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Capacity Management in Agricultural Commodity Processing and Application in the Palm Industry

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Abstract

This paper studies the capacity investment decisions of an agri-processor that uses a commodity input to produce a commodity output and a byproduct. Using a multi-period model, we study the one-time processing and (output) storage capacity investment decisions, and the periodic processing and inventory decisions in the presence of input and output spot price uncertainties and uncertain production yield. We identify three capacity investment strategies, investing in storage-dominating, processing-dominating or mixed portfolio, and provide conditions under which each strategy is optimal. Using a calibration based on the palm industry, we provide rules of thumb for capacity management: The processor should decrease its processing capacity with an increase in price correlation; and with an increase (a decrease) in input or output price volatility when this volatility is low (high). The storage capacity should be adjusted in a similar fashion only as a response to a change in output price volatility, otherwise it should not be altered. We find that not accounting for the byproduct revenue or inventory holding possibility in capacity planning leads to sizeable profit loss. Ignoring production yield uncertainty has a significant negative impact on profitability if the capacity planning is made based on the maximum yield possible, as often done in practice; but it has an insignificant impact if the planning is made based on the average yield.

Key Words: Capacity Management, Multi-product Firm, Commodity Risk Management, Spot Market, Agriculture.

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1 Introduction

This paper studies the capacity investment decisions of a processor that uses a primary commodity input to produce a commodity output and a byproduct in the context of agricultural industries. Specifically, we analyze the input processing and output storage capacity investment decisions of the agri-processor. Our analysis is applicable to several agricultural industries including the oilseed (such as palm, soybean, rapeseed, sunflower seed, coconut) and grain (such as corn and wheat) industries.

Consider, for example, the palm industry. In this industry, palm oil mills produce crude palm oil (commodity output) and palm kernel (byproduct) from fresh fruit palm bunches (commodity input). As reported in Table 11 of the U.S. Department of Agriculture’s Foreign Agriculture Service Report\(^1\), palm is the largest oilseed industry with 59.29 million tonnes of crude palm oil production volume between 2013 and 2014, which account for an estimated market value of $49.09 billion. In a palm oil mill, the fresh fruit palm bunches go through several processing stations (reception, sterilization, threshing, pressing and centrifuge) to produce palm kernel and crude palm oil, which is transferred to storage tanks prior to dispatch from the mill. The processing volume of the fresh fruit palm bunches is constrained by the joint capacity of the processing stations, whereas the crude palm oil production and inventory volume is constrained by the capacity of the storage tank. Therefore, choosing the right levels for the input processing and output storage capacity is critical for the mill’s profitability. Similar capacity investment decisions are relevant for other oilseed processors, which produce crude vegetable oil (commodity output) and meal or cake (byproduct) from the oilseed (commodity input), and grain processors, which produce biofuel (commodity output) and animal feed (byproduct) from the grain (commodity input).

There are unique characteristics of the processors in these agricultural industries that present challenges for capacity management. First, since the input and the output are commodities, there exist regional exchange (spot) markets (Devalkar et al. 2011). In buying and selling of these commodities, the processors are exposed to the prevailing spot prices when these transactions are carried out either through the exchange market, or through bi-lateral contracts with prices benchmarked on the price in the exchange market. The spot prices for the input and the output are closely linked and show considerable variability, as depicted in Panel (a) of Figure 1 for the palm industry. The uncertainty in these spot prices

\(^1\)http://www.fas.usda.gov/psdonline
plays a key role in capacity management because, at any given time, the processing profit depends on these prices; and the processor may choose to hold output inventory to sell at a later date to benefit from output spot price fluctuations. Second, there is uncertainty in production yield (extraction rate) from each input, as depicted in Panel (b) of Figure 1, again, for the palm industry.\textsuperscript{2} This uncertainty is driven by several factors including the weather conditions\textsuperscript{3} and the infestation of pests and diseases during the growing period of the input (Boyabath and Wee 2013), the harvest timing of the input\textsuperscript{4}, and the processing technology used (Chang et al. 2003). The production yield uncertainty impacts the capacity investment decisions because the processing profit depends on this yield. Moreover, because the output goes through the storage capacity before dispatching from the processor, when the realized output yield after processing is larger than the unoccupied storage capacity, a disposal cost is incurred for the excess yield beyond this available capacity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(a) Daily Spot Prices of FFB and CPO (b) Monthly Average of CPO production yield}
\end{figure}

Figure 1: The characteristics of fresh fruit palm bunches (FFB) and crude palm oil (CPO) in Malaysian Peninsula for the period January 2006 to December 2013 as reported by Malaysian Palm Oil Board. The prices are in Malaysian Ringgit per metric ton, whereas the production yield (extraction rate) is reported in percentage.

Against the background of these characteristics, we consider a multi-period optimization

\textsuperscript{2}Based on Table 11 of USDA’s report discussed before, the difference between the lowest and highest production yield observations in Panel (b) of Figure 1 corresponds to nearly 6 million tonnes of crude palm oil production that account for an estimated $5 billion.

\textsuperscript{3}For example, in the soybean industry, insufficient rainfall decreases the size of the bean, which in turn, decreases the extraction rate of soybean oil.

\textsuperscript{4}For example, in the palm industry, the ripeness of the fresh fruit palm bunches is an important determinant of the crude palm oil extraction rate, and this ripeness changes with the timing of the harvest.
problem in which a firm procures an input commodity, with the marginal procurement cost equal to the spot price of this commodity, and sells an output commodity, with the marginal sales revenue equal to the spot price of this commodity, and a byproduct from a fixed price. The output can also be procured from the spot market, with the marginal procurement cost equal to the output spot price, to be stored and sold at later date. The firm maximizes its expected total profit over a finite planning horizon. At the beginning of the planning horizon, the firm chooses the input processing and output storage capacity levels. In the rest of the planning horizon, the firm periodically makes the processing volume and output inventory decisions constrained by these capacity levels. In particular, in each period, the processing volume is chosen with respect to output production yield uncertainty, and the output inventory is chosen after this uncertainty is realized.

With this model, we answer the following research questions: 1) What are the main characteristics of the optimal capacity investment policy? 2) How does the spot price uncertainty impact this optimal policy and the profitability? 3) What is the benefit of using the optimal policy relative to heuristic capacity investment policies which have practical and theoretical significance? In answering the last two questions, we focus on the palm industry, and calibrate our model to represent a typical palm oil mill. We conduct numerical experiments using realistic instances. These instances are chosen based on the publicly available data from Malaysian Palm Oil Board (including the spot price and production yield data presented in Figure 1), complemented by publicly available and proprietary data from palm oil mills located in Malaysia. We summarize our main findings below:

**Characteristics of The Optimal Capacity Investment Policy.** We characterize the optimal input processing and output storage capacity investment levels and the periodic processing and inventory decisions in closed form. We identify two key constructs that crucially affect these decisions: the *storage margin per output*, which is the margin from holding inventory at a given period and selling it at a later period, and the *processing margin per input*, which is the margin from sourcing and processing the input, and selling the byproduct and selling or disposing (if the unoccupied storage capacity is insufficient for the production yield) the output at a given period. The optimal processing volume in each period is determined by considering the different forms of the processing margin based on the disposal of the output under different yield realizations. The optimal inventory volume in each period is determined by comparing the storage margin with the the opportunity cost
of holding inventory, which is the relevant processing margin per output, due to limiting the subsequent period’s processing volume through decreasing the unoccupied storage capacity.

We identify three optimal capacity investment strategies: When the processing capacity cost relative to the storage capacity cost is sufficiently high, the firm invests in storage-dominating portfolio, where the storage capacity is strictly larger than what is required for production (with full utilization of the processing capacity) under all yield realizations. When the relative cost is sufficiently low, the firm invests in processing-dominating portfolio, where the processing capacity is strictly larger than what is required for production (with full utilization of the storage capacity) under all yield realizations. Otherwise, the firm invests in mixed portfolio. The storage-dominating portfolio is driven by the storage margin (it does not exist when the firm does not hold inventory), whereas the processing-dominating portfolio is driven by the byproduct revenues (it does not exist when the byproduct revenues are not accounted for). We find that with these two optimal portfolios, the investment cost of one capacity does not have an impact on the other capacity. However, with the mixed portfolio, higher processing (storage) capacity investment cost decreases the storage (processing) capacity level. This result underlines the need for the firm to evaluate the impact of investment cost of each capacity in a holistic fashion: not accounting for the cost-interdependencies in choosing the right capacity portfolio can be a detrimental strategy.

**Impact of Spot Price Uncertainty.** We conduct sensitivity analysis, both analytically and numerically, to investigate the impact of input and output spot price volatility and spot price correlation. We find that the impact of these parameters crucially depends on their effect on the expected marginal revenue from jointly using both capacities, i.e. the maximum of the storage and the processing margins. The processor benefits from a lower correlation, and a lower (higher) input or output spot price volatility when this volatility is low (high). These changes increase the variability of the processing margin in each period, and while higher processing margin is desirable, lower processing margin is less consequential because the storage margin dominates. We also provide some rules of thumb for capacity management: The processor should decrease its processing and storage capacity investment with an increase (a decrease) in output spot price volatility when this volatility is low (high). The processor should decrease its processing capacity with an increase in correlation, and with an increase (a decrease) in input spot price volatility when this volatility is low (high); however, the storage capacity should not be altered with these changes.
Comparison with Heuristic Capacity Investment Policies. We find that ignoring the byproduct revenue in capacity planning leads to very significant profit loss. This result is interesting because byproduct revenue constitutes a small portion of the total revenues. However, when this revenue is not accounted for, processing margin becomes insignificant. Ignoring the inventory holding possibility in capacity planning leads to lower profit loss, but the loss is still sizeable. Comparing our results with the two heuristic policies demonstrates that processing margin is more crucial than the storage margin for profitability. In practice, capacity planning is often carried out by ignoring the production yield uncertainty and considering the maximum yield possible. We find that this heuristic policy significantly decreases the profitability because the processing margin is overestimated. We propose a new heuristic policy where the production yield uncertainty is ignored but the capacity planning is made based on the average yield. In this case, the storage margin is underestimated but the processing margin is not impacted. We find that the profit loss with the proposed policy is insignificant. This is because, the processing margin often dominates the storage margin at a given period, and thus, the underestimation of the storage margin is less consequential.

The remainder of this paper is organized as follows: §2 surveys the related literature and discusses the contribution of our work. §3 describes the model and the basis for our assumptions. §4 derives the optimal strategy. §5 studies the impact of spot price uncertainty on the optimal capacity investment policy and the profitability; and the performance of the optimal policy in comparison with heuristic policies in the context of the palm industry. §6 concludes with a discussion of the limitations of our analysis and future research directions.

2 Literature Review

This paper contributes to the operations management literature on commodity processing by analyzing the capacity investment decisions of a processor in the context of agricultural industries. The papers in this literature study operating decisions (such as procurement, processing, inventory and production) of a commodity processor in a variety of models, but they do not consider the capacity investment decisions. These papers capture the idiosyncratic features of different commodity markets including the electricity (Zhou et al. 2014), electronic equipment (Pei et al. 2011), metals (Plambeck and Taylor 2013), natural gas (Secomandi 2010 Lai et al. 2011), petroleum (Dong et al. 2014), semi-conductors (Wu and Kleindorfer 2005); and within agricultural industries, beef (Boyabath et al. 2011), cocoa
(Boyabath 2014), corn (Goel and Tanrısever 2013) and soybean (Devalkar et al. 2011). We refer the reader to Kleindorfer and Wu (2003) and Goel and Tanrısever (2013) for a review of papers in this literature. Because the focus of these papers are on operating decisions, they either (often implicitly) assume abundant processing and storage resources, or consider fixed capacity levels for these resources. Our contribution is to study how these capacity levels are chosen. We now compare our work with two most related papers in detail.

Devalkar et al. (2011) consider a multi-period optimization problem in which a firm procures an input commodity and sells an output commodity using forward contracts. The firm operates under fixed procurement and processing capacity constraints. They characterize the optimal periodic processing and (input) inventory decisions. Using a calibration based on the soybean industry, they demonstrate that employing the optimal policy provides significant benefits over heuristic policies that have practical and theoretical significance. The characterization of the periodic processing and inventory decisions are more complex than the one studied in this paper. However, we extend their analysis to consider the capacity investment decisions, and the impact of spot price uncertainty on these decisions.

Plambeck and Taylor (2013) model a firm that uses a commodity input to produce a commodity output facing input and output spot price uncertainties. They do not consider capacity investment decisions, and because they focus on a single-period model, inventory decision is irrelevant. They study three different process improvement investments, increasing the production yield per input or the throughput and increasing the former in the expense of the latter; and investigate the impact of spot price uncertainty on these investments. In their setting, as highlighted on page 630, the impact of spot price uncertainty is driven by the ability of the firm to stop processing when the margin is negative. In agricultural industries, the processing margin is unlikely to be negative in practice (which we provide empirical support in §5 in the context of the palm industry). Therefore, when we study the impact of spot price uncertainty, we assume that the processing margin is always positive, and thus, the main driver of their results is not applicable in our setting.

