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The Size of Venture Capital and Private Equity Fund Portfolios

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Abstract

We propose a model that examines the optimal size of venture capital and private equity fund portfolios. The relationship between a VC and entrepreneurs is characterized by double-sided moral hazard, which causes the VC to trade off larger portfolios against lower values of portfolio companies. We analyze the structural relations between the VC's optimal portfolio structure and entrepreneurs' and VC's productivities, their disutilities of effort, the value of a successful project, and the required initial investment in a venture. We also test the model's predictions using a small proprietary dataset collected through a survey targeted to VC and private equity funds worldwide.

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1 Introduction

Venture capital provides an opportunity for young, innovative firms to develop and grow. Unlike regular investors, venture capitalists (VCs) are actively involved in the management of their portfolio companies. VCs provide assistance with strategic and operational planning, management recruitment, marketing, and obtaining additional capital.[1] However, the relationship between VCs and entrepreneurs is characterized by double-sided moral hazard. In most cases, the parties' effort levels are unobservable/nonverifiable and, thus, noncontractible.

Unlike the optimal contracting between VCs and entrepreneurs,[2] the optimal size of VCs' portfolios has received little attention in the theoretical and empirical venture capital literature. Investing in multiple ventures, as opposed to a single project, allows a VC to make the best use of her funds, while reducing the risk of her investment. However, because the VC invests time and effort in advising her portfolio firms, increasing the number of firms in the portfolio dilutes the quantity and quality of managerial advice to each entrepreneurial venture.

Because the number of experts is limited, venture capital is not easily scalable. The higher the number of ventures financed and advised by a VC, the lower the amount of advice provided to each venture.[3] If entrepreneurs' and VC's efforts are complementary, reduced VC's advice can also lead to a lower commitment of entrepreneurs to their ventures. On the other hand, if VC's wealth is unconstrained, investing in more ventures increases the expected total dollar return of her portfolio. Thus, the choice of the optimal portfolio size involves a trade-off between the number

[1] See Sahlman (1990) for a discussion of the nature of the relationship between VCs and entrepreneurs, and Gorman and Sahlman (1989) and Wright and Lockett (2003) for some survey evidence of VCs' activities.

[2] There is a large literature concerned with the design of financial contracts that mitigate the agency problems between VCs and entrepreneurs. See Bascha and Walz (2001), Bergemann and Hege (1998), Cornelli and Yosha (2003), Garmaise (2001), Houben (2002), Kaplan and Strömberg (2004), Marx (1998), Repullo and Suarez (2004), and Schmidt (2003), among others, for models of optimal structure of contracts between VCs and enterpreneurs. See Gompers (1995) and Gompers and Lerner (1999b) for empirical investigations of venture capital contracts.

[3] In addition, Cestone and White (2003) show that it may be beneficial for a VC to commit to not fund too many ventures because potential competition among ventures in the VC's portfolio may diminish expected returns.

of projects and each project's expected value net of the cost of VC's effort devoted to it.

Our paper builds on a few recent important papers in the VC literature. Kanniainen and Keuschnigg (2003, 2004), KK henceforth, provide seminal theoretical analysis of the optimal size of VCs' portfolios. (In what follows, we use the terms "portfolio size" and "number of portfolio firms" interchangeably.) Fulghieri and Sevilir's (2004) model considers a VC's incentives to concentrate on a single venture versus investing in two ventures. Cumming (2006) provides an empirical analysis of VCs' portfolio sizes using a Canadian dataset. Kaplan and Strömberg (2003) and Hege, Palomino, and Schwienbacher (2003) provide some evidence on the allocation of cash flows between VCs and entrepreneurs.

Our paper contributes to the literature in two ways. First, we extend the theoretical literature by simultaneously examining VC's portfolio size and the allocation of cash flow rights between a VC and entrepreneurs. We show that endogenizing the profit sharing between the parties is instrumental in analyzing the determinants of the optimal VC's portfolio structure. Second, we test the predictions of our model using a unique international VC and private equity dataset.

The essential elements of our model are as follows. A VC chooses not only the number of firms in her portfolio, but also the shares of the ventures' profits that the entrepreneurs retain. Profit sharing affects the optimal effort levels exerted by the VC and the entrepreneurs, which, in turn, determine the expected values of entrepreneurial ventures.

The complexity of the model allows us to derive only the equilibrium effort levels of the VC and the entrepreneurs in a general form. We make two compromises while solving for the optimal portfolio structure. First, we assume specific functional forms for the probability of a venture's success, as well as for the costs of VC's and entrepreneurs' efforts. Second, we assume that VC's and entrepreneurs' efforts are additive and, thus, not complementary to the probability of projects' success.[4] The model results in unambiguous predictions regarding the effect of the parameters on the optimal portfolio size only when the sharing rule is held constant. However, when we endogenize the sharing rule, the signs of the reduced-form relations between

[4]This assumption is made for analytical tractability only and it does not affect the relation between the optimal portfolio structure and the model's parameters.

most of the parameters and the optimal portfolio size become ambiguous. Thus, we demonstrate that one should account for the endogeneity of both the profit sharing rule and the portfolio size while examining these relations empirically.

The framework of our model of effort choices is most similar to Bhattacharyya and Lafontaine (1995), who analyze franchising agreements, which also pose a double-sided moral hazard problem for a principal facing multiple agents.[5] The framework of our model in the portfolio structure choice stage is more similar to that in Casamatta (2003), who examines the rationale for the joint provision of funds and advice by VCs to entrepreneurs.

To test the predictions of the model we obtained comprehensive data for 42 generalist venture capital and private equity funds based in Europe and North America through a survey and follow-up phone interviews in 2004.[6] We analyze the determinants of VCs' portfolio sizes and entrepreneurs' profit shares while accounting for the inherent endogeneity of the profit sharing rule using instrumental variables. The results of the analysis provide some support for the predictions of our model. In particular, we find strong support for the prediction that VC's portfolio size varies non-monotonically with the profit shares held by entrepreneurs. In addition, consistent with the model, entrepreneurs' profit shares are found to be positively related to the number of firms in VC's portfolio. Overall, while the data do not support every specific detail of the model, the evidence is consistent with the notion that VC's portfolio size and profit sharing rule are determined jointly.

The remainder of the paper is organized as follows. The next section lays out the framework of the model. We derive the relations between the optimal VC's and entrepreneurs' effort levels and the model's parameters in a general setting in Section 3. Section 4 analyzes the optimal VC's portfolio structure. In Section 5 we establish empirical implications for the system of relations among the portfolio size, the profit shares given to entrepreneurs, and the model's parameters, and present the data and

[5]The difference between our model and that of Bhattacharyya and Lafontaine (1995) is that in their setting, the assumption of a constant marginal cost of effort of a franchizor (the party choosing the number of "companies" in her portfolio of franchisees) makes the optimal choice of the number of portfolio firms irrelevant. Double-sided moral hazard in the venture capital setting has been analyzed, most recently, by Casamatta (2003), Casamatta and Haritchabalet (2004), Cassiman and Ueda (2006), Lerner and Schoar (2005), Schmidt (2003), and Ueda (2004).

[6]Generalist funds are those investing in a wide array of firms, both in early and late stages.

the empirical tests of the model's predictions. We conclude and discuss the limitations of this study and possible directions for future research in Section 6. All proofs are provided in the Appendix.

2 The setup of the model

Consider a single venture capitalist, who finances multiple risky entrepreneurial ventures. There is an infinitely elastic supply of identical projects, each requiring an initial investment of I and providing a (discounted) cash flow of R if successful, and zero if unsuccessful. Assuming that the value of a project in an unfavorable state is zero simplifies the analysis, but prevents us from analyzing security design issues – for example, the expected payoff from holding a fraction x of a firm's equity is identical to a debt position with a promised repayment of xR. However, optimal security design is not the focus of this paper.[7] This setting assumes one financing round.[8]

Entrepreneurs are assumed to be identical. This assumption may seem unrealistic, since VCs' screening ability is limited, and they typically have both "good" and "bad" projects in their portfolios. However, the distinction between "good" and "bad" projects is made ex-post, while the portfolio structure decisions are based on entrepreneurial ventures' ex-ante characteristics. In addition, this assumption reflects the largely documented specialization of the majority of VCs, who tend to invest in similar projects.[9]

The probability of venture i's success, p_i, depends on the effort exerted by entrepreneur i, e_i, the effort (the amount of advice to venture i) exerted by the VC,

[7] Security design is the focus of recent papers by Casamatta (2003) and Cestone and White (2003) among others. They assume that the project value can take two realizations: R_{high} and R_{low}, and are able to examine the effects of the form of financing on entrepreneurs' and VCs' actions.

[8] The no-staging assumption precludes dynamic considerations. With multiple rounds of financing, the optimal structure of VC's portfolio would be affected by her past decisions, as well as by past shocks to projects' values. While staging is an important phenomenon, it is beyond the scope of our analysis. See Fulghieri and Sevilir (2004) for a model in which a VC can re-allocate resources from a failing venture to a successfull one in the second financing stage.

[9] Moreover, the results of our model for the case of identical entrepreneurs can be easily extended to a case in which a VC faces entrepreneurs with different characteristics, as long as there is no information asymmetry. This extension is available upon request.

E_i, the productivity (quality) of the entrepreneurs, α, and that of the VC, β. The modelling of an entrepreneur's effort level as a continuous variable is one of the differences between our approach and that of KK, where entrepreneurs' effort choices are binary, and the profit shares given to the entrepreneurs are the lowest ones that satisfy their participation constraint. Because VC's and entrepreneurs' effort levels only affect the probabilities of projects' success, and are not verifiable, they are not contractible.

We assume that all agents in the model are risk-neutral. This assumption has the following implications. First, it prevents the portfolio structure from being affected by risk sharing considerations, which are interesting, but are beyond the scope of our model. Second, VC's expected utility depends exclusively on the expected value of her portfolio of entrepreneurial ventures net of her effort costs, and no assumptions regarding the joint probability of the ventures' success are necessary.

We assume that all the functions in the model are twice continuously differentiable. The probability of venture i's success is assumed to be strictly increasing and weakly concave in both entrepreneur i's and VCs' effort levels: $p_{i_{e_i}} > 0$, $p_{i_{E_i}} > 0$, $p_{i_{e_i e_i}} \leq 0$, $p_{i_{E_i E_i}} \leq 0$. The effort levels of entrepreneur i and the effort of the VC devoted to project i are assumed to be weakly complementary: $p_{i_{e_i E_i}} \geq 0$. The probability of project i's success is positively related to the qualities of both the entrepreneur and the VC: $p_{i_\alpha} > 0$ and $p_{i_\beta} > 0$. The marginal contributions of entrepreneur's and VC's efforts to the project's success are also positively related to the respective agents' qualities, i.e. $p_{i_{e_i \alpha}} > 0$ and $p_{i_{E_i \beta}} > 0$. To ensure the existence of an equilibrium in effort levels, we assume that $p_{i_{e_i e_i}} + p_{i_{e_i E_i}} < 0$ and $p_{i_{E_i E_i}} + p_{i_{e_i E_i}} < 0$.[10] In addition, in order to interpret $p_i(e_i, E_i, \alpha, \beta)$ as a probability function, we assume that $0 \leq p \leq 1$ for all e_i, E_i, α and β.

