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Search for Optimal CEO Compensation: Theory and Empirical Evidence

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Abstract

We integrate an agency model with dynamic search equilibrium to study three important issues concerning executive compensation. We show that 1) the equilibrium pay-to-performance sensitivity depends positively on a firm's specific risk, and negatively on its systematic risk, which offers a plausible explanation for the inconclusive empirical relationship between the pay-to-performance sensitivity and a firm's total risk; 2) a growing economy simultaneously induces the growth in executive compensation and firm size; 3) the faster growth of executive compensation relative to the growth of firm size in the past decade is mostly due to the increase in firms' specific risks.

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1. Introduction

Executive compensation in a typical US firm has increased by a large amount in the last few decades. This large increase has generated an intense debate in the public and the academia on whether CEOs are over-compensated for firms' performance that is beyond their efforts. Although the increase in firm value contributed to the increase in executive pay, a closer look at the data reveals two interesting facts (see section 4 for a detailed description of the data). First, incentive pay, as the predominant component of executive pay, has increased more rapidly than the increase in firm value. From 1993 to 2005, median incentive pay increased by 187.2%, as opposed to 54.5% increase in median firm value, and its share in total pay increased from 64.3% to 78.2%. Second, and partly as a result of the first fact, total executive pay has outpaced firm value. The ratio between CEO pay (in millions) and firm value (in billions) increased from 1.36 in 1993 to 2.08 in 2005. These facts suggest that an important step to understanding the increase in executive compensation is to understand the pay to performance sensitivity, which is abbreviated as the PPS. What are the main factors that determine the PPS?

Two factors are intuitively important for the PPS, both arising from the notion that executive contracts should be designed to maximize a firm's value in a market economy. One is job mobility of CEOs. When different firms compete for CEOs, each firm has incentive to design contracts to increase the probability of retaining a CEO. Thus, changes in the market conditions can affect the PPS by affecting the severity of competition in the CEO market. Another factor is the risks faced by a firm. By switching from one firm to another, a CEO can change the amount of firm-specific risks that he is exposed to, but cannot change the amount of systematic risks. Thus, the PPS should depend on the two types of risks differently.

To address these issues regarding executive pay, we integrate a principal-agent model into search theory to determine incentive contracts in a market equilibrium, and then empirically evaluate the model. The use of search theory endogenizes CEOs' outside options and enables our principal-agent model to emphasize the distinction between firm-specific and systematic risks. The integrated model captures the intuitive mechanism that the competition among firms for CEOs affects optimal incentive contracts in the equilibrium by affecting a CEO's incentive to participate in a firm.

In our model, there are many firms and CEOs in the economy. In each period, a firm offers a compensation package, which includes salary plus a profit-sharing payment. The CEO decides to accept the offer only if it yields an expected utility higher than his reservation utility. The utility from a compensation scheme partly depends on a match-specific risk whose realization is only observable to the CEO. The match-specific risk can be understood as the match quality between the CEO and the particular firm. Because only the CEO observes the match-specific shock and because the CEO can search for another job, an optimal contract in this paper cannot induce the CEO to participate in all states of the world, in contrast to standard principal agent models. If the CEO accepts the contract, he chooses the effort level. If he rejects the contract, he will search for a new job. After his acceptance decision, a publicly observable economy-wide shock occurs. The firm's output depends on the match-specific risk, the aggregate productivity shock, and the CEO's effort. The incentive contract can be contingent on the firm's output and the productivity shock, but not directly on the unobservable match-specific risk and the CEO's effort.

One distinct feature of our model is that a CEO's reservation utility is endogenous. Due to the competition among firms for CEOs, a CEO's reservation utility depends on the possibility of getting a new job in the future and the compensation of the new job. This link between a CEO's reservation utility and other firms' compensation contracts implies that a market equilibrium must determine all firms' contracts and CEOs' reservation utilities simultaneously. We focus on a stationary and symmetric equilibrium in which all firms offer the same type of contracts.

The equilibrium incentive contract exhibits new and important features, which are confirmed by our empirical tests using executive compensation data from 1992 to 2005. First, the equilibrium PPS depends positively on a firm's specific risk, and negatively on the systematic risk. For example, when the gross domestic product (GDP) growth is used to proxy aggregate productivity, an increase of one standard deviation in firms' specific risks raises the incentive of the current year stock and option grants by $\$535,000$ while an increase of one standard deviation in firms' systematic risks decreases that by $\$119,476$. The intuition for the positive effect of a firm's specific risk on the PPS is as follows. A CEO will only work for the firm if the realized match quality is higher than a cut-off level. This implies that, to a CEO, a firm's profit is analogous to a call option written on the match quality and thus increases with the match-specific risk. The CEO prefers a positive dependence of the PPS on the match-specific risk since he can receive a higher profit-sharing payment. In contract, the systematic risk is common to all firms. Given a fixed total compensation, the CEO prefers a contract with a relatively high salary and a relatively low profit-sharing ratio, so as to reduce the downside effect of a large systematic risk.

Second, total compensation, as well as firm value, increases with aggregate productivity and systematic risks, and decreases with firm-specific risks. Third, the relative pace of growth in total compensation to firm value depends positively on firm-specific risks and negatively on systematic risks. The reason is that the improved aggregate conditions make a firm more profitable, hence the opportunity cost of leaving a CEO position vacant is higher than in normal circumstances. To retain the existing CEO or to attract a new CEO, a firm needs to increase the pay. Therefore, a growing economy induces both the pay and firm value to increase. On the other hand, a higher firm specific risk leads to a higher profit-sharing payment, which, in turn, leads to a lower firm value. Thus, in a growing economy with a higher specific risk, total compensation will grow faster than firm value. Our empirical analysis suggests that, from 1992 to 2005, the growing economy is the contributing factor for the growth in firm value and total pay while the increase in firm-specific risks is the driving force for total compensation to outpace firm value.

Our paper contributes to the labor search literature (e.g., Mortensen and Pissarides, 1994) by integrating incentive contracts into a search model. To the principal-agent literature (e.g., Holmstrom, 1982), our paper contributes in three dimensions. First, we explicitly model CEOs' quitting decisions and study the incentive contracts that induce both the optimal effort and optimal retention. Second, we endogenously determine the effects of aggregate productivity on a CEO's reservation utility. Third, we analyze the optimal contract in a dynamic equilibrium in which firms interact in the CEO job market. This dynamic equilibrium structure contrasts with a typical principal-agent model that analyzes the optimal contract for a single firm in a static setting. With the current set-up, we show that a firm's specific and systematic risks have opposite effects on the PPS. This result is different from a simple negative effect of a firm's total risk on the PPS predicted by a standard principal-agent model. Our result offers a possible explanation to the mixed evidence on the empirical relationship between a firm's total risk and the PPS.[1]

A more specific comparison is with Oyer (2004), who also recognizes that an agent may choose not to participate in a contract in certain states of the world. However, he does not study a market equilibrium; instead, he assumes that the reservation utility is exogenous. Moreover, Oyer (2004) studies broad-based stock option plans for lower-ranked employees. On the ground that such plans have only limited incentive effects on employees, he abstracts from the effort-inducing mechanism.

This paper is broadly related to Murphy and Zabojnik (2007) and Gabaix and Landier (2008) in the attempt to explain the observed increase in executive pay. Murphy and Zabojnik (2004) argue that, as CEOs' general managerial skills become more important, firms are more likely to hire outside CEOs and, hence, the pay has to increase. Gabaix and Landier (2008) use a standard assignment model (e.g., Becker, 1973) to show that an increase in firm size leads to an increase in executive pay because there is positive assortative matching between firm size and CEO quality. Although our paper also explains the increase in executive pay, our focus is on how systematic risks and firm-specific risks affect the PPS through the effort-inducing mechanism.[2]

The rest of the paper is organized as follows. Section 2 describes the model and analyzes an individual firm's optimal compensation while taking other firms' contracts as given. Section 3 characterizes the market equilibrium, the optimal compensation polices and the equilibrium firm size. Section 4 presents the empirical analysis, and Section 5 concludes the paper. Proofs and tables are relegated to the Appendix.

2. Model environment and individual firms' compensation contracts

2.1. Model environment

Consider a discrete-time economy with many firms and many CEOs who are infinitely-lived with a discount factor $\beta \in (0, 1)$. We normalize the measure of CEOs to 1 and the measure of firms to N. In each period, a CEO is either employed and producing or unemployed and searching, while

[1] Core and Guay (1999) and Oyer and Shaefer (2005) find a positive relationship between a firm's total risk and the PPS while Aggarwal and Samwick (1999) document a negative one. Prendergast (2002) and Guo and Ou-Yang (2006) provide alternative explanations for the observed positive relation. A few other studies focus on the relationship between a firm's risk and the PPS from different perspectives. For example, Jin (2002) studies a CEO's portfolio diversification effect on the PPS. Shi (2009) differentiates respondable and non-respondable risk.

[2] Although the effort-inducing mechanism is present in Edmans, Gabaix and Landier (2008), they do not analyze how PPS depends on systematic and specific risks of a firm. They intend to explain the negative relationship between the CEO's effective equity stake and firm size.

a firm is either filled with a CEO or has a vacant CEO position.

Each CEO is assumed to be effort averse. His utility function is characterized by

$$U(W, e) = W - \frac{c}{2} e^2, \tag{2.1}$$

where W is the CEO's income in the period, e the CEO's effort, and $c > 0$ a constant.

A firm is assumed to be risk-neutral. For a firm with a CEO, its profit depends on the aggregate productivity shock y, the match-specific risk x to be described in detail later, and the CEO's effort e. For tractability, we assume that a firm's profit is given by

$$\pi \equiv \pi(y, x, e) = y\sqrt{x}e. \tag{2.2}$$

Note that profits are correlated among firms through the aggregate productivity shock.

Each firm chooses a contract to maximize the expected firm value while taking other firms' incentive contracts and aggregate economic conditions as given. As is standard in the contract literature, we assume that profits and aggregate productivity of the economy are verifiable and contractible. In contrast, the effort level and the match-specific risk are observed only by the CEO and not verifiable, therefore not contractible. To facilitate the description of the market and the decisions, we depict the timing of events in each period in Fig. 1.

At the beginning of each period, firms with vacant CEO positions and job-searching CEOs enter the search market. To gain access to this market, a firm must pay a hiring cost, $H > 0$. Search is modeled as in Mortensen and Pissarides (1994). Denote v and u as, respectively, the numbers of vacancies and job-searching CEOs in the market. The market will yield an aggregate number of matches at the end of the period. For simplicity, this number is assumed to be

$$m(v, u) = Auv/(u + v), \tag{2.3}$$

where the positive constant A measures the matching efficiency in the economy.[3] Denote the tightness of the job market as $\theta = u/v$, the matching probability of a searching CEO as $\lambda \equiv m(v, u)/u$, and the matching probability of a vacancy as $q \equiv m(v, u)/v$. It is easy to verify that

$$\lambda = A/(1 + \theta) \text{ and } q = A\theta/(1 + \theta) = A - \lambda.$$

[3]The specific matching function has constant returns to scale and is strictly concave in the two arguments, u and v. The intuition for the main results of our paper should hold for more general matching functions, but the algebra becomes more complicated.

These expressions reflect the intuitive property that, when there are more searching CEOs per vacancy, the matching probability falls for a searching CEO and increases for a vacancy. Each CEO or firm takes the tightness and matching probabilities as given, because these characteristics depend only on the aggregate numbers of vacancies and searching CEOs.

A firm with a CEO offers an incentive contract at the beginning of the period. Then, a match-specific shock, x, occurs to the particular firm-CEO pair, which is only observable to the CEO. This risk can be understood as the match quality between the CEO and the firm in this period, rather than a permanent characteristic of the firm, the CEO, or the match.[4] After observing the matching quality, the CEO decides whether to accept the contract. After the CEO makes the choice, an aggregate productivity shock, y, occurs. Observing the aggregate shock, the CEO who accepted the contract chooses the effort level to carry out production. He will be paid at the end of the period according to the incentive contract. If the CEO rejects the contract, he must quit, in which case he derives utility B from all benefits and leisure in the current period.

