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Farm-yield Management When Production Rate is Yield Dependent

Onur BOYABATLI
Singapore Management University, oboyabatli@smu.edu.sg

Kwan Eng WEE
Singapore Management University, kewee@smu.edu.sg

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Abstract

In agricultural industries, unfavorable weather conditions, high infestation of pests and diseases lead to not only a lower farm-yield available for processing, but also a lower production rate in processing due to the inferior quality of the crop, a feature that is largely ignored by the academic literature. This paper studies the role of the yield-dependent production rate influencing the insights coming from traditional models that (often implicitly) assume yield-invariant production rate. We consider a firm that reserves farm space for an agricultural input under the yield and the open market price uncertainties; and, after these uncertainties are realized, processes the yield from the farm space and the input sourced from the open market to sell through different sales contracts. The production rate from each input is yield dependent and non-decreasing in the realized yield. We show that, contrary to common intuition, the firm may benefit from increasing yield variability, specifically, when the probability of achieving a higher production rate is moderate. Furthermore, a lower farm space dependency always better responds to the increasing yield variability, whereas this response crucially depends on the size of the reserved farm space when the production rate is yield invariant. We find that the cost of ignoring the yield-dependent nature of the production rate in procurement planning can be substantial, and this cost is very sensitive to the sales contract used. Our results have important implications about the procurement strategy and the sales contract choice of processors in agricultural industries.

Key Words: Farm, Yield, Contracting, Risk Management, Agriculture, Spot Market.

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1 Introduction

In agricultural industries, the processors often rely on farm space to grow crops for sourcing of their inputs. The farm space is leased or reserved in advance of the growing season, and the realized amount of farm-yield fluctuates because of the weather conditions, pests and diseases. In practice, the farm-yield shows considerable variability (Kazaz and Webster, 2011), and presents unique challenges for the procurement strategy of these processors.

One such challenge is driven by the impact of the farm-yield on the crop quality. In the majority of agricultural industries, unfavorable weather conditions, high infestation of pests and diseases result in not only a lower farm-yield available for processing, but also a lower production rate in processing due to the inferior quality of the crop. For example, in sugar industry, cold weather reduces the quantity of sugarcane produced, and at the same time, decreases the sugar content by sapping energy from the sugarcane (Pearson 2011). In cocoa industry, high infestation of cocoa pod borer, which is one of the main fatal pests affecting the cocoa plantations, reduces the quantity of cocoa beans produced, and at the same time, decreases the cocoa butter production rate from the beans by reducing their fat content (Misnawi and Teguh 2008). In cocoa and soybean industries, insufficient rainfall decreases the quantity of beans produced, and since the resulting beans are smaller in size, the butter content of the cocoa beans and the meal content of the soybeans are lower. In all of these industries, the production rate from the crop is yield-dependent, and a lower farm-yield is associated with a lower production rate. Therefore, it is important for the procurement managers to understand the impact of yield-dependent production rate when choosing the right procurement strategy to adapt. The analysis and the insights presented in this paper helps managers achieve this goal.

In the operations management literature, a vast amount of papers study procurement decisions in the presence of yield uncertainty, often in the form of supply disruptions, in a variety of settings. However, as highlighted in Noparumpa et al (2011), the relationship between the farm-yield and the crop quality have received very limited attention. There is no work in the literature that explicitly models the crop quality in terms of its effect on the production rate. The papers in this literature (often implicitly) assume yield-invariant production rate, and provide insights on how the procurement strategy should respond to a change in yield uncertainty. Therefore, it is an open question whether the insights coming
from these papers are applicable when the production rate is yield dependent.

Our objective in this paper is to develop that knowledge base. We investigate how the procurement strategy and the profitability of the processors respond to changing yield variability. We study the impact of yield-dependent production rate on this response by making a comparison with the yield-invariant production rate benchmark case. To understand the cost of ignoring the yield-dependent nature of the production rate, we analyze the reduction in profitability when a yield-invariant production rate is used for procurement planning. Motivated by the variety of contracts used in practice (Boyabatlı, 2011), we also study whether there exist any structural differences in our results based on the sales contract type used by the processors.

To analyze this problem, we propose a stylized model in which we consider a firm (processor) that procures an agricultural input, produces and sells an output in a single period so as to maximize its expected profit. The input can be sourced from the farm space at the beginning of the growing season, where the realized amount of input fluctuates due to yield uncertainty, and from the open market at the end of the growing season. The production rate from each input is yield dependent, and non-decreasing in the yield realization. The processed input is sold through a sales contract that is characterized by its unit price. We focus on two sales contracts that are commonly used in practice: A pass-through sales contract where the unit price is given by the sum of the open market price of the input and a fixed processing margin, and a fixed-price sales contract. The firm decides the reserved farm space under the yield and the open market price uncertainties, and the processing volume, including the input sourced from the open market, after these uncertainties are realized.

With this model, we investigate the role of yield-dependent production rate and obtain the following insights:

1) We refine the common intuition prevalent in the academic literature that a higher yield variability is detrimental for the firm’s profitability. We show that this intuition is correct if the production rate is yield invariant. The intuition can be wrong, however, if the production rate is yield dependent, specifically, when the probability of achieving a higher production rate is moderate. Furthermore, we show that a lower farm space dependency always better responds to the increasing yield variability with the yield-dependent production rate, whereas this response crucially depends on the size of the reserved farm space.
with the yield-invariant production rate. Managerially, these results are important because they imply that the optimal procurement strategy adopted as a response to a change in the business environment should differ depending on the relationship between the farm-yield and the production rate. Thus, employing the response with the yield-invariant production rate can be a detrimental strategy when the true production rate is yield dependent.

2) We show that if the firm uses a yield-invariant production rate for procurement planning when the true production rate is yield dependent, the reduction in the expected profit can be substantial. The implication is that the cost of ignoring the yield-dependent nature of the production rate is significant. We also show that this cost is very sensitive to the sales contract used. This underlines the need for processors to take a holistic view of their procurement strategy to manage it together with their sales contract choice.

3) We show that with the pass-through sales contract, in comparison with the fixed-price sales contract, the benefit of increasing yield variability for the firm’s profitability is less likely to be observed, and the cost of ignoring the yield-dependent nature of the production rate is higher. Our results indicate that pass-through sales contracts, despite providing risk management benefit by transferring the open market price exposure to the consumer, are prone to other risks, either in the form of missed opportunities or higher cost of mis-management practices.

The remainder of this paper is organized as follows: Section 2 surveys the related literature and discusses the contribution of our work. §3 describes the general model, and derives the optimal strategy for this model. §4 illustrates the specific model that we use for further analysis, which is obtained by imposing more structure on our general model. Focusing on this specific model, §5 studies the impact of yield-dependent production rate on the management of yield variability, and the cost of ignoring the yield-dependent nature of the production rate in procurement planning. We conclude in §6 with a discussion of main insights and future research directions.

2 Literature Review

Our paper’s contribution is to the operations management literature on supply management in the presence of yield uncertainty. Within this literature, the yield uncertainty can be categorized into three forms. There is a vast amount of classical papers that consider production yield caused by the defective items in the manufacturing process. We refer the
readers to Yano and Lee (1995) for a review of the early literature, and Sobel and Babich (2012) for a review of the recent papers in this area. A recent stream of papers consider supply disruptions that are driven by the natural and the man-made catastrophic events, where the yield takes the form of all or nothing delivery. We refer the readers to Tomlin and Wang (2012) for a review of the papers in this stream. Similar to our work, a stream of papers consider farm-yield uncertainty in agricultural industries, where the amount of crop obtained from the farm space at the end of the growing season fluctuates due to a variety of reasons. We refer the readers to Kazaz and Webster (2011) for a review of the papers in this stream.

The papers in the supply management literature investigate the interplay between the procurement decisions in the presence of yield uncertainty with other key operational issues. In the context of agricultural industries, these issues include multiple production opportunities (Jones et al 2001), yield-dependent processing cost (Kazaz 2004), yield-dependent open market price (Kazaz and Webster 2011), pricing flexibility and downward substitution among multiple products (Noparumpa et al 2011). In the context of other industries, these issues include upstream supplier competition (Babich et al 2007), downstream buyer competition (Tang and Kouvelis 2011), investment in supplier process improvement (Wang et al 2010), imperfect information about the supplier reliability (Gumus et al 2012), co-production technology (Tomlin and Wang 2008), and product line design (Chen et al 2013). The majority of the papers in this literature establish that a higher yield variability is detrimental for the firm’s profitability. Babich et al (2007), Tang and Kouvelis (2011), and Chen et al (2013) are notable exceptions. In particular, Babich et al. (2007), considering two competing suppliers with correlated reliability uncertainty, demonstrate that a higher correlation, in other words, a higher aggregate supply uncertainty, may increase the firm’s profit by reducing the equilibrium procurement cost due to intensified supplier competition. Extending this model to include two competing retailers, Tang and Kouvelis (2011) show that a higher yield variability can be beneficial for the firm that sources from both suppliers when its competitor sources only from a single supplier in equilibrium. Chen et al (2013) find that the firm may benefit from a higher yield variability in the presence of co-production technology, where multiple outputs are produced in a single production run with random yields. Our analysis demonstrates that a higher yield variability can also be beneficial in the absence of strategic interactions with other supply chain agents, or co-
production technology, specifically, when the yield has an impact on the production rate due to its effect on the quality of the input.

Although a vast amount of papers study supply management in the presence of yield uncertainty, the relationship between the yield and the quality has received limited attention in this literature. Motivated by the semi-conductor industry, Chen et al (2013) consider a co-production system with vertically differentiated co-products in terms of their quality. They assume that a high quality co-product is only attainable in the production run if the processing yield is sufficiently high. Motivated by the wine industry, Noparumpa et al (2011) consider a firm that uses a high and a low quality input to produce a high and a low quality output, where the high quality output can only be produced from the high quality input. The input can be sourced from the open market and the leased farm space. The firm leases the farm space under the yield uncertainty, which determines the total input volume available for processing, and the quality uncertainty, which determines the proportion between the high and low quality input. They investigate the value of downward product substitution flexibility, pricing flexibility and the open market access for the firm’s profitability. In Chen et al (2013) and Noparumpa et al (2011), the relationship between the yield and the quality determines the ability to produce a higher quality output in a multi-product setting. In the agricultural industries that have motivated our paper, this relationship manifests itself by affecting the production rate of a single output. In our paper, we explicitly consider this relationship. This is the first paper that investigates the role of yield-dependent production rate in supply management.

