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Social Networks among Auction Bidders: The Role of Key Bidders and Structural Properties on Auction Prices

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Abstract

Auctions have been studied extensively as an economic marketplace. The economist's focus is on modeling final sales prices, but the processes that give rise to those outcomes are rarely studied in great detail. This research is intended to provide that complementary perspective. We show how the interactions between bidders in an auction unfold in a dynamic pattern of bids and counter-bids, and thereby over the duration of an auction, create a network structure. The auction network contributes significantly to models of price dynamics and the network predicts final sales prices better than economic (non-network) indicators alone. In addition, network analyses are useful in identifying the key bidders whose actions seem to exert disproportionate influence on other bidders and the final sales prices. Furthermore, the key bidders may be identified very early in an auction process, which has practical implications for the auction house managers and for other bidders.

Keywords: online auctions, dynamic pricing, bidders, networks

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Auctions are an important means of selling many types of products, from antiques to real estate to thoroughbreds. They have served as an important vehicle for commerce for hundreds of years. Sotheby's and Christie's were founded in 1744 and 1766, respectively, and they continue to thrive today, with annual sales exceeding five billion dollars each. Tokyo's famed fish market was founded in 1923, and its annual sales also exceed five billion dollars.

Newer and larger still is eBay, established in 1995, with annual revenue around $12 billion dollars. Online auctions like eBay are important because of their size, but they interest researchers particularly because they provide rich sources of relatively novel data.

In the short time since eBay's founding, the auctions literature has also grown as researchers pose numerous questions to examine this means of market exchange. The literature to date has focused on studying aspects of auction design to achieve maximum final sales prices. For example, economic theories suggest that the auctions that attract more bidders will experience more competition, therefore resulting in higher final sales prices, compared to auction items that are bid upon by few bidders.

Economic theory tests have provided a reasonable understanding of auction outcomes—final prices. Yet what is lacking in the literature is an understanding of the processes underpinning auctions that yield those resulting sales outcomes. For example, what happens between the first and final bids?, how do bidders behave?, are all bidders the same?, do prices increase steadily over the course of an auction?, if not, what is occurring amongst bidders to drive any differential effects? In this paper, we use network analyses to model interactions among bidders. Following the bidding and counter-bidding exchanges in a continuous manner over time illuminates both structural and sequential aspects of auction processes.

The remainder of this paper is organized as follows: 1) the auctions literature is briefly reviewed, along with some relevant social networks research. We show how interactions among bidders may be viewed and analyzed as a network, and derive predictions about auction structures and bidder behaviors. 2) The methods and measures used to study auction data as networks are described, and a model is presented to test predictions about the network effects. 3) Results on the bidder network are presented, including models of price dynamics and key bidder profiles. 4) A discussion follows of the results of the study and their implications.

1. Literature Review and Theoretical Background

The question that has received the most attention in the auctions literature is, "How may we achieve the most profitable sales prices?" As a result, the research focuses on modeling auction outcomes, including the winning auctions prices and the highest bids per bidder (Lucking-Reiley 1999). Final sales prices have been studied as a function of numerous design elements e.g., English (ascending) auctions tend to generate larger surpluses than second-price auctions or Dutch (descending) auctions (Milgrom and Weber 1982).

Auction outcomes have also been studied as a function of characteristics of the products being sold. For example, higher prices occur when bidders can observe the product quality or when there are offers with money back guarantees. Without objective information providing assurances of quality, bidders make inferences from other cues, such the sellers' reputations or price cues such as the posted reservation prices. Bajari and Hortacsu (2003) found that sellers requesting higher minimum bids created auction environments that discouraged bidders from bidding. As a result, fewer bidders participated, resulting in less competition, and yielding lower than expected profits. Chernomaz and Levin (2012) and Kagel and Levin (2005) showed that package bidding, i.e. bidding on multiple items simultaneously improves auction efficiency particularly when the products auctioned have high synergy among them.

While many such facets of auctions have been studied with regard to their effects on sales price outcomes, we seek to generalize our understanding of auctions along two dimensions to contribute to the literature. We begin by examining the sequential nature of auctions as intricate processes that yield the sales price outcomes. We then examine the auction exchanges among bidders as a social network enterprise to understand the structure and nature of the interactions.

1.1 Longitudinal Processes of Auctions

Regarding the longitudinal nature of auctions, scholars have commented that there is a very real need for research that systematically looks at the processes that give rise to the auction outcomes (Bapna et al. 2008). The sequential nature of the series of bids and counterbids would seem to be intriguing theoretically and pragmatically, yet it is rarely discussed and woefully under-studied, in favor of the simpler economic focus on sales outcomes.

Certainly some process information has been studied, e.g., revealing that sales prices are affected by the order of the items bid upon, and the number of bidders in the auction. In addition, some sequential analyses have been used to show, e.g., that bidders seem to be strategic in planning for future bids for items to be sold subsequently. Bidders seem sensitive to the sequential auction environment, and place demonstrably greater price increases at the beginning and end of auctions, especially for expensive items (Reddy and Dass 2006). Furthermore, as bids unfold, real time forecasting has been shown to accurately predict and even maximize final sales prices (Dass, Jank, and Shmueli 2011).

The call to study more longitudinal aspects of auctions is echoed in many disciplines, including the social networks literature. De Nooy (2011) argues that part of the reason for the increased interest in longitudinal networks is that such data are increasingly available that capture when ties form and are modified. Thus, research questions of interpersonal and group dynamics are no longer merely theoretical inquiries, but rather may be addressed empirically. In addition, when new forms of data become available, researchers begin to pose altogether new classes of research questions.

For example, Spiro, Acton, and Butts (2013) write about brokeraging, which is a mediative process in which the broker facilitates interactions between two parties. They maintain that a brokerage is best understood as a process that unfolds via actor interactions within networks. While this current research is not about brokers, it similarly highlights the importance of both the interpersonal structures, and their changing form over time.

Brandes, Indlekofer, and Mader (2012) also acknowledge the growth in longitudinal network data and heightened interest in studying such dynamics. They focused on offering means of visualizing the representation of the development of connections. They also urge researchers to incorporate the sequential nature of the network interactions into predictive stochastic actor models. This paper contains several visual representations of the network dynamics, and the focus of the paper is indeed a statistical model to test predictions about network structures and the roles of various actors.

Stochastic models are also useful in forecasting temporal patterns. Hasan (2012) considered relational connectivity as temporal trajectories, in that actors who occupy different structural positions are likely to proceed following different paths. These trajectories comprise an element of our modeling as well. We seek to build on the auctions outcomes literature by explicitly analyzing bids to understand price development, and to do so, in part we will model the velocity and acceleration of bidding patterns as they converge toward the ultimate auction sales prices.

1.2 Relational Connectivity of Auctions

In addition to the longitudinal nature of bidding, in this paper, social networks techniques are shown to discern structural elements of the bidding that nicely complement the traditional linear and economic foci on outcomes. To study the dynamics of auctions requires that we model the interactions among bidders in the form of their bid-counterbid actions and responses.

Bidders track their own behavior over time, as well as the actions and reactions of the parties against whom they are bidding. Subsequent bids are placed directly in response to those competitive bidders (Cheema 2008; Lavi and Oren 2012). That is, bidders do not rationally post multiple bids simultaneously for the same item, and in many auctions are precluded from doing so (Malmendier and Lee 2011; Zheng 2012). Rather, a bidder posts a subsequent bid only in response to a competitor having posted a dominating bid. Bidders respond to bids in the developing sequences, and therefore to other bidders, even if the other bidders are strangers (as is true in many game theory formulations). The fact that bids are submitted contingent upon other bids by other bidders indicates that bidders' behaviors are interdependent.

Understanding how bidders compete against each other and interact with each other in auctions is frequently pointed to as an area of research that is both very important and understudied (Milgrom and Weber 1982; Roth and Ockenfels 2002). In this paper, we analyze the details of bidders' interconnections, and this greater depth of knowledge illuminates auctions in ways they have not been considered to date, offering both new views on auctions and new phenomena as elements of auctions, including but not limited to final sales prices.

The notion that a bidder responds to other bidders may be easier to understand in a traditional in-person auction (e.g., Christie's), in which bidders identify each other in the audience by face or by a paddle number. Yet online auctions are less anonymous than one might think. Bidders become familiar with other bidders by their online auction username, and the actions they take (e.g., other objects they are bidding on, frequency of their bidding, etc.). The comparison between online and offline phenomena is not unusual, given the still relative novelty of many online activities. For example, in their comparison of interpersonal connections within organizations, Johnson, Kovács, and Vicsek, (2012) found that offline (or, "in real life"), social networks are affected by actor attributes such as gender, tenure, and positions in organizational hierarchies, whereas online social networks were less affected by these individual characteristics.

To seek validation in this regard—that online auctions may resemble social networks, we conducted a small pilot study, in which we interviewed eight managers of major art auction houses and four known art collectors. We also conducted an online survey of contemporary art collectors and dealers who are regular bidders at the auction house used in this paper. One of the leading providers of art auction price information promoted the survey to their clients by offering a free customized art report (a value of $50) for their participation. Forty-one respondents participated in the study in which they were asked about their bidding behavior.

The results indicate that most bidders (80%) browse through the web pages to investigate who is bidding on what items (mean = 5.54, $p < .01$ compared to the mid-point of 4.0 on a 7-point scale). They also generally (78%) recognize nicknames of other bidders if they competed against each other for more than one item (mean = 5.29, $p <.01$). While this study is small in scale, it seems to strengthen the argument for the relevance of networks in auctions research.

A natural implication of the pursuit of studying auctions as networks is to study the positions of the bidders within the network. An important and popular means of characterizing actors within networks actor is via their centralities. We will track the centrality of all actors on all auction lots over the entire course of the auction event, and we will use these indices to identify key bidders in the network. The auction house managers we interviewed all agreed that the identification of such bidders would be highly desirable information. Currently these auction houses only collect essentially contact information (i.e., a bidder's name, address, financial institution). In some instances, a particular manager may have personal knowledge about a particular bidder's preferences, but in most cases, they rely on bidder activities during auctions for more information, such as what items they bid on, etc.

Network methods are easily used to identify key actors. Furthermore, we will show how important the key bidders are in affecting other bidders' behaviors, and ultimately the auction prices. It is not simply the number of bidders that contributes to auction prices, as economists would predict. Specifically, while statistically controlling for this known effect of number of bidders or competition, we will clearly attribute the movement of auctions to small numbers of key bidders whose influence over the bidding processes also translate into greater profitability.

That network structures and dynamics may be attributable to a small portion of actors is not an unusual finding. For example, in studies of inter-firm competitive networks, Braha, Stacey, and Bar-Yam (2011) show that any given company tends to consider only a small set of other companies as their main competitors. Yet there are usually a few companies that many consider to be competitors. In auctions, we shall show that such key bidders begin to separate from the others very early on, and other bidders are influenced by their bidding, e.g., in bidding more on items that key bidders are considering.

