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Understanding the Benefit of Taxi Ride-Sharing - A Case Study of Singapore

Abstract—This paper assesses the potential benefit of ride-sharing for serving more taxi requests and reducing city traffic flow. It proposes a simple yet practical framework for taxi ride-sharing and scheduling. The proposed framework recommends ride-sharing plans with limited waiting time and limited extra travel time to minimize the discomfort of travellers; it also helps travellers who share ride with other travellers achieve desired taxi fare saving; and helps taxi drivers who serve multiple taxi requests via one single trip to gain more earnings. Therefore, both travellers and drivers are economically motivated to participate in the ride-sharing practice. Comprehensive simulation study is conducted based on real taxi booking data in the city of Singapore and evaluates the effect of various factors in the ride-sharing practice e.g., waiting time, extra travel time, and taxi fare saving. The results demonstrate ride-sharing could serve 20%-25% more taxi booking requests during peak hours with limited number of taxis and hence greatly improve the taxi shortage problem of certain hot spot areas. It also indicates there is around 2-3km travel distance saving per taxi trip on average, which is around 20%-30% of the average ride distance. This finding suggests ride-sharing may have the potential to help reduce the traffic flow, gas consumption, and air pollution of a modern big city.

I. INTRODUCTION

Taxis provide flexible point-to-point service to general public. They are a vital element in a city public transport system functioning in accordance with public demand. In a modern metropolis like Singapore with over 5 million people, there are approximately 28,000 taxis and 99,000 licensed taxi drivers, providing more than 1 million taxi trips daily according to the statistics of Singapore Land Transport Authority (the regulatory agency for land transportation)¹. Despite the large number of taxis, people still suffer from the difficulty of hailing a taxi, especially during peak hours. Figure 1 shows the temporal distribution of taxi booking requests and the booking success rate of a normal working day in Singapore from a very popular taxi booking app GrabTaxi. The figure shows the booking success rate is extremely low during the peak hours, e.g., only around 30% of the bookings are successful during the morning peak at 8 am. On the other hand, despite many unfulfilled taxi requests, vehicles (including both taxis and private cars) on road are traveling with many empty seats. For example, the mean occupancy rate of private car trips in US is only 1.6 person per mile [1].

With the newly emerging concept of “sharing economies”, people start to hope that ride-sharing may represent an important opportunity to improve the usage of empty car seats and satisfy people’s increasing travel demand without increasing the number of vehicles, therefore, reducing traffic congestion, fuel consumption, and air pollution in the city. Traditionally, ride-sharing/carpooling is mainly pre-arranged

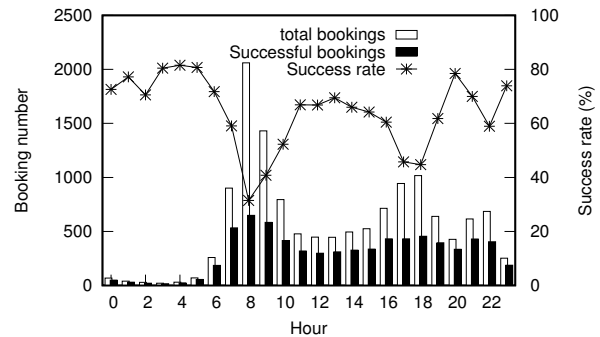


Fig. 1. Average hourly booking statistics on weekdays based on real taxi booking data of Singapore in June 2014.

between a small group of travellers with the same origin and/or destination, e.g., airport. With today’s rapid deployment of information and communication technologies, e.g., geolocating smartphones and mobile network, large-scale and real-time ride-sharing becomes more and more popular. A lot of mobile apps and systems, e.g., Lyft, Avego, and Zimride, are developed for private carpooling, which match private car drivers who have fixed trip schedules with riders who have similar demands. While some recent research works [2], [3] discuss efficient scheduling algorithms for real-time taxi ride-sharing. Traditional taxi service provides traveller(s) an exclusive usage of the taxi, i.e, the taxi cannot pick up other travellers before dropping off the current traveller(s) on board. On the other hand, the shared taxi service can have multiple travellers on board with different origins and/or destinations at the same time. The proposed algorithms try to efficiently assign the most convenient taxis to pick up ad-hoc bookings on their way and plan the optimal route schedule to reduce the total travel distance. They provide service quality constraints to minimize traveller discomfort based on waiting time and service delay caused by detour.

Although the existing works [2], [3] enable ride-sharing to certain degree, they are not very practical because of the following reasons. 1) Some algorithms depend on very complicated index structures with relatively high implementation and maintenance cost. 2) Many algorithms may cause discomfort to travellers because the algorithms may force taxis to constantly change travel route and board new travel companions. 3) Some algorithms are not taxi-driver-friendly because the drivers are required to strictly follow the planned routes, and they need to constantly monitor new coming pick up assignments and the changes of travel routes, which will disturb the driving and cause great danger to both drivers and travellers. 4) The design and evaluation of the algorithms do not emphasize the economy perspective of taxi ride-sharing.

¹<http://www.lta.gov.sg/content/ltaweb/en/publications-and-research.html>

For example, there is no guaranteed taxi fare saving for travellers and profit increase for taxi drivers, which are very important enablers of a successful taxi ride-sharing system.

This paper seeks to understand the outcome of ride-sharing. A simple yet practical taxi ride-sharing framework is proposed, which is very easy to implement and also easy to be embedded into current taxi booking apps. The framework forms a shared trip plan before assigning the trip plan to a taxi. Therefore, the travellers are informed with a fixed trip route/schedule so that they will not feel the discomfort of changing the travel route and travel companions unexpectedly. On the other hand, the taxi driver is given a fixed trip plan before he/she starts the journey. Therefore, the driver can focus on the driving and does not need to worry about changing the travel route which can greatly improve driving safety. In addition, the framework design considers the economy aspect of taxi ride-sharing by offering travellers a constraint on taxi fare saving and drivers a shared trip surcharge. Consequently, travellers can choose to share a trip with others only when the sharing can save them satisfying amount of taxi fare (e.g., 20%). At the same time, the framework entitles taxi drivers to charge a fixed shared trip surcharge (e.g., 10% of the normal taxi fare), which serves as the incentive to motivate them to take shared trip jobs.

Extensive simulation study is performed to investigate the benefit of taxi ride-sharing based on real taxi booking data in the city of Singapore. The real booking data are obtained from GrabTaxi, a very popular taxi booking app in Singapore. To make the study comprehensive, various factors in the taxi ride-sharing practice is considered e.g., waiting time, extra travel time limitation, taxi fare saving, and driver's surcharge of ride-sharing. The findings show that taxi ride-sharing could serve 20%-25% more taxi requests during peak hours with limited number of taxis and effectively address the issue of taxi shortage of certain hot spot areas. The simulation also indicates there is around 2 to 3 km travel distance saving per taxi trip on average, which is around 20%-30% of the average ride distance. This suggests that taxi ride-sharing may have the potential to help reduce the traffic flow, gas consumption, and air pollution.

II. RELATED WORK

Taxi ride-sharing has become more and more popular owing to its potential benefit. There are mainly two classes of taxi ride-sharing studies, namely *static taxi ride-sharing* and *dynamic taxi ride-sharing*. Static taxi ride-sharing requires all taxi trips to be known beforehand. Therefore, a globally optimal sharing plan could be derived to maximise the collective benefits of sharing, e.g., cumulative trip length reduction. For instance, Santi et al. [4] propose a shareability network structure to model the spatial and temporal proximity of the taxi trips and apply a classical graph algorithm to determine the best trip sharing strategy. Their study reveals that a large amount of taxi trips are routinely shareable while keeping the traveler discomfort low in terms of prolonged travel time in big cities (e.g., New York).

