09	0.01	0.00**	0.00**
svar	1.09	5.09	0.12	0.08**	0.07**
ik	0.65	5.09	0.15	0.11**	0.09**
cay	1.62	5.09	0.07	0.02**	0.02**
			Panel B: Long-run risks		
dp	0.23	5.09	0.10	0.05**	0.04**
tbl	0.23	5.09	0.08	0.03**	0.03**
lty	0.02	5.09	0.06	0.02	0.02
ltr	0.90	5.09	0.10	0.05**	0.05**
tms	0.50	5.09	0.10	0.05**	0.04**
dfy	0.26	5.09	0.09	0.05**	0.04**
dfr	0.36	5.09	0.01	0.00**	0.00**
svar	1.09	5.09	0.20	0.20**	0.17**
ik	0.65	5.09	0.16	0.13**	0.11**
cay	1.62	5.09	0.08	0.03**	0.02**
			Panel C: Rare disaster		
dp	0.23	5.09	0.08	0.03**	0.03**
tbl	0.23	5.09	0.06	0.02**	0.01**
lty	0.02	5.09	0.06	0.02	0.02
ltr	0.90	5.09	0.10	0.05**	0.05**
tms	0.50	5.09	0.07	0.02**	0.02**
dfy	0.26	5.09	0.09	0.04**	0.04**
dfr	0.36	5.09	0.01	0.00**	0.00**
svar	1.09	5.09	0.12	0.07**	0.06**
ik	0.65	5.09	0.15	0.11**	0.09**
cay	1.62	5.09	0.04	0.01**	0.01**

Table 2 Bounds on Market Predictability with Quarterly Data

This table reports the R^2 from the market predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is over 1952Q1–2012Q4. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. $\phi_{x,rz}$ is the key coefficient that determines the improvement of our bounds over Ross's. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	$\phi_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
			Panel A: Habit formation		
dp	1.45	16.7	0.04	0.03**	0.03**
tbl	1.00	16.7	0.02	0.01**	0.01**
lty	0.28	16.7	0.06	0.05**	0.06**
ltr	1.18	16.7	0.30	1.47	1.59
tms	0.29	16.7	0.09	0.15**	0.16**
dfy	0.31	16.7	0.14	0.33	0.36
dfr	1.66	16.7	0.25	1.06**	1.15**
svar	1.76	16.7	0.07	0.09**	0.09**
ik	2.78	16.7	0.08	0.11**	0.12**
cay	4.77	16.7	0.13	0.29**	0.31**
			Panel B: Long-run risks		
dp	1.45	16.7	0.24	0.93**	1.01**
tbl	1.00	16.7	0.04	0.03**	0.03**
lty	0.28	16.7	0.12	0.24	0.26
ltr	1.18	16.7	0.17	0.50**	0.54**
tms	0.29	16.7	0.14	0.32	0.35
dfy	0.31	16.7	0.10	0.18**	0.20**
dfr	1.66	16.7	0.17	0.51**	0.55**
svar	1.76	16.7	0.09	0.14**	0.16**
ik	2.78	16.7	0.17	0.51**	0.55**
cay	4.77	16.7	0.14	0.35**	0.38**
			Panel C: Rare disaster		
dp	1.45	16.7	0.18	0.54**	0.58**
tbl	1.00	16.7	0.02	0.01**	0.01**
lty	0.28	16.7	0.02	0.01**	0.01**
ltr	1.18	16.7	0.16	0.42**	0.46**
tms	0.29	16.7	0.09	0.13**	0.14**
dfy	0.31	16.7	0.10	0.18**	0.20**
dfr	1.66	16.7	0.15	0.40**	0.43**
svar	1.76	16.7	0.08	0.11**	0.12**
ik	2.78	16.7	0.11	0.19**	0.20**
cay	4.77	16.7	0.12	0.22**	0.24**

