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## Pricing for a Last-Mile Transportation System

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### ABSTRACT

The Last-Mile Problem refers to the provision of travel service from the nearest public transportation node to a home or other destination. Last-Mile Transportation System (LMTS), which has recently emerged, provide on-demand shared transportation. We consider an LMTS with multiple passenger types—adults, senior citizens, children, and students. The LMTS designer determines the price for the passengers, last-mile service vehicle capacity, and service fleet size (number of vehicles) for each last-mile region to maximize the social welfare generated by the LMTS. The level of last-mile service (in terms of passenger waiting time) is approximated by using a batch arrival, batch service, multi-server queueing model. The LMTS designer's optimal decisions and optimal social welfare are obtained by solving a constrained nonlinear optimization problem. Our model is implemented in numerical experiments by using real data from Singapore. We show the optimal annual social welfare gained is large. We also analyze a counterpart LMTS in which the LMTS designer sets an identical price for all passenger types. We find that in the absence of price discounts for special groups of passengers, social welfare undergoes almost no change, but the consumer surplus of passengers in special groups suffers significantly.

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## 1. Background and literature survey

The Last-Mile Problem (LMP)—that is, the design and provision of travel service from a public transportation node to a passenger's final destination—has attracted growing attention in recent years, for several reasons. First, the lack of last-mile service is the main deterrent to use of public transportation services. At the same time, with population rapidly increasing in many cities, how to motivate people to use public transport, and in turn reduce road congestion and air pollution, is challenging. Second, the impaired mobility of certain demographic groups, such as senior citizens, children and students traveling alone with safety concerns, and people with physical disabilities, increases demand for last-mile service and may even be required by law if they use a public transportation mode.<sup>1</sup> Third, with more business models and services arising from the sharing economy, last-mile service, which offers on-demand transportation that utilizes a shared resource (i.e., a shared vehicle fleet), serves as an important testbed and breaks ground for generic on-demand shared services.

A specific last-mile region in a *Last-Mile Transportation System* (LMTS) is illustrated schematically in Fig. 1. LMTS serves a public transportation node, such as a rapid transit metro station at which trains discharge passengers. Passengers' final destinations (homes, workplaces, public institutions, etc.) are spatially distributed in the urban area served by the node, and

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<sup>1</sup> In Singapore, many people in these groups prefer a public transportation mode because it is much cheaper than personalized travel services. One key reason they sometimes cannot take public transportation (and must therefore rely on personalized travel services) is the lack of an LMTS.

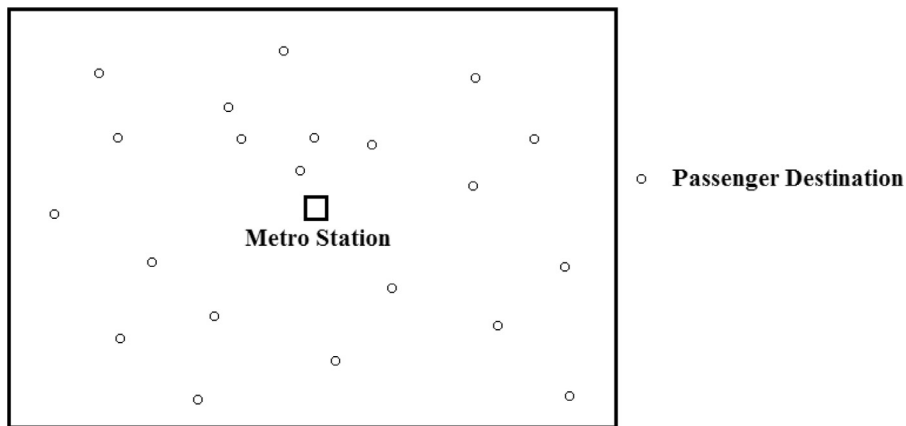


Fig. 1. Schematic of a last-mile region around a metro station.

a fleet of vehicles is available to transport each passenger to her final destination. The routes and schedules of LMTS vehicles are flexible, and can adjust to specific last-mile service requests.

Any passenger needing last-mile service is required to provide advance notice to the LMTS of her impending arrival at the alighting station and her specific final destination. Once this information is received, the LMTS assigns her to one of the vehicles in the LMTS fleet, plans the vehicle's route so that it includes a stop at her destination, estimates the vehicle's departure time, and notifies her accordingly. Once all of the passengers assigned to a vehicle are on board, the vehicle executes a delivery route with stops at each passenger's destination and returns to the station to pick up passengers for its next delivery tour. Detailed LMTS settings for the area around the last-mile region of one metro station can be found in Wang and Odoni (2014).

Many papers address various models and case studies of the LMP and LMTS. With the high penetration of services such as Uber worldwide, most people are aware of the benefits of on-demand transportation services and request even more specialized forms, including last-mile service. Several case studies analyze LMTS in different contexts, including Liu et al. (2012) study of a bicycle-sharing program for an LMTS in Beijing. Many studies have also examined the design and operation of an LMTS. Wang and Odoni (2014) address the planning side by focusing on LMTS from a stochastic and planning perspective and provide closed-form approximations for the performance of an LMTS as a function of the system's fundamental design parameters. Addressing the operational side, Wang (2017) focuses on LMTS from an operational perspective and provides efficient strategies for passenger assignment, vehicle routing and scheduling based on a set of last-mile demand information. Personal rapid transit (PRT), which refers to a variety of transportation systems with characteristics that are similar, in some ways, to LMTS, has also attracted significant attention in recent years, such as Anderson (1998), Bly and Teychenne (2005), Lees-Miller et al. (2009), Berger et al. (2011), and Mueller and Sgouridis (2011).

The pricing problem of urban transportation systems has been studied in diverse contexts. The relevant literature mainly focuses on dynamic pricing and congestion pricing in transportation network. The most influential papers in this area include Yang & Bell (1997), Yang & Huang (1998, 2005), Yang & Meng (2000), Lindsney & Verhoef (2001), Mookherjee & Friesz (2008), Lu et al. (2008), Lou et al. (2010), de Palma & Lindsey (2011), Wu et al. (2011), Lawphongpanich & Yin (2012), Do Chung et al. (2012), and Wang et al. (2016). None of these papers addresses LMTS pricing, which is the subject of this paper.

We study LMTS pricing with multi-type passengers—adults, senior citizens, children, and students. Given each type's last-mile service demand in each last-mile region, the geometric route configuration, discounts for specific passenger types, and the vehicle operating cost, we solve a constrained nonlinear optimization problem to determine the price for the passengers, the vehicle capacity, and the service fleet size (number of vehicles) in each last-mile region to maximize the social welfare generated by LMTS.

The main body of the paper is organized as follows: In Section 2, we propose a constrained nonlinear optimization model for LMTS pricing. Section 3 implements the model in a set of numerical experiments by using real data from Singapore; we also discuss our results and insights. Section 4 contains summary and concluding remarks.

## 2. Model

We now present the main model for the LMTS pricing. We first introduce the settings and notation in Section 2.1. We then describe two important measures—passenger waiting time and passenger utility—in Section 2.2 and Section 2.3, respectively. We propose the LMTS designer's problem—a constrained nonlinear optimization model in Section 2.4, and finally discuss how to solve it in Section 2.5.

