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Platform pricing with endogenous network effects

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Abstract

This paper examines a monopoly platform's two-sided pricing strategy through modeling the trades between the participating sellers and buyers. In this approach, the network effects emerge endogenously through the equilibrium trading strategies of the two sides. We show that platform pricing depends crucially on the characteristics associated with market liquidity, including both sides' entry costs, the buyers' preferences, and the distribution of the sellers' quality. The platform may subsidize sellers if the market is sufficiently liquid, whereas buyer subsidy can be optimal given an illiquid market. We also illustrate the impact of the sellers' quality heterogeneity on the platform's optimal fees and the two sides' entry scales. These findings provide guidance for platform pricing based on specific market and user characteristics, which are directly applicable to managerial decisions and deepen theoretical understanding of two-sided platforms.

Keywords: *Two-sided platforms, subsidy, variety, quality, network effects, vertical differentiation*

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1 Introduction

Platform business models are creating vibrant online marketplaces. For instance, smartphone application markets bring together third-party applications and end users. In the sharing economy, Airbnb and HomeAway provide listings of vacancies from hosts to renters. Also, the dominant online retailer Amazon is shifting to the platform model by welcoming an increasing number of third-party sellers on board. All these platforms facilitate a high volume of transactions between two groups of users, buyers and third-party sellers, and profit from their transactions by imposing fees on one or both groups.

In this paper, we examine platform pricing, focusing on the transactions between the sellers and buyers. The literature often links platform pricing to the relative strengths of cross-side network effects, which refer to the benefits one group of users derive from an increase in the number of users in the other group. However, this abstracts out the specific economic mechanisms that generate the network effect. For example, as more sellers join the platform, their competition is altered, which then leads to a new set of product prices that affect both the sellers' and the buyers' surplus. Moreover, the existing findings may not be directly applicable to managerial problems, in which the strengths of network effects are often not readily measurable or observable. To address these issues, we build a microfoundation to model the transactions on the platform. The microfoundation incorporates important user characteristics such as seller heterogeneity and buyers' preferences, which determine their trading decisions and benefits. In addition, these characteristics affect the users' entry decisions that are based on the trading benefits and are, therefore, critical to the platform owner's profit and pricing problem.

Our approach has two powerful merits. First, it allows network effects to emerge endogenously through users' decisions, which uncovers new properties of the network effects. In the existing research, the cross-side network effect is often specified exogenously. Moreover, such specification is often linear and constant, and tends to assume away interactions within the same side.[1] We endogenize network effects by modeling entry decisions with trading choices of the platform users

[1] In an earlier debate, Liebowitz and Margolis (1994) also point out the limitations of assumptions of network effects. They raise methodological concerns regarding the assumption that "the benefits of an activity depend upon the number of participants" (p. 149) in describing network externality. Their concern motivates a stronger theoretical foundation for network effects studied in the two-sided context.

and by characterizing the closed-form trading equilibrium. The economic mechanisms give rise to both cross-side and same-side network effects. The results show that the endogenous cross-side network effects are not always linear as commonly assumed. Also, these network effects are not constant due to the changes in the competition intensity as more sellers enter the platform.

Second, the microfoundation allows us to examine the impact of the characteristics of platform users on the platform owner's pricing decisions. In particular, we are interested in seller variety and quality variation. Through the users' trading decisions, these dimensions of heterogeneity determine the platform's optimal pricing strategies. The managerial relevance of such heterogeneity on a platform is ubiquitous. Amazon, eBay, and Alibaba are known as online platforms for trading a rich diversity of products, spanning hundreds of categories and varying quality levels. Mobile application markets, such as App Store and Google Play, have also grown to offer applications that have varying ratings and suit a multitude of purposes. In addition, some platforms have a global presence, such as Craigslist which has more than 700 local sites in 70 countries and Airbnb with an even wider coverage. While serving multiple regions, these platforms face varying quality heterogeneity as a result of economic, cultural, and political characteristics of different regions. Thus, it is inevitable for platform owners to strategize contingent on heterogeneity of their sellers.

Building on the microfoundation, we study how a monopoly platform's two-sided pricing strategy depends on quality heterogeneity among sellers and other market characteristics. More specifically, should the platform owner subsidize buyers or sellers? How does quality heterogeneity play into this decision? And how do other aspects of the market environment guide the platform's strategy? In our study, the platform collects entry fees from both sides. Entry on both sides is endogenous such that it depends on the entry fees as well as on the surplus generated from the interactions after entry. We model the microfoundation of the interactions by introducing quality heterogeneity to the circular city model to capture both horizontal and vertical differentiation. Both cross-side and same-side network effects then emerge endogenously through sellers' and buyers' trading decisions. Based on this equilibrium, the platform's optimal strategy in offering subsidies is established. We then identify the condition that leads to optimal decisions of subsidizing each side. We also illustrate the impact of quality variation on the platform's optimal fees and profits.

Our major finding is that a platform's subsidy choice depends crucially on *market liquidity*. A liquid market possesses characteristics that lead to a large network on the platform. In our setting, a number of features are associated with a more liquid market, including higher numbers of potential buyers and sellers, lower entry cost on either side,[2] weaker horizontal preferences of buyers, higher average value of products, and more variation in product quality. These features reduce trading friction, attract more users, and lead to a higher trading volume. Unlike the magnitude of network effects that varies with the buyers', sellers', and the platform's endogenous choices, the features related to market liquidity are exogenous and readily identifiable and measurable in practice. Thus, our findings provide direct and practical guidance to platform pricing strategies.

More specifically, we find that the platform tends to subsidize sellers when the market is more liquid and subsidize buyers when the market is less liquid. Subsidizing the seller side is optimal when the marginal seller subsidized creates an overall gain for the platform that offsets the subsidy provided. The platform can realize such a gain from the buyer side because admitting the marginal seller improves the horizontal preference match for the buyers. In a more liquid market, the equilibrium numbers of sellers and buyers are high. As buyers' horizontal preferences are well served by the sellers, sellers' quality plays a more prominent role in the buyers' purchasing choices and create additional surplus for the buyers. This sharpens the platform's gain from the buyer side. In the same spirit, subsidizing the buyer side is optimal when the marginal buyer subsidized creates an overall gain for the platform. The seller side can generate a gain through the additional demand from the marginal buyer subsidized. However, if the market is highly liquid, the intense competition among the sellers makes it difficult for the platform to obtain sufficient gains. Therefore, the buyer-side subsidy is more likely in an illiquid market.

We also derive several important findings regarding quality heterogeneity among the sellers. More quality variation among the sellers allows the platform to admit more sellers in equilibrium, which establishes the relationship between vertical and horizontal differentiation. An increase in quality variation allows higher quality sellers to attract more buyers and charge higher prices; meanwhile, lower quality sellers lose some buyers and suffer a price cut. Overall, the gains outweigh

[2] The entry cost is different than the endogenous entry fee set by the platform. This term is more specifically defined in Section 3.

the losses, so the total seller surplus increases with quality variation. Similarly, the total surplus of buyers is also higher. Given that the platform internalizes its users' gains through entry fees, it benefits from increased quality variation. More importantly, this gain is sharpened if more sellers are present, which reduces horizontal differentiation and therefore allows quality to play a more dominant role in the two sides' choices. Thus, it is optimal for the platform to admit more sellers when their products are more vertically differentiated. As a higher number of sellers leads a more crowded market with reduced horizontal differentiation, this suggests that the two dimensions of differentiation are substitutes for the platform in equilibrium.

We also show that quality heterogeneity has an impact on the platform's fees and profit. First, more variation in the sellers' quality levels leads to a lower seller-side fee and a higher buyer-side fee. This is consistent with the results that greater quality variation makes seller-side subsidy more likely and buyer-side subsidy less likely. Here, it allows us to also understand the magnitude of fees (inclusive of subsidies) conditional on quality heterogeneity. Furthermore, greater quality variation increases the platform's profit and shifts the surplus away from the seller side and toward the buyer side. The intuitions for the previous results also support these findings. As the platform reduces the seller-side fee (and possibly subsidizes the sellers), it will rely more on the buyer side for its profits. The increase in quality variation creates an overall positive effect that allows the platform to generate more profits from both sides combined.

The remaining of the paper is organized as follows: In Section 2, we discuss the related literature. We describe the model setup in Section 3 and analyze the model in Section 4. And then, Section 5 explores the relationship between horizontal and vertical differentiation in equilibrium. Section 6 presents the platform's optimal prices and its subsidization strategies. Finally, we conclude the paper in Section 7.

2 Related Literature

One of the contributions of our paper is to offer a practical theory on platform pricing. In the seminal studies (Rochet and Tirole 2003, Yoo et al. 2003, and Parker and Van Alstyne 2005, Armstrong 2006), a consistent insight is that the strength of network effect exerted by users on one side to the other side (i.e., the cross-side network effect) is a key determinant of the platform's pricing strategies and that, by symmetric construction, the platform tends to subsidize the side that contributes a stronger network effect. Through endogenizing these network effects, we show that the platform's subsidization decision depends on the features related to market liquidity, such as the two sides' entry costs, the potential buyer demand, the sellers' quality, and the buyers' preferences.

Several recent studies also endogenize network effects. In Bakos and Katsamakas (2008), the network effect is modeled as the platform's choice of design investments. They point out the asymmetry in the platform's decisions on the two sides. We endogenize the network effect through deriving the surplus created by interactions between sellers and buyers, an approach also taken by Hagiu (2009). Hagiu (2009) finds that when consumers have stronger preferences for variety, the platform relies more on the seller side for generating profits. The finding suggests that mitigated seller competition may allow the platform to subsidize buyers (or generate more revenues from the seller side), which is broadly consistent with one of our findings. An important difference in our work is that we link platform pricing to several measurable economic parameters, whereas buyers' preference for variety in Hagiu (2009) is subject to model interpretation.

Another important development in the growing literature is the attention on users' interactions on the platform (Economides and Katsamakas 2006, Hagiu 2009, Lin et al. 2011, Cheng et al. 2011, Halaburda and Piskorski 2013, Hagiu and Wright 2013, and others). Economides and Katsamakas (2006) model pricing decisions of application developers (i.e., sellers) on operating system platforms and compare industry profits and market shares between proprietary and open platforms. Lin et al. (2011) examine the competition of quality-differentiated sellers on a platform where seller entry is driven by an innovation race. They find that the platform may subsidize the buyers when their valuation for quality is more dispersed. Cheng et al. (2011) consider a net neutrality problem by modeling content providers' pricing decisions in interacting with consumers through an Internet Service Provider (ISP) platform. In the context of a matching platform, Halaburda and Piskorski (2013) consider the tradeoffs between choice and competition based on a microfoundation. Hao et al. (2014) study the advertising contract of a two-sided platform by analyzing the complex interactions among the platform owner, app developers, advertisers, and users. Edelman and Wright (2013) examine the transactions on and off a platform. By exploring the impact of platform price coherence (i.e., sellers are restricted to charge the same price on and off the platform) on consumer surplus and social welfare, they illustrate the harm of platform intermediation.