Kazaz and Webster (2011) is the closest to our work in terms of its objective. Motivated by the lack of academic literature that considers the relationship between the yield and the open market price, they study the role of yield-dependent open market price influencing the insights coming from models that assume yield-invariant open market price; and quantify the cost of ignoring the yield-dependent nature of the open market price in procurement planning focusing on the olive oil industry. They do not consider the relationship between the yield and the quality, and thus, assume a yield-invariant production rate. In our paper, we consider the yield-dependent open market price as a part of our model, and in the spirit of their work, we study the role of yield-dependent production rate influencing the insights coming from traditional models that assume yield-invariant production rate. We demonstrate that the traditional insights may not continue to hold, and explain
why. We quantify the cost of ignoring the yield-dependent nature of the production rate in procurement planning focusing on the cocoa industry, and show that this cost can be significant.

Another stream of literature is related to our paper due to their incorporation of optimal contracting in the presence of spot (open) markets. We refer the reader to Kleindorfer and Wu (2003) and Boyabath et al. (2011) for a review of papers related to this theme. Majority of the papers in this stream focus on spot markets where the spot price is not sensitive to the actions of the agents (buyers or sellers) who participate in this market. Mendelson and Tunca (2007) and Chod et al. (2010) are two notable exceptions that analyze the equilibrium price formation in the spot markets. In line with the agricultural markets, our target application, we do not consider such strategic spot market interactions. This stream of literature studies the procurement decisions in a variety of models capturing the idiosyncratic features of different commodity markets. The examples include beef (Boyabath et al. 2011), cocoa, wheat and sugar (Boyabath 2011), electricity (Zhou et al 2013), metals (Swinney and Netessine 2011), petroleum (Dong et al. 2010), oilseeds (Boyabath and Dang 2011), and semi-conductors (Wu and Kleindorfer 2005). In the absence of yield uncertainty, these papers provide insights on how the procurement strategy should respond to changes in the business environment, and the differences in this response based on the procurement contract type. Unlike these papers, we consider yield uncertainty in studying the procurement decisions, and paralleling these papers, we investigate the differences in our insights based on the sales contract type.

The following representation is used throughout the text: A realization of the random variable $\tilde{y}$ is denoted by $y$. $\chi(.)$ denotes the indicator function, $E$ denotes the expectation operator, and $(x)^+ = \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated. “F-input” denotes the input sourced from the farm space and “O-input” denotes the input sourced from the open market.

3 General Model Description and The Optimal Strategy

In this section, we describe our general that model that captures the important characteristics of processors facing farm-yield uncertainty in the agricultural markets (§3.1), and the optimal decisions with this model (§3.2).
3.1 Model Description

We consider a firm (processor) that procures an agricultural input, produces and sells an output in a single selling season so as to maximize its expected profit. Paralleling the common feature of the processors in agricultural industries, we assume that the input can be sourced from the farm space at the beginning of the growing season, and from the open market at the end of the growing season. We model the firm’s decisions as a two-stage problem: The firm decides the reserved farm space under the yield and open market price uncertainties (stage 1); and the processing volume, including the input sourced from the open market, after the resolution of these uncertainties (stage 2).

At the beginning of the growing season (stage 1), the firm determines the amount of farm space \( Q \) to be reserved. Hereafter, we denote the input sourced from the farm space as F-input, and \( Q \) as the F-input contract volume. At the end of the growing season (stage 2), only \( vQ \) units of F-input are available for processing. Here, \( \tilde{v} \) denotes the proportional farm-yield uncertainty with mean \( \mu_v \), standard deviation \( \sigma_v \) and cdf \( f(.) \) defined in the support of \( [l, u] \) with \( 0 \leq l \leq u \leq 1 \). Each unit of F-input has a production rate \( a^F(v) > 0 \) that is non-decreasing in the farm-yield realization \( v \). The firm pays only for the F-input volume delivered at the end of the growing season. The unit procurement cost is given by the sum of a fixed price \( e > 0 \) and a yield-dependent price \( \eta(v) \) that is non-decreasing in the farm-yield realization \( v \). The latter captures the quality premium paid for the yield-dependent production rate.

At the end of the growing season (stage 2), the firm may choose to purchase additional input from the open market, which is denoted by O-input hereafter, from a unit price \( \omega > 0 \). The open market price \( \tilde{\omega} \) is unknown at the beginning of the growing season. Let \( \tilde{\omega}|v \) denote the open market price uncertainty conditional on the yield realization. Paralleling Kazaz and Webster (2011), we assume that \( \tilde{\omega} \) is negatively correlated with the yield uncertainty \( \tilde{v} \). This is because, in stage 2, when the yield is low, the open market price is high due to the low availability of O-input. We will make specific assumptions on \( (\tilde{v}, \tilde{\omega}) \) distribution in §4. Each unit of O-input has a production rate \( a^O(v) > 0 \) that is non-decreasing in the farm-yield realization \( v \). We assume that, as a result of the better care of the farmer, the fluctuations in the farm-yield can be driven by the uncontrollable factors such as weather conditions, or the controllable factors such as pests, diseases, and farming technology. We do not explicitly model these factors, and thus, do not consider their potentially different impacts on the farm-yield uncertainty.
crop quality is higher when it is sourced from the farm space, and thus, the production rate of F-input is no less than the production rate of O-input, i.e. \( a^F(v) \geq a^O(v) \ \forall v \).

The firm has a fixed contracted demand \( D \) for its output, and incurs a unit processing cost \( c \). The processed input is sold through a sales contract that is characterized by its unit price. In practice, as also highlighted in Kazaz (2004), the processors often charge, fully or partially, the open market procurement cost of the input to the consumer. Paralleling this observation, we assume that the unit price is given by \( \theta \omega + P_\theta \) for \( \theta \geq 0 \), where \( \theta \omega \) denotes the revenue indexed on the open market price of the input, and \( P_\theta > 0 \) denotes the fixed revenue. For \( \theta > 0 \), the sales price is unknown at the beginning of the growing season (stage 1) due to the open market price uncertainty \( \tilde{\omega} \). We do not consider the optimal choice among different sales contracts, but analyze how our results would change when the processor uses different sales contracts.

We close this section with an important remark. In traditional models, output yield is represented by a single random variable \( \tilde{v} \). One may think that the output yield in our model can also be represented by a single random variable \( \tilde{\omega}a(\tilde{v}) \), and thus, our insights would be very similar to the insights coming from these traditional models. Such representation is not accurate because the procurement cost is charged based on the farm-yield realization \( v \), whereas the processing cost is charged based on the production rate realization \( a(v) \); and the exposure to the yield uncertainty is different for the input that is sourced from the open market (\( a^O(\tilde{v}) \)) and the farm space (\( \tilde{\omega}a^F(\tilde{v}) \)). Regardless of these representation issues, the focus of our paper is to understand the role of \( a(\tilde{v}) \), a feature that is largely ignored in the academic literature. We will show that the yield-dependent production rate has indeed a very significant impact on the insights coming from the traditional models that assume yield-invariant production rate, and explain why.

### 3.2 The Optimal Strategy with the General Model

In this section, we describe the optimal solution for the firm’s procurement and processing decisions. We solve the firm’s problem using backward induction starting from stage 2. All the proofs are relegated to the Technical Appendix.

In stage 1, the firm procured \( Q \) units of F-input (reserved \( Q \) units of farm space).

\(^2\)A similar argument can be made for beef markets (Boyabatlı et al 2011). Because the processor (meat-packer) can interfere with the finishing operations (such as the feeding policy) in the supplier (feedlot), the contracted input (cattle) has a higher production rate than the open market input.
In stage 2, the firm observes the yield and the open market price realizations \((v, \omega)\), and constrained by the available F-input volume \(vQ\), decides the F- and O-input processing volume \(z^* = (z^F, z^O)\) to maximize the profit:

\[
\max_{z \geq 0} -\omega z^O - c(z^F + z^O) + (\theta \omega + P_0) \min \left( a^F(v)z^F + a^O(v)z^O, D \right) \\
\text{s.t. } z^F \leq vQ.
\]  

(1)

In (1), the first term is the procurement cost of the O-input, whereas the second term denotes the total processing cost. The last term is the sales revenue from the product market where \(a^F(v)z^F + a^O(v)z^O\) denotes the output volume available after processing. Since F-input does not have any procurement cost at this stage, the firm optimally processes O-input only after all available F-input is used:

**Proposition 1** The optimal F- and O-input processing volumes are given by

\[
z^{F*} = \min \left( vQ, \frac{D}{a^F(v)} \right) \chi \left( \theta \omega + P_0 \geq \frac{c}{a^F(v)} \right), \\
z^{O*} = \left( \frac{D - a^F(v)vQ}{a^O(v)} \right)^+ \chi \left( \theta \omega + P_0 \geq \frac{\omega + c}{a^O(v)} \right).
\]

When the sales margin is positive, the firm optimally satisfies the demand from the F-input processing unless constrained by the available F-input volume \(vQ\). The remaining demand (if any) is satisfied from the O-input processing only when it is profitable to source from the open market.

In stage 1, the firm determines the optimal F-input contract volume (farm space to reserve) \(Q^*\) with respect to the yield \(\tilde{v}\) and the open market price \(\tilde{\omega}\) uncertainties so as to maximize the expected profit. Using Proposition 1, the firm’s expected profit for a given F-input volume \(Q \geq 0\) is given by

\[
V(Q) = -\mathbb{E} [\tilde{v} (e + \eta(\tilde{v}))] Q + \mathbb{E} \left[ \min \left( a^F(\tilde{v})\tilde{v}Q, D \right) \left( \theta \tilde{\omega} + P_0 - \frac{c}{a^F(\tilde{v})} \right)^+ \right] \\
+ \mathbb{E} \left[ \left( D - a^F(\tilde{v})\tilde{v}Q \right)^+ \left( \theta \tilde{\omega} + P_0 - \frac{\tilde{\omega} + c}{a^O(\tilde{v})} \right)^+ \right],
\]  