Table 3 Bounds on Market Predictability with Liquidity Cost

This table reports the R^2 from the market predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from August 1962 to December 2012. Pastor and Stambaugh (2003) liquidity factor is used as the proxy of liquidity cost. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. $\phi_{x,rz}$ is the key coefficient that determines the improvement of our bounds over Ross's. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	$\phi_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
			Panel A: Habit formation		
dp	0.23	5.19	0.06	0.02**	0.01**
tbl	0.25	5.19	0.07	0.03**	0.02**
lty	0.03	5.19	0.07	0.03	0.02
ltr	0.95	5.19	0.11	0.06**	0.06**
tms	0.47	5.19	0.08	0.04**	0.03**
dfy	0.25	5.19	0.06	0.02	0.02
dfr	0.42	5.19	0.07	0.02**	0.02**
svar	1.09	5.19	0.12	0.07**	0.06**
ik	0.79	5.19	0.18	0.17**	0.15**
cay	1.64	5.19	0.10	0.06**	0.05**
			Panel B: Long-run risks		
dp	0.23	5.19	0.11	0.07**	0.06**
tbl	0.25	5.19	0.09	0.04**	0.04**
lty	0.03	5.19	0.08	0.03	0.03
ltr	0.95	5.19	0.14	0.10**	0.08**
tms	0.47	5.19	0.11	0.06**	0.05**
dfy	0.25	5.19	0.10	0.05**	0.05**
dfr	0.42	5.19	0.07	0.02**	0.02**
svar	1.09	5.19	0.20	0.21**	0.19**
ik	0.79	5.19	0.20	0.20**	0.17**
cay	1.64	5.19	0.10	0.06**	0.05**
			Panel C: Rare disaster		
dp	0.23	5.19	0.09	0.04**	0.04**
tbl	0.25	5.19	0.07	0.03**	0.02**
lty	0.03	5.19	0.08	0.03	0.03
ltr	0.95	5.19	0.14	0.10**	0.08**
tms	0.47	5.19	0.08	0.03**	0.03**
dfy	0.25	5.19	0.09	0.05**	0.04**
dfr	0.42	5.19	0.07	0.02**	0.02**
svar	1.09	5.19	0.11	0.06**	0.06**
ik	0.79	5.19	0.18	0.17**	0.15**
cay	1.64	5.19	0.08	0.04**	0.03**

Table 4 Bounds on Market Predictability with Leverage Constraint

This table reports the R^2 from the market predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is over 1968Q1–2009Q4. Adrian, Etula and Muir (2013) leverage factor is used as the proxy of leverage constraint. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. $\phi_{x,rz}$ is the key coefficient that determines the improvement of our bounds over Ross's. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

z	$R^2(\%)$	$\bar{R}^2_{Ross}(\%)$	$\phi_{x,rz}$	$\bar{R}^2_{RA}(\%)$	$\bar{R}^2_{SR}(\%)$
			Panel A: Habit formation		
dp	0.93	18.5	0.15	0.40**	0.19**
tbl	0.16	18.5	0.08	0.13	0.06*
lty	0.11	18.5	0.13	0.29	0.14
ltr	0.77	18.5	0.43	3.44	1.60
tms	0.47	18.5	0.16	0.45	0.21**
dfy	0.57	18.5	0.20	0.72	0.33**
dfr	3.50	18.5	0.36	2.35**	1.09**
svar	1.23	18.5	0.14	0.37**	0.17**
ik	1.82	18.5	0.19	0.65**	0.30**
cay	5.10	18.5	0.19	0.66**	0.31**
			Panel B: Long-run risks		
dp	0.93	18.5	0.26	1.21	0.56**
tbl	0.16	18.5	0.20	0.76	0.35
lty	0.11	18.5	0.21	0.82	0.38
ltr	0.77	18.5	0.29	1.51	0.70
tms	0.47	18.5	0.15	0.43	0.20**
dfy	0.57	18.5	0.29	1.51	0.70
dfr	3.50	18.5	0.35	2.27**	1.05**
svar	1.23	18.5	0.12	0.27**	0.13**
ik	1.82	18.5	0.18	0.62**	0.29**
cay	5.10	18.5	0.20	0.71**	0.33**
			Panel C: Rare disaster		
dp	0.93	18.5	0.19	0.64**	0.30**
tbl	0.16	18.5	0.08	0.12	0.06*
lty	0.11	18.5	0.16	0.46	0.21
ltr	0.77	18.5	0.27	1.36	0.63
tms	0.47	18.5	0.11	0.21**	0.10**
dfy	0.57	18.5	0.23	1.01	0.47*
dfr	3.50	18.5	0.25	1.17**	0.54**
svar	1.23	18.5	0.11	0.21**	0.10**
ik	1.82	18.5	0.16	0.45**	0.21**
cay	5.10	18.5	0.18	0.62**	0.29**