Simpson, Markovsky, and Steketee (2011a) found that actors primed with low power tended to provide more accurate perceptions of network ties than more powerful actors. They argue that this knowledge is also more useful to less powerful actors, but only if it is not pervasive among low power actors (Simpson, Markovsky, and Steketee 2011b). In auctions, we might expect that the key bidders are so preoccupied in their bidding that they may be relatively unaware of their impact on others' bidding patterns. In contrast, less powerful bidders may be not only relatively accurately aware of the key bidders' bids, but they may be seeking the knowledge of the key bidders' bids as a signal regarding which auction lots to bid on or avoid.

In addition to identifying key bidders in this auction, we will also show that more generally, the bidders' centralities follow a typical skewed distribution, with few bidders of high centrality, and many more bidders of lower centrality. Shore, Chu, and Bianchi (2013) described such skewed networks in the realms of economics and ecology as more often sustainable, rather than diminishing because the actors are tied together by the flow of resources throughout the network, not the consumption of the resource.

The auctions are not driven entirely by a small number of key bidders; as influential as they are, other bidders must be present to fulfill all auction bidding and purchasing. Schnettler (2009) reviewed 50 years of research small world networks, and provided different means of defining them. A popular structural definition is that a small world network tends to show a good deal of clustering, and on average, short geodesics. Friedkin (2011) points to several efficiencies that small world networks created in their interactions, e.g., that while there is clustering, there are usually some path redundancies. In auctions, if several bidders have become involved in bidding for some item, the redundancies in roles may suggest that when bidder A bids, and bidder B counterbids, if A does not return a bid, it is likely that bidder C might.

Harrigan, Achananuparp, and Lim (2012) argue that these small clusters of reciprocal dyadic and triadic interactions are extremely important in enhancing social contagion. These small scale interactive effects contained within the greater network substantially increase the speed of something "going viral," to use the vernacular, or of being influential. The small sets of dyadic interactions, or bids and counter-bids in the auctions, create an important effect much like a collective action (cf., Flores, Koster, Lindner, and Molina 2012). In their inquiry into the question of why some contagions grow more rapidly than others, Barash, Cameron, and Macy (2012) point to the need for the presence of bridge actors who connect the clusters. This effect may be particularly important in auctions because the bridging mechanism may facilitate bidding effects across auction lots. That is, a simple bid-counterbid pattern might be studied within the auction of a single item, but bidding patterns are likely to be more complex when bidders can bid across multiple auctions for multiple items.

While the consideration of an auction as a social network may not surprise social networks researchers, the use of the network paradigm is new to auctions researchers. Furthermore, even social networks researchers will be pleased to discover, as we shall show shortly, that the usefulness of network indices in predicting such effects as final sales outcomes dominate the predictive effects of non-network (e.g., economic) variables.

2. Methods, Measures, and Models

In this section, the methods used to study auction data as networks are first described: 1) a small example is used to illustrate how an auction may yield network data, 2) the mechanisms of the large, online auction from which the study's data were obtained are described, and 3) the examination of auctions as networks is shown to enable the examination of qualities of auctions that complement the extant literature. Next, the measured variables are described, specifically: 4) showing how to define centrality on auction bidders, 5) illustrating the multivariate nature of simultaneous auctions, with bidding interactions being conducted both within and between sale lots, 6) a series of control variables are described that are included in the modeling to enhance both the precision of the network estimates and the generalizability of the results for having taken into account several practical matters. Finally, in section 2.7) the model is presented that will be used to test predictions about the network effects.

2.1 Methods: Small Auction Example

Figure 1 illustrates the process underlying a simultaneous online auction. The larger auction is described in detail shortly, but for now we note that a simultaneous auction simply means that many items are available for bidding, and each bidder can bid for several items at once. Figure 1a (the first table) presents a portion of a bid history for one auction item (Lot 1). The excerpts of sequential bids unfold in real time, from December 5th at 10:30 p.m. through 3 a.m. the following morning. There are three bidders with online nicknames *Poker*, *Kyozaan*, and *Anonymous3*. The bidding begins (at the bottom of the first table) with a bid from *Anonymous3*, a counterbid by *Kyozaan*, etc.

--- Insert Figure 1 about here ---

Figure 1c (to the right) shows how graph theory may be used to represent the bids and counterbids as a network link between any pair of bidders, who place a bid or react to another's bid. This particular excerpt begins with *Anonymous3* and *Kyozaan* repeatedly bid and counter-bid three times, so in graph theory these bidder-nodes would be linked with a value of 3. In the same bid history excerpt, *Poker* and *Kyozaan* also bid back and forth three times, so the graph requires another node to represent the third party, *Poker*, and a link between that node and *Kyozaan* of strength of 3.

For single-item auctions, the graph would be complete. Data representing simultaneous auctions are more complex, thus, there appears a second table, Figure 1b, which shows the bidding and counter-bidding for a different auction item (Lot 2). Separate graphs may be created, one for each item, particularly if our interest was focused on the items themselves. Our interest is more in the bidders, how they bid and counterbid, to see how they compete for items, both within a lot as well as across auction lots. Thus, in Figure 1d, we aggregate the bid-counterbid exchanges collectively resulting in the addition of more nodes and more links to represent the growing auction bidding patterns.

Even the simple patterns depicted in Figure 1d show distinct roles among the bidders. For example, compared to the others, *Anonymous3* seems to be the least involved, whereas *Kyozaan* stands out because the frequencies of bids and counter-bids with the others are greater than for any other bidder. We will use these differences to identify and characterize key bidders in auctions.

2.2 Methods: Online Auction Data

In this section, we describe the full auction (from which Figure 1 was sampled). The data we model have many rich and complex qualities, which make the modeling more challenging but also enhances the relevance and external validity of the study. The data in this study are from first price English online auctions (i.e., they have an ascending bid format) organized by a company that specializes in a line of contemporary international art that is a leading emerging art market ($450mm last year). In this particular art market, this online auction house sold more ($10.1mm, 212 items) than Sotheby's ($5.2mm, 67 items) or Christie's ($8.7mm, 108 items) last year. Hence the auction is sizable and important.

The auction house organizes four to five annual sale events (like Christies and Sotheby's) where 100-200 art items are bid upon simultaneously in a three-day online auction. Prior to the auction, a catalog of available items is prepared (like those of Christies and Sotheby's), with information about the items to be auctioned including the title of the artwork, its size, media of the artwork, and the artist. In terms of value information, only pre-auction estimates are provided. There may be situations where a reserve price is set, but they are not disclosed to the buyers. The auction house contacts potential bidders via email and courier service, and sends them event invitations and catalogs.

At the same time, bidders register to pre-qualify. Pre-qualification includes a credit check, and a selection of an online nickname to be used throughout the auction.

The auction house consults with the consignor, seller, art experts, and professional art appraisers to derive an estimate for the value of each item to be sold. Auction houses evaluate their performance in part by comparing final realized sales prices with the lower bound pre-auction estimates, i.e., predicted sales prices. It is common to use this ratio as a measure of the extent of successful performance of the items and the auctions, thus we use this ratio as an outcome variable, after log-transforming it due to the predictable skew, and refer to it as *seller profit*.

These auctions start with bids at a value lower than the low estimate value. Bidders can only bid the incremental value that is preset by the auction house. If they decide on posting a higher bid than the pre-set value, an automated bidding system will place proxy bids on his/her behalf, but at the pre-set value. (Given the pre-sets in this auction, which are common among auctions, we chose to model the number of bids rather than bidding increments, but our results do not change if values are substituted.) The auction has a fixed ending time and date set by the auction house (our data come from a three-day auction), and the ending time for each lot is automatically extended until no new bid is submitted for three consecutive minutes so as to preclude sniping (i.e., last minute frenetic bidding).

2.3 Methods: Construing Auctions as Networks

One of the reasons that process details on bidding sequences and bidder interactions have not yet been commonly studied is because such micro auction data have inherent qualities that render many traditional analytical techniques inappropriate. For example, while auction outcomes such as final sales price may be modeled using statistical techniques such as regression, auction process data capture bidders' contingent interconnections. Many standard statistical models assume independent observations, thus immediately rendering them inapplicable given that the very phenomena of interest here are the bidder interactions which violate assumptions of independence.

Network analytical techniques are a natural fit for studying these auction bidding behaviors because they were created for the express purpose of analyzing observations that are interdependent and for modeling the patterns of those interconnections. From this perspective, the graph in Figure 1d may be considered to be a network. While many networks are social in nature, in our auction data, two bidders may not know each other. However, it has been shown that bidders do become familiar with each other (e.g., through their online nicknames), and their bidding strategies (i.e., timing, frequency, bid increments, etc.), as they mutually bid on some common lots over the course of the auction even if they do not communicate directly (Brusco and Lopomo 2002; Konrad and Kovenock 2009; Kwasnica and Sherstyuk 2007). Thus the links in the network in Figure 1d comprise a set of behavioral transactions, expressed by the number of times any pair of bidders bid consecutively on the same lot. We will denote actor *j* placing a bid, which is surpassed by *k*, and after *k* bids, *j* responds, and so forth, across all lots in the auction. The link is a bid-counterbid mechanism that transfers information regarding bidder strategy, preference, and valuation of the auctioned item.

In addition, certainly networks have been used to study many contexts of interconnectivity that are not social, including transportation routes between cities, electrical routes over circuits, parts flowing in supply chains, etc., as well as other commercial contexts (Bohman 2012; Chua 2010; Pauksztat, Steglich, and Wittek 2011). As such, network analysis is a class of analytical tools that may be applied to numerous substantive data, like any other modeling technique. In the abstract, a network is comprised of nodes and the links that connect them. In our case the nodes are bidders and the links capture the patterns of bids and counterbids. The emergent network forms as bidders influence each other's behavior, in turn affecting both auction processes and outcomes. As such, referring to bidding processes and bidder interactions as networks will allow our leveraging the numerous network methodologies to reveal effects in the bidding data. Network methods will enable the analyses of both the sequential and interactive qualities of auctions that have been identified in the auctions literature as important yet under-studied.

2.4 Measures: Centrality of Bidders

We define the bidder network as the set of *g* bidders whose relationship is based on whether bidder n_j and bidder n_k bid sequentially on a lot where n_j, $n_k \in N$; $N = \{1, 2, \ldots, g\}$. The $g \times g$ symmetric matrix X_m contains elements $(X_m)_{jk} = p$, the set of ordered pairs recording the tie of type *m* between pairs of bidders, in this case, the number of times *p*, that bidders n_j and n_k bid sequentially on lots in the auction, $p = \{ 0, 1, 2, \ldots, P\}$, where *P* is the maximum number of consecutive bids placed in the auction.[1]

Bids arrive at unequal time points for different items, so it is difficult to examine the network evolution with a common time scale. Therefore, we model the emergence and evolution of the network by dividing the auction duration into 100 equal time intervals, and constructing the bidder network matrix X_m for each time period. This scaling allows far greater detail than say, early, middle and later bids, or quintiles, etc., approximating continuity. Analyzing these dynamics will allow us to track how the bidders influence each other and the auction outcomes

In networks, an important means of capturing structures of connections is to compute centrality indices for the actors embedded in the network (Wasserman and Faust 1994). In a standard social network, the relational ties may reflect friendships, and high scores reflect actors who are central or popular amidst many friends. For our purposes, the index may be interpreted as not representing popularity so much as being a measure of the extent of a bidder being key to the auction—a bidder is connected to other bidders via deep bid-counterbid actions and dyadic engagement. In this regard, the label centrality conveys that highly interactive bidders will be important to, or central to, the bidding action and we will show, to the auction outcomes as well.