Our paper also focuses on the transactions on the platform. We offer three distinctive features: price competition, endogenous network effects, and specific user characteristics. The microfoundation in our work characterizes price competition among sellers, which is absent in some studies (Halaburda and Piskorski 2013). Price competition is a natural way to capture interactions on marketplace-type platforms. It also enables us to model the two sides' decisions in a fairly general setting. The equilibrium outcome characterizes buyers' and sellers' surplus from trading on the platform, which in turn connects to the two sides' entry decisions.

The endogenous network effects in our work are not only derived from the transactions on the platform, but also from the numbers of sellers and buyers determined by their entry decisions. Many studies with endogenous entry take cross-side network effects as given (Rochet and Tirole 2003, Parker and Van Alstyne 2005, Armstrong 2006), while some other studies endogenize cross-side network effects but not entry. Both Edelman and Wright (2013) and Lin et al. (2011) model seller entry, but the number of sellers on the platform is limited to two. We allow multiple sellers to enter the platform, which enables network effects to arise from sellers' entry and price competition. Economides and Katsamakas (2006) and Wu and Lin (2013) both consider competition among multiple sellers; however, the network size on the seller side is exogenous and fixed. In their studies, the focus is on the platform's strategy facing the existing user base. Our interest lies in how the platform attracts users' participation on both sides with consideration for equilibrium trades on the platform; incorporating both entry and transactions in the model allows us to fully endogenize both cross-side and same-side network effects.

Another novelty in our paper is to consider specific characteristics, such as quality heterogeneity of sellers, buyers' preferences, and two sides' entry costs, in the platform's pricing strategies. In fact, the platform research is showing increasing interest in user characteristics in addition to the network effect parameters. Bhargava and Choudhary (2004) consider the buyers' heterogeneous preference for quality and analyze a platform's versioning strategy. They find that the cross-side and same-side network effects lead to stronger incentives for providing services of differentiated quality. Boudreau (2012) empirically shows that it is the heterogeneity of sellers that generates the variety of software they produce. Chao and Derdenger (2013) emphasize that consumer heterogeneity is a primary driver of the platform's bundling strategies. Anderson et al. (2014) observe that game development costs tend to be lower for lower-performance consoles. They reveal that offering a lower-performance platform, which attracts more developers, may be an optimal strategy. Relative to these studies, the characteristics we examine are particularly meaningful in contributing to the presence and the magnitude of network effects, which then connect to the platform's pricing problem.

3 Model Setup

Three types of players make decisions in this game: a platform, N potential sellers, and z potential buyers. The platform charges each seller and buyer an entry fee, denoted by R_s and R_b, respectively.[3] In addition to the entry fees, sellers and buyers incur entry costs. Buyers are heterogeneous in the entry cost c which follows a uniform distribution on $[0, C]$. We assume that all sellers have the same entry cost, f.[4] Entry fees and entry costs differ in three aspects. First, entry fee is a transfer payment between a user and the platform, whereas entry cost is a deadweight loss, to the three parties combined. Second, entry fees are endogenous and chosen by the platform, whereas entry costs are exogenous. Third, unlike positive entry costs, the entry fees can be negative, which suggests a subsidy provided by the platform.

On the platform, sellers' products are differentiated both horizontally and vertically. We introduce quality heterogeneity to Salop's circle to model these two dimensions of differentiation and interactions between the two sides (Salop 1979). Horizontal differentiation is represented by sellers' locations on a unit circle, and buyers' horizontal preferences are uniformly distributed along this

[3] In practice, platforms often charge a two-part tariff on the seller side. By assuming that all players are risk neutral and that, prior to entry, sellers face the same expectation regarding transactions, the lump sum R_s is equivalent to a two-part tariff.

[4] We assume heterogeneous entry cost on the buyer side to ensure an interior solution of buyer entry scale such that the scale adjusts marginally as the buyer entry fee changes. Such heterogeneity is unnecessary on the seller side because a seller's expected profit decreases as more sellers enter the platform, which leads to an interior entry scale on the seller side even though their entry costs are identical. If sellers also differ in their entry costs, our major findings continue to hold.

circle. We adopt the common assumption that each seller offers only one product; thus, *seller* and *product* are conceptually equivalent. We also normalize sellers' production cost to zero. Vertical differentiation refers to quality heterogeneity among sellers. Sellers are uncertain about their quality before entering the platform. Many reasons, such as the serendipitous nature of R&D and unpredictable market environment, may lead to such uncertainties. Each seller's quality is modeled as an independent draw from the identical distribution on $[\underline{v}, \overline{v}]$ with mean μ and variance σ^2.

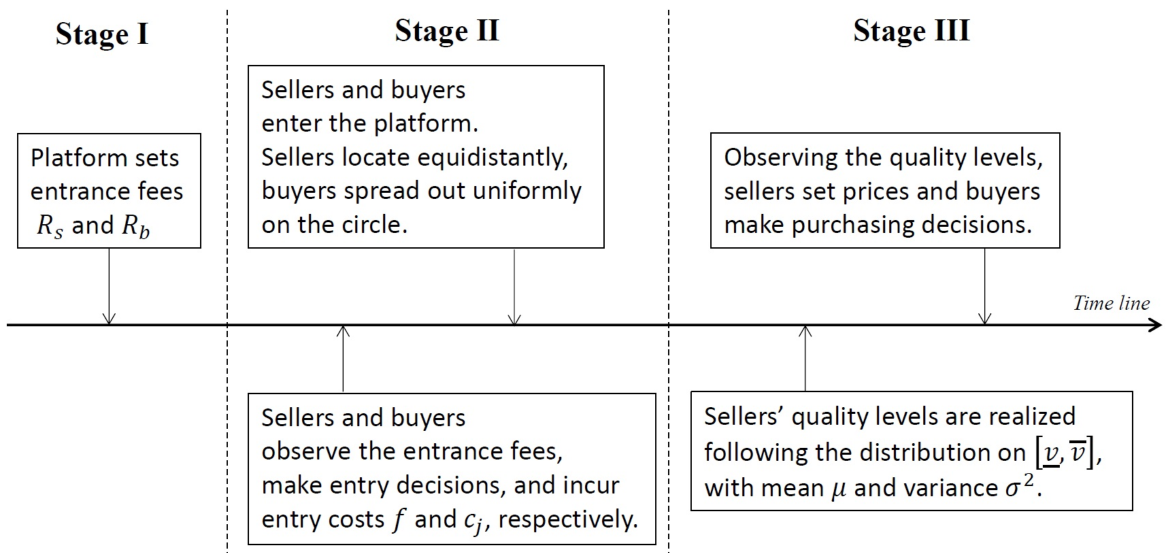

Figure 1: Timeline of the Game

The game unfolds in three stages (Figure 1). In stage I, the platform sets the two entry fees R_s and R_b. In stage II, before qualities are realized, sellers and buyers simultaneously make entry decisions by paying their respective entry fees and incurring entry costs. Upon entry, sellers are located equidistantly as they face the same expected quality,[5] and buyers are assigned uniformly on the circle. In stage III, sellers' quality levels are realized and become public information, based on which the two sides trade. Sellers compete by setting prices, and buyers have unit demand. Buyer j receives surplus $v_i - p_i - td_{ij}$ by purchasing from seller i that has quality v_i, where p_i is the price set by seller i, d_{ij} is the distance between buyer j and seller i, and t is buyers' unit

[5]Typically, product features are determined first, and they reflect sellers' locations on the circle upon entry. Quality is revealed later after product is developed and introduced to the market.

transportation cost. The distance can be interpreted as the degree of misfit between the buyer's horizontal preference and the seller's product.

4 Analysis

We solve the subgame-perfect Nash equilibrium by backward induction.

4.1 Stage III: Trading

We analyze the equilibrium trading decisions, taking the number of sellers and buyers who enter the platform, n_s and n_b, as given. We focus on the equilibrium in which market is fully covered (i.e., every buyer buys from some seller). Without loss of generality, let the location of the ith seller be $\frac{i}{n_s}$ for $i = 0, 1, \cdots, n_s - 1$. A buyer who is located between sellers i and $i+1$ at distance x from seller i is indifferent between buying from either seller if $v_i - p_i - t(x - \frac{i}{n_s}) = v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n_s} - x\right)$. To ensure that all sellers obtain a positive market share (i.e., every seller sells to some buyers), we assume $\overline{v} - \underline{v} < \frac{t}{N}$, which is consistent with the condition for localized competition in Alderighi and Piga (2012). The location of the marginal buyer between the sellers i and $i+1$ is then solved as $x^*_{i,i+1} = \frac{(v_i - p_i) - (v_{i+1} - p_{i+1})}{2t} + \frac{i}{n_s} + \frac{1}{2n_s}$. Similarly, the location of the marginal buyer between sellers i and $i-1$ is $x^*_{i-1,i} = \frac{(v_{i-1} - p_{i-1}) - (v_i - p_i)}{2t} + \frac{i-1}{n_s} + \frac{1}{2n_s}$. Then seller i's quantity sold is

$$q_i = n_b(x^*_{i,i+1} - x^*_{i-1,i}) = n_b\left[\frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n_s}\right].$$

Given that seller i's revenue is $\pi_i = q_i p_i$, the first-order condition (FOC) with respect to p_i gives,

$$p_i = \frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{4} + \frac{t}{2n_s}.$$

From here, notice that $q_i = n_b\left[\frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{2t} + \frac{1}{n_s} - \frac{p_i}{t}\right] = \frac{n_b p_i}{t}$.

The optimal price of seller i depends on the prices charged by its two neighboring sellers, $i-1$ and $i+1$. In equilibrium, the prices of all n_s sellers must be solved simultaneously. Wu and Lin (2013) provides a full analysis of the equilibrium solution, based on which the equilibrium price for seller i is

$$p_i^* = \frac{t}{n_s} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}, \tag{1}$$

where $b_j = \frac{\delta^{n_s-j}+\delta^j}{\sqrt{3}(\delta^{n_s}-1)}$ and $\delta = 2+\sqrt{3}$. Seller i's equilibrium quantity is then $q_i = \frac{n_b}{t} p_i^*$ and revenue is $\pi_i = \frac{n_b}{t} p_i^{*2}$.

