11-2016

Cost sensitive online multiple kernel classification

Doyen SAHOO
Singapore Management University, doyens@smu.edu.sg

Peilin ZHAO
Ant Financial

Steven C. H. HOI
Singapore Management University, chhoi@smu.edu.sg

Citation

SAHOO, Doyen; ZHAO, Peilin; and HOI, Steven C. H.. Cost sensitive online multiple kernel classification. (2016). *JMLR: Workshop and Conference Proceedings: 8th Asian Conference on Machine Learning: Hamilton, New Zealand, 2016 November 16-18*. 63, 65-80.
Available at: https://ink.library.smu.edu.sg/sis_research/3442

JMLR: Workshop and Conference Proceedings 63:65–80, 2016 ACML 2016

Cost-Sensitive Online Multiple Kernel Classification

Doyen Sahoo doyensahoo.2014@smu.edu.sg
School of Information Systems, Singapore Management University, Singapore

Peilin Zhao peilinzhao@hotmail.com
Artificial Intelligence Department, Ant Financial

Steven C.H. Hoi chhoi@smu.edu.sg
School of Information Systems, Singapore Management University, Singapore

Editors: Robert J. Durrant and Kee-Eung Kim

Abstract

Learning from data streams has been an important open research problem in the era of big data analytics. This paper investigates supervised machine learning techniques for mining data streams with application to online anomaly detection. Unlike conventional machine learning tasks, machine learning from data streams for online anomaly detection has several challenges: (i) data arriving sequentially and increasing rapidly, (ii) highly class-imbalanced distributions; and (iii) complex anomaly patterns that could evolve dynamically. To tackle these challenges, we propose a novel Cost-Sensitive Online Multiple Kernel Classification (CSOMKC) scheme for comprehensively mining data streams and demonstrate its application to online anomaly detection. Specifically, CSOMKC learns a kernel-based cost-sensitive prediction model for imbalanced data streams in a sequential or online learning fashion, in which a pool of multiple diverse kernels is dynamically explored. The optimal kernel predictor and the multiple kernel combination are learnt together, and simultaneously class imbalance issues are addressed. We give both theoretical and extensive empirical analysis of the proposed algorithms.

Keywords: Cost-Sensitive Learning; Online Learning; Multiple Kernel Learning;

1. Introduction

With an increasing interest in mining large data streams, there is a need to design scalable and effective learning algorithms that can comprehensively address emerging big data analytics challenges. In this paper we focus our attention on supervised learning for data streams with imbalanced labels with application to anomaly detection. Real world examples include intrusion detection (Roesch et al., 1999), anomaly detection in video surveillance (Xiang and Gong, 2008), fraud detection in markets (Donoho, 2004), and many others (Chandola et al., 2009). Despite extensive studies this remains a challenging problem due to a number of issues including: (i) *high complexity (nonlinearity)* of the (anomaly) patterns; (ii) *high class-imbalanced distributions* where the number of anomaly examples could be significantly less than normal ones; (iii) *high variety* of patterns dynamically changing due to a variety of anomaly behaviors, and *high variety* of data to be processed (heterogeneous data sources, multi-modal data etc.); (iv) *high volume and velocity* of sequentially arriving data; and (v) *pattern evolution* or concept drifts.

Most existing strategies only partly address the challenges posed by imbalanced data streams, and as a result the current state of the art is not able to provide a comprehensive solution to mining imbalanced data streams or for anomaly detection. We design Cost-Sensitive Online Multiple Kernel Classification (CSOMKC) algorithms, which provide a novel method to address all the above challenges of cost-sensitive online classification (and online anomaly detection) from big data streams, including (i) highly complex patterns via kernel methods; (ii) high class imbalance via cost-sensitive learning; (iii) high variety and data heterogeneity via multiple kernel learning; (iv) high volume and velocity via online learning algorithms; as well as (v) dealing with the concept drifting via online multi-kernel learning. Designing such a technique is a significantly challenging, and would have several applications. To the best of our knowledge, this is the first learning method that can simultaneously address all these issues in a simple, efficient, scalable yet effective framework.

Traditional classification algorithms aim to maximize accuracy. However, if the data exhibits imbalanced label distribution, accuracy becomes a poor measure of performance. As a result, we consider alternate metrics *sum* (weighted combination of specificity and sensitivity) and *cost* (weighted cost of misclassification of positive and negative instances) for evaluation of algorithms on imbalanced data streams. We develop a cost-sensitive multiple kernel formulation for the problem to be solved based on maximizing *sum* or minimizing *cost* and then design an online solution. We split the learning procedure in each online learning iteration - by first updating the kernel based prediction function for each of the kernels in the predefined pool (each kernel can be a different function, or different modality of data source), followed by dynamically exploring the multiple kernels, and updating the kernel combination. Both are done in an online manner, thus dealing with scalability concerns and concept drift. In addition, both the updates account for the cost-sensitive nature of the data streams. Further, we derive theoretical guarantees, and obtain the lower bound and upper bound of *sum* and *cost* respectively, obtained by our algorithms. We conduct extensive empirical analysis and show how our proposed methods outperform other state of the art cost-sensitive algorithms.

2. Related Work

Our work is primarily related to online learning and cost-sensitive learning and their intersecting studies. Online Learning refers to a family of scalable learning methods that incrementally update the model from a stream of data (Cesa-Bianchi and Lugosi, 2006; Hoi et al., 2014). Many of these techniques are based on maximum-margin classification, starting from the classical Perceptron Algorithm (Rosenblatt, 1958) to the more recent Online Gradient Descent (Zinkevich, 2003), Relaxed Online Maximum Margin Algorithm (ROMMA) (Li and Long, 2002), Approximate Maximal Margin Algorithm (ALMA) (Gentile, 2002), Margin Infused Relaxed Algorithm (MIRA) (Crammer and Singer, 2003), Passive Aggressive (PA) algorithms (Crammer et al., 2006), etc. Most of these methods do not consider imbalanced data distribution and hence are not suitable for imbalanced data streams or online anomaly detection. The most closely related work is the class of cost-sensitive online learning methods that directly try to optimize over a cost-sensitive metric. These include PAUM Li et al. (2002) which is a Perceptron based algorithm for uneven margins, cost-sensitive variant of Passive-Aggressive algorithms CPA_{PB} (Crammer et al., 2006); and CSOC - cost-sensitive online classification (Wang et al., 2014). There also have been some efforts in attempting to maximize the Area Under the Curve in an online manner (Zhao et al., 2011; Ding et al., 2015). Yet, none of these methods explore how to address the complexity of data through other methods (e.g. kernels or multi-kernel solutions), thus limiting their applicability in real world settings.

Another closely related area of work is one that deals with learning with kernels. Kernel methods have shown tremendous success in detecting complex nonlinear patterns owing to their ability to induce a high-dimensional reproducible kernel Hilbert space, and learning linear patterns in this space (Schölkopf and Smola, 2002). Unfortunately, there are several types of kernels, and the appropriate kernel function is not known, particularly while mining data streams where the kernel function could evolve. To address this, learning the kernel function was proposed. Some of the popular techniques include marginalized kernels (Kashima et al., 2003), idealized kernels (Kwok and Tsang, 2003), graph-based spectral learning (Bousquet and Herrmann, 2003) and non-parametric kernel learning (Zhuang et al., 2011). Multiple Kernel Learning (MKL) (Lanckriet et al., 2004; Sonnenburg et al., 2006; Gönen and Alpaydın, 2011) evolved as one of the most popular kernel learning technique. Many techniques have been proposed to solve the MKL optimization including SimpleMKL (Rakotomamonjy et al., 2008), Extended Level Method (Xu et al., 2008) and Mirror Descent (Aflalo et al., 2011). Despite this MKL was plagued with computational challenges and high retraining costs. Online Learning with Kernels (Kivinen et al., 2004) and Online Multiple Kernel Learning (Jin et al., 2010; Martins et al., 2010; Hoi et al., 2013)have also been proposed to address scalability, but they do not generalize well with imbalanced data streams.