In the first stage, the VC chooses the number of projects to finance, n, the profit share in each venture offered to entrepreneurs, x_i for project i, and makes an irreversible investment of I in each project. We assume that VC's wealth is unbounded (the supply of financing by limited partners is perfectly elastic). This assumption is supported by the empirical evidence that wealth constraints are not binding for most

[10] This condition is similar to the one ensuring uniqueness and global stability of the solution to a Cournot duopoly game: $\pi_{i,i} + \pi_{i,-i} < 0$, where $\pi_{i,i}$ is the second partial derivative of firm i's profit with respect to its own quantity, and $\pi_{i,-i}$ is the cross-partial derivative of firm i's profit with respect to its own and its rival's quantities. (See Chapter 4 in Vives (2000)).

VCs. For example, Cumming (2006) reports that, on average, venture capital funds have about half of their resources invested in entrepreneurial ventures at a given point in time.

Because a project generates zero cash flows when it fails, a linear profit sharing rule is as good as any other sharing rule.[11] The VC makes a take-it-or-leave-it (TIOLI) offer to each entrepreneur, specifying the share of the profits she retains in exchange for her investment and future advice ($1-x_i$ for venture i). Each entrepreneur accepts the TIOLI offer if his participation constraint is satisfied. We assume, for simplicity, that if an entrepreneur rejects the offer, his reservation utility is zero. Hence, an entrepreneur will accept any offer that leaves him with a positive profit share.[12]

In the second stage, the entrepreneurs and the VC choose their nonverifiable effort levels. Effort is costly. Its cost to entrepreneur i, K_i, is increasing and convex in the effort level: $K_{i_{e_i}} > 0$ and $K_{i_{e_i e_i}} > 0$. The cost of effort and the contribution of a marginal unit of effort to the cost of effort are assumed to be increasing in an entrepreneur's "disutility of effort" parameter, γ: $K_{i_\gamma} > 0$ and $K_{i_{e_i \gamma}} > 0$. Similarly, the cost of effort of the VC, L, is increasing and convex in the total amount of advice provided to the projects: $L_{\sum_{i=1}^{n} E_i} > 0$ and $L_{\sum_{i=1}^{n} E_i \sum_{i=1}^{n} E_i} > 0$, for $i = 1, ..., n$. Moreover, we assume that because of "coordination costs", the total cost and the marginal cost of VC's effort are increasing and weakly convex in the number of firms she invests in, holding the total effort level constant: $L_n > 0$, and $L_{nn} \geq 0$. The cost of VC's effort, the contribution of the marginal unit of effort to the total cost of effort, and the coordination costs are increasing in the VC's disutility of effort parameter, δ:

[11] A large body of literature argues that linear profit sharing contracts are not optimal in the venture capital setting (see, for example, Cornelli and Yosha (2003), and Schmidt (2003)), and that convertible securities should be used instead. Kaplan and Strömberg (2003) find that in their sample of 213 financing rounds, convertible securities are used in 206 rounds, and 170 rounds use solely convertible preferred stock. They find, however, that the state contingencies of cash flow rights are not very large. Bhattacharyya and Lafontaine (1995) show that the optimal sharing rule can be obtained with a linear contract in a setting with double-sided moral hazard, but no asymmetric information and risk neutral agents.

[12] An entrepreneur does not commit to exert any effort by accepting the offer. Therefore, the worst he can do is receive zero expected value, since the expected value of a project is non-negative, and the entrepreneur does not invest any funds in the project.

$L_\delta > 0$, $L_{\sum\limits_{i=1}^{n} E_i \delta} > 0$ and $L_{n\delta} > 0$. The VC and the entrepreneurs maximize the expected values of their respective shares of the projects net of effort costs, U_{VC} and U_{en_i} respectively, given the sharing rules, x_i for project i, and the size of the VC's portfolio, n:

$$U_{en_i} = x_i p_i(e_i, E_i, \alpha, \beta) R - K_i(e_i, \gamma) \text{ for } i = 1, ..., n, \tag{1}$$

$$U_{VC} = \sum_{i=1}^{n} [[1 - x_i] p_i(e_i, E_i, \alpha, \beta) R - I] - L(\sum_{i=1}^{n} E_i, n, \delta). \tag{2}$$

The game is solved by finding a sub-game perfect equilibrium. In the second stage, n entrepreneurs and the VC maximize their expected values, net of effort costs, with respect to their own effort levels, given the size of VC's portfolio and the (linear) profit sharing rules. In the first stage, the VC chooses the number of portfolio companies and the profit shares given to the entrepreneurs, while accounting for her own and entrepreneurs' second-stage optimal effort levels choices.

In order to ensure interior solutions for the parties' optimal effort levels, we assume that the marginal benefit of VC's and entrepreneurs' efforts are higher than their respective marginal effort cost, as the effort levels approach zero: $x_i p_{i_{e_i}} R > K_{i_{e_i}}$ as $e_i \to 0$ for any $x_i > 0$, $E_i > 0$, $\alpha > 0$, $\beta > 0$, and $\gamma > 0$, and $[1 - x_i] p_{i_{E_i}} R > L_{E_i}$ as $E_i \to 0$ for any $x_i > 0$, $n > 0$, $e_i > 0$, $\alpha > 0$, $\beta > 0$, and $\delta > 0$.

3 Optimal effort levels

In this section we derive the second-stage optimal effort levels of the VC and the entrepreneurs, taking the portfolio structure as given. Maximizing each entrepreneur's expected value, U_{en_i} in (1), with respect to his effort level, e_i, and VC's expected value, U_{vc} in (2), with respect to the level of advice given to each entrepreneur, E_i, results in a system of $2n$ first-order conditions that have to hold simultaneously in equilibrium. The n F.O.C.'s of entrepreneurs are

$$x_i p_{i_{e_i}} R - K_{i_{e_i}} = 0 \text{ for } i = 1, ..., n, \tag{3}$$

indicating that each entrepreneur chooses his effort so that its marginal benefit equals its marginal cost. The n F.O.C.'s of the VC are

$$[1 - x_i] p_{i_{E_i}} R - L_{E_i} = 0 \text{ for } i = 1, ..., n, \tag{4}$$

ensuring that in equilibrium the marginal cost of VC's effort equals the marginal increase in the expected value of venture i.

While multiple asymmetric equilibria may exist, in order to obtain tractable results, we restrict our attention to symmetric equilibria. The following lemma establishes the existence of a symmetric equilibrium in the second stage, in which all entrepreneurs choose identical effort levels, and the VC devotes the same effort to each project. The symmetric equilibrium in effort levels is conditional on the VC offering identical profit shares to all entrepreneurs in the first stage. We prove, in the next section, that there exists an equilibrium in the first stage of the game, in which the VC gives equal profit shares to all entrepreneurs.

Lemma 1 *If the profit shares given to all entrepreneurs are equal, $x_i = x$ for all $i = 1, ..., n$, then there exists a symmetric equilibrium in entrepreneurs' and VC's effort levels, where $e_i^*(x, n) = e^*(x, n)$ and $E_i^*(x, n) = E^*(x, n)$ for all $i = 1, ..., n$.*

In this symmetric equilibrium, VC's effort is evenly distributed across n firms in her portfolio, and all entrepreneurs exert identical efforts. Therefore, for the symmetric case, the system of F.O.C.'s in (3) and (4) may be rewritten as the following system of two equations:

$$xp_e R - K_e = 0, \tag{5}$$

$$[1 - x]p_E R - L_E = 0, \tag{6}$$

where p denotes the probability of success of each entrepreneurial project, e is each entrepreneur's effort level, and E is the effort exerted by the VC on each project.

Differentiating this system of equations with respect to the number of portfolio firms, n, while assuming fixed profit share, x, equating the resulting expressions to zero, and solving the system of two equations leads to the following proposition:

Proposition 1 *For any given $x = \overline{x}$, VC's equilibrium effort devoted to each project, $E^*(n, \overline{x})$, is strictly decreasing in the number of firms in her portfolio, n. If there are no complementarities between entrepreneurs' and VC's efforts, $p_{eE} = 0$, then the equilibrium effort level of each entrepreneur, $e^*(n, \overline{x})$, is independent of n. If efforts are complementary, $p_{eE} > 0$, then $e^*(n, \overline{x})$ is strictly decreasing in n.*

We illustrate the relations between E^*, e^*, and n, described in Proposition 1 in Figure 1. Figure 1A describes the case of no complementarities between entrepreneurs' and VC's efforts, while Figure 1B presents the case of complementary efforts. Increasing the number of portfolio firms stretches VC's effort over more projects. If, as we assumed, the total cost of effort is convex in n, then the optimal level of VC's advice to each firm decreases as n increases. In addition, if entrepreneurs' and VC's efforts are complementary, reduced advice by the VC induces entrepreneurs to optimally exert less effort. This latter effect, in turn, reinforces VC's incentive to exert less effort. Therefore, the direct and the indirect effects of a change in n on the effort levels of the entrepreneurs and the VC are reinforcing. The indirect effect is zero in the absence of complementarities, in which case the optimal effort of a typical entrepreneur, e^*, is not affected by n.

We now turn to the relation between the equilibrium effort levels and the profit share offered to the entrepreneurs, x. Differentiating the system in (5) and (6) with respect to x, while assuming that the number of firms in the portfolio is fixed, equating the resulting expressions to zero, and solving the system of two equations, leads to the following result:

Proposition 2 *1) If VC's and entrepreneurs' efforts are not complementary, $p_{eE} = 0$, then for any given $n = \overline{n}$, the equilibrium effort of each entrepreneur, $e^*(\overline{n}, x)$, is strictly increasing in the share given to entrepreneurs, x, and the equilibrium effort of the VC devoted to each project, $E^*(\overline{n}, x)$, is strictly decreasing in x.*
2) If VC's and entrepreneurs' efforts are complementary, $p_{eE} > 0$, then for any given $n = \overline{n}$, $e^*(\overline{n}, x)$, and $E^*(\overline{n}, x)$ do not vary monotonically with the share given to entrepreneurs, x. For $x \to 1$, both $e^*(\overline{n}, x)$ and $E^*(\overline{n}, x)$ are decreasing in x. For $x \to 0$, both $e^*(\overline{n}, x)$ and $E^*(\overline{n}, x)$ are increasing in x.

The profit shares given to entrepreneurs, x, have a direct and, possibly, an indirect effect on VC's and entrepreneurs' optimal effort levels. The direct effect of an increase in x is the reduced (increased) incentive of the VC (entrepreneurs) to exert effort for any given effort level of the entrepreneurs (VC). If entrepreneurs' and VC's efforts are complementary, then there is also an indirect effect of increasing x. Increased entrepreneurial effort raises the expected value of each venture given the VC's level of effort, increasing the VC's marginal benefit of exerting effort. Because the two effects have opposite implications for the equilibrium VC's effort level, the sign of the relation between E^* and x depends on their relative magnitudes. The same discussion applies, of course, to entrepreneurs' effort levels, which are affected by x directly and indirectly (through the change in the optimal VC's effort).