2.1. Settings and notation

Consider a city that plans to implement LMTS at a set of metro stations indexed by  $1, \dots, J$ . Trains dynamically arrive at station  $j$  over time and the inter-arrival time between consecutive arriving trains is assumed to be a constant, denoted by  $h^j$ . There are  $I$  types of passengers, indexed by  $1, \dots, I$ , who take trains and, potentially, use the LMTS. Without loss of generality, we define type-1 passengers as *regular passengers*, such as adults. This type of passengers are typically not eligible to enjoy any discount while taking the public transportations. We define passengers with other types  $2, \dots, I$  as special passengers, such as senior citizens, children, and students. These passengers are typically eligible to enjoy the discounted fares while taking the public transportations. Upon each train's arrival at station  $j$ ,  $N_i^j$  of type  $i \in \{1, \dots, I\}$  passengers are discharged. We assume that  $N_1^j, \dots, N_I^j$  are independent random variables. For each passenger type  $i$ ,  $N_i^j$  is also independent and identically distributed for all trains that arrive at station  $j$ . We denote by  $\mu_{N_i^j}$  and  $\sigma_{N_i^j}^2$  the mean and variance of  $N_i^j$ , respectively. At each station  $j$ , passenger destinations are independent and identically distributed in the last-mile region, which is convex and compact with known dimensions.

The LMTS designer needs to determine the full price  $p \in P$  for the regular passengers, i.e., type-1 passengers who buy the tickets at the full price without being eligible to enjoy any discount. The fare for any specific type passengers with  $i \in \{1, \dots, I\}$ ,  $\mathbf{p}_i$ , is restricted to be a predetermined fraction  $\theta_i \in [0, 1]$  of the full price, i.e.,  $\mathbf{p}_i = \theta_i p$ .<sup>2</sup> Note that we have  $\theta_1 = 1.0$  so the price  $\mathbf{p}_1$  for the regular passengers equals to the full price  $p$ . Therefore, the fares for all types of passengers are automatically determined after the full price  $p$  is determined. We assume that the feasible set of full price,  $P$ , is finite. This is reasonable in the sense that the fare in the public transportation system cannot be arbitrarily high and the minimum price unit is one cent. We also assume that the price for each passenger type  $i$ ,  $\mathbf{p}_i$ , is identical at all stations and does not depend on a passenger's travel distance and travel time. This assumption is reasonable in the sense that in many public transportation systems around the world—such as Singapore, Beijing, Boston, and New York—the fare is a fixed rate if the travel distance is less than a threshold level (typically, at least several miles).

The designer needs to determine the capacity of vehicles used for the last-mile service,  $c \in C$ . We assume that the feasible set of vehicle capacity,  $C$ , is finite. This is reasonable in the sense that a vehicle cannot be arbitrarily large and the capacity must be an integer.

The designer also needs to determine the number of vehicles that provide the last-mile service at each metro station  $j$ ,  $m^j \in \{0, 1, \dots, M\}$ . We assume each vehicle incurs per unit of time operating cost  $q$ .

At each station  $j$ , we denote by  $\hat{N}_i^j$  the number of type- $i$  passengers who are willing to use the LMTS. We denote by  $\hat{N}^j = \sum_{i=1}^I \hat{N}_i^j$  the total number of passengers who are willing to use the LMTS. We denote by  $\mu_{\hat{N}^j}$  and  $\sigma_{\hat{N}^j}^2$  the mean and variance of  $\hat{N}^j$ , respectively. Service times for passengers who use the LMTS are independent and identically distributed. We denote by  $\mu_{S_{\hat{N}^j}}$  and  $\sigma_{S_{\hat{N}^j}}^2$  the mean and variance of travel time, respectively, of one service trip (serving no more than  $c$  passengers) at station  $j$  if  $\hat{N}^j$  passengers from each train are willing to use the LMTS. Note that both  $\mu_{S_{\hat{N}^j}}$  and  $\sigma_{S_{\hat{N}^j}}^2$  depend on  $c$ ,  $\mu_{\hat{N}^j}$  and passenger destination topologies in the last-mile region around station  $j$ .

**Example 1.** From Wang and Odoni (2014), for the last-mile region in Fig. 1 (assumed to be a square area with side travel time  $b$  and uniformly distributed destinations),  $\mu_{S_{\hat{N}^j}} \approx \frac{0.57c}{\sqrt{\mu_{\hat{N}^j}}} b + 0.764$ ,  $\sigma_{S_{\hat{N}^j}}^2 \approx \frac{\beta_2}{(c+1)\beta_1^2} \mu_{S_{\hat{N}^j}}^2$ , where  $\beta_1 \approx 0.7124$  and  $\beta_2 \approx 0.1385$ .

Before we present the optimization model for the LMTS pricing problem, we provide the glossary of notation in Table 1.

2.2. Passenger waiting time

Every passenger shares an LMTS vehicle with other passengers. A passenger who gets off the train and intends to use the LMTS is either (1) assigned to an idle vehicle that has an available seat or (2) directed to wait in a queue until a vehicle is available. Note that at each station  $j$ , the passenger expected waiting time for the last-mile service,  $\mathbf{w}^j$ , depends on the passenger destination topologies. We assume that  $\mathbf{w}^j$  in the LMTS satisfies the following properties.

**Assumption 1.** At each station  $j \in \{1, \dots, J\}$ , given passenger destination topologies, the passenger expected waiting time for last-mile service,  $\mathbf{w}^j$ , is increasing in  $\mu_{\hat{N}^j}$  and  $\sigma_{\hat{N}^j}^2$ , respectively.

Under this assumption, a passenger waits longer if more passengers request the service or the number of passengers who request the service is more uncertain. Many passenger destination topologies that commonly occur in cities satisfy this

<sup>2</sup> In this paper, we bold all variables that are functions of the LMTS designer's decisions. We also bold all optimal decisions and optimal outputs.