(2)

where the expectation is taken with respect to \((\tilde{v}, \tilde{\omega})\). In (2), the first term is the expected F-input procurement cost, whereas the remaining terms denote the expected profit from F- and O-input processing, respectively.
Proposition 2 Let \( R(\hat{v}) \doteq \mu_v c + \mathbb{E} [\hat{\eta}(\hat{v})] \) denote the unit expected F-input procurement cost. When \( R(\hat{v}) < \mathbb{E} \left[ \hat{v} \left( a^F(\hat{v}) \min \left( \frac{\tilde{\omega}^* + c}{a^O(\hat{v})}, \theta \tilde{\omega} + P_\theta \right) - c \right) \right] \), the optimal F-input contract volume \( Q^* \in \left( \frac{\partial V}{\partial a^F(\hat{v})}, \frac{\partial V}{\partial a^O(\hat{v})} \right) \) is the solution to \( \frac{\partial V}{\partial Q} \big|_{Q^*} = 0 \) where

\[
\frac{\partial V}{\partial Q} = -R(\hat{v}) + \int_{\hat{v}}^{\hat{v}(Q)} v a^F(v) \mathbb{E}_{\hat{\eta}(v)} \left[ \left( \min \left( \frac{\tilde{\omega}^* + c}{a^O(v)}, \theta \tilde{\omega} + P_\theta \right) - \frac{c}{a^F(v)} \right)^+ \right] f(v) \, dv, \tag{3}
\]

with \( \hat{v}(Q) \) denotes the unique yield realization such that \( \lim_{v \to \hat{v}(Q)^-} v a^F(v) Q \leq D \) and \( \lim_{v \to \hat{v}(Q)^+} v a^F(v) Q \geq D \). In this case, the optimal expected profit \( V^* \) is given by

\[
D \left[ \int_{\hat{v}}^{\hat{v}(Q^*)} \mathbb{E}_{\hat{\eta}(v)} \left[ \left( \theta \tilde{\omega} + P_\theta - \frac{\tilde{\omega}^* + c}{a^O(v)} \right)^+ \right] f(v) \, dv + \int_{\hat{v}(Q^*)}^\mu \mathbb{E}_{\hat{\eta}(v)} \left[ \left( \min \left( \frac{\tilde{\omega}^* + c}{a^O(v)}, \theta \tilde{\omega} + P_\theta \right) - \frac{c}{a^F(v)} \right)^+ \right] f(v) \, dv \right]. \tag{4}
\]

When \( R(\hat{v}) \geq \mathbb{E} \left[ \hat{v} \left( a^F(\hat{v}) \min \left( \frac{\tilde{\omega}^* + c}{a^O(\hat{v})}, \theta \tilde{\omega} + P_\theta \right) - c \right) \right] \), \( Q^* = 0 \) and the optimal expected profit is given by \( V^* = \mathbb{E} \left[ \left( \theta \tilde{\omega} + P_\theta - \frac{\tilde{\omega}^* + c}{a^O(\hat{v})} \right)^+ \right] D \).

The optimal F-input contract volume is characterized by comparing the expected procurement cost \( R(\hat{v}) \) with the expected marginal revenue of an additional unit of F-input. At stage 2, the additional unit of F-input is valuable only when the available output volume \( v a^F(v) Q \) is not sufficient to satisfy the demand. In (3), \( \hat{v}(Q) \) denotes the demand-matching yield threshold. When there is unsatisfied demand, i.e. \( v < \hat{v}(Q) \), the marginal revenue is given by the additional profit margin from the F-input processing over the O-input processing per \( v a^F(v) \) units of output. When \( \frac{\tilde{\omega}^* + c}{a^O(v)} \leq \theta \omega + P_\theta \), it is profitable to source from the open market to satisfy the demand. Therefore, the additional profit margin is a function of the opportunity gain of not using the O-input. Otherwise, O-input processing is not profitable, and thus, the additional profit margin is a function of the sales price \( \theta \omega + P_\theta \).

The optimal expected profit of the firm is characterized by the expected profit margin per output. As depicted in (4), this expected profit margin is given by the profit margin from the O-input processing (the first term in (4)), and the additional profit margin from the F-input processing over the O-input processing (the second term in (4)). At stage 2, the additional profit margin from the F-input processing is relevant only when the F-input volume is sufficient to satisfy the demand, i.e. when \( v \geq \hat{v}(Q^*) \).

4 Specific Model Description and The Optimal Strategy

In §3, we introduced our general model. In this section, we introduce our specific model by imposing more structure on the general model. In particular, we make additional assump-
tions on the production rate with the F- and the O-input, the procurement and the sales contract parameters, and the distribution of the yield and the open market price uncertainties. §4.1 discusses these additional assumptions and §4.2 provides the characterization of the optimal procurement decision. We will focus on this specific model to analyze the impact of yield-dependent production rate in the next section.

### 4.1 Additional Assumptions for the Specific Model

For the production rate, we assume 

\[ a^O(v) = a \forall v, \quad a^F(v) = a \text{ for } v \leq v_0 \quad \text{and} \quad a^F(v) = a + \delta \text{ for } v > v_0, \]

where \( v_0 \in [l, u] \) and \( \delta > 0 \). In other words, O-input has a yield-invariant production rate, whereas F-input has a yield-dependent production rate with a larger rate when the yield is higher than the threshold \( v_0 \).\(^3\) When \( v_0 = l \) or \( v_0 = u \), the production rate of F-input is also yield-invariant. We assume that the firm pays an additional unit premium \( \tau \) for the F-input when the production rate is high, i.e.

\[ \eta(v) = 0 \text{ for } v \leq v_0 \quad \text{and} \quad \eta(v) = \tau \text{ for } v > v_0. \]

We also assume that the fixed part of the sales contract \( P_\theta \) is sufficiently large such that, ignoring the procurement cost, there is a positive processing margin with the F- and the O-input, i.e. \( P_\theta \geq \frac{c}{\alpha} \left( > \frac{c}{\alpha + \rho} \right) \) for \( \theta \geq 0 \).

For the procurement contract, we assume that the unit procurement cost of F-input is given by the expected open market price, i.e. \( e = E[\tilde{\omega}] \). This has two important implications. First, the firm optimally uses F-input only when

\[
\int_{v_0}^{u} v \left[ \delta E[\omega|v] \left[ \min \left( \frac{\omega + c}{a}, \theta \omega + P_\theta \right) \right] - \tau \right] f(v) \, dv > -Cov(\tilde{v}, \tilde{\omega}) + \int_{l}^{u} v \delta E[\omega|v] \left[ \left( \frac{\omega + c}{a} - \theta \omega - P_\theta \right)^+ \right] f(v) \, dv, \quad (5)
\]

where all three terms are non-negative.\(^4\) As can be observed from (5), the firm optimally does not use F-input when the yield threshold \( v_0 \) and the premium \( \tau \) are sufficiently high, and the additional production rate \( \delta \) is sufficiently low. This is because the only benefit of the F-input over the O-input is the additional production rate for high yield realizations \((v > v_0)\). Second, the optimal F-input volume is such that \( Q^* < \frac{D}{(a + \delta)v_0} \). In other words,

\[ Q = \frac{D}{(a + \delta)v_0} \]

\(^3\)This definition of \( a^F(v) \) implies that, for a given F-input contract volume \( Q \), the demand-matching yield threshold \( \tilde{v}(Q) \) of Proposition 2 is given by max \( \left( \frac{D}{(a + \delta)Q}, \min \left( v_0, \frac{D}{aQ} \right) \right) \).

\(^4\)In (5), the right-hand (left-hand) side denotes the additional expected marginal cost (revenue) of the first unit of F-input over O-input. The cost is driven by the advance commitment requirement of F-input. In particular, since \( e = E[\tilde{\omega}] \) is a sunk cost at stage 2, the F-input has an additional cost when it is not profitable to process the O-input. Moreover, F-input does not benefit from the negative correlation between the yield and open market price uncertainties. The revenue is driven by the higher production rate of F-input. In particular, more output is generated with F-input when \( v > v_0 \) at an additional cost of \( \tau \).
when the firm optimally uses F-input, i.e. when the condition (5) is satisfied, the demand-matching yield threshold of Proposition 2 in the optimal solution is given by \( \hat{v}(Q^*) = \frac{D}{(a+\delta)Q^*} > v_0 \). This is because, for larger \( Q \) such that \( \hat{v}(Q) \leq v_0 \), when there is unsatisfied demand, i.e. \( v < \hat{v}(Q) \), since the production rate benefit of the F-input is not observed, the additional profit margin from the F-input processing over the O-input processing is absent.

For the yield \( \tilde{v} \) and the open market price \( \tilde{\omega} \) uncertainties, we assume \( \tilde{v} \) to follow a uniform distribution. We define \( \tilde{\omega} = \tilde{\xi} \gamma (\tilde{v} \mu_v) \) for \( \gamma > 0 \), where \( \tilde{\xi} \) represents the yield-independent part of the open market uncertainty. We assume \( \tilde{\xi} \) to follow a normal distribution with mean \( \mu_\xi \) and standard deviation \( \sigma_\xi \), which is independent of the \( \tilde{v} \) distribution. Our representation of the open market price has two important implications. First, since \( E[\tilde{\omega}] = \mu_\xi \), the yield uncertainty does not have an impact on the expected open market price. Second, \( Cov(\tilde{v}, \tilde{\omega}) = -\gamma \sigma^2_v \leq 0 \), i.e. the yield and the open market price uncertainties are non-positively correlated. To guarantee that the open market price realization is almost always positive in the relevant \( \tilde{\xi} \) and \( \tilde{v} \) support, we assume that the coefficient of variation of \( \tilde{\xi} \) is sufficiently small and \( \gamma < \frac{2(\mu_\xi - 3\sigma_\xi)}{u-l} \).

For the sales contract, we consider two commonly used sales contracts in practice: A pass-through sales contract \((\theta = \frac{1}{a})\), where the realized sales price is given by \( \tilde{\omega} + P_{\hat{v}} \), and a fixed-price sales contract \((\theta = 0)\), where the sales price is given by \( P_0 \). With the pass-through contract, the firm charges a fixed processing margin over the open market procurement cost of the input used per unit output. In the next section, when we study the differences in the impact of yield-dependent production rate based on the sales contract, to be able to make comparisons between the two contracts, we will assume that they have the same expected sales price, i.e. \( E[\tilde{\omega}] + P_{\hat{v}} = P_0 \).

### 4.2 The Optimal Strategy with the Specific Model

In this section, we provide the characterization of the optimal procurement decision with each sales contract. It is innocuous for the readers who are not interested in these technical details to skip directly to the next section.