Table 5 Bounds on Size Portfolio Predictability

This table reports the R^2 from the size portfolio predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from January 1959 to December 2012. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

	Small size portfolio			Median size portfolio			Large size portfolio		
z	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$
Panel A: Habit formation									
dp	0.12	0.01**	0.01**	0.26	0.00**	0.00**	0.22	0.00**	0.00**
tbl	0.46	0.03**	0.02**	0.33	0.02**	0.02**	0.23	0.01**	0.01**
lty	0.14	0.03**	0.03**	0.05	0.04	0.03	0.02	0.02	0.02
ltr	0.95	0.02**	0.02**	1.40	0.03**	0.03**	0.86	0.02**	0.02**
tms	0.52	0.07**	0.06**	0.61	0.06**	0.05**	0.51	0.03**	0.03**
dfy	0.50	0.02**	0.02**	0.68	0.02**	0.02**	0.24	0.02**	0.02**
dfr	0.33	0.01**	0.01**	0.25	0.00**	0.00**	0.37	0.00**	0.00**
svar	0.60	0.05**	0.04**	0.48	0.03**	0.03**	1.03	0.07**	0.06**
ik	0.29	0.10**	0.08**	0.59	0.13**	0.11**	0.64	0.12**	0.10**
cay	0.61	0.02**	0.02**	0.94	0.02**	0.02**	1.67	0.03**	0.02**
Panel B: Long-run risks									
dp	0.12	0.16	0.13	0.26	0.11**	0.09	0.22	0.05**	0.04**
tbl	0.46	0.04**	0.04**	0.33	0.04**	0.03**	0.23	0.03**	0.03**
lty	0.14	0.04**	0.03**	0.05	0.04	0.03	0.02	0.02	0.02
ltr	0.95	0.08**	0.07**	1.40	0.09**	0.08**	0.86	0.05**	0.04**
tms	0.52	0.02**	0.02**	0.61	0.01**	0.01**	0.51	0.05**	0.04**
dfy	0.50	0.04**	0.04**	0.68	0.06**	0.05**	0.24	0.04**	0.04**
dfr	0.33	0.01**	0.00**	0.25	0.00**	0.00**	0.37	0.00**	0.00**
svar	0.60	0.16**	0.13**	0.48	0.13**	0.11**	1.03	0.19**	0.16**
ik	0.29	0.11**	0.09**	0.59	0.15**	0.12**	0.64	0.14**	0.12**
cay	0.61	0.01**	0.00**	0.94	0.01**	0.01**	1.67	0.03**	0.03**
Panel C: Rare disaster									
dp	0.12	0.13	0.11	0.26	0.09**	0.08**	0.22	0.03**	0.02**
tbl	0.46	0.04**	0.03**	0.33	0.04**	0.03**	0.23	0.01**	0.01**
lty	0.14	0.03**	0.03**	0.05	0.04	0.03	0.02	0.02	0.01
ltr	0.95	0.08**	0.07**	1.40	0.09**	0.08**	0.86	0.05**	0.04**
tms	0.52	0.01**	0.01**	0.61	0.01**	0.01**	0.51	0.03**	0.02**
dfy	0.50	0.04**	0.04**	0.68	0.06**	0.05**	0.24	0.04**	0.03**
dfr	0.33	0.01**	0.00**	0.25	0.00**	0.00**	0.37	0.00**	0.00**
svar	0.60	0.03**	0.03**	0.48	0.02**	0.02**	1.03	0.07**	0.06**
ik	0.29	0.08**	0.07**	0.59	0.12**	0.10**	0.64	0.12**	0.10**
cay	0.61	0.00**	0.00**	0.94	0.00**	0.00**	1.67	0.01**	0.01**