In these data then, centrality scores reflect the extent of the bidders' interactions, and their level of direct competition with the other bidders. A central bidder, with the advantage of being in the midst of the action in the network, is naturally playing an influential role on other bidders. This analysis will enable the identification of bidders who are important in these networks, playing key roles in the auction, and coupled with the sequential nature of the data and analyses, we will track when their centrality appears or diminishes.

In every time period, three actor descriptors are computed for to examine how the actors' positions stabilize or change. First, we computed and normalized degree centrality:

$$C_D(n_i) = d(n_j) / (g-1), \qquad (1)$$

where $C_D(n_i)$ = degree centrality of bidder *j*, $d(n_j)$ is the total number of bidders that are connected to bidder n_j, and *g* is, as before, the total number of nodes in the network. The bidders with higher degrees are more central,

[1] We also considered other ways to identify the bidder network, such as (1) number of items on which the pair of bidders bid jointly, and (2) the proportion of times any pair of bidders appeared (bid) together across all lots in the auction. Both of these measures do not capture the sequential nature of bids, but they yield similar results. Similarly, we also tried creating a directional bidder network, which capture who follows whom in the auction. The key bidders identified using the directional network are same as the ones obtained using a non-directional network. We present only the results using a non-directional network in this paper. Results from a directional network may be obtained from the authors.

and bidders with low degrees are those who reside on the social-periphery of the network.

Next, we computed the average degree indices of the overall network at each time period:

$$C_D = \sum_{j=1}^{g}\left[C_D(n^*) - C_D(n_j)\right] / \left[(g-1)(g-2)\right]. \quad (2)$$

$C_D(n^*)$ is the largest observed degree in the network, to normalize the degree measure, as a percentage of the network's maximum centrality.

Lastly, we computed Bonacich's (1987) power index, essentially a weighted form of degree centrality. Whereas degree centralities (above) treat all of an actor's connections to others equally, weighted centralities consider the connections of the others to whom the focal actor is connected. For example, two actors n_j and n_k may have the same number of connections to other actors, but if the actors to whom n_j is connected are themselves highly central compared to actor n_k's connections, then actor n_j is more central according to the weighting captured in this index. Indices of weighted centralities capture averages of an actor's direct and indirect links (cf., Google's Page Rank algorithm). The weighted centrality for bidder n_j, $C_W(n_j)$ is derived by iteratively solving: $C_W(n_j) = \alpha\,(I\text{-}\beta X_m)^{-1} X_m 1$, where α normalizes the sum of squares of the indices to equal the number of ties in the network, $\beta > 0$ weighs higher scores for actors tied to other central actors, and I and 1 are the identity matrix and a vector of ones.

2.5 Measures: Multivariate Bidder Networks—Within and Between Lots

Most extant research has focused on single-item auctions, where bidders compete for one item at a time. This study involves a more complex auction format, known as a simultaneous auction (Brusco and Lopomo 2002). In these auctions, a large number of items are sold concurrently to the same group of bidders over a fixed duration of time. Compared with single-item auctions (e.g., eBay, Amazon), simultaneous online auctions offer additional opportunities for competitive behavior in that bidders compete against each other within a lot (for a particular item), and also across lots (for multiple items), all at the same time. Simultaneous auctions are usually more profitable than sequential auctions even though they are relatively inefficient (Bulow and Klemperer 2009). Therefore, another form of simultaneous auctions, called pooled is typically implemented as they generate more revenue than the ascending auction while achieving equivalent efficiency levels (Salmon and Iachini 2007).While simultaneous auctions have been discussed in the literature, they have not been empirically tested until now and they should be studied more because they have become the preferred format in selling a wide range of objects, including FCC radio spectrum, U.S. treasury bills, timber and cars (Kwasnica and Sherstyuk 2007).

Thus far, we have defined network patterns of connectivity as bid and counter-bids for a single item as the most direct network relational exchanges, and then aggregated up for the auction-level analyses (cf., Willer, van Assen, and Emanuelson 2011). Within-lot and between-lot interactions further distinguishes the network patterns into two identifiable measures, and thus allowing better scrutiny of the bidder network. Thus, we now operationalize within-lot interactions between two bidders in a lot as the number of times the two bidders bid against each other sequentially. To capture the intensity of direct rivalry between two individual bidders for an item, we consider the *maximum* number of sequential bids between any bidder pairs within a lot. Formally, the within-lot interaction index for lot i is given by:

$$wl_i = \max(f_{jk}) \text{ for } j = 1,\ldots B_i\text{-}1, \text{ and } k = j\text{+}1,\ldots B_i, \tag{3}$$

where B_i denotes the number of bidders in lot i, f_{jk} the number of bids between bidders n_j, and n_k.

Analogously, we operationalize between-lot dyadic interaction as the number of lots in which two individual bidders have both submitted bids. For all possible bidder pairs, we count the number of lots in which the pair has competed against each other. For a particular lot, we take the average of these pair-wise measures as an indicator of the item-specific between-lot bidder interaction (bl_i):

$$bl_i = \frac{1}{N_i} \sum_{j=1}^{B_i - 1} \sum_{k=j+1}^{B_i} cl_{jk} \ , \tag{4}$$

where B_i denotes the number of bidders in lot i, cl_{jk} the number of common lots bid by bidders n_j and n_k, N_i the number of bidder pairs in lot i.

2.6 Measures: Control Variables

The model will also contain several control variables to statistically capture extraneous heterogeneity. In art auctions, artist reputations naturally play a role in the prices their art items command, so one control variable captured whether the artist was just “emerging” or already “established” (c.f., Reddy and Dass 2006). In addition, to control for unobserved artist heterogeneity of particular artists, we include a random effect in our model to represent the specific individual artists who created each lot. Product characteristics also affect prices, so one variable captures the lot medium characteristics (e.g., paper vs. canvas; cf. Reddy and Dass 2006), and one variable captures the (log) size of the lot (cf., Beggs and Graddy 1997).

To be able to conclude that bidder interactions affect seller profits, we included several traditional measures to capture competition, including: the number of unique bidders participating in a lot and the number of bids per bidder in a lot (Bapna et al. 2008). In addition, we included the average number of lots bid by bidders in the lot (cf., Brusco and Lopomo 2002; given that bidders can bid in multiple lots simultaneously, we were concerned that budget constraints could lower the average budget for each lot and affect the competition in the auction, hence this covariate).

2.7 The Model

The model we fit to test our predictions about the importance of bidder interactions follows:

$$\ell\text{n (Seller Profit)}_i = \beta_0 + \Sigma_j\, \beta_j x_{ji} + b_1\, u_{1i} + e_i, \tag{5}$$

for j = 1 to 9. This model is fit across lots i = 1, 2, … 199, x_{1i} = within-lot dyadic bidder interaction and x_{2i} = between-lot dyadic bidder interaction (both defined shortly); x_{3i} = number of bidders; x_{4i} = number of bids per bidder; x_{5i} = average number of lots bid by bidders; x_{6i} = dummy variable to indicate if the lot belonged to an established artist (= 1 if an established artist); x_{7i} = dummy variable to indicate if the lot belonged to an emerging artist (= 1 if an emerging artist); x_{8i} = dummy variable to indicate medium (=1 if paper, 0= if canvas); x_{9i} = log(size of art work in square inches); u_{1i} = artist of lot *i* and $b_1 \sim N(0,\psi^2)$ where ψ^2 is the variance of the random effect.

3. Results

The results of this study are presented in the six sections that follow: 1) the sample is described, 2) basic statistics on the bidder network are presented, 3) price dynamics are modeled, 4) key bidders are identified and profiled, and their impact on prices and price development are modeled, and 5) competition is analyzed both within and between auction items. 6) Finally, we ran several tests of our model's robustness.

3.1 Sample Description

This study captures the bid history data from a complete auction event. During the three-day simultaneous online auction, 256 bidders competed against each other for 199 lots (paintings, drawings), by 70 artists (each of whom presented an average of 2.8 lots). A total of 3080 bids were placed.

Proxy votes or votes by agents were allowed, however their use was infrequent—most of the bidders did not (67.06%), and of those who did, 76.19% of these bidders used 8 or fewer proxy votes. The mean number of proxy votes placed was 10.98.

Table 1 contains the basic descriptive statistics: on average, 6.35 bidders bid on each lot, each bidder bid on 4.93 lots, and each lot received 15.47 bids. The average realized price per lot was $62,065.

--- Insert Table 1 about here ---

The sample descriptors indicate that these art bidders are not the average eBay bidder looking for a good deal on a modestly priced item. Rather, these art bidders are spending a lot of money, and many of them are experienced and come to the auction well-informed about the art items and the auction process.

3.2 Results on the Overall Network

The number of bidders increased from 61 in the first time period to 256 by the end of the auction. As more bidders arrived, the bidding activity increased resulting in more bid-counterbid links, from 309 at the beginning of the auction to 1463 at the end of the auction. Figure 2 presents the visual representation of the network at the beginning of auction, the mid-point of the auction, and at the end.

--- Insert Figure 2 about here ---

The structure of the bidder network may be compared to regular and random networks. It appears distinct from being statistically regular (where all degrees of all bidders are equal) or random (where most of the bidders degrees are concentrated around the mean). The skewed distribution in Figure 3 depicts most bidders as having a low degree and only a few bidders as having a large degree, a result very different from the degree homogeneity that would be observed in a random network. Overlaying the plot with the theoretical cumulative distribution function of *p(c)*, the probability that a randomly selected bidder will have *c* links, thus following a power-law implying that the bidder network is dominated by only a few highly central bidders.

--- Insert Figure 3 about here ---

Accordingly, we might say that the bidder network exhibits properties of a "small world" network. This quality is discerned by measuring the geodesic length between bidders (l) and the clustering coefficient (τ) of the network is the average proportion of links between the vertices within a neighborhood over the total number of possible links in the network (where $\tau = [3k(k-1)] / 2k(2k-1) + 8pk^2 + 4p^2k^2$, for k = the number of connected nodes, and p = 0 for regular networks and p = 1 for random networks). Small-world networks are graphs that are highly clustered like a regular graph ($\tau_{real} >> \tau_{random}$), but possess small path lengths like a random graph ($l_{real} \approx l_{random}$). The bidder network has a high clustering coefficient ($\tau_{bidder} = 0.881$) compared to a random network with same number of bidders and bidder relationships (1463) (τ_{random}=0.022), but similar path length to a random network (l_{bidder}=3.097 $\approx$ l_{random}=3.391). The properties of this bidder network suggest the fast spread of information compared to a random network.