Dynamic taxi ride-sharing matches real-time trip requests with running taxis. The trip requests are not required to

be known beforehand, and the taxis are allowed to re-route and pick up new trip requests on the fly. Because of the dynamic nature of the problem, it requires the system to be able to respond quickly while receiving a new trip request. Ma et al. [2] propose a T-Share service model with an efficient spatio-temporal taxi index structure to quickly retrieve a candidate taxi to serve the new trip request which satisfies the pick-up and drop-off delay constraints of the request and meanwhile incurs the minimum additional travel distance. The taxi searching algorithm of T-share follows an incremental approach which expands the search area from the origin and destination of a trip request step by step until the nearest available taxi is found. Shemshadi et al. [3] improve the efficiency of the taxi searching algorithm by replacing the incremental search approach with a decremental search approach. They introduce an economic search margin to limit the expansion of search area and gradually reduce the search area. Later, Huang et al. [5] design effective kinetic tree algorithms for optimal taxi scheduling to serve dynamic requests with guaranteed service quality (i.e. pick-up and drop-off time delay). Given a trip request and a candidate taxi, the algorithm finds the best way to accommodate the trip to the current route plan of the candidate taxi. Both ride sharing algorithms discussed above rely on very complicated index structure which incurs high maintenance cost, and the success of the algorithms requires all taxis in the system to follow the complicated and dynamic travel plan strictly at all times, which is highly unrealistic in the real-world scenarios. In terms of the effect of taxi ride-sharing, both methods only consider the pick-up and drop-off time delay as the impact factors. In the real-world, there could be many other factors which can affect travelers or drivers for adopting taxi ride-sharing application, for example, the travelers' discomfort of changing trip routes and trip companions on the fly, the travelers' incentive in the form of reduced taxi fare, and the taxi drivers' incentive in the form of increased profit. This paper proposes a more practical taxi ride-sharing framework which relies on simple scheduling protocol and easy-to-maintain data structure. It considers more practical factors such as minimizing travelers' discomfort of changing trip routes and trip companions on the fly and also economy factors for both travelers and taxi drivers. Recently, Javier et al. [6] proposed a dynamic trip sharing algorithm based on request-trip-vehicle shareability graph (RTV-graph), which computes all possible combinations of trips that could share a vehicle and vehicles that can serve them. Then, solving an integer linear assignment problem to find the optimal assignment of vehicles to trips. The construction of the RTV-graph incurs a large amount of shortest path computation. To compute the RTV-graph efficiently for dynamic ride-sharing, the work relies on a precomputed static shortest path and travel time look-up table. However, in real-world, the vehicle travel time in an urban road network is hardly static. The framework proposed in this paper does not depend on static shortest path look-up table and could incorporate real-time traffic condition, thus is more practical. Gidofalvi et al. [7] design a trip grouping algorithm for dynamic ride sharing. It assumes trip requests are queued to be scheduled for certain time. Then, it groups "closeby" requests as a rider group

to be served by a taxi using some heuristics, for example, expiration time, estimated combination saving, and greedy grouping. It utilizes space partitioning and parallelisation to optimize the algorithm performance. However, this heuristic-based algorithm does not guarantee any waiting time and cost constraints as previous work does.

There are recent works on applying ride-sharing to shared autonomous vehicles (SAV) [8], [9]. The framework proposed in this paper is readily applicable to scheduling SAV fleets. However, some of the benefits of the framework may not be very obvious comparing with applying to taxi fleets, e.g., the benefit of avoiding frequently updating driving routes on the fly. Unlike human drivers, SAVs can strictly follow the scheduled routes and will not be distracted by frequent route updates during driving. This paper implements another ride-sharing strategy which is similar to the ones proposed for SAV ride-sharing [8] that allows the vehicle to change route at most once while it is serving a trip request to pick up another ad-hoc request. This strategy is compared with the proposed practical ride-sharing framework in the simulation study.

In operation research, early research studies a dial-a-ride problem (DARP) which refers to the matching between one driver and multiple riders with specified trip requests. The main objective is to plan minimum cost driver routes to accommodate as many riders as possible under a set of constraints (e.g. departure and arrival time). Exact dynamic programming algorithms have been developed for this problem [10]. Later, a multiple driver version of this problem is formulated using mixed-integer programming, and a branch-and-cut algorithm is proposed to solve this problem [11]. The traditional DARP considers static scenario where the set of rider requests and drivers are known ahead of time, and optimal solution can only be found for small to medium size instances. This is unrealistic for large-scale and ad-hoc taxi service. Recent research studies dynamic DARP problem [12], [13]. The main idea is to divide the whole time span into a set of short time periods and to solve a static version of the problem based on the drivers and riders that are known at the time of execution. Therefore, these algorithms are not able to provide immediate responses to ad-hoc rider requests. Most of the proposed applications above only consider departure time and arrival time as constraints and/or have not been tested for real-life large-scale datasets. A comprehensive survey of this problem has been presented in [14].

There are some other variants of the general ride-sharing problem. For example, the slugging problem assumes the pick-up and drop-off points are pre-determined by the driver while the riders are required to walk to the meeting point for picking up and then to walk to their destinations from the drop-off point [15], [16]. The commute ride sharing problem mines historical mobility data of daily commuters and groups the commuters with similar travel patterns together to share ride [17], [18]. These problem formulations are different from the taxi ride-sharing problem studied in this paper, and thus are out of the scope of discussion. Interested readers can refer to [19] for a comprehensive survey.

III. PRACTICAL TAXI RIDE-SHARING FRAMEWORK

In this section, a practical dynamic taxi ride-sharing framework is proposed which is simple and respects both travelers' and drivers' convenience and economic interests. The framework involves a *taxi dispatch server*, *travelers* who submit taxi booking requests to the server via a mobile app, and *taxi drivers* who constantly update the server their locations and status and receive job assignments from the server. The framework makes sure that both the traveler and the taxi driver are aware of the (share) trip route and schedule before the start of a trip to minimize traveler's discomfort and to reduce potential inconvenience and risk of the driver. Therefore, the framework maintains a cache at the server side which records the incoming taxi booking requests for a short while. It forms valid shared trip plans among the cached booking requests based on certain constraints which will be explained later. When a shared trip plan is formed, the server informs the travelers involved and try to recommend a suitable taxi to serve the shared trip plan. Once the travelers and the driver agree on the shared trip plan, the driver goes to pick up the travelers based on the fixed schedule, and there is no change of route or schedule during the trip.

Five constraints are defined when forming a valid shared trip plan with respect to the service quality (e.g., waiting time, departure/arrival delay, taxi capacity) and economical interests of travelers and taxi drivers (e.g., taxi fare saving and shared trip surcharge).

- **Waiting time** defines the maximum time allowed between the time when a traveler submits a request and the time when a taxi is assigned to serve the trip. It consists of the time that a traveler's request being cached in the system for the formation of the shared trip plan, and the time for a taxi being assigned to serve the trip. This constraint guarantees a fast response of the taxi ride-sharing system to a traveler's taxi booking request.
- **Departure delay constraint** defines the maximum travel time of the taxi from its current location to the pick-up location of the traveler after a taxi is being assigned to serve the trip. This constraint guarantees that each traveler can enjoy a taxi service within a short time period after the system confirms that a taxi has been assigned to serve him/her.
- **Arrival delay constraint** is the maximum time caused by the detour as a shared trip needs to serve two requests that is acceptable to the travelers. It refers to the extra time that a traveler has to spend on the shared trip route, as compared with normal trip route without sharing. This constraint prevents the system from assigning unreasonable shared trip plan which takes long detour and causes unacceptable delay to the travelers.
- **Capacity constraint** defines the maximum number of passengers per trip request. This constraint avoids forming shared trips that exceed taxi capacity.
- **Share trip surcharge** refers to the extra surcharge collected for each shared trip, represented in the form of a fixed percentage of the total taxi fare (e.g., 10% of the total taxi fare). This extra amount of fare serves as the

incentive to encourage taxi drivers to serve shared trips as shared trips are more economic and drivers earn more for each kilometer they drive, as compared with the normal taxi trips that only serve one request.