We illustrate the two parts of Proposition 2 in Figure 2. Figure 2A presents the case of no complementarities, while Figures 2B and 2C describe the case in which efforts are complementary for $x \to 1$ and $x \to 0$ respectively. If efforts are complementary, then when the share given to a typical entrepreneur is close to one, a further increase in his share does not (directly) increase his optimal effort level enough to offset the indirect negative effect of the reduced VC's effort. Hence, for x close to one, the indirect effect more than offsets the direct effect. This results in a negative combined effect of increasing x on both VC's and entrepreneurs' effort levels. On the other hand, when x is close to zero, the direct negative effect of increasing x on the optimal VC's effort level is more than offset by the indirect positive effect of increased entrepreneur's effort level. Hence, for x close to zero, both entrepreneurs' and VC's effort levels are increasing in x.

In addition to studying the effects of changes in x and n on the optimal efforts of the entrepreneurs and the VC, our general framework allows us to analyze the effect of changes in the model's parameters on the equilibrium efforts. The set of parameters, Φ, includes the quality parameters of the VC and entrepreneurs, their disutility of effort parameters, the value of a successful project, and the required initial investment: $\Phi = \{\alpha,\ \beta,\ \gamma,\ \delta,\ R,\ I\}$. The next proposition establishes these relations.

Proposition 3 *For any given $n = \overline{n}$ and $x = \overline{x}$, the equilibrium effort level of each entrepreneur, $e^*(\overline{n}, \overline{x}, \Phi)$, and the equilibrium effort devoted to each project by the VC, $E^*(\overline{n}, \overline{x}, \Phi)$, are monotonically increasing in their own quality; are monotonically decreasing in their own disutility of effort; are monotonically increasing in the value of a successful project; and are independent of the required investment. In addition, if efforts are complementary, $p_{eE} > 0$, $e^*(\overline{n}, \overline{x}, \Phi)$ and $E^*(\overline{n}, \overline{x}, \Phi)$ are monotonically increasing in entrepreneurs'/VC's counterparty's quality and are monotonically decreasing in their counterparty's disutility of effort.*

This result is intuitive. VC's and entrepreneurs' efforts are increasing (decreasing) in the value of a successful project and in their own quality (disutility of effort) because of the trade-off between the marginal costs and benefits of exerting effort. If efforts are complementary, the same logic holds for VC's and entrepreneurs' counterparty's quality and disutility of effort. In the absence of complementarities, each party's effort level is independent of the counterparty's characteristics. The equilibrium effort levels do not depend on the cost of initial investment because in the effort choice stage this investment is sunk.

4 Optimal portfolio size and profit sharing rule

In this section, we examine the first stage of the game, in which the size of VC's portfolio and the profit sharing rule are chosen. First, we solve a partial equilibrium model, in which the profit sharing rule is pre-determined (and identical across projects), and the VC maximizes her net expected value with respect to the number of projects she invests in. This allows us to derive the partial effects of the model's parameters on the optimal portfolio size. We perform a similar analysis for the profit sharing rule. That is, we assume a fixed portfolio size, and study the partial effect of the parameters on the optimal sharing rule. Then, we integrate the two partial equilibrium models to derive the relations between the optimal portfolio size and the model's parameters, when both the portfolio size and the profit sharing rule are endogenous.

For reasons of analytical tractability, in this section we make specific assumptions regarding the functional forms of the probability of a project's success and entrepreneurs' and VC's costs of effort. In particular, we assume that the probability of project i's success equals

$$p_i = \frac{\alpha e_i + \beta E_i}{\overline{p}_i}. \tag{7}$$

The functional form of p_i in (7) weakly satisfies the assumptions of the general model. The probability of the project's success is linearly increasing in VC's and entrepreneur's effort levels. Their efforts are not complementary, in the sense that the cross-partial derivative of p_i with respect to efforts is zero. The value of $\overline{p}_i$ in (7) is chosen so that the equilibrium probability of a project's success is lower than one.[13]

[13] One possible value of $\overline{p}_i$ is $\alpha\widehat{e}_i + \beta\widehat{E}_i$, where $\widehat{e}_i$ and $\widehat{E}_i$ are social welfare-maximizing effort levels (which maximize the expected value of venture i net of the costs of parties' efforts devoted to the venture). The proof of this statement is available upon request.

The effort cost function of entrepreneur i, K_i, takes the following form:

$$K_i = \gamma e_i^2, \tag{8}$$

and the total cost of effort of a VC, L, is given by:

$$L = \delta \left[\left[\sum_{i=1}^{n} E_i \right]^2 + n^2 \right]. \tag{9}$$

The last element in (9) reflects the assumption that coordination costs are increasing and convex in the number of portfolio firms. This functional form is parallel to that in Bhattacharyya and Lafontaine (1995), where the effort is separated into "private" and "public". In our framework, $\delta \left[\sum_{i=1}^{n} E_i \right]^2$ is the total cost of private effort, while δn^2 is the cost of public effort. Before solving the model, we establish that there exists an equilibrium in which the profit shares that the VC offers to the n entrepreneurs are identical. We focus on a symmetric equilibrium in profit shares thereafter.[14]

Lemma 2 *There exists a symmetric equilibrium in the profit shares given to the entrepreneurs, in which* $x_i^*(n, \Phi) = x^*(n, \Phi)$ *for all* $i = 1, ..., n$.

The expected values of a typical entrepreneur and the VC, net of effort costs, in (1) and (2) may be rewritten as

$$U_{en_i} = xR \left[\frac{\alpha e_i + \beta E_i}{\overline{p}} \right] - \gamma e_i^2, \tag{10}$$

$$U_{VC} = \sum_{i=1}^{n} \left[[1 - x] R \left[\frac{\alpha e_i + \beta E_i}{\overline{p}} \right] \right] - nI - \delta \left[\left[\sum_{i=1}^{n} E_i \right]^2 + n^2 \right], \tag{11}$$

where x is a venture's share given to each entrepreneur. Applying the F.O.C.'s in (5) and (6) results in the following equilibrium effort levels devoted to each venture by the VC and the entrepreneur:

$$e^* = \frac{x\alpha R}{2\gamma \overline{p}}, \tag{12}$$

$$E^* = \frac{[1 - x]\beta R}{2\delta n \overline{p}}. \tag{13}$$

[14] To ensure that $0 \leq x^*(n, \Phi) \leq 1$, we also require the following restriction to hold: $\frac{\alpha^2}{\gamma} - \frac{\beta^2}{\delta} \geq 0$. (The reason for this restriction becomes clear in Section 4.2.)

The effect of the model's parameters on the equilibrium effort levels of entrepreneurs and of the VC are consistent with those derived in the general framework discussed in the previous section for the case of no complementarities. The optimal effort levels of the VC and the entrepreneurs are independent of the characteristics of their counterparty. Moreover, the equilibrium effort level of the entrepreneurs is unrelated to the number of firms in VC's portfolio, while VC's optimal per-project effort is decreasing in n. Finally, as shown in Section 3, VC's effort devoted to each venture is decreasing in x, while entrepreneurs' optimal efforts are increasing in x.

4.1 Optimal portfolio size when the profit sharing rule is fixed

In this subsection, we assume that the profit sharing rule, x, is fixed, and examine the effects of x and the model's parameters on the optimal number of firms in VC's portfolio, $n^*(x, \Phi)$. Focusing on a symmetric equilibrium, we can rewrite the expected value of VC's portfolio in (11) as:

$$U_{VC} = n\left[[1-x]\frac{R^2}{\overline{p}^2}\left[\frac{x\alpha^2}{2\gamma} + \frac{[1-x]\beta^2}{2\delta n}\right] - I - \delta\left[\frac{[1-x]^2\beta^2R^2}{4\delta^2 n\overline{p}^2} + n\right]\right]. \tag{14}$$

Differentiating VC's expected value in (14) with respect to n, and equating the resulting expression to zero, gives the optimal size of VC's portfolio:

$$n^*(x, \Phi) = \frac{1}{4}\frac{[1-x]\, xR^2\alpha^2}{\gamma\delta\overline{p}^2} - \frac{1}{2}\frac{I}{\delta}. \tag{15}$$

Analyzing the expression in (15) leads to the following proposition:

Proposition 4 *The optimal number of firms in VC's portfolio, $n^*(x, \Phi)$, does not vary monotonically with the profit share given to entrepreneurs, x. For $x < \frac{1}{2}$, $n^*(x, \Phi)$ is increasing in x. For $x > \frac{1}{2}$, $n^*(x, \Phi)$ is decreasing in x. Moreover, holding the profit share given to entrepreneurs, x, constant, the optimal number of firms in VC's portfolio, $n^*(x, \Phi)$, is increasing in the quality of entrepreneurs; is independent of the quality of the VC; is decreasing in the disutilities of effort of entrepreneurs and the VC; is increasing in the value of a successful project; and is decreasing in the required investment in each project.*

The direct effect of increasing entrepreneurs' profit shares on the value of VC's portfolio is negative. In addition, increasing x reduces the optimal VC's per-project effort level, which further reduces the expected value of each project. However, it also increases the optimal entrepreneurs' efforts, increasing the value of VC's portfolio. VC's optimal reaction to the change in x in terms of portfolio size may take two forms. The VC can invest in fewer projects and increase his level of advice to each venture, thus increasing each project's value. Alternatively, the VC can decide to invest in a larger number of ventures, possibly increasing the total value of his portfolio. Which of these two alternatives is optimal depends on the profit shares retained by the entrepreneurs. For large x, the positive (indirect) effect of decreasing n on VC's expected profits through her increased effort level is larger in absolute magnitude than the negative (direct) effect of the reduced number of projects (i.e., $\frac{dn^*}{dx} < 0$). However, for small enough x, the reverse is true: the positive direct effect of increasing n dominates the negative indirect effect (i.e., $\frac{dn^*}{dx} > 0$). This non-monotonicity plays a crucial role in the next subsection, in which we investigate the total effects of changes in the model's parameters on the optimal VC's portfolio size.

Intuitively, the portfolio size is positively related to the profitability of each venture. Thus, a higher quality of entrepreneurs, lower disutility of the parties' efforts, higher value of a successful project, and lower required initial investment result in a higher value of each venture and in a larger optimal portfolio size.[15]

Proposition 4 is generally consistent with the results in KK. The crucial assumption underlying Proposition 4, however, is that the profit sharing rule is determined exogenously. In Subsection 4.3 we derive the relations between the optimal portfolio size and the model's parameters when the profit sharing rule is a choice variable. We show that in such setting one cannot make unambiguous predictions regarding the relations between the optimal portfolio size and the model's parameters.

[15]VC's quality does not affect the optimal portfolio size in (15). A higher VC's quality increases the value of each venture, given VC's per-project effort level. This, in turn, leads the VC to fund a larger number of projects. However, the optimal VC's per-project effort level decreases as the number of firms in his portfolio increases, which reduces the value of each venture. Holding everything else constant, this would reduce the optimal number of projects to be funded. In our model, the two effects exactly offset each other. Thus, the optimal number of firms chosen by the VC is independent of her quality parameter. This result follows from the specific functional forms in the model. In a more general setting, the effect of the VC's quality on the optimal number of portfolio firms is likely to be positive.