Note that when a CEO quits a firm, the CEO is allowed to search in the current period immediately. However, the corresponding firm cannot immediately enter the search market in the same period, because the firm missed the opportunity to incur the hiring cost H at the beginning of the period to enter the market. Instead, the firm must wait for the next period to enter the search market. This assumption is intended to capture the idea that it takes time for a firm to advertise a job vacancy.

To simplify the analysis, we assume that the match-specific shock is i.i.d. across time and firms, that the aggregate productivity shock is i.i.d. across time, and that the two shocks are independent of each other. To simplify further, we assume that the match-specific risk is uniformly distributed in the interval $[\underline{x}, \bar{x}]$, where $\bar{x} > \underline{x} > 0$. This implies that the mean of x is $\mu_x = (\underline{x} + \bar{x})/2$ and the standard deviation is $\sigma_x = (\bar{x} - \underline{x})/(2\sqrt{3})$. Denote the cumulative distribution function of x as $F_1(x)$ and the cumulative distribution function of y as $F_2(y)$. Note that y need not be uniformly distributed.

In this environment, we first analyze a single firm's optimal incentive contract while taking

[4]A high match quality means that a CEO's talent, experience, education, and personal objective match well in the particular period with the firm's size, the nature of the business, the strategic direction and the organizational culture, and so on. A CEO who is well matched with a firm at one point of time may not be well matched with the firm at another time if the CEO's feature or the firm's situation has changed.

other firms' contracts as given. Later, we will analyze the equilibrium in the market.

2.2. An individual firm's optimal incentive contract

Denote an individual firm i's profit as π_i. The CEO's compensation from firm i is assumed to be a linear contract consisting of a fixed salary and a profit-sharing payment. Denote the contract as $D \equiv (a,\, b)$, where a is the fixed salary and b the profit-sharing ratio. Then the compensation can be stated as $W_i = W(D, \pi_i) = a + b\pi_i$.[5]

Taking other firms' contracts as given, firm i chooses a contract to maximize its expected residual profit, i.e., the profit after paying its CEO. We solve the firm's optimal contracting problem recursively. First, given any contract, we determine the CEO's best response, i.e., the CEO's acceptance decision, and the optimal effort in the case of accepting the contract. Second, we solve for the firm's optimal contract that anticipates the CEO's best response to the contract.

Let us first examine the CEO's optimal choice of effort under an arbitrary contract, D. Denote the value function of an employed CEO who accepts the contract D as $J_E\,(x, D)$, which is measured after the CEO observes the match-specific risk x but before observing the aggregate productivity shock y (see Fig. 1 for the timing). Denote J_S as the value function of a CEO who either does not have a match when exiting the previous period or who has just rejected a contract in the current period. The value function J_S is determined as:

$$J_S = B + \beta \left[\lambda \int_{x'} \max[J_E(x', D'), J'_S] dF_1(x') + (1-\lambda) J'_S \right], \tag{2.4}$$

where β is the discount factor and B is the utility of unemployment benefits and leisure that such a CEO receives. The sum inside the brackets $[\cdot]$ is the CEO's expected value as a searching CEO in the next period. With probability λ, the CEO will get a match in the next period and will then choose whether or not to accept the contract. With probability $(1-\lambda)$, the CEO will fail to get a match in the next period, in which case his value function will be given by J'_S. Throughout this paper, the prime $'$ indicates next-period variables.

The value function for an employed CEO, $J_E(x, D)$, is given by the following Bellman equation:

$$J_E(x, D) = \int \left\{ \max_e \left[W(D, \pi) - \frac{c}{2} e^2 \right] + \beta \int_{x'} \max \left[J_E(x', D'), J'_S \right] dF_1(x') \right\} dF_2(y). \tag{2.5}$$

[5]For the purpose of direct comparison to the standard principal-agent model, we use a linear contract here. Dynamic contracts or renegotiation can be done in future studies.

Here, $\pi = \pi(y, x, e)$ is described by (2.2). The first term in the braces is the CEO's current utility, which is maximized by the choice of effort after observing aggregate productivity, y. The second term is the CEO's continuation payoff in the next period, in which he will choose whether to accept next period's contract, D', or to reject it. Since J_E is defined as the CEO's expected value before observing y in the current period, the expectation with respect to y is taken on the sum of the current and future utilities.

When choosing the effort level (as in the first maximization problem in (2.5)), the CEO takes the profit function in (2.2) as given. After substituting (2.2), we can solve for the optimal level of effort under the given contract $D = (a, b)$ as

$$e^*(D, x, y) = by\sqrt{x}/c. \tag{2.6}$$

Intuitively, the optimal effort depends positively on the profit-sharing ratio b, the realized match quality x, and aggregate productivity y, but negatively on the effort-aversion coefficient c.

Substituting the optimal effort into (2.5) and intergrating over y yields:

$$J_E(x, D) = a + \frac{x}{2c} b^2 \mathbb{E}\left(y^2\right) + \beta \int_{x'} \max[J_E(x', D'), J_S'] dF_1(x'). \tag{2.7}$$

Standard techniques show that the right side of this equation is a continuous and monotone contraction mapping for the function J_E (see Stokey and Lucas with Prescott, 1989). By the contraction mapping theorem, there exists a unique function J_E that solves the above equation. Moreover, since the right-hand side of (2.7) maps functions $J_E\left(., D'\right)$ that are (weakly) increasing in the first argument into functions that are strictly increasing in the first argument, the solution $J_E(x, D)$ is strictly increasing in x. Similarly, the solution $J_E\left(x, D\right)$ is concave in x.

Now we turn to the CEO's acceptance decision, still taking the arbitrary contract D as given. For a CEO with a contract, after seeing the match-specific risk, x, he accepts the contract if and only if $J_E(x, D) > J_S$. Because $J_E(x, D)$ is strictly increasing in x and J_S is independent of x, there exists a unique cut-off match quality, denoted as $x_d\left(D\right)$, such that $J_E(x, D) > J_S$ if and only if $x > x_d(D)$. That is, the CEO's optimal acceptance decision obeys a reservation rule: he accepts the contract if the match-specific quality x exceeds the cut-off match quality $x_d(D)$, and quits otherwise. The cut-off match quality $x_d(D)$ is defined as the solution to the equation

$J_E(x_d, D) = J_S$. To express the cut-off match quality explicitly, let us denote the expected future value for a CEO who accepts the current contract as

$$I \equiv \int_{x'} \max[J_E(x', D'), J'_S] dF_1(x').$$

Note that I is taken as given by both the agent and the firm for the contracting problem in the current period, since it depends only on the future contract and future market conditions. Substituting J_E from (2.7) into the defining equation for x_d, we obtain:

$$x_d(D) = \frac{2c}{b^2 \mathbb{E}\left(y^2\right)} \left(J_S - \beta I - a\right). \tag{2.8}$$

We now analyze the firm's optimal contract. Denote the value function of a firm with a CEO as J_F and the value function of a hiring firm with a vacant CEO position as J_H, both being measured at the beginning of the period (see Fig. 1 for the timing). We first determine J_H as:

$$J_H = -H + q\beta J'_F + (1-q)\beta J'_H. \tag{2.9}$$

The term H is the recruiting cost. With probability q, the firm will be matched by the end of the period, in which case the firm will enter the next period with a CEO. With probability $(1-q)$, the firm will be unmatched, in which case the firm will enter the next period without a CEO.

For a firm with a matched CEO, the CEO's optimal acceptance decision is $x_d(D)$ given by (2.8), and the optimal choice of effort is $e^*(D, x, y)$ given by (2.6). Anticipating such best responses to a contract, the firm chooses the contract $D = (a, b)$ as follows:

$$J_F = \max_{a,b} \int \left\{ \int_{x_d(D)}^{\bar{x}} \left(\hat{\pi} - \hat{W} + \beta J'_F\right) dF_1(x) + \int_{\underline{x}}^{x_d(D)} \beta J'_H dF_1(x) \right\} dF_2(y), \tag{2.10}$$

where

$$\hat{\pi}(D, x, y) \equiv \pi(y, e^*(D, x, y), x) \quad \text{and} \quad \hat{W}(D, x, y) \equiv W(D, \hat{\pi}(D, x, y)) = a + b^2 y^2 x / c.$$

The two integrals inside $\{\cdot\}$ in (2.10) give the value of the firm when the contract is accepted and rejected, respectively. When $x > x_d(D)$, the contract is accepted. The firm obtains the residual profit $(\hat{\pi} - \hat{W})$ in the current period plus $\beta J'_F$ which is the firm's continuation value in the future as a firm with a CEO. If the contract is rejected, the firm enters the next period without a CEO, in which case the value is given by $\beta J'_H$. Since the firm does not observe the match-specific risk, x, and since the contract is offered before y is realized, J_F is independent of x and y.

When choosing the contract for the current period, $D = (a,\, b)$, the firm takes J_H and the future values $(J'_H,\, J'_F)$ as given. Also, the firm anticipates that the CEO's effort e^* and acceptance rule $x_d(D)$ will depend on the contract. Solving the the maximization problem in (2.10) leads to the following optimal contract:

$$b = \frac{1}{2}\left(1 + \frac{x_d}{\bar{x}}\right) \qquad \text{and} \qquad a = \beta(J'_F - J'_H) - b(1-b)^2 \frac{\bar{x}\mathbb{E}(y^2)}{c}. \tag{2.11}$$

2.3. Some properties of a CEO's optimal choices and the optimal contract

The CEO's optimal acceptance decision is given by (2.8), which generates the following probability of contract acceptance:

$$prob\,(x > x_d) = 1 - F_1(x_d) = \frac{\bar{x} - x_d}{\bar{x} - \underline{x}}.$$

Thus, a reduction in the cut-off level x_d translates into an increase in the retention probability of the CEO. For any given I and J_S, suppose $J_S - \beta I > a$, so that the cut-off level x_d is positive. In this case, the optimal cut-off level x_d and the retention probability depend on the contract as follows. First, the cut-off level decreases with the fixed salary a and the profit-sharing ratio b. This is because a higher a or b makes the compensation more generous to the CEO, thus increasing the retention probability. Second, $\partial^2 x_d/\partial b^2 > 0$ and $\partial^2 x_d/\partial a^2 = 0$. The result $\partial^2 x_d/\partial b^2 > 0$ indicates that the marginal benefit of increasing the profit-sharing ratio on retention is diminishing. This result arises because a higher b induces higher effort but the marginal disutility of effort to the CEO is increasing. In contrast, the marginal benefit of increasing the fixed salary on retention is constant, as indicated by the result $\partial^2 x_d/\partial a^2 = 0$, because an increase in a increases the CEO's compensation independently of the effort level. Thus, when b is already high, increasing the fixed salary is more efficient in achieving retention than increasing b.

Moreover, the cut-off level x_d and the retention probability depend on the market conditions through J_S and I. If the market is good for CEOs, the value of search, J_S, is high, in which case x_d is high and the retention probability is low. On the other hand, if staying on the job gives the CEO a high payoff in the future, i.e., if I is high, then x_d is low and the CEO is likely to stay with the firm. In the equilibrium analysis later, we will link these market conditions to other firms' contracts and basic parameters of the economy.

The optimal contract, given by (2.11), has the following features. First, b is less than 1 in general, which is different from the standard result of a textbook agency model with one firm and one agent (e.g., pages 27-28 in Murphy 1999). That is, $b = 1$ for a risk-neutral agent and $b < 1$ for a risk-averse agent. Therefore, for a risk-neutral agent, it is optimal for the firm to sell it to the CEO provided that the latter is not liquidity constrained. This standard result for a risk-neutral agent does not hold in our model because of the moral hazard problem associated with quitting. The CEO can unilaterally decide to quit after observing the match quality that is not observed by the firm. Even if the firm offers $b = 1$ to the CEO, the CEO can still reject the contract and avoid the consequence of a bad realization of x. In this sense, setting $b = 1$ is not equivalent to selling the firm to the CEO in our model. To induce the CEO to accept a contract with $b = 1$, the amount of payment the firm receives (i.e., $-a$) would be too low to be optimal.[6]

Second, the fixed salary increases with the firm's opportunity cost of leaving the CEO position vacant in the next period, which is given by $\beta\left(J'_F - J'_H\right)$. This opportunity cost depends on the market conditions, and hence is linked to other firms' contracts.