Two other streams of literature are also related to our paper. First, there is a vast amount of research in the operations management literature that studies capacity investment decisions in a variety of settings. Our paper is most relevant to the stream of papers that models the idiosyncratic features of specific industries for capacity management. The examples include Karabuk and Wu (2003) for the semi-conductor industry and Sönmez et
al. (2013) for the liquified natural gas industry. Our consideration of the characteristics of commodity processors in agricultural industries is the distinguishing feature of our work with respect to this literature. Second, our paper is related to the stream of papers that study production planning in agricultural industries. In this stream, Kazaz (2004) considers a processor facing farm-yield uncertainty in the olive industry. Mehrotra et al. (2011) analyze a food processor that manufacture a large number of products on several production lines over multiple periods. Motivated by the citrus industry, Kazaz and Webster (2011) consider a fruit processor and investigate the impact of yield-dependent spot prices on the procurement decisions. Motivated by the cocoa industry, Boyabatlı and Wee (2013) study the impact of the farm-yield uncertainty on the production yield of an agri-processor. Our consideration of the capacity management and our focus on the palm industry are the distinguishing features of our work with respect to this stream of papers.

3 Model Description

Notation and Preliminaries. A realization of the random variable \( \tilde{y} \) is denoted by \( y \).

\[
(u)^+ = \max(u, 0), \quad -j = S \setminus j \quad \text{for} \quad j \in S \quad \text{and} \quad \Omega_{12} = \Omega_1 \cup \Omega_2.
\]

Pr denotes probability, \( E \) denotes the expectation operator. Bold face letters represent row vectors. The monotonic relations (increasing, decreasing) are used in the weak sense. Subscript \( t \) denotes the parameters and decision variables in period \( t \). Superscript \( I \) denotes the input related parameters and decisions variables, whereas \( O \) (\( B \)) denotes the same for the output (byproduct).

We consider a firm that uses a commodity input to produce a commodity output and a byproduct to maximize its expected total profit over a finite planning horizon. The firm operates under input processing and output storage capacity constraints. At the beginning of the planning horizon, the firm chooses the capacity investment levels. In the rest of the planning horizon, the firm periodically makes the processing volume and output inventory decisions subject to these capacity levels.

Let \( K = (K^I, K^O) \) denote the capacity investment portfolio of the firm, where \( K^I \) is the input processing, and \( K^O \) is the output storage capacity.\(^5\) In agricultural industries, the processors are located around the plantations of the input, and because of the limited land

\(^5\)Our model implicitly assumes that the input is not stored. This is consistent with several agricultural industries where processing of the stored input creates production inefficiencies. For example, in the palm industry, the input (palm fruit) is processed within 24 hours from harvesting, otherwise fatty acids are accumulated which decrease the production yield (extraction rate) of the output (crude palm oil).
availability around these plantations, the marginal capacity investment cost increases in the capacity level. Paralleling this observation, we assume that the capacity investment cost $C(K)$ is convex increasing in $K$. Specifically, we consider a quadratic capacity cost function with parameters $\beta^I$ and $\beta^O$, i.e. $C(K) = \beta^I (K^I)^2 + \beta^O (K^O)^2$. The structural analysis in §4 continues to hold for a general convex increasing $C(K)$ except for the closed-form characterization of the capacity portfolio which requires a specific functional form.

We assume that the marginal procurement cost of the commodity input is given by the spot price of this commodity. As discussed in the Introduction, in practice, this case is relevant when the input is procured through an exchange (spot) market, or when the input is procured through bi-lateral contracts where the unit price is benchmarked on the price in the exchange market. Similarly, we assume that the marginal sales revenue of the commodity output is given by the spot price of this commodity. Input and output spot prices follow correlated stochastic processes with Markovian property, i.e. the current spot price realizations are sufficient to characterize the distribution of the future spot prices. We defer the specification of these stochastic processes to §5; it does not affect the structural analysis is §4 as long as they satisfy the Markovian assumption. The firm can also procure output from the spot market to store and the marginal procurement cost is given by the spot price. The output storage incurs a per period unit holding cost $h$.

We consider a per-period unit processing cost $c > 0$. For each unit of the processed input, the production yield of the output (byproduct) is given by $a (a^B)$. We assume $a^B \in (0, 1)$ is constant and byproduct is sold immediately after processing at a fixed unit price $p^B$. Therefore, $c = c - a^B p^B$ denotes the effective processing cost, which can be negative if the byproduct revenue is sufficiently high. The output production yield $a$ is uncertain. For tractability, we assume that $\tilde{a}$ follows a Bernoulli distribution: $a = a^L$ (low yield scenario) with probability $q \in [0, 1]$ and $a = a^H$ (high yield scenario) with probability $1 - q$ for $0 < a^L < a^H < 1 - a^B$ where the last inequality follows because the overall production yield cannot be larger than 1. Let $\bar{a} = qa^L + (1 - q)a^H$ denote the mean production yield. We assume that $\tilde{a}$ is independent and identically distributed across periods. This assumption also implies that $\tilde{a}$ is statistically independent of the spot price processes, which is a reasonable assumption in the palm industry as we empirically verify in §5.

The storage capacity $K^O$ also has an impact on the processing activities because output goes through the storage capacity before dispatching from the processing facility. In par-
ticular, when the realized output yield after processing is larger than the available storage capacity, the excess output volume (beyond this available capacity) cannot be used; and the firm incurs a unit disposal cost $d \geq 0$ for this excess volume.\footnote{If the excess output volume is not disposed but it is sold to the spot market, our model can be modified in the following manner: We assume $d < 0$, and thus, it is a disposal revenue; and $d$ is a linear function of the output spot price. This modification does not affect the structural analysis in §4 as long as $|d|$ is not larger than the output spot price, which is satisfied when the excess volume is sold to the spot market.} In the palm industry, the palm oil mills operate under no excess of output policy. This is a special case of our model where $d \to \infty$, which we will assume throughout §5.

We formulate the firm's problem as a finite horizon stochastic dynamic program. The per-period processing capacity $K^I$ (in units of, for example, tonnes of input per day), and the storage capacity $K^O$ (tonnes of output) are chosen at period $t = 0$, and they are fixed in the remaining periods. In each period $t \in [1, T]$, the sequence of events is as follows:

1. At the beginning of period $t$, the firm observes the input and the output spot prices $P_t = (p^I_t, p^O_t)$, and the output inventory level $s_{t-1}$ (carried from period $t - 1$); and chooses the input processing volume $z_t$ within the processing capacity level $K^I$.

2. The production yield $a$ is realized. The output yield beyond the available storage capacity $K^O - s_{t-1}$ is disposed.

3. The firm decides the output inventory level $s_t$ within the storage capacity $K^O$. The available output volume that is not stored is sold to the spot market, whereas the excess inventory beyond the available output volume is procured from the spot market.

The firm's immediate payoff in period $t \in [1, T]$ is given by

$$L(z_t, s_t|s_{t-1}, P_t) \doteq -p^I_t z_t - c z_t + E_{\tilde{a}} \left[-d (\tilde{a} z_t - (K^O - s_{t-1}))^+ + p^O_t (s_{t-1} + \min (\tilde{a} z_t, K^O - s_{t-1}) - s_t)^+ \right.$$  
$$- p^O_t (s_t - (s_{t-1} + \min (\tilde{a} z_t, K^O - s_{t-1})))^+ - h s_t] . \tag{1}$$

In (1), the first two terms capture the effective processing and procurement cost, whereas the remaining terms capture the cash flows based on the production yield realization. Within these cash flows, the first term denotes the disposal cost, the second term denotes the spot sale revenue for the available output volume that is not stored, whereas the third term is the spot procurement cost for the inventory level beyond the available output volume. The last term denotes the inventory holding cost incurred.
Let $V_t(s_{t-1}, P_t)$ for $t \in [1, T]$ denote the optimal value function from period $t$ onwards given $s_{t-1}$ and $P_t$, which satisfies

$$V_t(s_{t-1}, P_t) = \max_{0 \leq z_t \leq K^I} \{ L(z_t, s_t|s_{t-1}, P_t) + \mathbb{E}_t[V_{t+1}(s_t, \tilde{P}_{t+1})] \},$$

(2)

with a boundary condition $V_{T+1}(s_T, P_{T+1}) = 0$ and initial inventory level $s_0 = 0$, where $\mathbb{E}_t[.]$ is the short hand notation for $\mathbb{E}[.|P_t]$. At period $t = 0$, the firm observes $P_0$ and choose $K = (K^I, K^O)$ incurring the capacity investment cost $C(K) = \beta^I (K^I)^2 + \beta^O (K^O)^2$. The firm’s optimal expected profit is given by $\Pi^* = \max_{K \geq 0} \mathbb{E}_0[V_1(0, \tilde{P}_1)] - C(K)$.

4 The Optimal Strategy

In this section, we describe the optimal solution for the periodic input processing and output inventory decisions and the capacity investment decisions at the beginning of the planning horizon. All the proofs are relegated to the Technical Appendix.

To facilitate the analysis, we make the following key observation. Sourcing the inventory from the output available in-house (the realized output yield after processing and the inventory available from the previous period) and from the output spot market have identical cost, which is the prevailing output spot price: For the former, it is the opportunity cost of not selling to the spot market (spot sale revenue), and for the latter, it is the spot procurement cost. Therefore, the firm’s immediate payoff in period $t \in [1, T]$, as given by (1), can be decoupled into two components:

$$L_{Pr}(z_t|s_{t-1}, P_t) = -(p^I_t + c)z_t + \mathbb{E}_a \left[ p^O_t \min (\bar{a}z_t, K^O - s_{t-1}) - d (\bar{a}z_t - (K^O - s_{t-1}))^+ + p^O_t s_{t-1} \right],$$

$$L_{Sc}(s_t|s_{t-1}, P_t) = -(p^O_t + h)s_t,$$

where $Pr$ refers to "Processing return" and $Sc$ refers to "Storage cost." This decoupling suggests that the processing and inventory decisions in the same period can be viewed as independent: the firm chooses the processing volume to sell in the output spot market together with the inventory carried over from the previous period generating processing return $L_{Pr}(z_t|s_{t-1}, P_t)$, and chooses the output inventory level $s_t$ to source from the spot market incurring storage cost $L_{Sc}(s_t|s_{t-1}, P_t)$. The processing decision does not impact any other decisions, whereas the inventory decision has an impact only on the subsequent
period’s processing decision through limiting the available storage capacity. Therefore, the optimization problem in (2) can be written based on a series of independent two-stage optimization problems by grouping the inventory decision in period $t-1$ with the processing decision in period $t$ as illustrated in Figure 2.

$$\begin{align*}
\text{Information} & \quad \text{P}_0 \quad \text{P}_1 \quad \tilde{a} \quad \text{P}_2 \quad \tilde{a} \quad \text{P}_3 \quad \tilde{a} \quad \ldots \quad \text{P}_T \quad \tilde{a} \\
\text{Decision} & \quad K = (K^I, K^O) \quad z_1 \quad s_1 \quad z_2 \quad s_2 \quad z_3 \quad s_3 \quad \ldots \quad z_T \quad s_T \\
\text{Period} & \quad 0 \quad 1 \quad 2 \quad 3 \quad \ldots \quad T \\
\text{Cash Flow} & \quad -C(K) \quad L_{Pr}(z_1) \quad L_{Sc}(s_1) \quad L_{Pr}(z_2) \quad L_{Sc}(s_2) \quad L_{Pr}(z_3) \quad L_{Sc}(s_3) \quad \ldots \quad L_{Pr}(z_T) \quad L_{Sc}(s_T) \\
& \quad \underbrace{G_1(P_1)}_{z_1} \quad \underbrace{G_2(P_2)}_{z_2} \quad \underbrace{G_3(P_3)}_{z_3} \quad \ldots \quad \underbrace{G_{T-1}(P_{T-1})}_{z_T}
\end{align*}$$

Figure 2: Schematic representation of the formulations in (2) and (3).

In this new formulation, using $L_{Sc}(s_{T-1}, P_T) = 0$ (because inventory is not needed in period $T$), the optimal value function in period $t \in [1, T - 1]$ can be written as

$$V_t(s_{t-1}, P_t) = \max_{0 \leq z_t \leq K^I} L_{Pr}(z_t | s_{t-1}, P_t) + \sum_{\tau = t}^{T-1} E_t \left[ G_\tau(\tilde{P}_\tau) \right], \quad (3)$$

where the expected optimal profit $G_t(P_t)$ for the two-stage problem in period $t$ is given by

$$G_t(P_t) = \max_{0 \leq s_t \leq K^O} \left\{ L_{Sc}(s_t | P_t) + E_t \left[ \max_{0 \leq z_{t+1} \leq K^I} L_{Pr}(z_{t+1} | s_t, \tilde{P}_{t+1}) \right] \right\}. \quad (4)$$

### 4.1 Periodic Input Processing and Output Inventory Decisions

In this section, we provide the optimal solution for (4). In particular, we first characterize the optimal processing volume $z^*_{t+1}(s_t, P_{t+1})$ for a given output inventory level $s_t$, followed by the characterization of the optimal output inventory level $s^*_t(P_t)$.