- **Taxi fare saving rate constraint** refers to the minimum saving a traveler expects to have via sharing his/her trip with another traveler, again represented in the form of a fixed percentage of the total taxi fare (e.g., 15% of the total taxi fare). This constraint guarantees that travelers actually pay less by sharing trips with others, and hence it serves as an incentive, in addition to higher taxi booking success rate, to encourage more travelers to participate into taxi ride-sharing.

A. Preliminaries

The proposed taxi ride-sharing framework is built based on a road network $G = \{V, E, W\}$. The vertex set V represents road junctions, and the edge set E represents road segments. Each edge $e(v_i, v_j) \in E$ ($v_i, v_j \in V$) is associated with a weight $W(v_i, v_j) \in W$ which is the travel cost along the edge e . In the framework, the travel cost is set to the travel time. However, the framework is readily applicable to the case where the travel cost is measured by distance. Given two vertices s and e on the road network, a path $p(s, e)$ from s to e is an edge sequence $\langle e(v_0, v_1), e(v_1, v_2), \dots, e(v_{k-1}, v_k) \rangle$ where $s = v_0$, $e = v_k$ and $\forall e(v_{i-1}, v_i) \in p(s, e)$, $e(v_{i-1}, v_i) \in E$. The travel cost of the path $p(s, e)$ is denoted as $W(p(s, e)) = \sum_{i=0}^{k-1} W(v_i, v_{i+1})$. For a given pair of vertices s and e , their shortest path, denoted as $sp(s, e)$, refers to the path from s to e with minimum cost, $sp(s, e) = \arg \min_{p(s, e)} W(p(s, e))$, and $W(sp(s, e))$ refers to the travel cost of the shortest path from s to e . A meter function $M(p(s, e))$ is used to calculate the normal taxi fare of a trip from s to e on the road network based on the length of path $p(s, e)$.

Definition 3.1 (Taxi Booking Request): A taxi booking request $r = \{s, e, t, n, w, dd, ad, \delta\}$ with respect to a road network $G = \{V, E, W\}$ is defined by a pick-up point $s \in V$, a drop-off point $e \in V^2$, a submit time t , a passenger number n , and four constraint parameters including a maximum waiting time w , a maximum departure delay time dd , a maximum arrival delay time ad , and a taxi fare saving rate δ . \square

It is assumed all requests r_i s that have not yet been served are queued at a cache C maintained by the server, based on ascending order of their submitted time $r_i.t$. A taxi booking request r_i can be satisfied by either a shared-trip or a single trip (normal taxi trip that only serves r_i). The taxi ride-sharing framework gives shared trip a higher priority, and hence the server will try to satisfy a booking request r_i using a shared trip. Given a request r_i , the waiting time w_i is partitioned into two parts denoted as $r_i.w_1$ and $r_i.w_2$ with $r_i.w_1 + r_i.w_2 = r_i.w$. It is assumed the server has up to $r_i.w_1$ time to find another taxi booking request $r_j \in C$ that can be shared with r_i ; and has up to $r_i.w_2$ time to locate a suitable taxi to serve

r_i by either a shared trip or a single trip, which are formalised in Definition 3.2 and Definition 3.3 respectively.

Note if the server fails to locate a suitable request r_j that can share its trip with r_i within $r_i.w_1$, r_i will take a single trip. To be more specific, the longer the $r_i.w_1$ is, the higher the chance that a trip r_j that can share the trip with r_i can be found; on the other hand, the longer the $r_i.w_2$ is, the higher the chance that a free taxi can be identified to serve r_i (and maybe r_j together). A system parameter θ in the range of $(0, 1)$ is introduced which is the ratio of $r_i.w_1$ to $r_i.w$, i.e., $r_i.w_1 = \theta \cdot r_i.w$. By default, θ is set to 0.5, while its value can be tuned.

Definition 3.2 (Single Trip Plan): A valid single trip plan $T^{sin}(t_f, n, p = \langle v_1, v_2 \rangle, e, t_{exp}, d)$ w.r.t. a booking request r contains a formation time t_f when the single trip plan is formed, a passenger number n , a path p from v_1 to v_2 with $v_1 = r.s$ to $v_2 = r.e$ to serve r , an expiration time $t_{exp} = r.t + r.w$, which is the latest time that a taxi shall be located and assigned to serve the trip plan T^{sin} , and a departure delay constraint $d = r.dd$. Note only taxis that can reach v_1 by d are considered as qualified taxis which might be assigned to serve T^{sin} . \square

For a single trip, there is no need to consider the arrival delay constraint ad and taxi fare saving rate constraint γ as there is no taxi ride-sharing involved. In other words, a single trip does not take any detour, and the traveler only needs to pay the normal taxi fare $M(p(s, e))$ based on the path $p(s, e)$ taken by the taxi. Without loss of generality, this paper assumes the taxi driver takes the shortest path, i.e., $T^{sin}.p(s, e) = sp(s, e)$.

Definition 3.3 (Shared Trip Plan):³ A valid shared trip plan $T^{sha}(t_f, n, p = \langle v_1, v_2, v_3, v_4 \rangle, t_{exp}, d)$ with respect to two booking requests r_1 and r_2 , contains a formation time t_f when the shared trip plan is formed, a passenger number $n = r_1.n + r_2.n$, an ordered route p that the taxi driver will take to serve the requests, passing by locations v_1, v_2, v_3 , and v_4 in order, an expiration time t_{exp} that is the latest time a taxi needs to be assigned to serve T^{sha} , and a departure delay time constraint dd . To be more specific, given $v_1, v_2 \in \{r_1.s, r_2.s\}$ and $v_3, v_4 \in \{r_1.e, r_2.e\}$ which means the shared trip needs to reach those two pickup locations first before sending the travelers to the drop-off locations to guarantee r_1 and r_2 do share part of the journey. Without loss of generality, this paper assumes the taxi driver always takes the shortest path, i.e., $\forall v_i, v_{i+1} \in T^{sha}.p$, $p(v_i, v_{i+1}) = sp(v_i, v_{i+1})$, i.e., $T^{sha}.p = sp(v_1, v_2) + sp(v_2, v_3) + sp(v_3, v_4)$. The expiration time t_{exp} is set to $\min(r_1.t + r_1.w, r_2.t + r_2.w)$, and the departure delay d is set to $\min(r_1.dd, r_2.dd) - W(p)$. \square

A valid shared trip plan needs to satisfy the following constraints.

- **Location order constraint.** The condition $v_1, v_2 \in \{r_1.s, r_2.s\}$ and $v_3, v_4 \in \{r_1.e, r_2.e\}$ in above definition is to guarantee that i) the taxi route p serves both requests; i) for booking request r_1 or r_2 , the planned shared trip reaches its pick-up location before the drop-off

²To simplify the discussion, this paper assumes s and e are located at vertices of V , but it can be simply extended to support cases where s or e is located at any edge $e \in E$ of the road network.