**Corollary 1** With the pass-through sales contract \((\theta = \frac{1}{a})\), when \( \int_{v_0}^{a} v \left[ \delta \left( \frac{\mu_v - \gamma (v - \mu_v) + c}{a} \right) - \tau \right] f(v) \, dv > \gamma \sigma^2_v \), the optimal F-input contract volume \( Q^* \in \left( \frac{D}{(a+\delta)u}, \frac{D}{(a+\delta)v_0} \right) \) is the solution to

\[
\int_{v_0}^{u} v f(v) \, dv = \int_{v_0}^{u} v (\mu_v - \gamma (v - \mu_v)) f(v) \, dv + \int_{v_0}^{\frac{D}{(a+\delta)v_0}} v (a+\delta)(\mu_v - \gamma (v - \mu_v) + c) - c \right) f(v) \, dv,
\]

The upper bound on \( \gamma \) uses the fact that almost all the probability mass of \( \tilde{\xi} \) is located above \( \mu_\xi - 3\sigma_\xi \).
and the optimal expected profit of the firm is given by

\[ V^* = D \left[ \frac{P_1}{\pi} - \frac{\xi}{a} + \int_0^\infty \rho \left( \frac{\mu \xi - \gamma(v - \mu)}{a} + \frac{\delta c}{a + \delta} \right) f(v) \, dv \right] . \tag{6} \]

Otherwise, \( Q^* = 0 \) and the optimal expected profit is given by

\[ V^* = \left[ \frac{P_1}{\pi} - \frac{\xi}{a} \right] D . \]

With the pass-through sales contract, the optimal procurement volume and the optimal expected profit are independent of the open market variability \( \sigma_\xi \). Since the O-input procurement cost is completely passed to the customer, when the firm optimally does not use F-input, the profit margin is given by \( P_1 - \frac{\xi}{a} \). As can be observed from (6), the F-input processing brings an additional profit margin when the optimal F-input volume is sufficient to satisfy the demand, i.e. when \( v > \hat{v}(Q^*) = \frac{D}{(a + \delta)\xi} \). This additional profit margin is given by the sum of the expected open market procurement cost conditional on the yield realization (since the firm gains from the open market price passed to the customer in the sales price) and the reduction in the processing cost per unit output (since each output requires \( \frac{1}{a + \delta} \) units of F-input rather than \( \frac{1}{a} \) units of O-input).

**Corollary 2** With the fixed-price sales contract \( (\theta = 0) \), when \( \int_{v_0}^u v \left[ \delta - \frac{\sigma_c}{\pi} L \left( \frac{\xi(v) - \mu_c}{\sigma_c} \right) \right] f(v) \, dv > \int_1^u v \frac{\sigma_c}{\pi} \left[ L \left( \frac{\xi(v) - \mu_c}{\sigma_c} \right) - \left( \frac{\xi(v) - \mu_c}{\sigma_c} \right) \right] f(v) \, dv + \gamma \sigma_c^2 \), the optimal F-input contract volume \( Q^* \in \left( \frac{D}{(a + \delta)u}, \frac{D}{(a + \delta)v_0} \right) \) is the solution to

\[ \mu_\delta + \tau \int_{v_0}^u v f(v) \, dv = \int_{v_0}^u v \left( aP_0 - \sigma_\xi L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right) - c \right) f(v) \, dv \\
+ \int_{v_0}^u \frac{D}{(a + \delta)v} \left( a + \delta \right) \left( P_0 - \frac{\sigma_\xi}{\pi} L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right) \right) - c \right) f(v) \, dv, \]

where \( \xi(v) = aP_0 - c + \gamma(v - \mu_v) \), and \( L(y) = \int_y^\infty (y - z) \phi(z) \, dz \) is the standard normal loss-function. In this case, the optimal expected profit of the firm is given by

\[ V^* = D \left[ \int_1^u \frac{\sigma_\xi}{\pi} L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right) f(v) \, dv + \int_0^u \left( \frac{P_0 - c}{a + \delta} - \frac{\sigma_\xi}{a} \frac{L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right)}{\sigma_\xi} \right) f(v) \, dv \right] . \tag{7} \]

Otherwise, \( Q^* = 0 \) and the optimal expected profit is given by

\[ V^* = D \left[ \int_1^u \frac{\sigma_\xi}{\pi} L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right) f(v) \, dv \right] . \]

With the fixed-price sales contract, the optimal procurement volume and the optimal expected profit depend on the open market variability \( \sigma_\xi \). As can be observed from (7), when the optimal F-input volume is sufficient to satisfy the demand, the additional profit margin of the F-input processing is given by the fixed processing margin \( P_0 - \frac{\xi}{a + \delta} \).
5 The Impact of Yield-Dependent Production Rate

In this section, we study the impact of yield-dependent production rate using the specific model developed in §4. In §5.1, we investigate the impact on the management of yield variability. In particular, we analyze how the farm space dependency and the profitability of the firm respond to increasing yield variability in comparison with the yield-invariant production rate benchmark case. In §5.2, we illustrate the cost of ignoring the yield-dependent nature of the production rate. In particular, we analyze the reduction in the expected profit when the firm uses a yield-invariant production rate for procurement planning when the true production rate is yield dependent. Motivated by the variety of contracts used in practice, we also study whether there exist any structural differences in our results with the fixed-price and the pass-through sales contracts.

Throughout this section, when analytical results are not attainable, we resort to numerical experiments. The parameter levels for these experiments are chosen based on our interactions with the cocoa processors in Southeast Asia. In cocoa industry, cocoa beans are first processed (by cleaning, roasting and grinding) into cocoa liquor, which is further processed (by pressing and milling) into final products, cocoa butter and cocoa powder. Since there is 20% yield loss in the cocoa liquor processing, we use $a + \delta = 0.8$, and set $\delta = 0.05$. For the processing cost, we assume $c \in \{2.5\%, \ 5\%, \ 7.5\%\}$ of the mean open market price $\mu$. For the unit premium paid for the high-production rate F-input, we assume $\tau \in \{0\%, \ 2.5\%, \ 5\%\}$ of the processing cost $c$. For the yield uncertainty $\tilde{v}$ parameters, we set $l = 0.1$ and $u = 0.9$. To estimate the open market price uncertainty $\tilde{\omega}$ parameters, we use the monthly average price for cocoa beans from the International Cocoa Organization website\(^6\) pertaining to the period January 2011 through December 2012. We obtain $\mu_\omega = \$2686/ton$, and $\sigma_\omega = 15\%$ of $\mu_\omega$, and also use $\sigma_\omega \in \{5\%, \ 10\%\}$ of $\mu_\omega$. Since $\tilde{\omega} = \tilde{\xi} - \gamma(\tilde{v} - \mu_\omega)$ in our model, we obtain the yield-independent open market uncertainty $\tilde{\xi}$ parameters based on the estimated $\tilde{\omega}$ parameters. In particular, $\mu_\xi = \mu_\omega$ by definition, and for $\sigma_\xi$, we assume $\gamma \in \{0, \ 100, \ 200\}$, and for each $\gamma$, we obtain $\sigma_\xi = \sqrt{\sigma^2_\omega - \gamma^2 \sigma^2_\tilde{v}}$ for $\sigma_\omega \in \{5\%, \ 10\%, \ 15\%\}$ of $\mu_\omega$. For the pass-though sales contract, we assume $P_{1_\alpha} = \frac{\xi}{\alpha} + \Delta$, where $\Delta \in \{50, \ 70, \ 90\}$ denotes the processing margin. Because we assume the same expected price for both sales contracts, $P_{\frac{1}{2}}$ implicitly defines the fixed-price sales contract price through $P_{\frac{1}{2}} = \frac{P_{1_\alpha}}{\alpha} + P_{1_\frac{2}{1}}$. We assume $D \in \{20000, \ 30000, \ 40000\}$ tonnes which is representative of cocoa butter (and

\(^6\)http://www.icco.org/statistics/cocoa-prices/monthly-averages.html
cocoa powder) demand for a medium-sized cocoa processor during the growing period of cocoa beans. The appropriate values for the yield threshold \( v_0 \) are specified later in this section. In summary, for a given \( v_0 \) value, we focus on 729 numerical instances for the fixed-price and the pass-through sales contracts separately.

5.1 The Impact on Management of Yield Variability

In this section, we investigate the impact of yield variability on the optimal F-input contract volume (reserved farm space) and the optimal expected profit of the firm. For these comparative statics analysis, we focus our attention on the interior optimal solution, where the firm optimally uses F-input, i.e. \( Q^* > 0 \). Since the yield uncertainty \( \tilde{v} \) is uniformly distributed, to operationalize the yield variability, we define the support of \( \tilde{v} \) as \([l - \epsilon, u + \epsilon]\) for \( \epsilon \in [0, l] \). An increase in \( \epsilon \) leads to a mean-preserving spread of \( \tilde{v} \) distribution, and thus, corresponds to an increase in yield variability.

To understand the impact of yield-dependent production rate on our results, we first analyze the benchmark case where the production rate of F-input is yield invariant. Because we assume \( a^F(v) = a \) for \( v \leq v_0 \) and \( a^F(v) = a + \delta \) for \( v > v_0 \), where \( v_0 \in [l - \epsilon, u + \epsilon] \), there are two cases that represent this benchmark. In particular, when \( v_0 = u + \epsilon \), F-input always has a low production rate, i.e. \( a^F(v) = a \forall v \); and when \( v_0 = l - \epsilon \), F-input always has a high production rate, i.e. \( a^F(v) = a + \delta \forall v \). When \( v_0 = u + \epsilon \), the production rate advantage of F-input over O-input does not exist; and, as follows from (5), the firm optimally does not use F-input, i.e. \( Q^* = 0 \). Therefore, we discuss the yield-invariant production rate benchmark using the \( v_0 = l - \epsilon \) case.

**Proposition 3** Let \( v_0 = l - \epsilon \), i.e. F-input has a yield-invariant production rate of \( a^F(v) = a + \delta \). For \( Q^* \geq \frac{D}{(a+\delta)\sqrt{(u+\epsilon)(l-\epsilon)}} \), \( \frac{\partial Q^*}{\partial \epsilon} \geq 0 \). For \( Q^* \leq \frac{D}{(a+\delta)\sqrt{(u+\epsilon)(l-\epsilon)}} \), \( \frac{\partial Q^*}{\partial \epsilon} \leq 0 \) when the open market price is yield independent (\( \gamma = 0 \)).

Proposition 3 demonstrates that, with the yield-invariant production rate, the impact of yield variability on the firm’s farm space dependency is crucially affected from the reserved farm space: With a sufficiently large reserved farm space, a higher farm space dependency better combats the increasing yield variability, whereas with a sufficiently small reserved farm space, a lower farm space dependency may better combat the same.

Since Proposition 3 only partially characterizes the impact of yield variability when the open market price is yield dependent (\( \gamma > 0 \)), to increase our understanding, we conduct
numerical experiments. Using the numerical instances with $\gamma > 0$ as described above, we calculate $Q^*$ for $v_0 = l - \epsilon$ as $\epsilon \in [0, 0.2, 0.4, 0.6, 0.8]$ increases. We focus only on the interior optimal solutions with strictly positive F-input contract volume. We observe that, when $Q^* \leq \frac{D}{(a+\delta)\sqrt{(u+\epsilon)(l-\epsilon)}}$, paralleling the $\gamma = 0$ case in Proposition 3, the optimal F-input contract volume decreases in the yield variability.

When the production rate is yield dependent, the impact of yield variability on the optimal F-input contract volume is different from the yield-invariant benchmark:

**Proposition 4** Let $v_0 \in (l - \epsilon, u + \epsilon)$, i.e. F-input has a yield-dependent production rate, and $v_0$ is sufficiently small such that $Q^* > 0$. In this case, $\frac{\partial Q^*}{\partial \epsilon} \leq 0$.