Table 6 Bounds on Value Portfolio Predictability

This table reports the R^2 from the value portfolio predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from January 1959 to December 2012. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

	Low bm portfolio			Median bm portfolio			High bm portfolio		
z	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$
				Panel A: Habit formation					
dp	0.19	0.00**	0.00**	0.15	0.00**	0.00**	0.26	0.00**	0.00**
tbl	0.36	0.02**	0.02**	0.20	0.02**	0.01**	0.06	0.01	0.01
lty	0.07	0.03	0.02	0.02	0.02	0.02	0.00	0.03	0.02
ltr	0.74	0.02**	0.02**	1.46	0.02**	0.02**	1.13	0.04**	0.04**
tms	0.58	0.03**	0.03**	0.48	0.04**	0.03**	0.17	0.04**	0.03**
dfy	0.41	0.01**	0.01**	0.26	0.03**	0.03**	0.17	0.03**	0.02**
dfr	0.43	0.01**	0.01**	0.27	0.00**	0.00**	0.20	0.01**	0.01**
svar	0.61	0.04**	0.04**	1.06	0.06**	0.05**	1.84	0.17**	0.15**
ik	0.57	0.12**	0.11**	0.62	0.08**	0.07**	0.39	0.06**	0.05**
cay	1.41	0.03**	0.02**	1.46	0.02**	0.02**	0.68	0.02**	0.02**
				Panel B: Long-run risks					
dp	0.19	0.04**	0.03**	0.15	0.15	0.13	0.26	0.15**	0.13**
tbl	0.36	0.04**	0.04**	0.20	0.03**	0.03**	0.06	0.03	0.03
lty	0.07	0.02	0.02	0.02	0.04	0.03	0.00	0.04	0.03
ltr	0.74	0.06**	0.05**	1.46	0.06**	0.05**	1.13	0.06**	0.05**
tms	0.58	0.06**	0.05**	0.48	0.02**	0.02**	0.17	0.02**	0.02**
dfy	0.41	0.06**	0.05**	0.26	0.05**	0.04**	0.17	0.05**	0.04**
dfr	0.43	0.01**	0.01**	0.27	0.01**	0.01**	0.20	0.01**	0.01**
svar	0.61	0.12**	0.10**	1.06	0.19**	0.16**	1.84	0.19**	0.16**
ik	0.57	0.14**	0.12**	0.62	0.13**	0.11**	0.39	0.13**	0.11**
cay	1.41	0.03**	0.03**	1.46	0.01**	0.01**	0.68	0.01**	0.01**
				Panel C: Rare disaster					
dp	0.19	0.02**	0.02**	0.15	0.02**	0.02**	0.26	0.02**	0.02**
tbl	0.36	0.02**	0.02**	0.20	0.02**	0.02**	0.06	0.02	0.02
lty	0.07	0.01	0.01	0.02	0.01	0.01	0.00	0.01	0.01
ltr	0.74	0.06**	0.05**	1.46	0.06**	0.05**	1.13	0.06**	0.05**
tms	0.58	0.03**	0.03**	0.48	0.03**	0.03**	0.17	0.03**	0.03**
dfy	0.41	0.04**	0.04**	0.26	0.04**	0.04**	0.17	0.04**	0.04**
dfr	0.43	0.00**	0.00**	0.27	0.00**	0.00**	0.20	0.00**	0.00**
svar	0.61	0.05**	0.04**	1.06	0.05**	0.04**	1.84	0.05**	0.04**
ik	0.57	0.13**	0.11**	0.62	0.13**	0.11**	0.39	0.13**	0.11**
cay	1.41	0.01**	0.01**	1.46	0.01**	0.01**	0.68	0.01**	0.01**