3. CSOMKC: Cost-Sensitive Online Multiple-Kernel Classification

3.1. Problem Setting

Consider a binary classification task. Here, our goal is to learn a function $f : \mathbb{R}^d \to \mathbb{R}$ based on a sequence of training examples $\mathcal{D} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_T, y_T)\}$, where $\mathbf{x}_t \in \mathbb{R}^d$ is a d-dimensional instance representing the features and $y_t \in \mathcal{Y} = \{-1, +1\}$ is the class label assigned to $\mathbf{x}_t$. We use $\hat{y} = \text{sign}(f(\mathbf{x}))$ to predict the class assignment for any $\mathbf{x}$, and the magnitude of $f(\mathbf{x})$ to measure the classification confidence. Performances of the learnt functions are usually evaluated based on accuracy A:

$$A = \frac{\sum_{t=1}^{T} \mathbb{I}_{(\hat{y}_t = y_t)}}{T}$$

Here $\mathbb{I}$ is the indicator function resulting in 1 if the condition is true, and 0 otherwise. Unfortunately, many real world datasets present imbalanced labels. For a dataset with 99% labels as -1, a model that classifies all instances as -1 has an accuracy of 99%, which prima facie seems good, but it is obviously not a good performance. Clearly, Accuracy is not a good performance indicator for (imbalanced) classification. Accordingly, for imbalanced labels, we evaluate algorithms on cost-sensitive measures: *sum* and *cost*.

Sum is the weighted sum of *sensitivity* and *specificity* of the algorithm. Let $T_p = \{t \,|y_t = +1\}$ and $T_n = \{t \,|y_t = -1\}$ denote the set of positive and negative instances respectively. $\mathcal{M} = \{t \,|y_t \neq \hat{y}_t\}$ is the set of indexes that correspond to a mistake. Similarly, we have

$\mathcal{M}_p = \{t\ |y_t \neq \hat{y}_t;\ y_t = +1\}$, and $\mathcal{M}_n = \{t\ |y_t \neq \hat{y}_t;\ y_t = -1\}$ for mistakes on positive and negative instances. Lastly, $|S|$ denotes the number of instances in any set S. Sensitivity (S_e) and Specificity (S_p) are defined as: $S_e = \frac{|T_p| - |\mathcal{M}_p|}{|T_p|}$ and $S_p = \frac{|T_n| - |\mathcal{M}_n|}{|T_n|}$. The weighted *sum* parameterized by $\alpha \in [0, 1]$ is given as:

$$sum = \alpha(S_e) + (1 - \alpha)(S_p)$$

For $\alpha = 0.5$, *sum* is reduced to balanced accuracy.

Cost is the weighted sum of mistakes on positive and negative instances, and is parameterized by $c \in [0, 1]$:

$$cost = c(|\mathcal{M}_p|) + (1 - c)(|\mathcal{M}_n|)$$

Here, the aim is to tradeoff the cost of wrongly classifying a positive instance against the cost of wrongly classifying a negative instance using tradeoff parameter c.

Our objective is to either maximize *sum* or minimize *cost*. We transform both to the following objective:

$$\min_f \sum_{y_t=+1} \rho \mathbb{I}_{\hat{y_t} \neq y_t} + \sum_{y_t=-1} \mathbb{I}_{\hat{y_t} \neq y_t} \tag{1}$$

where $\rho = \frac{\alpha |T_n|}{(1-\alpha)|T_p|}$ for maximizing *sum*, and $\rho = \frac{c}{1-c}$ for minimizing *cost*.

3.2. Cost-Sensitive Multiple Kernel Classification

Data streams may exhibit complex nonlinear patterns. Kernels have evolved as popular tools to detect nonlinearity by mapping a low dimensional feature space to a high dimensional space. We aim to learn a kernel-based prediction function to optimize the cost-sensitive measure in Eq. (1), in order to detect nonlinear patterns. We propose to use Multiple Kernel Learning (MKL) so that: (i) prior knowledge of appropriate kernel is not required; (ii) model's learning capacity increases when multiple kernels complement each other; and (iii) heterogeneous data sources can be combined into one prediction model (e.g. using different kernels for numeric and text data, or different kernels for different modalities of data). This way we are able to learn a powerful model which can detect complex nonlinear patterns and handle a variety of data.

To do this, we first define a loss function as the convex surrogate of the indicator function (which is not convex and has been used in Eq. (1)), and we get:

$$\ell^{\rho}(f, (\mathbf{x}, y)) = (\rho \mathbb{I}_{y=1} + \mathbb{I}_{y=-1}) * \max(0, 1 - y(f \cdot \mathbf{x})) \tag{2}$$

Using this loss function, Eq. (1) can be cast into the following regularized optimization (C is the regularization tradeoff parameter):

$$\min_f \ \frac{1}{2}\|f\|^2 + C \sum_{t=1}^{T} \ell^{\rho}(f, (\mathbf{x}, y)) \tag{3}$$

Our goal is to solve this using MKL. Consider a collection of m different predefined kernel functions $\mathcal{K} = \{\kappa_i :\ \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}, i = 1, \ldots, m\}$. MKL aims to learn a kernel-based prediction model by identifying the best convex combination of the m kernels, that is, a weighted combination $\theta = (\theta_1, \ldots, \theta_m)$. The proposed cost-sensitive multiple kernel machine can be cast into the following optimization:

$$\min_{\theta \in \Delta} \min_{f \in \mathcal{H}_{K(\theta)}} \frac{1}{2} \|f\|^2_{\mathcal{H}_{K(\theta)}} + C \sum_{t=1}^{T} \ell^{\rho}(f, (\mathbf{x}_t, y_t)) \tag{4}$$

where $\Delta = \{\theta \in \mathbb{R}^m_+ \, | \theta^T \mathbf{1}_m = 1\}$, $K(\theta)(\cdot, \cdot) = \sum_{i=1}^{T} \theta_i \kappa_i(\cdot, \cdot)$; $\mathcal{H}_{K(\theta)}$ is the Reproducible Kernel Hilbert Space induced by the multiple kernel combination; and $\ell^{\rho}(f, (\mathbf{x}_i, y_i))$ is a convex loss function as defined in Eq. (2). The optimization can be solved by adapting existing techniques (see section on Related Work). However, it is computationally challenging and not suitable for large data streams, and data with temporal properties. Most of the techniques suffer from extremely high retraining cost and expensive memory requirements. To tackle this challenge, we propose online learning based CSOMKC algorithms.

3.3. CSOMKC Algorithms

We design Cost-Sensitive Online Multiple Kernel Classification (CSOMKC), which learns the model in an online learning (hence scalable and adaptive to temporal patterns) setting. Instances are sequentially processed, and in each iteration we aim to update the kernel prediction model and the kernel combination. Doing both simultaneously in an online manner is significantly challenging, and due to imbalanced data, traditional methods can not be directly applied. We update the model via a 2-step approach: updating each kernel predictor, and updating the kernel combination.