**Proposition 1** The optimal processing volume $z^*_{t+1}(s_t, P_{t+1})$ is given by

$$z^*_{t+1}(s_t, P_{t+1}) = \begin{cases} 
0 & \text{if} \quad -p_{t+1}^I - c + \bar{a}p_{t+1}^O \leq 0, \\
\min \left( \frac{k^O_{s_t}}{\bar{a}^L}, K^I \right) & \text{if} \quad -p_{t+1}^I - c + qa^L_{t+1} - (1 - q)a^H d \leq 0 < -p_{t+1}^I - c + \bar{a}p_{t+1}^O, \\
\min \left( \frac{k^O_{s_t}}{\bar{a}^L}, K^I \right) & \text{if} \quad -p_{t+1}^I - c - \bar{a}d \leq 0 < -p_{t+1}^I - c + qa^L_{t+1} - (1 - q)a^H d, \\
K^I & \text{if} \quad -p_{t+1}^I - c - \bar{a}d > 0.
\end{cases}$$

The optimal processing volume is characterized by the *processing margin per input*, that is the expected processing revenue minus the effective processing and input spot procurement
cost. The expected processing revenue depends on the sufficiency of the available storage capability \( K^O - s_t \) under each yield realization: when \( K^O - s_t \) is sufficient, it is given by the output spot price; otherwise it is given by the disposal cost. Therefore, \(-p_{t+1}^I - c + \bar{a} p_{t+1}^O, -p_{t+1}^I - c + q a^L p_{t+1}^O - (1-q)\alpha H d \) and \(-p_{t+1}^I - c - \bar{a} d \) denote the processing margin when \( K^O - s_t \) is sufficient under both yield realizations, only under the low yield realization, and under neither yield realizations, respectively. When \(-p_{t+1}^I - c + \bar{a} p_{t+1}^O \leq 0\), it is not profitable to process. Otherwise, \( z_t^*(s_t, \mathbf{P}_{t+1}) \) is determined by the smallest positive processing margin. For example, when \(-p_{t+1}^I - c - \bar{a} d \leq 0 < -p_{t+1}^I - c + q a^L p_{t+1}^O - (1-q)\alpha H d \), it is profitable to process if \( K^O - s_t \) is sufficient under the low yield realization but it is not profitable to process if \( K^O - s_t \) is insufficient under both yield realizations. Therefore, the firm optimally processes up to \( \frac{K^O - s_t}{a^H} \), the largest processing volume for which \( K^O - s_t \) is sufficient under the low yield realization, unless constrained by the processing capacity \( K^I \). When \(-p_{t+1}^I - c - \bar{a} d > 0\), the firm optimally processes up to \( K^I \) because the processing margin is positive due to the byproduct revenue (which is subsumed in \( c = c - a^R p^R \)).

Using Proposition 1, the optimal expected processing profit \( L^*_p(s_t, \mathbf{P}_{t+1}) \) is given by

\[
L^*_p(s_t, \mathbf{P}_{t+1}) = \min \left( K^I, \frac{K^O - s_t}{a^H} \right) (-p_{t+1}^I - c + \bar{a} p_{t+1}^O)^+ + \left( K^I - \frac{K^O - s_t}{a^L} \right)^+ (-p_{t+1}^I - c - \bar{a} d)^+ + s_t p_{t+1}^O + \left( \min \left( K^I, \frac{K^O - s_t}{a^L} \right) - \min \left( K^I, \frac{K^O - s_t}{a^H} \right) \right) (-p_{t+1}^I - c + q a^L p_{t+1}^O - (1-q)\alpha H d)^+. \tag{5}
\]

The optimal expected processing profit is determined by the effective production capacity, which is given by the processing capacity \( K^I \) and the available storage capacity \( K^O - s_t \) per input under each yield realization, and the corresponding expected marginal revenue of this production capacity, which is given by the processing margin when it is profitable to process. For example, for \( \min \left( K^I, \frac{K^O - s_t}{a^H} \right) \), \( K^O - s_t \) is sufficient for processing up to \( K^I \) under both yield realizations, and thus, the expected marginal revenue for these units is given by \( (-p_{t+1}^I - c + \bar{a} p_{t+1}^O)^+ \). For \( \left( K^I - \frac{K^O - s_t}{a^L} \right)^+ \), \( K^O - s_t \) is not sufficient under either yield realization, and thus, the expected marginal revenue is given by \( (-p_{t+1}^I - c - \bar{a} d)^+ \).

We now turn to the optimal inventory decision which, following (4), is the solution to

\[
\max_{0 \leq s_t \leq K^O} L_{S_c}(s_t|\mathbf{P}_t) + \mathbb{E}_t \left[ L^*_p(s_t, \mathbf{P}_{t+1}) \right], \tag{6}
\]

where \( L_{S_c}(s_t|\mathbf{P}_t) = -(p_t^O + h)s_t \) and \( L^*_p(s_t, \mathbf{P}_{t+1}) \) is as given in (5).
Proposition 2 The optimal output inventory level $s^*_t(P_t)$ is characterized by

$$s^*_t(P_t) = \begin{cases} 0 & \text{if } X_t \leq 0, \\
(K^O - a^H K^I)^+ & \text{if } X_t - Y_t \leq 0 < X_t, \\
(K^O - a^L K^I)^+ & \text{if } X_t - Y_t - Z_t \leq 0 < X_t - Y_t, \\
K^O & \text{if } 0 < X_t - Y_t - Z_t, \end{cases}$$

(7)

where

$$X_t = -p_t^O - h + E_t[p_{t+1}^O],$$

(8)

$$Y_t = \frac{E_t\left((-p_{t+1}^I - c + \bar{a} p_{t+1}^O)^+ - (-p_{t+1}^I - c + q a^L p_{t+1}^O - (1 - q)a^H d)^+\right)}{a^H},$$

$$Z_t = \frac{E_t\left((-p_{t+1}^I - c + q a^L p_{t+1}^O - (1 - q)a^H d)^+ - (-p_{t+1}^I - c - \bar{a} d)^+\right)}{a^L}.$$

In (8), $X_t$ denotes the storage margin per output, whereas $Y_t$ ($Z_t$) denotes the expected processing benefit per output under the high (low) yield realization, which is due to selling the output rather than disposing it under this yield realization. When $X_t \leq 0$, it is not profitable to hold inventory. Otherwise, it is profitable to hold inventory, and the optimal inventory level is determined by the trade off between the storage margin $X_t$ and the opportunity cost of holding inventory due to limiting the subsequent period’s processing volume through decreasing the unoccupied storage capacity. In particular, $Y_t$ denotes this opportunity cost when the subsequent period’s processing volume is constrained only under the high yield realization, whereas $Y_t + Z_t$ denotes the same when it is constrained under both yield realizations. When $X_t - Y_t \leq 0 < X_t$, the firm stores up to $(K^O - a^H K^I)^+$ which does not affect the subsequent period’s processing volume. When $X_t - Y_t - Z_t \leq 0 < X_t - Y_t$, it is profitable to hold inventory when the subsequent period’s processing volume is constrained under the high yield realization, but it is not profitable to hold inventory when the processing volume is constrained under both yield realizations. Therefore, the firm stores up to $(K^O - a^L K^I)^+$. When $X_t - Y_t - Z_t > 0$, holding inventory is profitable regardless of its impact on the processing volume, and thus, $s^*_t(P_t)$ is given by $K^O$.

4.2 Capacity Investment Decisions

In this section, we solve for the firm’s optimal capacity investment decision. At period $t = 0$, the firm observes $P_0$ and chooses the capacity portfolio $K = (K^I, K^O)$ incurring the capacity investment cost $C(K) = \beta^I(K^I)^2 + \beta^O(K^O)^2$ to maximize its expected profit in
the entire planning horizon: \( \max_{K \geq 0} V(K) - C(K) \), where \( V(K) \equiv \mathbb{E}_0[V_1(0, \tilde{P}_1)] \) denotes the expected profit for a given capacity portfolio \( K \). It follows from (3) that

\[
V_1(0, P_1) = \max_{0 \leq z_1 \leq K^I} L_{PR}(z_1|0, P_1) + \sum_{\tau=1}^{T-1} \mathbb{E}_\tau \left[ G_\tau(\tilde{P}_\tau) \right],
\]

and the expected profit for a given \( K \) can be written as \( V(K) = M_1 K^I + M_2 \min (a^L K^I, K^O) + M_3 \left[ \min (a^H K^I, K^O) - \min (a^L K^I, K^O) \right] + M_4 (K^O - a^H K^I)^+ \), where

\[
M_1 \doteq \mathbb{E}_0 \left[ \sum_{t=1}^T (-p^I_t - c - \tilde{a}d)_+ \right],
\]

\[
M_2 \doteq Y_0 + Z_0 + \mathbb{E}_0 \left[ \sum_{t=1}^{T-1} \max (X_t, Y_t + Z_t) \right],
\]

\[
M_3 \doteq Y_0 + \mathbb{E}_0 \left[ \sum_{t=1}^{T-1} \max (X_t, Y_t) \right],
\]

\[
M_4 \doteq \mathbb{E}_0 \left[ \sum_{t=1}^{T-1} X_t^+ \right],
\]

and \( X_t, Y_t \) and \( Z_t \) for \( t \in [0, T - 1] \) are as given in (8) of Proposition 2. In (9), \( M_1 \) (\( M_4 \)) denotes the expected marginal revenue of the processing capacity \( K^I \) (storage capacity \( K^O \)) in the absence of the other capacity. In particular, \( M_1 \) is the total expected processing profit when only byproduct is sold over the entire planning horizon, and it is relevant for all processing capacity units \( K^I \). \( M_4 \) denotes the total expected storage profit over the entire planning horizon, and it is relevant for the excess storage capacity units \( (K^O - a^H K^I)^+ \) which do not have an impact on the processing activities. \( M_2 \) and \( M_3 \) capture the expected marginal revenue from jointly using \( K^I \) and \( K^O \) as a production capacity over the entire planning horizon. Because holding inventory in each period may limit the subsequent period’s processing volume, this production capacity can be used for holding inventory or for processing. Therefore, in each period, the expected marginal revenue is given by the maximum of the storage margin \( X_t \) and the relevant expected processing benefit per output. For the first \( \min (a^L K^I, K^O) \) capacity units, the relevant processing benefit is given by \( Y_t + Z_t \), whereas for the remaining \( \left[ \min (a^H K^I, K^O) - \min (a^L K^I, K^O) \right] \) capacity units, because holding inventory constrains the subsequent period’s processing volume only under the high yield realization, the relevant processing benefit is given by \( Y_t \). Because \( K^O \) is unoccupied at the beginning of the planning horizon, the first period’s processing volume is not constrained by the inventory, and thus, the expected marginal revenue only depends
on the relevant processing benefit per output \((Y_0 + Z_0\) or \(Y_0\)).

**Proposition 3** The optimal capacity portfolio \(K^* = (K^I, K^O)\) is characterized by

\[
(K^I, K^O) = \begin{cases}
    \left(\frac{M_1 + a^H(M_3 - M_4) + a^L(M_2 - M_3)}{2\beta^I}, \frac{M_4}{2\beta^O}\right) & \text{if } \beta \in \Omega_1 \\
    \left(\frac{M_1 + a^H M_3 + a^L(M_2 - M_3)}{2\beta^I + 2(a^H)^2\beta^O}, \frac{a^H [M_1 + a^H M_3 + a^L(M_2 - M_3)]}{2\beta^I + 2(a^H)^2\beta^O}\right) & \text{if } \beta \in \Omega_2 \\
    \left(\frac{M_1 + a^L M_3}{2\beta^I}, \frac{M_4}{2\beta^O}\right) & \text{if } \beta \in \Omega_3 \\
    \left(\frac{M_1 + a^L M_2}{2\beta^I + 2(a^L)^2\beta^O}, \frac{a^L [M_1 + a^L M_2]}{2\beta^I + 2(a^L)^2\beta^O}\right) & \text{if } \beta \in \Omega_4 \\
    \left(\frac{M_1}{2\beta^I}, \frac{M_2}{2\beta^O}\right) & \text{if } \beta \in \Omega_5
\end{cases}
\]

where \(\beta = (\beta^I, \beta^O)\) and \(M_i\) for \(i \in \{1, \ldots, 4\}\) is as given in (9) and

\[
\Omega_1 = \left\{ \beta : \beta^I \geq \frac{a^H [M_1 + a^H (M_3 - M_4) + a^L (M_2 - M_3)]}{M_4} \right\},
\]

\[
\Omega_2 = \left\{ \beta : \frac{a^H [M_1 + a^H (M_3 - M_4) + a^L (M_2 - M_3)]}{M_4} > \frac{\beta^I}{\beta^O} \geq \frac{a^H [M_1 + a^L (M_2 - M_3)]}{M_3} \right\},
\]

\[
\Omega_3 = \left\{ \beta : \frac{a^L [M_1 + a^L (M_2 - M_3)]}{M_3} > \frac{\beta^I}{\beta^O} \geq \frac{a^L [M_1 + a^L (M_2 - M_3)]}{M_3} \right\},
\]

\[
\Omega_4 = \left\{ \beta : \frac{a^L [M_1 + a^L (M_2 - M_3)]}{M_3} > \frac{\beta^I}{\beta^O} \geq \frac{a^L M_1}{M_2} \right\},
\]

\[
\Omega_5 = \left\{ \beta : \frac{\beta^I}{\beta^O} < \frac{a^L M_1}{M_2} \right\}.
\]

The optimal processing and storage capacity levels are characterized by the ratio of the expected marginal revenue of an additional capacity unit to its marginal investment cost.

The marginal investment cost of each capacity is given \(2\beta^I\) for \(j \in \{I, O\}\) unless both capacities are linked in the optimal solution, i.e. \(K^O = \hat{a}K^I\) where \(\hat{a} \in \{a^L, a^H\}\), in which case it is given by \(2\beta^I + 2(\hat{a})^2\beta^O\). The expected marginal revenue of each capacity depends on the joint revenue of both capacities as a production capacity, and the individual revenue of the processing capacity due to byproduct profit and the individual revenue of the excess storage capacity due to the total storage profit. Intuitively, the expected marginal revenue takes different forms based on the capacity investment costs \(\beta' = (\beta^I, \beta^O)\).