³To simplify the discussion, this paper assumes a shared trip plan is to serve two taxi booking requests, while the definition can be easily generalized to serve multiple requests.

location; and iii) there is at least one part of the journey (e.g., $sp(v_2, v_3)$) that the planned shared trip serves both requests (i.e., the travelers of r_1 and travelers of r_2 are in taxi simultaneously) to make sure this is indeed a shared trip.

- **Departure delay constraint.** The constraint $d = \min(r_1.dd, r_2.dd) - W(p)$ in above definition is to guarantee that the planned shared trip can reach $r_1.s$ by $r_1.dd$ and reach $r_2.s$ by $r_2.dd$.
- **Arrival delay constraint.** Without loss of generality, this paper assumes $r_1.e = v_3$ and $r_2.e = v_4$. The planned shared trip will reach r_1 's drop-off location latest by $d + W(p) - W(sp(v_3, v_4))$, and reach r_2 's drop-off location latest by $d + W(p)$. The arrival delay constraint is satisfied iff $r_1.ad \leq d + W(p) - W(sp(v_3, v_4))$ and $r_2.ad \leq d + W(p)$ to make sure the detour taken by the shared trip is still able to send the travelers to their drop-off locations by the specified arrival delay.
- **Capacity constraint.** The constraint $n \leq c_{max}$ is to guarantee that total number of passengers of the planned shared trip does not exceed the maximum taxi capacity c_{max} .
- **Taxi fare constraint.** Given a planned shared trip T^{sha} , the taxi fare f is derived based on $M(T^{sha}.p) \times (1 + \alpha)$. Here, $M(T^{sha}.p)$ captures the total normal taxi fare based on the travel distance of the shared trip, and α stands for share trip surcharge. The fare f will be distributed between r_1 and r_2 according to their respective normal taxi fare without sharing, i.e., $f_1 = \frac{M(sp(r_1.s, r_1.e)) \cdot f}{M(sp(r_1.s, r_1.e)) + M(sp(r_2.s, r_2.e))}$ is the amount of fare paid by r_1 , and $f_2 = \frac{M(sp(r_2.s, r_2.e)) \cdot f}{M(sp(r_1.s, r_1.e)) + M(sp(r_2.s, r_2.e))}$ is the amount of fare paid by r_2 . Only if $f_1 \leq M(sp(r_1.s, r_1.e)) \times (1 - r_1.\delta)$ and $f_2 \leq M(sp(r_2.s, r_2.e)) \times (1 - r_2.\delta)$, the taxi fare constraint is satisfied to make sure both r_1 and r_2 can save the specified minimum amount of taxi fare and hence T^{sha} is valid.

Given a formed shared trip plan, all the taxis that can reach the first pick up location v_1 within the departure delay time d and with a capacity $c \geq n$ are qualified taxis, and any of them could be assigned to serve the shared trip.

B. Framework Overview

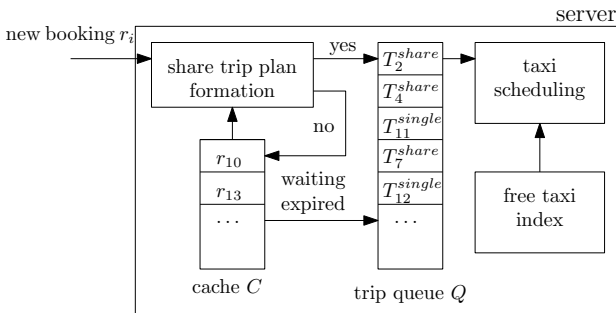


Fig. 2. The workflow of the practical taxi ride-sharing framework.

Figure 2 visualizes the workflow of the proposed taxi ride-sharing framework. When a traveler submits a new booking request r_i , r_i first goes through the shared trip formation process. The process browses all the active booking requests pending in the server cache C to find another booking request $r_j \in C$ that can form a valid shared trip plan with r_i . Recall that r_i and r_j can form a valid shared trip plan only if all the five constraints discussed previously are satisfied, including location order constraint, departure delay constraint, arrival delay constraint, capacity constraint, and taxi fare constraint. If such r_j could be found, the system generates a shared trip plan T_x^{sha} with regard to r_i and r_j , inserts T_x^{sha} into a trip queue Q waiting for the scheduling process to assign a free taxi to serve this trip, and removes r_j from the cache. Otherwise, r_i will be inserted into the cache C , under the assumption that r_i might be able to form a valid shared trip with another request r_l that will be submitted in the near future. As mentioned above, each request r_i specifies a maximum waiting time w . It will only spend up to $\theta \cdot r_i.w$ time in the cache C . In other words, if r_i cannot form a valid shared trip plan with another request r_j within $\theta \cdot r_i.w$, the server will form a single trip plan T_y^{sin} for r_i , insert T_y^{sin} into the trip queue Q , and remove r_i from C .

To sum up, this framework emphasise on system simplicity and the capability of forming valid shared trip plans in real-time, rather than finding the optimal solutions. When generating the shared trip plan, a greedy strategy is used to find the locally best booking request in the system cache for the new incoming booking, but not try to find a globally optimal share plans for all cached bookings. The locally best booking request means the booking request which can form a shared trip plan with the new incoming booking with the lowest travel cost. This will reduce the computational complexity and improve the system response time. When generating the shared trip plan, this framework also does not consider the taxi status in the system (e.g., the framework does not consider generating the shared trip plan which is easier to find an available taxi nearby based on the current location of the taxis in the system) for the same reason. Because in a real system, the simplicity and short response time is often more important than optimality. The implementation detail of the framework is provided in supplementary materials.

IV. EXPERIMENTS

In this section, experiments are conducted to study the benefit of the proposed taxi ride-sharing strategy based on real-world taxi booking data in the city of Singapore. The effect of different constraints in taxi booking service is evaluated, including *available taxi number*, *waiting time*, *departure delay*, *arrival delay*, *taxi ride-sharing surcharge*, and *taxi fare saving constraint*⁴. The results indicate taxi ride-sharing could significantly improve taxi booking success rate during peak

⁴Because the passenger number information is not available in the booking data, the capacity constraint cannot be implemented in the simulation. However, in the real world, travelers who use the taxi ride sharing service usually travel alone or not with many others. Therefore, combining two bookings normally will not exceed the maximum capacity of a normal taxi (i.e., 4).

hours when the demand is high, reduce travelers' travel cost, and improve taxi drivers' profit.

A. Experiment Setup

Three simulation programmes are implemented including the proposed taxi ride-sharing framework, denoted as SHARE₁, a variant taxi ride-sharing algorithm, demoted as SHARE₂, using a similar strategy used for SAV ride-sharing [8], and a normal taxi service strategy, denoted as NORMAL. SHARE₂ uses the same constraints as SHARE₁, but it does not cache booking requests to form shared trip plans beforehand. A booking request is immediately processed for taxi scheduling, and an occupied taxi can change route on-the-fly to pick up another qualifying booking to share the trip.

The experiment uses real taxi booking requests of all working days from 1 June to 30 June, 2014 in Singapore in total of 22 days from GrabTaxi, a popular taxi booking app in the market, as the input of the simulation. It generates the road network based on the real Singapore map retrieved from OpenStreetMap. The road network contains 49,593 vertices and 109,251 edges. It obtains the travel cost of the edges by analysing the real taxi trajectory data collected in a duration of six months. For each edge, the average taxi travel time for every hour of a day is calculated, and this real travel time information is used through out the experiment.