**Table 1**  
Glossary of notation.

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*Parameters:*  
*J*: number of stations (last-mile regions);  
*I*: number of passenger types;  
*h<sup>j</sup>*: inter-arrival time between consecutive arriving trains at station *j*;  
*N<sub>i</sub><sup>j</sup>*: random variable to indicate the number of type-*i* passengers discharged from each train at station *j*;  
*μ<sub>N<sub>i</sub><sup>j</sup></sub>*: mean of *N<sub>i</sub><sup>j</sup>*;  
*σ<sub>N<sub>i</sub><sup>j</sup></sub><sup>2</sup>*: variance of *N<sub>i</sub><sup>j</sup>*;  
*θ<sub>i</sub>*: fraction of the price for type-*i* passengers relative to the full price for regular passengers (type-1 passengers);  
*q*: operating cost per vehicle per unit of time;  
*v*: random variable to indicate a passenger's valuation on the last-mile service;  
*v<sub>i</sub>*: upper limit of a type-*i* passenger's valuation on the last-mile service;  
*f<sub>i</sub>(•)*: P.D.F. of a type-*i* passenger's valuation on the last-mile service;  
*F<sub>i</sub>(•)*: C.D.F. of a type-*i* passenger's valuation on the last-mile service;  
*F<sub>i</sub><sup>̄</sup>(•)*: complementary C.D.F. of a type-*i* passenger's valuation on the last-mile service;  
*α<sub>i</sub>*: type-*i* passenger's per unit of time disutility from waiting to use the LMSTs;  
*Decision variables:*  
*p*: full price for the regular passengers (i.e., type-1 passengers);  
*c*: capacity of vehicles used for the last-mile service;  
*m<sup>j</sup>*: number of vehicles that provide the last-mile service at station *j*, *j* ∈ {1, ..., *J*};  
*Endogenously determined variables:*  
*p<sub>i</sub>*: price for type-*i* passengers, *i* ∈ {1, ..., *I*};  
*N̂<sub>i</sub><sup>j</sup>*: random variable to indicate the number of type-*i* passengers who are willing to use the LMSTs at station *j*;  
*N̂<sup>j</sup>*: random variable to indicate the total number of passengers who are willing to use the LMSTs at station *j*;  
*μ<sub>N̂<sub>i</sub><sup>j</sup></sub>*: mean of *N̂<sub>i</sub><sup>j</sup>*;  
*σ<sub>N̂<sub>i</sub><sup>j</sup></sub><sup>2</sup>*: variance of *N̂<sub>i</sub><sup>j</sup>*;  
*μ<sub>S<sub>N<sub>i</sub><sup>j</sup></sub></sub>*: mean of the travel time of one last-mile service trip at station *j*;  
*σ<sub>S<sub>N<sub>i</sub><sup>j</sup></sub></sub>*<sup>2</sup>: variance of the travel time of one last-mile service trip at station *j*;  
*w<sup>j</sup>*: passenger's expected waiting time for the last-mile service at station *j*;  
*U<sub>i</sub><sup>j</sup>(v)*: a type-*i* passenger's utility if she values the last-mile service at *v* and uses this service at station *j*;  
*CS<sup>j</sup>(p, c, m<sup>j</sup>)*: per unit of time expected consumer surplus at station *j*;  
*Profit<sup>j</sup>(p, c, m<sup>j</sup>)*: per unit of time expected vehicle profit at station *j*;  
*SW<sup>j</sup>(p, c, m<sup>j</sup>)*: per unit of time expected social welfare at station *j*;  
*SW(p, c, {m<sup>j</sup>})*: per unit of time expected social welfare over all stations;  
*Optimal outputs:*  
*m<sup>j</sup>(p, c)*: optimal number of vehicles at station *j*, given the full price for regular passengers is *p* and vehicle capacity is *c*;  
*p<sup>\*</sup>(c)*: optimal full price for regular passengers, given the vehicle capacity is *c*;  
*p<sup>\*</sup>*: optimal full price for regular passengers;  
*c<sup>\*</sup>*: optimal vehicle capacity;  
*m<sup>j\*</sup>*: optimal number of vehicles at station *j*;  
*SW<sup>\*</sup>*: optimal per unit of time expected social welfare over all stations;

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assumption. For the last-mile region in Fig. 1, Wang and Odoni (2014) provide an expression to approximate *w<sup>j</sup>* as:

$$\begin{aligned}
 w^j \approx & \frac{6m^j \mu_{N_j} \sigma_{S_{N_j}}^2 + 6c \left| \frac{\sqrt{\mu_{N_j}}}{c} \right|^2 \mu_{S_{N_j}}^2 + m^{j2} c \mu_{S_{N_j}}^2 - c \mu_{S_{N_j}}^2}{12m^j (h^j m^j c - \mu_{N_j} \mu_{S_{N_j}})} \\
 & \cdot \exp \left[ - \frac{4(h^j m^j c - \mu_{N_j} \mu_{S_{N_j}}) \mu_{N_j} \mu_{S_{N_j}}}{6m^j c \mu_{N_j} \sigma_{S_{N_j}}^2 + 6c^2 \left| \frac{\sqrt{\mu_{N_j}}}{c} \right|^2 \mu_{S_{N_j}}^2 + m^{j2} c^2 \mu_{S_{N_j}}^2 - c^2 \mu_{S_{N_j}}^2} \right] \\
 & + \frac{\left( 6c^2 \left| \frac{\sqrt{\mu_{N_j}}}{c} \right|^2 + m^{j2} c^2 + 6\mu_{N_j}^2 - 6m^j c \mu_{N_j} - c^2 \right) \mu_{S_{N_j}}}{12m^j c \mu_{N_j}}, \tag{1}
 \end{aligned}$$

where *μ<sub>S<sub>N<sub>i</sub><sup>j</sup></sub></sub>* and *σ<sub>S<sub>N<sub>i</sub><sup>j</sup></sub><sup>2</sup></sub>* are functions of *c*, *μ<sub>N<sub>i</sub><sup>j</sup></sub>* and *σ<sub>N<sub>i</sub><sup>j</sup></sub>*<sup>2</sup>. Assumption 1 is satisfied for most realistic input values.

For analytic purposes, we express passenger expected waiting time *w<sup>j</sup>* as an explicit function of *μ<sub>N<sub>i</sub><sup>j</sup></sub>*, *σ<sub>N<sub>i</sub><sup>j</sup></sub>*<sup>2</sup>, *c*, *m<sup>j</sup>*, and features of passenger destination topologies:

$$w^j = w^j(\mu_{N_j}, \sigma_{N_j}^2, c, m^j, \text{topology}^j), \forall j \in \{1, \dots, J\}. \tag{2}$$

### 2.3. Passenger utility

Each passenger is endowed with a valuation (willingness-to-pay) for using the LMTS. For each passenger type  $i$ , passenger valuations are heterogeneous and supported on  $[0, \bar{v}_i]$ . The fraction of type- $i$  passengers whose valuations are no greater than  $v$  is denoted by  $F_i(v) = v/\bar{v}_i$ . We denote  $f_i(v) \triangleq dF_i(v)/dv$  and  $\bar{F}_i(v) \triangleq 1 - F_i(v)$ .

At station  $j$ , if a type- $i$  passenger with valuation  $v$  uses the LMTS, she garners the expected utility (measured in dollars)  $U_i^j(v) = v - \mathbf{p}_i - \alpha_i \mathbf{w}^j$ , where  $\alpha_i \geq 0$  is type- $i$  passenger's per unit of time disutility from waiting to use the LMTS. A type- $i$  passenger with valuation  $v$  uses the LMTS if and only if doing so allows her to get non-negative expected utility, i.e.,  $U_i^j(v) \geq 0$ . Therefore, for each passenger type  $i$ , only passengers with valuation  $v \geq \mathbf{p}_i + \alpha_i \mathbf{w}^j$  request to use the LMTS.

Therefore, at each station  $j$ , the mean of the total number of passengers who get off from one train and request to use the LMTS is given by

$$\mu_{\hat{N}^j} = \sum_{i=1}^I E(\hat{N}_i^j) = \sum_{i=1}^I \mu_{N_i^j} \cdot \bar{F}_i(\mathbf{p}_i + \alpha_i \mathbf{w}^j). \tag{3}$$

The variance of the total number of passengers who get off from one train and request to use the LMTS is given by

$$\sigma_{\hat{N}^j}^2 = \sum_{i=1}^I \text{Var}(\hat{N}_i^j) = \sum_{i=1}^I \sigma_{N_i^j}^2 \cdot (\bar{F}_i(\mathbf{p}_i + \alpha_i \mathbf{w}^j))^2, \tag{4}$$

where the first equality follows from the assumption that passenger arrivals and valuations over different passenger types are independent.

Therefore, at each station  $j$ , given the full price  $p$  for regular passengers, the number of service fleet  $m^j$ , the vehicle capacity  $c$  and passenger destination topologies, Eqs. (2)–(4) allow us to compute passenger average waiting time  $\mathbf{w}^j$ , and the mean  $\mu_{\hat{N}^j}$  and variance  $\sigma_{\hat{N}^j}^2$  of the total number of passengers who use the LMTS.

### 2.4. The LMTS designer's problem

The LMTS designer aims to determine the full price  $p$  for regular passengers, the vehicle capacity  $c$ , and the number of operating vehicles  $m^j$  at each station  $j$  with the objective of maximizing per unit of time expected social welfare for the city or region.