Degree centralities decline as the auction progresses, indicating network fragmentation. Figure 4 shows an exponential decay of *p(c),* the probability that a randomly selected bidder will have *c* links with increasing number of bidders, as more bidders enter the auction. This form indicates that the speed and efficiency of the counter-bidding activities among bidders in the network decline over the duration of the auction. (Differences between actors' in-degree and out-degree centralities were negligible, and therefore, a symmetric network was considered.)

--- Insert Figure 4 about here ---

The average weighted centrality index of the network increases as the auction progresses almost four-fold from 3.06 at the end of the first period to 11.33 at the end of the auction. This finding suggests that as the network evolves and becomes fragmented with increasing numbers of bidders in the auction, either the centrality in the network is being distributed among more bidders, or some key bidders are becoming increasingly and disproportionately dyadically engaged in frequent bidding and counter-bidding—two very different possibilities. These competing explanations are tested shortly.

Table 2 contains the standardized parameters and standard errors resulting from fitting model (5) to estimate auction sales prices. The final realized price was calculated as a percentage over the pre-auction low estimate obtained from the auction house. As predicted, within-lot interactions increased sales prices, whereas between-lot interactions kept prices lower. The number of bidders and bids, while serving as controls, also contributed significantly to higher prices.

--- Insert Table 2 about here ---

3.3 Results of Network on Auction Price Dynamics

In addition to modeling final prices, we sought to understand how prices develop toward the realized sale outcome. In particular, as price moves in the online auction, an important question to ask is whether the network evolution influences the price dynamics. To answer it, we employ an emerging hybrid modeling technique, "functional regression analysis" (Ramsay and Silverman 1997), incorporating covariates to capture any heterogeneity of the price paths (Bapna et al. 2008; Reddy and Dass 2006; see the Appendix for details).

Using these techniques, we estimated the price curve, $f_i(t)$ for lot *i*, for each lot in the auction using penalized splines. In addition, such smoothing splines yield derivatives of the price curve, thereby providing a detailed view of the underlying dynamics such as velocity (i.e., the first derivative, $f_i'(t)$) and acceleration (i.e., the second derivative, $f_i''(t)$).

The 199 different lots have different realized prices, so the price curves were first scaled from 0 to 1, where each data point represents the ratio of the lot's final realized value to its value at that instant in the auction. Functional regression tracked whether the bidder network characteristics significantly affect the price dynamics. Unlike standard regressions, where explanatory variables are scalars or vectors, functional regression allows us to model the response variables—price, velocity, and acceleration. These additional insights are important as researchers have shown interest in predicting which bidders will bid at what points in time, e.g., in showing that price velocity can help price forecasts (Dass et al. 2011).

The estimated parameter curves (and 95% confidence bands) are displayed in Figure 5. The results plotted are those for the weighted centrality indices, but all network predictors showed significant and similar effects on price dynamics. The first curve is positive throughout the auction, implying a positive relationship between centrality and price. The relationship is statistically significant throughout the auction except at the very beginning and the end of the auction. The parameter increases and peaks during the middle of the auction and declines as the auction progresses towards the end. This result implies that the centrality exhibited by bidders has the maximum effect on price in the middle of the auction with diminishing or no impact by the end of the auction.

--- Insert Figure 5 about here ---

Price velocity is reflected by the second parameter curve, and it is positive (and significant) at the beginning of the auction, becoming insignificant during the rest of the auction and negative (and significant) towards the end of the auction. This curve indicates that higher (lower) centrality in the network contributes to faster (slower) price-increases (velocity) at the beginning of the auction and slower (faster) price-increases at the end of the auction. The third parameter curve is associated with price acceleration, and it is not significant throughout the duration of the auction.

3.4 Results on the Key Bidders

Network centrality measures can be used to identify key bidders, based on the bidders' interactions. The mean (and standard deviation) for the bidders' centrality indices were 0.169 (0.307), and for weighted centrality, 11.334 (22.471). Recall the skewed distribution of bidder centrality and power, whereby most bidders have low scores, and a few bidders have large scores. The five bidders with the highest centrality scores were the bidders named: *Kyozaan, Poker, Anonymous 3, Lord of the Rings* and *Anonymous 118*.

Kyozaan emerged as a highly central bidder in this auction. This bidder entered the auction event early, bid on many lots (56) and bid the most (182 bids). *Kyozaan* had the highest number of bidding relationships (59), also reflected in the high centrality measures. This bidder won four of the 56 lots bid on (two were by major established artists) and spent $351,600. Figure 6 depicts the network characteristics of *Kyozaan*, illustrating that weighted centrality increased by 147%, whereas (simple) centrality reduced by 77% during the auction. These results suggest *Kyozaan*'s growing influence during the auction—simple connectivity did not increase but this bidder was increasingly bidding against other highly engaged (central) bidders.

--- Insert Figures 6, 7, 8 about here ---

The changing centrality statistics for the key bidders over time are presented in Figure 7. For a conservative comparison, the five bidders with the next highest centrality indices are also plotted. The key bidders distinguish themselves clearly and very early in the auction. They begin to differentiate themselves from the others as soon as time point 1—that is, within 7 hours after the auction started. A more comprehensive characterization of the key bidders is displayed in Figure 8.

To test the effect of the key bidders on prices, and to ascertain whether it is truly their network behaviors driving their influence, or other bidder attributes, we created three indices: the first based on bidder connectivity (i.e., network-based centrality indices), the second based on bidder wealth (i.e., depth of pocket, or the maximum amount a bidder would need to pay at any given time if the bidder won all items currently bid upon), and the third based on simple bidder activity (i.e., the number of bids placed). Like model (5) for the entire network, in this model for the key bidders, we also controlled for other determinants, including aggregate competitive measures (number of bidders, bids per bidder, average number of lots bid by bidders), etc. Together, these data were analyzed using the following mixed effect model:

$$\ell n(Seller\ Profit)_i = \beta_0 + \sum_{j=1}^{12} \beta_j x_{ji} + b_1 u_{1i} + e_i. \qquad (6)$$

This model was fit for i = 1, 2, … 199 lots, x_{1i} = number of key bidders based on dyadic interactions; x_{2i} = number of key bidders based on depth of pocket; x_{3i} = number of key bidders based on bidding intensity; x_{4i} = wl_i, within-lot dyadic bidder interaction; x_{5i} = bl_i, between-lot dyadic bidder interaction; x_{6i} = number of bidders; x_{7i} = number of bids per bidder; x_{8i} = average number of lots bid by bidders; x_{9i} = dummy variable to indicate if lot belonged to an established artist (= 1 if an established artist); x_{10i} = dummy variable to indicate a lot by an emerging artist (= 1 if an emerging artist); x_{11i} = dummy variable for medium (=1 if paper, 0= if canvas); x_{12i} = log(size of art work in square inches); u_{1i} = artist of lot i; and $b_1 \sim N(0,\psi^2)$ where ψ^2 is the variance of the random effect.

Table 3A represents the key bidders' behaviors, fitting the model at times t = 0.1, t = 0.5, and t = 1.0, to show how the coefficients of the model variables change over time. These results show that the centrality measure of identifying key bidders has a significant impact on seller profit (β = 0.234 for t = 0.1, β = 0.274 for t = 0.5, and β = 0.076 for t = 1.0, all p<.01). Note that the non-network definitions of key bidders, based on depth of pocket or bidding intensity were not significant predictors. (The predictors are modestly correlated r = 0.310.)

--- Insert Table 3A about here ---

To check on any possible problem with multicollinearity, the table also reports variance inflation factors. Most of the VIF indices may be deemed reasonable (cf., in not exceeding 6, per Maruyama 1998, p.64). Nevertheless, several of the VIFs were relatively large, e.g., 4.95 and 5.11 for the average number of lots bid by bidders, and potentially more problematic for this networks-oriented research were the 3.48 and 3.74 and 3.02 for the between-lot dyadic interactions. Thus to statistically control for multivariate associations, we present in Table 3B a simultaneous equations version of the models across these time points. The focal equation for seller profit remains the same, and an equation was inserted to capture the possibly correlated between-lot interactions. This simultaneous estimation may be worthwhile, given that several of the predictors on the between-lot interactions equations were significant. Yet overall, the effects on the profit dependent measure largely converge with the results in Table 3A—the patterns of results are almost identical, with the exception that the number of bids per bidder is a significant predictor throughout the auction when the three time cuts are modeled simultaneously.

--- Insert Table 3B about here ---

The positive and significant effect of network-based key bidders on seller profit suggests that the presence of a key bidder enhances the seller profit, and across all lots, the more key bidders were bidding on more of the lots, realized profits would be enhanced further still. On average, lots for which at least one key bidder was present (n = 131), realized a higher (3.45) seller profit compared to lots where no key bidders participated (2.84).[2]

In addition to estimating these effects of embedded actors, it would also be useful to examine the effects of extended reciprocity or mutuality beyond the bid-counterbid dyads. That is, once several dyadic ties are forged, other actors in a loosely bounded set may enter the bidding exchanges. We sought to test whether the phenomenon of "Simmelian" ties may address this question. Simmelian ties are localized effects of dyads within triadic structures, whereby a dyad of bidders respond to each other and a common third party bidder (Krackhardt and Kilduff, 2002). It is quite plausible that triadic relationships may function categorically differently from dyadic interactions, e.g., perhaps by reducing the bargaining power of individual bidders, in turn affecting realized prices, so to test this proposition, we also fit a model that included the triadic factor. As indicated in Table 4, there was in fact a significant triadic effect above and beyond the dyadic effects.

--- Insert Table 4 about here ---

There were also dramatic differences between these key bidders and other with respect to the speed of the price formation. On average, prices of lots in which at least one key bidder participated exceeded the pre-auction high estimate value in one day, compared to nearly two days for lots with no key bidders bidding.

Figure 9a and 9b presents the results on price and velocity for bidder *Kyozaan* and the other key bidders. For example, on average, the 56 lots that *Kyozaan* bid on realized a price 359% over the pre-auction

[2] We were able to test our model and predictions in another auction and our findings replicated, e.g., on average, the lots on which key bidders bid realized a higher price (89%) over the pre-auction low estimate, and the time taken to exceed the pre-auction high estimate was significantly faster. (Details may be obtained from the authors.)

low estimates for these lots compared to 271% for the other lots. Although the *Kyozaan* lots had higher average pre-auction estimates mean ($46,414), they nevertheless realized a still significantly higher price over the high estimates than those realized for non-*Kyozaan* lots (mean = $16,948). In addition, the time taken to cross the pre-auction high estimate was significantly faster (0.283; i.e., shortly after the first quarter of the auction) for *Kyozaan* lots compared to the non-*Kyozaan* lots (0.423; closer to the midpoint of the auction). Similar results were obtained for the lots bid on by the other key bidders.