3.3.1. Cost-Sensitive online kernel classification

We first develop a single-kernel cost-sensitive online kernel classification method. In every iteration of the online learning procedure, a kernel classifier with the prediction function $f(\mathbf{x})$ is updated by gradient descent (Kivinen et al., 2004; Wang et al., 2014) when the classifier suffers a nonzero loss. Using the cost-sensitive loss from Eq. (2) we can obtain the cost-sensitive gradient descent update by taking the derivative. The update rule for each individual kernel-based model is given by:

$$\begin{aligned} f_{t+1}(x) &= f_t(x) - \eta \nabla_f \ell_t^{\rho}(f_t, (\mathbf{x}_t, y_t)) \\ &= f_t(x) + \eta \rho_t y_t \kappa(\mathbf{x}_t, \mathbf{x}) \end{aligned}$$

where ρ_t manages the cost-sensitivity, by setting $\rho_t = \mathbb{I}_{(y_t=+1)} + \rho \mathbb{I}_{(y_t=-1)}$, and η is the learning rate parameter. At the end of each online learning round, we can express the prediction function as a kernel expansion (Schölkopf and Smola, 2002) $f_{t+1}(\mathbf{x}) = \Sigma_{i=1}^{t} \lambda_i \kappa(\mathbf{x}_i, \mathbf{x})$ where the λ_i coefficients are computed based on the update rule. For non-zero loss on the i^{th} instance, $\lambda_i \neq 0$ (the instance becomes a support vector) otherwise $\lambda_i = 0$.

3.3.2. Online multi-kernel combination learning

All $i = 1, \ldots, m$ kernel predictions (denoted by f_t^i) are combined to make a final weighted prediction on each iteration:

$$\hat{y}_t = \text{sign}\Big(\sum_{i=1}^{m} w_t^i \big(f_t^i(\mathbf{x}_t) \big) \Big)$$

We propose two new cost-sensitive weight combination learning schemes: 1) Based on Exponentiated Gradient; and 2) Based on Online Gradient Descent.

EG Combination: We aim to learn the optimal cost-sensitive convex combination of weights $\mathbf{w} = (w^1, \dots, w^m)^\top$, where w^i is set to $1/m$ at the beginning of the learning task. In our approach, we modify and adapt the *EG* algorithm (Kivinen and Warmuth, 1997) to update the cost-sensitive weights. We define

$$\mathbf{f}_t(\mathbf{x}_t) = (f_t^1(\mathbf{x}_t), \dots, f_t^m(\mathbf{x}_t))^\top,$$

and formulate the rule of updating $\mathbf{w}$ as follows:

$$\mathbf{w}_{t+1} = \arg\min_{\mathbf{w}\in\Delta} D_{KL}(\mathbf{w}\|\mathbf{w}_t) + \eta_{eg}\ell^\rho(\mathbf{w}, (\mathbf{f}_t(\mathbf{x}_t), y_t)),$$

where $\Delta = \{\mathbf{w} \in \mathbb{R}_+^m \,|\mathbf{w}^\top\mathbf{1}_m = 1\}$, $D_{KL}(\mathbf{u}\|\mathbf{v}) = \sum_i u_i \ln(\frac{u_i}{v_i})$ is the KL-divergence.

This optimization trades off two major concerns: (i) minimizing weight distribution between new weights and old weights (measured by KL-divergence); and (ii) new weights should suffer a small loss on the instance in the current iteration. The trade-off parameter is $\eta_{eg} > 0$. To obtain a closed-form solution for the above optimization, we approximate the loss function by using its first-order Taylor expansion at $\mathbf{w}_t$, and we get:

$$\begin{aligned}\mathbf{w}_{t+1} =& \arg\min_{\mathbf{w}\in\Delta} D_{KL}(\mathbf{w}\|\mathbf{w}_t) + \eta_{eg}\ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t)) \\ &+ \eta_{eg}\partial_{\mathbf{w}}\ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t)) \cdot (\mathbf{w} - \mathbf{w}_t)\end{aligned} \tag{5}$$

For this problem, we can derive a closed-form solution as:

$$\mathbf{w}_{t+1} \;=\; \frac{\mathbf{w}_t \odot \exp(-\eta_{eg}\nabla_{\mathbf{w}}\ell^\rho(\mathbf{w}_t; (\mathbf{f}_t(\mathbf{x}_t), y_t))}{\|\mathbf{w}_t \odot \exp(-\eta_{eg}\nabla_{\mathbf{w}}\ell^\rho(\mathbf{w}_t; (\mathbf{f}_t(\mathbf{x}_t), y_t))\|_1},$$

where $\odot$ is element-wise product. It is easy to check that $\nabla_{\mathbf{w}}\ell^\rho(\mathbf{w}_t; (\mathbf{f}_t(\mathbf{x}_t), y_t)) = -\rho_t y_t \mathbf{f}_t(\mathbf{x}_t)$ when $\ell^\rho(\mathbf{w}; (\mathbf{f}(\mathbf{x}), y)) > 0$, and 0 otherwise, where ρ_t is set in the same manner as for learning a single kernel predictor. We refer to this approach as CSOMKC(EG).

OGD combination: Since CSOMKC(EG) learns a convex combination, we aim to increase the generality by learning the optimal cost-sensitive linear combination of multiple kernel predictors. Similar with the EG update (5), this can be cast into the following optimization:

$$\mathbf{w}_{t+1} = \arg\min_{\mathbf{w}} \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|_2^2 + \eta_{ogd}\ell^\rho(\mathbf{w}, (\mathbf{f}_t(\mathbf{x}_t), y_t))$$

where $\mathbf{f}_t(\mathbf{x}_t)$ is the vector representing the predictions made by each individual kernel classifier. After replacing the loss function with its first order Taylor expansion, the update rule based on Online Gradient Descent can be derived as (Zinkevich, 2003; Wang et al., 2014):

$$\mathbf{w}_{t+1} \;=\; \mathbf{w}_t - \eta_{ogd}\nabla_{\mathbf{w}}\ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t)),$$

where the weights are updated only when the combined prediction suffers a loss, i.e., $\ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t)) > 0$; η_{ogd} represents the learning rate for the weight update of the combination; and ρ_t regulates the update to account for cost-sensitivity. This approach referred to as CSOMKC(OGD).

Both approaches are similar in the problem being addressed. However, EG uses multiplicative updates, whereas OGD uses additive updates. As a result, while EG may converge to a solution faster, in the long run OGD will outperform it. Both the approaches are outlined in Algorithm 1.