4.2 Stage II: Entry

Seller i's expected profit from entering the platform is $E(\pi_i) - R_s - f$, where $E(\pi_i)$ is its expected trading surplus (or expected revenue) gross of the entry fee and entry cost. For simplicity, in the remaining of the paper, we refer to $E(\pi_i)$ as a seller's *expected surplus*. Since all sellers are ex ante identical, they have equal expected surplus, denoted by $E(\pi)$.

Lemma 1 *A seller's expected surplus is:*

$$E(\pi) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s)\right], \tag{2}$$

where $g_s(n_s) = n_s\left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3} \frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) > 0$. *The following network effects emerge:*

1. *a positive cross-side network effect from buyers to sellers (i.e.,* $\frac{\partial E(\pi)}{\partial n_b} > 0$*), and*

2. *a negative same-side network effect (i.e.,* $\frac{\partial E(\pi)}{\partial n_s} < 0$*) among sellers.*

Proof. All proofs are relegated to the appendix. ■

The network effects that are commonly assumed in most related studies emerge endogenously in our model. Cross-side network effect is defined by that more participation on one side increases the surplus of those participating on the other side. Here, a seller's expected surplus is proportional to the number of buyers on the platform, which suggests a positive cross-side network effect exerted by the buyer side to the seller side; this is consistent with the common assumption in the literature. More interestingly, the equilibrium generates a negative same-side network effect; that is, a seller's expected surplus is decreasing in the number of sellers. The same-side network effect is rarely studied in the literature of two-sided platforms. One exception is Hagiu (2009), in which a seller's surplus is assumed to decrease in the number of participating sellers. In our model we derive this feature endogenously by explicitly modeling sellers' price competition.

If buyer j enters the platform, her expected payoff is $E(u) - R_b - c_j$, where c_j is her entry cost, and $E(u)$ is her expected surplus from trading on the platform, with the expectation taken on her location and the product quality of the seller from whom she makes the purchase.

Lemma 2 *A buyer's expected surplus is:*

$$E(u) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s), \tag{3}$$

where $g_b(n_s) = n_s \left(\frac{1}{6\sqrt{3}} \frac{\delta^{n_s}+1}{\delta^{n_s}-1} - \frac{n_s}{3} \frac{\delta^{n_s}}{(\delta^{n_s}-1)^2} \right) > 0$*.* $E(u)$ *shows a positive cross-side network effect (i.e.,* $\frac{\partial E(u)}{\partial n_s} > 0$*) from sellers to buyers; however, the buyer side exhibits no direct same-side network effect (i.e.,* $\frac{\partial E(u)}{\partial n_b} = 0$*).*

Network effects are also shown in the buyer surplus. $E(u)$ increases with n_s, which indicates again a positive cross-side network effect in the reverse direction of that in Lemma 1. The manifestation of positive cross-side network effects in both directions echoes the standard network effect assumptions. However, our model's microfoundation demonstrates that the effect is not always linear, contrary to the assumption in many existing models. Meanwhile, the buyer surplus is independent of the number of buyers, showing no direct same-side network effect among buyers. This result follows naturally because rivalry among buyers is often less severe than that among sellers or is non-existent. In some scenarios, additional buyers may lead to higher market prices and in turn reduce buyers' surplus; however, the opposite can also occur: The additional buyers can create benefits for the existing buyers (e.g., networked video games, group-buying, and customer reviews). This positive same-side network effect is likely to only further sharpen the economic forces in the current model.

Corollary 1 $\frac{\partial E(\pi)}{\partial \sigma^2} > 0$ *and* $\frac{\partial E(u)}{\partial \sigma^2} > 0$*; that is, an increase in quality variation leads to higher surplus for both sellers and buyers.*

Quality heterogeneity benefits both sides.[6] Taking the number of buyers and sellers as given, increased quality variation allows the higher-quality sellers to gain additional market share and charge higher prices; meanwhile, the opposite applies to the lower-quality sellers. Since the number of transactions made at a lower price is reduced, and those made at a higher price are expanded, the total profit gains overweigh the total profit loss. Therefore, a seller's expected surplus increases with quality variation (i.e., $g_s(n_s) > 0$). Buyers' expected surplus also increases. Because the impact of quality on price is partly absorbed by competition, the buyers' additional utility from the increased quality exceeds the increase in price. Given that the higher-quality sellers hold larger market shares to begin with, more buyers realize this gain. Even when the increased quality variation leads to diminished quality, some of the buyers who suffer a loss by purchasing from the lower-quality sellers have the opportunity to reduce the loss by choosing the neighboring high-quality seller. A buyer's expected surplus is, therefore, also higher as quality variation increases (i.e., $g_b(n_s) > 0$).

Having derived sellers' and buyers' expected trading surplus, we are now ready to analyze the two sides' entry decisions. A seller enters the platform as long as its expected profit is non-negative. Therefore, the equilibrium number of the sellers on the platform, n_s, must satisfy the free-entry condition, $E(\pi) - R_s - f = 0$, which, after substituting in Eq. (2), can be written as:

$$R_s = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s)\right] - f. \tag{4}$$

A buyer with entry cost c_j enters the platform if and only if her expected payoff is non-negative: $E(u) - R_b - c_j \geq 0$, or equivalently $c_j \leq E(u) - R_b$. Given the uniform distribution of c_j, the equilibrium number of buyer entering the platform is $n_b = \frac{z}{C}(E(u) - R_b)$, or

$$n_b = \frac{z}{C}\left[\mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s) - R_b\right]. \tag{5}$$

[6] Both effects are clear from the equilibrium solution (e.g., Eq. (1)). These effects are examined in more detail in Wu and Lin (2013). Here we briefly explain the intuitions.

4.3 Stage I: Platform Pricing

The platform's optimization problem is,

$$\max_{R_s, R_b} \Pi(R_s, R_b) = n_s R_s + n_b R_b. \tag{6}$$

Given the one-to-one mapping between R_s and n_s, an equivalent formulation is to let the platform choose n_s instead of R_s. After substituting in Eq. (4) and Eq. (5), the optimization problem becomes:

$$\max_{n_s, R_b} \Pi = \frac{z}{C}\left\{\mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s) - R_b\right\}\left\{R_b + \frac{t}{n_s} + \frac{\sigma^2}{t} g_s(n_s)\right\} - n_s f.$$

By taking the FOC with respect to R_b, we have,

$$R_b = \frac{1}{2}\left[\mu - \frac{9t}{4n_s} + \frac{\sigma^2}{t}(g_b(n_s) - g_s(n_s))\right].$$

Given this choice, the platform solves

$$\max_{n_s} \Pi(n_s) = \frac{z}{4C}\left[\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} g(n_s)\right]^2 - n_s f, \tag{7}$$

where $g(n_s) = g_b(n_s) + g_s(n_s) = n_s\left(1 - \frac{7}{6\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right)$. Let the platform's objective function be well-behaved such that the second-order condition is satisfied, the following first-order condition then defines a unique, interior equilibrium solution, n_s^*:

$$\Pi'(n_s) = \frac{z}{2C}\left[\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} g(n_s)\right]\left[\frac{t}{4n_s^2} + \frac{\sigma^2}{t} g'(n_s)\right] - f = 0. \tag{8}$$

The remaining variables are expressed as functions of n_s^* as shown in Table 1.

Table 1: Endogenous Variables as Functions of n_s

Variable	Expression
Number of sellers	n_s^*
Number of buyers	$n_b^* = \frac{z}{2C}\left(\mu - \frac{t}{4n_s^*} + \frac{\sigma^2}{t}g(n_s^*)\right)$
Seller-side fee	$R_s^* = \frac{z}{2Cn_s^*}\left(\frac{t}{n_s^*} + \frac{\sigma^2}{t}g_s(n_s^*)\right)\left(\mu - \frac{t}{4n_s^*} + \frac{\sigma^2}{t}g(n_s^*)\right) - f$
Buyer-side fee	$R_b^* = \frac{1}{2}\left(\mu - \frac{9t}{4n_s^*} + \frac{\sigma^2}{t}(g_b(n_s^*) - g_s(n_s^*))\right)$
Platform's profit	$\Pi^* = \frac{z}{4C}\left(\mu - \frac{t}{4n_s^*} + \frac{\sigma^2}{t}g(n_s^*)\right)^2 - n_s^* f$

5 Vertical and Horizontal Differentiation

This model features both vertical and horizontal differentiation. The degree of vertical differentiation is represented by σ^2; the degree of horizontal differentiation is captured by the distance between neighboring sellers, $\frac{1}{n_s^*}$, so that a higher number of equilibrium sellers implies a lower degree of horizontal differentiation. Note that the degree of vertical differentiation is exogenous, whereas the degree of horizontal differentiation is endogenously determined by the platform's pricing strategy. The following proposition depicts the relationship between the two dimensions of differentiation in equilibrium:

Proposition 1 $\frac{dn_s^*}{d\sigma^2} > 0$*; that is, when sellers are more differentiated vertically, the platform admits more sellers. This reduces horizontal differentiation.*

Proposition 1 states that, given an exogenous increase in the quality variation, it is optimal for the platform to admit more sellers. To understand the intuition, first consider how the platform determines the optimal number of sellers for any given level of quality variation. The marginal seller incurs costs for the platform, but it also brings benefits; at the optimal number of sellers, the marginal benefit equals the marginal cost. When the platform admits one more seller, it incurs the seller's entry cost f because the platform internalizes all of the sellers' gains and losses (if sellers incur a lower entry cost, the platform will be able to collect a higher seller fee). Thus, sellers' entry cost may be regarded as the platform's marginal cost of expanding the seller-side network (Eq. (8)). The marginal benefit is the additional surplus the platform extracts from the two sides with the one more seller admitted. Clearly, buyers benefit from the improved matching between their horizontal preferences and sellers' products. Such benefits attract more buyers to join the platform, which leads to more sales for sellers.

