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The Impact of Budget Constraints on Flexible versus Dedicated Technology Choice

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Abstract

This paper studies the flexible-versus-dedicated technology choice and capacity investment decisions of a multi-product firm under demand uncertainty in the presence of budget constraints. The firm operates under a capital budget for financing the capacity investment, and an operating budget, which is uncertain in the capacity investment stage, for financing the production. We investigate how the tightening of the capital budget and a lower financial flexibility in the production stage (the likelihood of having sufficient operating budget) shape the optimal technology choice. We find that the dominant regime is one where dedicated technology should be adopted for a larger investment cost range, and thus, is the best response to the tighter capital budget, whereas flexible technology is the best response to lower financial flexibility. We identify the key roles that the capacity intensity (the ratio of unit capacity cost to total unit capacity and production cost) of each technology and the pooling value of operating budget with dedicated technology, which brings this technology closer to flexible technology in terms of the resource network's flexibility, play in a budget-constrained environment. Managerially, our results underline that in the presence of financial constraints, firms should manage technology adoption together with plant location, which shapes capacity intensity, or product portfolio, which shapes financial flexibility.

Key Words: Capacity, Flexibility, Technology, Budget, Multi-product Newsvendor, Financial Constraints, Capital Market.

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1 Introduction

Multi-product firms often use product-flexible resources (flexible technology) to cope with demand uncertainty. Compared to product-dedicated resources (dedicated technology), these flexible resources can manufacture multiple products on the same capacity, and provide the ability to reallocate this capacity between products in response to demand realizations. This capacity-pooling benefit of the flexible technology is a hedge against demand uncertainty. Flexible technology investment is prevalent in many industries, including automotive, pharmaceutical, shipbuilding, aerospace and defense. Given the capital-intensive nature of these industries, one of the key determinants of the technology investment is the availability of financial resources to cover the costs incurred for capacity investment and production. In the operations management literature, as also highlighted in Van Mieghem (2003), the majority of papers focusing on technology investment (often implicitly) assume that firms have abundant financial resources, and study the flexible versus dedicated technology choice and stochastic capacity investment in the absence of financial constraints.

In theory, as follows from Modigliani and Miller (1958), financial constraints do not exist when there are no frictions (such as agency costs due to asymmetric information, taxes, financial distress cost etc.) in the capital markets. In this case, operational decisions can be made safe in the knowledge that sufficient financial resources to support these investments can be provided by raising funds from the external capital markets. However, in practice, there are frictions in the capital markets (Harris and Raviv 1991) and thus, the operational decisions are made in the presence of financial constraints, as empirically well-documented (see, for example, Whited and Wu 2006). It is therefore important for operations managers to understand the impact of financial constraints when choosing the right technology to adopt and capacity investment to make.

In this paper, we aim to develop that knowledge base by analyzing the flexible-versusdedicated technology choice and stochastic capacity investment in the presence of budget constraints. In practice, the capital expenditure and the operating expenditures are funded by different budgets. The capacity investment is financed through a capital budget which covers the procurement cost of the physical assets such as land and machinery. The subsequent production activities are financed through an operating budget, which covers the factory related costs such as overhead, raw material procurement, machining and labor.

The operating budget remains uncertain at the time of the capacity investment and may become constraining in the production stage for two main reasons. *First*, the operating budget may be lower than anticipated due to *tighter external financing conditions*. As commonly observed in practice, banks frequently limit credit to firms as a response to changes in economic conditions (Chava and Purnanandam 2011). For example, in the aftermath of the global financial crisis in 2007, the unavailability of external capital became an issue for manufacturing firms all around the world (Pimlott 2009); "credit terms have tightened or available credit has disappeared altogether." (Matson 2009). The resulting lower operating budget may become insufficient to finance the production at its full scale. For example, in the automotive industry, Saab scaled down its production volume, and even suspended its production on several occasions between 2010 and 2011 owing to problems in financing the operating costs including the procurement cost (Hamilton 2011) and the wages for the employees (Kinnader 2011). In the furniture manufacturing industry, Norwalk shut down its production facility due to not being able to meet its payroll after Comerica, its main lender, canceled its credit line (Byrnes 2009). In the shipbuilding industry, China Rongsheng Heavy Industries Group experienced problems in financing their (ship) production, and laid off as many as 8000 workers due to tighter external financing conditions (Jourdan and Wallis 2013).¹ In all manufacturing industries, the Chinese small and medium enterprises scaled down their production volume or entirely closed their production facilities because they were not able to finance their production due to tighter external financing conditions (Shih-huang 2011). Second, the operating budget may be lower than anticipated due to a decrease in internal financing. When the focal unit is a business unit which relies on internal financing from its parent company, the company may fund this unit at a lower level than anticipated due to a reallocation to a more profitable business unit (Scharfstein and Stein 2000). For example, facing budget constraints, Boeing decided to close down its strategic airlifter assembly plant in California to reallocate the funds elsewhere (Butler 2013).

In summary, the operating budget may be lower than anticipated due to tighter external financing conditions or due to lower internal financing, and the resulting operating budget may become insufficient to finance the production at its full scale. Such production scaledowns have a lasting and significant impact on production volume when the production lead time is high such that the cash conversion cycles (the cycle time of sales revenues funneled back to production) are long, as in the case of the shipbuilding and the aerospace and defense industries; or, regardless of the production lead time, when these production scale-downs result in the loss of production resources or expertise (for example, due to laying off workers

¹In the shipbuilding industry, the tighter external financing conditions have also decreased the demand for ships, as the ship-buyers have also experienced financing problems. However, according to a Rongsheng executive, the main problem has been the operating budget constraints: "We have been getting orders but can't seem to get construction loans from banks to build these projects" (Jourdan and Wallis 2013).

or plant closures). We conclude that it is important for the operations manager to jointly consider capital- and operating-stage budget constraints in choosing which technology to adopt. Moreover, which technology should be favored in the face of tighter capital budget or operating budget conditions are open questions.

To shed light on these questions, we propose a two-stage model that - in a stylized manner - captures the significance of the operating budget in the technology choice decision of a firm. The firm produces and sells two products under demand uncertainty to maximize its expected profit. In the first stage, the firm makes the flexible-versus-dedicated technology choice and the capacity investment decision with respect to a capital budget constraint and in the face of demand and operating budget uncertainty. In the second stage, it determines the production quantities after these uncertainties are resolved. We refer to the likelihood that the firm is able to find an operating budget that is sufficient to fully cover the operating costs for any capacity level (i.e. the likelihood that it is budget-unconstrained in the production-stage for any given capacity level) as its "financial flexibility."

We solve for the optimal capacity level and production quantities with each technology. Using a reformulation of the sum of unit capacity and production costs as a unit (aggregate) investment cost with an associated capacity intensity level (the proportion of capacity cost in the investment cost), we identify the unique unit investment cost threshold that determines the optimal technology choice. In presenting our insights, to better delineate the intuition, we first focus on the special case with identical capacity intensities, and then discuss how our results are impacted as the capacity intensity of flexible technology increases.

Technology Choice. Our *first research objective* is to study the impact of accounting for budget constraints on the optimal technology choice. We carry out this analysis by making a comparison with the budget-unconstrained benchmark case, which has been the main focus of the literature to date. To this end, we compare the unit investment cost threshold we derive with its counterpart in a budget-unconstrained environment. The former captures the capacity-pooling value of the flexible technology and the relative impact of the capital budget constraint and the operating budget uncertainty on each technology, while the latter only captures the capacity-pooling value of the flexible technology mis-specification, but what is the direction of technology mis-specification? What are the main drivers that are unique to a budget-constrained environment that influence this direction of mis-specification?