Social welfare is the summation of consumer surplus and vehicle profit over all stations. At each station  $j$ , the per unit of time expected consumer surplus (measured in dollars) is given by

$$\begin{aligned} \mathbf{CS}^j(p, c, m^j) &= \frac{1}{h^j} \sum_{i=1}^I \mu_{N_i^j} \cdot E\left(\left(U_i^j(v)\right)^+\right) \\ &= \frac{1}{h^j} \sum_{i=1}^I \mu_{N_i^j} \cdot \int_{v=\mathbf{p}_i+\alpha_i\mathbf{w}^j}^{\bar{v}_i} (v - \mathbf{p}_i - \alpha_i \mathbf{w}^j) f_i(v) dv \\ &= \frac{1}{h^j} \sum_{i=1}^I \mu_{N_i^j} \cdot \left( \int_{v=\mathbf{p}_i+\alpha_i\mathbf{w}^j}^{\bar{v}_i} v f_i(v) dv - (\mathbf{p}_i + \alpha_i \mathbf{w}^j) \bar{F}_i(\mathbf{p}_i + \alpha_i \mathbf{w}^j) \right). \end{aligned} \tag{5}$$

At each station  $j$ , the per unit of time expected vehicle profit is given by

$$\begin{aligned} \mathbf{Profit}^j(p, c, m^j) &= \frac{1}{h^j} \sum_{i=1}^I \mathbf{p}_i \cdot E(\hat{N}_i^j) - m^j q \\ &= \frac{1}{h^j} \sum_{i=1}^I \mathbf{p}_i \cdot \mu_{N_i^j} \cdot \bar{F}_i(\mathbf{p}_i + \alpha_i \mathbf{w}^j) - m^j q. \end{aligned} \tag{6}$$

Therefore, the per unit of time expected social welfare generated by the LMTS at each station  $j$  is given by

$$\begin{aligned} \mathbf{SW}^j(p, c, m^j) &= \mathbf{CS}^j(p, c, m^j) + \mathbf{Profit}^j(p, c, m^j) \\ &= \frac{1}{h^j} \sum_{i=1}^I \mu_{N_i^j} \cdot \left( \int_{v=\mathbf{p}_i+\alpha_i\mathbf{w}^j}^{\bar{v}_i} v f_i(v) dv - \alpha_i \mathbf{w}^j \bar{F}_i(\mathbf{p}_i + \alpha_i \mathbf{w}^j) \right) - m^j q. \end{aligned} \tag{7}$$

The per unit of time expected social welfare generated by the LMTS over all stations is given by

$$\mathbf{SW}(p, c, \{m^j\}) = \sum_{j=1}^J \mathbf{SW}^j(p, c, m^j). \tag{8}$$

**Table 2**  
Algorithm for solving LMTS designer's problem.

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```

For  $p \in P, c \in C$ 
For  $j = 1: J$ 
For  $m^j = 0: M$ 
Compute  $\mathbf{SW}(p, c, m^j)$ 
End
Get  $\mathbf{m}^j(p, c)$ 
End
 $\mathbf{SW}(p, c, \{\mathbf{m}^j(p, c)\}) = \sum_{j=1}^J \mathbf{SW}^j(p, c, \mathbf{m}^j(p, c))$ 
End
 $(\mathbf{p}^*, \mathbf{c}^*) = \operatorname{argmax}_{p \in P, c \in C} \mathbf{SW}(p, c, \{\mathbf{m}^j(p, c)\})$ 
For  $j = 1: J$ 
 $\mathbf{m}^{j*} = \mathbf{m}^j(\mathbf{p}^*, \mathbf{c}^*)$ 
End
 $\mathbf{SW}^* = \mathbf{SW}(\mathbf{p}^*, \mathbf{c}^*, \{\mathbf{m}^j(\mathbf{p}^*, \mathbf{c}^*)\})$ 

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The LMTS designer determines the full price  $p$  for regular passengers, the vehicle capacity  $c$ , and the number of operating vehicles  $m^j$  (upper bounded by  $M$ ) at each station  $j$  to maximize the LMTS's per unit of time expected social welfare. Therefore, the LMTS designer solves the following constrained nonlinear optimization problem:

$$\begin{aligned} & \max_{p \in P, c \in C, m^j \in \{0, 1, \dots, M\}} \mathbf{SW}(p, c, \{m^j\}) \\ & \text{subject to: } \frac{\mu_{\hat{N}j} \mu_{S_{\hat{N}j}}}{h^j m^j c} < 1, \forall j \in \{1, \dots, J\} \end{aligned}$$

We denote by  $(\mathbf{p}^*, \mathbf{c}^*, \{\mathbf{m}^{j*}\})$  the optimal solution and  $\mathbf{SW}^*$  the optimal value to the optimization problem above. Full price for regular passengers ( $p$ ), vehicle capacity ( $c$ ), and number of vehicles at each station ( $m^j$ ) are decision variables exogenously determined by the LMTS designer. Among the three types of decisions, full price for regular passengers ( $p$ ) and vehicle capacity ( $c$ ) are coupled decisions across all stations, i.e., price and vehicle capacity for the last-mile service around all the metro stations are identical. After  $p$  as the full price is decided, the price for type- $i$  passengers can be determined as  $p_i = \theta_i p$  for all  $i$ .

The number of vehicles ( $m^j$ ) could be different at different station  $j$ . For each station, the constraint on each station's LMTS utilization ratio,  $\mu_{\hat{N}j} \mu_{S_{\hat{N}j}} / h^j m^j c < 1$ , is necessary to avoid unbounded increase over time in passenger waiting time. Passenger waiting time ( $w^j$ ), mean ( $\mu_{\hat{N}j}$ ) and variance ( $\sigma_{\hat{N}j}^2$ ) of passengers who use the LMTS are then endogenously determined through Eqs. (2)–(4). Specifically, Eq. (2) specifies that passenger waiting time ( $w^j$ ) depends on mean ( $\mu_{\hat{N}j}$ ) and variance ( $\sigma_{\hat{N}j}^2$ ) of passengers who use the LMTS, vehicle capacity ( $c$ ), and the number of vehicles at that station ( $m^j$ ). Eqs. (3) and (4) specify that the mean ( $\mu_{\hat{N}j}$ ) and variance ( $\sigma_{\hat{N}j}^2$ ) of passengers who use the LMTS at each station depend on price ( $p$ ) and passenger waiting time ( $w^j$ ) at that station.

2.5. Algorithm for solving the LMTS designer's problem

In this subsection, we present an algorithm to solve the LMTS designer's problem defined in Section 2.4. To state our algorithm, we denote by  $\mathbf{m}^j(p, c) = \operatorname{argmax}_{m^j \in \{0, 1, \dots, M\}} \mathbf{SW}^j(p, c, m^j)$  the optimal number of vehicles at station  $j$ , given full price  $p$  for regular passengers and vehicle capacity  $c$ . Our algorithm is described in Table 2.

The key idea of this algorithm is to decompose the LMTS designer's optimization problem into a two-layer optimization problem: an inner-layer optimization problem to optimize the number of vehicles at each station ( $m^j$ ) with fixed full price ( $p$ ) and fixed vehicle capacity ( $c$ ) (i.e., fixed coupled decisions across stations); and an outer-layer optimization problem to optimize the coupled decisions price ( $p$ ) and vehicle capacity ( $c$ ). We then explain in details how to solve the inner-layer and the outer-layer optimization problems.