An increase in quality variation changes the balance between the platform's marginal cost and marginal benefit at the optimal number of sellers (before-increase). The marginal cost of admitting sellers is the constant entry cost f, whereas the marginal benefit increases with quality variation. To see the latter, recall that increased quality variation exerts a positive impact on both sellers' and buyers' surplus ($g_s(n_s) > 0$ and $g_b(n_s) > 0$, Corollary 1). Admitting more sellers intensifies the positive impact of quality heterogeneity on both sides (i.e., $g'(n_s) > 0$, which arises from $g'_s(n_s) > 0$ and $g'_b(n_s) > 0$). Intuitively, as more sellers enter the platform, sellers are located more densely, so the degree of horizontal differentiation is reduced. Seller quality then becomes a more important factor in buyers' purchasing decisions. Thus, a buyer is less captive to a particular seller and is more inclined to switch to a different seller facing a higher quality variation. This sharpens the shifts in sellers' market shares and, in turn, magnifies the effects on their surplus that are ultimately internalized by the platform. To summarize, when quality variation increases, the platform's marginal benefit of expanding the seller side increases, while the marginal cost remains constant. Therefore, it is optimal for the platform to admit more sellers.

Proposition 1 suggests that the vertical and horizontal dimensions of differentiation are substitutes in equilibrium. Understanding the quality heterogeneity of products can then help a platform owner to more efficiently plan for its capacity, such as space acquisition for a shopping mall, technical resource allocation for serving end users, IT staffing for managing application developers, and so on. More importantly, the relationship between the two dimensions of differentiation guides the platform's strategy in managing sellers' entry scale. In some cases, platform owners exercise limited or no control over the degree of quality heterogeneity on the seller side. For example, as Airbnb expands to serve over 33,000 cities in 192 countries, differences in income distribution, culture, and government regulations across these regions may create variations in quality of hosts. Hosts in certain regions might be more consistent in terms of lodging space quality and hospitality than those in other regions. Our finding implies that the platform owner may be better off tailoring the network size of its hosts in different regions for quality heterogeneity based on regional characteristics. In other cases, the platform has access to various instruments to influence the outcomes of sellers' quality. Smartphone platform owners such as Apple and Google employ various contests and even directly screen applications to craft the distribution of application qualities. Our finding then offers insights into how platforms might consider balancing the number of applications while controlling quality variation.

6 Platform Subsidy

In this section, we examine the platform's pricing strategies. In particular, we look for the conditions under which the platform subsidizes the entry of either side. We proceed by first characterizing the pricing strategy without vertical differentiation (i.e., $\sigma^2 = 0$); and then, we study how a positive σ^2 affects the platform's subsidization strategies.

6.1 No Vertical Differentiation

When $\sigma^2 = 0$, the FOC Eq. (8) is reduced to the following:

$$\Pi'(n_s) = \frac{zt}{8Cn_s^2}\left(\mu - \frac{t}{4n_s}\right) - f = 0. \tag{9}$$

Proposition 2 *Without vertical differentiation, the platform never subsidizes sellers; it subsidizes buyers ($R_b^* < 0$) if and only if*

$$\frac{Ctf}{z} > k\mu^3, \tag{10}$$

where k is a positive constant.[7] *Therefore, the platform is more likely to subsidize buyers if there are fewer potential buyers (z is lower), the average product quality is lower (μ is lower), buyers' or*

[7] The value of k comes from a general feature of the circular city model. See the proof for the derivation.

sellers' entry is more costly (C or f is higher), or buyers have stronger horizontal preferences (t is higher).

The platform's pricing strategies for the two sides are in sharp contrast. Whereas it can be optimal for the platform to pay for buyers' participation, such subsidy is never optimal on the seller side absent vertical differentiation. We can explore the driving forces of subsidy by examining the benefit of admitting the marginal seller or buyer. Imagine the platform chooses n_b and n_s simultaneously instead of choosing R_b and n_s sequentially (the two approaches are mathematically equivalent). Given n_b and n_s, the two sides' entry conditions then determine their respective entry fees: $R_s = E(\pi) - f = \frac{t}{n_s^2} n_b - f$, and $R_b = E(u) - \frac{C}{z} n_b = \mu - \frac{5t}{4n_s} - \frac{C}{z} n_b$. The platform's optimization problem then becomes $\max_{n_s, n_b} \Pi(R_s, R_b) = n_s R_s + n_b R_b$, and the FOCs with respect to n_s and n_b are:

$$\frac{\partial \Pi}{\partial n_s} = n_s \frac{\partial R_s}{\partial n_s} + n_b \frac{\partial R_b}{\partial n_s} + R_s^* = 0, \tag{11}$$

$$\frac{\partial \Pi}{\partial n_b} = n_b \frac{\partial R_b}{\partial n_b} + n_s \frac{\partial R_s}{\partial n_b} + R_b^* = 0. \tag{12}$$

Let us first examine the seller-side subsidy by considering the effect of admitting one more seller (Eq. (11)). Admitting an additional seller changes the platform's profit through three effects. The first one is through the fee[8] collected from the marginal seller (R_s^*). Second, to expand the seller side, the platform must lower the seller fee that applies to all sellers ($n_s \frac{\partial R_s}{\partial n_s} < 0$). We call this the *same-side loss* (SSL) (see Table 2), which is the platform's loss on the side of the additional entry. Third, all buyers benefit from the increased number of sellers, thus, when holding the number of buyers fixed, it is optimal for the platform to raise the buyer-side fee ($n_b \frac{\partial R_b}{\partial n_s} > 0$). This is termed as the *cross-side gain* (CSG), which is the platform's gain on the side *across* from the side of the additional entry. When the platform makes optimal decisions, the three effects add up to zero. Therefore, the platform subsidizes sellers (i.e., $R_s^* < 0$) if and only if the same-side loss is outweighed by the cross-side gain.

[8] We use the term *fee* in the general sense to include subsidy, which is a negative "fee."

Table 2: Breakdown of the Net Benefit from an Additional Entry (No Vertical Differentiation)

Side of Entry	**Same-Side Loss (SSL)**	**Cross-Side Gain (CSG)**
Seller side	$n_s \frac{\partial R_s}{\partial n_s} = -\frac{2tn_b}{n_s^2}$	$n_b \frac{\partial R_b}{\partial n_s} = \frac{5tn_b}{4n_s^2}$
Buyer side	$n_b \frac{\partial R_b}{\partial n_b} = -\frac{Cn_b}{z}$	$n_s \frac{\partial R_s}{\partial n_b} = \frac{t}{n_s}$

We now explain that, without vertical differentiation, the same-side loss resulting from the additional seller is always greater than the corresponding cross-side gain. When an additional seller joins the platform, pre-existing sellers suffer two losses. First, the additional seller intensifies seller competition, which reduces each seller's equilibrium price. This loss is fully recovered on the buyer side as buyers pay lower prices and obtain higher surplus. Each seller also suffers a second loss: its market share shrinks to accommodate the new seller. Part of this loss is again recovered on the buyer side, as some buyers' horizontal preferences are better served. However, the remaining part of this loss is a pure loss, because not all buyers experience an improved product match. Thus, the SSL is always greater than the CSG, which implies that subsidizing sellers is never optimal.

We now turn to the discussion of the buyer-side subsidy. Similar to the seller side, an additional *buyer* changes the platform's profits through three channels (Eq. (12)), and subsidizing buyers is optimal when the cross-side gain from the additional buyer outweighs the same-side loss. The entry of an additional buyer has no direct impact on the other buyers (without the rivalry that the sellers face among one another), so the platform does not need to lower the buyer fee by much. In particular, the SSL is less severe when fewer buyers are on the platform (See Table 2). Also, the CSG is greater with fewer sellers because their competition is then less intense and greater gains can be obtained from the increased number of buyers. If market conditions are such that a low number of buyers and sellers enter the platform in equilibrium, it is indeed possible for the CSG to exceed the SSL, in which case it is optimal for the platform to subsidize buyers.

The condition that governs buyer-side subsidy is satisfied when entry costs on both sides and buyers' transportation cost are high, the potential buyer market is small, and/or the expected quality of sellers is low. Higher costs incurred by buyers and a lower expected quality both reduce buyers' surplus and discourage their participation on the platform. Meanwhile, a higher entry cost for sellers and a smaller potential buyer market imply a lower expected surplus for sellers, which reduces seller-side participation. Therefore, an *illiquid* market does lead to smaller networks on both sides, which may allow the platform to subsidize buyers.

6.2 Vertical Differentiation

We now consider the general case with vertical differentiation among sellers. When $\sigma^2 > 0$, $g_s(n_s)$ and $g_b(n_s)$ take more complex forms. For analytical tractability, we use approximation to simplify these functions while still capturing the main forces in the model (see Appendix B, where we also define constants δ_1, δ_2, and δ_3.). Numerical calculations indicate that the approximation does not affect the results. The platform's optimization problem (7) is then simplified to:

$$\max_{n_s} \Pi(n_s) = \frac{z}{4C}\left(\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}\delta_1 n_s\right)^2 - n_s f, \tag{13}$$

for which the FOC is,

$$\Pi'(n_s) = \frac{z}{2C}\left(\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}\delta_1 n_s\right)\left(\frac{t}{4n_s^2} + \frac{\sigma^2}{t}\delta_1\right) - f = 0. \tag{14}$$

Note that $\Pi''(n_s) = \frac{z}{2C}\frac{3t^4+16\sigma^4\delta_1^2 n_s^4 - 8\mu t^3 n_s}{16t^2 n_s^4} < 0$, meaning that the second-order condition is satisfied, and therefore the FOC is both necessary and sufficient to define a unique solution of n_s^*. The endogenous variables are expressed as functions of n_s in Table 3.

Proposition 3 *When sellers are vertically differentiated, the platform subsidizes the sellers ($R_s^* < 0$) if and only if*

$$\frac{Ctf}{z} < \sigma^2(\delta_5\mu + \delta_6\sigma), \tag{15}$$

where δ_5 and δ_6 are positive constants.[9] *Therefore, the platform is more likely to subsidize sellers when entry on either side is less costly (C or f is lower), horizontal preferences are weaker (t is*

[9] These constants are derived in the proof of this proposition.

Table 3: Endogenous Variables as Functions of n_s under Approximation

Variable	**Expression**
Number of sellers	n_s^*
Number of buyers	$n_b^* = \frac{z}{2C}\left(\mu - \frac{t}{4n_s} + \delta_1 \frac{\sigma^2}{t} n_s\right)$
Seller-side fee	$R_s^* = \frac{z}{2C}\left(\frac{t}{n_s^2} + \delta_2 \frac{\sigma^2}{t}\right)\left(\mu - \frac{t}{4n_s} + \delta_1 \frac{\sigma^2}{t} n_s\right) - f$
Buyer-side fee	$R_b^* = \frac{1}{2}\left(\mu - \frac{9t}{4n_s} - \delta_3 \frac{\sigma^2}{t} n_s\right)$
Platform's profit	$\Pi^* = \frac{z}{4C}\left(\mu - \frac{t}{4n_s} + \delta_1 \frac{\sigma^2}{t} n_s\right)^2 - n_s f$

lower), more potential buyers are present (z is higher), the average quality is higher (μ is higher) or sellers differ more in quality (σ^2 is higher).