Algorithm 1 Cost-Sensitive Online Multiple Kernel Classification

INPUTS: Kernels: $k_i(\cdot,\cdot) : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}, i = 1, \ldots, m$; Learning rates: $\eta > 0, \eta_{eg} > 0$; Cost Sensitive Parameter: $\rho > 0$
Initialization: $\mathbf{f}^1 = \mathbf{0}, \mathbf{w}^1 = \frac{1}{m}\mathbf{1}$
for $t = 1, 2, \ldots$ **do**
  Receive an instance: $\mathbf{x}_t$
  Predict $\hat{y}_t = \text{sign}\Big(\mathbf{w}_t \cdot \mathbf{f}_t(\mathbf{x}_t)\Big)$
  Receive the class label: y_t
  Set $\rho_t = \rho * \mathbb{I}(y_t = 1) + \mathbb{I}(y_t = -1)$
  for $i = 1, 2, \ldots, m$ **do**
    Set $\ell^\rho(f_t^i; (\mathbf{x}_t, y_t)) = \rho_t \max(0, 1 - y_t f_t^i(\mathbf{x}_t))$
    if $\ell^\rho(f_t^i; (\mathbf{x}_t, y_t)) > 0$ **then**
      Update $f_{t+1}^i(\mathbf{x}) = f_t^i(\mathbf{x}) + \eta_i \rho_t y_t \kappa_i(\mathbf{x}_t, \mathbf{x})$
    end if
  end for
  Set $\ell^\rho(\mathbf{w}_t; (\mathbf{f}_t(\mathbf{x}_t), y_t)) = \rho_t \max(0, 1 - y_t \mathbf{w}_t \cdot \mathbf{f}_t(\mathbf{x}_t))$
  if $\ell^\rho(\mathbf{w}_t; (\mathbf{f}_t(\mathbf{x}_t), y_t)) > 0$ **then**
    Update $\mathbf{w}_{t+1} = \frac{\mathbf{w}_t \odot \exp(\eta_{eg} \rho_t y_t \mathbf{f}_t(\mathbf{x}_t))}{\|\mathbf{w}_t \odot \exp(\eta_{eg} \rho_t y_t \mathbf{f}_t(\mathbf{x}_t))\|_1}$ for update by EG **OR**
    Update $\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_{ogd} \nabla_{\mathbf{w}} \ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t))$ for update by OGD
  end if
end for

3.4. Theoretical Analysis

In this section, we present the theoretical properties of the algorithm CSOMKC(EG). We derive the loss bound for Algorithm 1 when the kernel combination is learnt by EG algorithm. We assume $\kappa(\mathbf{x}, \mathbf{x}) \leq 1$ for all κ and $\mathbf{x}$. We define the optimal regularized objective value for the kernel $\kappa_i(\cdot,\cdot)$ denoted by $O(\kappa_i, \ell^\rho, \mathcal{D})$ with respect to the dataset $\mathcal{D}$ as:

$$\min_{f_i \in \mathcal{H}_{\kappa_i}} \left(\sum_{t=1}^{T} \ell^\rho(f_i, (\mathbf{x}_t, y_t)) + \|f_i\|_{\mathcal{H}_{\kappa_i}} \sqrt{\rho^2 L_i^p + L_i^n} \right)$$

where $L_i^p = \sum_{y_t=1} \mathbb{I}(y_t f_i(\mathbf{x}_t) \leq 1)$, $L_i^n = \sum_{y_t=-1} \mathbb{I}(y_t f_i(\mathbf{x}_t) \leq 1)$.

Lemma 1 *Assume $\|\mathbf{f}_t(\mathbf{x}_t)\|_\infty \leq R$, and the CSOMKC(EG) algorithm is run with learning rate $\eta_{eg} = \sqrt{\frac{2\ln m}{R^2 T}}$ on a sequence of examples $\mathcal{D} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_T, y_T)\}$. Then for any combination of function $\mathbf{f}_t = \sum_{i=1}^m w^i f_t^i$, $\mathbf{w} \in \Delta$ we have*

$$\sum_{t=1}^{T} \ell^\rho(\mathbf{w}_t, (\mathbf{f}(\mathbf{x}_t), y_t)) \leq \sum_{t=1}^{T} \ell^\rho(f_t, (\mathbf{x}_t, y_t)) + R\sqrt{\frac{T \ln m}{2}}.$$

Moreover, we have

$$\rho|\mathcal{M}_p| + |\mathcal{M}_n| \leq \min_{1 \leq i \leq m} O(\kappa_i, \ell^\rho, \mathcal{D}) + R\sqrt{\frac{T \ln m}{2}}.$$

Proof The idea of this proof follows the principle of similar proof in Cesa-Bianchi and Lugosi (2006). We denote $l_t(\mathbf{w}_t) = \ell^\rho(\mathbf{w}_t, (\mathbf{f}(\mathbf{x}_t), y_t))$. Then, by convexity of l_t, we get

$$l_t(\mathbf{w}_t) - l_t(\mathbf{w}) \leq -(\mathbf{w} - \mathbf{w}_t) \cdot \nabla l_t(\mathbf{w}_t). \tag{6}$$

Next, we would bound the right hand side of the above inequality. To facilitate the analysis, we denote $\mathbf{z} = \eta_{eg} \nabla l_t(\mathbf{w}_t)$ and $\mathbf{v} = \mathbf{w}_t \cdot \mathbf{z} - \mathbf{z}$, then

$$\begin{aligned}
&-(\mathbf{w} - \mathbf{w}_t) \cdot \mathbf{z} \\
&= -\mathbf{w} \cdot \mathbf{z} + \mathbf{w}_t \cdot \mathbf{z} - \ln(\sum_{i=1}^m w_t^i e^{v_i}) + \ln(\sum_{i=1}^m w_t^i e^{v_i}) \\
&= -\mathbf{w} \cdot \mathbf{z} - \ln(\sum_{i=1}^m w_t^i e^{-z_i}) + \ln(\sum_{i=1}^m w_t^i e^{v_i}) \\
&= \sum_{j=1}^m w^j \ln e^{-z_j} - \ln(\sum_{i=1}^m w_t^i e^{-z_i}) + \ln(\sum_{i=1}^m w_t^i e^{v_i}) \\
&= \sum_{j=1}^m w^j \ln(\frac{1}{w_t^j} \frac{w_t^j e^{-z_j}}{\sum_{i=1}^m w_t^i e^{-z_i}}) + \ln(\sum_{i=1}^m w_t^i e^{v_i}) \\
&= \sum_{j=1}^m w^j \ln \frac{w_{t+1}^j}{w_t^j} + \ln(\sum_{i=1}^m w_t^i e^{v_i}) \\
&= D_{KL}(\mathbf{w} \| \mathbf{w}_t) - D_{KL}(\mathbf{w} \| \mathbf{w}_{t+1}) + \ln(\sum_{i=1}^m w_t^i e^{v_i}).
\end{aligned}$$

Plugging the above equality into the inequality (6) and summing over t, we get

$$\sum_{t=1}^T [l_t(\mathbf{w}_t) - l_t(\mathbf{w})] \leq \frac{1}{\eta_{eg}} [D_{KL}(\mathbf{w} \| \mathbf{w}_1) + \sum_{t=1}^T \ln(\sum_{i=1}^m w_t^i e^{v_i})],$$

by omitting $-D_{KL}(\mathbf{w} \| \mathbf{w}_{T+1})$. To bound the right hand side of the inequality, note that $D_{KL}(\mathbf{w} \| \mathbf{w}_1) \leq \ln m$, since $\mathbf{w}_1 = (1/m, \ldots, 1/m)$. We need bound the second term in the right hand side. Since $\|\mathbf{f}_t(\mathbf{x}_t)\|_\infty \leq R$, then $|z_i| \leq \eta_{eg} R$, and applying Hoeffding's inequality:

$$\ln(\sum_{i=1}^m w_t^i e^{v_i}) \leq \eta_{eg}^2 R^2 / 2.$$

As a result, we get the following inequality,

$$\sum_{t=1}^T [l_t(\mathbf{w}_t) - l_t(\mathbf{w})] \leq \frac{\ln m}{\eta_{eg}} + \frac{\eta_{eg} R^2 T}{2} = R\sqrt{\frac{T \ln m}{2}}.$$

Re-arranging this concludes the first part of the theorem. If we set $w^i = 1$ and $w^j = 0$, for $j \neq i$, using $\ell^\rho(\mathbf{w}_t, (\mathbf{f}_t(\mathbf{x}_t), y_t)) \geq \rho_t$ when prediction is wrong, and combining with Lemma 1 of the paper (Wang et al., 2014), the above inequality will derive the second part. ■

Using this result, we now bound the *sum* incurred by CSOMKC.