Finally, both the firm's optimal contract and the CEO's optimal choices can be determined as functions of the market conditions. Fig. 2 depicts the unique solutions for $x_d\left(D\right)$ and b. The upward sloping curve is the firm's optimal choice of b, given by (2.11), and the downward sloping curve is the CEO's optimal choice x_d, given by (2.8). The intersection of the two curves is the equilibrium pair (b, x_d), as functions of (J_S, J_E, λ).

3. Optimal contracts in a market equilibrium

In the above analysis, market conditions, such as the matching rates and future payoffs, are taken as given. We determine them in a market equilibrium below.

[6] In the current setting, a CEO is risk-neutral in income and effort averse. We show that b is less than 1 in equilibrium. It is easy to show that b will be even smaller if the CEO is also risk averse in income.

3.1. Definition and existence of a market equilibrium

To begin with, let us determine the law of motion of the measure of searching CEOs. In a period, the measure of searching CEOs is u and the employed CEOs is $1-u$. Since each searching CEO gets a match with probability λ and accepts the contract with probability $[1-F_1(x_d)]$, the flow from searching CEOs to employed CEOs is $u\lambda[1-F_1(x_d)]$. Since each employed CEO quits with probability $F_1(x_d)$, the flow from employed CEOs to searching CEOs is $(1-u)F_1(x_d)$. Thus, the measure of searching CEOs at the beginning of the next period is

$$u' = u + (1-u)F_1(x_d) - u\lambda[1-F_1(x_d)]. \tag{3.1}$$

We focus on a stationary and symmetric market equilibrium. Such an equilibrium consists of individual firms' choices (a, b), other firms' choices $(\bar{a}, \bar{b})$, CEOs' choices (e^*, x_d), and value functions (J_E, J_S, J_F, J_H) such that the following requirements are satisfied:

(i) Given the firm's (a, b) and other firms' $(\bar{a}, \bar{b})$, the choices e^* and x_d are optimal for a CEO.

(ii) Given $(\bar{a}, \bar{b})$ and a CEO's best response functions, the firm's choices (a, b) are optimal.

(iii) The value functions satisfy (2.5), (2.4), (2.10) and (2.9).

(iv) Competitive entry of firms requires the benefit of hiring a CEO to be equal to the cost of hiring. That is, $\beta q(J'_F - J'_H) = H$, and hence $J_H = 0$.

(v) Symmetry requires $(a, b) = (\bar{a}, \bar{b})$ and $x_d = \bar{x}_d$.

(vi) Stationarity requires $u' = u$, $J'_F = J_F$, $J'_H = J_H$, $J'_E = J_E$ and $J'_S = J_S$.

It is important to note that the symmetric equilibrium only indicates firms and managers being homogeneous ex ante. They are heterogeneous ex post in productivity.[7] That is, after firms and managers are matched, each matched pair draws a level of match-specific productivity from a continuous distribution that generates heterogeneity across matches. After the match, a manager chooses whether to keep the match or to break up with the firm, using a reservation rule on the match-specific productivity. Such a reservation rule is quintessential in search theory.

[7] It is empirically important to distinguish ex post heterogeneity emphasized by our model from that in Gabaix and Landier (2008). As any assignment model, Gabaix and Landier (2008) focuses on heterogeneity in the characteristics of firms and managers that are observable and contractible ex ante (i.e., before firms and managers are paired together). We focus on match-specific heterogeneity occurring ex post that is non-contractible. It is well known that a large fraction of the wage differential among workers cannot be explained by observable heterogeneity but may be attributed to match-specific heterogeneity (see Mortensen 2005). This is also likely to be the case for managers. An excellent CEO in a mining firm may or may not be a good CEO in a software firm.

Moreover, on the firm's side, it is not just a matter of getting any manager. Instead, it is to keep a manager who has a high match-specific productivity with the firm. The firm designs contracts to retain such a good match and to induce effort.

Based on the equilibrium definition, we solve for equilibrium values of (a, b, x_d), (J_E, J_S, J_F, J_H), and (q, λ) through a set of equations presented in Appendix A. In particular, we show that there exists a unique non-zero solution for b^* if the unemployment benefit satisfies the condition, $B \in [B_1, B_2]$, where B_1 and B_2 (with $B_2 > B_1$) are constants given in Appendix A. We will maintain this condition throughout the analysis.

3.2. Equilibrium incentive contract and firm size

Given the unique solution b^*, we can express the equilibrium salary a^* and firm value J_F^* as

$$a^* = b^*(1-b^*)^2 \frac{\mathbb{E}(y^2)\bar{x}}{c\sqrt{3}\sigma_x}\left(\beta\bar{x} - \sqrt{3}\sigma_x\right) \qquad \text{and} \qquad J_F^* = b^*(1-b^*)^2 \frac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}. \tag{3.2}$$

Since J_F^* is the value of a producing firm with a filled CEO position, we can interpret it as the size of the firm. The following proposition states the effects of aggregate productivity, the systematic risk and the firm specific risk on the optimal contract $D^* = (a^*,\, b^*)$ and the equilibrium firm value (please see detailed comparative statics in Appendices B).

Proposition 3.1. *In equilibrium, the optimal incentive contract possesses the following features:*

1) The profit-sharing ratio, b, decreases with expected aggregate productivity $\mathbb{E}(y)$ and the systematic risk σ_y. It increases with the match-specific risk σ_x under $b < 2/3$.

2) The salary, a, increases with expected aggregate productivity $\mathbb{E}(y)$ and the systematic risk σ_y. It decreases with the match-specific risk σ_x under $b < 2/3$.

3) The equilibrium firm value, J_F^*, increases with expected aggregate productivity $\mathbb{E}(y)$ and the systematic risk σ_y. It decreases with the match-specific risk σ_x under $b < 2/3$.

Below we explain the equilibrium results in Proposition 3.1:

The effects of expected aggregate productivity, $\mathbb{E}(y)$. With high expected aggregate productivity, a firm has strong incentive to fill the CEO position since its expected profit from production is high. Thus, the opportunity cost of leaving a vacant CEO position is high. To induce the CEO's participation, the firm needs to offer a higher retention-inducing pay, i.e., the salary. Also, aggregate productivity y and the CEO's effort are complementary to each other in the firm's profit function. When aggregate productivity is higher, the CEO has stronger incentive to exert effort for a given profit-sharing ratio. Put differently, higher aggregate productivity reduces the firm's implicit cost of inducing effort. As a result, the firm can reduce the profit-sharing ratio and still induce the CEO to exert effort. Consequently, higher aggregate productivity, accompanied by a lower profit-sharing ratio, still leads to a higher firm value.

The effects of the firm's specific risk, σ_x, and systematic risk, σ_y. To explain why b increases with σ_x,[8] recall that a CEO works for a firm only if the match quality is higher than a reservation value. That is, the firm's profit is analogous to a call option written on the match quality with a strike price being the reservation value, and hence it increases with the volatility of the match-specific risk. Naturally, a CEO prefers a positive dependence of the profit-sharing ratio on the specific risk since he can receive more profit sharing payment. Also, a higher σ_x is more likely to result in a CEO accepting the contract with a higher realized match quality. To induce a better effort, the firm is willing to offer a higher profit-sharing ratio because better effort leads to a higher profit. However, the increase in profit due to the increased effort is smaller than the increased profit sharing payment due to the higher pay-to-performance ratio. Therefore, the firm value decreases with σ_x. As for the systematic risk, it is common to all firms and is taken as given by all CEOs. In order for a firm to induce the optimal effort from the CEO and at the same time to provide partial insurance to the effort-averse CEO, the firm offers a higher salary and a lower pay ratio when the aggregate risk is higher. The lower pay ratio leads to a higher firm value.

It is important to note that a traditional principal-agent model is unable to distinguish the opposite effects of the systematic risk and the firm-specific risk on the profit-sharing ratio. Instead, it predicts the profit-sharing ratio decreases as the firm's total risk increases.

3.3. Relative size of total compensation to firm value

Based on the equilibrium incentive contract and the firm value presented in Proposition 3.1, we now discuss the relative growth between the total compensation and firm size. To this end, we

[8] Bhattacharyya and Lafontaine (1995) show similar results in a franchising setting. Zabojnik (1996) also shows a possible positive relationship between the total risk embedded in a firm's production function and the pay-to-performance sensitivity if the agent's disutility of effort satisfies certain conditions.

express the equilibrium salary in terms of the equilibrium firm value as $a^* = J_F^* \left(\beta - \frac{\sqrt{3}\sigma_x}{\bar{x}}\right)$.[9] From the expression for a^*, we can obtain the expected total compensation as

$$W^* = a^* + b^* \mathbb{E}(\pi) = J_F^* \left[\frac{b^2}{1-b} + \left(\beta - \frac{\sqrt{3}\sigma_x}{\bar{x}}\right)\right].$$

To investigate the size of total compensation relative to firm value, J_F^*, we denote the ratio between the expected total pay and firm value as $R_{pay/size}$. It is easy to show that

$$R_{pay/size} = \frac{a^*+b^*\mathbb{E}(\pi)}{J_F^*} = \frac{b^2}{1-b} + \left(\beta - \frac{\sqrt{3}\sigma_x}{\bar{x}}\right) \qquad \text{and} \qquad \frac{\partial R_{pay/size}}{\partial b} = \frac{b(2-b)}{(1-b)^2} > 0.$$

Thus, a higher profit-sharing ratio leads to a higher ratio of the expected total pay to firm size. Given the dependence of $R_{pay/size}$ on the profit-sharing ratio, we can derive the following corollary (please see detailed comparative statics in Appendix B).

Corollary 3.2. *The equilibrium ratio of the total expected pay to firm size, $R_{pay/size}$, decreases with expected aggregate productivity $E(y)$ and the systematic risk σ_y. The effect of the match-specific risk σ_x on $R_{pay/size}$ is positive when $\partial b/\partial \sigma_x > \mu_x / \left[\sqrt{3}(\mu_x + \sqrt{3}\sigma_x)^2\right]$.*

The intuition for the above results can be obtained based on the intuition provided for the optimal contract in Proposition 3.1. For example, when expected aggregate productivity, $E(y)$, is high, a firm has strong incentive to fill the CEO position since its expected profit from production is high. Consequently, the firm offers a higher salary and a lower profit-sharing ratio. This lower profit-sharing ratio will increase the firm value and, at the same time, reduce the incentive pay to the CEO. In this case, the ratio $R_{pay/size}$ decreases because the increase in the expected total pay is smaller than the increase in the firm value. Also, when the match-specific risk is high, it is optimal for a firm to offer a higher profit-sharing ratio, as explained earlier. This higher profit-sharing ratio will increase the total pay through the increase in the incentive pay. At the same time, it reduces the firm value. Hence, the ratio $R_{pay/size}$ will be higher.

[9] Unlike Gabaix and Landier (2008) who take firm size as given and show that an increase in firm size can lead to the rise in the executive pay, we show that a growing economy, an increase in systematic risk or a decrease in firm specific risk can simultaneously increase the equilibrium salary and firm size.

4. Empirical analysis

The objective of our empirical analysis is three-fold: 1) to verify our theoretical predictions on the PPS, annual compensation and firm size; 2) to clarify the mixed evidence on the relationship between firms' risks and the PPS; 3) to provide new evidence on the relative growth between executive compensation and firm size. Specifically, we test the following three predictions based on Proposition 3.1 and Corollary 3.2.

> *Prediction 1:* The PPS, b, decreases with aggregate productivity and a firm's systematic risk, and increases with the firm's specific risk.
>
> *Prediction 2:* Annual compensation and firm size increase with aggregate productivity and a firm's systematic risk, and decrease with the firm's specific risk.
>
> *Prediction 3:* The relative growth of total pay to firm size increases with a firm's specific risk and decrease with the firm's systematic risk and aggregate productivity.

4.1. Data and definitions of empirical variables

Executive compensation data are retrieved from the ExecuComp for the period of 1992 to 2005. Firm characteristics and returns are obtained from the COMPUSTAT and CRSP. We exclude financial and utility firms. Our final sample consists of 10,837 firm-year for 2,432 firms and 4,010 executives.

To conduct the intended empirical analysis, we first identify the empirical measures for the PPS b, salary a, the total compensation, firm size and $R_{pay/size}$.