Proposition 3 suggests that, with vertical differentiation, the platform may subsidize sellers. Recall that the optimality of the seller-side subsidy depends crucially on whether the cross-side gain (CSG) exceeds the same-side loss (SSL) when the platform admits an additional seller (Section 6.1). Quality heterogeneity impacts the SSL and CSG differently. The effect of quality heterogeneity on SSL is only minuscule because of the *ripple effect* in sellers' competition.[10] More specifically, sellers' equilibrium price (Eq. (1)) shows that the effect of a seller's quality on other sellers is the strongest for the immediate neighbors and diminishes like ripples as it traverses to the neighbors' neighbors, and so on. In fact, the effect is reduced to nearly zero at the fourth neighbor on either side of a seller. Therefore, quality heterogeneity has a negligible impact on the SSL as the additional seller enters, given a few existing sellers on the platform. On the other hand, quality heterogeneity sharpens the CSG that the additional seller brings to the buyers. As explained in Section 5, an increase in quality variation raises buyers' surplus, and to a greater extent when more sellers are on the platform. In sum, quality heterogeneity enhances the CSG substantially with little effect on

[10] *Ripple effect* shows that the impact of a seller's quality on its immediate neighbors and, in turn, on their neighbors weakens at a rapid rate. For a detailed discussion on the ripple effect see Wu and Lin (2013).

the SSL, thus the CSG may outweigh the SSL, which implies seller subsidy may be optimal.

Notice that Proposition 3 is consistent with Proposition 2, which is a special case with $\sigma^2 = 0$. At this value, the right hand side of Condition (15) is zero, so the condition is never satisfied, meaning subsidizing sellers is never optimal without vertical differentiation.

The condition for seller-side subsidy includes less costly entry on either side, weaker horizontal preferences for buyers, more potential buyers, and a higher average quality, which jointly describe a highly *liquid* platform market with a larger network size on both sides. Given a high number of sellers, the platform's CSG from admitting an additional seller is further amplified. In such a crowded market, sellers are located more narrowly, which makes the effect of quality more pronounced. Thus, buyers are less captive to any particular seller due to its location advantage, because horizontal preferences are well served in this crowded market; instead, buyers are more inclined to switch to a seller who offers a higher quality. In brief, the value created by sellers' quality heterogeneity is enhanced by the intensity of their horizontal competition, allowing the platform to derive a higher net benefit from the marginal seller and provide subsidy to the seller side.

We now turn to the buyer-side subsidy and have the following result:

Proposition 4 *When sellers are vertically differentiated, the platform subsidizes the buyer side ($R_b^* < 0$) if and only if*

$$\frac{Ctf}{z} > \delta_7 (\mu + \theta) (\mu - \delta_8 \theta)(\mu - \delta_9 \theta), \tag{16}$$

where $\theta = \sqrt{\mu^2 - 9\delta_3 \sigma^2}$, and δ_7, δ_8, and δ_9 are positive constants.[11]

Proposition 4 specifies the condition for the buyer-side subsidy when sellers are vertically differentiated. Note that Condition (16) at $\sigma^2 = 0$ is identical to Condition (10) from Proposition 2. We can also show that the right hand side of Condition (16) increases with both μ and σ^2, meaning that the condition is less likely to be satisfied when σ^2 is higher. Other parameters have same effect on the condition as they do in Proposition 2.

The qualitative interpretation for Proposition 4 is consistent with that in Proposition 2; thus, with vertical differentiation, an illiquid platform market is still more conducive to buyer-side subsidy. Meanwhile, quality heterogeneity tightens Condition (16), working against the buyer-side

[11] These constants are derived in the proof of this proposition.

subsidy, because increased quality variation leads to a more liquid market. The intuition is the following. Based on Proposition 1, more quality variation leads to a larger seller-side network size in equilibrium. The buyer-side network size also becomes larger as a result of the positive cross-side network effect exerted by the expanded seller side. As the platform reduces the buyer-side fee to admit the additional buyer, the SSL increases with the buyer-side network. Furthermore, given a larger seller-side network, the CSG from admitting the additional buyer is reduced due to the intensity of seller competition. Thus, with increased quality variation, the platform suffers a greater SSL and obtains a smaller CSG when it admits more buyers, which makes the buyer-side subsidy less likely.

The driving forces for offering subsidies to the seller side and the buyer side, as illustrated in their respective conditions, are clearly reversed. Whereas characteristics of a liquid platform market more likely lead to a subsidy for sellers, the buyer-side subsidy relies on a less liquid platform market; quality heterogeneity works favorably for the seller-side subsidy, while tightening the condition for the buyer-side subsidy. Such asymmetry is curious because the well-known insight that the platform is more inclined to subsidize the side that exerts a stronger cross-side network effect (Parker and Van Alstyne 2005, Armstrong 2006) does not distinguish between the two sides. Without contradicting previous findings, the asymmetry identified in this work arises from the negative same-side network effect resulted from the sellers' price competition, which is not accounted for those earlier works. Given that the buyers often do not exhibit the same degree of rivalry among themselves, the two sides create different dynamics in the net benefit the platform internalizes from admitting the marginal seller/buyer.

It is also interesting that the sellers' and buyers' entry costs play the same role in the platform's subsidization strategy. A higher entry cost on either side encourages the buyer subsidy, and a lower entry cost on either side more likely implies a seller subsidy. This is surprising because subsidy is expected to mitigate the friction from the entry cost, which suggests that the two sides' entry costs should pull the optimality of subsidy to their respective side. In contrast, we find the opposite because the entry costs on both sides have a consistent effect on the network size of the platform: When the buyer side network is constrained by the buyer-side entry cost, it in turn reduces seller-side entry, and vice versa. Thus, either entry cost creates the same friction for market liquidity, and the two sides' entry costs impact the subsidization strategy in the same manner.

In practice, trading between buyers and sellers on a platform is ubiquitous, such as in online and offline marketplaces of wide varieties of goods and services, software platforms, gaming platforms, and many more.[12] Our findings provide a new angle in explaining subsidies on these platforms. Buyer-side subsidy is commonly observed, especially when the networks are not large. For example, Microsoft subsidized Xbox by offering the console at a very low price. Although the strategy was designed in part to compete with other console makers, not having high liquidity in the market of players and game developers allowed Microsoft to recover the consumer-side subsidy from the game developer side. Also, it is well known that public transportation systems, such as the Mass Transit Railway (MTR) in Hong Kong, often subsidize the passenger side. While collecting low fares from passengers, MTR profits from renting out limited retail spaces in the stations. Without a fierce competition, the differentiation among the stores allows them to generate profit from the passenger traffic that MTR brings through subsidization; the profits are ultimately absorbed by MTR.

Our findings suggest that subsidizing sellers can be optimal for the platform if the networks on both sides are large or the quality variation on the seller side is high. In contrast to game consoles' buyer-side subsidy, computer operating systems often subsidize the sellers (software developers). Microsoft Windows is known to subsidize developers by offering free or low-cost software development kit (SDK) and support, while charging high prices on the user side (Eisenmann et al. 2006). Part of the reason may be that the quality of console games is not as variable as quality of software programs – the quality of software programs critically depends on developers' expertise on design and development and, often, industry-specific knowledge. Trade fairs also come close to illustrating this case, including wedding expos, computer fairs, and many others. Some of these fairs, especially those that are large in scale and held in central locations (e.g., metropolitan areas) tend to charge an entrance fee on the buyer side and attract a large crowd nevertheless. Because of easy access for attendees and low setup costs for vendors, both sides incur low entry costs to participate. Although a reduced rental fee can still be collected on the seller side, the organizer may profit substantially

[12] Many platforms also do not fall under this umbrella, such as the dating platform, which facilitates matching rather than trades.

on the buyer side and subsidize the remainder of the sellers' costs.

6.3 Impact of σ^2

The previous discussion focuses on the platform's subsidization strategies. We now investigate how the entry fees change with quality variation, regardless of whether they are subsidies.

Proposition 5 *As quality variation, σ^2, increases, it is optimal for the platform to reduce the seller fee, R_s^*, and raise the buyer fee, R_b^*.*

It might seem intuitive to reason that more quality variation suggests mitigated competition, which may increase sellers' surplus and allow the platform to charge a higher seller fee; however, our finding shows the opposite. This is because increased quality variation affects the two sides' surplus through both a direct effect and an indirect effect. The direct effect is that, fixing the number of sellers and buyers, both sides benefit from the increase in quality variation (Corollary 1). The indirect effect is that increased quality variation induces the platform to attract more sellers (Proposition 1), which tends to reduce the sellers' surplus (Lemma 1) but raise the buyers' surplus (Lemma 2). On the buyer side, the two effects are both positive; thus, the platform raises the buyer fee. On the seller side, the two effects move in opposite directions. In the complex interaction between the two sides, the indirect effect dominates. As a result, the platform reduces the seller fee. Note that the contrast between the platform's pricing strategies on the two sides echoes the "seesaw principle" discussed in Rochet and Tirole (2006).

The impacts of quality variation on the fees on both sides are aligned with our findings on platform subsidies from Section 6.2. Proposition 3 shows that the platform is more likely to subsidize sellers when quality variation is more pronounced. Here, as quality variation increases, the platform reduces the seller fee, implying that seller subsidy is more likely. Whereas Proposition 3 states the conditions under which the optimal fees are positive/negative, here we focus more on the properties of the optimal fees regardless of whether they are negative. An additional implication is that the *extent* of the subsidy that the platform offers to sellers is greater when sellers are more heterogeneous in quality. Furthermore, as Proposition 4 shows, quality heterogeneity among sellers works against buyer subsidy; here we see that the platform either offers a smaller subsidy or charges a higher fee to buyers when quality varies more significantly.

These findings provide a basis for how the platform can adjust its pricing when market condition, such as quality heterogeneity of sellers, changes. For example, if some smartphone application developers succeed in a particular technology innovation and release games of superior visual experience or applications that more seamlessly integrate with the operating system, the newly introduced applications then effectively increase quality variation in the market. The platform may then consider reducing the fee on the developer side while raising the price charged on the user side. The same strategy may be implemented in reverse. For instance, if Airbnb decides to exclude hosts below a certain quality to avoid legal disputes, the reduced quality variation would suggest a higher fee for the hosts.