Theorem 1 *After receiving a sequence of T training examples $\mathcal{D} = \{(\mathbf{x}_t, y_t), t = 1, \dots, T\}$, the weighted sum $= \alpha(S_e) + (1-\alpha)(S_p)$ achieved by Algorithm 1 for kernel combination update by EG, with $\eta_i = \|f_i\|_{\mathcal{H}_{\kappa_i}} / \sqrt{\rho^2 M_i^p + M_i^n}$ and $\rho = \frac{\alpha T_n}{(1-\alpha)T_p}$, is bounded as:*

$$sum \geq \frac{(1-\alpha)}{T_n} \left[\min_{1 \leq i \leq m} O(\kappa_i, \ell^\rho, \mathcal{D}) + R\sqrt{\frac{T \ln m}{2}} \right].$$

Proof We know that $sum = 1 - \frac{1-\alpha}{|T_n|}\left[\frac{\alpha|T_n|}{(1-\alpha)|T_p|}|\mathcal{M}_p| + |\mathcal{M}_n|\right]$. Using $\rho = \frac{\alpha|T_n|}{(1-\alpha)|T_p|}$, and combining the above with Lemma 1 gives us the desired result. ■

We now derive a bound for the *cost* suffered, which unlike *sum* does not require the estimates of ratio $\frac{T_n}{T_p}$ (which may be unknown in advance). We set $\rho = \frac{1-c}{c}$, where $c \in (0, 1)$.

Theorem 2 *After receiving a sequence of T training examples $\mathcal{D} = \{(\mathbf{x}_t, y_t), t = 1, \dots, T\}$, the weighted cost $= c(|\mathcal{M}_p|) + (1-c)(|\mathcal{M}_n|)$ suffered by Algorithm 1 for kernel combiantion update by EG, with $\eta_i = \|f_i\|_{\mathcal{H}_{\kappa_i}} / \sqrt{\rho^2 M_i^p + M_i^n}$ and $\rho = \frac{c}{1-c}$, is bounded as:*

$$cost \leq (1-c) \left[\min_{1 \leq i \leq m} O(\kappa_i, \ell^\rho, \mathcal{D}) + R\sqrt{\frac{T \ln m}{2}} \right].$$

Proof From the definition of *cost*, we know that $cost = (1-c)(\frac{c}{1-c}|\mathcal{M}_p| + |\mathcal{M}_n|) = (1-c)(\rho|\mathcal{M}_p| + |\mathcal{M}_n|)$. Combining this with Lemma 1 proves this theorem. ■

Time Complexity: Traditional linear online algorithms execute in $O(T)$ where T is the number of instances. For an online kernel algorithm, in the worst case scenario, where every instance becomes a support vector, the algorithm would run in $O(T^2)$. However, applying budget techniques like the Randomized Budget Perceptron (Cavallanti et al., 2007) reduces the running time to $O(BT)$ where B is the user-specified budget, and $B << T$. The multiple kernel variants with m kernels require time complexity of running m online kernel algorithms. Therefore, the time to run CSOMKC is in $O(mBT)$, i.e., the time complexity with budget approximations is linear in number of instances.

4. Experimental Evaluation

We now present comprehensive empirical analysis of our proposed scheme, where we have evaluated algorithms' performance on imbalanced datasets, and anomaly detection tasks.

4.1. Datasets

We use a wide variety of datasets, across a wide spectrum of applications and varying number of instances, features, and imbalance ratios. All the datasets are publicly available and were retrieved from UCI repository, LIBSVM, and KDD Cup 2008. The datasets can be categorized into 6 regular imbalanced datasets, and 2 anomaly detection datasets. Among the imbalanced datasets we have: Spam and Webspam datasets which are self explanatory; Cod-rna is a bioinformatics dataset; Twitter dataset is about detecting buzz in social media, and is temporal in nature; Internet Ads is about predicting whether images in a given URL are ads or not; and Page-blocks attempts to classify page blocks into text or not. The anomaly detection datasets are KDD08 (from KDD Cup 2008 dataset on breast cancer); and Malware dataset built from Android Malware Genome Project which is about classifying apps as malware or not. The other details are given in Table 1.

Table 1: Details of the datasets used

Data ID	Name of Dataset	Instances	Features	$T_n : Tp$
Regular Imbalanced Datasets				
D1	Spam	4601	57	1.53
D2	Webspam	350000	254	1.54
D3	Cod-rna	59535	8	2.00
D4	Twitter	140707	77	4.07
D5	Internet Ads	3279	1556	6.14
D6	Page-blocks	21888	10	8.79
Highly Imbalanced Anomaly Detection Datasets				
D7	KDD08	102294	117	163.20
D8	Malware	208243	122	549.91

4.2. Kernels

Different kernels are suitable for different types of data. For example polynomial kernels which implicitly construct new polynomial features are more suited for NLP(among other tasks). Gaussian kernels have been the most widely used kernels for a variety of tasks. Additionally, depending on the data distribution, appropriate parameters need to be set, which is often done by validation techniques. To automatically select from a rich pool of kernels, we predefine a diverse set of 10 kernels which include three polynomial kernels $\kappa(x,y) = (x^T y)^p$ of degree parameter $p = 1, 2, 3, 4$, five RBF kernels ($\kappa(x,y) = e^{(\frac{-||x-y||^2}{2\sigma^2})}$) of kernel width parameter $\sigma = 2^{-2}, 2^{-1}, 2^0, 2^1, 2^2$, and a sigmoid kernel($\kappa(x,y) = \tanh(xy)$).

4.3. Algorithms Compared

Online Learning with Kernels suffers from an unbounded growth of support vectors. For large data, even with a good classifier, the number of support vectors keeps growing linearly. To make the computation realistically possible, a budget on the number of support vectors is required. We set a budget of 2000 support vectors per kernel classifier for all datasets with number of instances greater than $50,0000$, and apply the Randomized Budget Perceptron Cavallanti et al. (2007) by randomly discarding a support vector when the budget constraint is violated, and hence approximating the kernel predictions. This approximation is done for all algorithms that make kernel based predictions. In our experiments, OMKCSC-EG and OMKCSC-OGD are compared the following algorithms:
Linear Algorithms: We compare with the following linear algorithms:

1. Simple Linear Online Gradient Descent (Zinkevich, 2003)

2. PAUM (Li et al., 2002) Perceptron based method for uneven margins

3. CPA_{PB} (Crammer et al., 2006) which is a cost-sensitive variant of the popular online Passive Aggressive algorithms and

4. CSOL (Wang et al., 2014) that directly optimizes cost-sensitive measures.

Kernel Algorithms: We compare with the following online kernel methods:

1. Best Single Cost-Sensitive Kernel (CSC Kernel) determined by validation over first few samples of the data

2. Online Multiple Kernel Classification with hedge combination OMKC(H) (Hoi et al., 2013)

3. OMKC with linear combination OMKC(OGD) (Sahoo et al., 2014)

4. Finally, we also compare with OMKCSC(U), where a uniform combination of the multiple cost-sensitive kernel predictors is used, i.e., our strategies of combining multiple kernels via EG or OGD are ignored.

All algorithms are evaluated on the basis of *sum* with $\alpha = 0.5$ (which is essentially the balanced accuracy); and *cost* with $c = 0.95$. For a comprehensive study, we also evaluate the results of sensitivity and specificity of each algorithm.