To delineate the intuition behind Proposition 3, we fix \(\beta^I\), and discuss how the optimal portfolio changes as \(\beta^O\) increases. When \(\beta^O\) is sufficiently low, i.e. \(\beta \in \Omega_1\), there is excess storage capacity. Therefore, an additional storage capacity unit does not have production benefit, and its expected marginal revenue is given by the total expected storage profit \(M_4\). Because there is excess storage capacity, an additional processing capacity unit can be
used for output production, and thus, its marginal revenue is given by the sum of the joint revenue, i.e. the additional profit generated beyond the storage profit, and the individual revenue $M_1$ due to byproduct profit. For the joint revenue, for the first $a^L$ units, the additional profit is $M_2 - M_4$; and for the next $(a^H - a^L)$ units, it is $M_3 - M_4$. Therefore, the expected marginal revenue is given by $M_1 + a^H(M_3 - M_4) + a^L(M_2 - M_3)$. As $\beta^O$ increases, $K^{O*}$ decreases, and there is no excess storage capacity in the optimal solution. Therefore, $M_4$ does not impact the expected marginal revenue of each capacity investment. For example, when $\beta \in \Omega_2$, $K^{O*} = a^H K^I*$, the expected marginal revenue of an additional processing capacity unit is identical to $\beta \in \Omega_1$ case except for $M_4$ is not deducted. As $\beta^O$ further increases, the expected marginal revenue of an additional processing capacity unit is given by the sum of the relevant joint revenue (which is different in each $\Omega$ region because $K^{O*}$ decreases), and the individual revenue $M_1$. When $\beta^O$ is sufficiently high, i.e. $\beta \in \Omega_5$, there is excess processing capacity, and thus, the expected marginal revenue of an additional processing capacity unit only depends on its individual revenue $M_1$. In this case, because an additional storage capacity unit can be used for production under both yield realizations, its expected marginal revenue is given by $M_2$.

The optimal expected profit $\Pi^*$ over the entire planning horizon can be obtained by using Proposition 3, but it is omitted for brevity. We close this section with important observations about the optimal capacity investment strategy:

**Corollary 1** Let $\gamma^* \equiv \frac{K^{O*}}{K^I^*}$ denote the optimal capacity ratio.

- **i.** When $\beta \in \Omega_1$, $\gamma^* > a^H$ and $K^*$ is characterized by the “storage-dominating” portfolio. $\frac{\partial K^*}{\partial \beta_j} = 0$ for $j \in \{I, O\}$;

- **ii.** When $\beta \in \Omega_234$, $\gamma^* \in [a^L, a^H]$ and $K^*$ is characterized by the ”mixed” portfolio. $\frac{\partial K^*}{\partial \beta_j} \leq 0$ for $j \in \{I, O\}$ with strict inequality holding for $\beta \in \Omega_24$;

- **iii.** When $\beta \in \Omega_5$, $\gamma^* < a^L$ and $K^*$ is characterized by the ”processing-dominating” portfolio. $\frac{\partial K^*}{\partial \beta_j} = 0$ for $j \in \{I, O\}$.

Corollary 1 highlights three distinct optimal capacity investment strategies: When the processing capacity cost relative to the storage capacity cost is sufficiently high, the firm invests in storage-dominating portfolio, where the storage capacity is strictly larger than what is required for production (with full utilization of the processing capacity) under both
yield realizations. When the relative cost is sufficiently low, the firm invests in processing-dominating portfolio, where the processing capacity is strictly larger than what is required for production (with full utilization of the storage capacity) under both yield realizations. Otherwise, the firm invests in mixed portfolio. The storage-dominating portfolio is driven by the storage margin (it does not exist when the storage margin is zero, for example, when $h \to \infty$), whereas the processing-dominating portfolio is driven by the byproduct revenue (it does not exist when the byproduct revenue is insufficient to cover the processing cost, i.e. $c = c - aBp^B > 0$). With these two optimal portfolios, the investment cost of one capacity does not have an impact on the other capacity. However, with the mixed portfolio, higher investment cost of one capacity (weakly) decreases the other capacity level. Managerially, this result is important because it underlines the need for the firm to evaluate the impact of investment cost of each capacity in a holistic fashion: not accounting for the cost-interdependencies in the capacity portfolio can be a detrimental strategy.

5 Application in the Palm Industry

In the previous section, we characterized the firm’s optimal capacity investment portfolio based on a general framework. In this section, we study the impact of spot price uncertainty on this portfolio and the firm’s profitability, and the performance of the optimal capacity investment policy in comparison with heuristic policies in the context of the palm industry.

In the palm industry, the palm oil mill processes the palm fresh fruit bunches to produce crude palm oil and palm kernel. The palm fresh fruit bunches first go through reception and sterilization stations where they are applied high-pressure steam. Afterwards, the palm fruits are separated from the palm bunches in the threshing station. These palm fruits are then crushed in the pressing station to produce palm kernel and crude palm oil, which is cleared from water and waste using centrifuge. The crude palm oil is transferred to storage tanks\(^8\) prior to dispatch from the mill. In the context of our model, the palm fresh fruit bunch (FFB hereafter) is the input, whereas the crude palm oil (CPO hereafter) and the palm kernel are the output and the byproduct, respectively. The joint capacity of the reception, sterilization, threshing, pressing and centrifuge stations corresponds to the processing capacity $K^P$, whereas the capacity of the CPO storage tank corresponds to $K^O$.

\(^8\)These storage tanks are lined with suitable protective coating (otherwise, iron contamination may occur) and they have steam-heating coils to maintain a specific temperature (otherwise, solidification and fractionation may occur due to the oxidation).
The remainder of this section is organized as follows: §5.1 discusses the additional modeling assumptions introduced beyond our model in §3. §5.2 describes the data and calibration used for our numerical experiments in the subsequent sections. §5.3 investigates the impact of spot price uncertainty on the firm’s optimal capacity investment portfolio and profitability. §5.4 studies the performance of the optimal capacity investment policy in comparison with a variety of heuristic policies that have practical and theoretical significance.

5.1 Additional Modeling Assumptions

In practice, the palm oil mills do not dispose CPO because they process FFB only if the available storage capacity will always be sufficient for the CPO yield. In other words, they operate under no excess of output policy. To capture this policy, we assume infinite disposal cost, i.e. $d \to \infty$. Corollary 2 provides the characterization of the optimal capacity investment portfolio and the optimal expected profit for this special case of our model:

**Corollary 2** With no excess of output policy, i.e. $d \to \infty$, the optimal capacity investment portfolio $K^* = (K^I, K^O)$ is given by

$$
(K^I, K^O) = \begin{cases} 
\left( \frac{a_H(M_3-M_4)}{2\beta^I}, \frac{M_4}{2\beta^O} \right) & \text{if } \beta \in \Omega_1 = \left\{ \beta : \frac{\beta^I}{\beta^O} \geq \frac{(a_H)^2(M_3-M_4)}{M_4} \right\}, \\
\left( \frac{a_H M_3}{2\beta^I + 2(a_H)^2 \beta^O}, \frac{(a_H)^2 M_4}{2\beta^I + 2(a_H)^2 \beta^O} \right) & \text{if } \beta \in \Omega_2 = \left\{ \beta : \frac{\beta^I}{\beta^O} < \frac{(a_H)^2(M_3-M_4)}{M_4} \right\},
\end{cases}
$$

where $M_3 = Y_0 + \mathbb{E}_0 \left[ \sum_{t=1}^{T-1} \max(X_t, Y_t) \right]$ and $M_4 = \mathbb{E}_0 \left[ \sum_{t=1}^{T-1} X_t^+ \right]$ with $X_t = -p_t^O - h + \mathbb{E}_t[p_{t+1}^O]$ and $Y_t = \mathbb{E}_t[(p_{t+1}^I - c + a_H p_{t+1})^+]$. The optimal expected profit is given by

$$
\Pi^* = \begin{cases} 
\frac{(a_H(M_3-M_4))^2}{4\beta^I} + \frac{(M_4)^2}{4\beta^O} & \text{if } \beta \in \Omega_1, \\
\frac{(a_H M_3)^2}{4(\beta^I + \beta^O(a_H)^2)} & \text{if } \beta \in \Omega_2.
\end{cases}
$$

With the infinite disposal cost, $\Omega_{345} = \emptyset$ in Corollary 1, and the optimal capacity ratio $\gamma^* = \frac{K^O}{K^I} \geq a_H$. In other words, the optimal capacity investment is given either by “storage-dominating” portfolio (when $\beta \in \Omega_1$) or by “mixed” portfolio (when $\beta \in \Omega_2$), which we call “high-yield-balanced” portfolio because $K^O = a_H K^I$. In comparison with Proposition 3, because byproduct-only production is not feasible, $M_1 = 0$. Moreover, holding inventory in any period can limit the subsequent period’s processing volume only under the high yield realization, and thus, $Z_t = 0$ and $M_2 = M_3$. Because there is no byproduct-only profit, $Y_t$ is given by the processing margin per output under the high yield realization. Throughout this section, we use the characterization given in Corollary 2.
To study the impact of spot price uncertainty, we impose additional structure on the spot price process model. In particular, we use a single-factor bi-variate mean-reverting price process to describe the evolution of the input and the output spot prices. Specifically, input and output spot prices at time $t$, $P_t = (p_t^I, p_t^O)$, are modeled as

$$
dp_t^I = \theta^I (\bar{p}^I - p_t^I) + \sigma^I \, d\tilde{W}_t^I, $$
$$
dp_t^O = \theta^O (\bar{p}^O - p_t^O) + \sigma^O \, d\tilde{W}_t^O, $$

(11)

where $\theta^j > 0$ is the mean-reversion parameter, $\bar{p}^j$ is the long-term price level and $\sigma^j$ is the volatility for $j \in \{I, O\}$, whereas $(d\tilde{W}_t^I, d\tilde{W}_t^O)$ denotes the increment of a standard bi-variate Brownian motion with correlation $\rho$. Because FFB and CPO spot prices are positively correlated in practice, we assume $\rho > 0$ throughout our analysis. The price model in (11) implies that at time $\hat{t}$ with realized spot prices $P_{\hat{t}} = (p_{\hat{t}}^I, p_{\hat{t}}^O)$, the spot prices $\tilde{P}_t = (\tilde{p}_t^I, \tilde{p}_t^O)$ at a future time $t > \hat{t}$ follow a bi-variate Normal distribution with

$$
\mathbb{E}[\tilde{p}_t^j | P_{\hat{t}}] = e^{-\theta^j (t - \hat{t})} p_{\hat{t}}^j + \left( 1 - e^{-\theta^j (t - \hat{t})} \right) \bar{p}^j, $$
$$
\text{VAR}[\tilde{p}_t^j | P_{\hat{t}}] = \frac{1 - e^{-2\theta^j (t - \hat{t})}}{2\theta^j} (\sigma^j)^2, $$
$$
\text{COV}[\tilde{p}_t^I, \tilde{p}_t^O | P_{\hat{t}}] = \frac{1 - e^{-(\theta^I + \theta^O) (t - \hat{t})}}{\theta^I + \theta^O} \rho \sigma^I \sigma^O. $$

Throughout the analysis, we assume that the processing margin is non-negative for all price realizations, i.e. $-p_t^I - c + \bar{a} p_t^O \geq 0$ for $t = 1, \ldots, T$. In §5.2, based on the data used for calibrating our numerical experiments, we verify that this is a reasonable assumption in the palm industry. This assumption has two important implications. First, in the absence of this assumption, the processing margin per output in period $t$ ($Y_t$) is impacted by the spot price uncertainty parameters (volatility $\sigma^j$ and correlation $\rho$) because the margin $-p_{t+1}^I - c + \bar{a} p_{t+1}^O$ is censored below from zero: the firm optimally does not process if the margin is negative. Therefore, assuming non-negative margin preserves our sensitivity results from this theoretical impact which does not have practical relevance in the palm industry. Second, with this assumption, $Y_t$ is normally distributed, and $M_3$ in Corollary

\footnotetext{9}{As highlighted in Devalkar et al. (2011), single-factor mean-reverting price processes are commonly used in the literature to model the spot price processes for agricultural commodities.}

\footnotetext{10}{Another way to model the price process is to assume that the logarithm of the spot prices follow (11). In this case, for a given $P_t$, $\tilde{P}_t$ is log-normally distributed. We verify that this price model and our price model have similar fit to the data used for calibrating our numerical experiments in §5.2.}
2 can be written in closed form (using the moments and the pdf and cdf of the standard normal distribution). This closed-form characterization leads to an efficient and accurate numerical computation of $K^*$ and $\Pi^*$.

5.2 Data and Model Calibration for Numerical Experiments

In the next two sections, to illustrate the impact of the spot price uncertainty and the performance of the optimal and heuristic capacity investment policies, we resort to numerical experiments. In this section, we describe the data and model calibration used for these experiments. Our focal unit of analysis is a palm oil mill located in Southeast Asia. Within this region, Malaysia and Indonesia share common characteristics and they are the two largest players in the palm oil industry accounting for 86% of the world palm oil production between 2013 and 2014 as reported in Table 11 of U.S. Department of Agriculture's Foreign Agriculture Service Report on oilseeds. Our numerical experiments use publicly available data from Malaysian Palm Oil Board\textsuperscript{11} (MPOB hereafter), complemented by publicly available and proprietary data from palm oil mills located in Malaysia. Throughout this section, we use “RM” to denote Malaysian Ringgit and “mt” to denote metric ton.