We identify that the overall resource network's flexibility plays an important role. Because the operating budget can be allocated between the two products in response to the demand realizations, the operating budget can be interpreted as a flexible resource that is used in conjunction with the capacity investment. In a budget-constrained environment, the technology choice is determined by comparing a flexible system (flexible capacity and a flexible operating budget) with a partially-flexible system (dedicated capacities and flexible operating budget). In the absence of budget constraints, because the operating budget is not constraining, this comparison is between a flexible system (flexible capacity) and a nonflexible system (dedicated capacities). In other words, the flexibility of the operating budget brings dedicated technology closer to flexible technology in terms of the overall resource network's flexibility. We show that to what extent this flexibility is beneficial with dedicated technology, i.e. the pooling value of the operating budget with dedicated technology, is an important driver of the technology choice.

With identical capacity intensities, we establish that dedicated technology should be adopted for a larger unit investment cost range in comparison with the budget unconstrained benchmark due to the pooling value of the operating budget with this technology. When flexible technology has larger capacity intensity, the relative total capacity investment and production costs also matter, and the financial flexibility level plays a key role. In particular, flexible (dedicated) technology should be adopted for a larger unit investment cost range in comparison with the budget unconstrained benchmark when the financial flexibility is low (high). These results demonstrate that the capacity intensity of each technology, the pooling value of the operating budget with dedicated technology and the financial flexibility level have a pronounced effect on the direction of technology mis-specification.

Impact of a tighter capital budget. Our *second research objective* is to study how the tightening of the capital budget shapes the flexible-versus-dedicated technology choice. Intuitively, a tighter capital budget has a negative impact on profitability under either technology; however, it is an open question which technology is less negatively affected. To answer this question, we conduct sensitivity analysis to investigate how the unit investment cost threshold we derive changes in the capital budget. With identical capacity intensities, because dedicated technology has a lower total capacity investment cost, this technology should be adopted for a larger unit investment cost range, and thus, is the best response to the tightening of the capital budget. When flexible technology has a larger capacity intensity, the dominant regime is the same unless the capital budget is severely constraining and the financial flexibility in the production stage is moderate. In this case, the operating budget considerations become critical as we explain later in detail. Therefore, flexible technology should be adopted for a larger unit investment cost range, and thus, is the best response to the tightening of the capital budget. Managerially, these results are important because they imply that the optimal technology adopted should differ depending on the severity of the capital and operating budget constraints. Thus, indiscriminately adopting the same technology as financial constraints get tighter can be a detrimental strategy.

Impact of lower financial flexibility in the production stage. Our third research *objective* is to study how financial flexibility affects the flexible-versus-dedicated technology choice. To answer this question, we conduct sensitivity analysis to investigate how the unit investment cost threshold we derive changes in the financial flexibility level. With identical capacity intensities, the dominant regime is one where dedicated technology should be adopted for a larger unit investment cost range, and thus, is the best response to lower financial flexibility. This finding is reversed when the financial flexibility is sufficiently low. These results are driven by the impact of financial flexibility on the pooling value of the operating budget with dedicated technology: We establish that lower financial flexibility increases this pooling value unless the financial flexibility is sufficiently low. When flexible technology has a higher capacity intensity, the total production cost is lower with this technology, and thus, all else equal, this technology is less negatively impacted by lower financial flexibility. When the capacity intensity of flexible technology is sufficiently large, this effect outweighs the increasing pooling value of the operating budget with dedicated technology: The dominant regime is one where flexible technology should be adopted for a larger unit investment cost range, and thus, is the best response to lower financial flexibility. We conclude that financial flexibility and flexible technology are complements when higher financial flexibility favors adoption of flexible technology, and they are substitutes otherwise.

Because flexible technology has a higher capacity investment cost than dedicated technology, the comparison between the capacity intensity with either technology is determined by the relative production costs. The relative production costs depend on technology characteristics (e.g. automation level), the industry setting or the plant location. For example, Fine and Freund (1990) argue that when the two technologies are highly automated, because the labor content for either technology is very low, the material procurement cost will dominate, and thus, the production costs will be very similar for the two technologies. In this case, flexible technology has a strictly larger capacity intensity. Consider another example in the pharmaceutical industry. Pisano and Rossi (1994) argue that the production cost is higher with the flexible technology. This is because dedicated technology uses equipment and facilities optimized for a particular product, whereas flexible technology requires change-overs between products that impose additional costs. In this case, if the production cost with flexible technology is sufficiently high, then both technologies may have the same capacity intensity. Plant location matters as well: the same technology deployed in plant locations with different labor costs may incur different production costs and hence exhibit different capacity intensity levels. Regardless of which technology has a higher production cost, our results underline the importance of considering these costs rather than only capacity costs, which have been the main focus of the extant literature.

The remainder of this paper is organized as follows: §2 surveys the related literature and discusses the contribution of our work. §3 describes the model and discusses the basis for our assumptions. §4 derives the optimal strategy for a given technology. §5 analyzes the optimal technology choice and investigates the impact of budget constraints on this choice. §6 concludes with a discussion of the main insights and future research directions.

2 Literature Review

Our paper's contribution is to the literature on stochastic capacity and technology investment in multi-product firms. Papers in this stream consider investment in flexible and dedicated capacity, and barring two exceptions (Boyabath and Toktay 2011, Chod and Zhou 2014), they do so in the absence of financial constraints. Yet operations managers rely on limited budgets to make this capacity investment and subsequent production decisions. This is the first paper that studies how the capital budget constraint and the operating budget uncertainty jointly shape the flexible-versus-dedicated technology choice and the optimal capacity investment with each technology.

In the literature on stochastic capacity and technology investment in multi-product firms, Boyabath and Toktay (2011) and Chod and Zhou (2014) are the only papers that model flexible technology investment in financially-constrained environments. They focus on a budget constraint that applies to the capacity investment stage only, and unlike our paper, they assume this budget constraint can be relaxed by borrowing from external sources. They implicitly assume full financial flexibility in the production stage, i.e. there is no budget constraint in this stage. In their models, the capacity intensity of the technology does not matter either. In other words, two of the key drivers that our work highlights are not captured in these papers. More importantly, these papers focus on different research questions than ours. We compare our work with these two papers in detail.

Chod and Zhou (2014) examine the relation between a firm's resource flexibility and its financial leverage. They analyze the optimal mix of flexible and dedicated capacity investments in the presence of a capital budget. They assume that the firm can borrow from external capital markets, and that the cost of borrowing is determined by the agency cost of debt. Their model captures very different aspects of capacity investment in the presence of financial constraints. In particular, they investigate the link between flexible capacity investment and underinvestment or asset substitution problems raised by the agency cost of debt. Their main finding is that when the cost of flexible capacity investment is relatively low, lenders anticipate the benefit of flexible capacity in reducing the default risk, and provide more favorable credit terms, which in turn, increases the firm's financial leverage.

Boyabath and Toktay (2011) analyze flexible-versus-dedicated technology choice in the presence of a capital budget. They also assume that this budget can be relaxed by borrowing from external capital markets, and the equilibrium financing cost is endogenously determined by the operational decisions. They analyze the impact of demand uncertainty on the equilibrium technology choice. They normalize the production costs to zero, thus, the operating budget is irrelevant. Therefore, two key drivers of our results, the capacity intensity of each technology, and the financial flexibility in the production stage, do not exist in their model. In this paper, as summarized in §1, our results identify the critical roles that these two drivers play in the relative attractiveness of each technology.