In order to make the simulation more realistic, the programmes simulate the real taxi movement during their free time in the experiments. Initially, the taxis are distributed around the Singapore road network based on the real taxi distribution at the time of study which is extracted from the real taxi trajectory data. The roaming strategy proposed by [20] is adopted to simulate a taxi's movement when it is not assigned to serve any booking. The roaming strategy divides the road network into mutually exclusive zones (e.g., grids), and assumes the free taxi makes probabilistic moves toward different zones based on their relative attractiveness. The attractiveness of a zone i is defined as $\frac{b_i}{W(sp(i,p))^2}$, where b_i is the number of bookings issued from zone i in the hour of the study time, p is the current taxi location, and $W(sp(p,i))$ is the travel cost of the shortest path from p to the center point of i . The simulation assumes the taxi fare is estimated based on the travel distance and the trip start time. It applies the flag-down fare, distance rate, and time-based surcharge rate from the Land Transport Authority of Singapore (<http://www.lta.gov.sg>), and does not consider other charge such as waiting time rate and city area surcharge.

To evaluate the benefit of taxi ride-sharing, the following metrics, *taxi booking success rate*, *average travel distance saving per trip*, *average taxi fare*, and *average driver profit*, are considered. Table I lists the parameters to be evaluated with their default values and value ranges.

Simulation results are reported in the following subsections. Simulation is first conducted for one day time period to understand the general pattern of the effect of taxi ride-sharing at different time of a day, as reported in Section IV-B. Then, subsequent simulation focuses on the peak hours of a day, when the taxi shortage problem becomes the most severe and

TABLE I
PARAMETER SETTINGS. THE VALUES HIGHLIGHTED BY BOLD ARE THE DEFAULT VALUES.

Parameter	Notation	Value Range
Taxi number	n	{400, 500 , 600, 700, 800, 900, 1000}
Waiting time (sec.)	w	{60, 180, 300, 600 , 900, 1200, 1500}
Departure delay (sec.)	dd	{300, 600 , 900, 1200, 1500}
Arrival delay (sec.)	ad	{300, 600 , 900, 1200, 1500}
taxi ride-sharing surcharge	δ	{0.05, 0.1, 0.2 , 0.3, 0.4, 0.5}
Taxi fare saving rate	α	{0, 0.1, 0.2 , 0.3, 0.4}

the benefit of taxi ride-sharing is the most significant, and study the main benefits of taxi ride-sharing under different parameter settings, as reported in Section IV-C. Every simulation is repeated by 5 times, and the 5-time average is reported as results.

B. One-day simulation results

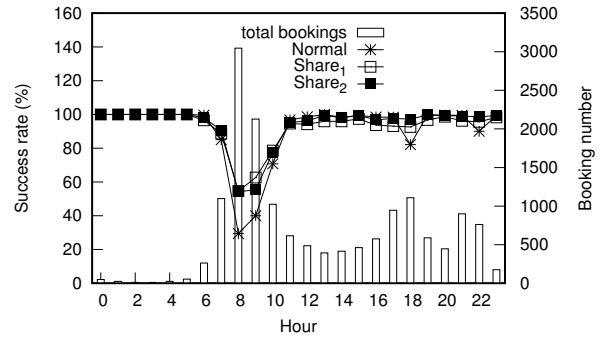


Fig. 3. Hourly booking success rate (3 June 2014).

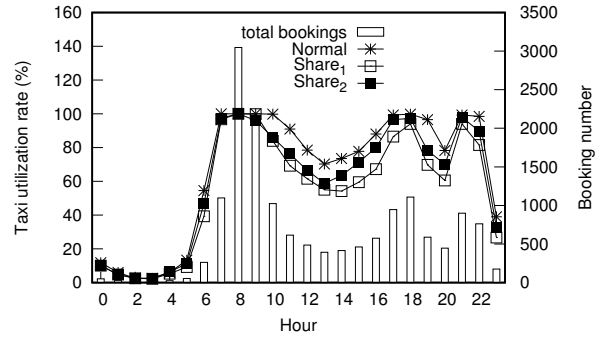


Fig. 4. Taxi utilised for serving at least one booking during each hour of a day.

Figure 3 demonstrates the hourly booking success rate of the three simulations of a normal working day in June 2014⁵. The default parameter setting is applied.

The figure shows the advantage of ride-sharing is more significant when the taxi demand is very high, especially during the peak hours (e.g., 8am, 9am, and 6pm). For example during the morning peak, NORMAL can only serve 30%-40%

⁵The simulation results based on other working days follow a very similar pattern and hence are omitted to avoid redundancy.

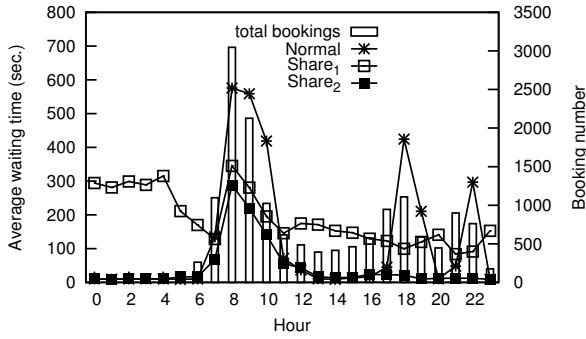


Fig. 5. The average waiting time of the served bookings during each hour of a day.

of the bookings, while $SHARE_1$ and $SHARE_2$ could serve 55%-62% of the bookings. On the other hand, during non-peak hours when the demand on taxi is not high, all three strategies can almost achieve 100% success rate. Consequently, there is no room for taxi ride-sharing to improve. It is also noticeable that the performance in terms of success rate of $SHARE_1$ and $SHARE_2$ is comparable.

To better understand the taxi demand and taxi supply during the non-peak hours, the hourly taxi utilization rate is demonstrated by Figure 4. The taxi utilization rate is the percentage of taxis that have been utilized to serve at least one booking during each hour. As shown by the figure, during most non-peak hours (e.g., 10am-4pm), the taxi utilization rate under $SHARE_1$ and $SHARE_2$ is lower than that under NORMAL. This observation demonstrates that the taxi ride-sharing strategies could serve almost the same number of booking requests using a smaller number of taxis, which contributes directly to the improvement of traffic congestion and air pollution. The figure also shows the taxi utilization rate of the $SHARE_1$ is slightly lower than that of $SHARE_2$. During peak hours when the taxi demand is high, the taxi utilization rate of all the three strategies is close to 100%. This means the demand during the peak hours is very high and it exceeds the taxi fleet capacity. In order to support the large number of taxi requests without increasing the number of taxis, taxi ride-sharing is an almost free solution.

The simulation also evaluates the average actual waiting time of the served bookings under three different strategies. Here, the actual waiting time of a booking refers to the duration from the time a booking is submitted to the time a taxi is assigned to serve the booking. It is found during the peak hours, the average actual waiting time under NORMAL is much longer than the average waiting time under $SHARE_1$ and $SHARE_2$. It indicates when the taxi demand is so high that exceeds the taxi fleet capacity, it is very hard for a traveler to get a taxi and even when the traveler is lucky to get a taxi, he/she has to wait long time for the taxi to come and pick her up. On the other hand, when $SHARE_1$ and $SHARE_2$ are applied, the travelers' actual waiting time could be greatly reduced. When the two different ride-sharing strategies are compared, it could be observed that the average actual waiting time under $SHARE_1$ is longer than that under $SHARE_2$, especially during the non-peak hour. This is because the original taxi ride-

sharing strategy $SHARE_1$ caches the submitted bookings in a cache C for a while (e.g., 300 sec.) to generate trip plans, and then schedules taxis for formed trip plans. Therefore, there is a time gap between the booking submission and the scheduling, which extends the actual waiting time. During the non-peak hours, when the demand is low, a booking in most cases has to be cached for longer time before it forms a valid shared trip plan with another booking or forms a single trip plan eventually, which results in longer waiting time. Different from $SHARE_1$, the variant taxi ride-sharing strategy $SHARE_2$ performs the taxi scheduling directly.