**Calibration for Price Process Parameters.** In our computational experiments, each period corresponds to a week-day in practice. We use the daily spot prices of FFB and CPO reported in MPOB from January 1, 2006 to December 31, 2013, corresponding to 1940 weekdays. While the daily CPO price is the same across Malaysian Peninsula, the daily FFB price depends on the sub-regions (North, South, West, and East) of Malaysian Peninsula and the quality (Grade A, B, and C) of the palm fruit. Therefore, we use the average FFB prices across sub-regions and grades. The daily spot prices used in our calibration, in unit of Malaysian Ringgit per metric ton (RM/mt), are plotted in Panel (a) of Figure 1. According to the price process specified in (11), the daily spot prices evolve as

\[
p_t^I = e^{-\theta_I} p_{t-1}^I + (1 - e^{-\theta_I}) \bar{p}^I + \sigma_I \sqrt{\frac{1 - e^{-2\theta_I}}{2\theta_I}} z^I, \tag{12}
\]

\[
p_t^O = e^{-\theta_O} p_{t-1}^O + (1 - e^{-\theta_O}) \bar{p}^O + \sigma^O \sqrt{\frac{1 - e^{-2\theta_O}}{2\theta_O}} z^O,
\]

where $(z^I, z^O)$ follows a standard bi-variate Normal distribution with correlation $\rho$. The expressions in (12) can be viewed as a system of simultaneous equations of $(p_t^I, p_t^O)$ on

\[\text{http://mpob.gov.my}\]
\( (p^I_t - p^O_t), \text{i.e. } p^j_t = \alpha^j p^j_{t-1} + \eta^j + \tilde{\epsilon}^j \) for \( j \in \{I, O\} \). Because the error terms \((\tilde{\epsilon}^I, \tilde{\epsilon}^O)\) are correlated, we use the Seemingly Unrelated Regression (SUR, see Zellner 1962) to estimate \(\alpha^j, \eta^j\) and the covariance matrix of \((\tilde{\epsilon}^I, \tilde{\epsilon}^O)\). Based on these estimates, using (12), we obtain the following price process parameters: \(\theta^I = 0.00345, \bar{p}^I = 532.75, \sigma^I = 8.60, \theta^O = 0.00437, \bar{p}^O = 2689.87, \sigma^O = 39.08\) and \(\rho = 0.734\). The root mean-squared errors (RMSE) between the observed and the estimated prices for FFB and CPO, \(R^2\) of the individual regression equations, and the McElroy’s \(R^2\) of the SUR are given by Table 1. According to the McElroy’s \(R^2\) measure, the SUR equations can explain 99.36% percent of the variation in the spot prices observed.

<table>
<thead>
<tr>
<th>Goodness of Fit</th>
<th>FFB</th>
<th>CPO</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>8.59</td>
<td>39.00</td>
</tr>
<tr>
<td>RMSE (% of (\bar{p}))</td>
<td>1.61%</td>
<td>1.45%</td>
</tr>
<tr>
<td>(R^2)</td>
<td>99.63%</td>
<td>99.62%</td>
</tr>
<tr>
<td>McElroy’s (R^2)</td>
<td>99.36%</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Goodness of fit test result of the Seemingly Unrelated Regression (SUR) estimation

**Calibration for production yield parameters.** The most granular data from MPOB is the monthly average production yield (extraction rate) in the Malaysian Peninsula. As plotted in Panel (b) of Figure 1, the CPO yield from January 2006 to December 2013 varies in the range of 18.51% to 20.37%, with mean 19.72% and standard deviation 0.43%. As follows from Corollary 2, the values for the mean production yield \(\bar{a}\) and the high yield realization \(a^H\) are required for numerical computation. We set \(\bar{a} = 19.72\%\), the average yield in our data set, and \(a^H = 20.37\%\), the highest yield recorded in our data set. As discussed in §3, we have assumed that production yield and spot price distributions are statistically independent. We verify that this is a reasonable assumption. In particular, we show that the correlation between CPO yield and the CPO price change with \(k\)-month lag for \(k \in [1, \ldots, 5]\) is not significantly different from zero (in the range of \([-0.02, -0.08]\)).

**Calibration for other operational parameters.** For processing cost, we set \(c = 40\)

\(^{12}\)SUR is a generalization of a linear regression model that consists of multiple regression equations with correlated error terms.
RM per mt of FFB, which is representative of the palm industry. For example, in the 2013 annual report of Sime Darby, a major palm producer in Malaysia, the average “mill cost” between 2008 and 2012 is reported as 199.75 RM per mt of CPO, which corresponds to 39.39 RM per mt of FFB by using the average production yield 19.72%. As discussed in §5.1, we have assumed positive processing margin for all price realizations. We verify that this is a reasonable assumption with \( c = 40 \). In particular, by using the daily FFB \( (p^I) \) and CPO \( (p^O) \) prices, and the monthly CPO \( (a) \) and palm kernel \( (a^B) \) production yield reported in MPOB, we show that the observed processing margin \( ap^O + a^Bp^B - c - p^I \) with \( c = 40 \) is almost always positive.\(^{13}\) For the inventory holding cost, we use \( h = 1 \) RM/day per mt of CPO, which is approximately 10% of the CPO value if held in storage for one whole year (based on the long-term CPO price level \( p^O = 2689.87 \) and 250 week-days per year). For the byproduct (palm kernel), we use the overall average of the data reported in MPOB within our time frame for both the price \( p^B = 1510.70 \) RM/mt, and the production yield \( a^B = 5.53\% \). This implies an effective processing cost of \( c = c - a^Bp^B = -39.47 \) RM per mt of FFB. Capacity cost parameters \( \beta^I \) and \( \beta^O \) are calibrated based on a quotation from an anonymous palm oil mill located in Malaysia. In this quotation, the cost of processing facilities (fruit reception, sterilization, threshing, pressing and centrifuge stations) with a capacity of 30 mt of FFB per hour (which is 300 mt of FFB per day in our model assuming 10 hour per day) is 6,723,940 RM, whereas the cost of storage tank with a capacity of 2000 mt of CPO is 969,570 RM. Using this quotation, we estimate \( \beta^I = 75 \) and \( \beta^O = 0.25 \).

**Numerical Computation.** As follows from Corollary 2, computation of the optimal capacity investment portfolio and the optimal expected profit requires computing \( \mathbb{E}_0[\max(X_t, Y_t)] \) and \( \mathbb{E}_0[X^+_t] \) for \( t \in [1, T - 1] \). It can be proven that at time 0, \( (X_t, Y_t) \) for \( t \in [1, T - 1] \) follow a bi-variate normal distribution, and thus, these expressions can be written in closed form (using the moments and the pdf and cdf of the standard normal distribution). Therefore, the numerical computation can be carried out in an efficient and accurate manner. In our experiments, we initialize the FFB and CPO prices at the beginning of the planning horizon to their last available values in the data set: \( p^I_0 = 528.5 \) RM/mt and \( p^O_0 = 2570.5 \) RM/mt. We consider a planning horizon of 10 years which corresponds to 2500 week-days in our data set, we only observe two days with negative processing margin, and the margins in these days are insignificantly lower than zero (-0.83 and -0.17).
days, i.e. $T = 2500$. In our baseline scenario, $K^I^* = 858.91$ mt/day, $K^O^* = 1653.66$ mt, and $\Pi^* = 56,012,483.86$ RM. The optimal capacity investment is given by the storage-dominating portfolio, paralleling the common practice in the palm industry.

5.3 The Impact of Spot Price Uncertainty

In this section, we conduct sensitivity analysis to study the impact of input and output spot price volatility ($\sigma^I$ and $\sigma^O$) and spot price correlation ($\rho$) on the firm’s optimal capacity investment portfolio $K^*$ and optimal expected profit $\Pi^*$.

**Proposition 4** (Price correlation $\rho$) \( \frac{\partial K^I^*}{\partial \rho} < 0, \frac{\partial K^O^*}{\partial \rho} \leq 0, \) and \( \frac{\partial \Pi^*}{\partial \rho} < 0, \) where \( \frac{\partial K^O^*}{\partial \rho} = 0 \) only when $K^*$ is given by the “storage-dominating” portfolio.

The impact of $\rho$ on $K^*$ and $\Pi^*$ is determined by its effect on $M_3$ in Corollary 2. In each period $t \in [1, T - 1]$, this expected marginal revenue is given by the maximum of the storage margin $X_t$ and the processing margin $Y_t$, i.e. $\mathbb{E}_0[\max(X_t, Y_t)]$. With a lower $\rho$, there will be a higher likelihood when the input spot price is low (high) that the output spot price will be high (low). Therefore, the variability of the processing margin increases, whereas the storage margin is not affected. While higher processing margin increases the marginal revenue, lower processing margin is less consequential because the storage margin dominates. Therefore, $\mathbb{E}_0[\max(X_t, Y_t)]$ for $t \in [1, T - 1]$, and in turn, $\mathbb{E}_0\left[\sum_{t=1}^{T-1} \max(X_t, Y_t)\right]$ increases. As a result, $K^*$ and $\Pi^*$ increase. Figure 3 illustrates this impact in our baseline scenario for $\rho \in [0.5, 0.975]$ with 0.025 increments. In all instances, $K^*$ is given by the storage-dominating portfolio, and thus, paralleling Proposition 4, $K^O^*$ is not impacted by $\rho$. 
Figure 3: Impact of spot price correlation ($\rho$) on the optimal processing ($K^I^*$) and storage ($K^O^*$) capacity levels and the optimal expected profit, where $\rho \in [0.5, 0.975]$ with 0.025 increments. $\ast$ denotes the baseline scenario. $K^*$ is given by the storage-dominating portfolio.

**Proposition 5 (Input price volatility $\sigma^I$)** There exist $\sigma^I < \sigma^I$ such that $\frac{\partial K^I^*}{\partial \sigma^I} < 0$, $\frac{\partial \Pi^*}{\partial \sigma^I} < 0$ for $\sigma^I < \sigma^I$; and $\frac{\partial K^I^*}{\partial \sigma^I} > 0$, $\frac{\partial \Pi^*}{\partial \sigma^I} > 0$ for $\sigma^I > \sigma^I$. When $K^*$ is given by the “storage-dominating” portfolio, $\frac{\partial K^O^*}{\partial \sigma^I} = 0$; otherwise $\frac{\partial K^O^*}{\partial \sigma^I} < 0$ for $\sigma^I < \sigma^I$ and $\frac{\partial K^O^*}{\partial \sigma^I} > 0$ for $\sigma^I > \sigma^I$.

Paralleling the $\rho$ effect, in each period $t \in [1, T - 1]$, the variability of the processing margin $Y_t$ is impacted by $\sigma^I$, whereas the storage margin $X_t$ is not. Therefore, the impact of $\sigma^I$ on $K^*$ and $\Pi^*$ is determined by its effect on $M_3$ in Corollary 2. Because $\rho > 0$, higher $\sigma^I$ decreases (increases) the variability of the processing margin when $\sigma^I$ is low (high). Therefore, there exists a unique $\hat{\sigma}^I$ threshold such that the expected marginal revenue in period $t \in [1, T - 1]$, i.e. $E^0[\max(X_t, Y_t)]$, decreases (increases) in $\sigma^I$ when $\sigma^I$ is lower (higher) than this threshold. Because this threshold is period dependent, the impact of $\sigma^I$ on $E^0[\sum_{t=1}^{T-1} \max(X_t, Y_t)]$ can only be proven partially for sufficiently low $\sigma^I$, where $\sigma^I < \sigma^I \doteq \min\{\hat{\sigma}^I\}_\forall t$, and for sufficiently high $\sigma^I$, where $\sigma^I > \sigma^I \doteq \max\{\hat{\sigma}^I\}_\forall t$. To investigate the impact of $\sigma^I$ in the remaining range, we conduct numerical experiments. Figure 4 illustrates this impact in our baseline scenario for $\sigma^I \in [1, 17]$ with 1 increments. Our numerical experiments reinforce the results in Proposition 5: $K^I^*$ and $\Pi^*$ decrease.
(increase) in $\sigma^I$ when $\sigma^I$ is low (high). In all instances, $K^*$ is given by the storage-dominating portfolio, and thus, paralleling Proposition 5, $K^{O^*}$ is not impacted by $\sigma^I$.

![Figure 4: Impact of input spot price volatility ($\sigma^I$) on the optimal processing ($K^I*$) and storage ($K^{O^*}$) capacity levels and the optimal expected profit, where $\sigma^I \in [1, 17]$ with 1 increments. * denotes the baseline scenario. $K^*$ is given by the storage-dominating portfolio.](image)

**Proposition 6 (Output price volatility $\sigma^O$)** There exist $\sigma^O < \sigma^O$ such that

i) When $K^*$ is given by the “high-yield-balanced” portfolio, $\frac{\partial K^I*}{\partial \sigma^O} < 0$, $\frac{\partial K^{O*}}{\partial \sigma^O} < 0$, $\frac{\partial \Pi^*}{\partial \sigma^O} < 0$ for $\sigma^O < \sigma^O$, and $\frac{\partial K^I*}{\partial \sigma^O} > 0$, $\frac{\partial K^{O*}}{\partial \sigma^O} > 0$, $\frac{\partial \Pi^*}{\partial \sigma^O} > 0$ for $\sigma^O > \sigma^O$;

ii) When $K^*$ is given by the “storage-dominating” portfolio, $\frac{\partial K^I*}{\partial \sigma^O} < 0$ for $\sigma^O < \sigma^O$ and $\frac{\partial K^{O*}}{\partial \sigma^O} > 0$.