2.5.1. Inner-layer optimization problem

In the inner-layer optimization problem, we fix price ( $p$ ) and vehicle capacity ( $c$ ), and optimize the number of vehicles at each station ( $m^j$ ). Although the total number of decision variables is equal to the number of stations  $J$ , we can decompose this multi-station optimization problem into a series of single-station optimization problems for each station with a single decision variable, the number of vehicles at that station  $m^j$ . For each single-station optimization problem, given price ( $p$ ) and vehicle capacity ( $c$ ), the LMTS designer solves the following problem:

$$\begin{aligned} & \max_{m^j \in \{0, 1, \dots, M\}} \mathbf{SW}^j(p, c, m^j) \\ & \text{subject to: } \frac{\mu_{\hat{N}j} \mu_{S_{\hat{N}j}}}{h^j m^j c} < 1 \end{aligned}$$

Now, we discuss how to solve this optimization problem. At each station  $j$ , we notice that, in practice,  $m^j$  may only take a finite number of values. Therefore, we can enumerate over all possible  $m^j \in \{0, 1, \dots, M\}$ . For each given  $m^j$  in  $\{0, 1, \dots, M\}$ ,

**Table 3**  
Bisection method to approximate  $\hat{w}^j$ .

---

Step 0: Set  $w_{LB}^j \leftarrow 0$  and  $w_{UB}^j \leftarrow \bar{w}$   
 Step 1: Set  $\hat{w}^j \leftarrow \frac{w_{LB}^j + w_{UB}^j}{2}$   
 Step 2: Use  $\hat{w}^j$  and Eq. (3) to approximate  $\mu_{\hat{N}^j}$ :  

$$\mu_{\hat{N}^j} \leftarrow \sum_{i=1}^I \mu_{\hat{N}_i^j} \cdot \bar{F}_i(\mathbf{p}_i + \alpha_i \hat{w}^j)$$
  
 Step 3: Use  $\hat{w}^j$  and Eq. (4) to approximate  $\sigma_{\hat{N}^j}^2$ :  

$$\sigma_{\hat{N}^j}^2 \leftarrow \sum_{i=1}^I \sigma_{\hat{N}_i^j}^2 \cdot (\bar{F}_i(\mathbf{p}_i + \alpha_i \hat{w}^j))^2$$
  
 Step 4: Plug estimated  $\mu_{\hat{N}^j}$  from Step 2 and  $\sigma_{\hat{N}^j}^2$  from Step 3 into Eq. (2), denoted by  $\tilde{w}^j \leftarrow w^j(\mu_{\hat{N}^j}, \sigma_{\hat{N}^j}^2, c, m^j, \text{topology}^j)$   
 Step 5:  
 While  $|\tilde{w}^j - \hat{w}^j| > \epsilon$   
 IF  $\tilde{w}^j > \hat{w}^j$   
 Set  $w_{LB}^j \leftarrow \hat{w}^j$   
 ELSE  
 Set  $w_{UB}^j \leftarrow \hat{w}^j$   
 END  
 Repeat Step 1–Step 4  
 END

---

verifying whether the utilization ratio constraint  $\mu_{\hat{N}^j} \mu_{S_{\hat{N}^j}} / h^j m^j c < 1$  is satisfied and computing the social welfare at station  $j$  requires us to compute passenger average waiting time  $w^j$ . Typically, there is no closed-form solution for the exact  $w^j$ . We can use the following approach to find an approximated  $w^j$ , denoted as  $\hat{w}^j$ , which can be arbitrarily close to the true value  $w^j$ .

We assume that  $w^j$  falls between 0 and  $\bar{w}$  and denote by  $\epsilon$  the maximum tolerable difference between  $\hat{w}^j$  and  $w^j$ . Following from Assumption 1, we can use the following bisection method in Table 3 to find such  $\hat{w}^j$ .

The passenger expected waiting time function  $w^j(\mu_{\hat{N}^j}, \sigma_{\hat{N}^j}^2, c, m^j, \text{topology}^j)$  could be approximately obtained as Eq. (1) following the method described by Wang & Odoni (2014).

### 2.5.2. Outer-layer optimization problem

In the outer-layer optimization problem, we aim to optimize price  $p$  and vehicle capacity  $c$ . In practice, both the feasible set of price  $P$  and the feasible set of vehicle capacity  $C$  are finite and typically not very large in terms of cardinality. We could enumerate over all the feasible prices and vehicle capacities to seek their optimal values. The LMTS designer uses each enumerated price  $p$  and vehical capacity  $c$  as input variables to solve the aforementioned inner-layer optimization problem and obtain the parameterized optimal social welfare,  $\mathbf{SW}(p, c, \{m^j(p, c)\})$ . In the outer-layer problem, the LMTS designer seeks the optimal price  $p \in P$  and vehical capacity  $c \in C$  to maximize the social welfare, i.e., the LMTS designer solves the following optimization problem:

$$\mathbf{SW}^* = \max_{p \in P, c \in C} \mathbf{SW}(p, c, \{m^j(p, c)\})$$

The feasible set of price  $P$  and vehicle capacity  $C$  are finite and typically not very large in terms of cardinality. For instance, if the feasible price is no more than \$10, the set of price  $P$  will be  $[0, 10]$  containing 1000 elements in unit of cent. If we use normal vehicles, the set of capacity  $C$  could be  $[1, 50]$  containing 50 integral elements. Therefore, we do not have any computational burden to enumerate all possible fares  $p \in P$  and all possible vehicle capacity  $c \in C$ .

## 3. Numerical experiments

In this section, we implement the pricing model described in our numerical experiments by using real data from Singapore for the LMTS at a sample set of metro stations. In this section, we first discuss the background and settings of the numerical experiments and then present the results, followed by detailed discussion.

### 3.1. Experimental background and settings

Singapore is a city-state with high utilization of public transportation, especially buses and mass rapid transit (MRT). The latter consists of metro and light urban rail service, with five lines and 102 stations currently in operation. In an effort to increase the use of public transport in response to Singapore's rise in affluence and steadily increasing population, the government imposed an extremely high tax for private vehicle ownership (e.g., the cost of a license plate that is valid for 10 years is approximately S\$ 50,000). Public transport is the leading transportation service mode in Singapore; several million residents (out of a total population around 5.6 million) use public transport—especially the MRT—daily.





Fig. 2. Singapore MRT map and sample stations for experiment.

Singapore also has a rapidly aging population. According to Ng (2015), the number of residents 65 and older doubled from 220,000 in 2000 to 440,000 in 2015, and is expected to rise to 900,000 by 2030. Public transport is this group’s primary transportation mode. Meanwhile, school children also rely on public transport to commute daily between home and school. These two large groups, therefore—senior citizens and children—are potential last-mile service users, and demonstrate that LMTS holds significant promise for Singapore. Both government agencies and transport service providers are committed to the establishment of this innovative transportation service.

In a set of numerical experiments, we implement the LMTS pricing model for 10 sample MRT stations located near residential areas well outside the downtown region. Distances between successive sample MRT stations are longer than the distances between downtown stations, which causes the last segment of a passenger’s trip to be longer; this renders LMTS more valuable—even necessary—in the last-mile region around these sample stations. Each station covers a last-mile region in which passengers’ final destinations are distributed. The side length of the region is around 1.5–2.5 km, which induces a travel time  $b$  of approximately 4–5 min if we assume that vehicle travel speed is around 30–40 km/hour in residential areas. Passenger expected waiting time is approximated by Eq. (1) described in Example 1. Fig. 2 shows the Singapore MRT line map and the 10 sample stations (in the red rectangle).