There is a stream of papers that study flexible capacity investment in the absence of financial constraints. Fine and Freund (1990) model an *n*-product firm that invests in n dedicated capacities and a single flexible capacity that can manufacture all products. They show that when the product demands are perfectly positively correlated, the firm does not invest in flexible capacity. Van Mieghem (1998), focusing on a similar model with two products, demonstrates that this argument does not continue to hold in the presence of asymmetric prices in the product markets. Bish and Wang (2004) extend the analysis and the result in Van Mighem (1998) by endogenizing the pricing decision in each product market. In a similar modeling framework, Chod and Rudi (2005) consider investment in a flexible capacity with endogenous pricing decision, and demonstrate that the optimal capacity investment level increases with a higher demand variability and a lower demand correlation assuming a bivariate normal demand uncertainty. A number of papers in this literature investigate the interplay between flexible capacity investment and other key operational issues. These key issues include supply chain network design (Jordan and Graves 1995), downward substitution among quality-differentiated products (Netessine et al. 2002), supply disruptions (Tomlin and Wang 2005), risk-aversion of the decision-maker (Van Mieghem 2007), competition in the product markets (Goyal and Netessine 2007), the ability to produce above/below the installed capacity (Goyal and Netessine 2011), and the productivity loss in the product launches (Gopal et al. 2013). None of these papers

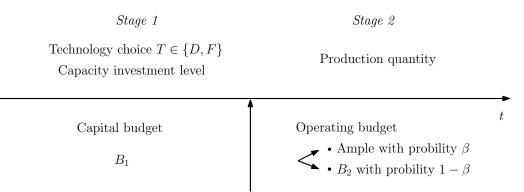
incorporate financial constraints.

Two other streams of literature are related to our paper due to their incorporation of financial constraints. Following the seminal work of Modigliani and Miller (1958), there is a vast amount of research in the Corporate Finance literature that investigates the interaction between a firm's operational investments and financing policies in a variety of settings. Since the focus of these papers is on financial issues, they do not consider separate operational decisions such as technology, capacity and production as considered in our paper, and the sequential nature of these decisions under demand uncertainty. In the Operations Management literature, Babich and Sobel (2004), Buzacott and Zhang (2004) and, more recently, Babich (2010) and Yang and Birge (2011) analyze similar issues with a stronger formalization of operational decisions. We refer the reader to Yang et al. (2012) for a review of papers in this stream. All these papers analyze the impact of an endogenous capital budget constraint in a single-product firm. The budget constraint is endogenous as it is determined by the interaction between the capacity investment and the external capital markets. These papers establish the value of integrating financing and capacity investment decisions. Our scope is a focal unit that makes the operational decisions based on the exogenously given capital-budget and an uncertain operating-budget. Therefore, unlike these papers, we do not study the value of integrating financing and capacity investment decisions. Instead, we extend the analysis of the impact of budget constraint in two significant ways: First, we consider an operating budget, which is uncertain in the capacity investment stage, and can be constraining in the production stage. Second, we consider a multi-product unit where the technology choice also matters.

3 Model Description and Assumptions

Notation and Preliminaries. The following mathematical representation is used throughout the text: The random variable $\tilde{\xi}$ has a realization ξ . Bold face letters represent column vectors of the required size and ' denotes the transpose operator. \mathbb{E} denotes the expectation operator, Ω^C denotes the complement of set Ω and $(x)^+ \doteq \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

We consider a firm that produces and sells two products in a single period so as to maximize its expected profit. We model the firm's decisions as a two-stage problem: The firm makes its technology choice (dedicated D versus flexible F) and capacity investment decision under the demand and operating budget uncertainties; and the production decision after the resolution of these uncertainties. Each technology $T \in \{D, F\}$ is characterized by a unit capacity cost c_T and a unit production cost y_T that is identical for both products. The unit (aggregate) investment cost is denoted by η_T and equals $c_T + y_T$. The sequence of events is presented in Figure 1.



Demand (ξ_1, ξ_2) and operating budget uncertainties are resolved

Figure 1: Timeline of Events

The firm is exposed to two budget constraints: a capital budget to finance the capacity investment, and an operating budget to finance the production volume. In practice, capital and operating budgets are separate. Therefore, we assume that any leftover budget from the capacity investment stage cannot be used to finance the production. The capital budget is denoted by B_1 . The operating budget, which is common for both products, is uncertain at the capacity investment stage, and it is realized before the production decisions are made. For tractability, we choose a two-point characterization: The operating budget is ample with probability $\beta \in [0, 1]$, and it is $B_2 > 0$ with probability $1 - \beta$.² By ample, we mean this budget is sufficient to finance the production at a level that fully utilizes the highest capacity investment level that can be made with the capital budget B_1 . B_2 is insufficient to finance this production volume, i.e. $\frac{B_2}{y_T} < \frac{B_1}{c_T}$ for $T \in \{D, F\}$.

Ample operating budget represents the case where the firm is financially flexible and is able to find sufficient resources (internally or from external capital markets) to fully fund the production activities regardless of the capacity investment level. B_2 represents the case where the firm is financially inflexible, and is not able to find resources beyond B_2 due to the reasons discussed in the Introduction. Depending on the capacity level chosen in stage 1, B_2 may or may not be sufficient to fully utilize this capacity level. If B_2 is insufficient, then the production is budget constrained with probability $(1 - \beta)$. As β increases, the firm is more likely to be financially flexible in the production stage, and it is always financially flexible

²A continuous distribution on the operating budget could be utilized, but is not tractable. This two-point characterization captures the budget uncertainty and yields closed-form solutions for the optimal capacity investment.

when $\beta = 1$. Therefore, β captures the expected financial flexibility level in the production stage. For brevity, hereafter we drop the term "expected," and denote β as the financial flexibility level in the production stage. In summary, $\beta < 1$ introduces the possibility of an operating budget constraint, and B_2 parameterizes the severity of this budget constraint.

Price-dependent demand for each product *i* is represented by the same iso-elastic inversedemand function $p_i(q_i; \xi_i) = \xi_i q_i^{1/b}$.³ Here, $b \in (-\infty, -1)$ is the constant price elasticity of demand, p_i denotes the price, q_i denotes the quantity, and ξ_i represents the demand risk in market *i*. The demand uncertainty $\tilde{\boldsymbol{\xi}}' = (\tilde{\xi}_1, \tilde{\xi}_2)$ has a positive support with probability density function $f(\xi_1, \xi_2)$. We assume that $\tilde{\boldsymbol{\xi}}$ follows a symmetric bivariate distribution with mean $\mathbb{E}[\xi_1] = \mathbb{E}[\xi_2] = \mu_{\xi}$ and covariance matrix $\boldsymbol{\Sigma}$ with $\boldsymbol{\Sigma}_{ii} = \sigma^2$, where σ denotes the standard deviation, and $\boldsymbol{\Sigma}_{ij} = \rho \sigma^2$ for $i \neq j$ where ρ denotes the correlation coefficient.

We assume that the firm adheres to a production clearance strategy, that is, choosing the production level so as to fully utilize the available production capacity. This is an assumption that is widely used in the literature for tractability (see for example, Chod and Rudi (2005), Goyal and Netessine (2007, 2011) and Swinney et al (2011)).⁴ In our model, the available production capacity is determined by the two resources required for production, the capacity invested at stage 1 and the realized operating budget. With the production clearance strategy, the firm optimally chooses how to allocate the maximum available production capacity between the two products.