With the yield-dependent production rate, a lower farm space dependency always better combats the increasing yield variability. The difference between Propositions 3 and 4 is driven by the impact of $\epsilon$ on $v_0 = l - \epsilon$ in the yield-invariant benchmark case. In particular, a higher $\epsilon$ decreases $v_0$ which incents the firm to increase $Q^*$. This positive impact may dominate the negative impact of a higher $\epsilon$ on $Q^*$ for a given $v_0$, and thus, $Q^*$ may increase (which is the case for a sufficiently large $Q^*$ as shown in Proposition 3).

We next analyze the impact of yield variability on the optimal expected profit.

**Proposition 5** Let $v_0 = l - \epsilon$, i.e. F-input has a yield-invariant production rate of $a^F(v) = a + \delta$. For a given $Q \in \left(\frac{D}{(a+\delta)u}, \frac{D}{(a+\delta)v_0}\right)$, $\frac{\partial V(Q)}{\partial \epsilon} \leq 0$. This comparative static result holds when $Q$ is adjusted optimally such that $Q^* > 0$.

The common intuition prevalent in the academic literature argues that a higher yield variability is detrimental for the firm’s profitability. Proposition 5 is consistent with this observation. With a higher yield variability, the firm suffers from low yield realizations due to lower availability of F-input for processing, whereas it does not benefit from high yield realizations as much since the sales volume is capped by the demand $D$.

As in Proposition 4, the impact of yield variability on the profitability of the firm is different from the yield-invariant benchmark case:

**Proposition 6** Let $v_0 \in (l - \epsilon, u + \epsilon)$, i.e. F-input has a yield-dependent production rate, and $v_0$ is sufficiently small such that $Q^* > 0$. With the yield-independent open market price

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7For the other yield-invariant benchmark case, i.e. $v_0 = u + \epsilon$ such that $a^F(v) = a \forall v$, a paralleling result to Proposition 5 can be obtained. In particular, for a given $Q \in \left(\frac{D}{\sigma^2}, \frac{D}{\sigma v_0}\right)$, $\frac{\partial V(Q)}{\partial \epsilon} \leq 0$. However, since the firm optimally does not use F-input, we do not consider this case.
(\gamma = 0), either \( \frac{\partial V^*}{\partial \epsilon} \leq 0 \) or there exists a unique threshold \( v_0 > \sqrt{(u + \epsilon)(l - \epsilon)} \) such that, 
\( \frac{\partial V^*}{\partial \epsilon} \leq 0 \) for \( v_0 \leq v_0 \) and \( \frac{\partial V^*}{\partial \epsilon} \geq 0 \) otherwise.

Proposition 6 demonstrates that the firm may benefit from a higher yield variability in the presence of yield-dependent production rate. We now explain the intuition behind this result. Let \( V(Q; v_0) \) denote the expected profit for a given F-input contract volume \( Q \) and yield threshold \( v_0 \). It is easy to establish that

\[
V(Q; v_0) = V(Q; l - \epsilon) - Q \int_{l-\epsilon}^{v_0} v \left[ \delta \mathbb{E}_{\omega|v} \left[ \min \left( \frac{\omega + c}{a}, \theta \omega + P_0 \right) \right] - \tau \right] f(v) \, dv, \tag{8}
\]

where the first term is the expected profit for a given \( Q \) when the F-input has a yield-invariant production rate of \( a^F(v) = a + \delta \), and the second term is the expected loss for a given \( Q \) due to the lower production rate \( a^F(v) = a \) for yield realizations \( v \leq v_0 \). The impact of \( \epsilon \) on the optimal expected profit is given by its impact on these two terms at the optimal solution, i.e. for \( Q = Q^* \). Because \( Q^* \leq \frac{D}{(a + b)v_0} \), it follows from Proposition 5 that the first term is decreasing in yield variability at the optimal solution. When \( v_0 = l - \epsilon \), as in the yield-invariant production rate benchmark case, the expected loss term in (8) does not exist, and thus, yield variability is always detrimental for the firm’s profitability as proven in Proposition 5.\(^8\) Therefore, the positive impact of a higher yield variability on the firm’s profitability is driven by its impact on the expected loss term in (8).

When the open market price is yield independent (\( \gamma = 0 \)), the expected loss for a given \( Q \) decreases in the yield variability for \( v_0 \leq \sqrt{(u + \epsilon)(l - \epsilon)} \) and increases otherwise. For a sufficiently large \( v_0 \), when the firm optimally uses F-input, the decrease in the expected loss may outweigh the decrease in the yield-invariant expected profit at the optimal solution, and thus, the firm may benefit from a higher yield variability. Because the firm may optimally decide not to use F-input for a sufficiently large \( v_0 \), the benefit of higher yield variability may not be observed, and the optimal expected profit decreases in \( \epsilon \) for all \( v_0 \) values with \( Q^* > 0 \) as depicted in Proposition 6.

When the open market price is yield dependent (\( \gamma > 0 \)), there exists a critical \( v_0' > \sqrt{(u + \epsilon)(l - \epsilon)} \) such that the expected loss for a given \( Q \) decreases in yield variability for \( v_0 \leq v_0' \) and increases otherwise. However, unlike the \( \gamma = 0 \) case, for a sufficiently large \( v_0 \), the decrease in the expected loss cannot be proven to outweigh the decrease in the yield-

\(^8\)When the F-input has a yield-invariant production rate of \( a^F(v) = a \), i.e. \( v_0 = u + \epsilon \), the firm optimally does not use F-input, and thus, the expected loss term in (8) does not exist at the optimal solution.
invariant expected profit at the optimal solution. Therefore, the impact of yield variability can only be characterized for $v_0 \leq \sqrt{(u + \epsilon)(l - \epsilon)}$, where the expected loss increases in yield variability, and thus, a higher yield variability is detrimental for the firm’s profitability:

**Proposition 7** Let $v_0 \in (l - \epsilon, u + \epsilon)$, i.e. F-input has a yield-dependent production rate, and $v_0$ is sufficiently small such that $Q^* > 0$. With the yield-dependent open market price ($\gamma > 0$), $\frac{\partial V^*}{\partial \epsilon} \leq 0$ for $v_0 \leq \sqrt{(u + \epsilon)(l - \epsilon)}$.

To increase our understanding about the impact of yield variability on the optimal expected profit with the yield-dependent production rate, we conduct numerical experiments. In these experiments, since $l = 0.1$ and $u = 0.9$ in our setting, we consider $v_0 \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$. For each $v_0$, using the 729 numerical instances for the fixed-price and the pass-through sales contracts, we calculate the optimal expected profit of the firm as $\epsilon \in [0, 0.2, 0.4, 0.6, 0.8]$ increases. We focus only on the instances that result in strictly positive F-input contract volume. We summarize our numerical results using a representative figure. Figure 1 plots the optimal expected profit for a given $\epsilon$ as a function of $v_0$ with the fixed-price (Panel a) and the pass-through (Panel b) sales contract, assuming yield-independent open market price ($\gamma = 0$).

![Figure 1: The impact of a higher yield variability on the optimal expected profit of the firm for changing yield-threshold $v_0$ with $\gamma = 0$, $\sigma_\xi = 15\%$ of $\mu_\xi$, $D = 20000$, $c = 2.5\%$ of $\mu_\omega$, $\tau = 2.5\%$ of $c$ and $\Delta = 50$.](image)

Two important observations can be made from Figure 1:
1) As depicted in each panel, there exists a critical \( v_0 \) threshold above (below) which the optimal expected profit increases (decreases) in yield variability, paralleling Proposition 6. In Figure 1’s numerical instance, the firm optimally uses F-input for all the \( v_0 \) values considered. As \( v_0 \) increases further beyond these values, the firm optimally does not use F-input. The implication is that the firm benefits from a higher yield variability when \( v_0 \) is not too low or too high. A similar pattern is observed in our numerical instances with the yield-dependent open market price (\( \gamma > 0 \)).

2) Comparing Panels a and b reveals that the critical \( v_0 \) threshold is smaller with the fixed-price sales contract. At each numerical instance, we observe that if the optimal expected profit increases in yield variability with the pass-through sales contract, it also increases in yield variability with the fixed-price sales contract. The implication is that the benefit of a higher yield variability for the firm’s profitability is more likely to be observed with the fixed-price sales contract.

In summary, the yield-dependent nature of the production rate has a significant role in influencing the insights on the management of yield variability coming from traditional models that assume yield-invariant production rate. When the production rate is yield invariant, a higher yield variability is detrimental for the firm’s profitability. In contrast, the firm may benefit from a higher yield variability in the presence of yield-dependent production rate, specifically, when the probability of achieving a higher production rate is moderate. This is intuitive because when this probability is either low or high, the decision model used is close to the traditional model. While the impact of yield variability on the firm’s farm space dependency is crucially affected from the size of the reserved farm space when the production rate is yield invariant, in contrast, a lower farm space dependency always better combats the increasing yield variability with the yield-dependent production rate.

5.2 The Cost of Ignoring The Yield-Dependent Production Rate

In this section, we illustrate the cost of ignoring the yield-dependent nature of the production rate in procurement planning, and investigate the differences between the fixed-price (\( \theta = 0 \))

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2If we consider another unit procurement cost \( e < \mathbb{E}[\omega] \) such that the firm optimally uses F-input for all the feasible \( v_0 \) values, we can prove that the optimal expected profit decreases in yield variability when \( v_0 = u + \epsilon \). In other words, the benefit of a higher yield variability is not observed for a sufficiently high \( v_0 \) even if the firm optimally uses F-input.
and the pass-through ($\theta = \frac{1}{a}$) sales contract based on this cost. To this end, we consider a firm that uses a yield-invariant decision-making model when the true production rate is yield dependent. For $\theta \in \{0, \frac{1}{a}\}$, let $Q_\theta^*$ denote the optimal F-input contract volume when the production rate is yield invariant, whereas $V_\theta(Q)$ and $Q_\theta^*$ denote the expected profit for a given F-input contract volume $Q$, and the optimal F-input contract volume with the true production rate, respectively. The cost of ignoring the yield-dependent nature of the production rate is given by $\left[V_\theta \left(Q_\theta^* \right) - V_\theta \left(Q_\theta^* \right) / V_\theta \left(Q_\theta^* \right)\right]$, i.e. the percentage reduction in the expected profit due to choosing suboptimal contract volume.

As discussed in §5.1, there are two cases that represent the yield-invariant production rate model: $a_F(v) = a \forall v$ and $a_F(v) = a + \delta \forall v$, or equivalently, $v_0 = u$ and $v_0 = l$. We analyze these two cases separately. In each case, we numerically calculate the percentage reduction in the expected profit focusing on the 729 numerical instances for a given true (yield-dependent) production rate. Paralleling the numerical analysis in §5.1, we consider $v_0 \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$ to capture the true production rate. Before discussing our findings, we first present a result that will be useful in explaining the intuition.