4.2 Optimal profit sharing rule when the portfolio size is fixed

In this subsection we assume that the number of firms in VC's portfolio is fixed and investigate the effects of the parameters on the optimal profit share given by the VC to entrepreneurs, $x^*(n,\Phi)$. Differentiating VC's expected value in (14) with respect to x, and equating the resulting expression to zero, provides the following expression for the optimal profit sharing rule, given the portfolio size:

$$x^*(n,\Phi) = \frac{\frac{\alpha^2}{\gamma}n - \frac{\beta^2}{\delta}}{2\frac{\alpha^2}{\gamma}n - \frac{\beta^2}{\delta}}. \tag{16}$$

The next proposition establishes how $x^*(n,\Phi)$ varies with the number of portfolio firms, n, and the set of the model's parameters, Φ.

Proposition 5 *The optimal profit share given to the entrepreneurs, $x^*(n,\Phi)$ is increasing in the number of portfolio firms, n. Moreover, holding the portfolio size, n, constant, $x^*(n,\Phi)$, is increasing in entrepreneurs' quality parameter and in VC's disutility of effort parameter; is decreasing in VC's quality parameter and in entrepreneurs' disutility of effort parameter; and is independent of the value of a successful project and of the required investment.*

Increasing the number of portfolio firms reduces the equilibrium per-firm effort level of the VC, thus reducing the expected value of each project. In order to mitigate the negative direct effect of the reduced VC's per-project effort, the VC optimally increases the profit shares offered to the entrepreneurs. This increases entrepreneurs' optimal efforts, which, in turn, positively affects the projects' expected values.

The relations between the model's parameters and $x^*(n,\Phi)$ follow from their direct effects on the expected value of the ventures and from the indirect effects through the optimal efforts of the VC and the entrepreneurs. The relations in Proposition 5 reflect the relative magnitudes of the direct and the indirect effects. Because both VC's and entrepreneurs' effort levels, and the expected values of the ventures are linear in the value of a successful project, R, the latter does not affect the optimal sharing rule. Because the parties' equilibrium effort levels, e^* and E^*, are independent of the required initial investment, I, the latter does not interact with x and, thus, does not affect the equilibrium sharing rule.

4.3 The total effects of the parameters on the optimal portfolio size and the profit sharing rule

The parameters of the model influence the choice variables in two ways. First, there is a direct effect, outlined in Propositions 4 and 5. In addition, there is an indirect effect, through changes induced into the other choice variable. Using the envelope theorem to examine the total effect of each of the parameters on the optimal profit sharing rule leads to the following result:

Proposition 6 *When both the profit sharing rule and the portfolio size are endogenous, the equilibrium profit shares given to the entrepreneurs are increasing in entrepreneurs' quality and in the value of a successful project; are decreasing in the quality of the VC, in entrepreneurs' disutility of effort, and in the required initial investment; and are independent of VC's disutility of effort.*

The difference between the relations in Proposition 6 and those in Proposition 5 are due to the indirect effects of the parameters on the optimal entrepreneurs' profit shares through changes in the optimal portfolio size. In the case of the quality of the entrepreneurs and their disutility of effort, the direct and the indirect effects are reinforcing. In the case of VC's quality, there is no indirect effect, since $n^*(x, \Phi)$ was shown in Proposition 4 to be independent of VC's quality. The optimal sharing rule is not affected by the quality of the VC because the direct and the indirect effect of δ exactly offset each other. Finally, the relations between $x^*(n, \Phi)$ and the value of a successful project and the required initial investment are due solely to the indirect effects of these parameters through the optimal portfolio size.

Proposition 6 results in testable empirical predictions regarding the reduced-form relations between proxies for the model's parameters and the profit shares given to entrepreneurs. Some of the predictions are consistent with the conclusions in Casamatta (2003), who analyzes effort exertion by a VC and entrepreneurs in a setting in which outside financing arises endogenously. For example, Casamatta shows that the less competent is the entrepreneur the higher the VC's profit share. This is consistent with the positive relation between entrepreneurs' quality and their shares in the ventures, demonstrated in Proposition 6.

Proposition 7 provides the relations between the optimal number of firms in VC's portfolio and the model's parameters, when their indirect effects through changes in the optimal profit sharing rule are taken into account.

Proposition 7 *When both the portfolio size and the profit sharing rule are endogenous, the relations between the portfolio size and the qualities and the disutilities of effort of the VC and the entrepreneurs are ambiguous. The optimal portfolio size is increasing in the value of a successful project, and is decreasing in the required initial investment.*

This is one of the main results of the paper. It is impossible to make empirical predictions regarding the reduced-form relations between most of VC's and entrepreneurs' characteristics and the optimal VC's portfolio size. The reason is that the relation between the optimal portfolio size and the profit shares retained by the entrepreneurs can not be signed unambiguously. Thus, it is unclear how these characteristics affect the optimal portfolio size. On the other hand, the indirect effect on $n^*(\Phi)$ through $x^*(\Phi)$ is absent in the case of changes in R and I. Thus, it is possible to make unambiguous predictions regarding these reduced-form relations.

The main conclusion from this subsection is that the testing of the determinants of the optimal VC's portfolio size can not be performed in a reduced form setting. It should be done in a framework that accounts for the simultaneous effects of various factors on the optimal profit sharing rule and on the interrelation between the latter and the optimal portfolio size. We discuss such framework in the next section.

5 Empirical analysis

A theory implies monotonic comparative statics, providing unambiguously signed predictions for the reduced-form regression coefficients if and only if the theory implies that the direct effect of a change in an exogenous variable on an endogenous variable and the indirect effect (through another endogenous variable) are reinforcing (see, for example, Barclay, Marx and Smith (2003)).

As demonstrated in Proposition 7, this is not the case in our model. Therefore, the relation between the portfolio size, the profit sharing rule, and VC's, entrepreneurs', and projects' characteristics can not be tested in a reduced form. A proper testing of our model's predictions would involve an estimation of the following system of two equations using an instrumental variables approach:

$$x^* = \zeta_0 + \zeta_1\alpha + \zeta_2\beta + \zeta_3\gamma + \zeta_4\delta + \zeta_5 R + \zeta_6 I + \zeta_7 n + \overline{\zeta_8\Psi} + \epsilon_x, \tag{17}$$

$$n^* = \eta_0 + \eta_1\alpha + \eta_2\beta + \eta_3\gamma + \eta_4\delta + \eta_5 R + \eta_6 I + \eta_7 f(x) + \overline{\eta_8}\overline{\Psi} + \epsilon_n, \tag{18}$$

where $\overline{\Psi}$ is a vector of factors that are likely to affect the VC's optimal portfolio structure, but were omitted from the model, such as staging, syndication, voting rights, and control rights, among others,[16] and $f(x)$ is a non-linear function of entrepreneurs' profit shares, which are expected to capture the non-monotonicity of the relation between n^* and x.[17]

The predictions of our model regarding the signs of coefficients $\widehat{\zeta}_i$ and $\widehat{\eta}_i$ are summarized in Table 1. Column 1 presents the signs of the predicted structural relations between n^*, x, and the model's parameters. For comparative purposes, column 2 shows the reduced-form relations obtained in a setup in which x is omitted from the n^* equation. Column 3 shows how x^* is expected to covary with n and the model's parameters, while column 4 presents the reduced-form relations between x^* and the parameters when n is omitted from the equation.

We describe the dataset used in the empirical tests in Subsection 5.1. Subsection 5.2 presents the summary statistics for the variables of interest. Empirical tests are presented in Subsection 5.3.

5.1 Data source

The data were obtained from a survey of and follow-up phone interviews with venture capital and private equity funds (VC funds hereafter), in Europe and North America. The survey and interviews were conducted in Spring and Summer of 2004. We were able to obtain reliable, private, and confidential data from 42 limited partnership VC funds from Canada (2 funds), Czech Republic (1 fund), Denmark (1 fund), France (3 funds), Germany (5 funds), Israel (2 funds), Italy (1 fund), the Netherlands (1 fund), Switzerland (1 fund), the U.K. (3 funds), and the U.S. (22 funds).

These 42 funds have financed a total of 668 entrepreneurial firms by June 2004. Two reasons motivated us to obtain data from a diverse set of countries in Europe and North America. First, there does not exist any international evidence on VCs' portfolio sizes. Second, by considering a diverse set of countries, we attempt to miti-

[16] While these factors may be determined endogenously, the theory is not rich enough to specify a structural model in which all VC's choice variables are endogenous. Thus, $\overline{\Psi}$ is simply included in the right-hand side of (17) and (18).

[17] We achieve identification by omitting some of the elements of $\overline{\Psi}$ from (17) and (18), as discussed below.

gate the effects of country-specific legal and institutional structures on VCs' portfolio composition.[18] Our data comprise details on each investment from each VC fund, which go beyond those available from VC associations.

The funds in our data almost invariably have the typical finite horizon of 10 years, with the option to continue for 2-3 years. The scope of the data is broadly similar to other venture capital datasets. For example, Lerner and Schoar (2005) examine a dataset consisting of 208 transactions by 23 private equity funds. Kaplan and Strömberg (2003) use a dataset of a similar size.

As discussed in Lerner and Schoar (2005), it is difficult to ascertain the representativeness of the data in view of the lack of completeness of any international private equity or venture capital dataset. However, it is important to note that the response rate to our survey was low. Specifically, three rounds of surveys were e-mailed to over 8,000 funds, indicating a response rate of about 0.5%. Because of the diverse characteristics of respondent funds and because the summary statistics discussed below are generally in line with other empirical VC studies, we do not think that our results are influenced by a self-selection bias. However, the low response rate precludes us from generalizing our results to the whole population of VC and private equity funds.

5.2 Summary statistics

We group the variables into three broad categories: (1) entrepreneurs' ownership percentage and VCs' portfolio sizes, (2) proxies for the model's parameters, and (3) control variables. Group (1) encompasses the endogenous variables in the model, and groups (2) - (3) contain the explanatory variables. Table 2 provides the definitions of the variables and basic summary statistics.[19]

The median number of entrepreneurial firms per VC fund (NUM_FIRMS) is 9.5, and the mean is 15.9. The profit share of entrepreneurs (ENT_SHARE) typically varies over the life of the investment, subject to entrepreneurs' performance

[18] We are aware of only one empirical study of VC portfolio sizes (Cumming, 2004), and that study is based on a Canadian-only sample. The Canadian VC market is dominated by government-sponsored funds, which could distort the results of examining VCs' portfolio structures.

[19] To conserve space, we do not report the correlation matrix among the variables used in the analysis. In general, the correlations among most explanatory variables are relatively low in absolute value, which reduces the likelihood of the results being influenced by multicollinearity.

(see, for example Kaplan and Strömberg (2003)). In gathering the data, we therefore asked VC fund managers to indicate the typical ownership percentage held in their investee companies. The median entrepreneurs' ownership percentage is 80% in our sample, and the mean is 70%. These numbers are somewhat higher than the typical ownership percentage reported by Kaplan and Strömberg (2003), whose study indicates that U.S. VCs typically hold about 50% equity in the investee companies. For U.S. VCs in our sample, the average entrepreneurs' ownership is 68%, and the median is 80%.