Flexible technology has a single resource with capacity level K_F that is capable of producing two products. Dedicated technology has two resources that can each produce a single product. Because we assume symmetric products, the firm optimally invests in identical capacity levels for each product with dedicated technology. Therefore, a single capacity level K_D is sufficient to characterize the capacity investment decision. For tractability, we assume $\frac{c_D}{c_D+y_D} > 1 - \frac{\mathbb{E}[\min(\tilde{\xi}_1, \tilde{\xi}_2)]}{\mu_{\xi}}$. This is a reasonable assumption for capacity-intensive industries, where the capacity cost is larger than the production cost.⁵

³The iso-elastic function is commonly used in the literature (see, for example, Chod and Rudi 2006).

⁴Another interpretation of the production clearance is as follows: In practice, the firm may receive regular price for some capacity units, discounted price for some others, and no revenues for the unused capacity. The price in our model can be interpreted as *average revenue* received from one unit of capacity, which satisfies two important properties: The average revenue increases in demand shock and decreases in capacity.

⁵For example, when $\tilde{\boldsymbol{\xi}}$ follows a symmetric bivariate normal distribution, this assumption is equivalent to $\frac{c_D}{c_D+y_D} > \frac{\sigma_{\xi}}{\mu_{\xi}} \sqrt{\frac{1-\rho}{\pi}}$. Even with a correlation coefficient ρ as low as -1, and a coefficient of variation $\frac{\sigma_{\xi}}{\mu_{\xi}}$ as high as 0.3, this implies $\frac{c_D}{c_D+y_D} > 0.24$, which is satisfied in capacity-intensive industries.

4 The Optimal Strategy for a Given Technology

In this section, we describe the optimal solution for the firm's capacity investment and production decisions with each technology. We solve the firm's problem using backward induction starting from stage 2. All the proofs are relegated to the Appendix.

4.1 Flexible Technology

In stage 1, the firm invested in capacity level K_F . In stage 2, the firm observes the demand and the operating budget; and determines the production volume in each market $\mathbf{Q}_{\mathbf{F}}' = (Q_F^1, Q_F^2)$ to maximize the profit. When the operating budget realization is B_2 , the firm's stage-2 profit maximization problem is given by

$$\max_{\mathbf{Q}_{F} \ge \mathbf{0}} \quad \xi_{1} Q_{F}^{1}^{(1+\frac{1}{b})} + \xi_{2} Q_{F}^{2}^{(1+\frac{1}{b})} - y_{F} (Q_{F}^{1} + Q_{F}^{2})$$
(1)
s.t.
$$Q_{F}^{1} + Q_{F}^{2} \le \min \left(K_{F}, \frac{B_{2}}{y_{F}} \right).$$

With the production clearance assumption, the firm chooses the production vector so as to fully utilize the available production capacity, thus, the constraint in (1) is binding at the optimal solution: The firm optimally allocates the maximum attainable production volume given the capacity and the budget constraints between the two products based on the demand realizations. The unique optimal production vector is given by $\mathbf{Q}_{\mathbf{F}}^{*'} =$ $\min\left(K_F, \frac{B_2}{y_F}\right)\left(\frac{\xi_1^{-b}}{\xi_1^{-b}+\xi_2^{-b}}, \frac{\xi_2^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right)$. When there is ample budget, the optimal production volume is determined by the capacity K_F .

In stage 1, the firm has a capital budget B_1 and determines the optimal capacity investment level K_F^* with respect to demand and operating budget uncertainties so as to maximize the expected profit. The firm's expected profit for a given feasible capacity investment level $K_F \in \left[0, \frac{B_1}{c_F}\right]$ can be written as

$$\Psi_{F}(K_{F}) = -c_{F}K_{F} + \beta \left[\mathbb{E} \left[\left(\tilde{\xi}_{1}^{-b} + \tilde{\xi}_{2}^{-b} \right)^{-\frac{1}{b}} \right] K_{F}^{(1+\frac{1}{b})} - y_{F}K_{F} \right] + (1-\beta) \left[\mathbb{E} \left[\left(\tilde{\xi}_{1}^{-b} + \tilde{\xi}_{2}^{-b} \right)^{-\frac{1}{b}} \right] \min \left(K_{F}, \frac{B_{2}}{y_{F}} \right)^{(1+\frac{1}{b})} - y_{F} \min \left(K_{F}, \frac{B_{2}}{y_{F}} \right) \right].$$
(2)

For $K_F \leq \frac{B_2}{y_F}$, the production volume in each market is effectively operating-budget unconstrained $\left(Q_F^{j^*} = \frac{\xi_j^{-b}}{\xi_1^{-b} + \xi_2^{-b}}K_F\right)$. For $K_F > \frac{B_2}{y_F}$, it is operating-budget constrained $\left(Q_F^{j^*} = \frac{\xi_j^{-b}}{\xi_1^{-b} + \xi_2^{-b}}\frac{B_2}{y_F}\right)$ with probability $1 - \beta$, and it is effectively budget unconstrained otherwise. **Proposition 1** Let $K_F^u \doteq \left(\frac{(1+\frac{1}{b})\mathbb{E}\left[\left(\tilde{\xi}_1^{-b}+\tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}\right]}{c_F+y_F}\right)^{-b}$ denote the budget-unconstrained capacity investment level, and $K_F^{\beta} \doteq \left(\frac{(1+\frac{1}{b})\mathbb{E}\left[\left(\tilde{\xi}_1^{-b}+\tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}\right]}{\frac{c_F}{\beta}+y_F}\right)^{-b}$ denote the β -flexible capacity investment level. When $\frac{B_2}{y_F} \ge K_F^{\beta}$, $K_F^* = \min\left(K_F^u, \frac{B_2}{y_F}\right)$. Otherwise, $K_F^* = \min\left(K_F^{\beta}, \frac{B_1}{c_F}\right)$ We first discuss the budget-unconstrained capacity level K_F^u , and the β -flexible capacity level K_F^{β} (by definition $K_F^{\beta} \le K_F^u$). K_F^u is the solution to $\frac{\partial\Psi_F(K_F)}{\partial K_F} = 0$ when $\min\left(K_F, \frac{B_2}{y_F}\right)$ in (2) is replaced by K_F . K_F^{β} is the same when $\min\left(K_F, \frac{B_2}{y_F}\right)$ is replaced by $\frac{B_2}{y_F}$. When $K_F > \frac{B_2}{y_F}$, since an additional unit of capacity is utilized in the production stage with probability β , for a given unit production cost y_F , the marginal capacity investment cost is $\frac{c_F}{\beta}$, as appears in the characterization of K_F^{β} . K_F^{β} strictly increases in the financial flexibility β and equals K_F^u at full financial flexibility ($\beta = 1$).

To delineate the intuition behind the optimal capacity investment strategy in Proposition 1, we first ignore the impact of the capital budget B_1 . When B_2 is sufficient to finance the production fully utilizing K_F^u (Case III in Figure 2), $K_F^* = K_F^u$. Otherwise, the firm considers whether to be conservative and invest in $\frac{B_2}{y_F}$, which is effectively budget unconstrained in the production stage, or to take an investment risk and invest in K_F^β which will not be fully utilized in the production stage with probability $1 - \beta$ due to the budget constraint B_2 . When B_2 is sufficiently low $\left(\frac{B_2}{y_F} < K_F^\beta\right)$, because the upside gain from the ample operating budget realization is significantly large, the firm chooses to take the investment risk (Case I), and $K_F^* = K_F^\beta$. When $\frac{B_2}{y_F} \ge K_F^\beta$, the upside gain is not large enough. Therefore, the firm chooses to be conservative (Case II), and $K_F^* = \frac{B_2}{y_F}$. Now consider the impact of B_1 . Since $\frac{B_1}{c_F} > \frac{B_2}{y_F}$ by assumption, B_1 can be constraining only when the firm chooses an investment level that is not always fully utilized in the production stage (Case I). In particular, when $\frac{B_1}{c_F} < K_F^\beta$, $K_F^* = \frac{B_1}{c_F}$. This is illustrated in Figure 2.

$$K_F^* = \min\left(\frac{B_1}{c_F}, K_F^\beta\right) \qquad \qquad K_F^* = \frac{B_2}{y_F} \qquad \qquad K_F^* = K_F^u$$

•				•		-
0	Ι	K_F^{eta}	II	K_F^u	III	$\frac{B_2}{y_F}$

Figure 2: The optimal capacity investment K_F^* , where K_F^u and K_F^β are as defined in Proposition 1.