Unlike the $\rho$ and $\sigma^I$ effects, in each period $t \in [1, T - 1]$, $\sigma^O$ has an impact on both the processing margin $Y_t$ and the storage margin $X_t$. Therefore, the impact of $\sigma^O$ on $K^*$ and $\Pi^*$ is determined by its effect on $M_3$ and $M_4$ in Corollary 2. Higher $\sigma^O$ increases the variability of the storage margin in period $t$. The firm benefits from high storage margin, whereas low storage margin is less consequential because the firm optimally chooses not to hold inventory. Therefore, the total expected storage margin $M_4$ increases in $\sigma^O$. The impact of $\sigma^O$ on $M_3$ parallels the $\sigma^I$ effect. In particular, there exists a unique $\sigma^O$ threshold such that the expected marginal revenue in period $t \in [1, T - 1]$ decreases (increases) in $\sigma^O$ when $\sigma^O$ is lower (higher) than this threshold. Because this threshold is period dependent, the impact
of $\sigma^O$ can only be partially characterized: $M_3$ decreases in $\sigma^O$ when $\sigma^O < \overline{\sigma}^O = \min\{\hat{\sigma}^O_t\} \forall t$, and increases in $\sigma^O$ when $\sigma^O > \overline{\sigma}^I = \max\{\hat{\sigma}^O_t\} \forall t$. When $K^*$ is given by the “high-yield-balanced” portfolio, the impact of $\sigma^O$ on $K^*$ and $\Pi^*$ is characterized by its impact on $M_3$. When $K^*$ is given by the “storage-dominating” portfolio, the impact of $\sigma^O$ on $M_3$ and $M_4$ are both relevant. In this case, $K^{O*}$ increases in $\sigma^O$ because $M_4$ increases, whereas $K^{I*}$ decreases in $\sigma^O$ when $\sigma^O$ is low because $M_3$ decreases and $M_4$ increases.

To further investigate the impact of $\sigma^O$, we conduct numerical experiments. Figure 5 illustrates this impact in our baseline scenario for $\sigma^O \in [1, 81]$ with 4 increments. Figure 5 reinforces the results in Proposition 6: $K^*$ and $\Pi^*$ decrease in $\sigma^O$ when $\sigma^O$ is low, otherwise increase in $\sigma^O$.

In summary, our results demonstrate that the palm oil mill benefits from a lower spot price correlation, and a lower (higher) FFB or CPO price volatility when this volatility is low (high). We provide some rules of thumb for capacity management of a typical palm oil mill observed in practice (that invests in storage-dominating portfolio): The palm oil mill should decrease its processing and storage capacity investment with an increase (a decrease)
in CPO volatility when this volatility is low (high). The storage capacity should not be altered with changes in the spot price correlation or FFB price volatility. However, the palm oil mill should decrease its processing capacity with an increase in spot price correlation, and with an increase (a decrease) in FFB price volatility when this volatility is low (high).

5.4 Performance Evaluation of Heuristic Capacity Investment Policies

In this section, we numerically compare the performance of the optimal capacity investment policy with that of heuristic capacity investment policies which have practical and theoretical significance. These comparisons enable us to highlight the important factors for capacity management in the palm industry. To this end, we define the percentage profit loss

$$
\Delta = \left[ \frac{\Pi^* - \Pi(K_{hp})}{\Pi^*} \right] \times 100\%, \text{ where } \Pi^* \text{ is the optimal expected profit and } \Pi(K_{hp}) \text{ denotes the expected profit with the capacity portfolio } K_{hp} = (K^I_p, K^O_p) \text{ chosen under the heuristic policy (hp).}
$$

We consider the following heuristic capacity investment policies:

“High-Yield-Balanced” Portfolio Heuristic: The firm chooses the capacity investment portfolio by assuming \( K^{O}_{hp} = a^H K^{I}_{hp} \). The optimal capacity investment with this policy is characterized by the high-yield-balanced portfolio (when \( \beta \in \Omega_2 \) in Corollary 2). Because storage-dominating portfolio is not feasible, considering this policy is beneficial in understanding the value of excess storage capacity (which is not used for production).

“Deterministic Yield (High)” Heuristic: The firm ignores the production yield uncertainty, and the capacity planning is made based on the maximum possible yield. The optimal capacity investment with this policy can be obtained from Corollary 2 by substituting \( \bar{a} \) with \( a^H \) in \( Y_t \). This policy is implemented by the palm oil mills in practice; and its consideration is beneficial in understanding the impact of yield uncertainty.

“No-Storage” Heuristic: The firm does not hold output inventory, and the storage capacity \( K^{O}_{hp} \) is only used as a production capacity in conjunction with \( K^{I}_{hp} \). The optimal capacity investment with this policy can be obtained from Corollary 2 by setting infinite holding cost, i.e. \( h \rightarrow \infty \). Considering this policy is beneficial in understanding the impact of inventory holding possibility on capacity management.

“No-Byproduct” Heuristic: The firm does not consider byproduct revenue \( (a^B p^B) \) in capacity planning. The optimal capacity investment with this policy can be obtained from Corollary 2 by substituting the effective processing cost \( c = s - a^B p^B \) with \( s \). Considering this policy is beneficial in understanding the impact of byproduct on capacity management.
Tables 2 and 3 summarize the percentage profit loss incurred under each of these heuristic policies with respect to different values of two parameters of interest, the output spot price volatility $\sigma^O$ and the mean production yield $\bar{a}$, respectively.

<table>
<thead>
<tr>
<th>Percentage Change in CPO Volatility $\sigma^O$ (%)</th>
<th>$\Delta=\text{Percentage Loss}~%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High-Yield-Balanced</td>
</tr>
<tr>
<td>-15</td>
<td>0.48</td>
</tr>
<tr>
<td>-10</td>
<td>0.62</td>
</tr>
<tr>
<td>-5</td>
<td>0.79</td>
</tr>
<tr>
<td>0</td>
<td>0.98</td>
</tr>
<tr>
<td>5</td>
<td>1.18</td>
</tr>
<tr>
<td>10</td>
<td>1.40</td>
</tr>
<tr>
<td>15</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Table 2: Performance of heuristics with respect to output spot price volatility

<table>
<thead>
<tr>
<th>Percentage Change in Mean Yield $\bar{a}$ (%)</th>
<th>$\Delta=\text{Percentage Loss}~%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High-Yield-Balanced</td>
</tr>
<tr>
<td>-3</td>
<td>1.63</td>
</tr>
<tr>
<td>-2</td>
<td>1.37</td>
</tr>
<tr>
<td>-1</td>
<td>1.15</td>
</tr>
<tr>
<td>0</td>
<td>0.98</td>
</tr>
<tr>
<td>1</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 3: Performance of heuristics with respect to mean production yield

We make the following observations from these results:

1) Ignoring the byproduct revenue in capacity planning leads to very significant profit loss (larger than 98%). This result is interesting because, by definition, the byproduct revenue constitutes a small portion of the total revenues.\textsuperscript{14} However, the processing margin becomes insignificant when the byproduct revenue is not accounted for, and the firm invests in a very conservative processing capacity. In our numerical experiments, we observe that the processing margin is non-positive in the absence of the byproduct revenue, and thus,

\textsuperscript{14}For example, when the production yield of the output is assumed to be at its mean level, and the output spot price is assumed to be at its long-term mean, the byproduct revenues are only 14\% of the total revenues.
\( K_{hp}^I = 0 \), i.e. the firm operates as a storage facility under this policy.

2) Ignoring the inventory holding possibility in capacity planning leads to a sizeable profit loss (up to 18.79%). In our numerical experiments, we observe that both capacity levels are significantly lower than those of the optimal policy. In comparison with the No-Byproduct policy, where processing possibility is absent, the profit loss is less significant with the No-Storage policy, where inventory holding possibility is absent. This demonstrates that the processing margin is more crucial than the storage margin for profitability.

3) Ignoring the production yield uncertainty in capacity planning leads to significant profit loss when planning is made based on the high yield realization (up to 34.63%). With this policy, the firm overestimates the processing margin per output \( (Y_t) \), and thus, overinvests in capacity. The magnitude of processing margin mis-specification decreases as \( \bar{a} \) gets closer to \( a^H \), and thus, the profit loss \( \Delta \) decreases in \( \bar{a} \), as depicted in Table 3. We conjecture that ignoring the production yield uncertainty would lead to a less significant profit loss when the capacity planning is made based on the mean production yield. The optimal capacity investment with this proposed policy can be obtained by substituting \( a^H \) with \( \bar{a} \) in Corollary 2. When storage-dominating portfolio is optimal, the production yield uncertainty impacts this portfolio through the maximum of the storage margin per \( a^H \), i.e. 
\[
a^H (-p_t^O - h + E_t[p_{t+1}^O]),
\]
and the processing margin per input, i.e. 
\[
E_t [-p_{t+1}^I - c + \bar{a}p_{t+1}^O].
\]

With the proposed policy, the firm underestimates the storage margin, but the processing margin is not impacted. Because processing margin is more crucial than the storage margin for profitability, the underestimation of the storage margin should not have a significant impact. Our numerical experiments validate this conjecture. We observe that the storage-dominating portfolio is optimal with the proposed policy, and the capacity levels are very close to those of the optimal policy, and thus, the profit loss is close to zero.

4) Making the capacity planning based on the high-yield-balanced portfolio strategy does not lead to significant profit loss (less than 1.63%). With this policy, the firm invests in a larger processing capacity, but a smaller storage capacity in comparison with the optimal policy. In our numerical experiments, we observe that \( K_{hp}^I \) is at most 1% higher than \( K_I^* \), whereas \( K_{hp}^O \) is 90% lower than \( K_O^* \). This is consistent with our earlier observation that processing margin is more crucial than storage margin for profitability: An additional small amount of processing capacity provides similar benefit with very large amount of excess storage capacity. This observation is important because it is not an uncommon practice for
the palm oil mills to invest in a storage-dominating portfolio where there is significant excess storage capacity. Our results show that this practice is not essential, and comparable profit can be generated when the capacity planning is made based on no excess storage capacity.

6 Conclusion

This paper contributes to the operations management literature on commodity processing by analyzing the capacity investment decisions of a processor in the context of agricultural industries. The papers in this literature either (often implicitly) assume abundant processing and storage resources, or consider fixed capacity levels for these resources.

We consider a multi-period optimization problem and characterize the optimal processing and output storage capacity levels and the periodic processing and inventory decisions in closed form. As summarized in the Introduction, we provide insights on i) the structure of the optimal capacity investment policy; ii) how the spot price uncertainty shape the capacity investment decisions and the profitability, and iii) the performance of the optimal capacity investment policy in comparison with heuristic policies that have practical and theoretical significance. To study the impact of spot price uncertainty and employing heuristic policies, we use a calibration based on the palm industry. Although the magnitude impact can be different, we expect our results to be structurally the same for the other oilseeds industries and grain industries. This is because in these industries, similar to the palm industry, processing capacity is significantly more expensive than the storage capacity, and thus, the processor is expected to invest in storage-dominating portfolio and the processing margin is expected be significantly higher than the storage margin. Moreover, because of the commodity nature of the input and the output, the processing margin is expected to be low when the byproduct revenue is not accounted for.

Relaxing the assumptions made on the processing environment gives rise to a number of interesting areas for future research. First, we assume fixed byproduct price. This is a reasonable assumption for some of the agricultural industries, but is a limitation for some others. For example, in the soybean industry, the byproduct of soybean processing is the meal, and because it is a commodity, it has an uncertain price correlated with the input and the output prices. Second, we focus on expected profit maximization, and do not consider the risk associated with the profit. Such risk considerations can be incorporated in our model by using real option valuation techniques (Birge 2000), by imposing a utility function to
the decision maker (Kouvelis et al. 2013) or by imposing risk constraints to the decision problem (Devalkar et al. 2014). Third, we assume that the marginal procurement cost and the marginal sales revenue of the output are given by the spot price of this commodity. Analyzing the case where there is a spread between these two prices due to deadweight transactions costs, as considered in Kazaz and Webster (2011), is beyond the scope of this paper, and should prove to be an interesting problem for future research. In this case, the processing and inventory decisions cannot be characterized in closed form, and thus, the optimal capacity investment decisions can only be evaluated numerically.

References


We use the following notation and results throughout the appendix. Let \( \phi(.) \) and \( \Phi(.) \) denote the p.d.f and c.d.f. of the standard normal random variable, respectively. \( \phi'(z) = -z\phi(z), \phi(z) = \phi(-z), \int_{-\infty}^{v} z\phi(z)dz = -\phi(v) \). The following result is from Cain (1994):

**Lemma 1** Let \( X = (X_1, X_2) \) follow a bivariate normal distribution with mean vector \( \mu = (\mu_1, \mu_2) \), and covariance matrix \( \Sigma \) where \( \Sigma_{jj} = \sigma_j^2 \) for \( j = 1, 2 \) and \( \Sigma_{12} = \rho \sigma_1 \sigma_2 \) and \( \rho \) denotes the correlation coefficient.

\[
\mathbb{E}[\max(X_1, X_2)] = \mu_1 \Phi \left( \frac{\mu_1 - \mu_2}{\theta} \right) + \mu_2 \Phi \left( \frac{\mu_2 - \mu_1}{\theta} \right) + \theta \phi \left( \frac{\mu_2 - \mu_1}{\theta} \right),
\]

where \( \theta = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2} \).