We use the mean number of years of post-high school education of entrepreneurs in a VC fund (ENT_EDU) as a proxy for entrepreneurs' quality, α. A proxy for entrepreneurs' disutility of effort, γ, is the average age of entrepreneurs (ENT_AGE). We proxy for VC fund managers' quality, β, by the mean number of years of the managers' post-high school education (MGR_EDU).[20] VC's disutility (cost) of effort, δ, is proxied by the average age of the managers (MGR_AGE). The average required investment in a project, I, is proxied by the average capital invested in a venture (CAP_INV) as of June 2004. The expected return in case the project is successful, R, is proxied by the VCs' perception of the proportion of projects with expected IRR above 100% (IRR_100).

Our control variables are as follows. First, VCs' portfolio size is expected to be positively related to the number of managers employed by the fund. VC funds employed a median of 5 fund managers (FND_MGR).[21] Second, we expect the number of firms in a VC fund to be positively related to the total capital raised by the fund (CAP_RAISED). Third, the number of firms is expected to be positively related to the total investment duration from the date of first investment to June 2004 (DUR). Fourth, we expect it to be affected by the amount of government guarantees for failed investments ($CGOVT$), both because of a potential negative effect of government guarantees on VCs' effort levels and because of the potentially different maximization function of a fund.[22] We expect the profit sharing rule to be affected by

[20] We also use an alternative proxy based on the mean number of years of managers' work experience and obtain results similar to those reported.

[21] Since the number of managers employed by a VC fund can be considered an endogenous variable, we repeat the analysis while excluding this variable from the regressions. Omitting the number of managers does not affect the qualitative results.

[22] See, for example, Keuschnigg (2004), Keuschnigg and Nielsen (2003), and Lerner (1999). None

whether a fund is in the early stage ($EARLY$). About 22% of firms in our sample are in the early stage. We also expect entrepreneurs' profit shares to be negatively related to the number of financing rounds ($FINANCE$). The VC fund managers' assessment of the average risk of their entrepreneurial ventures (AVG_RISK), measured on a scale of 1 to 10, is also expected to affect profit sharing. In addition, we expect the proportion of firms in which a VC is the lead investor ($PERCENT\_LEAD$) to be positively (negatively) associated with VC's (entrepreneurs') profit shares. Finally, since our data covers from different countries, we control for the country's institutional environment using a "legality index" ($LEGALITY$). The legality index, based on Berkowitz, Pistor, and Richard (2003), reflects the following factors: civil versus common law systems, efficiency of judicial system, rule of law, corruption, risk of expropriation, risk of contract repudiation, and shareholder rights.[23]

5.3 Empirical tests

Our model suggests that the relation between the portfolio size and various VCs', entrepreneurs', and projects' characteristics should be examined while accounting for the endogeneity of the profit sharing rule. Thus, we estimate regressions of VCs' portfolio sizes while instrumenting for entrepreneurs' profit shares by their predicted values from first-stage regressions, and estimate regressions of entrepreneurs' profit shares, instrumenting for VCs' portfolio sizes by their predicted values from first-stage regressions.[24] Table 3 presents the results of the second-stage regressions.

The model predicts a non-monotonic relation between entrepreneurs' profit shares, x, and the optimal number of portfolio firms, n^*. n^* is expected to be increasing in x when x is low, while it is expected to be decreasing in x when it is high. We rely on two alternative specifications to model this non-monotonicity.

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of the funds in our sample are "pure" government funds with 100% government support; however, 11 funds did receive some capital from government bodies, and 2 of the funds received more than 50% of their capital from government sources.

[23]We also considered GNP per capita and country dummy variables as alternative controls in our multivariate tests. However, GNP per capita is highly correlated with $LEGALITY$ (the correlation coefficient is 0.85), and including country dummy variables does not affect any of the results.

[24]The two first-stage regressions include the variables that are expected to affect portfolio size and profit sharing, discussed above. The results of the first-stage regressions are available upon request.

In column 1 of Table 3, we model the non-monotonicity of the relation between n^* and x by regressing the number of firms in a VC fund (NUM_FIRMS) on the predicted first-stage entrepreneurs' ownership percentage ($INST\_ENT\_SHARE$) and on the squared predicted entrepreneurs' ownership percentage ($INST\_ENT\_SHARE^2$). The coefficient on entrepreneurs' ownership percentage is positive and significant. The coefficient on the squared ownership percentage is negative and marginally significant, indicating that n is increasing in x when it is low, while it is decreasing in x when x becomes high.

The other predictions of the model are partially supported by the data. Consistent with the model, the number of years of entrepreneurs' education, proxying for their quality, is positively and marginally significantly related to the number of portfolio firms. Managers' quality, as proxied by the number of years of their education is significantly positively related to the portfolio size. Although the model predicts no relation between these two variables, this is likely to be an artifact of the specific functional forms in the model. Consistent with the model, the coefficient on managers' average age, proxying for their disutility of effort, is negative and significant at the 10% level. The average capital investment in a venture is negatively and significantly related to n^*.

The coefficients on control variables are mostly insignificant. This is not caused by multicollinearity, since the (unreported) correlations among the independent variables are small and, in most cases, insignificant. However, some of the variables exhibit very small variation. The legality index is a good example: the vast majority of the observations come from countries with developed legal systems, leading to almost no variation in $LEGALITY$. The exception is the amount of capital raised. Consistent with managers being pressured to invest funds that have been raised, n^* is positively associated with CAP_RAISED.

In column 2, we model the non-monotonicity of the relation between n^* and x by constructing a variable equalling the absolute value of the deviation of the predicted first-stage value of x from a threshold, below which the effect of increasing x on n^* is expected to be positive, and above which this effect is expected to be negative. We use 80% as the threshold value in Panel 2 because the median entrepreneur's profit share in our sample is 80%.[25] Consistent with the model, the coefficient on

[25] Various thresholds, ranging from 50% to 80%, provide results that are qualitatively similar to

the absolute deviation from the threshold is negative and significant, indicating that when entrepreneurs' ownership percentage is low, increasing it (reducing the absolute deviation from a threshold) would increase the number of portfolio firms, while when x is large, increasing it (increasing the deviation from a threshold) would reduce n^*. Increasing the deviation of entrepreneurs' ownership percentage from the threshold by one percentage point reduces the number of portfolio firms by about 0.6 firms. The associations between n^* and the other variables are broadly consistent with those in column 1.

Column 3 presents the results of the second-stage regressions of entrepreneurs' profit shares on the predicted first-stage portfolio size. First, and most importantly, the entrepreneurs' ownership percentage is positively and marginally significantly associated with the instrument for n ($INST\_NUM\_FIRMS$). In addition, consistent with the model, managers quality, as proxied by years of their education is negatively related to x^*. The relations between entrepreneurs' shares in their ventures and proxies for the other parameters of the model and the control variables are generally insignificant.

Overall, the data provide some support for our model. The strongest support is provided for the non-monotonic relation between VCs' portfolio sizes and entrepreneurs' ownership shares. The weakness of some of the other results is possibly attributable to the scope of the available data and to our ability to simultaneously control for numerous factors with a limited number of observations. It is important to emphasize that because of the small sample size we view our empirical results as preliminary and illustrative as to how to match the model with the data.

6 Conclusions

This paper develops a theoretical model in which a venture capitalist (VC) simultaneously chooses the profit sharing rule between herself and entrepreneurs and the size of her portfolio of entrepreneurial ventures. Profit sharing and portfolio size are both central to the venture capital finance problem, as VCs write contracts to mitigate double-sided moral hazard problem and undertake a limited number of investments to add value to entrepreneurial ventures they invest in. Prior theoretical models of

those reported.

venture capital finance have either considered portfolio size or contracts in isolation. A joint analysis of contracts and portfolio size illustrates an important non-monotonic relationship between the optimal VC's portfolio size and the profit shares given to entrepreneurs.

In our model, a VC maximizes her expected portfolio value net of her effort costs, with respect to the number of projects she invests in and to the share of the projects' expected values that she gives to entrepreneurs. The portfolio structure affects the unobservable effort levels of the entrepreneurs and the VC, thus influencing the value of VC's portfolio.

Our analysis generates predictions for the partial effects of various characteristics of VCs, entrepreneurs and projects on VC's portfolio size and profit sharing when both are chosen endogenously. The optimal VC's portfolio size is predicted to be positively related to the quality of entrepreneurs and to the value of a successful project, and to be negatively related to the disutilities that the VC and the entrepreneurs have from exerting effort, and to the required initial investment in the projects. The relation between the optimal portfolio size and the profit sharing rule is predicted to be non-monotonic: the optimal number of firms is first increasing and then is decreasing in the share of the profits retained by entrepreneurs

Furthermore, our analysis shows that when both the VC's portfolio size and the profit share given to entrepreneurs are determined endogenously, one can not make unambiguous predictions regarding the reduced-form relations between the optimal portfolio size and the model's parameters. Therefore, we conclude that the empirical analysis of VCs' portfolio size must be performed while accounting for the endogeneity of the profit sharing rule.

We test the predictions of the model using data collected through a survey of venture capital and private equity funds in Europe and North America. Our sample includes 42 VC funds. Using instrumental variables we find that a VC's portfolio size varies non-monotonically with the profit share retained by entrepreneurs. Another finding that is consistent with the model is that entrepreneurs' profit shares are increasing in the portfolio size. In addition, the relations among VCs' portfolio sizes and entrepreneurs' profit shares and proxies for various model's parameters are generally consistent with the model.

Given the small size of our sample of VC funds, we view the empirical results reported here as preliminary and believe that further research is warranted. In particular, although the model clearly has limitations, we think that the most interesting and potentially rewarding area of research is a further empirical examination of the determinants of venture capitalists' portfolio structures using larger and more representative datasets of VC funds. Such tests will greatly improve our understanding of the choices venture capitalists make.

A Appendix

Proof of Lemma 1

Without loss of generality, we restrict our attention to two entrepreneurs, i and j. $x_i = x_j = x$ by assumption. Assume first that VC chooses to devote identical efforts to both ventures: $E_i = E_j = E$. Then, the F.O.C.'s for entrepreneurs i's and j's effort level choices in (3) may be rewritten as

$$xp_{i_{e_i}}(e_i, E, \alpha, \beta)R - K_{i_{e_i}}(e_i, \delta) = 0, \tag{A.1}$$

$$xp_{j_{e_j}}(e_j, E, \alpha, \beta)R - K_{j_{e_j}}(e_j, \delta) = 0. \tag{A.2}$$

Assume $e_i = e^*$ solves (A.1). Then, since $p_{i_{e_i}}(e^*, E, \alpha, \beta) = p_{j_{e_j}}(e^*, E, \alpha, \beta)$ and $K_{i_{e_i}}(e^*, \delta) = K_{j_{e_j}}(e^*, \delta)$, $e_j = e^*$ also solves (A.2). Therefore, $e_i^*(E) = e_j^*(E)$, implying that if the VC devotes the same effort level to each project, then the optimal effort levels of entrepreneurs are identical.