The three main types of metro cards are for adults, senior citizens, and children/students ( $i = \{1, 2, 3\}$ ). We take adults as the regular passengers, i.e., adults pay the full price  $p$  (i.e.,  $\theta_1 = 1.0$ ). The current discount for senior citizens is around 30% (i.e.,  $\theta_2 = 0.7$ ), and for children/students is around 50% (i.e.,  $\theta_3 = 0.5$ ). The headway between metro trains ( $h^j$  in the model) can easily be obtained from the public train schedule. The expectation ( $\mu_{N_i}$ ) and the variance ( $\sigma_{N_i}^2$ ) of each passenger type discharged from trains can be obtained from metro card transaction data.

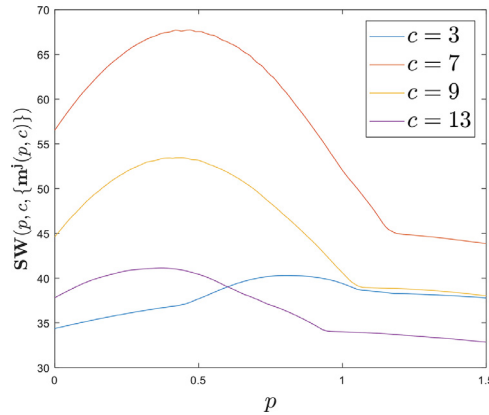
Table 4 lists the corresponding statistics for all sample MRT stations.

Considering the inter-arrival time between trains, the average total number of passengers discharged from all sample stations in each minute is 196.11.

As for the monetary value of transportation waiting time  $\alpha_i$ , we use two references: (1) according to Trading Economics (2016), the average hourly wage in Singapore is around S\$ 20; and (2) according to Gómez-Ibañez et al. (1999), for work trips in San Francisco, the monetary value per unit transfer waiting time is 195% of the user’s after-tax wages. Therefore, as an approximation, we estimate  $\alpha_1 = S\$ 20 \times 195\% / \text{hour} \approx S\$ 2/3 / \text{minute}$  for adults. The monetary value of

**Table 4**  
Information on sample stations.

| j: station | $h^j$ (min) | 1: Adults     |                    | 2: Senior citizens |                    | 3: Children/students |                    |
|------------|-------------|---------------|--------------------|--------------------|--------------------|----------------------|--------------------|
|            |             | $\mu_{N_1^j}$ | $\sigma_{N_1^j}^2$ | $\mu_{N_2^j}$      | $\sigma_{N_2^j}^2$ | $\mu_{N_3^j}$        | $\sigma_{N_3^j}^2$ |
| Station 1  | 6           | 143.6         | 1705.69            | 13.6               | 19.36              | 24.7                 | 100.00             |
| Station 2  | 6           | 53.1          | 533.61             | 5.0                | 2.89               | 8.8                  | 10.89              |
| Station 3  | 6           | 131.4         | 665.64             | 17.3               | 37.21              | 38.1                 | 73.96              |
| Station 4  | 6           | 71.3          | 1169.64            | 12.2               | 37.21              | 27.9                 | 309.76             |
| Station 5  | 6           | 135.9         | 2550.25            | 15.1               | 37.21              | 35.1                 | 187.69             |
| Station 6  | 5           | 55.7          | 466.56             | 8.0                | 20.25              | 22.6                 | 146.41             |
| Station 7  | 5           | 102.3         | 1608.01            | 9.3                | 34.81              | 31.2                 | 190.44             |
| Station 8  | 6           | 60.7          | 272.25             | 10.0               | 5.76               | 20.7                 | 132.25             |
| Station 9  | 6           | 13.4          | 31.36              | 2.7                | 2.89               | 3.3                  | 5.29               |
| Station 10 | 6           | 47.9          | 210.25             | 6.2                | 9                  | 3.3                  | 1.44               |



**Fig. 3.** Relationship between  $SW(p, c, \{m^j(p, c)\})$  and  $p$ .

waiting time for senior citizens and children/students are approximated as  $\alpha_2 = S\$ 1/6$  /minute and  $\alpha_3 = S\$ 1/20$  /minute, respectively. These numerical values could be adjusted, given more accurate estimations from the survey.  $\bar{v}_1$  for adults is set to be S\$ 1.5 (the average metro cost), and  $\bar{v}_2$  and  $\bar{v}_3$  for senior citizens and children/students is set to be S\$ 3.0 (base price to take an ordinary taxi).

On the service supply side, we evaluate the possibilities with different types of vehicles for outsourcing transportation service in the market: (1) If we use ordinary taxis as LMTS service fleets, the vehicle capacity is  $c=3$  and the per unit time cost is  $q \approx S\$ 24/\text{hour} = S\$ 0.4/\text{minute}$ ; (2) if we use large van taxis, the vehicle capacity is  $c=7$  and the per unit time cost is  $q \approx S\$ 30/\text{hour} = S\$ 0.5/\text{minute}$ ; (3) if we use 9-seater vans, the vehicle capacity is  $c=9$  and the per unit time cost is  $q \approx S\$ 40/\text{hour} \approx S\$ 0.67/\text{minute}$ ; and (4) if we use 13-seater minibuses, the vehicle capacity is  $c=13$  and the per unit time cost is  $q \approx S\$ 50/\text{hour} \approx S\$ 0.83/\text{minute}$ .

### 3.2. Experimental results and discussion

In this subsection, we explore the impact of the LMTS designer’s decisions on price and vehicle capacity on social welfare, compute the LMTS designer’s optimal price and vehicle capacity that maximizes social welfare, analyze passenger behaviors under the LMTS designer’s optimal decisions, and use Singapore as an example to show how an LMTS can potentially improve social welfare.

#### 3.2.1. Impact of prices on social welfare

We begin with exploring the impact of LMTS price on social welfare. In Fig 3, each curve that corresponds to a vehicle capacity  $c$  plots the relationship between each feasible price  $p$  and the corresponding optimal social welfare  $SW(p, c, \{m^j(p, c)\})$ .

We make the following observations:

- Given every feasible vehicle capacity decision  $c$ , the optimal LMTS price that maximizes social welfare,  $p^*(c) = \text{argmax}_{p \in P} SW(p, c, \{m^j(p, c)\})$ , cannot be too small. If the LMTS price is extremely small, an extremely large number of passengers might be willing to use it. A large number of vehicles would be required to serve these passengers, and as a result, the negative effect of prohibitively high vehicle operating costs would dominate the positive effect of benefiting a large group of passengers.