**Proof of Proposition 1:** For notational convenience, we define

\[
A_t = -p_t - c + \bar{a} p_t^O;
\]

\[
B_t = -p_t - c + q a^L p_t^O - (1-q)a_H d;
\]

\[
C_t = -p_t - c - \bar{a} d.
\]

For a given inventory level from the previous period \( s_{t-1} \) and the observed price \( P_t \), the optimal processing volume \( z^*_t(s_{t-1}, P_t) \) is the solution to \( \max_{0 \leq z_t \leq K^I} L_{Pr}(z_t) = p_t^O s_{t-1} + H_t(z_t) \), where \( H_t(z_t) \) is given by

\[
\left\{ \begin{array}{ll}
A_t z_t & \text{if} \quad 0 \leq z_t \leq \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right), \\
B_t z_t + (A_t - B_t) \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right) & \text{if} \quad \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right) < z_t \leq \min \left( K^I, \frac{K^O - s_{t-1}}{a_L} \right), \\
C_t z_t + (A_t - B_t) \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right) + (B_t - C_t) \min \left( K^I, \frac{K^O - s_{t-1}}{a_L} \right) & \text{if} \quad \min \left( K^I, \frac{K^O - s_{t-1}}{a_L} \right) < z_t \leq K^I.
\end{array} \right.
\]

The first order derivative is

\[
\frac{\partial L_{Pr}(z_t)}{\partial z_t} = \left\{ \begin{array}{ll}
A_t & \text{if} \quad 0 \leq z_t \leq \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right), \\
B_t & \text{if} \quad \min \left( K^I, \frac{K^O - s_{t-1}}{a_H} \right) < z_t \leq \min \left( K^I, \frac{K^O - s_{t-1}}{a_L} \right), \\
C_t & \text{if} \quad \min \left( K^I, \frac{K^O - s_{t-1}}{a_L} \right) < z_t \leq K^I.
\end{array} \right.
\]

Because \( A_t > B_t > C_t \), \( L_{Pr}(z_t) \) is piece-wise linear and concave in \( z_t \). It is easy to verify
that $L_{Pt}(z_t)$ is continuous at the boundaries. Therefore,

$$z^*_t(s_{t-1}, P_t) = \begin{cases} 
0 & \text{if } A_t \leq 0; \\
\min \left( \frac{K^O - s_t}{a^H}, K^I \right) & \text{if } B_t \leq 0 < A_t; \\
\min \left( \frac{K^O - s_t}{a^L}, K^I \right) & \text{if } C_t \leq 0 < B_t; \\
K^I & \text{if } 0 < C_t.
\end{cases}$$

Expanding $A_t, B_t$ and $C_t$ with their definitions in (13) and replacing $t$ with $t+1$ give us the characterization of the optimal processing volume $z^*_t(s_t, P_{t+1})$ in Proposition 1.

**Proof of Proposition 2:** The optimal inventory level $s^*_t(P_t)$ is given by the solution to (6). Using the definitions of $X_t, Y_t, Z_t$ as given in (8), and $A_t, B_t, C_t$ as given in (13) of the proof of Proposition 1, the objective function in (6) can be written as

$$X_t s_t + K^I \mathbb{E}_t [A^+_t]$$

$$+ (X_t - Y_t) s_t + \frac{K^O}{a^H} \mathbb{E}_t [A^+_t] + \left( \frac{K^O}{a^H} \right) \mathbb{E}_t [B^+_t]$$

$$+ (X_t - Y_t - Z_t)s_t + \frac{K^O}{a^L} \mathbb{E}_t [A^+_t] + \left( \frac{K^O}{a^L} \right) \mathbb{E}_t [B^+_t] + \left( \frac{K^O}{a^L} \right) \mathbb{E}_t [C^+_t]$$

The first order derivative is

$$X_t$$

$$\begin{cases} 
X_t & \text{if } 0 \leq s_t \leq (K^O - a^H K^I)^+; \\
(X_t - Y_t) & \text{if } (K^O - a^H K^I)^+ < s_t \leq (K^O - a^L K^I)^+; \\
(X_t - Y_t - Z_t) & \text{if } (K^O - a^L K^I)^+ < s_t \leq K^O.
\end{cases}$$

Because $X_t \geq (X_t - Y_t) \geq (X_t - Y_t - Z_t)$, the objective function is piece-wise linear and concave in the inventory level $s_t$. It is easy to verify that it is continuous at the boundaries. Therefore,

$$s^*_t(P_t) = \begin{cases} 
0 & \text{if } X_t \leq 0; \\
(K^O - a^H K^I)^+ & \text{if } X_t - Y_t \leq 0 < X_t; \\
(K^O - a^L K^I)^+ & \text{if } X_t - Y_t - Z_t \leq 0 < X_t - Y_t; \\
K^O & \text{if } 0 < X_t - Y_t - Z_t.
\end{cases}$$

**Proof of Proposition 3:** The expected profit at period 0 is $\Pi(K^I, K^O) = M_1 K^I + M_2 \min (a^L K^I, K^O) + M_3 \left[ \min (a^H K^I, K^O) - \min (a^L K^I, K^O) \right] + M_4 (K^O - a^H K^I)^+ - \beta I(K^I)^2 + \beta O(K^O)^2$, where $M_1, M_2, M_3, M_4$ are as given in (9). In order to solve for the
optimal capacity investment portfolio \((K^I, K^O^*)\), we first characterize the optimal storage capacity \(K^O^*(K^I)\) for a given processing capacity \(K^I\). For a given \(K^I\), \(\Pi(K^O|K^I)\) is

\[
\begin{cases}
-\beta^I(K^I)^2 - \beta^O(K^O)^2 + M_1 K^I + M_2 K^O & \text{if } 0 \leq K^O < a^L K^I, \\
-\beta^I(K^I)^2 - \beta^O(K^O)^2 + (M_1 + a^L(M_2 - M_3))K^I + M_3 K^O & \text{if } a^L K^I \leq K^O < a^H K^I, \\
-\beta^I(K^I)^2 - \beta^O(K^O)^2 + (M_1 + a^L(M_2 - M_3) + a^H(M_3 - M_4))K^I + M_4 K^O & \text{if } K^O \geq a^H K^I.
\end{cases}
\]

It is easy to verify that \(\Pi(K^O|K^I)\) is continuous in \(K^O\). The first order derivative is

\[
\frac{\partial \Pi(K^O|K^I)}{\partial K^O} = \begin{cases}
g_1(K^O) = -2\beta^O K^O + M_2 & \text{if } 0 \leq K^O < a^L K^I, \\
g_2(K^O) = -2\beta^O K^O + M_3 & \text{if } a^L K^I \leq K^O < a^H K^I, \\
g_3(K^O) = -2\beta^O K^O + M_4 & \text{if } K^O \geq a^H K^I.
\end{cases}
\]

Because \(M_2 \geq M_3 \geq M_4\), \(g_1(a^L K^I) \geq g_2(a^L K^I)\), and \(g_2(a^H K^I) \geq g_3(a^H K^I)\). Therefore, \(\Pi(K^O|K^I)\) is concave in \(K^O\). Let \(\hat{K}^O_i\) denote the solutions to \(g_i(K^O) = 0\) (for \(i = 1, 2, 3\)), where \(\hat{K}^O_1 = \frac{M_2}{2\beta^O}, \hat{K}^O_2 = \frac{M_3}{2\beta^O},\) and \(\hat{K}^O_3 = \frac{M_4}{2\beta^O}\). The optimal solution \(K^O^*(K^I)\) depends on the ordering among \(\hat{K}^O_1, a^L K^I\) and \(a^H K^I\). Since \(\hat{K}^O_1 \geq \hat{K}^O_2 \geq \hat{K}^O_3\), we have the following cases:

1. \(\hat{K}^O_1 < a^L K^I\): \(\Pi(K^O|K^I)\) increases for \(K^O \leq \hat{K}^O_1\), and then decreases afterwards. Thus, \(K^O^*(K^I) = \hat{K}^O_1\).

2. \(\hat{K}^O_1 \geq a^L K^I\): \(\Pi(K^O|K^I)\) increases for \(K^O \leq a^L K^I\), its behavior after \(a^L K^I\) depends on the ordering between \(\hat{K}^O_2\) and \(a^L K^I\):

   2.1. \(\hat{K}^O_2 < a^L K^I\): \(\Pi(K^O|K^I)\) decreases for \(K^O \geq a^L K^I\). Thus, \(K^O^*(K^I) = a^L K^I\).

   2.2. \(a^L K^I \leq \hat{K}^O_2 < a^H K^I\): \(\Pi(K^O|K^I)\) continues to increase for \(a^L K^I < K^O \leq \hat{K}^O_2\), but decreases afterwards. Therefore \(K^O^*(K^I) = \hat{K}^O_2\).

   2.3. \(\hat{K}^O_2 \geq a^H K^I\): \(\Pi(K^O|K^I)\) continues to increase for \(a^L K^I < K^O \leq a^H K^I\), its behavior after \(a^H K^I\) depends on the ordering between \(\hat{K}^O_3\) and \(a^H K^I\):

   2.3.1. \(\hat{K}^O_3 < a^H K^I\): \(\Pi(K^O|K^I)\) decreases for \(K^O > a^H K^I\), and \(K^O^*(K^I) = a^H K^I\).

   2.3.2. \(\hat{K}^O_3 \geq a^H K^I\): \(\Pi(K^O|K^I)\) increases for \(a^H K^I < K^O \leq \hat{K}^O_3\) and decreases afterwards, thus \(K^O^*(K^I) = \hat{K}^O_3\).
Combining these arguments yields

\[
K^{O*}(K^I) = \begin{cases} 
\dot{K}_1^O & \text{if } a^L K^I \geq \dot{K}_1^O, \\
 a^L K^I & \text{if } \dot{K}_1^O > a^L K^I \geq \dot{K}_2^O, \\
\dot{K}_2^O & \text{if } a^L K^I < \dot{K}_2^O \leq a^H K^I, \\
a^H K^I & \text{if } \dot{K}_2^O < a^H K^I \leq \dot{K}_2^O, \\
\dot{K}_3^O & \text{if } a^H K^I \leq \dot{K}_3^O.
\end{cases}
\]

By substituting \(K^{O*}(K^I)\) in \(\Pi(K^I, K^O)\), we obtain

\[
\Pi(K^I) = \begin{cases} 
-\beta^I (K^I)^2 - \beta_2 (\dot{K}_2^O)^2 + (M_1 + a^L(M_2 - M_3) + a^H (M_3 - M_4))K^I + M_4 \dot{K}_3^O & \text{if } 0 \leq K^I < \frac{\dot{K}_2^O}{a^H}, \\
-(\beta^I + \beta_2 (a^H)^2)(K^I)^2 + (M_1 + a^L(M_2 - M_3) + a^H M_3)K^I & \text{if } \frac{\dot{K}_3^O}{a^H} \leq K^I < \frac{\dot{K}_2^O}{a^H}, \\
-\beta^I (K^I)^2 - \beta_2 (\dot{K}_2^O)^2 + (M_1 + a^L(M_2 - M_3))K^I + M_3 \dot{K}_2^O & \text{if } \frac{\dot{K}_2^O}{a^H} \leq K^I < \frac{\dot{K}_2^O}{a^L}, \\
-(\beta^I + \beta_2 (a^L)^2)(K^I)^2 + (M_1 + a^L M_2)K^I & \text{if } \frac{\dot{K}_3^O}{a^L} \leq K^I < \frac{\dot{K}_2^O}{a^L}, \\
-\beta^I (K^I)^2 - \beta_2 (\dot{K}_1^O)^2 + M_1 K^I + M_2 \dot{K}_1^O & \text{if } K^I \geq \frac{\dot{K}_3^O}{a^L}.
\end{cases}
\]

It is easy to verify that \(\Pi(K^I)\) is continuous in \(K^I\). The first order derivative is

\[
\frac{\partial \Pi(K^I)}{\partial K^I} = \begin{cases} 
f^1(K^I) \doteq -2\beta^I K^I + M_1 + a^L(M_2 - M_3) + a^H (M_3 - M_4) & \text{if } 0 \leq K^I < \frac{\dot{K}_3^O}{a^H}, \\
f^2(K^I) \doteq -2(\beta^I + \beta_2 (a^H)^2)K^I + M_1 + a^L(M_2 - M_3) + a^H M_3 & \text{if } \frac{\dot{K}_3^O}{a^H} \leq K^I < \frac{\dot{K}_3^O}{a^H}, \\
f^3(K^I) \doteq -2\beta^I K^I + M_1 + a^L(M_2 - M_3) & \text{if } \frac{\dot{K}_2^O}{a^H} \leq K^I < \frac{\dot{K}_2^O}{a^H}, \\
f^4(K^I) \doteq -2(\beta^I + \beta_2 (a^L)^2)K^I + M_1 + a^L M_2 & \text{if } \frac{\dot{K}_3^O}{a^L} \leq K^I < \frac{\dot{K}_2^O}{a^L}, \\
f^5(K^I) \doteq -2\beta^I K^I + M_1 & \text{if } K^I \geq \frac{\dot{K}_3^O}{a^L}.
\end{cases}
\]

It is easy to verify that \(\frac{\partial \Pi(K^I)}{\partial K^I}\) is continuous in \(K^I\). Moreover, using \(M_2 \geq M_3 \geq M_4\), we can obtain that \(\Pi(K^I)\) is a smooth concave function of \(K^I\). Let \(\dot{K}_1^I\) be the solutions to \(f^i(K^I) = 0\) (for \(i = 1, 2..5\)):

\[
\begin{align*}
\dot{K}_1^I & \doteq \frac{M_1 + a^H(M_3 - M_4) + a^L(M_2 - M_3)}{2\beta^I}; \\
\dot{K}_2^I & \doteq \frac{M_1 + a^H M_3 + a^L(M_2 - M_3)}{2\beta^I + 2(a^H)^2 \beta_2^O}; \\
\dot{K}_3^I & \doteq \frac{M_1 + a^L(M_2 - M_3)}{2\beta^I}; \\
\dot{K}_4^I & \doteq \frac{M_1 + a^L M_2}{2\beta^I + 2(a^L)^2 \beta_2^O}; \\
\dot{K}_5^I & \doteq \frac{M_1}{2\beta^I}.
\end{align*}
\]

Using a similar approach as in the previous part of the proof, we obtain
1. $\hat{K}_1^I < \frac{K_O}{a^H}$: $\Pi(K^I)$ increases for $K^I \leq \hat{K}_1^I$, and then decreases afterwards. Thus, 
\[ (K^{I*}, K^{O*}) = \left( \hat{K}_1^I, \hat{K}_3^O \right) \]. This case corresponds to $\beta \in \Omega_1$.