--- Insert Figures 9 and 10 about here ---

These results indicate the importance of key bidders in affecting auction prices. Figure 10 presents the results of the functional regression showing the impact of the presence of five key bidders on auction price dynamics. A dummy variable x_j took the value of 1 if any of the key bidders (*Kyozaan*, *Poker*, *Anonymous3*, *Lord of the Rings* and *Anonymous118*) bid on lot *j*, and 0 if they did not. Controlling the same covariates, the results showed that while the key bidders had no significant effect on the current auction price, they significantly enhanced price velocity. The (95%) confidence bounds show the effect as positive and significant at the beginning of the auction, and turning negative as the auction progresses. The curve indicates that the presence (absence) of key bidders contributes to faster (slower) price increases or velocity at the beginning of the auction and slower (faster) price increases near the end of the auction. The parameter curve associated with price acceleration is significant and negative throughout the auction, indicating that lots where key bidders are present slow the price acceleration throughout the auction, especially at the end.

Next, we tested whether the key bidders moderate the effects on other, more established factors such as the number of bids. For instance, it has been shown in many papers (e.g., Reddy and Dass 2006) that the number of bids has a positive effect on price, but might the effect be even stronger in the presence of key bidders? To test this proposition, we examined the highest bids of non-key bidders who bid on items where key bidders participated and also on other similar items where the key bidders did not participate. We found that non-key bidders bid more (on average 1 bid more, $t=2.43$, $p<0.05$) and pay higher prices (on average $14,488 more, $t=2.02$, $p<0.05$) on items where key bidders participate compared to items where they do not. Thus key bidders drive up price (and profit), and also make other (non-key bidders) reach deeper into their wallets.

Finally, we also tested the proposition that key bidders might only bid on lots where there is most potential profit to gain (e.g. undervalued lots). To test this argument, we identified artists and comparable lot pairs where one includes a key bidder, and the other did not. Comparable lots were identified based on information available about the artists' works, their pre-auction estimates, opening bid, and type of artwork. Comparing these lots, we find that items where a key bidder participated obtained higher profit (148%) than those without any key bidder participation (51%) and the price took half the time to cross the pre-auction high estimate (speed=0.35 vs. 0.70). This result seems to converge in support of the view that key bidders have real effects on auction outcomes.

3.5 Results Within and Between Lots

Tables 3A and 3B also contain the parameter estimates of the model testing the within- and between-lot effects on seller profit. As anticipated, high intensity within-lot dyadic interactions increased seller profit (β = 0.392, 0.313, 0.141, all p's < 0.01). Early in the auction, the between-lot interaction effects are negligible, but with time, high intensity between-lot interactions decreased seller profit (β = -0.096, p < 0.01).

Similarly, early in the auction, the covariates of number of bidders and bids per bidder were not significant, but their effects were positive and significant as the auction network momentum built (β's = 0.653 and 0.533, both p < 0.01). We also controlled for the possible effect of the average number of lots per bidder, in case budgets constrained prices. By the end of the auction, the adjusted R^2 = 0.92 for the model suggests a strong fit.

3.6 Measure Robustness

To check the operationalizations used in this study, we examined several alternative measures. We developed a multi-dimensional index to include the "width" and "depth" of influence of bidders. The inputs included: a) the number of bids per lot by a bidder whereby larger values represent deeper influence; b) the number of lots bid on which the bidder bid, where larger values demonstrate wider influence; c) the type of items bid on which the bidder bid, in particular capturing the bidders bidding on higher value items (per the pre-auction estimates; d) the number of bidders a bidder had to compete against, with more suggesting wider impact; and e) the willingness to pay, as measured by the ratio of the final bid to the final price of the item. The measures were normalized and summed, but subsequent analyses were not conducted, because it was highly correlated with the weighted centrality measure already in the model (r = 0.932).

Another alternative intended to capture valuation. While our bidder networks had deep pockets, one might argue that early bids are low and therefore they can be outbid easily, compared to later, higher bids. Thus we sought to examine whether the influence of a bidder was due to it being a relatively high bid or an early one. Therefore, we developed a new score for bidders termed "value influence" of bidder n_j, or vi_j, computed as the ratio of the final bid by a bidder for a lot over the final price of all the lots bid by the bidder, normalized by the total lots auctioned: $vi_j = \sum_{i=1}^{I} \frac{max\ bid\ by\ n_j\ in\ lot\ i}{final\ price\ lot\ i}$, where $i = \{1,2,...I\}$. If a bidder only bids early in the auction, those bids will have less influence than if bid later in the auction. We also considered other bidder behavior specific measures such as the number of lots bid, the number of bids per lot, the number of competitors, etc., but all were highly correlated with the valuation measure and not considered further. This index correlated with weighted centrality r = 0.600; i.e., the value index is not completely redundant with the network measure we have been modeling, but the correlation is sufficiently substantial as to be indicative that our results are at least implicitly also capturing monetary dynamics of network interactions.

We also tested alternative variables that would likely be correlated but whose effects have already been established so as to estimate precisely the effects of the network connection variables, and to better make the argument that networks matter, above and beyond traditional variables and views. Thus, for example, in the between lot dyadic interactions, we control for the number of common lots that the bidders bid on. We also computed the average of the low pre-estimates of those lots to capture value, as well as the average number of bids the bidders placed to reflect the overall level of bidder participation on these other auctions. In addition, we considered multiple indicators of influence on price, including bid value rather than number of bids. Given that the bid increments are set by the auction house, we anticipated that the number of bids would be equally meaningful to using the actual incremental value. To verify such measurement assumptions, we explored modifications to our analyses, using these different measures, but changes were negligible. The consistency of the results suggests that our methods should be equally applicable to auctions of expensive items as well as less expensive items, given that counts of bids comprise a constant scale in either setting.

4. Discussion

Auctions researchers recognize the importance of bidder interactions, but the challenges to studying such ties have been the availability of data and the means of analyzing them. With the current research, we hope to have contributed to the literature by introducing a network analysis approach to examine bid-counterbid interaction networks.

The evolving bidder network was measured and studied as a form of a bidder behavior model. Over the duration of the auction, mean degree centrality trended downward, implying that the central role played by the average bidder diminishes as more bidders join the auction. As the networks become more heavily populated, any given bidder no longer plays as large a role in being the competitive anchor against which others bid; instead, the competition begins to diffuse across more actors. This pattern need not have been the case; instead, a strident and determined bidder could have maintained a central network position by continuing to counter-bid any posted bid.

This research also indicated that there is an effect of bidder network characteristics on price dynamics in an online auction. The network effect is significant throughout the auction on price and price velocity, and the nature of either network effect is not constant throughout the auction. It is important (as auction theory suggests) in the beginning, when the excitement of a lot of bidding-counter-bidding activity plays a signaling role for the popularity of the art items. Regarding the negative effect of the network near the end, the dyadic effect on auction prices slows, perhaps due to some bidders folding as the auction has become too rich for them to continue. This particular online art auction house tends to attract rather sophisticated bidders and deep-pocketed buyers. However, our findings should be more broadly applicable for two reasons: first, there was certainly bidder variability, and second, the bidding process resembles those in any simultaneous auction, such as in the government procurement of flowers, cars, oil, and so on. Finally, the effects on price acceleration might have been stronger if this auction were designed with a stated end time; then last minute bidding might have increased rapidly. Hence acceleration may be stronger in auctions such as eBay, in which the bidding duration has a hard stop.

Network methods were shown to be ideally suited for identifying key bidders, which can help auction houses allocate resources to these select bidders deserving of extra attention. Recall the managers of Sotheby's, Christie's, and other auction houses admitting their dearth of bidder information (name, address, banking information). Network indices can be computed and re-computed, essentially govern real time, to complement the auction managers' sparse information and help them very quickly discern, with reasonably strong accuracy, the identification of key bidders. In our data, the amount that bidders were willing to pay was highly correlated with the number of lots on which a bidder bid (r = 0.98), but that number would not be known to auction house managers until the end of the auction. In contrast, our weighted centrality index was nearly as highly correlated (r = 0.89), but its advantage is that it can be computed extremely early in the auction event and it can be traced nearly continuously to see if it changes and new key bidders require greater attention.

Alternatively one might argue that the bidders the auction houses should pay most attention to are not the key bidders—the people in the network who we have been illustrating have an effect over other bidders, series of bids, build-ups of prices, etc.—but instead, to those bidders who spend the most. It should not be surprising that the final amount bid by each bidder for different lots at the end of the auction is correlated with how much they spend in the first half of the auction (β = 0.233, p < 0.0001). The network descriptors predict total expenditure even better than the first half sub-totals (β = 0.310, p < 0.0001). When both network centrality and first half sub-totals are in the same model predicting overall expenditure, the network predictor is still significant (β = 0.390, p < 0.0001) and the first half sub-totals are insignificant. If the auction hosts wish to identify those likely big spenders sooner than when half of the auction is past, they can use the weighted centrality index, which may be computed in an ongoing manner.

Furthermore, the centrality measures on the key bidders explained significantly more variance in the model than simple heuristics such as the number of bids that bidders place, or the depths of their pockets. Influential actors in networks are of interest in many domains, such as recommendations by opinion leaders or word of mouth generated by innovators. In the auction context, when key bidders bid on certain items, those items sold for significantly higher prices than sales of auction items in which they did not participate. In addition, the presence of key bidders in the auction of a given item drew non-key bidders to bid more and pay higher prices than for items where the key bidders do not participate.

These results also demonstrated that dyadic bidder interactions are significant in explaining the variation in prices observed across lots, offering clear empirical evidence that dyadic bidder interactions matter in simultaneous online auctions. Specifically, the results indicate that, everything else being equal: 1) in auction lots where bidder pairs bid on more lots together (i.e., higher "between-lot interaction"), seller profit tends to be lower; and 2) in lots where two bidders directly outbid each other more frequently (i.e., higher "within-lot interaction"), seller profit tends to be significantly higher.

Implications

An important implication of our research is the manner in which an auctioneer might make use of this information of the key bidder influence on the auction price dynamics. An auctioneer or online auction host could use our approach to obtain quicker realizations of their reserve prices, for example, the key bidders accounted for $3 million in purchases and 33 lots (17% of the total lots auctioned) directly, not even considering their indirect effects through their influences on non-key bidders. Our techniques would be helpful to auction house managers to identify key bidders as early as possible during an auction. Note that identifying key bidders is possible if there are multiple auctions with some common bidder participation or within an auction on a real-time basis. Before online auctions are held, auction houses typically arrange a viewing event where bidders can attend and inspect items. Special invitation to the key bidders to these events should provide great payoff. Auction houses could use the key bidder information to expand on their "preferred" customer lists to generate activity in future auctions.