**Lemma 1**

i) $\frac{\partial Q_\theta^*}{\partial v_0} \leq 0$, ii) $\frac{\partial V_\theta(Q_\theta^*)}{\partial v_0} \leq 0$.

Lemma 1 demonstrates that the firm overestimates (underestimates) the F-input contract volume when the yield-invariant model $a_F(v) = a + \delta \forall v$ is used. Moreover, as $v_0$ increases, since it is less likely to observe a higher production rate with F-input, the optimal expected profit decreases.

When the firm uses the yield-invariant model of $a_F(v) = a \forall v$, or equivalently, $v_0 = u$, $Q_\theta^* = 0$ for $\theta \in \{0, \frac{1}{a}\}$, and the percentage reduction in the expected profit is given by $1 - \frac{V_\theta(0)}{V_\theta(Q_\theta^*)}$. Two important observations can be made from our numerical results, as summarized in Table 1.

1) The cost of ignoring the yield-dependent production rate can be substantial. In our numerical experiments, the average percentage reduction in the expected profit (for all the $v_0$ values considered) is 14.5% (49.6%) with the fixed-price (pass-through) sales contract. As depicted in in Table 1, the percentage reduction decreases in $v_0$ with each sales contract. This is because, as $v_0$ increases, $V_\theta(0)$ is not affected, whereas $V_\theta(Q_\theta^*)$ decreases, as follows from Lemma 1. For a large enough $v_0$, the firm may choose not to use F-input, i.e. $Q_\theta^* = 0$, and the percentage reduction is zero.
Fixed-price Sales Contract ($\theta = 0$) 

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<td>0.000%</td>
<td>3.544%</td>
</tr>
</tbody>
</table>

Pass-through Sales Contract ($\theta = \frac{1}{2}$) 

<table>
<thead>
<tr>
<th>$v_0$</th>
<th>Average</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>63.034%</td>
<td>55.860%</td>
<td>70.986%</td>
</tr>
<tr>
<td>0.3</td>
<td>61.319%</td>
<td>54.028%</td>
<td>69.443%</td>
</tr>
<tr>
<td>0.4</td>
<td>58.645%</td>
<td>51.209%</td>
<td>67.002%</td>
</tr>
<tr>
<td>0.5</td>
<td>54.636%</td>
<td>47.065%</td>
<td>63.252%</td>
</tr>
<tr>
<td>0.6</td>
<td>48.572%</td>
<td>40.986%</td>
<td>57.376%</td>
</tr>
<tr>
<td>0.7</td>
<td>38.963%</td>
<td>31.810%</td>
<td>47.542%</td>
</tr>
<tr>
<td>0.8</td>
<td>22.164%</td>
<td>17.062%</td>
<td>28.711%</td>
</tr>
</tbody>
</table>

Table 1: Percentage reduction in the expected profit when the F-input procurement decision is based on $v_0 = u = 0.9$ (yield-invariant production rate) and when the true production rate is yield dependent with $v_0 \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$. For both sales contract types, “Maximum” (“Minimum”) column illustrates the highest (lowest) percentage reduction, whereas “Average” column illustrates the average percentage reduction over 729 numerical instances considered for each $v_0$.

2) The cost of ignoring the yield-dependent nature of the production rate is higher with the pass-through contract. In all of our numerical instances, we observe that the percentage reduction in the expected profit with the pass-through contract is larger than the percentage reduction with the fixed-price sales contract.

To delineate the intuition behind the second observation, since $V_{\frac{1}{2}}(Q) > V_{\frac{1}{2}}(Q')$ from the optimality of $Q'$, it is sufficient to show $V_{\frac{1}{2}}(Q) / V_{\frac{1}{2}}(Q') > V_{\frac{1}{2}}(0) / V_{\frac{1}{2}}(0)$. It is easy to establish that, for a given F-input contract volume $Q$, the expected profit with the fixed-price sales contract is higher than the expected profit with the pass-through contract. In particular, $V_{\frac{1}{2}}(Q) = V_{\frac{1}{2}}(Q) + X(Q)$, where

$$X(Q) = \int_{l}^{\hat{v}(Q)} (D - a^{F}(v)\phi Q) \mathbb{E}_{\omega|v} \left[ \left( \frac{\omega + c}{a} - P_{0} \right)^{+} \right] f(v) \, dv > 0,$$

with $\hat{v}(Q) = \max \left( \frac{D}{(\alpha + D)^{Q}}, \min \left( v_{0}, \frac{D}{4Q} \right) \right)$. As can be observed from (9), the additional expected profit $X(Q)$ of the fixed-price sales contract is decreasing in $Q$, and thus, $V_{\frac{1}{2}}(Q)$ is closer to $V_{\frac{1}{2}}(0)$ for $Q = Q_{0}'$ than for $Q = 0$.

When the firm uses a yield-invariant model of $a^{F}(v) = a + \delta \forall v$, or equivalently, $v_{0} = 1$, as follows from Lemma 1, $Q_{\theta}' > Q_{\theta}'$ for $\theta \in \{0, \frac{1}{2}\}$. In this case, the cost of ignoring the yield-dependent nature of the production rate is lower than the $v_{0} = u$ case. In particular, the average percentage reduction in the expected profit (for all the $v_{0}$ values considered)
is 4.8% (0.4%) with the fixed-price (pass-through) sales contract. The most significant consequence of this result is that the optimal expected profit of the firm is more robust to deviations from the optimal F-input contract volume (due to using yield-invariant decision model) when this deviation yields a higher contract volume in comparison with the case when this deviation yields a lower contract volume. Paralleling the \( v_0 = u \) case, we observe that the cost of ignoring the yield-dependent nature of the production rate is higher with the pass-through sales contract when the firm optimally uses F-input. In particular, in all of our numerical instances yielding \( Q^*_\theta > 0 \) for \( \theta \in \{0, \frac{1}{2}\} \), the percentage reduction in the expected profit with the pass-through sales contract is larger than the percentage reduction with the fixed-price sales contract. In the remaining numerical instances, we observe \( Q^*_0 = 0 \) and \( Q^*_1 > 0 \), and the percentage reduction is higher with the fixed-price sales contract.

In summary, the cost of ignoring the yield-dependent nature of the production rate can be substantial; and when the firm uses farm space for procurement, this cost is higher with the pass-through sales contract in comparison with the fixed-price sales contract. Our results have important implications about the sales contract choice of processors in agricultural industries as we discuss in the next section.

### 6 Conclusion

This paper contributes to the operations management literature on supply management in the presence of yield uncertainty. In the agricultural industries, unfavorable weather conditions, high infestation of pests and diseases result in not only a lower farm-yield available for processing, but also a lower production rate in processing due to the inferior quality of the crop. Yet, the majority of the papers in the literature (often implicitly) ignore the relationship between the farm-yield and the crop quality when they provide insights on how the procurement strategy should be tailored as a response to yield uncertainty. Among the few papers that consider this relationship, there is no work that explicitly models the crop quality in terms of its effect on the production rate. Therefore, it is important to develop an understanding of the role of yield-dependent production rate influencing the insights coming from traditional models that assume yield-invariant production rate. This paper attempts to achieve this goal.

To analyze this problem, we consider a firm (processor) that procures an agricultural input, produces and sells an output in a single period so as to maximize its expected profit.
The firm decides on its procurement volume (farm space to reserve) under the yield and the open market price uncertainties, and the processing volume, including the input sourced from the open market, after these uncertainties are resolved. We characterize the optimal procurement decision in a general model, and investigate the impact of yield-dependent production rate by imposing more structure on this general model. As a benchmark case, we also consider a firm that operates under the yield-invariant production rate. Our analysis shows that the yield-dependent nature of the production rate has a significant impact on the management of yield variability, as summarized in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Optimal Expected Profit</th>
<th>Optimal Reserved Farm Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield-dependent</td>
<td>Increases when the probability of achieving a high production rate is moderate, and decreases otherwise.</td>
<td>Decrease the reserved farm space.</td>
</tr>
<tr>
<td>Production Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield-invariant</td>
<td>Decreases.</td>
<td>Increase the reserved farm space when it is sufficiently high; and decrease it otherwise.</td>
</tr>
<tr>
<td>Production Rate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Impact of a higher yield variability with the yield-dependent and the yield-invariant production rate.

Managerially, these results are important because they imply that blindly following the popular belief that suggests to decrease the variability in the farm-yield can be a perilous strategy, because the benefit depends critically on the relationship between the farm-yield and the crop quality. Moreover, the optimal procurement strategy adopted as a response to a change in the business environment should differ depending on the relationship between the farm-yield and the production rate.

To understand the cost of ignoring the yield-dependent nature of the production rate, we analyze the reduction in profitability when a yield-invariant production rate is used for procurement planning. We numerically quantify this cost using a calibration based on the cocoa industry. We find that this cost can be significantly high, and is very sensitive to the sales contract used. This underlines the need for processors to take a holistic view of their procurement strategy to manage it together with their sales contract choice.
Motivated by different contracts used in practice in the agricultural industries, we also study whether there exist any structural differences in our results based on the sales contract type. We find that with the pass-through sales contract, in comparison with the fixed-price sales contract, the cost of ignoring the yield-dependent nature of the production rate is higher, and the benefit of increasing yield variability for the firm’s profitability is less likely to be observed. These results indicate that pass-through sales contract, despite decreasing the exposure to open market price risk of the input by transferring it to the consumer, increases the exposure to other risks, either in the form of missed opportunities or higher cost of mis-management practices.

Our work comes with several limitations. Relaxing the assumptions we made in our model gives rise to a number of important generalizations. We assume that the open market is only used for procurement but not for the resale of the input sourced from the farm space. In processing, there may exist capacity constraints, the processing cost can be yield dependent (Kazaz 2004), and this cost may differ based on the source of the input (Boyabath et al 2011). In procurement, we do not consider the reservation cost of the farm space, and the possibility of procurement cost to be tied up to the open market price of the input (which parallels the pass-through sales contract). It would be interesting to incorporate these issues in characterizing the optimal procurement decisions. In §4, to be able to investigate the impact of yield-dependent production rate, we impose more structure on our model by making additional assumptions. For example, we assume a specific functional form of the production rate, where the open market input has a yield-invariant production rate, and the input sourced from the farm space has a yield-dependent production rate that is represented by a step function. A similar analysis can be carried out by using a different functional form of the production rate.