**Table 5**  
Relationship between  $\mathbf{p}^*(c)$ ,  $\mathbf{OC}(c)$ ,  $\mathbf{SW}(\mathbf{p}^*(c), c, \{\mathbf{m}^j(\mathbf{p}^*(c), c)\})$ , and  $c$ .

| $c$  | 3     | 7     | 9     | 13    |
|--|-------|-------|-------|-------|
| $\mathbf{p}^*(c)$ (\$\$)   | 0.80  | 0.47  | 0.44  | 0.37  |
| $\mathbf{OC}(c)$ (\$\$/served passenger)   | 0.88  | 0.61  | 0.66  | 0.71  |
| $\mathbf{SW}(\mathbf{p}^*(c), c, \{\mathbf{m}^j(\mathbf{p}^*(c), c)\})$ (\$\$/min) | 40.31 | 67.73 | 53.45 | 41.12 |

**Table 6**  
Optimal LMTS fares and the impacts on each passenger type.

| Passenger type $i$                      | 1: Adults | 2: Senior citizens | 3: Children/students |
|---|-----------|--------------------|----------------------|
| $\mathbf{p}_i^*$ (\$\$)                 | 0.47      | 0.33               | 0.24                 |
| $\mathbf{PCT}_i$                        | 50%       | 87%                | 91%                  |
| $\mathbf{CS}_i$ (\$\$/served passenger) | 0.39      | 1.30               | 1.37                 |

- Given every feasible vehicle capacity decision  $c$ , the optimal LMTS price  $p$  that maximizes social welfare,  $\mathbf{p}^*(c)$ , also cannot be too large. If the LMTS price becomes too large and using the LMTS becomes too expensive, only a small number of passengers will be willing to use it. Fewer vehicles would be required to serve these passengers, but the negative effect of losing passengers would dominate the positive effect of lower vehicle operating costs.

3.2.2. *Impact of vehicle capacity on social welfare*

Next, we aim to understand the impact of vehicle capacity,  $c$ , on the optimal LMTS price,  $\mathbf{p}^*(c)$ ; the vehicle operating cost per served passenger,  $\mathbf{OC}(c)$ ; and the optimal social welfare,  $\mathbf{SW}(\mathbf{p}^*(c), c, \{\mathbf{m}^j(\mathbf{p}^*(c), c)\})$ . Results are reported in Table 5.

We make the following observations:

- The optimal price  $\mathbf{p}^*(c)$  declines as vehicle capacity increases. When a vehicle has more seats, due to economy of scale, the operating cost per passenger is reduced. In turn, the LMTS designer can charge passengers cheaper prices.
- Vehicle capacity  $c$  cannot be too small or too large. When  $c$  is too small, delivering service to passengers requires the LMTS to operate too many vehicles. Therefore, a large number of vehicles leads to prohibitively high operating costs. When  $c$  is too large, it is more likely that some seats on a vehicle for a given trip will be empty. Therefore, vehicle operating efficiency is too low. As a result, social welfare suffers when vehicle capacity is either too small or too large.

We observe from Table 5 that the social welfare is maximized if the LMTS uses vehicles with capacity  $\mathbf{c}^* = \operatorname{argmax}_{c \in C} \mathbf{SW}(\mathbf{p}^*(c), c, \{\mathbf{m}^j(\mathbf{p}^*(c), c)\}) = 7$  to deliver service. In the rest of this subsection, all analyzes will be performed under  $\mathbf{c}^* = 7$ .

3.2.3. *Impact of prices on each type of passenger consumer surplus*

Next, we analyze passenger behaviors and consumer surplus under the optimal LMTS decisions  $(\mathbf{p}^*, \mathbf{c}^*) \in \operatorname{argmax}_{p \in P, c \in C} \mathbf{SW}(p, c, \{\mathbf{m}^j(p, c)\})$ . In Table 6, for each passenger type  $i \in \{\text{adults, senior citizens, children/students}\}$ , we report the optimal LMTS price  $\mathbf{p}_i^*$ ; the percentage of passengers who purchase fares to use the LMTS,  $\mathbf{PCT}_i$ ; and the expected consumer surplus that a served passenger receives from using the LMTS,  $\mathbf{CS}_i$ .

We make the following observations:

- Roughly 90% of seniors, children, and students and half of the adults benefit from the LMTS. Therefore, our designed LMTS allows a majority of people—and, in particular, most of the people in special groups (senior citizens, children/students)—to benefit from the LMTS.
- The consumer surplus an adult passenger receives is as high as  $0.39/1.5 = 26\%$  of her willingness-to-pay. The consumer surplus a senior, child, or student receives from using the LMTS is more than three times of an adult’s consumer surplus. Therefore, our designed LMTS can substantially improve people’s welfare; in particular, such improvements are significant for senior citizens, children and students.

3.2.4. *Necessity of offering discounts to passengers in special groups*

Next, we explain why offering discounts to special groups of passengers—in this case, 30% off for senior citizens and 50% off for children and students—is beneficial. We consider a counterpart LMTS with no discount for any passenger type, i.e.,  $\theta_1 = \theta_2 = \dots = \theta_l = 1$ . In this counterpart, the LMTS charges an optimal identical price, denoted by  $\mathbf{p}'$ , for all passengers. Let  $\mathbf{PCT}'_i$  denote the percentage of type- $i$  passengers who use the LMTS,  $\mathbf{CS}'_i$  the expected consumer surplus that a type- $i$  passenger receives from using the LMTS, and  $\mathbf{SW}'$  the optimal total social welfare per minute over all sample stations. In Table 7, we report the optimal price in the counterpart LMTS with identical prices; the relative change in the percentage of each passenger type that uses the LMTS from the primary LMTS with type-dependent prices to the counterpart LMTS with identical prices,  $\Delta \mathbf{PCT}_i = (\mathbf{PCT}'_i - \mathbf{PCT}_i) / \mathbf{PCT}_i$ ; the relative change in consumer surplus that a type- $i$  passenger receives from using the primary LMTS to the counterpart LMTS,  $\Delta \mathbf{CS}_i = (\mathbf{CS}'_i - \mathbf{CS}_i) / \mathbf{CS}_i$ ; and the relative total social welfare change from the primary LMTS to the counterpart LMTS,  $\Delta \mathbf{SW}^* = (\mathbf{SW}' - \mathbf{SW}^*) / \mathbf{SW}^*$ .

**Table 7**  
LMTS with type-dependent prices vs. LMTS with identical prices.

| Passenger type $i$<br>$p'$ (S\$) | 1: Adults<br>0.44 | 2: Senior citizens | 3: Children/students |
|----------------------------------|-------------------|--------------------|----------------------|
| $\Delta PCT_i$                   | 0.03              | -0.04              | -0.07                |
| $\Delta CS_i$                    | 0.04              | -0.05              | -0.07                |
| $\Delta SW^*$                    | 0.004             |                    |                      |

We make the following observations:

- If the LMTS designer does not offer discounts to special groups of passengers, the decrement in the percentage of special groups of passengers who use the LMTS is much higher than the increment in the percentage of adults who are willing to use the LMTS. In addition, by eliminating discount offers, the percentage of consumer surplus loss for a special-group passenger who uses the LMTS is much higher than the percentage of consumer surplus gain for an adult who uses the LMTS.
- Although removing discount offers can increase social welfare, the percentage of social welfare increment is almost negligible relative to the negative effects of removing discount offers on special groups of passengers.

The above analysis indicates that it is essential that special groups of passengers be allowed cheaper prices for using the LMTS.