In general, from the perspective of bidders, in general they would desire less intense competition to keep prices lower (Dass et al. 2011). More specifically, bidders could use our method of identifying key bidders as a mechanism to respond during auction in one of two ways. First, a segment of committed bidders may seek the item for its intrinsic value. The presence of key bidders for the item of choice for this segment may provide a positive signal and prompt the bidders in this category to reevaluate the value of the item. In such cases, as was evident in our study, the price realized tend be significantly higher than the auction house estimates. Although during the course of the auction, the key bidders may be implicitly identified by savvy bidders in this segment, auction houses may provide real-time access to information about all bidding activity by other bidders. This transparency of information and ease of access would help bidders to uncover key bidders. Online auction houses could provide real time information on the items which have high bidding activity, with the highest bid amounts, which bidders are most active and which items they are bidding on, etc. A different segment of buyer may seek the item for its investment value, and these bidders may choose to withdraw from bidding races in which key bidders are present because their presence will likely result in quick escalation of prices beyond initial valuation and budget constraints. Without additional data, these motivations cannot be teased apart, but in either case the motives would result in monitoring key bidder interactive behavior. The presence and easy access to information about bidder activity would likely help both these groups of bidders.

We conducted this research to better understand several theoretical and methodological issues. We hope that we have provided a good basis on which future research might continue to build, regarding the process of an auction and bidder interactions, and how they unfold and contribute to price formation, and the recognition of key bidders whose behavior influences other bidders and final auction prices.

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Table 1: Summary Data Description

	Mean	SD	Min.	Max.
No. of Bidders per Lot	6.35	2.47	2	14
No. of Unique Lots Bid per Bidder	4.93	7.95	1	65
No. of Bids per lot	15.47	7.46	2	48
Opening Bid	$19,343	$36,663	$650	$300,000
Pre-Auction Low Estimates of the Lots	$24,128	$45,747	$795	$375,000
Pre-Auction High Estimates of the Lots	$31,065	$60,351	$1,025	$475,000
Realized Sales Value of the Lots	$62,065	$133,198	$3,135	$1,486,100
Realized Price of Lots per Sq. Inch	$108.77	$225.49	$1.40	$1,865.42

Note: These summary statistics, particularly on prices, suggest our art bidders are not casual browsers.

Table 2: Effect of Dyadic Bidder Interaction on Seller Profit

Variables	Standardized Coefficient	(s.e.)	
Within-lot Dyadic Interaction	0.146	(0.013)	**
Between-lot Dyadic Interaction	-0.068	(0.007)	*
Number of Bidders	0.686	(0.005)	**
Number of Bids/ Bidder	0.539	(0.020)	**
Average Number of Lots bid by Bidders	0.038	(0.030)	
Established Artist	-0.027	(0.028)	
Emerging Artist	0.023	(0.030)	
Works of Paper	-0.023	(0.026)	
Size of the Artwork	0.024	(0.100)	

* p < .05; ** p < .01

Table 3A: Effect of Key Bidders on Auction Profit

Variables	Time=0.10 Standardized Coefficient	(s.e.)	VIF	Time=0.5 Stdz. Coef.	(s.e.)	VIF	Time=1.0 Stdz. Coef.	(s.e.)	VIF
Key Bidders by Dyadic Interactions	0.234*	(0.395)	1.99	0.274**	(0.189)	1.78	0.076**	(0.018)	1.83
Key Bidders by Depth of Pocket	0.014	(0.411)	2.19	0.104	(0.218)	2.58	0.047	(0.023)	2.89
Key Bidders by Bidding Intensity	0.125	(0.349)	1.70	0.115	(0.184)	1.92	-0.007	(0.018)	2.00
Within-lot Dyadic Interaction	0.392**	(0.169)	1.27	0.313**	(0.092)	1.82	0.141 **	(0.013)	2.92
Between-lot Dyadic Interaction	0.184	(0.104)	3.48	0.007	(0.066)	3.74	-0.096 *	(0.007)	3.02
Number of Bidders	0.003	(0.123)	1.51	0.033	(0.057)	1.47	0.653 **	(0.006)	1.78
Number of Bids per Bidder	-0.097	(0.304)	1.32	0.180	(0.178)	1.84	0.533 **	(0.020)	2.78
Average # Lots bid by Bidders	-0.271	(0.043)	4.95	-0.225	(0.021)	5.11	-0.017	(0.036)	3.57
Established Artist	0.094	(0.590)	1.45	0.350**	(0.293)	1.43	-0.023	(0.028)	1.36
Emerging Artist	-0.045	(0.703)	1.72	-0.090	(0.329)	1.59	0.029	(0.031)	1.57
Works of Paper	-0.125	(0.589)	1.47	-0.130	(0.283)	1.45	-0.017	(0.026)	1.35
Size of the Artwork	-0.089	(2.160)	1.76	0.224**	(1.078)	1.62	0.019	(0.090)	1.58
Adjusted R-Square		0.17			0.45			0.92	

* p < .05; ** p < .01; n.s. = not significant, VIF = variance inflation factor. Bidder interactions within-lots enhance final prices, and bidders competing between-lots lower final prices (controlling for overall activity per #bidders and bids per bidder).

Table 3B: Effect of Key Bidders on Auction Profit (Simultaneous Equations)

Variables	Time=0.10				Time=0.5				Time=1.0			
	Coef. DV=BLI	(s.e.)	Coef. DV=Profit	(s.e.)	Coef. DV=BLI	(s.e.)	Coef. DV=Profit	(s.e.)	Coef. DV=BLI	(s.e.)	Coef. DV=Profit	(s.e.)
Key Bidders by Dyadic Interactions			0.059*	(0.023)			0.060*	(0.020)			0.049*	(0.018)
Key Bidders by Depth of Pocket			0.018	(0.026)			0.013	(0.023)			0.029	(0.020)
Key Bidders by Bidding Intensity			0.013	(0.021)			-0.009	(0.019)			-0.004	(0.017)
Within-lot Dyadic Interaction			0.026*	(0.009)			0.024*	(0.009)			0.051**	(0.012)
Between-lot Dyadic Interaction (BLI)			-0.009	(0.007)			-0.007	(0.010)			-9.571**	(0.448)
#Bidders	-0.217	(0.128)	0.151**	(0.008)	-0.010	(0.085)	0.150**	(0.006)	-0.126*	(0.042)	1.351**	(0.430)
#Bids per Bidder			0.387**	(0.187)			0.351**	(0.019)			0.307**	(0.019)
Avg. #Lots bid by Bidders	4.585**	(0.545)	-0.048	(0.026)	3.714**	(0.362)	-0.053*	(0.019)	3.403**	(0.116)	-3.26*	(1.180)
Established Artist			-0.043	(0.034)			-0.015	(0.031)			-0.026	(0.027)
Emerging Artist			0.027	(0.041)			0.041	(0.034)			0.034	(0.030)
Works of Paper			-0.054	(0.033)			-0.035	(0.029)			-0.018	(0.025)
Size of the Artwork			-0.053	(0.123)			-0.020	(0.112)			0.074	(0.099)
GFI	0.986				0.974				0.973			

* p < .05; ** p < .01; n.s. = not significant. Note: Bidder interactions within-lots enhance final prices, and bidders competing between-lots lower final prices (controlling for overall activity per #bidders and bids per bidder). Estimated Model: $(Between\ Lot\ Interactions)_i = \beta_0 + \beta_1(No.of\ Bidders)_i + \beta_2(Avg.No.of\ Lots\ Bid\ by\ Bidders)_i + u,\ \ell n(Seller\ Profit)_i = \beta_0 + \sum_{j=1}^{12}\beta_j x_{ji} + e_i$.

Table 4

Effects of Actors, Dyads, and Triads on Auction Profit

Variables	Stdz Coef.		(s.e.)
Key Bidders by Dyadic Interactions	0.069	*	(0.024)
Key Bidders by Depth of Pocket	0.045		(0.026)
Key Bidders by Bidding Intensity	0.001		(0.034)
→Number of Triadic Ties	0.069	**	(0.011)
Within-lot Dyadic Interaction	0.155	**	(0.013)
Between-lot Dyadic Interaction	-0.097	**	(0.006)
Number of Bidders	0.622	**	(0.006)
Number of Bids per Bidder	0.503	**	(0.020)
Avg. # Lots bid by Bidders	-0.023		(0.037)
Established Artist	-0.020		(0.029)
Emerging Artist	0.038		(0.031)
Works on Paper	-0.014		(0.026)
Size of the Artwork	0.013		(0.098)

* $p < .05$; ** $p < .01$

Figure 1: Formation of a Bidder Network

Figures 1(a) and 1(b) track the bids and counter bids of two different auction lot items. Figures 1(c) and 1(d) are graphical representations of the links that emerge from the bid-counterbid exchanges.

Date & Time (U.S. EST)	Nickname	Amount $
Dec6 3:03:48 AM	Poker	71,500
Dec6 1:27:43 AM	Kyozaan	66,500
Dec6 1:26:17 AM	Poker	61,500
Dec6 1:26:10 AM	Kyozaan	56,500
Dec6 1:25:58 AM	Anonymous3	51,500
Dec6 1:25:58 AM	Kyozaan	49,000
Dec5 10:30:00 PM	Anonymous3	46,500
Dec5 10:30:00 PM	Start Price	44,000

Figure 1(a) - Lot 1

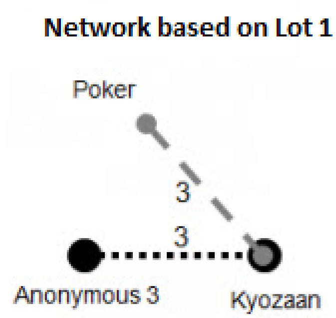


Figure 1(c)

Date & Time (U.S. EST)	Nickname	Amount $
Dec6 9:53:24 AM	Kyozaan	121,500
Dec6 9:52:25 AM	Socrates	111,500
Dec6 9:52:25 AM	Kyozaan	101,500
Dec6 4:42:35 AM	Socrates	94,000
Dec6 4:16:25 AM	Poker	86,500
Dec6 4:14:37 AM	Anonymous38	79,000
Dec5 1:16:56 AM	Kyozaan	74,000
Dec5 10:40:07 PM	Anonymous38	69,000