Proposition 6 *As quality variation, σ^2, increases, the platform's profit also increases.*

A natural question that follows is, how does quality heterogeneity affect the platform's profit? Proposition 6 shows that the platform's profit increases as sellers become more heterogeneous in quality. Thus, by reducing the seller fee and raising the buyer fee, the platform can boost its profit in response to an increase in quality variation. At the optimal fees, the platform can benefit from including more types of sellers to create a more diverse marketplace. This advises against mechanisms to screen out lower quality sellers for the sole reason of improving overall seller quality. Understandably, some platform might still exclude sellers that may lead to other undesirable consequences (e.g., by reviewing applications for security problems). Our result raises cautions regarding such exclusion and suggests an evaluation of screening criteria to avoid unintended narrowing of quality variation among sellers.

We also find that *as quality variation increases, the platform's revenue source shifts away from the seller side and toward the buyer side*; that is, $\frac{n_s^* R_s^*}{n_b^* R_b^*}$, decreases with σ^2. This follows intuitively from our previous results that the platform focuses less on charging sellers and more on charging buyers at an increased quality variation. The implications are meaningful regarding the design of the platform's business model and fee structure. Understanding the quality heterogeneity of its sellers, the platform can more effectively allocate its marketing and operating resources for a seller-focus or buyer-focus strategy.

7 Discussion and Conclusion

This paper examines a monopoly platform's two-sided pricing problem, using an approach that endogenizes network effects through the microfoundation of trades between sellers and buyers. We capture both horizontal and vertical dimensions of differentiation among competing sellers and derive the platform's optimal strategies related to these types of seller differentiation. We find that market liquidity plays a key role in the platform's subsidization strategy. Subsidizing seller is optimal when the market is sufficiently liquid. On the buyer-side, the platform may provide subsidy when the market is illiquid; however, vertical differentiation reduces the platform's incentive to subsidize buyers. We also discuss the impact of seller quality variation on the platform's optimal fees, level and composition of profits, as well as the two sides entry scale in equilibrium.

We suggest several future research directions. One natural extension is to study competing platforms with a microfoundation for transactions on each platform. Competing platforms may differ in their technologies and designs so that platforms are asymmetric in terms of horizontal differentiation and quality heterogeneity of products traded within their own markets. The asymmetry can yield new insights into how platforms set entry fees depending on both its own and its competitor's market characteristics. Also, platforms may be able to strategically inform buyers of sellers' products to reduce information asymmetry in the interaction between the two sides. For example, platforms' investments in technologies that improve buyers' shopping interface, facilitate product recommendations, or empower product search are instrumental for determining information and trading surplus for both sides. Endogenizing the related variables can be a fruitful direction.

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A Appendix: Proofs

Proof of Lemma 1

Proof. Recall that $q_i = n_b\left[\frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n_s}\right]$ and the FOC of seller i's revenue gives

$$p_i = \frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{4} + \frac{t}{2n_s}.$$

By substituting p_i into the expression of q_i, we have $q_i = \frac{n_b}{t} p_i$. Thus, seller i's equilibrium revenue is $\pi_i = \frac{n_b}{t} p_i^{*2}$ and $E(\pi_i) = \frac{n_b}{t} E(p_i^{*2})$, where $p_i^* = \frac{t}{n_s} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}$, $b_j = \frac{\delta^{n_s-j}+\delta^j}{\sqrt{3}(\delta^{n_s}-1)}$ and $\delta = 2+\sqrt{3}$. We will show below that $E(p_i^{*2}) = \frac{t^2}{n_s^2} + \sigma^2 \left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3} \frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right)$.

$$
\begin{aligned}
E(p_i^{*2}) &= E\left[\left(\frac{t}{n_s} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}\right)^2\right] \\
&= \frac{t^2}{n_s^2} + \frac{2t}{n_s}\left(E(v_{i-j}) - \sum_{j=0}^{n_s-1} b_j E(v_{i-j})\right) + E\left[\left(v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}\right)^2\right] \\
&= \frac{t^2}{n_s^2} + \frac{2t}{n_s} E(v)\left(1 - \sum_{j=0}^{n_s-1} b_j\right) + E(v_{i-j}^2)\left(1 - 2b_0 + \sum_{j=0}^{n_s-1} b_j^2\right) \\
&\quad + \sum_{j\neq k} b_j b_k E(v_{i-j})E(v_{i-k}) - 2\sum_{j=1}^{n_s-1} b_j E(v_i)E(v_{i-j}) \\
&\quad \left(\textit{Notice that } \textstyle\sum_{j=0}^{n_s-1} b_j = 1.\right) \\
&= \frac{t^2}{n_s^2} + \left(E^2(v) + Var(v)\right)\left(1 - 2b_0 + \sum_{j=0}^{n_s-1} b_j^2\right) \\
&\quad + \sum_{j\neq k} b_j b_k E(v_{i-j})E(v_{i-k}) - 2\sum_{j=1}^{n_s-1} b_j E(v_i)E(v_{i-j}) \\
&= \frac{t^2}{n_s^2} + Var(v)\left(1 - 2b_0 + \sum_{j=0}^{n_s-1} b_j^2\right) + E^2(v)\left(1 + \sum_{j=0}^{n_s-1}\sum_{k=0}^{n_s-1} b_j b_k - 2\sum_{j=0}^{n_s-1} b_j\right) \\
&\quad \left(\textit{The last term is zero, again because } \textstyle\sum_{j=0}^{n_s-1} b_j = 1.\right) \\
&= \frac{t^2}{n_s^2} + Var(v)\left(1 - 2b_0 + \sum_{j=0}^{n_s-1} b_j^2\right) \\
&= \frac{t^2}{n_s^2} + Var(v)\left(1 - 2\frac{\delta^{n_s}+1}{\sqrt{3}(\delta^{n_s}-1)} + \sum_{j=0}^{n_s-1} \frac{\delta^{2n_s-2j} + \delta^{2j} + 2\delta^{n_s}}{3(\delta^{n_s}-1)^2}\right) \\
&= \frac{t^2}{n_s^2} + Var(v)\left(1 - 2\frac{\delta^{n_s}+1}{\sqrt{3}(\delta^{n_s}-1)} + \frac{1}{3(\delta^{n_s}-1)^2}\left(\left(1+\delta^2\right)\sum_{j=0}^{n_s-1} \delta^{2j} + 2n_s\delta^{n_s}\right)\right) \\
&= \frac{t^2}{n_s^2} + Var(v)\left(1 - 2\frac{\delta^{n_s}+1}{\sqrt{3}(\delta^{n_s}-1)} + \frac{1}{3(\delta^{n_s}-1)^2}\left(\frac{\left(1+\delta^2\right)\left(\delta^{2n_s}-1\right)}{\delta^2-1} + 2n_s\delta^{n_s}\right)\right) \\
&= \frac{t^2}{n_s^2} + \sigma^2\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right)
\end{aligned}
$$

Since all sellers have the identical expected profit, the subscript i is dropped, and we have

$$E(\pi) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s)\right], \tag{17}$$

where $g_s(n_s) \equiv n_s \left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3} \frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right)$. It is straightforward to show that $g_s(n_s) > 0$ and that $E(\pi)$ decreases in n_s. Therefore, the cross-side network effect from buyers to sellers is positive, and the same-side network effect among sellers is negative. ■

Proof of Lemma 2

Proof. Since sellers are equidistantly located around the circle, the expected surplus of buyers between any pair of sellers i and $i+1$ is the following (note that the marginal buyer is denoted by $x^*_{i,i+1} = \frac{(v_i - p_i) - (v_{i+1} - p_{i+1})}{2t} + \frac{i}{n_s} + \frac{1}{2n_s}$):

$$\begin{aligned}
& E\left[u|_{x \in \left[\frac{i}{n_s}, \frac{i+1}{n_s}\right]}\right] \\
= \; & E\left[\int_{\frac{i}{n_s}}^{x^*_{i,i+1}} \left(v_i - p_i - t\left(x - \frac{i}{n_s}\right)\right) dx + \int_{x^*_{i,i+1}}^{\frac{i+1}{n_s}} \left(v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n_s} - x\right)\right) dx\right] \\
= \; & E\left[(v_i - p_i)\left(x^*_{i,i+1} - \frac{i}{n_s}\right) - \frac{t}{2}\left(x^*_{i,i+1} - \frac{i}{n_s}\right)^2 + (v_{i+1} - p_{i+1})\left(\frac{i+1}{n_s} - x^*_{i,i+1}\right)\right. \\
& \left. - \frac{t}{2}\left(\frac{i+1}{n_s} - x^*_{i,i+1}\right)^2\right] \\
= \; & E\left[\frac{-i}{n_s}(v_i - p_i) + \frac{i+1}{n_s}(v_{i+1} - p_{i+1}) - \frac{t}{2n_s^2}\left(i^2 + (i+1)^2\right)\right. \\
& \left. + x^*_{i,i+1}\left(v_i - p_i - v_{i+1} + p_{i+1} + \frac{t\,(2i+1)}{n_s}\right) - t\left(x^*_{i,i+1}\right)^2\right] \\
= \; & E\left[\frac{-i}{n_s}(v_i - p_i) + \frac{i+1}{n_s}(v_{i+1} - p_{i+1}) - \frac{t}{2n_s^2}\left(i^2 + (i+1)^2\right)\right. \\
& + \left(\frac{1}{2t}(v_i - p_i - v_{i+1} + p_{i+1}) + \frac{i}{n_s} + \frac{1}{2n_s}\right)\left(v_i - p_i - v_{i+1} + p_{i+1} + \frac{t\,(2i+1)}{n_s}\right) \\
& \left. - t\left(\frac{1}{2t}(v_i - p_i - v_{i+1} + p_{i+1}) + \frac{i}{n_s} + \frac{1}{2n_s}\right)^2\right]
\end{aligned}$$