Hereafter, we focus on a budget-constrained environment where B_1 and B_2 are insuf-

ficient to finance the capacity investment and the production volume with the budgetunconstrained capacity level respectively, i.e. $\frac{B_1}{c_F} < K_F^u$ and $\frac{B_2}{y_F} < K_F^u$. In this case, $K_F^* = \min\left(\frac{B_1}{c_F}, \max\left(\frac{B_2}{y_F}, K_F^\beta\right)\right)$, and substituting it in (2), the optimal expected profit Ψ_F^* can be obtained, which is omitted for brevity. Corollary 1 characterizes the impact of the financial flexibility β and the capital budget B_1 on K_F^* and Ψ_F^* :

Corollary 1 There exist unique $0 < \overline{\beta}_F^0 < \overline{\beta}_F^1 < 1$ such that i) $K_F^* = \frac{B_2}{y_F}$ and $\frac{\partial \Psi_F^*}{\partial \beta} = 0$ if $\beta \leq \overline{\beta}_F^0$; $K_F^* = \min\left(K_F^\beta, \frac{B_1}{c_F}\right)$, where $\frac{\partial K_F^\beta}{\partial \beta} > 0$, and $\frac{\partial \Psi_F^*}{\partial \beta} > 0$ otherwise;

ii) $K_F^* < \frac{B_1}{c_F}$ and $\frac{\partial \Psi_F^*}{\partial B_1} = 0$ if $\beta \le \overline{\beta}_F^1$; $K_F^* = \frac{B_1}{c_F}$ and $\frac{\partial \Psi_F^*}{\partial B_1} > 0$ otherwise.

In summary, two observations can be made with the flexible technology. First, a lower financial flexibility β reduces profitability when this flexibility is sufficiently high. Otherwise, the firm limits its capacity investment so that even B_2 is not constraining and β has no further impact on the profitability. Second, a tighter capital budget B_1 reduces profitability when the financial flexibility is sufficiently high. Otherwise, the optimal capacity investment remains below this budget, and it does not have any impact. These observations have important implications for the optimal technology choice as analyzed in §5.

4.2 Dedicated Technology

In stage 1, the firm invested in capacity level K_D for each market. In stage 2, the firm observes the demand realizations $\boldsymbol{\xi}' = (\xi_1, \xi_2)$ and the operating budget and determines the production volume in each market $\mathbf{Q}_{\mathbf{D}}' = (Q_D^1, Q_D^2)$ to maximize the profit. Because the operating budget can be allocated between the two products, it constrains the total production volume, whereas the capacity K_D constrains the production volume separately in each market. When the operating budget realization is B_2 , the firm's stage-2 profit maximization problem is given by

$$\begin{array}{l}
\max_{\boldsymbol{Q}_{D} \geq \boldsymbol{0}} \quad \xi_{1} Q_{D}^{1} {}^{(1+\frac{1}{b})} + \xi_{2} Q_{D}^{2} {}^{(1+\frac{1}{b})} - y_{D} (Q_{D}^{1} + Q_{D}^{2}) \\
\text{s.t.} \quad Q_{D}^{1} + Q_{D}^{2} \leq \frac{B_{2}}{y_{D}}, \quad Q_{D}^{j} \leq K_{D} \text{ for } j = 1, 2.
\end{array} \tag{3}$$

With the production clearance assumption, the firm chooses the production vector so as to fully utilize the available production capacity, which is determined by the capacity and the operating budget. The set of binding constraints in (3) take three different forms in the optimal solution. **Proposition 2** Under budget realization B_2 , the unique optimal production vector $\mathbf{Q}_{\mathbf{D}}^*$ is characterized by

$$\mathbf{Q}_{\mathbf{D}}^{*}{}' = \begin{cases} (K_D, K_D) & \text{for } \frac{B_2}{y_D} \ge 2K_D \\ \frac{B_2}{y_D} \left(\frac{\xi_1^{-b}}{\xi_1^{-b} + \xi_2^{-b}}, \frac{\xi_2^{-b}}{\xi_1^{-b} + \xi_2^{-b}}\right) & \text{for } K_D > \frac{B_2}{y_D}, \end{cases}$$

and for $2K_D > \frac{B_2}{y_D} \ge K_D$,

$$\mathbf{Q}_{\mathbf{D}}^{*}{}' = \begin{cases} \left(K_{D}, \frac{B_{2}}{y_{D}} - K_{D}\right) & \text{if } \xi_{2} \leq \xi_{1} \left[\frac{\frac{B_{2}}{y_{D}} - K_{D}}{K_{D}}\right]^{-\frac{1}{b}} \\ \frac{B_{2}}{y_{D}} \left(\frac{\xi_{1}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}\right) & \text{if } \xi \in \Omega_{D} \doteq \left\{\boldsymbol{\xi} : \left[\frac{\frac{B_{2}}{y_{D}} - K_{D}}{K_{D}}\right]^{-\frac{1}{b}} \xi_{1} \leq \xi_{2} \leq \xi_{1} \left[\frac{K_{D}}{\frac{B_{2}}{y_{D}} - K_{D}}\right]^{-\frac{1}{b}} \right\} \\ \left(\frac{B_{2}}{y_{D}} - K_{D}, K_{D}\right) & \text{if } \xi_{2} \geq \xi_{1} \left[\frac{K_{D}}{\frac{B_{2}}{y_{D}} - K_{D}}\right]^{-\frac{1}{b}}. \end{cases}$$

Proposition 2 is illustrated in Figure 3. When B_2 is sufficient to produce to capacity in both markets (Panel a), this is the optimal production strategy. When B_2 is not even sufficient to produce to capacity in one market (Panel c), the firm optimally allocates this budget between the two products based on the demand realizations. Otherwise (Panel b), the optimal production strategy takes two forms depending on the demand realizations. When the demands are sufficiently close to each other (region Ω_D), the solution is the same as in Panel c. When the demand in one market is sufficiently larger than the other (regions other than Ω_D), the firm optimally produces up to capacity in the former and uses the remaining budget in the latter.

$$\begin{cases} \xi_{2} \\ (K_{D}, K_{D}) \\ (K_{D}, K_{D}) \\ \vdots \\ a) S_{D} \ge 2K_{D} \end{cases}$$

$$\begin{cases} \xi_{2} \\ (S_{D} - K_{D}, K_{D}) \\ (S_{D} - K_{D}, K_{D}) \\ (S_{D} - K_{D}, K_{D}) \\ S_{D} \left(\frac{\xi_{1}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \\ S_{D} \left(\frac{\xi_{1}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{1}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{1}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{1}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}}, \frac{\xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b}} \right) \\ S_{D} \left(\frac{\xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b} - \xi_{2}^{-b}}{\xi_{1}^{-b} + \xi_{2}^{-b} - \xi_{2}^{$$

Figure 3: The optimal production decisions with dedicated technology for a given demand realization (ξ_1, ξ_2) , capacity level K_D , and the maximum production volume that can be financed by the operating budget, $S_D \doteq \frac{B_2}{y_D}$.