Now, we relax the assumption of VC devoting the same effort to each venture, and assume instead that the two entrepreneurs choose identical effort levels: $e_i = e_j = e$. Then, the two F.O.C.'s of VC in (4) may be rewritten as

$$[1-x]\, p_{i_{E_i}}(e, E_i, \alpha, \beta)R - L_{i_{E_i}}(E_i, E_j, n, \delta) = 0, \tag{A.3}$$

$$[1-x]\, p_{i_{E_j}}(e, E_j, \alpha, \beta)R - L_{i_{E_j}}(E_i, E_j, n, \delta) = 0. \tag{A.4}$$

Assume that $E_i = E^*$ solves (A.3). Then, similar to the arguments above, $E_j = E^*$ also solves (A.4). Thus, $E_i^*(e) = E_j^*(e)$, implying that if the two entrepreneurs choose identical effort levels, then the VC chooses to devote identical efforts to the two ventures. Given the symmetric equilibrium above, the F.O.C.'s of the entrepreneurs and the VC in (3) and (4) respectively may be rewritten as

$$xp_e(e, E, \alpha, \beta)R - K_e(e, \delta) = 0, \tag{A.5}$$

and

$$[1-x]\, p_E(e, E, \alpha, \beta)R - L_E(E, n, \delta) = 0. \tag{A.6}$$

Totally differentiating the F.O.C. in (A.5) with respect to E results in

$$\frac{de^*(E)}{dE} = -\frac{xp_{i_{eE}}R}{xp_{i_{ee}}R - K_{i_{ee}}}.$$

Given the assumptions on the derivatives, this implies that $\frac{de^*(E)}{dE} \geq 0$. Similarly, differentiating the F.O.C. in (A.6) with respect to e results in $\frac{dE^*(e)}{de} \geq 0$. In addition, it follows from the assumptions that $e^*(E) > 0$ when $E \to 0$. Also, $p_{i_{ee}} + p_{i_{eE}} < 0$ implies $\frac{de^*(E)}{dE} < 1$. Similarly, $E^*(e) > 0$ when $e \to 0$, and $\frac{dE^*(e)}{de} < 1$. Therefore, the reaction functions $e^*(E)$ and $E^*(e)$ intersect at least once, and there exists a symmetric equilibrium in effort levels, in which the two equations

$$e^* = \arg\max_e U_{en}(e, E^*, \alpha, \beta, \delta),$$
$$E^* = \arg\max_E U_{vc}(e^*, E, \alpha, \beta, \gamma)$$

hold simultaneously. ■

Proof of Proposition 1

Totally differentiating the F.O.C.'s in (5) and (6) with respect to n, while holding x constant, gives the following system of two equations:

$$p_{ee}xR\frac{de^*(E)}{dn} + p_{eE}xR\frac{dE^*(e)}{dn} - K_{ee}\frac{de^*(E)}{dn} = 0, \tag{A.7}$$
$$p_{EE}[1-x]R\frac{dE^*(e)}{dn} + p_{eE}[1-x]R\frac{de^*(E)}{dn} - L_{En} - L_{EE}\frac{dE^*(e)}{dn} = 0. \tag{A.8}$$

Solving the system of (A.7) and (A.8), while substituting in the F.O.C.'s in (5) and (6) results in

$$\frac{dE^*}{dn} = -\frac{[K_{ee} - p_{ee}xR]L_{En}}{\Gamma},$$
$$\frac{de^*}{dn} = -\frac{p_{eE}xRL_{En}}{\Gamma},$$

where

$$\Gamma = [L_{EE} - p_{EE}[1-x]R][K_{ee} - p_{ee}xR] - [1-x]xR^2p_{eE}^2. \tag{A.9}$$

Given the assumptions on derivatives, it is straightforward to show that $\Gamma > 0$ and that $\frac{dE^*}{dn} < 0$. If there are no complementarities ($p_{eE} = 0$), then $\frac{de^*}{dn} = 0$, while, if $p_{eE} > 0$, then $\frac{de^*}{dn} < 0$. ■

Proof of Proposition 2

Totally differentiating the F.O.C.'s in (5) and (6) with respect to x, while holding n constant, provides the following system of equations:

$$p_e R + p_{ee} x R \frac{de^*(E)}{dx} + p_{eE} x R \frac{dE^*(e)}{dx} - K_{ee} \frac{de^*(E)}{dx} = 0, \tag{A.10}$$

$$-p_E R + p_{EE}[1-x] R \frac{dE^*(e)}{dx} + p_{eE}[1-x] R \frac{de^*(E)}{dx} - L_{EE} \frac{dE^*(e)}{dx} = 0. \tag{A.11}$$

Solving the system in (A.10) and (A.11), while substituting in the F.O.C.'s in (5) and (6) gives

$$\frac{dE^*}{dx} = \frac{\Delta}{\Gamma},$$
$$\frac{de^*}{dx} = \frac{\Theta}{\Gamma},$$

where Γ is defined in (A.9) and was shown to be positive, and

$$\Delta = R[K_e p_{eE} \frac{[1-x]}{x} - p_E[K_{ee} - p_{ee} x R]],$$
$$\Theta = -R[L_E p_{eE} \frac{x}{[1-x]} - p_e[L_{EE} - p_{EE}[1-x]R]].$$

If there is no complementarities between the effort of the entrepreneur and that of the VC ($p_{eE} = 0$), then, given the assumptions on derivatives, it is easy to show that Θ is strictly positive, while Δ is strictly negative, which proves the first part of this proposition.

If there are complementarities ($p_{eE} > 0$), using the assumptions on derivatives, it is straightforward to show that $\lim_{x\to 0} \Delta = \infty$, $\lim_{x\to 0} \Theta = R p_e [L_{EE} - p_{EE} R] > 0$, $\lim_{x\to 1} \Delta = -R p_E [K_{ee} - p_{ee} R] < 0$, $\lim_{x\to 1} \Theta = -\infty$. It follows that $\lim_{x\to 0} \frac{dE^*}{dx} > 0$, $\lim_{x\to 1} \frac{dE^*}{dx} < 0$, $\lim_{x\to 0} \frac{de^*}{dx} > 0$, $\lim_{x\to 1} \frac{de^*}{dx} < 0$, which proves the second part of the proposition. ■

Proof of Proposition 3

Totally differentiating the F.O.C.'s in (5) and (6) with respect to α, while holding n and x constant, yields the following system of equations:

$$p_{ee} x R \frac{de^*(E)}{d\alpha} + p_{eE} x R \frac{dE^*(e)}{d\alpha} + p_{e\alpha} x R - K_{ee} \frac{de^*(E)}{d\alpha} = 0, \tag{A.12}$$

$$p_{EE}[1-x] R \frac{dE^*(e)}{d\alpha} + p_{eE}[1-x] R \frac{de^*(E)}{d\alpha} + p_{E\alpha}[1-x] R - L_{EE} \frac{dE^*(e)}{d\alpha} = 0. \tag{A.13}$$

Solving the system in (A.12) and (A.13), while substituting in the F.O.C.'s in (5) and (6) gives

$$\frac{dE^*}{d\alpha} = \frac{[1-x]R}{\Gamma}[p_{E\alpha}[K_{ee} - p_{ee}xR] + p_{e\alpha}p_{eE}xR],$$
$$\frac{de^*}{d\alpha} = \frac{xR}{\Gamma}[p_{e\alpha}[L_{EE} - p_{EE}[1-x]R] + p_{E\alpha}p_{eE}[1-x]R],$$

where Γ is defined in (A.9) and was shown to be positive. Using the assumptions on derivatives, it is straightforward to show that, for any α, $\frac{de^*}{d\alpha} > 0$. In addition, if efforts are complementary, $p_{eE} > 0$, then $\frac{dE^*}{d\alpha} > 0$, otherwise $\frac{dE^*}{d\alpha} = 0$.

By following the same approach, it can be easily shown that the derivatives of E^* and e^* with respect to β, γ, δ, and R are equal to the following expressions:

$$\frac{dE^*}{d\beta} = \frac{(1-x)R}{\Gamma}[p_{E\beta}[K_{ee} - p_{ee}xR] + p_{e\beta}p_{eE}xR],$$
$$\frac{de^*}{d\beta} = \frac{xR}{\Gamma}[p_{e\beta}[L_{EE} - p_{EE}[1-x]R] + p_{E\beta}p_{eE}[1-x]R],$$
$$\frac{dE^*}{d\gamma} = -\frac{p_{eE}[1-x]R}{\Gamma}K_{e\gamma},$$
$$\frac{de^*}{d\gamma} = -\frac{[L_{EE} - p_{EE}[1-x]R]}{\Gamma}K_{e\gamma},$$
$$\frac{dE^*}{d\delta} = -\frac{[K_{ee} - p_{ee}xR]}{\Gamma}L_{E\delta}$$
$$\frac{de^*}{d\delta} = -\frac{p_{eE}xR}{\Gamma}L_{E\delta},$$
$$\frac{dE^*}{dR} = \frac{[1-x]}{\Gamma}[p_E[K_{ee} - p_{ee}xR] + p_e p_{eE}xR],$$
$$\frac{de^*}{dR} = \frac{x}{\Gamma}[p_e[L_{EE} - p_{EE}[1-x]R] + p_E p_{eE}[1-x]R].$$

Using the assumptions on derivatives, it is easily shown that $\frac{dE^*}{d\beta} > 0$, $\frac{de^*}{d\beta} > 0$ if $p_{eE} > 0$ and $\frac{de^*}{d\beta} = 0$ if $p_{eE} = 0$, $\frac{dE^*}{d\gamma} < 0$ if $p_{eE} > 0$ and $\frac{dE^*}{d\gamma} = 0$ if $p_{eE} = 0$, $\frac{de^*}{d\gamma} < 0$, $\frac{dE^*}{d\delta} < 0$, $\frac{de^*}{d\delta} < 0$ if $p_{eE} > 0$ and $\frac{de^*}{d\delta} = 0$ if $p_{eE} = 0$, $\frac{dE^*}{dR} > 0$, $\frac{de^*}{dR} > 0$. In addition, $\frac{dE^*}{dI} = 0$, $\frac{de^*}{dI} = 0$. This concludes the proof. ■

Proof of Lemma 2

Without loss of generality, we focus on two entrepreneurs, i and j. It was shown in Lemma 1 that $x_i = x_j = x$ is consistent with $e_i^*(x) = e_j^*(x)$ and $E_i^*(x) = E_j^*(x)$. We now show that, given the best responses, $e^*(E^*, x)$ and $E^*(e^*, x)$, obtained in the second stage of the game, $x_i = x_j = x^*$ is indeed a symmetric equilibrium. Given the best responses e_i^* and e_j^*, maximizing U_{VC} with respect to x_i and x_j, yields the following 2 F.O.C.'s:

$$p_{i_E}[1-x_i]R\frac{dE_i^*(e_i)}{dx_i} + p_{i_{e_i}}[1-x_i]R\frac{de_i^*(E_i)}{dx_i} - L_{E_i} - p_i R = 0, \tag{A.14}$$

$$p_{j_E}[1-x_j]R\frac{dE_j^*(e_j)}{dx_j} + p_{j_{e_j}}[1-x_j]R\frac{de_j^*(E_j)}{dx_j} - L_{E_j} - p_j R = 0. \tag{A.15}$$

Because of the symmetry in effort levels, if x^* solves (A.14), then it must solve (A.15) as well, which proves the claim. ■

Proof of Proposition 4

Differentiating $n^*(x,\Phi)$ in (15) with respect to x results in

$$\frac{\partial n^*(x,\Phi)}{\partial x} = \frac{\alpha^2 R^2 [1-2x]}{4\gamma\delta\overline{p}^2}.$$

For $x < \frac{1}{2}$, $\frac{\partial n^*(x,\Phi)}{\partial x} > 0$, and for $x > \frac{1}{2}$, $\frac{\partial n^*(x,\Phi)}{\partial x} < 0$.