Define $a \equiv v_i - p_i - v_{i+1} + p_{i+1}$, then

$$\begin{aligned}
E\left[u|_{x\in\left[\frac{i}{n_s},\frac{i+1}{n_s}\right]}\right] &= E\left[\frac{-i}{n_s}(v_i - p_i) + \frac{i+1}{n_s}(v_{i+1} - p_{i+1}) - \frac{t}{2n_s^2}\left(i^2 + (i+1)^2\right)\right. \\
&\quad \left. + \left(\frac{1}{2t}a + \frac{i}{n_s} + \frac{1}{2n_s}\right)\left(a + \frac{t\,(2i+1)}{n_s}\right) - t\left(\frac{1}{2t}a + \frac{i}{n_s} + \frac{1}{2n_s}\right)^2\right] \\
&= E\left[\frac{-i}{n_s}(v_i - p_i) + \frac{i+1}{n_s}(v_{i+1} - p_{i+1}) - \frac{t}{2n_s^2}\left(i^2 + (i+1)^2\right)\right. \\
&\quad \left. + \frac{a^2}{4t} + \frac{2i+1}{2n_s}a + \frac{t}{4n_s^2}(2i+1)^2\right] \\
&= E\left[\frac{-i}{n_s}(v_i - p_i) + \frac{i+1}{n_s}(v_{i+1} - p_{i+1}) - \frac{t}{4n_s^2} + \frac{a^2}{4t} + \frac{2i+1}{2n_s}a\right] \\
&= \frac{-i}{n_s}(E(v_i) - E(p_i)) + \frac{i+1}{n_s}(E(v_{i+1}) - E(p_{i+1})) \\
&\quad - \frac{t}{4n_s^2} + \frac{1}{4t}E(a^2) + \frac{2i+1}{2n_s}E(a) \\
&= \frac{1}{n_s}(E(v) - E(p)) - \frac{t}{4n_s^2} + \frac{1}{4t}E(a^2)
\end{aligned}$$

Thus, $E\left(u|_x\right) = n_s \cdot E\left[u|_{x\in\left[\frac{i}{n_s},\frac{i+1}{n_s}\right]}\right] = E(v) - \frac{t}{n_s} - \frac{t}{4n_s} + \frac{n_s}{4t}E(a^2) = E(v) - \frac{5t}{4n_s} + \frac{n_s}{4t}E(a^2)$.

$$\begin{aligned}
E(a^2) &= E[(p_{i+1} - p_i - v_{i+1} + v_i)^2] \\
&= E[(v_i - p_i)^2 + (v_{i+1} - p_{i+1})^2 - 2(v_i - p_i)(v_{i+1} - p_{i+1})] \\
&= 2(E[(v_i - p_i)^2] - E[(v_i - p_i)(v_{i+1} - p_{i+1})])
\end{aligned}$$

As $p_i = \frac{t}{n_s} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}$, $v_i - p_i = \sum_{j=0}^{n_s-1} b_j v_{i-j} - \frac{t}{n_s}$. Since $\sum_{j=0}^{n_s-1} b_j = 1$, we can derive the following:

$$
\begin{aligned}
E[(v_i - p_i)^2] &= E\left[\left(\sum_{j=0}^{n_s-1} b_j v_{i-j} - \frac{t}{n_s}\right)^2\right] \\
&= E\left[\left(\sum_{j=0}^{n_s-1} b_j v_{i-j}\right)^2\right] - \frac{2t}{n_s}E(v)\sum_{j=0}^{n_s-1} b_j + \frac{t^2}{n_s^2} \\
&= E\left[\sum_{j=0}^{n_s-1} b_j^2 v_{i-j}^2 + \sum_{j\neq k} b_j b_k v_{i-j} v_{i-k}\right] - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
&= E(v^2)\sum_{j=0}^{n_s-1} b_j^2 + E^2(v)\sum_{j\neq k} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
&= \big(E^2(v) + Var(v)\big)\sum_{j=0}^{n_s-1} b_j^2 + E^2(v)\sum_{j\neq k} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
&= Var(v)\sum_{j=0}^{n_s-1} b_j^2 + E^2(v)\sum_{j=0}^{n_s-1}\sum_{k=0}^{n_s-1} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
&= Var(v)\sum_{j=0}^{n_s-1} b_j^2 + E^2(v)\sum_{j=0}^{n_s-1} b_j \sum_{k=0}^{n_s-1} b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
&= Var(v)\sum_{j=0}^{n_s-1} b_j^2 + E^2(v) - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2}
\end{aligned}
$$

Meanwhile

$$
\begin{aligned}
& E[(v_i - p_i)(v_{i+1} - p_{i+1})] \\
= \; & E\left[\left(\sum_{j=0}^{n_s-1} b_j v_{i-j} - \frac{t}{n_s}\right)\left(\sum_{j=0}^{n_s-1} b_j v_{i+1-j} - \frac{t}{n_s}\right)\right] \\
= \; & E\left[\sum_{j=0}^{n_s-1} b_j v_{i-j} \cdot \sum_{j=0}^{n_s-1} b_j v_{i+1-j}\right] - \frac{t}{n_s}\left[\sum_{j=0}^{n_s-1} b_j E(v_{i-j}) + \sum_{j=0}^{n_s-1} b_j E(v_{i+1-j})\right] + \frac{t^2}{n_s^2} \\
= \; & E\left[\sum_{j=0}^{n_s-1} b_j b_{j+1} v_{i-j}^2 + \sum_{j \neq k-1} b_j b_k v_{i-j} v_{i+1-k}\right] - \frac{t}{n_s}\left(E(v)\sum_{j=0}^{n_s-1} b_j + E(v)\sum_{j=0}^{n_s-1} b_j\right) + \frac{t^2}{n_s^2} \\
= \; & E(v^2)\sum_{j=0}^{n_s-1} b_j b_{j+1} + E^2(v)\sum_{j \neq k-1} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
= \; & (E^2(v) + Var(v))\sum_{j=0}^{n_s-1} b_j b_{j+1} + E^2(v)\sum_{j \neq k-1} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
= \; & Var(v)\sum_{j=0}^{n_s-1} b_j b_{j+1} + E^2(v)\sum_{j=0}^{n_s-1}\sum_{k=0}^{n_s-1} b_j b_k - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2} \\
= \; & Var(v)\sum_{j=0}^{n_s-1} b_j b_{j+1} + E^2(v) - \frac{2t}{n_s}E(v) + \frac{t^2}{n_s^2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E(a^2) &= 2(E[(v_i - p_i)^2] - E[(v_i - p_i)(v_{i+1} - p_{i+1})]) \\
&= 2Var(v)\left(\sum_{j=0}^{n_s-1} b_j^2 - \sum_{j=0}^{n_s-1} b_j b_{j+1}\right) \\
&= Var(v)\left(\sum_{j=0}^{n_s-1} b_j^2 - \sum_{j=0}^{n_s-1} 2 b_j b_{j+1} + \sum_{j=0}^{n_s-1} b_{j+1}^2\right) \\
&= Var(v)\sum_{j=0}^{n_s-1} (b_j - b_{j+1})^2
\end{aligned}
$$

As $b_j = \frac{\delta^{n_s-j}+\delta^j}{\sqrt{3}(\delta^{n_s}-1)}$ with $\delta = 2+\sqrt{3}$, and $b_{n_s} = b_0$,

$$\begin{aligned}
\sum_{j=0}^{n_s-1}(b_j - b_{j+1})^2 &= \frac{1}{3\left(\delta^{n_s}-1\right)^2}\sum_{j=0}^{n_s-1}(\delta^j + \delta^{n_s-j} - \delta^{j+1} - \delta^{n_s-j-1})^2 \\
&= \frac{1}{3\left(\delta^{n_s}-1\right)^2}\sum_{j=0}^{n_s-1}(-\delta^j(1+\sqrt{3}) + \delta^{n_s-j}(\sqrt{3}-1))^2 \\
&= \frac{1}{3\left(\delta^{n_s}-1\right)^2}\sum_{j=0}^{n_s-1}(2\delta^{2j+1} + 2\delta^{2n_s-2j-1} - 4\delta^{n_s}) \\
&= \frac{2}{3\left(\delta^{n_s}-1\right)^2}\left[\sum_{j=0}^{n_s-1}\delta^{2j+1} + \sum_{j=0}^{n_s-1}\delta^{2n_s-2j-1} - 2n_s\delta^{n_s}\right] \\
&= \frac{2}{3\left(\delta^{n_s}-1\right)^2}\left[\delta\sum_{j=0}^{n_s-1}\delta^{2j} + (2-\sqrt{3})\sum_{j=0}^{n_s-1}\delta^{2n_s-2j} - 2n_s\delta^{n_s}\right]
\end{aligned}$$

Notice that $(2-\sqrt{3})\sum_{j=0}^{n_s-1}\delta^{2n_s-2j} = \frac{1}{\delta}\sum_{j=0}^{n_s-1}\delta^{2(n_s-j)}$, then by reindexing, we have: $\frac{1}{\delta}\sum_{j=0}^{n_s-1}\delta^{2(n_s-j)} = \frac{1}{\delta}\sum_{j=1}^{n_s}\delta^{2j} = \delta\sum_{j=1}^{n_s}\delta^{2(j-1)} = \delta\sum_{j=0}^{n_s-1}\delta^{2j} = \delta\frac{\delta^{2n_s}-1}{\delta^2-1} = \frac{1}{2\sqrt{3}}(\delta^{2n_s}-1)$. Also, $\sum_{j=0}^{n_s-1}(b_j-b_{j+1})^2 = \frac{2}{3(\delta^{n_s}-1)^2}\left[\frac{1}{\sqrt{3}}(\delta^{2n_s}-1) - 2n_s\delta^{n_s}\right]$. Therefore,

$$\begin{aligned}
E\left(u|_x\right) &= E(v) - \frac{5t}{4n_s} + \frac{n_s}{4t}Var(v)\frac{2}{3\left(\delta^{n_s}-1\right)^2}\left[\frac{1}{\sqrt{3}}(\delta^{2n_s}-1) - 2n_s\delta^{n_s}\right] \\
&= \mu - \frac{5t}{4n_s} + \frac{n_s\sigma^2}{6t\left(\delta^{n_s}-1\right)^2}\left[\frac{1}{\sqrt{3}}(\delta^{2n_s}-1) - 2n_s\delta^{n_s}\right] \\
&= \mu - \frac{5t}{4n_s} + \frac{n_s\sigma^2}{t}\left[\frac{\delta^{n_s}+1}{6\sqrt{3}\left(\delta^{n_s}-1\right)} - \frac{n_s\delta^{n_s}}{3\left(\delta^{n_s}-1\right)^2}\right]
\end{aligned}$$

■

Proof of Corollary 1

Proof. $\frac{\partial E(\pi)}{\partial\sigma^2} = \frac{n_b}{tn_s}g_s(n_s)$ and $\frac{\partial E(u)}{\partial\sigma^2} = \frac{1}{t}g_b(n_s)$. Given $g_s(n_s), g_b(n_s) > 0$, the results follow immediately. ■