In the technology choice analysis, the *pooling value* provided by the common operating stage budget will emerge as an important driver. Figure 3 delineates when pooling value

exists. When the realized operating budget B_2 is sufficient to produce to capacity in both markets (Panel a), the same profit can be generated if B_2 is equally shared between the two products. Therefore, flexibility in how to allocate this budget does not have pooling value. Otherwise (Panels b and c), a higher profit can be generated if B_2 is flexibly allocated between the two products based on the demand realizations. In this case, the operating budget has pooling value. When the realized operating budget is ample, this budget is sufficient to produce to capacity in both markets, and again there is no pooling value.

Let $\Psi_D(K_D)$ denote the expected profit for a given $K_D \in \left[0, \frac{B_1}{2c_D}\right]$ in stage 1. For $K_D \leq \frac{B_2}{2y_D}$, the production volume in each market is effectively budget unconstrained $(Q_D^{j^*} = K_D)$, and $\Psi_D(K_D) = 2\mu_{\xi}K_D^{(1+\frac{1}{b})} - 2(c_D + y_D)K_D$. For $K_D > \frac{B_2}{2y_D}$, it is effectively budget unconstrained with probability β , and it is constrained by the operating budget otherwise:

$$\Psi_D(K_D) = -2c_D K_D + \beta \left[2\mu_{\xi} K_D^{(1+\frac{1}{b})} - 2y_D K_D \right] + (1-\beta) \left[\iint_{\Omega_D} \left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \left(\frac{B_2}{y_D} \right)^{(1+\frac{1}{b})} f(\xi_1,\xi_2) \, d\xi_1 \, d\xi_2 \quad (4) \right] \\ + \iint_{\Omega_D^C} \left(\max(\xi_1,\xi_2) K_D^{(1+\frac{1}{b})} + \min(\xi_1,\xi_2) \left(\frac{B_2}{y_D} - K_D \right)^{(1+\frac{1}{b})} \right) f(\xi_1,\xi_2) \, d\xi_1 \, d\xi_2 - B_2 \right].$$

When the realized operating budget is B_2 , if $\boldsymbol{\xi} \in \Omega_D$, the firm optimally allocates the budget between the two products; and if $\boldsymbol{\xi} \in \Omega_D^C$, the firm optimally produces up to the capacity level in the more profitable market and uses the remaining budget in the other market. For $K_D \geq \frac{B_2}{y_D}$, $\Omega_D^C = \emptyset$, and only the former production strategy is relevant.

To better delineate the intuition behind the optimal capacity investment strategy, we first analyze the benchmark case where the operating budget is product-specific, (and the realized budget is equally allocated between the two products due to their symmetry). In this case, for $K_D > \frac{B_2}{2y_D}$, the production volume in each market is constrained by the operating budget $(Q_D^{j^*} = \frac{B_2}{2y_D})$ with probability $1 - \beta$, and it is effectively budget unconstrained otherwise $(Q_D^{j^*} = K_D)$.

Proposition 3 (Product-specific Operating Budget) Let $K_D^u \doteq \left(\frac{(1+\frac{1}{b})\mu_{\xi}}{c_D+y_D}\right)^{-b}$ denote the budget-unconstrained capacity investment level, and $K_D^\beta \doteq \left(\frac{(1+\frac{1}{b})\mu_{\xi}}{\frac{C_D}{\beta}+y_D}\right)^{-b}$ denote the β -flexible capacity investment level. When $\frac{B_2}{2y_D} \ge K_D^\beta$, $K_D^* = \min\left(K_D^u, \frac{B_2}{2y_D}\right)$. Otherwise, $K_D^* = \min\left(K_D^\beta, \frac{B_1}{2c_D}\right)$.

The optimal capacity investment strategy parallels the one with the flexible technology except for one modification: Because the capacity is separate for each product, the budgetunconstrained capacity level K_D^u and the β -flexible capacity level K_D^β do not depend on the demand uncertainty in both markets, but depend on the expected demand in each market. Proposition 4 (Common Operating Budget) When $\frac{B_2}{2y_D} \ge \widehat{K}_D \doteq \left(\frac{(1+\frac{1}{b})(\mu_{\xi}-(1-\beta)\mathbb{E}[\min(\widehat{\xi}_1,\widehat{\xi}_2)])}{c_D+\beta y_D}\right)^{-b}$, $K_D^* = \min\left(K_D^u, \frac{B_2}{2y_D}\right)$, where K_D^u is as given in Proposition 3. Otherwise, $K_D^* = \min\left(K_D^P, \frac{B_1}{2c_D}\right)$, where the pooling-optimal capacity investment level $K_D^P > \frac{B_2}{2y_D}$ is the unique solution to $\frac{\partial \Psi_D}{\partial K_D}\Big|_{K_D^P} = 0$. When $\frac{B_2}{2y_D} < \frac{K_D^\beta}{2}$, $K_D^P = K_D^\beta$, where K_D^β is as given in Proposition 3. Figure 4 juxtaposes K_D^* from Proposition 4 and Proposition 3. When B_2 is sufficiently high or very constraining, how this budget is allocated makes no difference for the optimal capacity investment. At the intermediate values $\left(\frac{K_D^\beta}{2} < \frac{B_2}{2y_D} < \widehat{K}_D\right)$, K_D^P is chosen unless the capital budget B_1 is constraining. In this B_2 range, we can show $K_D^P > \max\left(\frac{B_2}{2y_D}, K_D^\beta\right)$: The firm takes a larger investment risk and chooses a higher capacity level than the same with the product-specific budget due to the pooling value of the operating budget.

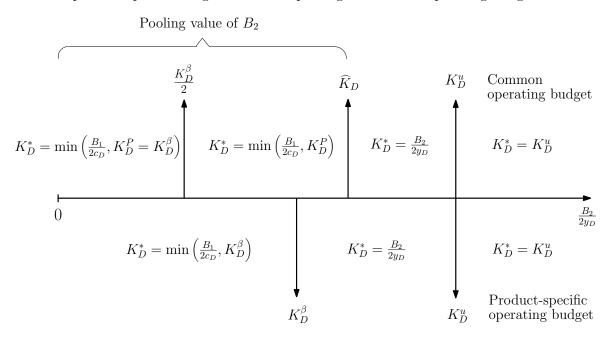


Figure 4: The optimal capacity investment K_D^* when the operating budget is common or product-specific, where $\hat{K}_D \doteq \left(\frac{\left(1+\frac{1}{b}\right)\left(\mu_{\xi}-(1-\beta)\mathbb{E}[\min(\tilde{\xi}_1,\tilde{\xi}_2)]\right)}{c_D+\beta y_D}\right)^{-b}$ such that $\frac{\partial \Psi_D}{\partial K_D}\Big|_{\left(\frac{B_2}{2y_D}\right)^+} \leq 0$ if $\frac{B_2}{2y_D} \geq \hat{K}_D$, and K_D^u , K_D^β are as defined in Proposition 3.