4.4. Parameters

There are 5 parameters required to be selected: Learning rate η for each kernel, cost-sensitive parameter ρ, and the combination learning rate for EG η_{eg} and for OGD η_{ogd}. For a fair comparison with OMKC, we set $\eta = 1$ for all algorithms. We set the combination parameters $\eta_{eg} = 0.1$, and $\eta_{ogd} = 0.1$ for all cases, and also perform sensitivity analysis for them. The cost-sensitive parameter ρ is set according to the objective being optimized. While trying to maximize *sum*, we set $\alpha = 0.5$, and accordingly ρ is set as $\rho = \frac{(1-\alpha)T_p}{\alpha T_n}$. While trying to minimize the *cost*, we set $c = 0.95$, and accordingly ρ is set as $\rho = \frac{1-c}{c}$.

4.5. Results and Discussion

All results are reported as the average over multiple permutations, except the Twitter dataset which is temporal in nature (random permutations are meaningless). The details are in Table 2. Further, the *sum* and *cost* of all algorithms, as the number of instances grows can be visualized in Figure 1.

From Table 2, we see that our proposed CSOMKC(EG) and CSOMKC(OGD) almost always secure the highest *sum* (balanced accuracy), and the lowest *cost*. For *sum*, which in our case is a measure of balanced accuracy, CSOMKC(EG) and CSOMKC(OGD) get superior performances in all datasets with the exception CSOMKC(EG) in InternetAds. For the anomaly detection datasets, the proposed algorithms achieve excellent results, beating the benchmarks by a significant margin. In *cost* performance, with the exception of CSOMKC(EG) in InternetAds, we get a significant outperformance of the proposed techniques in all cases. However, for all other cases, the performance is phenomenal. InternetAds has a very high dimensionality, and with the usage of multiple kernels, suffers from a relatively slow convergence for a small number of instances. An interesting insight can be drawn

Table 2: Evaluation of all the algorithms on all datasets based on the *sum* and *cost*.

SUM performance of algorithms				
	Spam	Webspam	Cod-rna	Twitter
Linear	0.879 ± 0.002	0.903 ± 0.000	0.835 ± 0.003	0.904 ± 0.000
CPA_{PB}	0.809 ± 0.002	0.853 ± 0.000	0.801 ± 0.000	0.901 ± 0.000
PAUM	0.883 ± 0.002	0.902 ± 0.000	0.836 ± 0.002	0.904 ± 0.000
CSOL	0.870 ± 0.003	0.905 ± 0.000	0.854 ± 0.001	0.915 ± 0.000
CSC Kernel	0.846 ± 0.083	$\mathbf{0.945 \pm 0.001}$	0.910 ± 0.003	0.882 ± 0.000
OMKC(H)	0.866 ± 0.002	0.934 ± 0.000	0.885 ± 0.001	0.913 ± 0.000
OMKC(OGD)	0.825 ± 0.008	0.900 ± 0.000	0.863 ± 0.002	0.889 ± 0.000
CSOMKC(U)	0.850 ± 0.005	0.899 ± 0.000	0.874 ± 0.000	0.915 ± 0.000
CSOMKC(EG)	$\mathbf{0.902 \pm 0.004}$	$\mathbf{0.945 \pm 0.001}$	$\mathbf{0.912 \pm 0.000}$	$\mathbf{0.935 \pm 0.000}$
CSOMKC(OGD)	$\mathbf{0.896 \pm 0.005}$	$\mathbf{0.944 \pm 0.001}$	$\mathbf{0.914 \pm 0.001}$	$\mathbf{0.932 \pm 0.000}$
	Internet-Ads	Page-Blocks	KDD08	Malware
Linear	0.970 ± 0.001	0.743 ± 0.003	0.564 ± 0.003	0.466 ± 0.007
CPA_{PB}	0.971 ± 0.000	0.747 ± 0.003	0.625 ± 0.001	0.524 ± 0.005
PAUM	0.974 ± 0.002	0.780 ± 0.000	0.558 ± 0.006	0.472 ± 0.005
CSOL	0.976 ± 0.002	0.819 ± 0.001	0.684 ± 0.006	0.804 ± 0.001
CSC Kernel	0.967 ± 0.096	0.904 ± 0.014	0.649 ± 0.083	0.688 ± 0.000
OMKC(H)	0.972 ± 0.002	0.845 ± 0.002	0.612 ± 0.007	0.814 ± 0.000
OMKC(OGD)	0.975 ± 0.001	0.793 ± 0.001	0.616 ± 0.010	0.781 ± 0.003
CSOMKC(U)	0.979 ± 0.000	0.859 ± 0.002	0.696 ± 0.008	0.771 ± 0.004
CSOMKC(EG)	0.976 ± 0.000	$\mathbf{0.913 \pm 0.000}$	$\mathbf{0.748 \pm 0.001}$	$\mathbf{0.836 \pm 0.018}$
CSOMKC(OGD)	$\mathbf{0.985 \pm 0.002}$	$\mathbf{0.912 \pm 0.001}$	0.733 ± 0.007	$\mathbf{0.84 \pm 0.007}$
COST performance of algorithms				
	Spam	Webspam	Cod-rna	Twitter
Linear	205.2 ± 11.172	17233.975 ± 88.282	4560.625 ± 86.373	4275.9 ± 0.000
CPA_{PB}	367.625 ± 4.914	18856.15 ± 81.388	4002.85 ± 31.466	4112.05 ± 0.000
PAUM	113.175 ± 2.369	12611.25 ± 57.134	3669.075 ± 55.402	4177.05 ± 0.000
CSOL	109.35 ± 3.041	5601.2 ± 29.628	1349.675 ± 3.147	2449 ± 0.000
CSC Kernel	114.05 ± 1.131	9055.8 ± 3.041	2489.375 ± 63.534	3690 ± 0.000
OMKC(H)	282.725 ± 5.409	11183.925 ± 8.309	3055.125 ± 15.167	3872.65 ± 0.000
OMKC(OGD)	351.25 ± 17.607	16265.4 ± 7.637	3391.325 ± 71.17	4883.6 ± 0.000
CSOMKC(U)	116.45 ± 1.131	7805.025 ± 36.946	1950.975 ± 4.066	2889.25 ± 0.000
CSOMKC(EG)	$\mathbf{95.725 \pm 3.359}$	$\mathbf{4407.325 \pm 29.097}$	1157.225 ± 0.530	$\mathbf{2141 \pm 0.000}$
CSOMKC(OGD)	$\mathbf{95.975 \pm 4.207}$	$\mathbf{4412.903 \pm 30.618}$	$\mathbf{1141.525 \pm 0.813}$	2210.7 ± 0.000
	Internet-ads	Page-blocks	KDD08	Malware
Linear	15.275 ± 0.672	1069.425 ± 12.127	535.9 ± 3.323	1973.6 ± 4.596
CPA_{PB}	13.45 ± 0.849	940.525 ± 5.48	528.975 ± 6.116	1977.8 ± 6.223
PAUM	11.875 ± 1.591	848.525 ± 22.451	550.175 ± 5.551	1970.95 ± 5.091
CSOL	10.9 ± 1.344	645.85 ± 6.435	674.975 ± 11.208	1924.85 ± 4.525
CSC Kernel	14.125 ± 2.934	445.35 ± 260.569	607.175 ± 165.852	208.525 ± 3.076
OMKC(H)	22.075 ± 1.52	623.4 ± 9.546	480.575 ± 8.945	141.925 ± 0.247
OMKC(OGD)	18.85 ± 0.778	819.575 ± 5.48	483.775 ± 12.763	169.1 ± 2.546
CSOMKC(U)	7.75 ± 1.344	425.425 ± 11.49	444.05 ± 6.223	1725.525 ± 2.157
CSOMKC(EG)	12.5 ± 1.414	$\mathbf{262.15 \pm 0.919}$	$\mathbf{419.6 \pm 15.556}$	136.675 ± 4.137
CSOMKC(OGD)	$\mathbf{6.000 \pm 1.909}$	266.4 ± 2.192	$\mathbf{421.625 \pm 9.016}$	$\mathbf{129.3 \pm 4.313}$

from 1. Our proposed scheme usually picks up the best pattern, and achieves superior performance right from the very beginning. It is also worth noting that in some cases, single kernels give a good performance on the first few samples of the data, but eventually are not able to match OMKCSC algorithms. This demonstrates difficulty in kernel selection by validation, and hence it is imperative to have a dynamic technique that can automatically choose a combination of multiple kernels.