Moreover, partially differentiating $n^*(x,\Phi)$ in (15) with respect to α, β, γ, δ, R, and I respectively gives

$$\frac{\partial n^*(x,\Phi)}{\partial \alpha} = \frac{\alpha R^2 x[1-x]}{2\gamma\delta\overline{p}^2} > 0,$$

$$\frac{\partial n^*(x,\Phi)}{\partial \beta} = 0,$$

$$\frac{\partial n^*(x,\Phi)}{\partial \gamma} = -\frac{\alpha^2 R^2 x[1-x]}{4\gamma^2\delta\overline{p}^2} < 0,$$

$$\frac{\partial n^*(x,\Phi)}{\partial \delta} = -\frac{1}{\delta}\left[\frac{1}{4}\frac{[1-x]\,xR^2\alpha^2}{\gamma\delta\overline{p}^2} - \frac{1}{2}\frac{I}{\delta}\right] = -\frac{1}{\delta}n^*(x,\Phi) < 0,$$

$$\frac{\partial n^*(x,\Phi)}{\partial R} = \frac{\alpha^2 Rx[1-x]}{2\gamma\delta\overline{p}^2} > 0,$$

$$\frac{\partial n^*(x,\Phi)}{\partial I} = -\frac{1}{2\delta} < 0.$$

This concludes the proof. ■

Proof of Proposition 5

Partially differentiating the expression for $x^*(n,\Phi)$ in (16) with respect to n results in

$$\frac{\partial x^*(n,\Phi)}{\partial n} = \frac{\alpha^2\beta^2\gamma\delta}{\left[2\alpha^2\delta n - \beta^2\gamma\right]^2} > 0.$$

Furthermore, partially differentiating $x^*(n,\Phi)$ in (16) with respect to α, β, γ, and δ gives

$$\frac{\partial x^*(n,\Phi)}{\partial\alpha} = \frac{2\alpha\beta^2\gamma\delta n}{\left[2\alpha^2\delta n-\beta^2\gamma\right]^2} > 0,$$
$$\frac{\partial x^*(n,\Phi)}{\partial\beta} = -\frac{2\alpha^2\beta\gamma\delta n}{\left[2\alpha^2\delta n-\beta^2\gamma\right]^2} < 0,$$
$$\frac{\partial x^*(n,\Phi)}{\partial\gamma} = -\frac{\alpha^2\beta^2\delta n}{\left[2\alpha^2\delta n-\beta^2\gamma\right]^2} < 0,$$
$$\frac{\partial x^*(n,\Phi)}{\partial\delta} = \frac{\alpha^2\beta^2\gamma n}{\left[2\alpha^2\delta n-\beta^2\gamma\right]^2} > 0.$$

Because $x^*(n,\Phi)$ does not depend on R and I, its derivative with respect to these variables is equal to zero. ■

Proof of Proposition 6

Using the envelope theorem, we can write

$$\frac{dx^*(\Phi)}{d\alpha} = \frac{\partial x^*(n,\Phi)}{\partial\alpha} + \frac{\partial x^*(n,\Phi)}{\partial n}\frac{\partial n^*(x,\Phi)}{\partial\alpha}, \tag{A.16}$$

and similarly for the case of other parameters. Using the relations derived in Propositions 4 and 5, it is straightforward to show that $\frac{dx^*(\Phi)}{d\alpha} > 0$, $\frac{dx^*(\Phi)}{d\beta} < 0$, $\frac{dx^*(\Phi)}{d\gamma} < 0$, $\frac{dx^*(\Phi)}{dR} > 0$, $\frac{dx^*(\Phi)}{dI} < 0$. In addition, using an expression analogous to (A.16), we can write that at n^*

$$\frac{dx^*(\Phi)}{d\delta} = \frac{\alpha^2\beta^2\gamma n^*}{\left[2\alpha^2\delta n^*-\beta^2\gamma\right]^2} + \frac{\alpha^2\beta^2\gamma\delta}{\left[2\alpha^2\delta n^*-\beta^2\gamma\right]^2}\left[-\frac{1}{\delta}n^*\right] = 0.$$

This concludes the proof. ■

Proof of Proposition 7

Using the envelope theorem and the results in Propositions 4 and 5, we can write

$$\frac{dn^*(\Phi)}{d\alpha} = \frac{\alpha R^2 x\left[1-x\right]}{2\gamma\delta\overline{p}^2} + \frac{\alpha^2 R^2\left[1-2x\right]}{4\gamma\delta\overline{p}^2}\frac{2\alpha\beta^2\gamma\delta n}{\left[2\alpha^2\delta n-\beta^2\gamma\right]^2}. \tag{A.17}$$

For $x\to 0$, (A.17) is positive. For $x\to 1$, (A.17) is negative. Similarly, for $x\to 0$, $\frac{dn^*(\Phi)}{d\beta} < 0$, $\frac{dn^*(\Phi)}{d\gamma} < 0$, $\frac{dn^*(\Phi)}{d\delta} > 0$, while for $x\to 1$, $\frac{dn^*(\Phi)}{d\beta} > 0$, $\frac{dn^*(\Phi)}{d\gamma} > 0$,

$\frac{dn^*(\Phi)}{d\delta} < 0$. Thus, the relations between $n^*(\Phi)$ and α, β, γ, and δ can not be signed unambiguously. For the cases of R and I, we use Proposition 4 to write

$$\frac{dn^*(\Phi)}{dR} = \frac{\partial n^*(x,\Phi)}{\partial R} > 0,$$
$$\frac{dn^*(\Phi)}{dI} = \frac{\partial n^*(x,\Phi)}{\partial I} < 0.$$

This concludes the proof. ■

References

Barclay, Michael, Leslie Marx and Clifford Smith, 2003, The joint determination of leverage and maturity, Journal of Corporate Finance 9, 149-167.

Bascha, Andreas and Uwe Walz, 2001, Convertible securities and optimal exit decisions in venture capital finance, Journal of Corporate Finance 7, 285-306.

Bergemann, Dirk and Ulrich Hege, 1998, Venture capital financing, moral hazard, and learning, Journal of Banking and Finance 22, 703-735.

Berkowitz, Daniel, Katharina Pistor and Jean-Francois Richard, 2003, Economic development, legality, and the transplant effect, European Economic Review 47, 165-195.

Bhattacharyya, Sugato and Francine Lafontaine, 1995, Double-sided moral hazard and the nature of share contracts, RAND Journal of Economics 26, 761-781.

Casamatta, Catherine, 2003, Financing and advising: optimal financial contracts with venture capitalists, Journal of Finance 58, 2059-2086.

Casamatta, Catherine and Carole Haritchabalet, 2004, Learning and syndication in venture capital investments, University of Toulouse working paper.

Cassiman, Bruno and Masako Ueda, 2006, Optimal project rejection and new firm start-ups, Management Science 52, 262-275.

Cestone, Giacinta and Lucy White, 2003, Anti-competitive financial contracting: the design of financial claims, Journal of Finance 58, 2109-2142.

Cornelli, Francesca and Oved Yosha, 2003, Stage financing and the role of convertible debt, Review of Economic Studies 70, 1-32.

Cumming, Douglas, 2006, The determinants of venture capital portfolio size: empirical evidence, Journal of Business 79, 1083-1126.

Dixit, Avinash, 1984, Comparative statics for oligopoly, International Economic Review 27, 107-122.

Fulghieri, Paolo and Merih Sevilir, 2004, Size and focus of a venture capitalist's portfolio, University of North Carolina working paper.

Garmaise, Mark, 2001, Informed investors and the financing of entrepreneurial projects, University of Chicago working paper.

Gompers, Paul, 1995, Optimal investment, monitoring, and the staging of venture capital, Journal of Finance 50, 1461-1489.

Gompers, Paul and Josh Lerner, 1999a, An analysis of compensation in the U.S. venture capital partnership, Journal of Financial Economics 51, 3-44.

Gompers, Paul and Josh Lerner, 1999b, The venture capital cycle, MIT Press.

Gorman, Michael and William Sahlman, 1989, What do venture capitalists do?, Journal of Business Venturing 4, 231-248.

Hege Ulrich, Frederic Palomino and Armin Schwienbacher, 2003, Determinants of venture capital performance: Europe and the United States, HEC School of Management working paper.

Hellmann, Thomas, 2000, A theory of strategic venture capital investing, Journal of Financial Economics 64, 285-314.

Houben, Eike, 2002, Venture capital, double-sided adverse selection and double-sided moral hazard, University of Kiel working paper.

Kanniainen, Vesa and Christian Keuschnigg, 2003, The optimal portfolio of start-up firms in venture capital finance, Journal of Corporate Finance 9, 521-534.

Kanniainen, Vesa and Christian Keuschnigg, 2004, Start-up investment with scarce venture capital support, Journal of Banking and Finance 28, 1935-1959.

Kaplan, Steven and Per Strömberg, 2003, Financial contracting theory meets the real world: an empirical analysis of venture capital contracts, Review of Economic Studies 70, 281-315.

Kaplan, Steven and Per Strömberg, 2004, Contracts, characteristics, and actions: evidence from venture capitalist analysis, Journal of Finance 59, 2173-2206.

Keuschnigg, Christian, 2004, Taxation of a venture capitalist with a portfolio of firms, Oxford Economic Papers 56, 285-306.

Keuschnigg, Christian and Soren Nielsen, 2003, Tax policy, venture capital and entrepreneurship, Journal of Public Economics 87, 175-203.

Lerner, Josh, 1999, The government as venture capitalist: the long-run effects of the SBIR program, Journal of Business 72, 285-318.

Lerner, Josh and Antoinette Schoar, 2005, Does legal enforcement affect financial transactions?: The contractual channel in private equity, Quarterly Journal of Economics 120, 223-246.

Marx, Leslie, 1998, Efficient venture capital financing combining debt and equity, Review of Economic Design 3, 371-187.

Mayer, Colin, Koen Schoors and Yishay Yafeh, 2005, Sources of funds and investment strategies of venture capital funds: evidence from Germany, Israel, Japan and the U.K., Journal of Corporate Finance 11, 586-608.

Repullo, Rafael and Javier Suarez, 2004, Venture capital finance: a security design approach, Review of Finance 8, 75-108.

Sahlman, William, 1990, The structure and governance of venture-capital organizations, Journal of Financial Economics 27, 473-521.