As discussed by Murphy (1999), a typical compensation package includes salary, bonus, and restricted stock and option grants. Since most incentive payments are related to a firm's equity, we therefore focus on the PPS related to stock and option grants.[10] Following Jensen and Murphy (1990), we define b as the change in the value of CEO compensation with respect to a change of \$1000 in shareholders' wealth. This measure is widely used in the existing literature (e.g. Aggarwal and Samwick 1999). For our empirical analysis, we calculate two versions of b. The first

[10]This interpretation is consistent with the current model because the equilibrium profit π is proportional to J_F (the value of an operating firm) and hence the payment $b\pi$ in the model is proportional to bJ_F (which is equivalent to an ownership sharing payment).

is calculated based on the current year stock and option grants. We call it the new equity incentive. Since ExecuComp data provide detailed information on these grants, it is straightforward to obtain b. The second version of b is computed from the accumulated stock and option grants up to the current year. We call it the total equity incentive. ExecuComp data offer no details on past option grants prior to 2005, which makes it difficult to calculate b. To overcome this, we use Core and Guay's (2002) one-year approximation method to compute the total equity incentive.

Salary, a, is set to be the annual salary paid to executives. Total compensation is taken as the flow compensation (TDC1) which consists of salary, bonus, other annual (short-term) compensation, total value of restricted stock granted, total value of stock options granted, long-term incentive payouts and other miscellaneous compensation. $R_{pay/size}$ is calculated as the ratio between annual total compensation and firm size, where firm size is proxied by either the firm's total assets or its market capitalization.

We then formulate three major explanatory variables: aggregate productivity, a firm's systematic risk and specific risk. Aggregate productivity is proxied by GDP and the commercial paper spread, the latter of which is defined as the difference between the annualized rate on three-month commercial paper and the three-month T-bill rate (please see Friedman and Kuttner 1993, Korajczyk and Levy 2003). Intuitively, a high GDP indicates a good economy. Also, Bernanke and Blinder (1992) suggest that a high commercial paper spread at the beginning of the year signals a bad economy since it tends to rise sharply during credit crunches. Therefore, in the regression analysis, we use the negative lagged commercial paper spread as a proxy for aggregate productivity. The annual GDP growth data and the commercial paper spreads are retrieved, respectively, from the websites of the Bureau of Economic Analysis (www.bea.gov/beahome.html) and the Federal Reserve Board (www.federalreserve.gov/).

Following Core and Guay (1999), a firm's risk is proxied by the volatility of its stock returns.[11] A firm's total risk is the volatility of stock returns over the 60 months prior to the fiscal year.

[11] We also consider the volatility of dollar returns as an alternative measure proposed by Aggarwal and Samwick (1999). The reason for them to use this measure is to ensure that risks are expressed in dollars since a firm's profit in their model is the sum of the executive's effort and the noise term. However, in our model, a firm's profit is the product of the aggregate variable, the firm's specific shock variable, and the executive's effort. If the executive's effort has the same measure as the profit, then the aggregate variable and the match-specific shock variable do not have to be measured in dollars. Therefore, stock return volatilities are proper measures for our test. Moreover, the correlations among the firm's total dollar risk, its systematic risk and specific risks are higher than 0.92. Such high correlations create multicollinearity problem for all regressions.

Its beta is obtained from the market model using the same set of monthly return data. A firm's systematic risk is equal to the firm's beta multiplied by the stock market risk, while the firm's specific risk is the square-root of total return variance minus the systematic return variance.

We take two additional steps to bring the model and the data together. First, in order to control for heterogeneity that exists in the data but not in our model, we include other control variables such as the executive's age and tenure, firm size, and firm growth. Tenure is defined as the number of years a person has been a CEO in the firm. A firm's growth is proxied by its sales growth while the size is proxied by its asset value or market capitalization. Second, it is well known that there are outliers in executive compensation. To reduce the effect of outliers on the empirical results, we winsorize the data of executive compensation and firm characteristics at the 1% and 99% levels.

Table 1 provides summary statistics for compensations and characteristics of the executives, characteristics of firms, and macroeconomic variables representing aggregate productivity. Panel A of Table 1 shows that the average annual salary for a CEO is about \$627,000, which is almost equal to the average annual bonus \$640,000. However, the median annual salary \$572,000 is much higher than that of the bonus \$375,000, indicating that bonuses are more skewed toward the high end. Similar patterns are observed for total pay. In particular, the average total compensation is \$3,991,000, which is about twice of the median total pay but about one-tenth of the maximum total pay. It is worth noting that the average total pay is more than six times of the average annual salary, indicating that the main income for an executive is from equity-related compensation. The average new equity incentives granted for a fiscal year is \$2.10 with respect to the \$1000 change in shareholders' wealth, compared to the average accumulated total equity incentives \$27.56. An average executive is almost 56 years old and stays with a firm for slightly more than eight years. The youngest executive is 29 years old while the oldest is 90. The longest tenure is 38 years, in contrast to the shortest job duration of five months.

Summary statistics of firms' characteristics suggest that firms in the sample are skewed toward large sizes. In particular, the average market capitalization is \$5,947 million, almost six times as large as the corresponding median value, \$1,196 million. The average asset value is \$4,283 million, almost four times as large as the median asset value, \$1,074 million. The average firm's total risk represented by the return volatility is 45%, which is slightly higher than the median 39%. The average firm's systematic risk is 15%, about one-third of the average total risk.[12]

During the sample period of 1992 to 2005, the average GDP in the United States is $9.09 trillion, compared to the minimum $6.34 trillion and maximum $12.49 trillion. The standard deviation is $1.93 trillion, indicating a small fluctuations in GDP during the sample period. On the other hand, the commercial paper spread is much more volatile. The commercial paper spread is averaged at 23 basis points with a standard deviation of 12 basis points.

Table 2 presents the correlations among the explanatory variables. Clearly, the commercial paper spread and GDP are negatively correlated with a correlation of -0.544. Also the correlation between asset value and market capitalization is 0.799, suggesting that the empirical results using either to proxy for firm size should be very similar. Note that most of the correlations among the explanatory variables are very small, suggesting that the regressions do not have the multicollinearity problem.

4.2. Test of Prediction 1: Effects of firm's specific and systematic risks on the PPS

To empirically verify that the PPS depends on firm specific and systematic risks differently, we run the following regression:

$$\begin{aligned} b = a_1 + a_2(\text{GDP \% } \mathit{or} \text{ NCP spread}) + a_3\text{Firm-specific risk} + a_4\text{Firm-systematic} \\ \text{risk} + a_5\text{Age} + a_6\text{Tenure} + a_7 \log(\text{Firm size}) + a_8\text{Firm growth} + \varepsilon, \end{aligned} \tag{4.1}$$

where aggregate productivity is proxied by the GDP growth (hereafter GDP %), or the negative lagged commercial paper spread (hereafter NCP).[13] The reason that we use GDP growth instead of GDP as the explanatory variable is because the PPS is a percentage variable, not a level variable. The executive's age and tenure, firm size, and firm growth are included as control variables. Equation (4.1) is performed with OLS and median regressions. As indicated in the previous subsection, a firm's growth is proxied by its sales growth while firm size is proxied by its asset value or market capitalization. Since the empirical results are qualitatively the same, to save space, we only report the results in Table 3 for the case in which the asset value is used as size proxy. Panel A presents the results of the OLS regression and median regression for new

[12]To reduce the impact of the skewed data, we use median regressions for our empirical tests, in addition to OLS.
[13]To simplify the language, the phrase "commercial paper spread" from this point on refers to the "negative lagged commercial paper spread".

equity incentives, while Panel B presents the corresponding results for total equity incentives. The main findings are as follows.

First, regardless of whether the GDP growth or NCP is used to represent aggregate productivity, the value of R^2 of the regression is similar. This suggests that the GDP growth and the commercial paper spread are equally good proxies for aggregate productivity.

Second, whether we use new equity incentives or total equity incentives, the regressions confirm that aggregate productivity has a negative effect on b. The coefficients are all significant at 1% level (see Panels A and B). The estimated coefficients are generally larger in the OLS regression than in the median regression due to the effect of outliers. The impact of aggregate productivity on the PPS is economically significant. For example, the OLS regression suggests that an increase of one standard deviation (or 1%) in GDP will reduce the total equity incentive by $\$935,522$ $(= 0.913 * \$5,947\text{million}*17.23\%/1000)$ under OLS while a decrease of one standard deviation (or 12 basis points) in commercial spread will reduce the total equity incentive by $\$2,028,842$ $(= 0.165 * 12 * \$5,947\text{million}*17.23\%/1000)$.[14]

Third, consistent with the model's predictions, b depends positively on firm-specific risk and negatively on firm-systematic risk in almost all regressions. Most coefficients are statistically significant. Given that b is determined in each period in our model, tests on the PPS of new equity grants are more direct. Thus, we use results in Panel A to discuss the impact of a firm's risks on the PPS. Based on the OLS regression with the GDP growth as the proxy for aggregate productivity, a rise of one standard deviation (19%) in firms' specific risk increases new equity incentives by $\$535,000$ $(= 2.748 * 19\% * \$5,947\text{million}*17.23\%/1000)$ while a rise of one standard deviation (10%) in firms' systematic risk decreases new equity incentives by $\$119,476$ $(= 1.166 * 10\% * \$5,947\text{million}*17.23\%/1000)$. These numbers show that the impacts of firms' specific risk and systematic risk on the PPS are economically significant.

Last, to contrast our predictions with those of a standard principal-agent model, we run regression (4.1) by replacing the "specific risk" and "systematic risk" with the firm's "total risk". For brevity, we only report the coefficient and t-value for the firm's "total risk", as well as the

[14] $\$5,947$ million is the average market value of equity, and 17.23% is the average stock return in our sample period. Therefore, $\$5,947$ million$*17.23\%$ is the average change in shareholder wealth during a year.

corresponding R^2. In general, the R^2 is smaller, indicating that b is better explained by separating the firm's systematic risk from its specific risk. More importantly, the relationship between b and a firm's total risk is positive and significant at 1% level for all regressions in Panel A and median regressions in Panel B. This finding is consistent with the results in Core and Guay (1999) but contradicts to the predicted negative relationship from a standard principal-agent model.

To summarize, our empirical results suggest that our model predictions are generally supported by the empirical analysis. In particular, the PPS b is negatively (positively) affected by the firm's systematic (specific) risks.

4.3. Test of Prediction 2 on annual compensation and firm size

To gain better understanding of the time trend in annual compensation and firm size, we report the median annual compensation and firm size in Table 4.[15] There is an upward trend in annual compensation, which is confirmed by Fig. 3. In particular, the median salary, salary plus bonus and total compensation increased from \$469,00, \$726,000 and \$1,315,000 in 1993 to \$677,000, \$1,304,000 and \$3,107,000 in 2005 respectively. The corresponding percentage increases are 44.35%, 79.61% and 136.27%.

Table 4 also shows a positive growth in the median firm size during the sample period, which is illustrated in Fig. 4. The percentage increases in the asset value and the market capitalization are 54.5% and 105.05% from 1993 to 2005, respectively. Since our theory shows a positive influence of aggregate productivity on compensation and firm size, we also plot the two aggregate proxies in Fig 4. Clearly, GDP has increased steadily while commercial paper spreads have decreased from 1993 to 2005.

Table 4 suggests that the percentage increases in the median total compensation are bigger than those in firm size. Therefore, we further document the ratio between total compensation and firm size. It is clear that the median ratio exhibits a positive time trend (please see Fig. 5). In particular, the median ratio has increased from 0.073% to 0.208% when using asset value as a proxy for firm size. A similar observation can be made when market capitalization is used for firm size. The median ratio based on asset value is more stable than the ratio based on

[15] In Table 4, we omit the median statistics for 1992 because there are only 27 observations for 1992 and the statistics are biased toward large firms.

market capitalization. Given the important influence of firm-systematic and firm-specific risks on compensation, firm size and $R_{pay/size}$, we also present median statistics for firms' risks in Table 4. The median firm-specific risk shows a positive time trend while the median firm-systematic risk presents a slightly downward trend (see Fig. 6). Specifically, the median firm-specific risk increased from 0.317 in 1993 to 0.406 in 2005, and the median firm-systematic risk dropped from 0.161 in 1993 to 0.151 in 2005.