Proof of Proposition 1

Proof. The first-order condition to the optimization problem (7) can be written as $\Pi'(n_s^*(\sigma^2),\sigma^2) \equiv 0$. Take differentiation with respect to σ^2, we get $\frac{\partial^2\Pi}{\partial n_s^2}\frac{dn_s^*}{d\sigma^2} + \frac{\partial^2\Pi}{\partial n_s\partial\sigma^2} = 0$. Given the expression of $\Pi'(n_s)$, Eq. (8), we have $\frac{\partial^2\Pi}{\partial n_s \partial\sigma^2} > 0$ because $g(n_s) > 0$ and $g'(n_s) > 0$. By the second-order condition, $\frac{\partial^2\Pi}{\partial n_s^2} = \Pi''(n_s) < 0$. Then $\frac{dn_s^*}{d\sigma^2}$ has the same sign as $\frac{\partial^2\Pi}{\partial n_s \partial\sigma^2}$. As a result, $\frac{dn_s^*}{d\sigma^2} > 0$. ■

Proof of Proposition 2

Proof. The FOC Eq. (9) implies that $\frac{zt}{C(n_s^*)^2}\left(\mu - \frac{t}{4n_s^*}\right) = 8f$, so the seller-side fee (from Table 1) $R_s^* = \frac{zt}{2C(n_s^*)^2}\left(\mu - \frac{t}{4n_s^*}\right) - f = 3f > 0$. For the buyer-side fee, $R_b^* = \frac{1}{2}\left(\mu - \frac{9t}{4n_s^*}\right) < 0$ if and only if $n_s^* < \frac{9t}{4\mu}$. Because $\Pi''(n_s) < 0$, $n_s^* < \frac{9t}{4\mu}$ if and only if $0 = \Pi'(n_s^*) > \Pi'(\frac{9t}{4\mu}) = \frac{16}{729}\frac{z\mu^3}{Ct} - f$. Define $k \equiv \frac{16}{729}$; notice that this numerical value is derived from the standard features of the circular city model. We then have Condition (10) for this the proposition. ■

Proof of Proposition 3

Proof. From the first-order condition (14), at the optimal n_s^*, $f = n_b^*\left(\frac{t}{4n_s^{*2}} + \frac{\sigma^2}{t}\delta_1\right)$. Plug this into the expression of R_s^* to get $R_s^* = n_b^*\left[\left(\frac{t}{n_s^{*2}} + \frac{\sigma^2}{t}\delta_2\right) - \left(\frac{t}{4n_s^{*2}} + \frac{\sigma^2}{t}\delta_1\right)\right]$. Then $R_s^* < 0$ if and only if $n_s^* > \frac{\delta_4 t}{\sigma}$, where $\delta_4 = \sqrt[4]{\frac{243}{4}}$. Because $\Pi''(n_s) < 0$, $n_s^* > \frac{\delta_4 t}{\sigma}$ if and only if $0 = \Pi'(n_s^*) < \Pi'(\frac{\delta_4 t}{\sigma})$, which leads to $\frac{Ctf}{z} < \sigma^2(\delta_5\mu + \delta_6\sigma)$, where $\delta_5 \equiv \frac{1}{2} - \frac{5}{27}\sqrt{3} > 0$ and $\delta_6 \equiv \frac{\sqrt[4]{12}}{324}(353 - 189\sqrt{3}) > 0$. ■

Proof of Proposition 4

Proof. Given the expression of $R_b^*(n_s)$, $R_b^* < 0$ if and only if $n_s < \frac{9t}{2(\mu+\theta)} \equiv \bar{n}$, where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$. Because $\Pi''(n_s) < 0$, $n_s^* < \bar{n}$ if and only if $0 = \Pi'(n_s^*) > \Pi'(\bar{n})$, which is reduced to $\frac{Ctf}{z} > \delta_7\,(\mu+\theta)\,(\mu - \delta_8\theta)(\mu - \delta_9\theta)$, where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$, $\delta_7 = \frac{4(35+12\sqrt{3})}{729}$, $\delta_8 = \frac{22+\sqrt{3}}{26}$, and $\delta_9 = \frac{29+4\sqrt{3}}{61}$. It can be shown that the right hand side is increasing in both μ and σ^2, so the condition is less likely to be satisfied when σ^2 is larger. ■

Proof of Proposition 5

Proof. From Eq. (8), we can write $\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}g(n_s) = \frac{2fC}{z\left[\frac{t}{4n_s^2} + \frac{\sigma^2}{t}g'(n_s)\right]}$. Thus, we can express R_s^* as the following:

$$R_s^* = \frac{f}{n_s}\frac{\frac{t}{n_s} + \frac{\sigma^2}{t}g_s(n_s)}{\frac{t}{4n_s^2} + \frac{\sigma^2}{t}g'(n_s)} - f = f\left[\frac{\frac{4t}{n_s^2} + \frac{4\sigma^2}{tn_s}g_s(n_s)}{\frac{t}{n_s^2} + \frac{4\sigma^2}{t}g'(n_s)} - 1\right] = f\left[\frac{3A + 4B(\frac{g_s(n_s)}{n_s} - g'(n_s))}{A + 4Bg'(n_s)}\right],$$

where $A \equiv \frac{t}{n_s^2}$ and $B \equiv \frac{\sigma^2}{t}$.

$$\begin{aligned} \frac{\partial R_s}{\partial A} &= \frac{f}{[A+4Bg'(n_s)]^2}\left[3(A+4Bg'(n_s)) - \left(3A + 4B\left(\frac{g_s(n_s)}{n_s} - g'(n_s)\right)\right)\right] \\ &= \frac{4fB}{[A+4Bg'(n_s)]^2}\left[4g'(n_s) - \frac{g_s(n_s)}{n_s}\right], \end{aligned}$$

$$\begin{aligned} \frac{\partial R_s}{\partial B} &= \frac{f}{[A+4Bg'(n_s)]^2}\left[4\left(\frac{g_s(n_s)}{n_s} - g'(n_s)\right)(A+4Bg'(n_s))\right. \\ &\quad \left. -4g'(n_s)\left(3A + 4B\left(\frac{g_s(n_s)}{n_s} - g'(n_s)\right)\right)\right] \\ &= \frac{4fA}{[A+4Bg'(n_s)]^2}\left[\frac{g_s(n_s)}{n_s} - 4g'(n_s)\right]. \end{aligned}$$

Define $D \equiv \frac{4f}{[A+4Bg'(n_s)]^2}\left[\frac{g_s(n_s)}{n_s} - 4g'(n_s)\right]$

$$\begin{aligned} \frac{\partial R_s}{\partial \sigma^2} &= \frac{\partial R_s}{\partial A}\cdot\frac{\partial A}{\partial \sigma^2} + \frac{\partial R_s}{\partial B}\cdot\frac{\partial B}{\partial \sigma^2} \\ &= -DB\cdot\frac{-2t}{n_s^3}\cdot\frac{\partial n_s}{\partial \sigma^2} + \frac{DA}{t} \\ &= D\left(\frac{2tB}{n_s^3}\cdot\frac{\partial n_s}{\partial \sigma^2} + \frac{A}{t}\right). \end{aligned}$$

Notice that $\frac{\partial n_s}{\partial \sigma^2} > 0$ and $D < 0$; therefore, $\frac{\partial R_s}{\partial \sigma^2} < 0$.

We derive numerically that the optimal buyer fee increases with σ^2. ■

Proof of Proposition 6

Proof. Look at the platform's optimization problem, Eq. (7). Let the solution be $n_s^*(\sigma^2)$. Then it must satisfy the FOC: $\Pi'(n_s^*(\sigma^2)) \equiv 0$. Plug $n_s^*(\sigma^2)$ into the objective function to obtain the value function $\Pi(n_s^*(\sigma^2), \sigma^2)$. Take total differentiation of this value function with respect to σ^2 to get $\frac{d\Pi(n_s^*(\sigma^2),\sigma^2)}{d\sigma^2} = \frac{\partial\Pi(n_s^*(\sigma^2),\sigma^2)}{\partial n_s^*}\frac{dn_s^*(\sigma^2)}{d\sigma^2} + \frac{\partial\Pi(n_s^*(\sigma^2),\sigma^2)}{\partial\sigma^2} = \Pi'(n_s^*(\sigma^2))\frac{dn_s^*(\sigma^2)}{d\sigma^2} + \frac{\partial\Pi(n_s^*(\sigma^2),\sigma^2)}{\partial\sigma^2} = \frac{\partial\Pi(n_s^*(\sigma^2),\sigma^2)}{\partial\sigma^2}$. Basically, this is just the Envelope Theorem. By direct observation, $\frac{\partial\Pi(n_s^*(\sigma^2),\sigma^2)}{\partial\sigma^2} = \frac{z}{2C}\left[\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}g(n_s)\right]\frac{g(n_s)}{t} > 0$. Therefore, $\frac{d\Pi(n_s^*(\sigma^2),\sigma^2)}{d\sigma^2} > 0$. ■

B Numerical Approximation

When n_s is not too small, $\frac{\delta^{n_s}+1}{\delta^{n_s}-1} \approx 1$ and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2} \approx 0$. For example, when $n_s = 6$, $\frac{\delta^{n_s}+1}{\delta^{n_s}-1} = 1.00074$ and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2} = 0.00222$. A larger n_s leads to closer approximations (Table 4).

Table 4: Numerical Examples

n_s	5	6	7	8	9	10	11
$\frac{\delta^{n_s}+1}{\delta^{n_s}-1}$	1.00277	1.00074	1.00020	1.00005	1.00001	1.00000	1.00000
$\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2}$	0.00693	0.00222	0.00069	0.00021	0.00006	0.00002	0.00001

Our numerical study without applying approximation shows that the equilibrium n_s is easily well above 10. Thus, approximating $\frac{\delta^{n_s}+1}{\delta^{n_s}-1}$ at 1 and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2}$ at 0, which requires n_s to be sufficiently high, is consistent with the actual equilibrium.

Based on this approximation, we can rewrite the following expressions:

$$g(n_s) = n_s\left(1 - \frac{7}{6\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) \approx n_s\delta_1 \tag{18}$$

$$g_s(n_s) = n_s\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) \approx n_s\delta_2 \tag{19}$$

$$g_s(n_s) - g_b(n_s) = n_s\left(1 - \left(\frac{4}{3\sqrt{3}} + \frac{1}{6\sqrt{3}}\right)\frac{\delta^{n_s}+1}{\delta^{n_s}-1} - n_s\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) \approx n_s\delta_3 \tag{20}$$

where $\delta_1 = 1 - \frac{7}{6\sqrt{3}}$, $\delta_2 = 1 - \frac{4}{3\sqrt{3}}$, and $\delta_3 = 1 - \frac{\sqrt{3}}{2}$.