### 3.2.5. Impact of total social welfare generated by the LMTS

Next, we analyze social welfare. As reported in the third column in Table 5, implementing the LMTS over all sample stations that we study in this paper can cause expected social welfare of as much as  $SW^* = S\$ 67.73/\text{min}$ . Recall that the average total number of passengers discharged from all sample stations in each minute is 196.11. Therefore, LMTS generates expected social welfare for the city or region of  $SW^*/196.11 = S\$ 0.35$  for each passenger. As reported by the Singapore Land Transport Authority (2012), average daily ridership for the rail system in Singapore is 2.525 million. Therefore, assuming a hypothetical scenario in which 10% riders may consider last-mile service, the estimated total social welfare the LMTS generates in Singapore per year, denoted by  $SW^*_{LMTS}$ , is estimated as:

$$SW^*_{LMTS} = S\$ 0.35 \times 2.525 \text{ million} \times 10\%/\text{day} \times 366 \text{ days/year} = S\$ 32.3 \text{ million/year}.$$

To evaluate how significant this is, we compare  $SW^*_{LMTS}$  with a benchmark defined as Value Added in industries in Passenger and Freight Rail Transport, Passenger Land Transport, & Aerial Cableways in Singapore in 2012, denoted by  $GDP_{Land\ Transport}$ . This benchmark is a proxy of the contribution of the Singapore land transportation service industry to Singapore's GDP. As reported by the Singapore Department of Statistics (2015),  $GDP_{Land\ Transport} = S\$ 1836.5$  million/year. We have

$$\frac{SW^*_{LMTS}}{GDP_{Land\ Transport}} = \frac{S\$ 32.3 \text{ million/year}}{S\$ 1,836.5 \text{ million/year}} = 1.76\%.$$

This result demonstrates that the LMTS can play a significant role in improving social welfare in Singapore.

### 3.2.6. Potential deployment in other places

In this numerical experiment, we evaluate the potential benefits of LMTS using Singapore as an example. In the optimal solutions with specific input parameters, the average operating cost to provide last-mile service for each served passenger is S\$ 0.61. The price for adults, senior citizens, and children/students is S\$ 0.47, S\$ 0.33, and S\$ 0.24, respectively. Therefore, the average subsidy for each trip for each user is S\$ 0.14, S\$ 0.28, and S\$ 0.37, respectively. Considering that the current actual subsidy for public transportation is 4 billion per year in Singapore (according to the Singapore Parliament (2015), which translates to S\$ 4.34 per public transportation ridership), this amount is promising. In addition, if we put a higher weight on the service provider's profit in the optimization model, the subsidy needed for LMTS will certainly be decreased further.

LMTS could potentially be deployed in other cities in Asia or Europe. First, many cities in Asia and Europe have huge populations, which could generate high utilization for public transportation systems such as subway and bus systems and, in turn, generate a large number of potential passengers for the LMTS. Second, many large cities in Asia and Europe—e.g., Hong Kong, Paris, and Tokyo—suffer from serious traffic congestion. Some governments attempt to control private-car ownership to mitigate congestion and reduce air pollution. Consequently, public transportation systems, including the LMTS, are becoming more and more important.

The LMTS could also be deployed in some U.S. cities. First, the density of public subway stations is much lower in many U.S. cities than in Asian cities such as Singapore and Hong Kong. Therefore, the actual travel distance for last-mile service in such U.S. cities is significantly longer than the travel distance for last-mile service in Asia. In other words, the so-called last-mile service in some U.S. cities could actually be the last-several-miles service. Second, conventional taxis in many U.S. cities are much more expensive than in Asia. These facts could allow the valuation ( $v$ ) of last-mile service in the optimization

model for potential passengers in the U.S. to be much higher than the valuation of last-mile service in Asia. It is also possible that LMTS service providers in the U.S. could charge a higher price, which might induce good overall social welfare without considerable subsidies.

### 3.2.7. Comparisons between LMTS and other ride-sharing systems

In many cities around the world, ride-sharing services are available through an online matching intermediary (e.g., Uber, Lyft, and Didi). Conventional taxis also operate in almost every city. We compare these ride-sharing services with the LMTS from the perspectives of pricing and usage. Our proposed LMTS is more suitable for last-mile service than other ride-sharing services, including the online matching intermediary and conventional taxis.

From the pricing perspective, the online matching ride-sharing intermediary has the following features: First, the price is dynamically adjusted in terms of when and where a service is requested (i.e., dynamic pricing). The algorithm for the dynamic pricing is a black box that is not transparent to the public, so the real-time price is unpredictable for passengers. Second, as a commercial company, when the intermediary sets the price, its ultimate objective is to maximize its own profit, rather than maximize social welfare. Therefore, the intermediary has an incentive to extract as much consumer surplus as possible. Third, the price not only affects the demand side—i.e., the number of passengers who request the service—but also affects the supply side dynamically—i.e., the number of drivers who are willing to provide the service.

The conventional taxi system has the following features: First, the price normally consists of a base component and an incremental component that depends on actual travel distance and actual congestion time. Second, the base rate and incremental rate are also time-dependent in some cities—e.g., a 25% surcharge during morning peak hours and a 50% surcharge at night in Singapore, according to [SMRT \(2017\)](#). Third, although the pricing formula is public information, a passenger can only learn the exact fare after completing the trip—i.e., there is uncertainty about how much a passenger will have to pay when requesting the service.

By comparing the pricing scheme to these common ride-sharing systems, the LMTS has the following features: First, the price for LMTS is fixed for each type of passenger at each metro station, and the price is independent of when and where the service is requested. This is public information, and passengers know how much they have to pay for the service before requesting it. Second, since the motivation to provide LMTS is to make public transportation more attractive, the objective in the pricing optimization model is to maximize social welfare, rather than maximizing the service provider's profit. Third, waiting time for last-mile service has been considered in LMTS pricing; in the paper, we describe our use of a batch arrival, batch service, multi-server queueing model to approximate the waiting time for last-mile service at each station.

From the usage perspective, the online matching ride-sharing intermediary has the following features: First, a service request can only be made through a smartphone app, and many children and older people do not have smartphones or are not accustomed to using a smartphone to request a ride-sharing service. Second, all service requests are dispatched and matched with vehicles through the centralized intermediary; hence, passengers must wait passively to be matched. The dispatch algorithm is a black box to passengers. The service sequence may be determined for a profit purpose, instead of following some fairness rules such as FCSF (first-come, first-served) in the LMTS. Third, the origin and destination for a travel request could be anywhere, and many drivers may not be willing to accept a last-mile request because of its short distance. Since drivers cruise around the city, rather than waiting at public transportation stations, a passenger who needs LMTS from a metro station typically has to wait for a driver who is willing to provide last-mile service.

A conventional taxi system has the following features: First, as with vehicles in the online matching ride-sharing platform, some taxis may not be willing to give rides for last-mile request. Second, since taxi stands are limited—or may not even be present at some public transportation stations—a passenger who needs last-mile service must typically walk to a main street or a shopping mall to hail a taxi. However, this is often difficult for core users of LMTS, such as children, seniors, and the disabled. Third, the waiting time for a taxi is unpredictable, which may be perceived as even longer when the passenger is only traveling a short distance.

By comparing usage to these common ride-sharing systems, LMTS offers the following features: First, a fleet of vehicles is dedicated to last-mile service in the LMTS, and vehicles are waiting for passengers at the single origin (the metro station). Second, if no vehicle is available for the time being, passengers know that vehicles are delivering last-mile service and will return after completing the current trip. Passengers join a single queue with an FCFS rule, and are able to reliably predict their waiting time.

## 4. Concluding remarks

In this paper, we consider a Last-Mile Transportation System (LMTS) with multi-type passengers—adults, senior citizens, children, and students. We propose a constrained nonlinear optimization model to determine the price for the passengers, the last-mile service vehicle capacity, and the service fleet size (number of vehicles) in each last-mile region to maximize the social welfare generated by the LMTS.

Our model is numerically implemented by using real data from Singapore. We show the optimal annual social welfare (measured in Singapore dollars) gained by implementing LMTS countrywide is large. We analyze a counterpart LMTS in which the LMTS designer sets the identical price for all types of passengers. We find that in the absence of price discounts for special groups of passengers, social welfare undergoes almost no change. Consumer surplus for LMTS passengers in special groups, however, suffers significantly.

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