To summarize, the median statistics in Table 4 exhibit two important features: (1) different components of executive compensation and firm sizes have increased; (2) the increase in total compensation has outpaced the increase in firm size.

To test *Prediction 2,* we run the following regression for annual compensation and firm size:

$$\begin{aligned}\log(\text{Compensation or firm size}) = a_1 + a_2(\log(\text{GDP})\ \textit{or}\ \text{NCP spread}) + a_3\text{Firm} \\ \text{-specific risk} + a_4\text{Firm-systematic risk} + a_5\text{Age} + a_6\text{Tenure} + a_7\text{Firm growth} + \varepsilon.\end{aligned} \tag{4.2}$$

Note that we use GDP as the explanatory variable because the dependent variable (e.g., compensation and firm size) has the same measure unit. Since the OLS results are qualitatively similar to those of the median regression, to save space, we only report the median regression results in Table 5. Panel A presents the results for annual compensations which are measured by salary, salary plus bonus and total compensation, while Panel B reports the results for firm size which is measured by a firm's asset value or its market capitalization. The following patterns emerge from Table 5.

First, executive pay (salary, salary plus bonus or total compensation), as well as firm size, increases with aggregate productivity. In other words, the growing macro-economy during the past decade has a positive and significant effect on executive pay and firm size. This is evident since all coefficients for GDP and the NCP spread are positive and significant at 1% level. For example, increasing GDP by 1% leads to an increase of 1.982% in total compensation and an increase of 2.224% in the firm's market capitalization.

Second, Table 5 confirms a negative impact of a firm's specific risk, as well as a positive effect of a firm's systematic risk, on compensation and firm size. All coefficients are significant at 1%. For example, when GDP proxies aggregate productivity, a 1% increase in the firm's specific risk leads to a 2.341% reduction in total compensation and a 5.606% reduction in the firm's market capitalization. On the other hand, a 1% increase in the firm's specific risk yields a 2.13% increase in total compensation and a 4.676% increase in the firm's market capitalization.

To determine the order of importance among aggregate productivity, the firm's specific risk and systematic risk on annual compensations and firm size, we first calculate the changes in these variables from 1993 to 2005. Based on Table 4, the percentage increase in GDP is 87.56% $(= 12.487/6.657 - 1)$. There is an increase of 8.9% $(= 0.406 - 0.317)$ in median firm specific risk while there is a decrease of 1% $(= 0.151 - 0.161)$ in median systematic risk. Also we calculate the percentage changes in total compensation and firm size. From 1993 to 2005, total compensation has increased by 136.27% $(= 3.107/1.315 - 1)$ while firm size measured by market capitalization has increased by 105.05% $(= 1.786/0.871 - 1)$.

Now we examine the overall effect of these three variables on total compensation and firm size. To do so, we take full derivatives to equation (4.2) and use the coefficients for the three variables in Table 5 to compute the predicted percentage changes for total compensation as

$$\begin{aligned}&1.982 * 87.56\%(\text{GDP}) - 2.341 * 8.9\%(\text{specific risk}) + 2.13 * (-1\%)(\text{systematic risk})\\&= 173.58\%(\text{GDP}) - 20.83\%(\text{specific risk}) - 2.13\%(\text{systematic risk}) = 150.61\%.\end{aligned}$$

The increase in the firm's specific risk and the decrease in the firm's systematic risk create negative effects on the total pay by 20.83% and 2.13%, respectively. However, the 87.56% increase in GDP is the main positive force which lifted up the total compensation by 173.58%. The changes in these three variables together induced total compensation to increase by 150.61%. The remaining −14.34% may be explained by other control variables such as the CEO's tenure, age and the firm's sales growth. A similar exercise shows that GDP growth is the driving force behind the 105.05% increase in market capitalization.

To summarize, our empirical evidence shows that the rapid growth of the macro-economy contributed to the increase in total compensation and firm size. The increase in the firm's specific risk and the decrease in its systematic risk actually dampened the growth in total compensation and firm size.

4.4. Test of Prediction 3 on ratio between total compensation and firm size

In this section, we address the following question: how did total compensation evolve relative to firm size over time? As shown in Table 4, total compensation increased faster than firm size in the sense that the ratio between total compensation and firm size exhibited a positive time trend. To explain this positive time trend, we test *Prediction 3* as follows:

$$R_{pay/size} = a_1 + a_2(\text{GDP } \% \textit{ or } \text{NCP spread}) + a_3\text{Firm-specific risk} + a_4\text{Firm-systematic risk} + a_5\text{Age} + a_6\text{Tenure} + a_7\text{Firm growth} + a_8\text{Year} + \varepsilon. \tag{4.3}$$

The variable "Year" is equal to the calendar year of the observation, which is intended to capture the possible time trend in $R_{pay/size}$. Table 6 reports the results.

The ratio $R_{pay/size}$ is affected positively by the firm's specific risk and negatively by the firm's systematic risk, confirming the theoretical prediction. All coefficients are significant at 1% level. For example, when the negative commercial paper spread is used to proxy aggregate productivity and when firm size is measured by asset value, a 1% increase in the firm's specific risk leads to a $7.526\% \times 10^{-3}$ increase in the ratio while a 1% reduction in the firm's systematic risk yields a $4.355\% \times 10^{-3}$ rise in the ratio.

However, the effects of aggregate productivity are mixed. The negative impact of aggregate productivity is confirmed when the negative commercial paper spread is used as the proxy but is somewhat rejected when the GDP growth is used. To be conservative when determining the order of importance among the firm's specific risk, its systematic risk and aggregate productivity, we use the estimated coefficients when the negative commercial paper is the explanatory variable. Based on Table 4, we know that the commercial paper spread has decreased by 23 basis points from 1993 to 2005. The change in $R_{pay/size}$ is $72\% \times 10^{-3}$. Recall that there is an 8.9% increase in the firm's specific risk and an 1% decrease in the firm's systematic risk. Using the coefficients estimated from (4.3), the change in $R_{pay/size}$ that can be explained by these three factors is

$$\begin{aligned}&[-0.01 \times 23(\text{NCP}) + 7.526 \times 8.9\%(\text{specific risk}) - 4.355 \times (-1\%)(\text{systematic risk})] \times 10^{-3} \\ &= [-23\%(\text{NCP}) + 66.98\%(\text{specific risk}) + 4.355\%(\text{systematic risk})] \times 10^{-3} = 48.34\% \times 10^{-3}.\end{aligned}$$

That is, the increase in the firm's specific risk and the decrease in its systematic risk create positive effects on $R_{pay/size}$ by $66.98\% \times 10^{-3}$ and $4.355\% \times 10^{-3}$, respectively. However, the reduction of 23 basis points in the commercial paper spread reduces $R_{pay/size}$ by $23\% \times 10^{-3}$. The overall impact of these three explanatory variables on $R_{pay/size}$ is a $48.34\% \times 10^{-3}$ increase, which accounts for about 67.13% of the $72\% \times 10^{-3}$ increase in $R_{pay/size}$. The remaining 32.87% may be explained by other control variables. Thus, our empirical evidence suggests that the increase in $R_{pay/size}$ is mainly due to the increase in firms' specific risks.

5. Conclusion

This paper addresses three issues regarding executive compensation: 1) how do systematic and specific risks of a firm affect the pay-to-performance sensitivity? 2) what are the key factors contributing to the increase in executive pay and firm size? and 3) what are the determinants of the relative growth rate between executive pay and firm size?

To address these questions, we propose a dynamic search equilibrium agency model which allows a CEO to search for outside options. The CEO can quit if his outside options exceed the utility derived from the existing incentive contract. In the model's setup with many firms and many agents, the contract offered by one firm depends on other firms' contracts through the CEO's outside options. Because of this link among different firms' contracts, all firms' contracts and CEOs' reservation utilities are determined simultaneously in a market equilibrium. The equilibrium compensation contract is aimed to induce not only the optimal effort but also the optimal participation, as opposed to the contract in a traditional model with a single firm where the participation constraint is always satisfied.

Our equilibrium analysis yields new results about the incentive contracts, which are confirmed by our empirical analysis. First, the equilibrium pay-to-performance sensitivity depends positively on a firm's specific risk, and negatively on a firm's systematic risk. These opposite effects of the two types of risks offer a possible theory to reconcile with the mixed empirical evidence on the relationship between the pay-to-performance sensitivity and firms' total risks. Second, a growing economy simultaneously induces the growth in executive compensation and firm size. Third, the faster growth of executive compensation relative to the growth of firm size in the past decade is mostly due to the increase in firms' specific risks.

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Appendix

A. Solution to the market equilibrium

Given the equilibrium definition in Section 3, we solve for the equilibrium values of (a, b, x_d), $(J_E, J_S, J_F,\ J_H)$, and (q, λ) through the following equations:

$$a = \beta(J_F - J_H) - b(1-b)^2 \tfrac{\bar{x}\mathbb{E}(y^2)}{c}, \tag{1}$$

$$b = \tfrac{1}{2}\left(1 + \tfrac{x_d}{\bar{x}}\right), \tag{2}$$

$$x_d(D) = \tfrac{2c[B-a+\beta(1-\lambda)(J'_S - I)]}{b^2\mathbb{E}(y^2)}, \tag{3}$$

$$J_F = \beta J_H + b(1-b)^2 \tfrac{\mathbb{E}(y^2)\bar{x}^2}{\sqrt{3}c\sigma_x}, \tag{4}$$

$$J_H = q\beta J_F + (1-q)\beta J_H - H, \tag{5}$$

$$J_S = B + \beta\lambda I + (1-\lambda)\beta J_S, \tag{6}$$

$$J_E(x) = a + \tfrac{b^2}{2c} x\mathbb{E}(y^2) + \beta I, \tag{7}$$

$$(1-u)F(x_d) = u\lambda[1 - F(x_d)], \tag{8}$$

$$\beta q(J_F - J_H) = H, \tag{9}$$

$$q = A - \lambda, \tag{10}$$

$$\text{with} \quad I \equiv \textstyle\int_{x'} \max[J_E(x'), J_S]dF(x').$$

First, we find the expressions for I and J_S based on J_E. To do so, we work with (6), (7) and $I \equiv \int_{x'} \max[J_E(x'), J_S]dF(x')$. Putting (7) into the expression for I, together with (6), we solve for J_S and I and further compute

$$J_S - I = -[1 - F(x_d)]\frac{b^2(1-b)}{2c}\mathbb{E}(y^2)\bar{x}^2.$$

After simplifying (3), we obtain

$$B - \beta(J_F - J_H) = \beta(1-\lambda)\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c(\bar{x}-\underline{x})} b^2(1-b)^2 + (3b^2 - 2b)\tfrac{\mathbb{E}(y^2)\bar{x}}{2c}. \tag{A.1}$$

Substituting the free entry condition in (9) into (5), we have

$$J_H = 0 \qquad \text{and} \qquad J_F = b(1-b)^2 \tfrac{2\mathbb{E}(y^2)\bar{x}^2}{c(\bar{x}-\underline{x})}.$$

Using (10) to simplify (5), we obtain

$$(A-\lambda)\beta b(1-b)^2 \tfrac{2\mathbb{E}(y^2)\bar{x}^2}{c(\bar{x}-\underline{x})} = H. \tag{A.2}$$

Since (A.1) and (A.2) only involve b and λ, we can solve for the two variables jointly. Once the optimal value of b is obtained, all other equilibrium outcomes as such a and x_d are solved since they are only functions of b.