To sum up, this set of simulation demonstrates taxi ride-sharing have the potential to improve taxi service quality by improving the taxi booking success rate, reducing the needed number of taxis running on the road, and reducing the travelers' waiting time. The advantage of taxi ride-sharing is more prominent during the peak-hours when the taxi demand is much higher and the taxi shortage problem is more severe. In addition, the original taxi ride-sharing strategy $SHARE_1$ relieves travelers from the inconvenience of changing the travel route and travel companion during their trip. However, it may result in a slightly longer waiting time, as compared with the variant ride-sharing strategy $SHARE_2$. On the other hand, $SHARE_2$ sacrifices a little on the traveler's convenience by allowing taxis to change trip routes for at most once to gain a better waiting time performance.

C. Peak-hour simulation results

As demonstrated by the previous subsection, the benefit of taxi ride-sharing is more prominent during the peak hours of a day when the demand on taxis is very high. This subsection further investigates the benefit of taxi ride-sharing by focusing on the peak hours. More simulation is conducted based on the booking data corresponding to the morning peak hours (i.e., 8am-9am), and the taxi booking success rate, average travel distance saving per trip, average taxi fare, and average driver profit with different parameter settings are examined.

The impact of number of taxis running. The experiment increases the taxi number from 400 to 1000 and keep all other parameters to their defaults, with the results shown in Figure 6. First, It is easily observed that the booking success rate increases as the taxi number increases, and the taxi ride-sharing strategies generally serve 20% more booking requests than the normal non-sharing strategy NORMAL (see Figure 6a). Second, the average travel distance saved per trip by applying taxi ride-sharing is investigated. The result in Figure 6b shows $SHARE_1$ can reduce about 2-3km travel distance per trip on average (e.g., about 20%–30% of average travel distance per trip), and $SHARE_2$ saves slightly more travel distance per trip. Third, the average taxi fare the travelers need to pay per trip is studied. The results show the taxi ride-sharing strategies $SHARE_1$ and $SHARE_2$ are able to save travelers 4 to 5 dollars per trip on average, which is around 30% of the normal taxi fare travelers usually need to pay. Finally, the average taxi driver profit is investigated. As shown by the result in Figure 6d, with the increase of taxi numbers, the

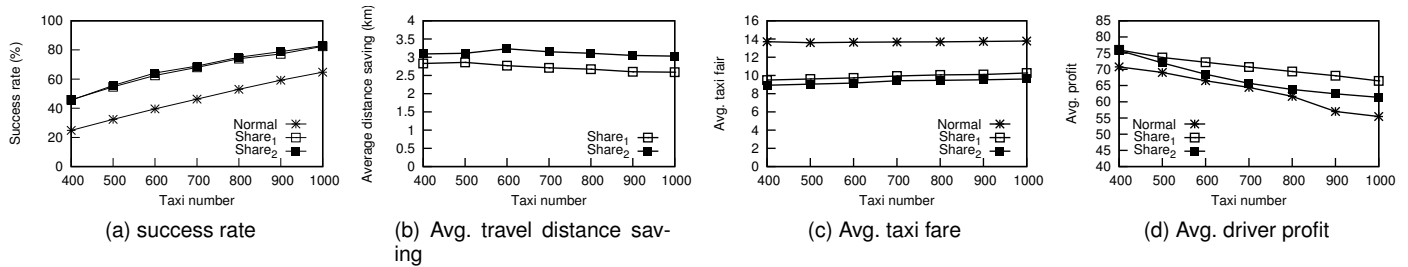


Fig. 6. Simulation results with increasing taxi number.

average taxi driver's profit decreases. However, the taxi ride-sharing strategies allow the taxi drivers to earn more than the normal non-sharing strategy. Generally, it could be observed from Figure 6c and Figure 6d that SHARE₁ makes the travelers pay a slightly higher taxi fare, and hence allows the taxi drivers to earn more profit, as compared with SHARE₂.

The impact of the waiting time constraint. The experiment increases the waiting time from 60 seconds to 1500 seconds and keep all other parameters as default. As shown by the simulation results in Figure 7, the waiting time constraint parameter has a very limited impact on the simulation results of both NORMAL and SHARE₂ because these two strategies do not keep the newly submitted booking request in the cache C for pairing with another booking request for taxi ride-sharing. However, the waiting time constraint has a more significant influence on the original taxi ride-sharing strategy SHARE₁ as the waiting time constraint directly affects the duration that a newly submitted booking request needs to wait in C for taxi ride-sharing, thus affecting the chance of the formation of shared trip plans. For SHARE₁, with the waiting time constraint increases, the booking success rate increases (see Figure 7a), the average travel distance reduced per trip also increases (see Figure 7b), while the average taxi fare per trip paid by the travelers and the taxi drivers' profit decrease.

The impact of the departure delay constraint and the arrival delay constraint. The experiment increases the two delay constraints from 300 seconds to 1500 seconds respectively and set the rest parameters to their defaults. The results are shown in Figure 8 and Figure 9 respectively. Figure 8 demonstrates with the increase of departure delay constraint, the success rate increases until the departure delay constraint reaches 900 seconds. After that, with the further increase of the departure delay, the success rate starts to decrease. The other metrics basically follow the same pattern. This is because a larger departure delay constraint indicates that the traveler is willing to wait for a longer duration for an available taxi to drive to the specified pick-up location. Therefore, when the system performs taxi scheduling, it can search taxis in a larger range, which increases the chance of finding a free taxi to serve a booking, thus increasing the booking success rate. However, if the departure delay constraint is too large, the system is more likely to assign distant taxis to serve booking requests. As a result, the taxis will waste more time on the way to reach the travelers' pickup locations, which reduces their

chance of serving more future booking requests. This is the reason that the success rate decreases with very large departure delay constraint. However, there is no obvious patterns while changing the arrival delay constraint. Therefore, the departure delay that the travelers can tolerant has a stronger impact on the taxi dispatch system, as compared with arrival delay.

The impact of the taxi fare saving rate constraint. The experiment increases the taxi fare saving rate constraint from 0% to 40% and set all other parameters as default. The simulation results are shown in Figure 10. As the taxi fare saving rate is only applicable to the taxi ride-sharing strategies SHARE₁ and SHARE₂, it does not affect the normal non-sharing strategy NORMAL. A higher taxi fare saving rate constraint means the travelers expect a higher saving by using taxi ride-sharing service, and hence the shared part of their trips is expected to be longer. Therefore, it is harder for the system to match two booking requests to form a valid shared trip plan. Consistent with the expectation, the booking success rate, travel distance saving, and taxi drivers' profit all drop significantly when the taxi fare saving rate constraint increases.

The impact of increasing the taxi ride-sharing surcharge parameter. The experiment increases the taxi ride-sharing surcharge from 5% to 50% and set all other parameters to the default setting. The results are demonstrated in Figure 11. As the taxi ride-sharing surcharge is only applicable to the taxi ride-sharing strategies, it does not affect the normal non-sharing strategy NORMAL. As shown in the figure, this parameter has a direct impact on the average taxi fare and the average profit of the drivers. As the taxi drivers charge more for taxi ride-sharing, the travelers' taxi fare on average increases as well as the taxi drivers' profit (see Figure 11c and Figure 11d). When the taxi ride-sharing surcharge is very high (i.e., more than 40%), the success rate and the average travel distance saving metrics drop (see Figure 11a and Figure 11b). This is because when the taxi ride-sharing surcharge is larger, it becomes more difficult to find two booking requests that can form a valid shared trip satisfying the travelers' taxi fare saving rate constraint.