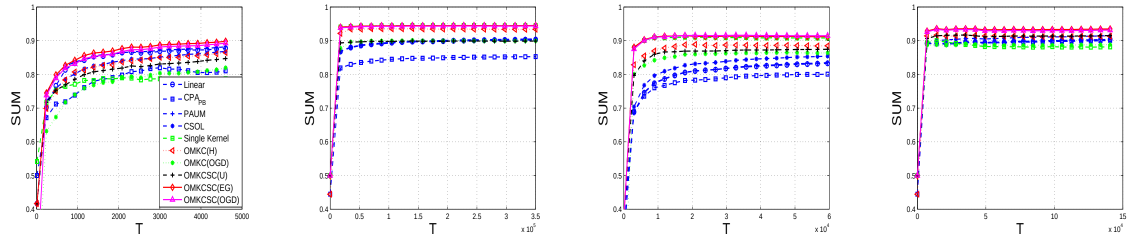
Linear vs Kernel Methods: The empirical results show that the introduction of kernels (and subsequently multiple kernels) have significantly increased the learning capacity of our models. In many cases, the balanced accuracy has improved by over 5% by using kernel methods as compared to the state of the art linear models. In the optimization of *cost*, have caused a great reduction of the cost e.g. in the Malware dataset, the cost suffered by kernel methods is a mere 7% of the cost suffered by the best linear methods.

Comparisons between kernel methods: Among the kernel methods our proposed techniques out performed the others, due to their ability to learn multiple cost-sensitive kernel prediction models, followed by their cost-sensitive combination. Firstly, it should be noted that the addition of multiple kernels has in fact helped increase the predictive power of the model as compared to one single kernel. However, this raises a further question: which of the algorithms CSOMKC(EG) and CSOMKC(OGD) is more suitable? CSOMKC(EG) has a a more limited predictive power as it learns only a convex combination of multiple kernels (as compared to CSOMKC(OGD). Further, if the data is described by a single kernel function, CSOMKC(EG) is able to quickly converge to that kernel predictor via multiplicative updates; but if the optimal prediction depends on a combination of several kernel functions, CSOMKC(OGD) slowly approaches the best solution, and will probably outperform CSOMKC(EG) in the long run. It should be noted that the results are very robust, as indicated by a very small standard deviation in most cases. In fact, in several cases, due to the low standard deviation, upon rounding up, it is reported as 0.

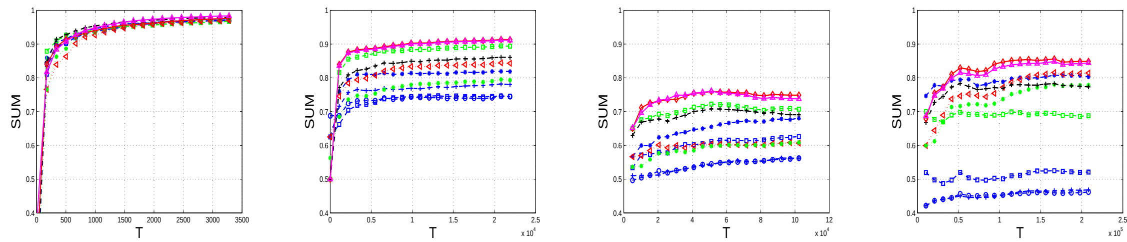
Sensitivity to learning rate parameters: We also analyzed the sensitivity of CSOMKC(EG) and CSOMKC(OGD) to their learning rates η_{eg} and η_{ogd} respectively. A sample of this analysis on the KDD08 dataset can be seen in Figure 2. As expected, both converge to the performance of OMKCSC(U) when the learning rates are set to 0. For a wide variety of other learning rates, CSOMKC(EG) and CSOMKC(OGD) substantially outperform OMKCSC(U) which for the KDD08 dataset is the next best performer. CSOMKC(EG) is more robust to the parameter choice and gives consistent results across a wide range, whereas CSOMKC(OGD) gives a good performance for small values of η_{ogd}, and when the learning rate is very large, its performance degrades. In our experiments, we had set $\eta_{ogd} = 0.1$ which is a conservative choice. A carefully chosen learning rate would further enhance the performance of CSOMKC(OGD).

5. Conclusion

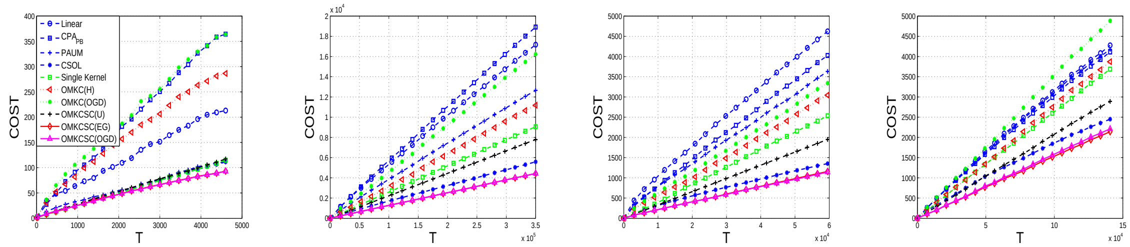
In this paper, we motivated the need for scalable techniques that can learn complex non-linear patterns in imbalanced data and proposed a novel scheme of Cost-Sensitive Online Multiple Kernel Classification, which dynamically explores a diverse set of predefined kernel functions, and simultaneously learns both the kernel predictions and their optimal combinations. Both the kernel predictors and their combinations are learnt in a cost-sensitive manner. The kernel predictors are learnt by gradient descent, and for the kernel combinations, we demonstrated two approaches - first by exponentiated gradient, and second by online gradient descent. We discussed the application of the proposed techniques to online anomaly detection. We derived theoretical properties of the algorithms and have conducted extensive empirical analysis on imbalanced datasets and anomaly detection datasets. Our proposed techniques significantly outperformed several state of the art methods designed for cost-sensitive classification.