To solve for b, we obtain an expression for λ from (A.2) and put it into (A.1). This yields the following equation which only involves b:

$$G(b) = (1-A)\beta\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}b^2(1-b)^2 + 2\beta\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}b(1-b)^2$$

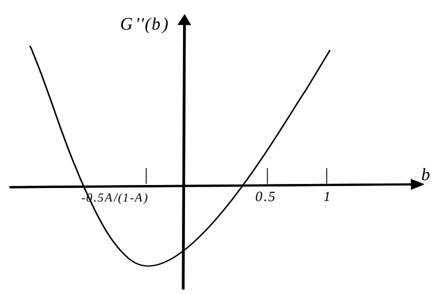
$$+\tfrac{\mathbb{E}(y^2)\bar{x}}{c}(3b^2-2b) + Hb - 2B = 0.$$

Given $x_d = (2b-1)\bar{x}$, the admissible b belongs to (0.5, 1). Therefore, we can show that

$$G''(b) = \tfrac{d^2G}{db^2} = 2\beta\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}\left[(1-A)(1-6b+6b^2)+6b-4\right] + 6\tfrac{\mathbb{E}(y^2)\bar{x}}{c}.$$

This quadratic function reaches its minimum at $-\frac{A}{2(1-A)}$, which is in the inadmissible range for b. Given $0<\beta<1$ and $0<A<2$, we can easily show that $G''(b=\frac{1}{2}) = \beta\frac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}(\frac{6\sqrt{3}\sigma_x}{\beta\bar{x}}+A-3)>0$ with reasonable parameters for σ_x and $\bar{x}$ and $G''(b=1) = 2\beta\frac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}(\frac{3\sqrt{3}\sigma_x}{\beta\bar{x}}+3-A)>0$. Since the coefficient in front of b in the quadratic function is positive, therefore, we can draw a diagram for $G''(b)$ below. The diagram indicates that $G'(b) = \frac{dG}{db}$ is increasing for $b \in (0.5, 1)$.

Fig. A1 : $G''(b)$



It is easy to show that

$$G'(b) = 2(1-A)\beta\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}b(1-b)(1-2b) + 2\beta\tfrac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}(1-b)(1-3b) + 2\tfrac{\mathbb{E}(y^2)\bar{x}}{c}(3b-1) + H.$$

We then obtain $G^{'}(b=1) = \frac{\mathbb{E}(y^2)\bar{x}^2}{c} + H > 0$ and $G^{'}(b=\frac{1}{2}) = \frac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x}(\frac{\sqrt{3}\sigma_x}{\bar{x}} - \frac{1}{2}\beta) + H$, whose sign is ambiguous. Hence, there are two possibilities to graph $G^{'}(b)$:

Fig. A2a : $G^{'}(b=\frac{1}{2}) > 0$

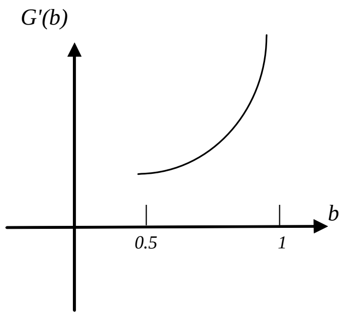
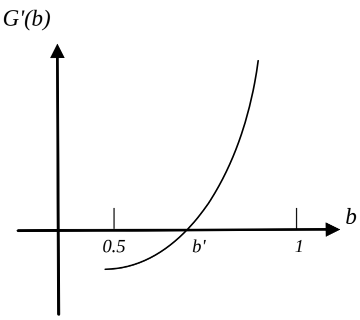


Fig. A2b : $G^{'}(b=\frac{1}{2}) < 0$



It is easy to show that

$$G(b=1) = \frac{\mathbb{E}(y^2)\bar{x}}{c} + H - 2B > G(b=0.5) = \frac{\mathbb{E}(y^2)\bar{x}^2}{16c}\left[\frac{(5-A)\beta\bar{x}}{\sqrt{3}\sigma_x} - 4\right] + \frac{1}{2}H - 2B.$$

In order to ensure existence of an equilibrium, we require $G(b=1) > 0$ and $G(b=0.5) < 0$. These two conditions imply that the unemployment benefit should satisfy $B \in [B_1, B_2]$, where

$$B_1 \equiv \frac{\mathbb{E}(y^2)\bar{x}^2}{32c}\left[\frac{(5-A)\beta\bar{x}}{\sqrt{3}\sigma_x} - 4\right] + \frac{1}{4}H, \quad B_2 \equiv \frac{\mathbb{E}(y^2)\bar{x}}{2c} + \frac{1}{2}H.$$

A sufficient condition for $B_2 > B_1$ is that H is sufficiently high.

Based on this restriction, we can depict the solution for b with respect to the two possibilities depicted in Fig. A2a and Fig. A2b:

Fig. A3a : $G^{'}(b=\frac{1}{2}) > 0$

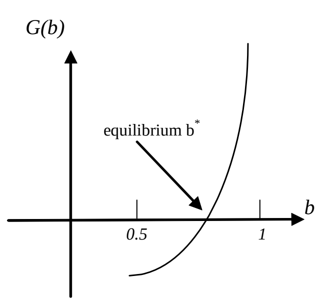
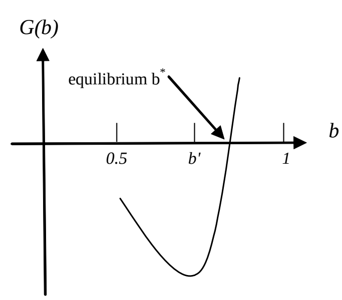


Fig. A3b : $G^{'}(b=\frac{1}{2}) < 0$



In either case, there exists a unique solution b^* and $\frac{\partial G}{\partial b}\mid_{b=b^*} > 0$.

B. Comparative statics for equilibrium incentive contract and firm size

B.1. Profit-sharing ratio and Fixed Salary

Given that $\frac{\partial G}{\partial b} \mid_{b=b^*} > 0$, it is easy to conduct comparative statics for the profit-sharing ratio b^* to various model parameters. We focus on the impact of expected aggregate productivity μ_y, the systematic risk σ_y and the specific risk σ_x. Since

$$\frac{\partial b^*}{\partial \mathbb{E}(y^2)} = -\frac{\partial G / \partial \mathbb{E}(y^2)}{\partial G / \partial b} = \frac{-(2B - Hb)}{\mathbb{E}(y^2) \partial G / \partial b} \mid_{b=b^*} < 0,$$

we can obtain

$$\frac{\partial b^*}{\partial \mu_y} = 2\mu_y \frac{\partial b^*}{\partial \mathbb{E}(y^2)} < 0 \qquad \text{and} \qquad \frac{\partial b^*}{\partial \sigma_y} = 2\sigma_y \frac{\partial b^*}{\partial \mathbb{E}(y^2)} < 0.$$

Also,we have

$$\frac{\partial b^*}{\partial \sigma_x} = -\frac{\partial G / \partial \sigma_x}{\partial G / \partial b} = \frac{b(2-3b)\mathbb{E}(y^2)\mu_x}{c\sigma_x \frac{\partial G}{\partial b}} + \frac{(2B - Hb)\underline{x}}{c\bar{x}\sigma_x \frac{\partial G}{\partial b}}.$$

It is easy to show that $\frac{\partial b^*}{\partial \sigma_x} \mid_{b=b^*} > 0$ when $b < \frac{2}{3}$, and ambiguous otherwise.

As for the equilibrium salary a stated in (3.2), we can rewrite it as

$$a^* = \left(\frac{\beta \bar{x}^2}{\sqrt{3}\sigma_x} - \bar{x} \right) \frac{\mathbb{E}(y^2)}{c} f(b^*) \qquad \text{with} \qquad f(b) = b(1-b)^2.$$

To ensure positive salary, we require $\beta > \frac{\sqrt{3}\sigma_x}{\bar{x}}$. Given the uniform distribution for x, it is easy to show $\frac{\sqrt{3}\sigma_x}{\bar{x}} < \frac{1}{2}$. That is, $\beta > \frac{\sqrt{3}\sigma_x}{\bar{x}}$ can be easily satisfied. To obtain the comparative statics, we first show

$$\frac{\partial f}{\partial b} \mid_{b=b^*} = (1-b^*)(1-3b^*) < 0,$$

given that $b^* > \frac{1}{2}$. Then we obtain

$$\frac{\partial a^*}{\partial \mu_y} = 2\mu_y \left(\frac{a^*}{\mathbb{E}(y^2)} + \frac{a^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} \right) > 0 \qquad \text{and} \qquad \frac{\partial a^*}{\partial \sigma_y} = 2\sigma_y \left(\frac{a^*}{\mathbb{E}(y^2)} + \frac{a^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} \right) > 0.$$

Also, for $b < \frac{2}{3}$,

$$\frac{\partial a^*}{\partial \sigma_x} = -\frac{1}{\sqrt{3}\sigma_x^2} \left[\left(\beta \bar{x} - \sqrt{3}\sigma_x \right) + \beta \bar{x}^2 (1-\beta) \right] \frac{\mathbb{E}(y^2)}{c} f(b^*) + \frac{a^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \sigma_x} < 0,$$

ambiguous otherwise.

B.2. Equilibrium Firm Value

We can rewrite the equilibrium firm value J_F^* as

$$J_F^* = \frac{\mathbb{E}(y^2)\bar{x}^2}{c\sqrt{3}\sigma_x} f(b^*).$$

As shown earlier, $\frac{\partial f}{\partial b^*} < 0$ for $b^* > \frac{1}{2}$. It is easy to obtain

$$\frac{\partial J_F^*}{\partial \mu_y} = 2\mu_y \left(\frac{J_F^*}{\mathbb{E}(y^2)} + \frac{J_F^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} \right) > 0$$

$$\frac{\partial J_F^*}{\partial \sigma_y} = 2\sigma_y \left(\frac{J_F^*}{\mathbb{E}(y^2)} + \frac{J_F^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} \right) > 0$$

$$\frac{\partial J_F^*}{\partial \sigma_x} = -\frac{\left(\mu_x - \sqrt{3}\sigma_x\right)}{\sqrt{3}\sigma_x^2} \frac{\mathbb{E}(y^2)}{c} f(b^*) + \frac{J_F^*}{f(b^*)} \frac{df}{db^*} \frac{\partial b^*}{\partial \sigma_x}.$$

The derivative $\partial J_F^*/\partial \sigma_x$ is negative for $b < 2/3$, and ambiguous otherwise.

B.3. Ratio between total expected pay and firm size

We know that

$$R_{pay/size} = \frac{a + b\mathbb{E}(\pi)}{J_F^*} = \frac{{b^*}^2}{1-b^*} + \beta - \frac{\sqrt{3}\sigma_x}{\bar{x}} \qquad \text{and} \qquad \frac{\partial R_{pay/size}}{\partial b} = \frac{b(2-b)}{(1-b)^2} > 0.$$

Thus, it is easy to show the following results:

$$\frac{\partial R_{pay/size}}{\partial \mu_y} = 2\mu_y \frac{b(2-b)}{(1-b)^2} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} < 0 \qquad \text{and} \qquad \frac{\partial R_{pay/size}}{\partial \sigma_y} = 2\sigma_y \frac{b(2-b)}{(1-b)^2} \frac{\partial b^*}{\partial \mathbb{E}(y^2)} < 0.$$

$$\frac{\partial R_{pay/size}}{\partial \sigma_x} = \frac{b(2-b)}{(1-b)^2} \frac{\partial b}{\partial \sigma_x} - \frac{\sqrt{3}\mu_x}{\bar{x}^2} \, |_{b=b^*} > 0$$

The derivative $\partial R_{pay/size}/\partial \sigma_x$ is positive when $\frac{\partial b}{\partial \sigma_x} > \frac{\mu_x}{\sqrt{3}(\mu_x + \sqrt{3}\sigma_x)^2}$, and ambiguous otherwise.

Table 1: summary statistics

This table reports the summary statistics on the executive compensation and characteristics, the firm characteristics, and macroeconomic variables for the period of 1992 to 2005 with a sample size of 10,837 firm-years. The executive compensation and characteristics data are retrieved from ExecuComp. New equity incentive is the pay-to-performance sensitivity of a CEO based on the stock and option grant for the fiscal year with respect to the $1,000 change in shareholders' wealth. Total equity incentive is the sensitivity for a CEO based on the cumulative stock and option grants with respect to the $1,000 change in shareholder's wealth. Firm characteristics data are from COMPUSTAT and CRSP. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. The annual GDP growth data are retrieved from the website of the Bureau of Economic Analysis at www.bea.gov/beahome.html. The commercial paper spread is defined as the difference between the annualized rate on three-month commercial paper and the three-month T-bill rate, which are retrieved from the website of the Federal Reserve Board at www.federalreserve.gov.