Table 7 Bounds on Momentum Portfolio Predictability

This table reports the R^2 from the momentum portfolio predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from January 1959 to December 2012. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

	Loser portfolio			Mediocre portfolio			Winner portfolio		
z	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$
Panel A: Habit formation									
dp	0.26	0.00**	0.00**	0.21	0.01**	0.01**	0.32	0.00**	0.00**
tbl	0.39	0.03**	0.03**	0.40	0.04**	0.04**	0.17	0.01**	0.00**
lty	0.12	0.04*	0.03*	0.06	0.05	0.04	0.02	0.01	0.01
ltr	1.29	0.06**	0.05**	1.80	0.04**	0.03**	0.38	0.00**	0.00**
tms	0.44	0.03**	0.02**	0.75	0.04**	0.03**	0.35	0.07**	0.06**
dfy	1.20	0.02**	0.02**	0.73	0.00**	0.00**	0.03	0.09	0.08**
dfr	0.09	0.04	0.03	0.14	0.01**	0.01**	0.51	0.02**	0.01**
svar	0.11	0.02**	0.02**	0.56	0.02**	0.02**	1.42	0.10**	0.09**
ik	1.05	0.08**	0.07**	1.00	0.09**	0.08**	0.21	0.13*	0.11*
cay	1.14	0.03**	0.02**	1.30	0.04**	0.03**	1.00	0.01**	0.01**
Panel B: Long-run risks									
dp	0.26	0.02**	0.01**	0.21	0.06**	0.05**	0.32	0.16**	0.14**
tbl	0.39	0.03**	0.03**	0.40	0.04**	0.03**	0.17	0.04**	0.03**
lty	0.12	0.02*	0.02*	0.06	0.03	0.03	0.02	0.04	0.04
ltr	1.29	0.07**	0.06**	1.80	0.08**	0.07**	0.38	0.02**	0.02**
tms	0.44	0.02**	0.02**	0.75	0.04**	0.04**	0.35	0.04**	0.03**
dfy	1.20	0.09**	0.08**	0.73	0.07**	0.06**	0.03	0.06	0.05
dfr	0.09	0.01*	0.01*	0.14	0.00*	0.00*	0.51	0.01**	0.01**
svar	0.11	0.09	0.08	0.56	0.10**	0.08**	1.42	0.20**	0.17**
ik	1.05	0.08**	0.07**	1.00	0.12**	0.10**	0.21	0.16	0.13*
cay	1.14	0.02**	0.01**	1.30	0.03**	0.03**	1.00	0.01**	0.01**
Panel C: Rare disaster									
dp	0.26	0.00**	0.00**	0.21	0.04**	0.03**	0.32	0.15**	0.13**
tbl	0.39	0.03**	0.03**	0.40	0.04**	0.03**	0.17	0.03*	0.02*
lty	0.12	0.02*	0.01*	0.06	0.03	0.03	0.02	0.04	0.04
ltr	1.29	0.07**	0.06**	1.80	0.08**	0.07**	0.38	0.02**	0.02**
tms	0.44	0.02**	0.02**	0.75	0.04**	0.03**	0.35	0.01**	0.01**
dfy	1.20	0.02**	0.02**	0.73	0.04**	0.04**	0.03	0.05	0.05
dfr	0.09	0.01	0.01	0.14	0.00*	0.00*	0.51	0.01**	0.01**
svar	0.11	0.03	0.03	0.56	0.04**	0.03**	1.42	0.04**	0.04**
ik	1.05	0.07**	0.06**	1.00	0.09**	0.08**	0.21	0.12*	0.10*
cay	1.14	0.01**	0.01**	1.30	0.01**	0.01**	1.00	0.00**	0.00**