Figure 1(b) - Lot 2

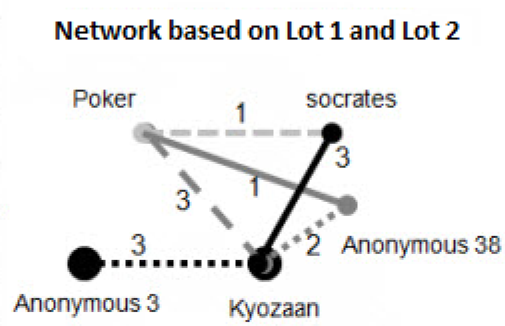
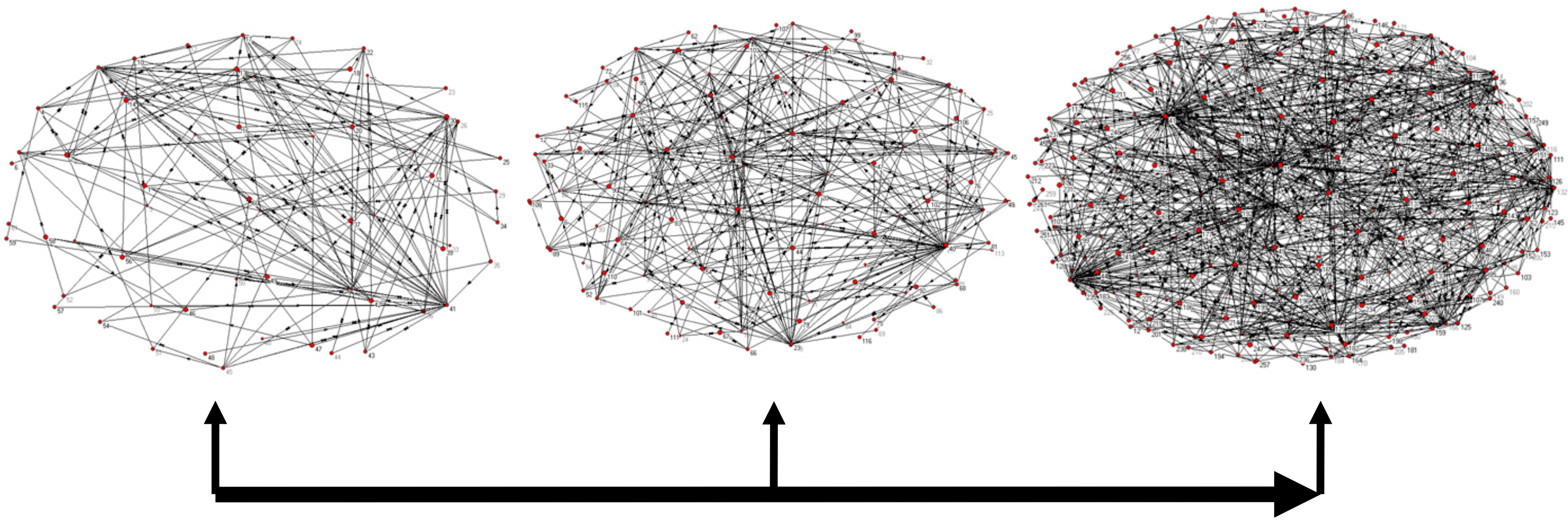


Figure 1(d)

Figure 2: Bidder Network Evolution over the Auction Duration

The three day auction event was divided into 100 time periods, and the bidding interactions between all pairs of bidders were collected to form 100 networks. The graphs below are only three of the 100 snapshots, but they are sufficient to illustrate the increased complexities of evolving interactions as the auction progresses.

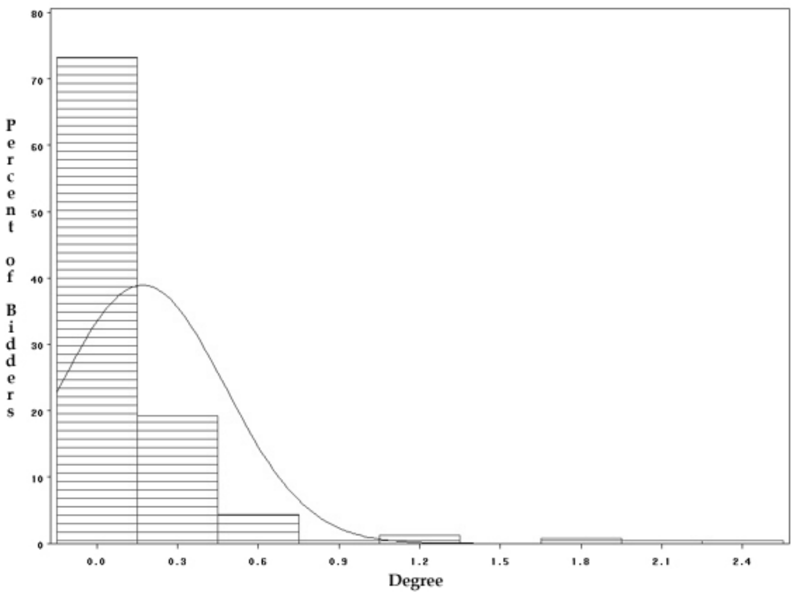


Beginning of the auction
- # Bidders: 61
- # Bids: 940
- # Bid Ties: 309
- Degree: 9.89
- Weighted Centrality: 3.06

Mid-point of the auction
- # Bidders: 122
- # Bids: 1567
- # Bid Ties: 599
- Degree: 5.84
- Weighted Centrality: 5.39

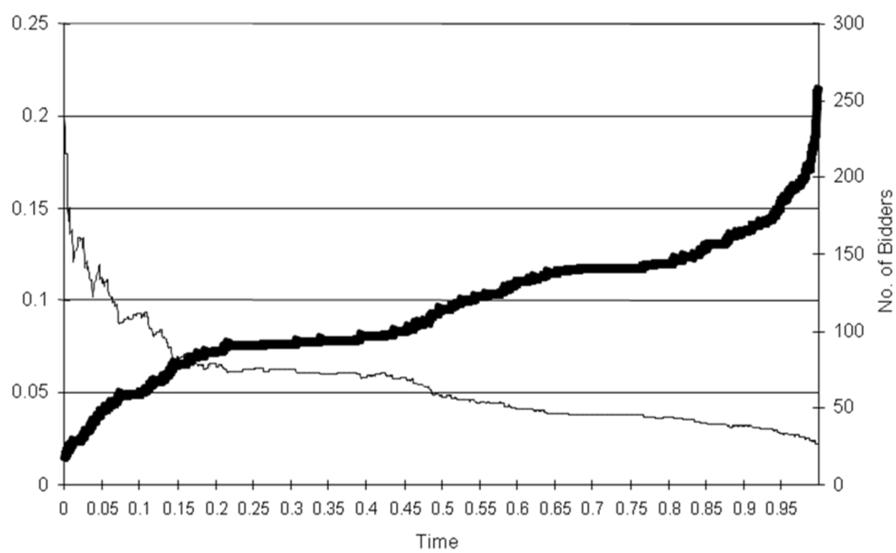
End of the auction
- # Bidders: 256
- # Bids: 3080
- # Bid Ties: 1463
- Degree: 2.91
- Weighted Centrality: 11.33

Figure 3: Degree Distribution



Note: This figure illustrates a typical Pareto characterization or power law (cf., Shore, Chu, and Bianchi 2013) of network actors; there are a few key actors who are central, and numerous players who have less impact on the bidding interactions and final sales prices.

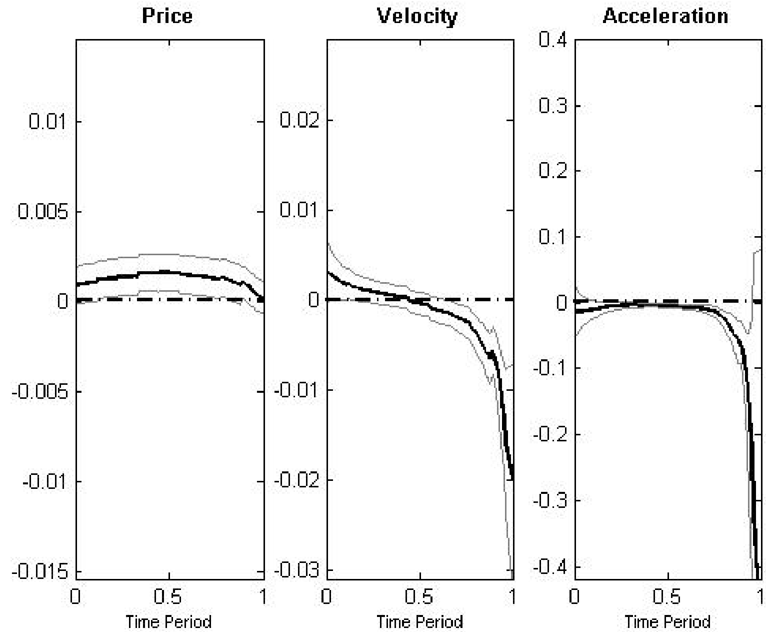
Figure 4: Increasing Number of Bidders and
Declining Distribution of *p(c)** over the Auction Duration



------ *p(c)* No. of Bidders

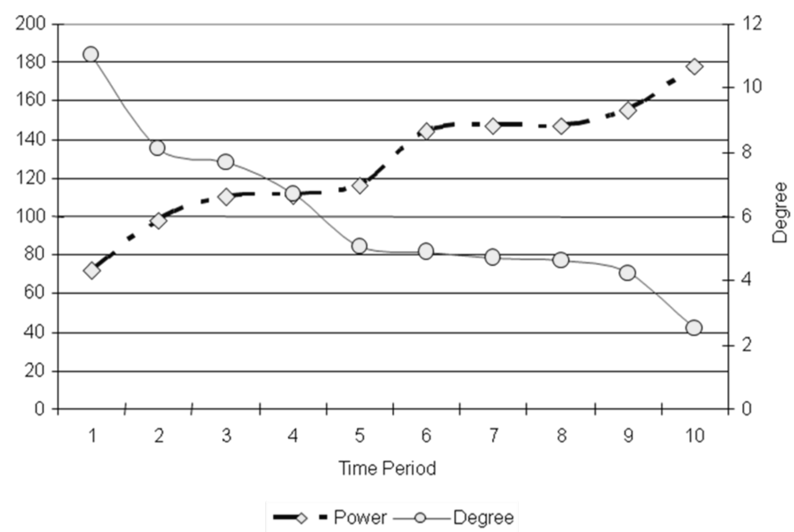
*Note: *p(c)* is the probability that a randomly selected bidder will have *c* links. This figure helps explain the Pareto or power distribution—as auctions proceed, additional new bidders join. As a result, bids are spread among a larger number of competitors, in turn reducing the average index of overall actor degrees.

Figure 5: Estimated Parameter Curves for Network Weighted Centrality Statistics



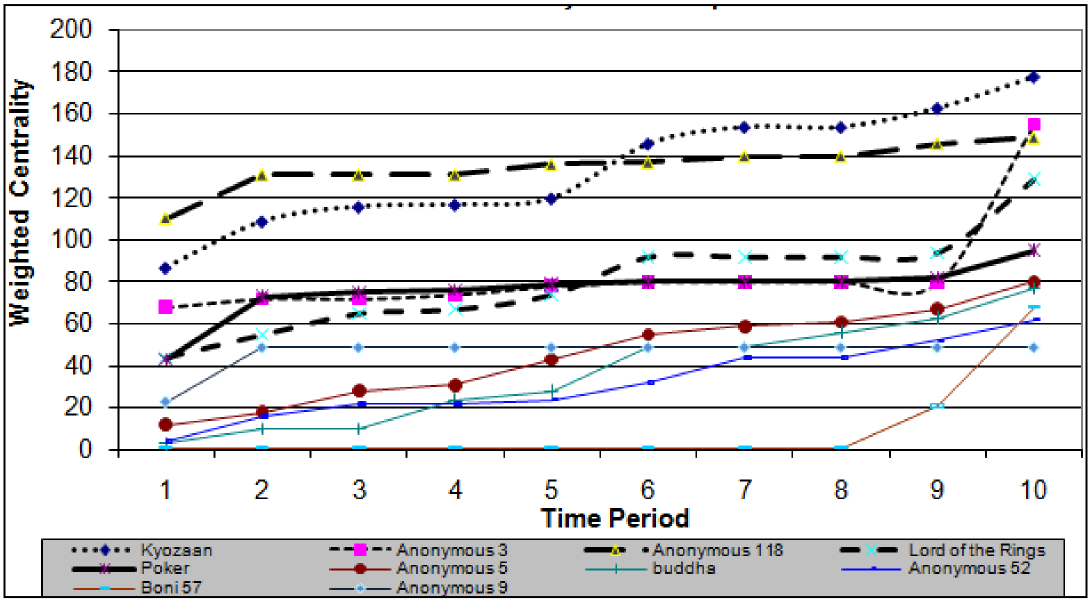
Note: The y-axes are the three standardized coefficients for price and its speeds. The upper and lower bounds of the confidence regions are also depicted, thus, for example, price is nearly always significant and positive throughout the time period, whereas velocity is first significant and positive, then significant and negative (passing through a brief transition period of flattened, non-significant velocity).