Variables	Mean	Std Dev	Min.	25% Percentile	Median	75% Percentile	Max.	Skewness	Kurtosis
	Panel A: Executive Characteristics and Compensation								
Salary (Thousand)	$627	$307	$29	$400	$572	$800	$1,700	0.96	4.11
Bonus (Thousand)	$640	$842	$0	$100	$375	$822	$4,901	2.59	11.25
Total Compensation (Thousand)	$3,991	$5,177	$210	$1,058	$2,145	$4,634	$30,835	2.95	13.10
New Equity Incentive (Per $1,000 Change in Shareholders' Wealth)	$2.10	$3.27	$0.00	$0.15	$1.00	$2.52	$19.58	3.12	14.51
Total Equity Incentive (Per $1,000 Change in Shareholders' Wealth)	$27.56	$58.63	$0.03	$2.29	$5.95	$18.87	$332.76	3.40	14.98
Executive Tenure	8.30	7.64	0.42	2.84	5.89	11.17	38.02	1.68	5.94
Executive Age	55.75	7.64	29.00	51.00	56.00	61.00	90.00	0.20	3.56
	Panel B: Firm Characteristics								
Total Firm Return Volatility (Annualized)	45%	21%	16%	30%	39%	55%	116%	1.27	4.53
Specific Firm Return Volatility (Annualized)	41%	19%	14%	27%	37%	51%	108%	1.20	4.39
Systematic Firm Return Volatility (Annualized)	15%	10%	1%	8%	13%	19%	54%	1.55	6.01
Market Capitalization (Million)	$5,947	$15,520	$42	$446	$1,196	$3,954	$108,684	4.86	28.78
Assets (Million)	$4,283	$9,795	$55	$416	$1,074	$3,330	$76,836	4.86	31.30
Sales Growth	13%	24%	-48%	1%	9%	20%	119%	1.49	8.00
	Panel C: Macroeconomic Variables								
GDP (Trillion)	$9.09	$1.93	$6.34	$7.82	$9.01	$10.13	$12.49	0.23	-1.01
Commercial Paper Spread (Basis Points)	24	13	1	15	26	33	43	-0.38	-0.87

Table 2: correlation

This table reports the correlations among explanatory variables and control variables for the period of 1992 to 2005 with a sample size of 10,837 firm-years. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. The dollar risks are obtained by multiplying the corresponding return volatilities to the market capitalization.

	GDP	Lagged CP Spread	Market Capitalization	Assets	Sales Growth	Tenure	Age	Total Firm Return Volatility	Specific Firm Return Volatility	Systematic Firm Return Volatility
GDP	1.000									
Lagged CP Spread	-0.544	1.000								
Market Capitalization	0.046	0.005	1.000							
Assets	0.044	-0.033	0.799	1.000						
Sales Growth	-0.023	0.030	0.034	-0.008	1.000					
Tenure	-0.031	0.028	-0.050	-0.075	0.058	1.000				
Age	-0.047	0.014	0.039	0.071	-0.055	0.420	1.000			
Total Firm Return Volatility	0.348	-0.270	-0.193	-0.225	0.087	0.013	-0.202	1.000		
Specific Firm Return Volatility	0.342	-0.245	-0.216	-0.249	0.092	0.012	-0.204	0.989	1.000	
Systematic Firm Return Volatility	0.249	-0.275	-0.019	-0.036	0.031	0.015	-0.121	0.667	0.557	1.000

Table 3: test of prediction 1 - effects of macroeconomic variable and firm risks on Pay-to-Performance Sensitivity (PPS)

This table reports the results for regression (4.1): *PPS b* = $a_1 + a_2$ (GDP %/NCP spread) + a_3 Firm-specific risk + a_4 Firm-systematic risk + a_5 Age + a_6 Tenure + a_7 log(Firm size) + a_8 Firm growth + ε. The sample size is 10,837 firm-years for the period of 1992 to 2005. The dependent variables in Panels A and B are, respectively, the new equity incentive calculated with stock and option grants for the fiscal year and the total equity incentive calculated with the cumulative stock and option grants, with respect to the $1,000 change in shareholders' wealth. GDP % is the GDP growth in the fiscal year. NCP spread is the negative lagged commercial paper spread. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. Firm size and firm growth are proxied by the firm's asset value and its sales growth, respectively. We also run regression (4.1) by replacing "specific " and "systematic" risks with "total risk". The coefficient and *t*-value for "total risk" are reported at the bottom of the table. For all regressions, we control for industry-fixed effects. For OLS, standard errors are clustered at firm level. For median regressions, standard errors are calculated by bootstrapping with 500 replications. *, **, and *** indicate significance levels at 10%, 5%, and 1%, respectively.

		Panel A: New Equity Incentive				Panel B: Total Equity Incentive			
	Prediction	OLS Regression		Median Regression		OLS Regression		Median Regression	
	This Model								
GDP (%)	-	-0.073 ***		-0.039 ***		-0.913 ***		-0.235 ***	
		(2.972)		(3.038)		(3.045)		(3.175)	
NCP Spread (basis points)	-		-0.020 ***		-0.006 ***		-0.165 ***		-0.035 ***
			(8.271)		(5.176)		(3.291)		(5.529)
Firm-Specific Risk (annualized)	+	2.748 ***	3.218 ***	2.167 ***	2.459 ***	6.908	11.096	3.877 ***	5.141 ***
		(8.505)	(9.575)	(12.409)	(12.212)	(0.927)	(1.395)	(5.347)	(6.530)
Firm-Systematic Risk (annualized)	-	-1.166 **	-0.759	-0.608 **	-0.551 **	-17.852	-14.688	-3.536 ***	-3.442 ***
		(2.280)	(1.496)	(2.221)	(1.987)	(1.618)	(1.331)	(3.085)	(2.805)
Age		-0.018 ***	-0.016 **	-0.011 ***	-0.011 ***	-0.337	-0.325	-0.080 ***	-0.072 ***
		(2.598)	(2.386)	(4.301)	(4.287)	(1.571)	(1.516)	(5.328)	(4.760)
Tenure		-0.022 ***	-0.023 ***	-0.019 ***	-0.021 ***	3.212 ***	3.201 ***	1.142 ***	1.135 ***
		(3.482)	(3.708)	(7.472)	(7.802)	(11.623)	(11.589)	(21.292)	(21.662)
log(Firm Size)		-0.419 ***	-0.384 ***	-0.185 ***	-0.167 ***	-6.723 ***	-6.424 ***	-1.786 ***	-1.715 ***
		(13.277)	(12.008)	(14.433)	(12.589)	(6.644)	(6.144)	(24.273)	(22.377)
Firm Growth		0.015	-0.090	-0.012	-0.055	2.139	1.035	0.697	0.476
		(0.091)	(0.551)	(0.140)	(0.662)	(0.830)	(0.406)	(1.509)	(1.082)
Adjusted R^2		0.115	0.121			0.248	0.249		
Pseudo R^2				0.067	0.067			0.090	0.091
	Traditional Model								
Firm Total Risk	-	2.068 ***	2.682 ***	1.736 ***	1.924 ***	-0.344	4.927	2.150 ***	3.565 ***
		(8.305)	(9.914)	(12.301)	(12.146)	(0.055)	(0.702)	(3.705)	(5.556)
Adjusted R^2		0.113	0.119			0.247	0.248		
Pseudo R^2				0.065	0.066			0.090	0.090

Table 4: median statistics for annual pay, firm Size, ratio between annual pay and firm size and firm risks during 1993–2005

This table reports the median statistics for annual compensation, firm size, ratio between annual pay and firm size, and firm risks. Firm size is either proxied by the firm's asset value or the firm's market capitalization. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. The sample size is 10,810 firm-years for the period of 1993 to 2005.

Year	Total	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Sample Size	10,810	126	678	770	818	861	907	901	890	949	1,005	1,016	1,051	838
Annual Pay (millions)														
Salary		0.469	0.500	0.500	0.517	0.538	0.536	0.550	0.551	0.575	0.600	0.636	0.650	0.677
Salary plus Bonus		0.726	0.800	0.800	0.835	0.916	0.881	0.931	0.941	0.856	0.979	1.039	1.212	1.304
Total Compensation		1.315	1.510	1.447	1.638	2.015	1.991	2.164	2.381	2.474	2.569	2.386	3.079	3.107
Firm Size (billions)														
Size 1 = Asset		0.967	1.048	0.998	1.057	1.158	1.094	1.110	1.145	1.184	1.189	1.217	1.459	1.494
Size 2 = Market Capitalization		0.871	1.000	1.041	1.204	1.449	1.144	1.121	1.145	1.201	1.066	1.437	1.759	1.786
$R_{pay/size}$ = Total Pay/Size														
Total Pay / Size 1		0.136%	0.144%	0.145%	0.155%	0.174%	0.182%	0.195%	0.208%	0.209%	0.216%	0.196%	0.211%	0.208%
Total Pay / Size 2		0.151%	0.151%	0.139%	0.136%	0.139%	0.174%	0.193%	0.208%	0.206%	0.241%	0.166%	0.175%	0.174%
Firm Risks														
Firm-Systematic Risk		0.161	0.143	0.139	0.100	0.081	0.084	0.127	0.134	0.137	0.156	0.167	0.160	0.151
Firm-Specific Risk		0.317	0.295	0.305	0.300	0.303	0.312	0.326	0.377	0.418	0.443	0.459	0.446	0.406
Macroeconomic Variables														
GDP (Trillion $)		6.657	7.072	7.398	7.817	8.304	8.747	9.268	9.817	10.128	10.470	10.971	11.734	12.487
Commerical Paper Spread (basis points)		24	15	29	27	26	38	43	40	31	17	5	8	1

Table 5: test of prediction 2 – effects of macroeconomic variable and firm risks on annual compensation and firm size

This table reports median regression results for (4.2): *log(annual compensation/firm size)* = a_1 + a_2 (log(GDP) / NCP spread) + a_3 Firm-specific risk + a_4 Firm-systematic risk + a_5 Age + a_6 Tenure + a_7 Firm growth + ε. The sample size is 10,837 firm-years for the period of 1992 to 2005. The dependent variables in Panels A and B are annual compensation and firm size, respectively. log(GDP) is the logarithemic of GDP in the fiscal year. NCP spread is the negative lagged commercial paper spread. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. Firm size is measured by its asset value or market capitalization while firm growth is proxied by its sales growth. We also run (4.2) by replacing "specific " and "systematic" risks with "total risk". The coefficient and *t*-value for "total risk" are reported at the bottom of the table. We control for industry-fixed effects. Standard errors are calculated by bootstrapping with 500 replications. *, **, and *** indicate significance levels at 10%, 5%, and 1%, respectively.