The optimal expected profit Ψ_D^* in a budget-constrained environment can be obtained by using K_D^* , and is omitted for brevity. The impact of the financial flexibility β and the capital budget B_1 on K_D^* and Ψ_D^* are structurally the same as the flexible technology, and results paralleling Corollary 1 can be obtained:

Corollary 2 There exist unique $0 \leq \overline{\beta}_D^0 < \widehat{\beta}_D^0 < 1$ and $\overline{\beta}_D^1 \in (\overline{\beta}_D^0, 1)$ such that *i*) $K_D^* = \frac{B_2}{2y_D}$ and $\frac{\partial \Psi_D^*}{\partial \beta} = 0$ if $\beta \leq \overline{\beta}_D^0$. Otherwise, $K_D^* = \min\left(K_D^P, \frac{B_1}{c_F}\right)$, where $\frac{\partial K_D^P}{\partial \beta} > 0$. In this case, $K_D^P = K_D^\beta$ if $\beta \ge \hat{\beta}_D^0$; ii) $K_D^* < \frac{B_1}{2c_D}$ and $\frac{\partial \Psi_D^*}{\partial B_1} = 0$ if $\beta \le \overline{\beta}_D^1$; $K_D^* = \frac{B_1}{2c_D}$ and $\frac{\partial \Psi_D^*}{\partial B_1} > 0$ otherwise.

Unlike Corollary 1, it is not analytically tractable to prove $\frac{\partial \Psi_D^*}{\partial \beta} > 0$ when $\beta > \overline{\beta}_D^0$ without introducing additional parametric assumption.⁶ However, in all our numerical experiments (that are described in §5), we observe that Ψ_D^* strictly increases in β for $\beta > \overline{\beta}_D^0$.

The pooling value of the (common) operating budget can be explicitly characterized by considering the difference between Ψ_D^* and the optimal expected profit $\widehat{\Psi}_D^*$ where this budget is product-specific. As intuition suggests, there is no pooling value if the demands are perfectly positively correlated. Otherwise, the pooling value crucially depends on the financial flexibility level β :

Corollary 3 Let $\triangle^P \doteq \Psi_D^* - \widehat{\Psi}_D^*$, and assume $\rho < 1$. $\triangle^P = 0$ if $\beta \leq \overline{\beta}_D^0$ or $\beta = 1$, and $\triangle^P > 0$ otherwise. When $\beta \geq \widehat{\beta}_D^0$, $\triangle^P \doteq (1-\beta) \left(\frac{B_2}{y_D}\right)^{(1+\frac{1}{b})} \left(\mathbb{E}\left[\left(\widetilde{\xi}_1^{-b} + \widetilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}\right] - 2^{-\frac{1}{b}} \mu_{\xi}\right)$.

When $\beta \leq \overline{\beta}_D^0$, $K_D^* = \frac{B_2}{2y_D}$ with the common or the product-specific budget, and the budget is always sufficient to finance the production with full capacity utilization. Therefore, there is no pooling value. At full financial flexibility, i.e. when $\beta = 1$, the operating budget is always ample, and thus, it does not matter if this budget is common or product-specific, i.e. $\Delta^P = 0$. The pooling value can be characterized in closed form for $\beta \geq \widehat{\beta}_D^0$ because K_D^* can be characterized in closed form here. It can be proven that $\mathbb{E}\left[\left(\widetilde{\xi}_1^{-b} + \widetilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}\right] \geq 2^{-\frac{1}{b}} \mu_{\xi}$ with equality holding only for $\rho = 1$, and thus, $\Delta^P > 0$ for $\rho < 1$ in this case.

We close this section with an important remark: \triangle^P first increases then decreases in β .⁷ For low β values, \triangle^P increases because the firm benefits from a higher β only with the common operating budget: $K_D^* > \frac{B_2}{2y_D}$ only with this budget due to its pooling value. For larger β values, the firm benefits from a higher β with the product-specific and the common operating budget. As β increases, because the firm is less likely to be budget constrained in the production stage, \triangle^P decreases and equals zero when $\beta = 1$. These observations have important implications for the optimal technology choice analyzed in §5.

⁶One sufficient condition for $\frac{\partial \Psi_D^*}{\partial \beta} > 0$ to hold when $\beta > \overline{\beta}_D^0$ is $\frac{c_D - y_D}{c_D + y_D} > 1 - \frac{\mathbb{E}[\min(\tilde{\xi}_1, \tilde{\xi}_2)]}{\mu_{\xi}}$. ⁷We can analytically perform the sensitivity analysis for sufficiently low β range, where Δ^P increases in

⁷We can analytically perform the sensitivity analysis for sufficiently low β range, where Δ^P increases in β , and for $\beta \geq \hat{\beta}_D^0$, where Δ^P decreases in β following from Corollary 3. In between, we resort to numerical experiments (that are described in §5). In all our numerical instances, we consistently observe that Δ^P first increases then decreases in β .

5 The Optimal Technology Choice and The Impact of Budget Constraints

In this section, we provide answers to our three main research questions. In particular, we analyze the optimal technology choice in a budget-constrained environment in comparison with the budget-unconstrained environment (§5.1), and how this choice is affected by a tighter capital budget (§5.2) and a lower financial flexibility in the production stage (§5.3). To provide analytical results and generate sharper managerial insights, we introduce a reformulation in our model. To characterize technology T, instead of (c_T, y_T) , i.e. the unit capacity and production costs, we use (η_T, α_T) where $\eta_T = c_T + y_T$ and $\alpha_T \doteq \frac{c_T}{c_T + y_T}$. In this formulation, η_T denotes the unit (aggregate) investment cost of technology T. We call $\alpha_T \in [0, 1]$ the capacity intensity and $(1 - \alpha_T)$ the production intensity of the technology T. Because the former measure uniquely defines the latter, we will focus on the capacity intensity.

It is easy to establish that the optimal expected profit strictly decreases in the unit investment cost with each technology. Therefore, for a given unit investment cost η_D of dedicated technology, there exists a unique unit investment cost threshold $\eta_F(\eta_D)$ for flexible technology such that it is optimal to invest in flexible technology when $\eta_F \leq \eta_F(\eta_D)$, and in dedicated technology otherwise. Let $\eta_F^u(\eta_D)$ denote this cost threshold in the absence of budget constraints, and $\overline{\eta}_F(\eta_D)$ denote the same in a budget-constrained environment.

In the absence of budget constraints, the firm optimally invests in budget-unconstrained capacity investment level K_F^u with flexible technology and K_D^u with dedicated technology as defined in Propositions 1 and 3 in §4. Therefore, for a given η_D , the flexible threshold is characterized by $\eta_F^u(\eta_D) \doteq \eta_D \left[\frac{\mathbb{E}\left[\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b} \right)^{-\frac{1}{b}} \right]}{2^{-\frac{1}{b}} \mu_{\xi}} \right]^{\frac{b}{b+1}}$, where the term $\mathbb{E}\left[\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b} \right)^{-\frac{1}{b}} \right]$ captures the capacity-pooling value of the flexible technology.⁸ In the presence of budget constraints, $\overline{\eta}_F(\eta_D)$ captures the capacity-pooling value of the flexible technology and the relative impact of the capital budget and the operating budget uncertainty on each technology. To answer our first research question, we compare $\overline{\eta}_F(\eta_D)$ with $\eta_F^u(\eta_D)$. To answer the remaining two questions, we conduct sensitivity analysis to investigate how $\overline{\eta}_F(\eta_D)$ changes in the capital budget B_1 and the financial flexibility β .

⁸This threshold is identical to the capacity cost threshold of Boyabath and Toktay (2011) ("the perfect capital market capacity cost threshold" with their terminology), where they normalize the production costs to zero. With our reformulation of the technology cost parameters, we are able to capture the non-zero production cost and obtain the same threshold as a function of the unit cost η_T .

Throughout this section, we assume that the capacity intensity is (weakly) larger with the flexible technology, i.e. $\alpha_F \geq \alpha_D$. Because flexible technology has a higher capacity investment cost than dedicated technology, it has a larger capacity intensity unless its production cost is significantly higher than dedicated technology. To better delineate the intuition behind our results, we first focus on the special case with identical capacity intensities, i.e. $\alpha_F = \alpha_D$. We then investigate how our results are impacted as α_F increases.