Schmidt, Klaus, 2003, Convertible securities and venture capital finance, Journal of Finance 58, 1139-1166.

Ueda, Masako, 2004, Banks versus venture capital: project evaluation, screening, and expropriation, Journal of Finance 59, 601-621.

Vives, Xavier, 2000, Oligopoly pricing - old ideas and new tools, MIT Press.

Wright, Mike and Andy Lockett, 2003, The Structure and Management of Alliances: Syndication in the Venture Capital Industry, Journal of Management Studies 40, 2073-2104.

Table 1 – Empirical predictions for the system of equations

Columns 1 and 3 summarize the empirical predictions for the signs of the structural relations between the model's parameters and the equilibrium VC's portfolio size (n^*) and the equilibrium entrepreneurs' profit shares (x^*) respectively. For comparative purposes, columns 2 and 4 summarize the reduced-form relations between the model's parameters and n^* and x^* respectively. α and β are the quality parameters of entrepreneurs and the VC respectively. γ and δ are the disutility of effort parameters of entrepreneurs and the VC respectively. R is the value of a successful project, and I is the initial investment required in each project. "+" indicates a positive relation, "–" indicates a negative relation, "0" means that the model predicts no relation between an exogenous and an endogenous variable, and "?" means that the sign of the predicted relation is ambiguous.

	n*		***x****	
	Structural relation	**Reduced-form relation**	**Structural relation**	**Reduced-form relation**
α	+	?	+	+
β	0	?	-	-
γ	-	?	-	-
δ	-	?	+	0
R	+	+	0	+
I	-	-	0	-
x	+ for small x - for large x			
n			+	

Table 2 – Summary statistics

This table presents the summary statistics for the variables used in the empirical tests. *NUM_FIRMS* is the number of firms in a VC fund. *ENT_SHARE* is the typical entrepreneur's ownership percentage, *ENT_EDU* is the mean number of years of entrepreneurs' post-high school education, *ENT_AGE* is the mean entrepreneurs' age, *MGR_EDU* is the mean number of years of VC fund managers' education, *MGR_AGE* is the mean VC fund managers' age, *CAP_INV* is the mean investment in each venture (in $MM), *IRR_100* is the percentage of projects that are expected by a VC to generate IRR above 100%. *FND_MGR* is the number of VC fund managers, *CAP_RAISED* is the total funds raised by a fund (in $MM), *DUR* is the duration of a fund, *CGOVT* is the percentage of government guarantees for failed ventures, *EARLY* is the percentage of firms in the early stage, *FINANCE* is the number of financing rounds, *AVG_RISK* is a subjective VC's assessment of the risk of a typical venture in his portfolio, *PERCENT_LEAD* is the percentage of firms in which a VC is a lead investor, and *LEGALITY* is an index of a country's legal conditions, based on Berkowitz, Pistor and Richard (2003).

Variable	Model's parameter	Mean	Median	Min	Max	Std Dev
Portfolio size and enterpreneurs' ownership percentage						
NUM_FIRMS	***n***	15.90	9.5	1	85	16.89
ENT_SHARE	***x***	70.26	80	0	97.5	24.02
Proxies for the model's parameters						
ENT_EDU	α	9.74	8.75	2.5	23	4.44
ENT_AGE	γ	44.08	45	33	50	3.92
MGR_EDU	β	7.02	6	4	12	2.04
MGR_AGE	δ	42.45	43	33	55	4.9
CAP_INV	***I***	11.79	4.10	0.22	103.80	20.35
IRR_100	***R***	12.08	12	0	38.10	10.26
Control variables						
FND_MGR		6.64	5	1	17	4.96
CAP_RAISED		264.90	101.50	11	3100	496.60
DUR		5.08	3.46	1	33.94	5.93
CGOVT		5.32	0	0	70	14.78
EARLY		22.26%	20.00%	0.00%	100.00%	26.28%
FINANCE		9.36	12	4	20	3.88
AVG_RISK		5.82	6	2	10	1.76
PERCENT_LEAD		58.36%	58.00%	0.00%	100.00%	28.43%
LEGALITY		20.31	20.85	14	21.91	1.50

Table 3 – Regressions of portfolio sizes and profit shares

Columns 1 and 2 present regressions of the number of portfolio firms, *NUM_FIRMS*, on non-linear functions of predicted first-stage values of entrepreneurs' ownership percentages, proxies for the model's parameters, and control variables, discussed in Table 2. In column 1 the regressions include predicted first-stage values of entrepreneurs' ownership percentages, *INST_ENT_SHARE*, and squared predicted values of ownership percentages, $INST\_ENT\_SHARE^2$. In column 2 the regressions include the absolute deviations of entrepreneurs' ownership percentage from 80% (|*INST_ENT_SHARE-80%*|). Column 3 presents a regression of entrepreneurs' ownership percentage, *ENT_SHARE*, on the predicted first-stage number of portfolio firms, *INST_NUM_FIRMS*, proxies for the model's parameters, and control variables, discussed in Table 2. T-statistics are presented in parentheses.

	Model's parameter	Dependent variable *NUM_FIRMS*	*NUM_FIRMS*	*ENT_SHARE*
Intercept		-75.245 (-1.28)	-28.349 (-0.87)	187.430 (2.54)
INST_NUM_FIRMS	***n***			0.582 (1.78)
INST_ENT_SHARE	***x***	2.042 (2.06)		
INST_ENT_SHARE^2	x^2	-0.016 (-1.61)		
|INST_ENT_SHARE-80%|	***|x-80%|***		-0.612 (-2.15)	
ENT_EDU	α	0.304 (1.92)	0.315 (1.94)	0.345 (0.66)
ENT_AGE	γ	-0.471 (-0.84)	-0.453 (-0.82)	-1.266 (-1.34)
MGR_EDU	β	0.288 (2.32)	0.301 (2.33)	-0.201 (-2.12)
MGR_AGE	δ	-0.650 (-1.77)	-0.632 (-1.72)	1.671 (1.59)
CAP_INV	***I***	-0.558 (-2.95)	-0.571 (-3.01)	0.862 (1.12)
IRR_100	***R***	0.162 (0.82)	0.169 (0.83)	0.152 (0.14)
FND_MGR		-0.201 (-0.22)	-0.177 (-0.20)	
CAP_RAISED		0.020 (2.55)	0.019 (2.45)	
DUR		0.652 (1.57)	0.666 (1.60)	
CGOVT		0.014 (0.34)	0.007 (0.19)	
EARLY				0.049 (0.61)
FINANCE				-0.612 (-0.88)
AVG_RISK				3.214 (1.24)
PERCENT_LEAD				-23.613 (-1.73)
LEGALITY		0.970 (0.65)	0.894 (0.59)	-2.691 (-1.07)
Adjusted R^2		0.528	0.519	0.396

Figure 1 – The effects of a change in n on the best response functions and the resulting equilibrium effort levels

This figure presents the best response functions of the VC (entrepreneur) before an increase in n, $E_0^*(e)$ $(e_0^*(E))$, and after the increase in n, $E_1^*(e)$ $(e_1^*(E))$. E_0^* (e_0^*) and E_1^* (e_1^*) are the resulting VC's (entrepreneur's) equilibrium efforts before and after the change in n. Figure 1A describes the case of no complementarities, while Figure 1B depicts the case of complementary efforts. The best responses for the case of complementary efforts are depicted as linear functions for illustrative purposes only.

Figure 1A – No complementarities

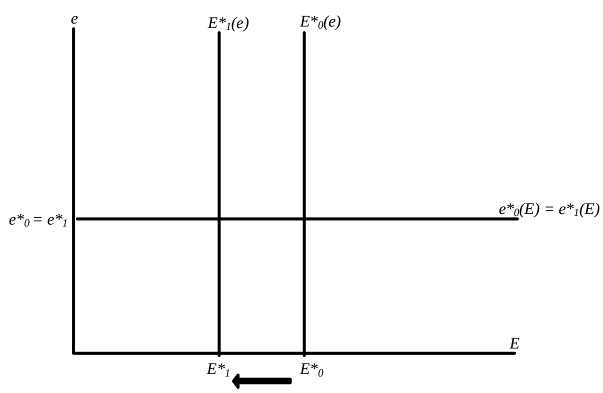

Figure 1B – Complementary efforts

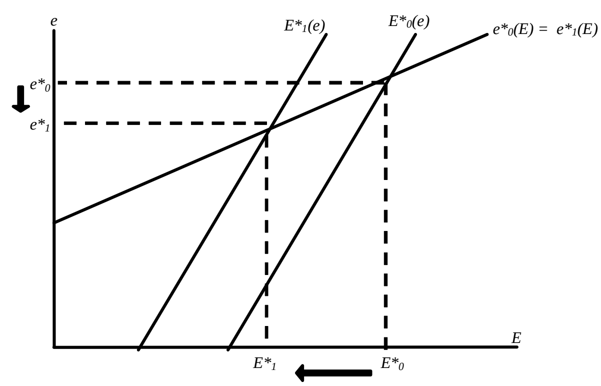

Figure 2 – The effects of a change in x on the best response functions and the resulting equilibrium effort levels

This figure presents the best response functions of the VC (entrepreneur) before an increase in x, $E_0^*(e)$ $(e_1^*(E))$, and after the increase in x, $E_1^*(e)$ $(e_0^*(E))$. E_0^* (e_0^*) and E_1^* (e_1^*) are the resulting VC's (entrepreneur's) equilibrium efforts before and after the change in x. Figure 2A depicts the case of no complementarities. Figure 2B presents the case of complementary efforts when $x \to 1$, while Figure 2C demonstrates the case of complementary efforts when $x \to 0$. The best responses for the case of complementary efforts are depicted as linear functions for illustrative purposes only.

Figure 2A – No complementarities

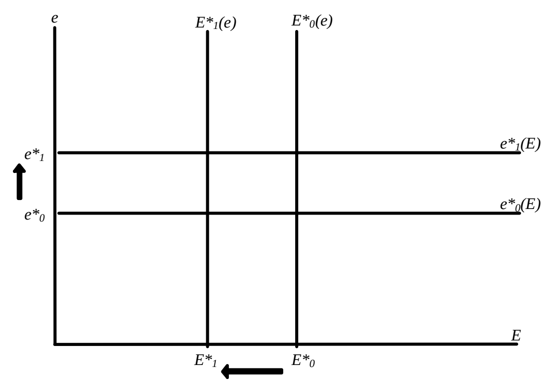

Figure 2B – Complementary efforts, $x \to 1$

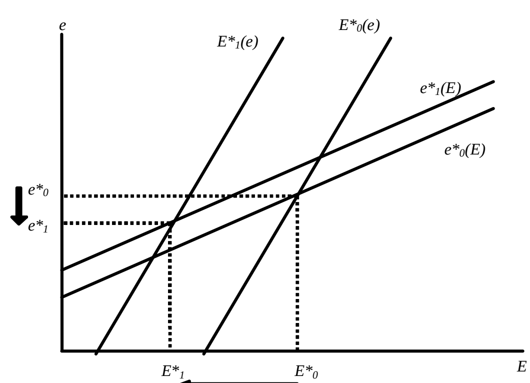

Figure 2C – Complementary efforts, $x \to 0$