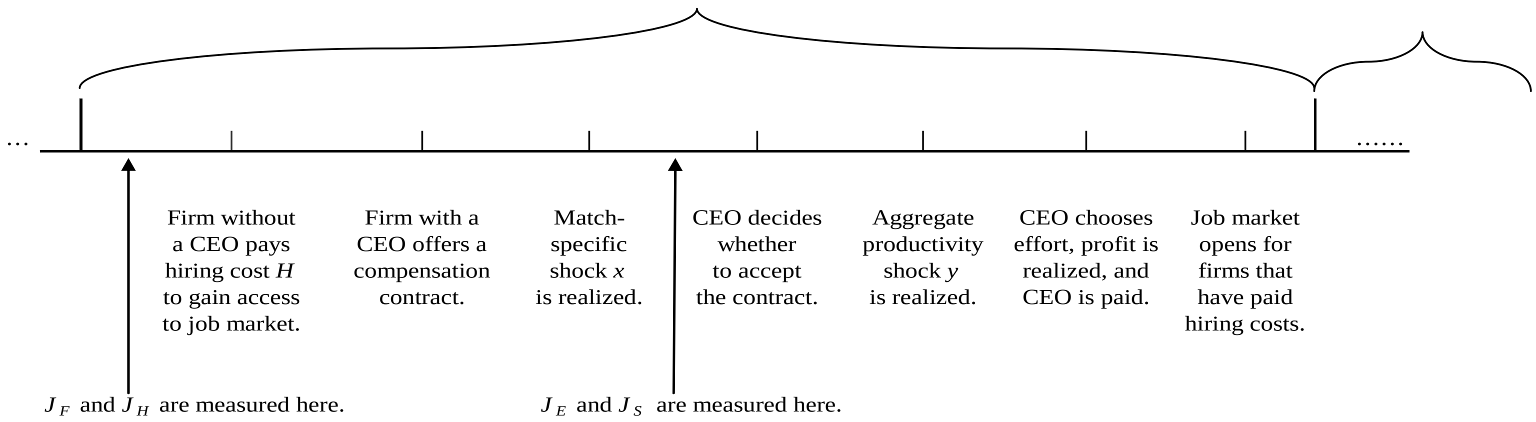
	Prediction	Panel A: Annual Compensation						Panel B: Firm Size			
	This Model	Salary		Salary plus Bonus		Total Compensation		Asset		Market Capitalization	
log (GDP)	+	1.003 ***		1.418 ***		1.982 ***		2.156 ***		2.224 ***	
		(29.48)		(27.94)		(27.45)		(22.28)		(23.40)	
NCP Spread (basis points)	+		0.007 ***		0.011 ***		0.011 ***		0.015 ***		0.018 ***
			(19.98)		(16.97)		(11.63)		(14.97)		(13.85)
Firm-Specific Risk (annualized)	-	-1.262 ***	-1.046 ***	-1.935 ***	-1.554 ***	-2.341 ***	-1.729 ***	-5.299 ***	-4.791 ***	-5.606 ***	-5.153 ***
		(32.08)	(27.53)	(30.93)	(27.14)	(24.76)	(18.49)	(39.79)	(42.17)	(39.18)	(42.11)
Firm-Systematic Risk (annualized)	+	0.712 ***	0.638 ***	1.056 ***	0.852 ***	2.13 ***	1.864 ***	4.385 ***	4.168 ***	4.676 ***	4.532 ***
		(11.41)	(9.94)	(10.33)	(7.44)	(13.34)	(10.26)	(21.44)	(20.47)	(20.31)	(19.25)
Age		0.010 ***	0.011 ***	0.011 ***	0.012 ***	0.005 **	0.005 **	0.014 ***	0.015 ***	0.005 **	0.005 **
		(11.33)	(12.43)	(7.80)	(8.04)	(2.52)	(2.57)	(5.84)	(5.38)	(1.96)	(2.02)
Tenure		-0.003 ***	-0.003 ***	-0.004 ***	-0.006 ***	-0.012 ***	-0.015 ***	-0.02 ***	-0.019 ***	-0.012 ***	-0.01 ***
		(3.16)	(3.67)	(3.34)	(4.89)	(6.53)	(7.86)	(10.55)	(9.67)	(4.84)	(4.43)
Firm Growth		-0.086 ***	-0.101 ***	0.324 ***	0.305 ***	0.485 ***	0.465 ***	0.206 ***	0.177 **	0.891 ***	0.907 ***
		(4.67)	(4.89)	(8.82)	(7.43)	(9.77)	(8.38)	(3.04)	(2.36)	(10.34)	(11.96)
Pseudo R^2		0.184	0.147	0.149	0.116	0.112	0.070	0.215	0.195	0.199	0.181
Firm Total Risk		-0.932 ***	-0.763 ***	-1.412 ***	-1.208 ***	-1.483 ***	-1.047 ***	-3.42 ***	-3.1 ***	-3.778 ***	-3.436 ***
		(29.02)	(25.71)	(30.09)	(27.20)	(20.16)	(13.53)	(29.17)	(32.57)	(28.04)	(27.27)
Pseudo R^2		0.165	0.133	0.130	0.105	0.091	0.055	0.162	0.149	0.149	0.137

Table 6: test of prediction 3 – effects of macroeconomic variable and firm risks on ratio between total compensation and firm size

This table reports median regression results for (4.3): $R_{pay/size}$ x 10^3 = a_1 + a_2 (GDP % / NCP spread) + a_3 Firm-specific risk + a_4 Firm-systematic risk + a_5 Age + a_6 Tenure + a_7 Firm growth + a_8 Year + ε. The sample size is 10,837 firm-years for the period of 1992 to 2005. The dependent variable in Panel A is the ratio between an executive's total compensation and the firm's asset value while the dependent variable in Panel B is the ratio between an executive's total compensation and the firm's market capitalization. GDP % is the GDP growth in the fiscal year. NCP spread is the negative lagged commercial paper spread. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. Firm growth is proxied by the firm's sales growth. We also run regression (4.3) by replacing "specific " and "systematic" risks with "total risk". The coefficient and *t*-value for "total risk" are reported at the bottom of the table. We control for industry-fixed effects. Standard errors are calculated by bootstrapping with 500 replications. *, **, and *** indicate significance levels at 10%, 5%, and 1%, respectively.

	Prediction	Panel A: $R_{pay/size}$ = Annual Total Pay/Asset Value		Panel B: $R_{pay/size}$ = Annual Total Pay/Market Cap	
	This Model				
GDP (%)	-	0.028 *		0.006	
		(1.704)		(0.345)	
NCP Spread (basis points)	-		-0.010 ***		-0.011 ***
			(6.682)		(7.474)
Firm-Specific Risk (annualized)	+	7.432 ***	7.526 ***	6.947 ***	6.878 ***
		(29.367)	(29.744)	(32.963)	(34.366)
Firm-Systematic Risk (annualized)	-	-4.605 ***	-4.355 ***	-4.360 ***	-4.086 ***
		(14.492)	(13.352)	(15.109)	(15.322)
Age		-0.014 ***	-0.016 ***	0.001	0.001
		(4.825)	(5.338)	(0.208)	(0.459)
Tenure		0.002	0.002	-0.003	-0.002
		(0.600)	(0.851)	(1.031)	(0.796)
Firm Growth		0.507 ***	0.532 ***	-0.816 ***	-0.873 ***
		(4.556)	(4.153)	(8.337)	(8.923)
Year		-0.056 ***	-0.038 ***	-0.052 ***	-0.032 ***
		(9.605)	(7.013)	(10.363)	(5.498)
Pseudo R^2		0.129	0.130	0.117	0.119
Firm Total Risk		5.474 ***	5.595 ***	5.046 ***	5.177 ***
		(29.463)	(27.806)	(23.751)	(25.825)
Pseudo R^2		0.108	0.111	0.095	0.099

Fig. 1 Timing of Events in Each Period



J_F is the value function of a firm with a filled CEO position.
J_H is the value function of a hiring firm with a vacant CEO position.
J_E is the value function of an employed CEO.
J_S is the value function of a CEO searching for a job.

Fig. 2 Relationship between b and x_d

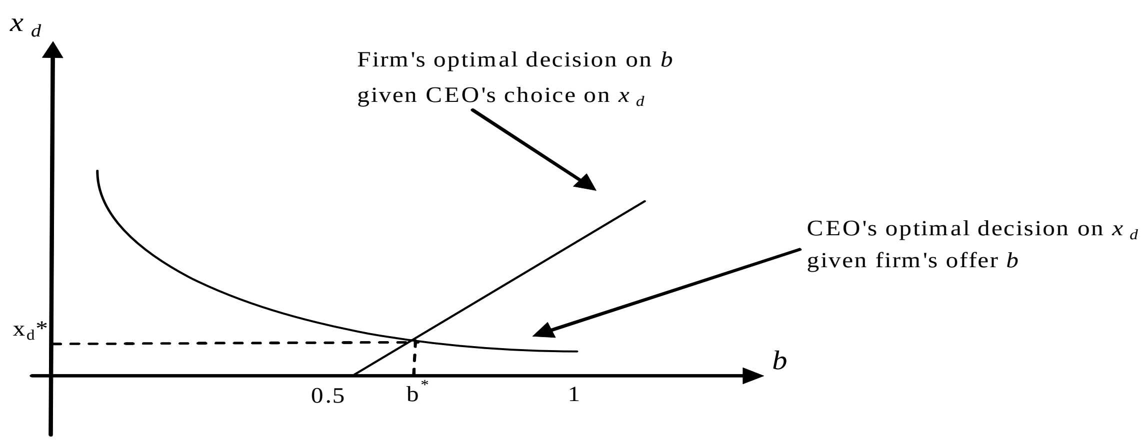


Fig. 3: Time Trend for Median Annual Pay during 1993–2005

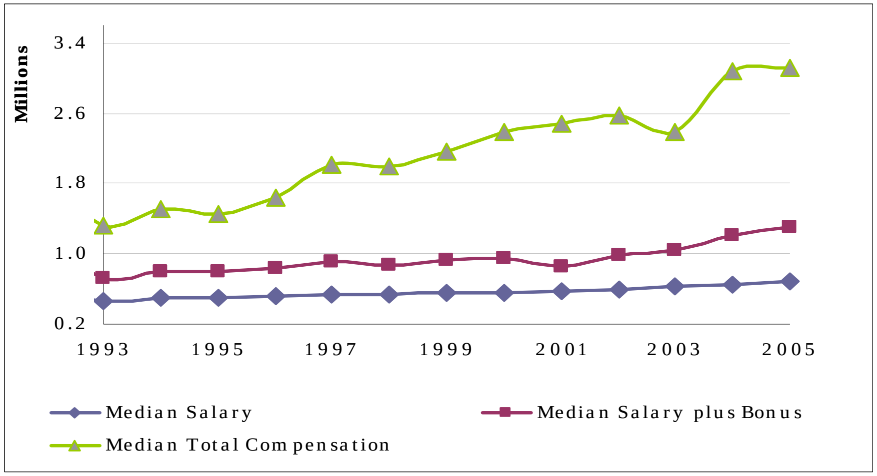


Fig. 4: Time Trend for Median GDP, CP Spread and Firm Size during 1993-2005

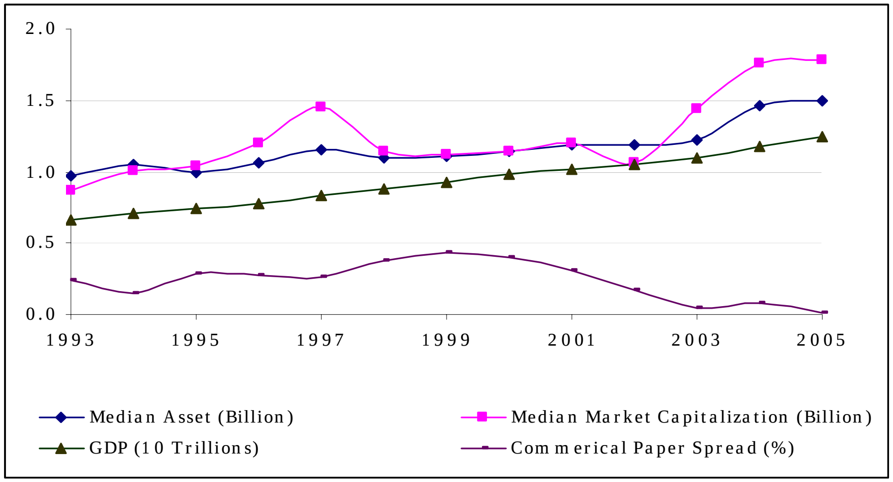


Fig. 5: Time Trend for Median Ratio between Pay and Size during 1993–2005

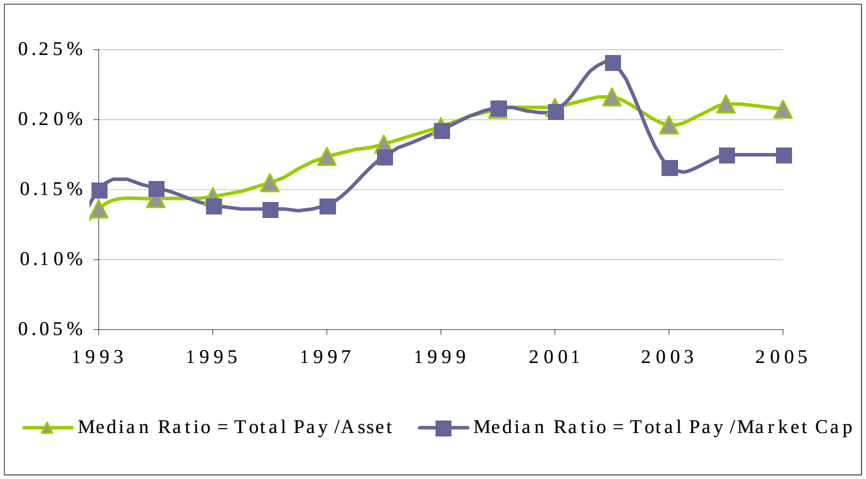


Fig. 6: Time Trend for Median Firm Risks during 1993-2005