In the path forward for future research, other interesting questions remain. First, we focus our attention on the differences based on the sales contracts. In practice, processors also use different types of procurement contracts. Two commonly used procurement contracts are the fixed-price contract, as considered in our paper, and the index-priced contract, where the procurement cost is a function of the open market price. It would be interesting to study whether there exist any structural differences in our results based on the procurement contract type. Second, we carry out comparative statics with respect to yield variability. Open market variability is another key determinant of the procurement strategy in the
agricultural markets. Analyzing the impact of open market variability on the farm space dependency and the profitability of the processor, and the role of yield-dependent nature of the production rate influencing these insights should prove to be an interesting problem. Finally, we assume that the cost of sourcing the input from the farm space is exogenous. In practice, the farmer may adjust the contract parameters, the unit procurement cost and the yield-dependent unit premium, as a response to the processor’s demand for the farm space. An interesting extension would be to analyze the strategic interaction between the farmer and the processor, and to characterize the contract parameters in equilibrium.

Acknowledgement. Financial support from Singapore Management University under project fund C207-MSS9B001 is gratefully acknowledged by the second author.

7 References


A Appendix

In this section, we provide the proofs for our technical statements in the paper. In particular, §A.1 contains the proofs for our technical statements in Section 3 with the general model, and §A.2 contains the proofs for the technical statements in Sections 4 and 5 with the specific model. We now provide definitions and results that we will use throughout §A.2 with the specific model. For expositional purposes, we define $K(v) = \mathbb{E}_x \left[ \min \left( \theta \left( \xi - \gamma (v - \mu_v) \right) + P_0, \frac{\xi - \gamma (v - \mu_v) + \epsilon}{a} \right) \right]$, where $\mu_v = \frac{w^l}{2}$. It follows that $K(v) = K$ when $\gamma = 0$, $K(v) \geq \frac{\xi}{a}$ by assumption, and $K(v)$ is decreasing in $v$. For a given $Q \leq \frac{D}{(a+\delta)\nu_0}$, the expected profit is given by

$$V(Q) = - \left[ \mu_v \mu_\xi + \tau \int_{v_0}^{u+\epsilon} v f(v) \, dv \right] Q + D \left[ \theta \mu_\xi + P_0 - \frac{c}{a} + \frac{\delta c}{a(a+\delta)} (1 - F(v_0)) \right]$$

$$- \int_{l-\epsilon}^{v_0} (D - avQ) \left( K(v) - \frac{c}{a} \right) f(v) \, dv - \int_{v_0}^{v_0} (D - (a+\delta)vQ) \left( K(v) - \frac{c}{a+\delta} \right) f(v) \, dv,$$

and the first-order-condition $H(Q) = \frac{\partial V(Q)}{\partial Q}$ is given by

$$H(Q) = -\mu_v \mu_\xi - \tau \int_{v_0}^{u+\epsilon} v f(v) \, dv + \int_{l-\epsilon}^{u} v aK(v) - c \right] f(v) \, dv + \int_{v_0}^{v_0} v \delta K(v) f(v) \, dv,$$

where $f(v) = \frac{1}{u-l+2\epsilon}$ is the cdf, and $F(v)$ is the pdf of the uniform distribution with support $[l-\epsilon, u+\epsilon]$. We use the following results in our proofs.

**Lemma A.1** If $Q^* > 0$, then $\delta K(v_0) > \tau$

**Proof of Lemma A.1**: As depicted in (5), if $Q^* > 0$, then the following condition must be satisfied:

$$\int_{v_0}^{u} v \left[ \delta K(v) - \tau \right] f(v) \, dv > -\text{Cov}(\hat{v}, \hat{\omega}) + \int_{l}^{u} v a \mathbb{E}_{\hat{\omega} \mid v} \left[ \left( \frac{\omega + c}{a} - \theta \omega - P_0 \right)^+ \right] f(v) \, dv,$$

where all three terms are non-negative. Since $K(v)$ is decreasing in $v$, $\int_{v_0}^{u} v \left[ \delta K(v) - \tau \right] f(v) \, dv \leq \left[ \delta K(v_0) - \tau \right] \int_{v_0}^{u} v f(v) \, dv$. If $\delta K(v_0) \leq \tau$, then the condition for $Q^* > 0$ cannot be satisfied. ■

**Lemma A.2** $\frac{\partial Q^*}{\partial v_0} < 0$ for $Q^* > 0$.

**Proof of Lemma A.2**: From the implicit function theorem, $\text{sgn} \left( \frac{\partial Q^*}{\partial v_0} \right) = \text{sgn} \left( \frac{\partial H(Q)}{\partial v_0} \right)_{Q^*}$, where $H(Q) = \frac{\partial \mathbb{V}(Q)}{\partial Q}$ as defined in (11). We obtain $\frac{\partial H(Q)}{\partial v_0} \bigg|_{Q^*} = v_0 (\tau - \delta K(v_0)) f(v_0) < 0$, where the inequality follows from Lemma A.1. ■

A.1 Proofs for the General Model: Section 3

**Proof of Proposition 1**: The proof is omitted. ■
Proof of Proposition 2: Using $\frac{c}{a^F(v)} \leq \frac{v + c}{a^F(v)} \forall v, \omega$ and $\min(x, y) = x - (x - y)^+$, it follows from (2) that

$$V(Q) = -R(\tilde{v})Q + D\mathbb{E} \left[ (\theta \tilde{v} + P_b - \frac{c}{a^F(\tilde{v})})^+ \right] - D\mathbb{E} \left[ (D - a^F(\tilde{v})\tilde{v})^+ \left( \min \left( \theta \tilde{v} + P_b, \frac{\tilde{v} + c}{a^F(\tilde{v})} \right) - \frac{c}{a^F(\tilde{v})} \right)^+ \right]. \quad (12)$$

The first-order condition in (3) is obtained by taking the partial derivative of (12) with respect to $Q$. Since $\hat{v}(Q)$ is decreasing in $Q$, we have $\frac{\partial V(Q)}{\partial Q} \leq 0$, thus, $V(Q)$ is concave in $Q$. For $Q^* > 0$, the optimal expected profit expression (4) is obtained by substituting $Q^*$ in (12), and using the optimality condition $R(\tilde{v}) = \int_1^{\hat{v}(Q^*)} v a^F(v)E_{\tilde{v}|v} \left[ \min \left( \frac{\tilde{v} + c}{a}, \theta \tilde{v} + P_b \right) - a^F(\tilde{v}) \right]^+ f(v) \, dv$. ■

A.2 Proofs for the Specific Model: Sections 4 and 5

Proof of Corollary 1: The proof follows Proposition 2 using the assumptions $a^O(v) = a \forall v, a^F(v) = a$ for $v \leq v_0$ and $a^F(v) = a + \delta$ for $v > v_0$, $P_\delta > \frac{\xi}{a}$, $R(\tilde{v}) = \mu_v \mu_\xi + \tau \int_{v_0}^u v f(v) \, dv$, and the definitions $\hat{v}(Q) = \max \left( \frac{D}{(a+\delta)Q}, \min \left( v_0, \frac{D}{\mu_\xi Q} \right) \right)$, and $K(v) = \mu_\xi - \gamma(v - \mu_v)$. ■

Proof of Corollary 2: The proof follows Proposition 2 using the assumptions $a^O(v) = a \forall v, a^F(v) = a$ for $v \leq v_0$ and $a^F(v) = a + \delta$ for $v > v_0$, $P_\delta > \frac{\xi}{a}$, $R(\tilde{v}) = \mu_v \mu_\xi + \tau \int_{v_0}^u v f(v) \, dv$, and the definitions $\hat{v}(Q) = \max \left( \frac{D}{(a+\delta)Q}, \min \left( v_0, \frac{D}{\mu_\xi Q} \right) \right)$, and $K(v) = P_\delta - \frac{\xi}{a} L \left( \frac{\xi(v) - \mu_\xi}{\sigma_\xi} \right)$ where $\xi(v)$ and $L(.)$ are as defined in the proposition. The expression for $K(v)$ is obtained by using $\xi = \mu_\xi + z \sigma_\xi$, where $z$ is the standard normal variable, and the identity $\int_{-\infty}^y z \phi(z) \, dz = -\phi(y)$, where $\phi(.)$ is the standard normal cdf. ■

Proof of Proposition 3: From the implicit function theorem, $\text{sgn} \left( \frac{\partial v}{\partial \epsilon} \right) = \text{sgn} \left( \frac{\partial H(Q)}{\partial \epsilon} \right)_{Q^*}$, where $H(Q) = \frac{\partial V(Q)}{\partial Q}$ as defined in (11) with $v_0 = l - \epsilon$.

We first prove the case with yield independent open market price ($\gamma = 0$). We obtain

$$\frac{\partial H(Q)}{\partial \epsilon} = \frac{(a + \delta)K - c}{(u - l + 2\epsilon)^2} \left[ (u + \epsilon)(l - \epsilon) - \frac{D}{(a + \delta)Q^*} \right]^2,$$

which has a negative sign when $Q^* \leq \frac{D}{(a+\delta)\sqrt{(l-\epsilon)(l+\epsilon)}}$, and positive sign otherwise.

With the yield-dependent open market price ($\gamma > 0$), for $Q \leq \frac{D}{(a+\delta)v_0}$, we obtain

$$\frac{\partial H(Q)}{\partial \epsilon} = \frac{1}{u - l + 2\epsilon} \left[ (l - \epsilon)[(a + \delta)K(l - \epsilon) - c] - 2 \int_{l-\epsilon}^{u+\epsilon} [(a + \delta)K(v) - c]v \frac{1}{u - l + 2\epsilon} \, dv \right] \geq \frac{(a + \delta)K(l - \epsilon) - c}{u - l + 2\epsilon} \left[ (l - \epsilon) - 2 \int_{l-\epsilon}^{u+\epsilon} v \frac{1}{u - l + 2\epsilon} \, dv \right]$$

(Since $K(v)$ is decreasing in $v$ and $K(l - \epsilon)$ is independent of $v$)

$$= \frac{(a + \delta)K(l - \epsilon) - c}{(u - l + 2\epsilon)^2} \left[ (l - \epsilon)(u + \epsilon) - \left( \frac{D}{(a + \delta)Q^*} \right)^2 \right].$$

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Since $K(l - \epsilon) \geq \frac{c}{a + \delta}$ by assumption, the last expression, when evaluated at $Q = Q^*$, has a positive sign for $Q^* \geq \frac{D}{(a + \delta)\sqrt{(l - \epsilon)(u + \epsilon)}}$. ■