Table 8 Bounds on Industry Portfolio Predictability

This table reports the R^2 from the industry portfolio predictive regression $r_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$, where z_t is the predictor and the time period is from January 1959 to December 2012. $\bar{R}^2_{Ross}$ is Ross's (2005) bound on the R^2, while $\bar{R}^2_{RA}$ and $\bar{R}^2_{SR}$ are the proposed bounds. Statistical significance is assessed by the Wald statistic for testing that the predictive R^2 is less than the theoretical upper bound. ** and * indicate significance at the 5% and 10% levels, respectively.

		Habit formation		Long-run risks		Rare disaster	
Portfolio	R^2	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$	$\bar{R}^2_{RA}$	$\bar{R}^2_{SR}$
Panel A: z_t is the dividend-price ratio (dp)							
NoDur	0.40	0.03**	0.03**	0.15**	0.12**	0.14**	0.12**
Durbl	0.19	0.01**	0.01**	0.04**	0.04**	0.02**	0.02**
Manuf	0.01	0.00	0.00	0.08	0.07	0.04	0.04
Enrgy	0.00	0.00	0.00	0.13	0.11	0.10	0.09
HiTec	0.10	0.01*	0.01*	0.06	0.05	0.03	0.02*
Telcm	0.49	0.01**	0.00**	0.01**	0.01**	0.01**	0.01**
Shops	0.28	0.00**	0.00**	0.08**	0.07**	0.08**	0.06**
Hlth	0.15	0.02*	0.02*	0.09	0.08	0.08	0.07
Utils	0.03	0.00	0.00	0.16	0.13	0.12	0.10
Other	0.31	0.01**	0.01**	0.08**	0.07**	0.05**	0.05**
Panel B: z_t is the term spread (tms)							
NoDur	0.38	0.04**	0.03**	0.02**	0.02**	0.02**	0.02**
Durbl	1.04	0.02**	0.01**	0.02**	0.02**	0.02**	0.02**
Manuf	0.69	0.03**	0.03**	0.02**	0.02**	0.01**	0.01**
Enrgy	0.12	0.03	0.03	0.01	0.01	0.00	0.00
HiTec	0.55	0.03**	0.02**	0.07**	0.06**	0.02**	0.02**
Telcm	0.21	0.01**	0.01**	0.01**	0.01**	0.00**	0.00**
Shops	0.44	0.03**	0.03**	0.02**	0.02**	0.02**	0.02**
Hlth	0.00	0.05	0.04	0.06	0.05	0.06	0.05
Utils	0.23	0.04**	0.03**	0.02**	0.02**	0.02**	0.02**
Other	0.30	0.05**	0.05**	0.04**	0.03**	0.03**	0.03**
Panel C: z_t is the consumption-wealth ratio (cay)							
NoDur	1.67	0.04**	0.04**	0.05**	0.04**	0.03**	0.02**
Durbl	0.89	0.00**	0.00**	0.00**	0.00**	0.00**	0.00**
Manuf	0.89	0.01**	0.01**	0.02**	0.02**	0.01**	0.01**
Enrgy	0.09	0.00	0.00	0.03	0.02	0.02	0.02
HiTec	0.91	0.01**	0.01**	0.01**	0.01**	0.00**	0.00**
Telcm	1.17	0.02**	0.02**	0.02**	0.02**	0.01**	0.01**
Shops	0.94	0.04**	0.03**	0.03**	0.03**	0.01**	0.01**
Hlth	1.01	0.05**	0.04**	0.03**	0.03**	0.02**	0.02**
Utils	0.64	0.03**	0.03**	0.07**	0.06**	0.06**	0.05**
Other	1.68	0.03**	0.02**	0.01**	0.01**	0.01**	0.01**