To sum up, in this subsection, comprehensive experiments are conducted to investigate the detailed benefit of the taxi ride-sharing strategies during the peak hours of a day. The results find generally taxi sharing strategies (i.e., SHARE₁ and SHARE₂) could achieve a much higher booking success rate and save more travel distance of the taxi trips, as compared

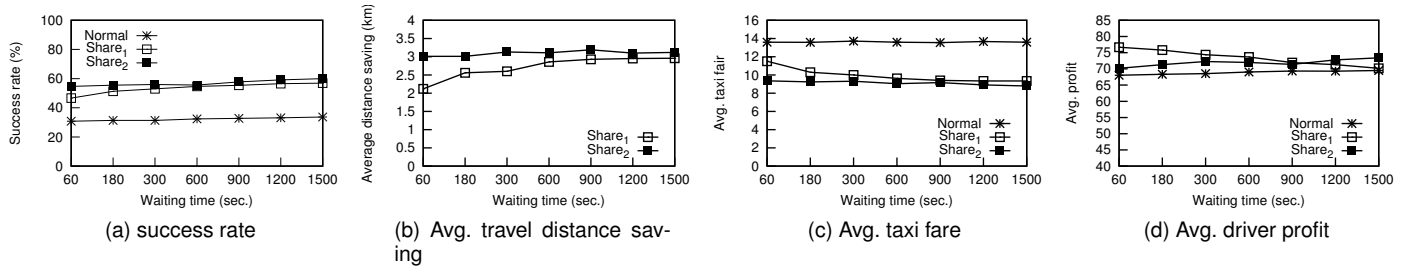


Fig. 7. Simulation results with increasing waiting time constraint.

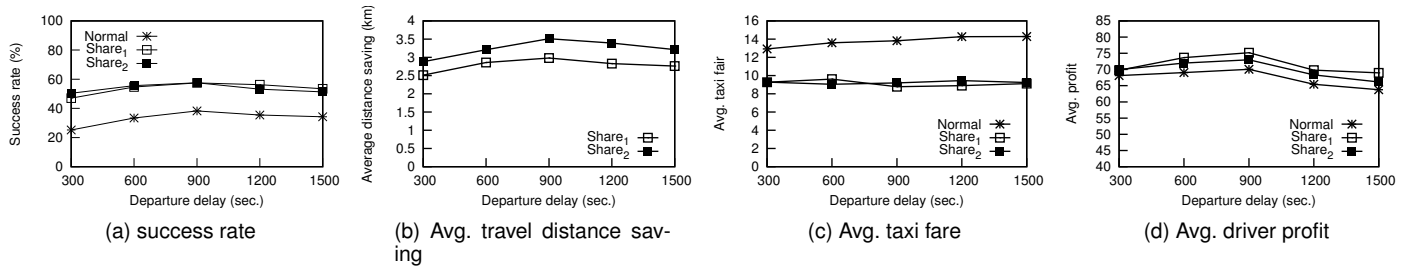


Fig. 8. Simulation results with increasing departure delay constraint.

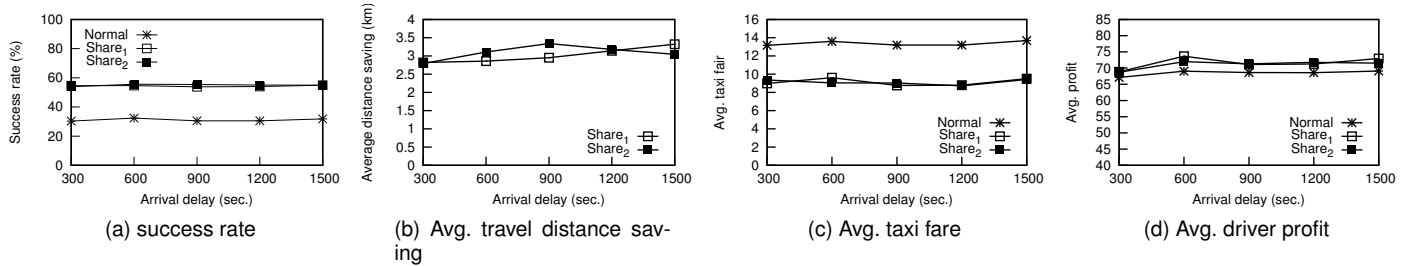


Fig. 9. Simulation results with increasing arrival delay constraint.

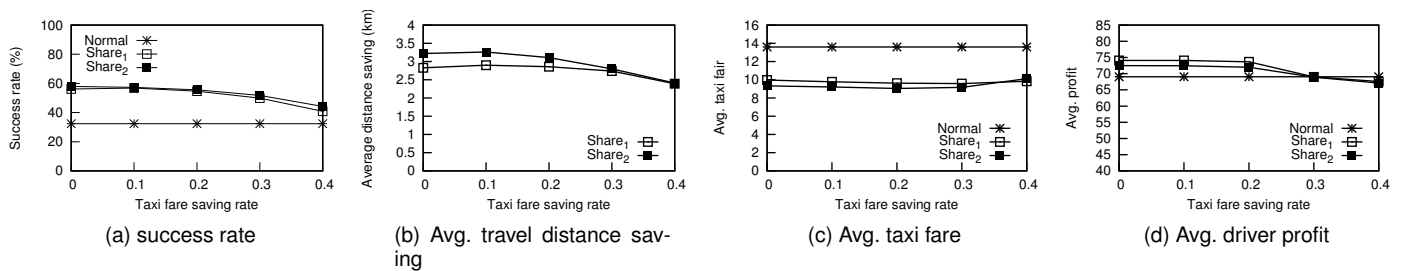


Fig. 10. Simulation results with increasing taxi fare saving rate constraint.

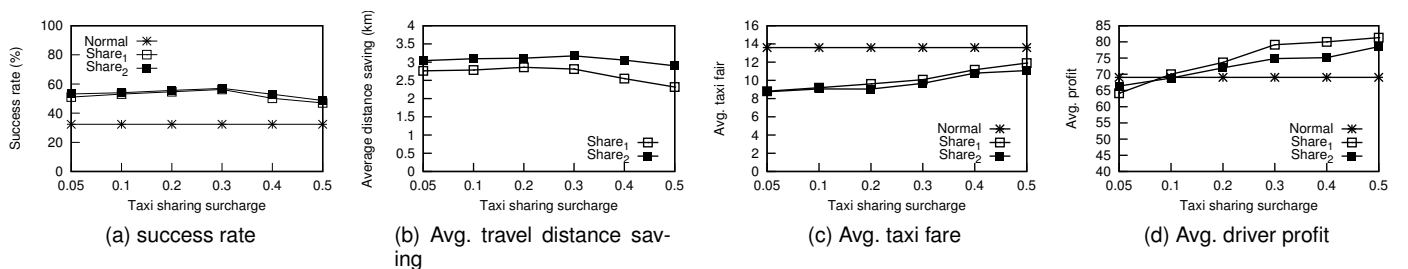


Fig. 11. Simulation results with increasing taxi ride-sharing surcharge constraint.

with the normal non-sharing strategy NORMAL. They enable the travelers to pay less taxi fare per trip and allow the taxi drivers to earn more profit for each kilometre they drive. The performances of SHARE₁ and SHARE₂ are comparable, but SHARE₁ has the additional advantage of releasing the drivers and the travelers from the discomfort of on-the-fly route changes.