**Proof of Proposition 4**: From the implicit function theorem, $\text{sgn} \left( \frac{\partial Q}{\partial \epsilon} \right) = \text{sgn} \left( \frac{\partial H(Q)}{\partial \epsilon} \right)_{Q^*}$, where $H(Q) = \frac{\partial V(Q)}{\partial \epsilon}$ as defined in (11). Taking the partial derivative of $H(Q)$ with respect to $\epsilon$, and using the optimality condition $H(Q^*) = 0$, we obtain

$$\frac{\partial H(Q)}{\partial \epsilon} \bigg|_{Q^*} = \frac{1}{u - l + 2\epsilon} \left[ -\tau(u + \epsilon) - \mu_\epsilon(u + l) + (l - \epsilon)[aK(l - \epsilon) - c] \right]$$

$$\leq \frac{1}{u - l + 2\epsilon} \left[ (\tau + \mu_\epsilon)(u + \epsilon) + \gamma(l - \epsilon) \left( \frac{u + l}{2} - (l - \epsilon) \right) \right],$$

where the last inequality follows from $K(l - \epsilon) \leq \frac{\mu_\epsilon - (l - \epsilon) - \frac{a\epsilon}{a + \delta}}{a + \delta}$. By assumption, we have $\gamma < \frac{2(\mu_\epsilon + \tau)(u + \epsilon)}{a + \delta (l - \epsilon)}$, which implies that $\gamma < \frac{2(\mu_\epsilon + \tau)(u + \epsilon)}{a + \delta (l - \epsilon)}$. It follows that the equation (13) has a negative sign, and thus, $\frac{\partial H(Q^*)}{\partial \epsilon} \bigg|_{Q^*} \leq 0$. ■

**Proof of Proposition 5**: We have $\frac{\partial V}{\partial \epsilon} = \frac{\partial V(Q)}{\partial \epsilon} \bigg|_{Q^*}$. Since $Q^* \leq \frac{D}{(a + \delta)\epsilon_0}$, it is sufficient to show that $\frac{\partial V(Q)}{\partial \epsilon} \leq 0$ for $Q \leq \frac{D}{(a + \delta)\epsilon_0}$. Using $v_0 = l - \epsilon$ in (10), we obtain

$$\frac{\partial V(Q)}{\partial \epsilon} = \frac{2}{u - l + 2\epsilon} \int_{l - \epsilon}^{u - \delta} (D - (a + \delta)vQ) \left( K(v) - \frac{c}{a + \delta} \right) \frac{1}{u - l + 2\epsilon} dv$$

$$- (D - (a + \delta)(l - \epsilon)Q) \left( K(l - \epsilon) - \frac{c}{a + \delta} \right) u - l + 2\epsilon$$

$$\leq \frac{K(l - \epsilon) - \frac{c}{a + \delta}}{u - l + 2\epsilon} \left[ \frac{2}{u - l + 2\epsilon} \int_{l - \epsilon}^{u - \delta} (D - (a + \delta)vQ) \frac{1}{u - l + 2\epsilon} dv - (D - (a + \delta)(l - \epsilon)Q) \right]$$

(Since $K(v)$ is decreasing in $v$ and $K(l - \epsilon)$ is independent of $v$)

$$= \frac{K(l - \epsilon) - \frac{c}{a + \delta}}{u - l + 2\epsilon} \left[ \frac{D}{(a + \delta)Q} - (u + \epsilon) \right] \left[ \frac{D}{(a + \delta)Q} - (l - \epsilon) \right] \leq 0,$$

where the last inequality follows from $l - \epsilon < \frac{D}{(a + \delta)Q} < u + \epsilon$, and $K(l - \epsilon) > \frac{c}{a + \delta}$. ■

**Proof of Proposition 6**: We have $\frac{\partial V^*}{\partial \epsilon} = \frac{\partial V(Q)}{\partial \epsilon} \bigg|_{Q^*}$. Since $Q^* \leq \frac{D}{(a + \delta)\epsilon_0}$, it is sufficient to focus on $Q \leq \frac{D}{(a + \delta)\epsilon_0}$ to analyze the impact of $\epsilon$. From (10), we obtain

$$\varphi(\epsilon) = \frac{(\delta K - \tau)^2}{a + \delta} [v_0^2 - (u + \epsilon)(l - \epsilon)] + \left( K - \frac{c}{a + \delta} \right) \left[ \frac{D}{(a + \delta)Q} - (u + \epsilon) \right] \left[ \frac{D}{(a + \delta)Q} - (l - \epsilon) \right].$$

(14)

It is sufficient to analyze the sign of $\varphi(\epsilon)_{|Q^*}$ to determine the sign of $\frac{\partial V^*}{\partial \epsilon}$.

For $v_0 \leq \sqrt{(u + \epsilon)(l - \epsilon)}$, since $\delta K > \tau$ for $Q^* > 0$ as follows from Lemma A.1, and $l - \epsilon < \frac{D}{(a + \delta)Q} < u + \epsilon$, we have $\varphi(\epsilon)_{|Q^*} \leq 0$.

For $v_0 > \sqrt{(u + \epsilon)(l - \epsilon)}$, the first term in (14), when evaluated at the optimal solution, is positive and increasing in $v_0$. To analyze the sign of the second term in (14), we define $g(x) = \left[ \frac{D}{(a + \delta)Q} - (u + \epsilon) \right] \left[ \frac{D}{(a + \delta)Q} - (l - \epsilon) \right]$. 29
\( (x - (l - \epsilon))(x - (u + \epsilon)) \) for \( x \in (l - \epsilon, u + \epsilon) \). It is easy to establish that \( g(x) \) is decreasing in \( x \) for \( x < \frac{u+l}{2} \), and is increasing in \( x \) for \( x > \frac{u+l}{2} \). Since \( \frac{\partial G}{\partial v_0} < 0 \), as follows from Lemma A.2, as \( v_0 \) increases, the second term in (14) first becomes more negative (for \( v_0 \) such that \( \frac{D}{(a+\delta)Q^*} \leq \frac{u+l}{2} \)), and then becomes more positive (for \( v_0 \) such that \( \frac{D}{(a+\delta)Q^*} \geq \frac{u+l}{2} \)). Therefore, for a sufficiently high \( v_0 \), both terms in (14) are increasing in \( v_0 \) at the optimal solution, and thus, \( \varphi(\epsilon)|Q^* \geq 0 \).

To prove the existence of a unique \( \tau_0 \) threshold, such that \( \varphi(\epsilon)|Q^* \leq 0 \) for \( v_0 \leq \tau_0 \), and \( \varphi(\epsilon)|Q^* \geq 0 \) otherwise, it is sufficient to show that if \( \frac{D}{(a+\delta)Q^*} \leq \frac{u+l}{2} \), then \( \varphi(\epsilon)|Q^* \leq 0 \). In other words, \( \varphi(\epsilon)|Q^* \geq 0 \) can only be observed for \( v_0 \) values, where the second term in (14) increases in \( v_0 \). Using \( H(Q^*) = 0 \) in (14), where \( H(Q) = \frac{\partial V(Q)}{\partial Q} \) as defined in (11), we obtain

\[
\varphi(\epsilon)|Q^* = \frac{(u-l+2\epsilon)}{a+\delta} [(aK-c)(l-\epsilon) - \tau(u+\epsilon) - \mu_\xi (u+l)] - 2 \left( K - \frac{c}{a+\delta} \right) \frac{D}{(a+\delta)Q^*} \left[ \frac{u+l}{2} - \frac{D}{(a+\delta)Q^*} \right].
\]

Since \( K = \mathbb{E}_\xi \left[ \min \left( \hat{\theta} + P_0, \hat{\xi} + c \right) \right] \leq \frac{\mu + c}{a} \), we have \((aK-c)(l-\epsilon) - \tau(u+\epsilon) - \mu_\xi (u+l) \leq -(\tau + \mu_\xi)(u+\epsilon) < 0\), and thus, if \( \frac{D}{(a+\delta)Q^*} \leq \frac{u+l}{2} \), then \( \varphi(\epsilon)|Q^* \leq 0 \).

In summary, there exists a unique \( \tau_0 > \sqrt{(u+\epsilon)(l-\epsilon)} \) such that \( \frac{\partial V^*}{\partial \epsilon} \leq 0 \) for \( v_0 \leq \tau_0 \), and \( \frac{\partial V^*}{\partial \epsilon} \geq 0 \) otherwise. We note here that for sufficiently large \( v_0 \) values, as follows from (5), we have \( Q^* = 0 \). Therefore, \( \tau_0 \) may not exist, and \( \frac{\partial V^*}{\partial \epsilon} \leq 0 \) for \( Q^* > 0 \).

**Proof of Proposition 7**: Let \( V(Q; v_0) \) denote the expected profit for a given F-input contract volume \( Q \) and yield threshold \( v_0 \). As depicted in (8),

\[
V(Q; v_0) = V(Q;l-\epsilon) - Q \int_{l-\epsilon}^{v_0} v \lbrack \delta K(v) - \tau \rbrack \frac{1}{u-l+2\epsilon} \, dv.
\]

Since \( V(Q;l-\epsilon) \) is decreasing in \( \epsilon \), as follows from Proposition 5, it is sufficient to show that \( G = \int_{l-\epsilon}^{v_0} v \lbrack \delta K(v) - \tau \rbrack \frac{1}{u-l+2\epsilon} \, dv \) increases in \( \epsilon \) for \( v_0 \leq \sqrt{(u+\epsilon)(l-\epsilon)} \) at the optimal solution. We obtain

\[
\frac{\partial G}{\partial \epsilon} = \frac{1}{u-l+2\epsilon} \left[ (l-\epsilon)[\delta K(l-\epsilon) - \tau] - 2 \int_{l-\epsilon}^{v_0} v \lbrack \delta K(v) - \tau \rbrack \frac{1}{u-l+2\epsilon} \, dv \right] \\
\geq \frac{\delta K(l-\epsilon) - \tau}{u-l+2\epsilon} \left[ \frac{1}{u-l+2\epsilon} \frac{1}{u-l+2\epsilon} \left( l-\epsilon - 2 \int_{l-\epsilon}^{v_0} v \frac{1}{u-l+2\epsilon} \, dv \right) \right] \\
= \frac{\delta K(l-\epsilon) - \tau}{u-l+2\epsilon} \left[ \frac{(u+\epsilon)(l-\epsilon) - v_0^2}{u-l+2\epsilon} \right],
\]

where the last term is non-negative for \( v_0 \leq \sqrt{(u+\epsilon)(l-\epsilon)} \) because \( \delta K(l-\epsilon) > \tau \), as \( K(v) \) is decreasing in \( v \) and \( \delta K(v_0) > \tau \) for \( Q^* > 0 \) as follows from Lemma A.1.

**Proof of Lemma 1**: The first part follows from Lemma A.2. For the second part, using \( \hat{v}(Q^*) = \frac{D}{(a+\delta)Q^*} \) in the optimal expected profit expression (4), we obtain \( \frac{\partial V^*}{\partial v_0} = \frac{\partial V^*}{\partial Q^*} \frac{\partial Q^*}{\partial v_0} \leq 0 \).