Figure 6: Network Characteristics of Kyozaan over the Auction Duration



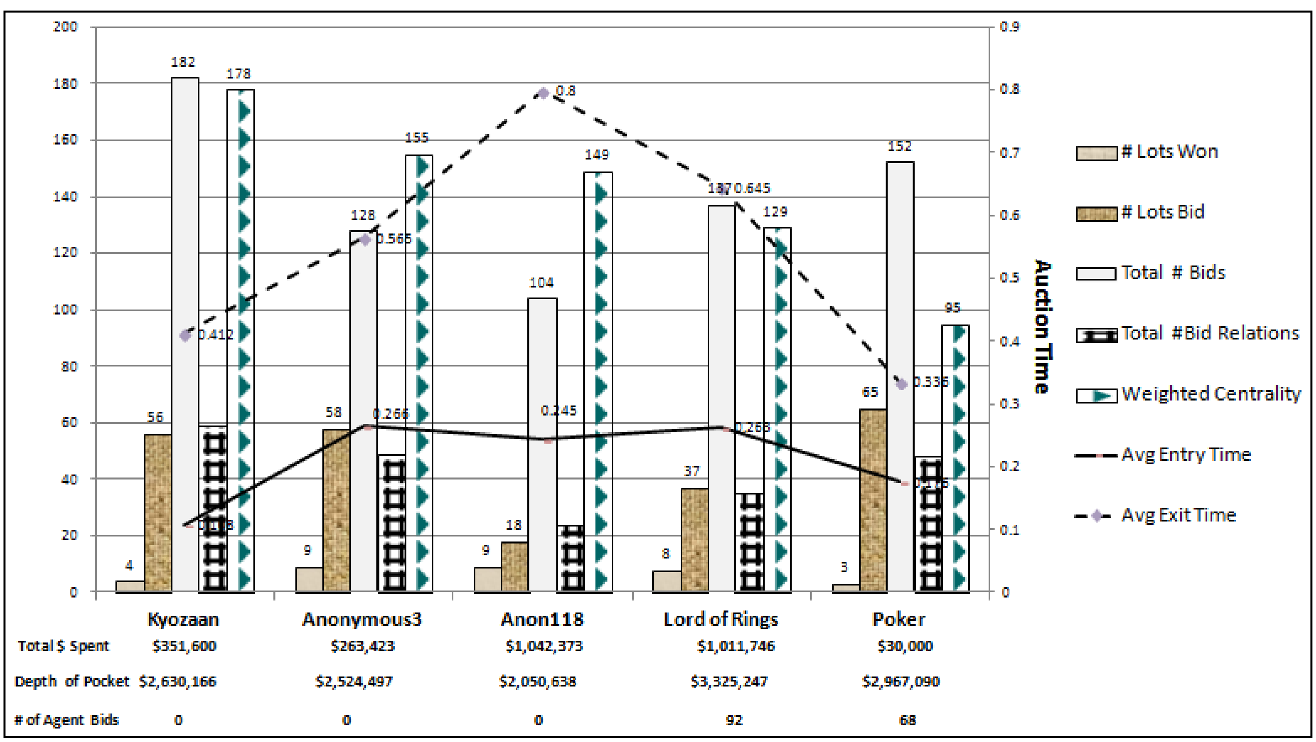
Note: While the simple centrality index ("degree") declines for this particular actor (online nickname, "Kyozaan"), the weighted centrality index continues to climb. This result is indicative of this key bidder's enhanced influence on bidders' interactions and auction prices.

Figure 7: Centralities of Top 5 Key Bidders and Next 5 Competitors



Note: The top 5 key bidders (*Kyozaan*, *Poker*, *Anonymous 3*, *Lord of the Rings* and *Anonymous 118*) begin to distinguish themselves in their interactions and influence even as early as the first time period. Their influence grows. As a conservative comparison, we also plot the next 5 bidders, and it is clear that they are not as central to the auction's interactions or price outcomes as the top 5 key bidders we identified.

Figure 8: Key Bidder Comparison



Note: These five bidders had the highest weighted centrality scores, information that was not redundant with bidding activity (e.g., Total # Bids), success (#Lots Won), expenditure (Total $ Spent).

Figure 9a: Impact of Kyozaan on Sales Results—Final Price Percentage Surplus

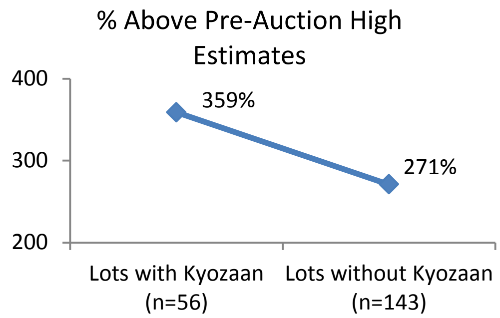


Figure 9b: Impact of Kyozaan on Sales Results—Speed to Exceed Pre-Auction Estimate

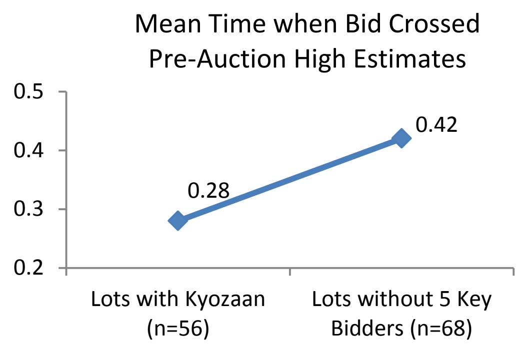
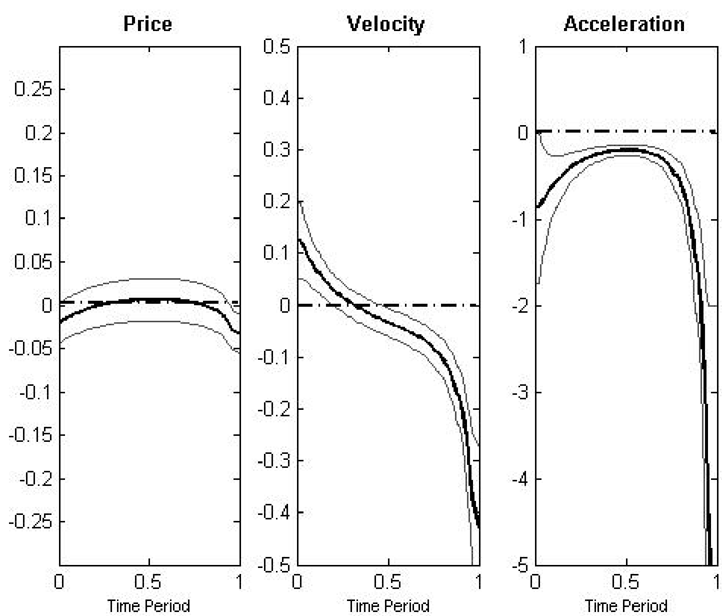


Figure 10: Estimated Parameter Curves for the Presence of 5 Key Bidders



Appendix: Analytical Details on Smoothing Splines and Functional Regression

Smoothing Splines

To recover the underlying price curves, we used penalized smoothing splines (Ramsay and Silverman 1997), which provide both small local variation and overall smoothness. For every lot auctioned, we fit a polynomial spline of degree *p:*

$$f(t) = \beta_0 + \beta_1 \times t \ + \beta_2 \times t^2 + \beta_3 \times t^3 \ldots + \beta_p \times t^p + \sum^{L} \beta_{pl} [(t-\tau)_+]^p$$

where $\tau_1\ \tau_2 \ldots \tau_L$ is a set of L knots and $u_+ = uI_{[u \geq 0]}$. The choice of *L* and *p* determines the departure of the fitted function from a straight line with higher values resulting in a rougher *f*, which may result in a potentially better fit but a poorer recovery of the underlying trend. A roughness penalty function of the following may be imposed to measure the degree of departure from the straight line:

$$PEN_m = \int \left[D^m f(t)\right]^2 dt$$

where $D^m f$, m =1, 2, 3 …., is the m^{th} derivative of the function *f*. The goal is to find a function $f^{(j)}$ that minimizes the penalized residual sum of squares:

$$PENSS_{\lambda,m}^{(j)} = \sum^{n} \left(y_i^{(j)} - f^{(j)}(t_i)\right)^2 + \lambda \times PEN_m^{(j)}$$

where the smoothing parameter λ provides the trade-off between fit $[(y_i^{(j)} - f^{(j)}(t_i))^2]$ and variability of the function (roughness) as measured by PEN_m . {Sensitivity tests were performed with different values of *p* (4, 5, 6 were used) and λ (14 different values between 0.001 and 100 were used). We found the model fit to be insensitive to different values of *p* and λ . However, the RMSE for the model was the lowest with *p*=4 and λ = 0.1. Thus, we use these smoothing parameters in recovering the price curves. }

Functional Regression

Unlike standard regression models where predictor and explanatory variables are scalars or vectors, functional regression allows one to have these variables to take on a functional form. For example, the response variable in our case is the price curve $f_j(t)$, $f_j'(t)$ (velocity) and $f_j''(t)$ (acceleration) that capture the price formation process during the auction. Potential explanatory variables are various network characteristics and other characteristics of the lot such as the type of artists (Established, emerging or other), medium of the painting (canvas or paper), and size of the painting (area in square inches). Functional regression models then allow us to understand the influence of covariates on price dynamics over time. As Ramsay and Silverman (1997) point out, this is achieved by estimating $\beta(t_i)$ for a finite number of points in time *t* (in our case t=100) and constructing a continuous parameter curve by simply interpolating between the estimated betas.

To capture the effects of the explanatory variables on each of the price dynamic variables $f_j(t)$, $f_j'(t)$ (velocity) and $f_j''(t)$ (acceleration), we run a regression for each time period (1-100) for data from all the lots (n=199). The parameter estimates associated with each explanatory variable are then plotted along with confidence bands to indicate the impact and its significance over the entire auction.