V. CONCLUSION

This paper seeks to understand the potential benefit of ride-sharing in a big urban city with very heavy transportation demand. A practical taxi ride-sharing strategy is proposed which fully considers travelers' comfort, drivers' convenience, and ease of implementation and maintenance. The proposed strategy also takes the economic factors into consideration. It motivates drivers to serve taxi ride-sharing trips by offering more profit, and encourages travelers to share taxis with others by enjoying attractive taxi fare discount.

Extensive simulation study is conducted based on real taxi booking data of Singapore city. The results indicate the effect of taxi ride-sharing is more prominent during peak hours when the taxi demand is very high and exceeds the taxi fleet capacity. Taxi ride-sharing enables the taxi fleet, to serve 20%-25% more booking requests and greatly reduces the traveler's waiting time during the peak hours. Therefore, it is highly recommend the taxi company to apply the taxi ride-sharing service during the peak hours of the city transportation. On the other hand, applying ride-sharing during non-peak hours could reduce the taxi utilization rate, which may cause some taxi drivers to loose their jobs.

In the future, simulation study based on the taxi booking data of other big cities is expected to make this study more general. In addition, the authors plan to find efficient optimization algorithms to maximize the global performance of the ride-sharing.

REFERENCES

- [1] T. Hodges and U. S. F. T. Administration, *Public Transportation's Role in Responding to Climate Change*. U.S. Federal Transit Administration, 2010. [Online]. Available: https://books.google.com.sg/books?id=JG_PZkRmv4cC
- [2] S. Ma, Y. Zheng, and O. Wolfson, "T-share: A large-scale dynamic taxi ridesharing service," in *Data Engineering (ICDE), 2013 IEEE 29th International Conference on*, April 2013, pp. 410–421.
- [3] A. Shemshadi, Q. Z. Sheng, and W. E. Zhang, "A decremental search approach for large scale dynamic ridesharing," in *Web Information Systems Engineering - WISE 2014 - 15th International Conference, Thessaloniki, Greece, October 12-14, 2014, Proceedings, Part I*, 2014, pp. 202–217.
- [4] P. Santi, G. Resta, M. Szell, S. Sobolevsky, S. H. Strogatz, and C. Ratti, "Quantifying the benefits of vehicle pooling with shareability networks," *Proceedings of the National Academy of Sciences*, vol. 111, no. 37, pp. 13 290–13 294, 2014. [Online]. Available: <http://www.pnas.org/content/111/37/13290.abstract>
- [5] Y. Huang, F. Bastani, R. Jin, and X. S. Wang, "Large scale real-time ridesharing with service guarantee on road networks," *Proc. VLDB Endow.*, vol. 7, no. 14, pp. 2017–2028, Oct. 2014. [Online]. Available: <http://dx.doi.org/10.14778/2733085.2733106>
- [6] J. Alonso-Mora, S. Samaranyake, A. Wallar, E. Frazzoli, and D. Rus, "On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment," *Proceedings of the National Academy of Sciences*, vol. 114, no. 3, pp. 462–467, 2017.
- [7] G. Gidofalvi, T. B. Pedersen, T. Risch, and E. Zeitler, "Highly scalable trip grouping for large-scale collective transportation systems," in *Proceedings of the 11th International Conference on Extending Database Technology: Advances in Database Technology*, ser. EDBT '08. New York, NY, USA: ACM, 2008, pp. 678–689. [Online]. Available: <http://doi.acm.org/10.1145/1353343.1353425>
- [8] D. J. Fagnant and K. M. Kockelman, "Dynamic ride-sharing and fleet sizing for a system of shared autonomous vehicles in austin, texas," *Transportation*, Aug 2016.
- [9] M. W. Levin, K. M. Kockelman, S. D. Boyles, and T. Li, "A general framework for modeling shared autonomous vehicles with dynamic network-loading and dynamic ride-sharing application," *Computers, Environment and Urban Systems*, vol. 64, pp. 373 – 383, 2017.
- [10] H. N. Psaraftis, "An Exact Algorithm for the Single Vehicle Many-to-Many Dial-A-Ride Problem with Time Windows," *TRANSPORTATION SCIENCE*, vol. 17, no. 3, pp. 351–357, Aug. 1983. [Online]. Available: <http://dx.doi.org/10.1287/trsc.17.3.351>
- [11] J.-F. Cordeau, "A branch-and-cut algorithm for the dial-a-ride problem," *Oper. Res.*, vol. 54, no. 3, pp. 573–586, May 2006. [Online]. Available: <http://dx.doi.org/10.1287/opre.1060.0283>
- [12] N. A. Agatz, A. L. Erera, M. W. Savelsbergh, and X. Wang, "Dynamic ride-sharing: A simulation study in metro atlanta," *Transportation Research Part B: Methodological*, vol. 45, no. 9, pp. 1450 – 1464, 2011, select Papers from the 19th {ISTTT}. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0191261511000671>
- [13] D. O. Santos and E. C. Xavier, "Dynamic taxi and ridesharing: A framework and heuristics for the optimization problem," in *Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence*, ser. IJCAI '13. AAAI Press, 2013, pp. 2885–2891. [Online]. Available: <http://dl.acm.org/citation.cfm?id=2540128.2540544>
- [14] N. Agatz, A. Erera, M. Savelsbergh, and X. Wang, "Optimization for dynamic ride-sharing: A review," *European Journal of Operational Research*, vol. 223, no. 2, pp. 295 – 303, 2012. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0377221712003864>
- [15] S. Ma and O. Wolfson, "Analysis and evaluation of the slugging form of ridesharing," in *Proceedings of the 21st ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, ser. SIGSPATIAL'13. New York, NY, USA: ACM, 2013, pp. 64–73. [Online]. Available: <http://doi.acm.org/10.1145/2525314.2525365>
- [16] Y. Li, R. Chen, L. Chen, and J. Xu, "Towards social-aware ridesharing group query services," *Services Computing, IEEE Transactions on*, vol. PP, no. 99, pp. 1–1, 2015.
- [17] B. Cici, A. Markopoulou, E. Frias-Martinez, and N. Laoutaris, "Assessing the potential of ride-sharing using mobile and social data: A tale of four cities," in *Proceedings of the 2014 ACM International Joint Conference on Pervasive and Ubiquitous Computing*, ser. UbiComp '14. New York, NY, USA: ACM, 2014, pp. 201–211. [Online]. Available: <http://doi.acm.org/10.1145/2632048.2632055>
- [18] N. Bicocchi and M. Mamei, "Investigating ride sharing opportunities through mobility data analysis," *Pervasive and Mobile Computing*, vol. 14, pp. 83 – 94, 2014, special Issue on Pervasive EducationSpecial Issue on The Social Car: Socially-inspired Mechanisms for Future Mobility Services. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S1574119214000868>
- [19] M. Furuhashi, M. Dessouky, F. Ordez, M.-E. Brunet, X. Wang, and S. Koenig, "Ridesharing: The state-of-the-art and future directions," *Transportation Research Part B: Methodological*, vol. 57, pp. 28 – 46, 2013. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0191261513001483>
- [20] S. F. Cheng and T. D. Nguyen, "Taxisim: A multiagent simulation platform for evaluating taxi fleet operations," in *Web Intelligence and Intelligent Agent Technology (WI-IAT), 2011 IEEE/WIC/ACM International Conference on*, vol. 2, Aug 2011, pp. 14–21.