2. $\hat{K}_1^I \geq \frac{K_O}{a^H}$: This case also implies that $\hat{K}_2^I \geq \frac{K_O}{a^H}$, because $\frac{\partial \Pi(K^I)}{\partial K^I}$ is continuous in $K^I$ (hence, $f^1 \left( \frac{K_O}{a^H} \right) = f^2 \left( \frac{K_O}{a^H} \right) \geq 0$). Hence, the firm’s optimal capacity investment depends on the ordering between $\hat{K}_2^I$ and $\frac{K_O}{a^H}$:

2.1. $\hat{K}_2^I < \frac{K_O}{a^H}$: $\Pi(K^I)$ increases for $K^I \leq \hat{K}_2^I$, and decreases afterward. Thus, 
\[ (K^{I*}, K^{O*}) = \left( \hat{K}_2^I, a^H \hat{K}_2^I \right) \]. This case corresponds to $\beta \in \Omega_2$.

2.2. $\hat{K}_2^I \geq \frac{K_O}{a^H}$: Similarly as above, this case implies that $\hat{K}_3^I \geq \frac{K_O}{a^H}$, and the firm’s optimal capacity investment depends on the ordering between $\hat{K}_3^I$ and $\frac{K_O}{a^H}$:

2.2.1. $\hat{K}_3^I < \frac{K_O}{a^H}$: $\Pi(K^I)$ increases for $K^I \leq \hat{K}_3^I$, and decreases afterward. Thus, 
\[ (K^{I*}, K^{O*}) = \left( \hat{K}_3^I, \hat{K}_2^O \right) \]. This case corresponds to $\beta \in \Omega_3$.

2.2.2. $\hat{K}_3^I \geq \frac{K_O}{a^H}$: Similarly as above, this case implies that $\hat{K}_4^I \geq \frac{K_O}{a^H}$, and the firm’s optimal capacity investment depends on the ordering between $\hat{K}_4^I$ and $\frac{K_O}{a^H}$:

2.2.2.1. $\hat{K}_4^I < \frac{K_O}{a^H}$: $\Pi(K^I)$ increases for $K^I \leq \hat{K}_4^I$, and decreases afterward. Thus, 
\[ (K^{I*}, K^{O*}) = \left( \hat{K}_4^I, a^H \hat{K}_4^I \right) \]. This case corresponds to $\beta \in \Omega_4$.

2.2.2.2. $\hat{K}_4^I \geq \frac{K_O}{a^H}$: $(K^{I*}, K^{O*}) = \left( \hat{K}_5^I, \hat{K}_1^O \right)$. This case corresponds to $\beta \in \Omega_5$.

Proof of Corollary 1: The proof is omitted. ■

Proof of Corollary 2: The optimal capacity investment levels $(K^{I*}, K^{O*})$ follow directly from Proposition 3 by using $d \to \infty$. In this case, $M_1 = 0$ and $M_2 = M_3$. The optimal expected profit is obtained by substituting $(K^{I*}, K^{O*})$ in $\Pi(K^I, K^O) = M_3 \min \left( a^H K^I, K^O \right) + M_4 \left( K^O - a^H K^I \right)^+ - (\beta^I (K^I)^2 + \beta^O (K^O)^2)$. ■

Proof of Proposition 4: Following Corollary 2, and using our assumption $-p_{t+1}^I - c + ap_{t+1}^O \geq 0$ for all price realizations, we have $M_3 = Y_0 + E_0 \left[ \sum_{t=1}^{T-1} \max \left( X_t, Y_t \right) \right]$ and 
\[ M_4 = E_0 \left[ \sum_{t=1}^{T-1} X_t^+ \right] \] with $X_t = -p_t^O - h + E_t \left[ p_{t+1}^O \right]$ and $Y_t = \frac{E_t \left[ -p_{t+1}^I - c + ap_{t+1}^O \right]}{a^H}$. Because $M_3$
and $M_4$ depend on the distribution of $(X_t, Y_t)$ for $t = 1, ..T - 1$ at time 0, we first establish this distribution based on our price model given in (11):

**Lemma 2** Let $\delta^I \equiv \exp(-\theta^I)$ and $\delta^O \equiv \exp(-\theta^O)$. At time 0, $(X_t, Y_t)$ for $t = 1, ..T - 1$ follow a bivariate normal distribution with

$$E_0[X_t] = -h - (1 - \delta^O)(\delta^O)^t(p_0^O - p^O),$$
$$E_0[Y_t] = \frac{1}{aH} (-c - [(\delta^I)^{t+1}p_0^I + (1 - (\delta^I)^{t+1})p^I] + \tilde{a} [(\delta^O)^{t+1}p_0^O + (1 - (\delta^O)^{t+1})p^O]),$$
$$VAR_0[X_t] = (1 - \delta^O)^2(aH)^2(\delta^O)^2 \frac{1}{2\theta^O},$$
$$VAR_0[Y_t] = \frac{1}{(aH)^2} \left( (1 - (\delta^I)^{2(t+1)})(\delta^I)^2 + a^2(1 - (\delta^O)^{2(t+1)})(\delta^O)^2 - 2\tilde{a}(1 - (\delta^I)^{t+1}(\delta^O)^{t+1}) \frac{\rho_{\delta^I\delta^O}}{\theta^I + \theta^O} \right),$$
$$COV_0(X_t, Y_t) = \frac{1 - \delta^O}{aH} \left[ \delta^I(1 - (\delta^I)^t(\delta^O)^t) \rho_{\delta^I\delta^O} \frac{\sigma^I}{\theta^I + \theta^O} - \tilde{a}\delta^O(1 - (\delta^O)^{2t}) \frac{(\delta^O)^2}{2\theta^O} \right].$$

Because the marginal distribution of $X_t$ is independent of $\rho$, so is $M_4$. Therefore, the impact of $\rho$ on $K^*$ (and $\Pi^*$) is characterized by its impact on $M_3$. Because $(X_t, Y_t)$ follow a bivariate normal distribution, we use Lemma 1 to establish $E_0[\max(X_t, Y_t)]$ for $t = 1, ..T - 1$, and after some algebra, we obtain

$$\frac{\partial E_0[\max(X_t, Y_t)]}{\partial \rho} = \phi \left( \frac{E_0[Y_t] - E_0[X_t]}{\theta} \right) \frac{\partial \theta}{\partial \rho},$$

where $\theta = \sqrt{VAR_0(X_t) + VAR_0(Y_t) - 2COV_0(X_t, Y_t)}$. The first term on the right-hand side is positive, and the second term is negative because, as follows from Lemma 2, $VAR_0[Y_t]$ is decreasing in $\rho$ and $COV_0[X_t, Y_t]$ is increasing in $\rho$. Therefore, $\frac{\partial \theta}{\partial \rho} < 0$, and thus, $\frac{\partial E_0[\max(X_t, Y_t)]}{\partial \rho} < 0$. Because $Y_0 = E_0[Y_t]$ for $t = 0$ is independent of $\rho$, $M_3 = Y_0 + E_0 \left[ \sum_{t=1}^{T-1} \max(X_t, Y_t) \right]$ is strictly decreasing in $\rho$. It is straightforward to verify that $K^I$, $K^{O*}$, and $\Pi^*$ in Corollary 2 are continuous in $\beta$. Therefore, $\frac{\partial K^I}{\partial \rho} < 0$, $\frac{\partial K^{O*}}{\partial \rho} \leq 0$, and $\frac{\partial \Pi^*}{\partial \rho} < 0$, where $\frac{\partial K^{O*}}{\partial \rho} = 0$ only when $\beta \in \Omega_1$, in which case $K^{O*}$ is independent of $M_3$. ■

**Proof of Proposition 5:** Similar to the $\rho$ impact, $M_4$ is independent of $\sigma^I$, and thus, the impact of $\sigma^I$ on $K^*$ (and $\Pi^*$) is characterized by its impact on $M_3$. Using similar steps with the proof of Proposition 4, we obtain $\frac{\partial E_0[\max(X_t, Y_t)]}{\partial \sigma^I} = \phi \left( \frac{E_0[Y_t] - E_0[X_t]}{\theta} \right) \frac{\partial \theta}{\partial \sigma^I}$, where

$$\frac{\partial \theta}{\partial \sigma^I} = \frac{1}{2\sqrt{aH}} \left[ (1 - (\delta^I)^{2(t+1)}) \frac{\sigma^I}{\theta^I} - 2\tilde{a}(1 - (\delta^I)^{t+1}(\delta^O)^{t+1}) \frac{\rho_{\sigma^I\delta^O}}{\theta^I + \theta^O} - 2\tilde{a}\delta^I(1 - \delta^I)(1 - \delta^O)(1 - (\delta^I)^t(\delta^O)^t) \frac{\rho_{\sigma^I\delta^O}}{\theta^I + \theta^O} \right].$$

The term inside the bracket can be written as $A\sigma^I - B$, where $A > 0$ and $B > 0$. Therefore, there exists a unique $\delta^I_t \equiv \frac{B}{A}$ threshold for $t = 1, ..T - 1$ such that $\frac{\partial E_0[\max(X_t, Y_t)]}{\partial \sigma^I} < 0$ for
\( \sigma^I < \bar{\sigma}^I \) and \( \frac{\partial E_0[\max(X_t,Y_t)]}{\partial \sigma^I} > 0 \) for \( \sigma^I > \bar{\sigma}^I \). Because \( Y_0 = E_0[Y_t] \) for \( t = 0 \) is independent of \( \sigma^I \), for \( M_3 = Y_0 + E_0 \left[ \sum_{t=1}^{T-1} \max(X_t,Y_t) \right] \), there exist \( \bar{\sigma}^I = \min\{\bar{\sigma}^I\} \) and \( \bar{\sigma}^I = \max\{\bar{\sigma}^I\} \) such that \( \frac{\partial M_3}{\partial \sigma^I} < 0 \) for \( \sigma^I < \bar{\sigma}^I \); and \( \frac{\partial M_3}{\partial \sigma^I} > 0 \) for \( \sigma^I > \bar{\sigma}^I \). This property also holds for \( K^I^*, K^O^*, \) and \( \Pi^* \) because they are either linear or quadratic functions of \( M_3 \). The only exception is that under storage-dominating portfolio \( (\beta \in \Omega_1) \), \( K^O^* \) is independent of \( M_3 \) and hence \( \sigma^I \). ■

**Proof of Proposition 6:** Following the similar steps with the proof of Proposition 5, we can show that there exist \( \sigma^O < \bar{\sigma}^O \) such that \( \frac{\partial M_3}{\partial \sigma^O} < 0 \) for \( \sigma^O < \sigma^O \); and \( \frac{\partial M_3}{\partial \sigma^O} > 0 \) for \( \sigma^O > \bar{\sigma}^O \). Different from Proposition 5, \( \sigma^O \) also impacts \( M_4 \). Because \( X_t \) follows a normal distribution with mean \( E_0[X_t] \) and variance \( \text{VAR}_0[X_t] \) as given in Lemma 2,

\[
E_0[(X_t)^+] = E_0[X_t]\Phi \left( \frac{E_0[X_t]}{\sqrt{\text{VAR}_0[X_t]}} \right) + \sqrt{\text{VAR}_0[X_t]} \phi \left( \frac{E_0[X_t]}{\sqrt{\text{VAR}_0[X_t]}} \right),
\]

and thus, \( \frac{\partial E_0[(X_t)^+]}{\partial \sigma^O} = \phi \left( \frac{E_0[X_t]}{\sqrt{\text{VAR}_0[X_t]}} \right) \frac{\partial \sqrt{\text{VAR}_0[X_t]}}{\partial \sigma^O} \). Because \( \text{VAR}_0[X_t] \) strictly increases in \( \sigma^O \) (as follows from Lemma 2), \( E_0[(X_t)^+] \) for \( t = 1, \ldots, T-1 \), and thus, \( M_4 = E_0 \left[ \sum_{t=1}^{T-1} X_t^+ \right] \) strictly increases in \( \sigma^O \).

Under high-yield-balanced portfolio \( (\beta \in \Omega_2) \), since \( K^I^*, K^O^*, \) and \( \Pi^* \) are linear or quadratic functions of \( M_3 \), we have \( \frac{\partial K^I^*}{\partial \sigma^I} < 0 \), \( \frac{\partial K^O^*}{\partial \sigma^O} < 0 \), \( \frac{\partial \Pi^*}{\partial \sigma^I} < 0 \), \( \frac{\partial \Pi^*}{\partial \sigma^O} > 0 \), \( \frac{\partial \Pi^*}{\partial \sigma^I} > 0 \), \( \frac{\partial \Pi^*}{\partial \sigma^O} > 0 \) for \( \sigma^O < \sigma^O \), and \( \sigma^O > \bar{\sigma}^O \). Under storage-dominating portfolio \( (\beta \in \Omega_1) \), \( K^O^* \) is increasing in \( \sigma^O \) because it is linear in \( M_4 \). \( K^I^* \) is decreasing in \( \sigma^O \) when \( \sigma^O < \sigma^O \), because \( M_3 \) is decreasing and \( M_4 \) is increasing. ■

**References**