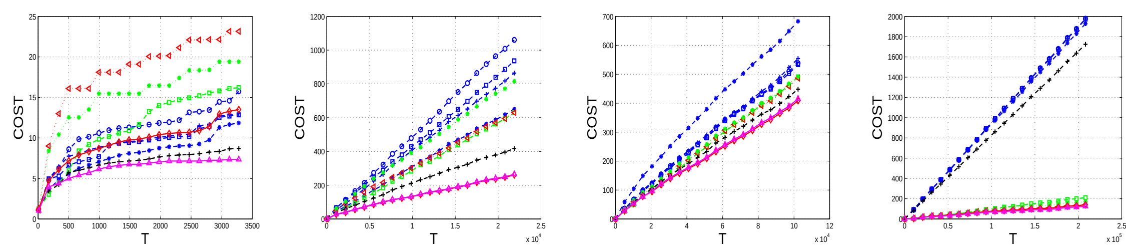
(a) SUM (Spam) (b) SUM (Webspam) (c) SUM (COD-RNA) (d) SUM (Twitter)



(e) SUM (InternetAds) (f) SUM (Page-blocks) (g) SUM (KDD08) (h) SUM (Malware)

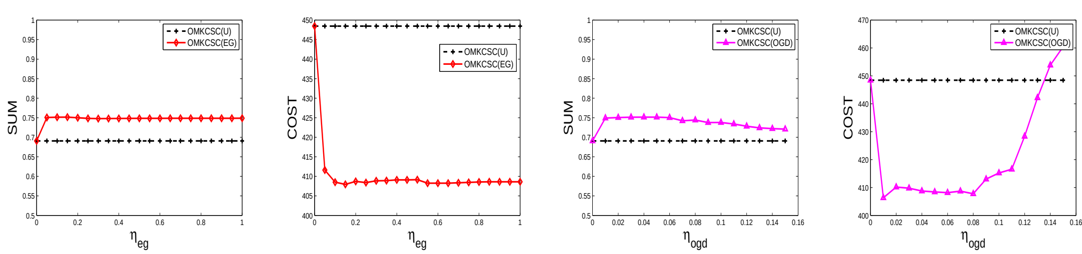


(i) COST (Spam) (j) COST (Webspam) (k) COST (COD-RNA) (l) COST (Twitter)



(m) COST (InternetAds) (n) COST (Page-blocks) (o) COST (KDD08) (p) COST (Malware)

Figure 1: Sum and Cost of different algorithms as the number of instances increases



(a) CSOMKC(EG) SUM (b) CSOMKC(EG) COST (c) CSOMKC(OGD) SUM (d) CSOMKC(OGD) COST

Figure 2: Parameter Sensitivity Analysis

Acknowledgements

This research is supported by the Singapore National Research Foundation under its International Research Centre @ Singapore Funding Initiative and administered by the IDM Programme Office, Media Development Authority (MDA).

References

Jonathan Aflalo, Aharon Ben-Tal, Chiranjib Bhattacharyya, Jagarlapudi Saketha Nath, and Sankaran Raman. Variable sparsity kernel learning. *JMLR*, 2011.

Olivier Bousquet and Daniel JL Herrmann. On the complexity of learning the kernel matrix. *Advances in NIPS*, 2003.

Giovanni Cavallanti, Nicolò Cesa-Bianchi, and Claudio Gentile. Tracking the best hyperplane with a simple budget perceptron. *Machine Learning*, 2007.

Nicolo Cesa-Bianchi and Gabor Lugosi. *Prediction, learning, and games*. Cambridge University Press, 2006.

Varun Chandola, Arindam Banerjee, and Vipin Kumar. Anomaly detection: A survey. *ACM Computing Surveys (CSUR)*, 41(3):15, 2009.

Koby Crammer and Yoram Singer. Ultraconservative online algorithms for multiclass problems. *JMLR*, 2003.

Koby Crammer, Ofer Dekel, Joseph Keshet, Shai Shalev-Shwartz, and Yoram Singer. Online passive-aggressive algorithms. *JMLR*, 7:551–585, December 2006.

Yi Ding, Peilin Zhao, Steven CH Hoi, and Yew-Soon Ong. An adaptive gradient method for online auc maximization. *AAAI*, 2015.

Steve Donoho. Early detection of insider trading in option markets. *ACM SIGKDD*, 2004.

Claudio Gentile. A new approximate maximal margin classification algorithm. *JMLR*, 2002.

Mehmet Gönen and Ethem Alpaydın. Multiple kernel learning algorithms. *JMLR*, 2011.

Steven C.H. Hoi, Rong Jin, Peilin Zhao, and Tianbao Yang. Online multiple kernel classification. *Machine Learning*, 90(2):289–316, 2013.

Steven CH Hoi, Jialei Wang, and Peilin Zhao. Libol: A library for online learning algorithms. *JMLR*, 15:495–499, 2014.

Rong Jin, Steven C.H. Hoi, and Tianbao Yang. Online multiple kernel learning: Algorithms and mistake bounds. In *Algorithmic Learning Theory*. Springer Berlin Heidelberg, 2010.

Hisashi Kashima, Koji Tsuda, and Akihiro Inokuchi. Marginalized kernels between labeled graphs. In *ICML*, volume 3, pages 321–328, 2003.

J. Kivinen, A.J. Smola, and R.C. Williamson. Online learning with kernels. *Signal Processing, IEEE Transactions on*, 52(8):2165–2176, 2004.

Jyrki Kivinen and Manfred K Warmuth. Exponentiated gradient versus gradient descent for linear predictors. *Information and Computation*, 132(1):1–63, 1997.

James T Kwok and Ivor W Tsang. Learning with idealized kernels. In *ICML*, 2003.

Gert R. G. Lanckriet, Nello Cristianini, Peter Bartlett, Laurent El Ghaoui, and Michael I. Jordan. Learning the kernel matrix with semidefinite programming. *JMLR*, 2004.

Yaoyong Li, Hugo Zaragoza, Ralf Herbrich, John Shawe-Taylor, and Jaz Kandola. The perceptron algorithm with uneven margins. In *ICML*, volume 2, pages 379–386, 2002.

Yi Li and Philip M Long. The relaxed online maximum margin algorithm. *ML*, 2002.

André Martins, M Figueiredo, P Aguiar, N Smith, and E Xing. Online multiple kernel learning for structured prediction. *arXiv preprint arXiv:1010.2770*, 2010.

Alain Rakotomamonjy, Francis Bach, Stéphane Canu, Yves Grandvalet, et al. Simplemkl. *JMLR*, 9:2491–2521, 2008.

Martin Roesch et al. Snort: Lightweight intrusion detection for networks. In *LISA*, volume 99, pages 229–238, 1999.

Frank Rosenblatt. The perceptron: a probabilistic model for information storage and organization in the brain. *Psychological review*, 65(6):386, 1958.

Doyen Sahoo, Steven CH Hoi, and Bin Li. Online multiple kernel regression. In *ACM SIGKDD*, 2014.

Bernhard Schölkopf and Alexander J Smola. *Learning with kernels*. The MIT Press, 2002.

Sören Sonnenburg, Gunnar Rätsch, Christin Schäfer, and Bernhard Schölkopf. Large scale multiple kernel learning. *JMLR*, 7:1531–1565, 2006.

Jialei Wang, Peilin Zhao, and Steven CH Hoi. Cost-sensitive online classification. *Knowledge and Data Engineering, IEEE Transactions on*, 26(10):2425–2438, 2014.

Tao Xiang and Shaogang Gong. Video behavior profiling for anomaly detection. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 30(5):893–908, 2008.

Zenglin Xu, Rong Jin, Irwin King, and Michael R Lyu. An extended level method for efficient multiple kernel learning. In *NIPS*, pages 1825–1832, 2008.

Peilin Zhao, Rong Jin, Tianbao Yang, and Steven C Hoi. Online auc maximization. In *Proceedings of the 28th International Conference on Machine Learning (ICML-11)*, 2011.

Jinfeng Zhuang, Ivor W Tsang, and Steven CH Hoi. A family of simple non-parametric kernel learning algorithms. *JMLR*, 12:1313–1347, 2011.

Martin Zinkevich. Online convex programming and generalized infinitesimal gradient ascent. *ICML*, 2003.