Throughout this section, we use the financial flexibility thresholds $\overline{\beta}_D^0$, $\widehat{\beta}_D^0$ and $\overline{\beta}_D^1$, as defined in Corollary 2 in §4.2. To represent the budget-constrained environment, we assume that both B_1 and B_2 are insufficient to finance the capacity investment and the production volume with K_D^u respectively. When analytical results are not attainable, we resort to numerical experiments. In these experiments, we assume that $\tilde{\xi}$ follows a symmetric bivariate normal distribution. We use the following baseline parameter values: $\mu_{\xi} = 10$, $\sigma_{\xi} = 4\%$ of μ_{ξ} , $\rho = 0$, $\eta_D = 3$, $\alpha_D = 0.7$, $\alpha_F = \alpha_D + l$, where $l = \frac{1-\alpha_D}{4}$, $B_2 = k *$ $2(1 - \alpha_D)\eta_D K_D^u$, where $k = \frac{1}{10}$, and b = -2. We consider a large set of values for the parameters of interest, β and B_1 . In particular, we choose 40 β values in [0, 1], and assume $B_1 = m * 2\alpha_D\eta_D K_D^u$, where m takes 40 values between 1.1k and 0.9, which satisfies our assumption $\frac{B_1}{\alpha_D} > \frac{B_2}{1-\alpha_D}$. To ensure robustness with respect to demand parameters, capacity intensity and operating budget, we vary the model parameters as follows: $\mu_{\xi} \in \{10, 20, 30\}$, $\sigma_{\xi} \in [4\%, 8\%]$ of μ_{ξ} , $\rho \in \{-0.45, 0, 0.45\}$, $\alpha_D \in \{0.7, 0.9\}$, $l \in \{0, \frac{1-\alpha_D}{8}, \frac{1-\alpha_D}{4}, \frac{1-\alpha_D}{2}\}$ and $k \in \{\frac{1}{1000}, \frac{1}{100}, \frac{1}{10}\}$. In summary, we focus on 691,200 instances to characterize $\overline{\eta}_F(\eta_D)$.

Our parameter choice is representative of the automotive industry along a number of dimensions. For example, Chod and Zhou (2014) document that the coefficient of variation of detrended annual sales of General Motors (Ford) in the period of year 1998 to year 2007 (preceding the global financial crisis) is 0.048 (0.046). If we consider the dedicated capacity investment, and calculate the first-best production volume for each product (in the absence of capacity or the operating budget constraints), the coefficient of variation of the sales for each product with our baseline parameters is 0.056. Moreover, the profit margin of each product is 3.23% of the revenues (net of the production cost) in the production stage. This is also consistent with the automotive industry. In particular, Chod and Zhou (2014) document that the average profit margin of General Motors (Ford) from 2005 to 2012 has been 4.73% (3.05%). Finally, because the automotive industry is capacity-intensive, the capacity-intensity level of each technology is chosen to be high.

5.1 Comparison with the Budget Unconstrained Benchmark

The majority of the papers in the literature have studied flexible versus dedicated technology choice in the absence of budget constraints. We now investigate the impact of accounting for these constraints on the optimal technology choice by making a comparison with the budget-unconstrained benchmark. In a budget-constrained environment, the technology choice is determined by comparing a flexible system (with flexible capacity and operating budget) with a *partially-flexible* system (with dedicated capacities and flexible operating budget). In the absence of budget constraints, because the operating budget is not constraining, this comparison is between a flexible system and a *non-flexible* system. In other words, the flexibility of the operating budget brings dedicated technology closer to flexible technology in terms of the overall resource network's flexibility. Therefore, the technology choice is impacted by to what extent this flexibility is beneficial with dedicated technology (the pooling value of the operating budget with dedicated technology). The technology choice is also impacted by to what extent the capacity investment is constrained by the capital budget and the production decisions are constrained by the realized operating budget with each technology. Therefore, the comparison between $\overline{\eta}_F(\eta_D)$ and $\eta_E^u(\eta_D)$ crucially depends on the relative total capacity investment cost with each technology, i.e. $\alpha_F \eta_F K_F^u |_{\eta_F^u(\eta_D)}$ versus $2\alpha_D \eta_D K_D^u |_{\eta_D}$, the relative total production cost with each technology, i.e. $(1 - \alpha_F)\eta_F K_F^u|_{\eta_E^u(\eta_D)}$ versus $2(1 - \alpha_D)\eta_D K_D^u|_{\eta_D}$, and the pooling value of the operating budget with dedicated technology.

With identical capacity intensities, the total capacity investment and production costs are the same with each technology⁹, and thus, only the pooling value of the operating budget with dedicated technology matters as demonstrated in Proposition 5.

Proposition 5 ($\alpha_F = \alpha_D$) $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$ if $\overline{\beta}_D^0 < \beta < 1$, and $\overline{\eta}_F(\eta_D) = \eta_F^u(\eta_D)$ otherwise.

As follows from Corollary 3 in §4, when the realized budget B_2 is always sufficient, i.e. $0 < \beta \leq \overline{\beta}_D^0$, or when the budget is always ample, i.e. $\beta = 1$, there is no pooling value of the operating budget with dedicated technology. Otherwise, there is pooling value. Because this pooling value brings the dedicated technology closer to the flexible technology in terms of the overall resource network's flexibility, flexible technology is adopted for a smaller unit

⁹In a budget-unconstrained environment, when the firm is indifferent between the two technologies, the total capacity investment level is lower with flexible technology due to its capacity-pooling benefit, i.e. $K_F^u|_{\eta_F^u(\eta_D)} \leq 2K_D^u|_{\eta_D}$; however, the investment cost is larger with flexible technology, i.e. $\eta_F^u(\eta_D) \geq \eta_D$. It turns out that the total investment cost is identical with each technology, i.e. $\eta_F K_F^u|_{\eta_D^u(\eta_D)} = 2\eta_D K_D^u|_{\eta_D}$.

investment cost range in comparison with the budget-unconstrained benchmark.

When $\alpha_F > \alpha_D$, the relative total capacity investment and production costs also matter:

Proposition 6
$$(\alpha_F > \alpha_D) \overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$$
 if $0 < \beta \leq \overline{\beta}_D^0$. There exists $\overline{\beta}_D^2 \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$ such that $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$ if $\overline{\beta}_D^2 \leq \beta \leq 1$.

When the financial flexibility β is sufficiently high, following Corollaries 1 and 2 in §4, both technologies are constrained by the capital budget. Because $\alpha_F > \alpha_D$, the total capacity investment cost is higher with flexible technology, and thus, this technology is more negatively affected by a limited capital budget. Moreover, as discussed above, there is the pooling benefit of the operating budget with dedicated technology. Therefore, flexible technology is adopted for a smaller unit investment cost range in comparison with the budget-unconstrained benchmark. When β is sufficiently low, following Corollaries 1, 2 and 3, the capital budget is not constraining with either technology, and there is no pooling value with dedicated technology. In this case, the optimal capacity investment level equals the production volume attainable with B_2 . Because $\alpha_F > \alpha_D$, the total production cost is higher with dedicated technology, and thus, this technology is more negatively affected by a limited operating budget. Therefore, flexible technology is adopted for a larger unit investment cost range in comparison with the budget-unconstrained benchmark.

To investigate the $\beta \in (\overline{\beta}_D^0, \overline{\beta}_D^2)$ range, we conduct numerical experiments. All instances in our numerical set reinforce the results in Proposition 6: We observe a unique $\overline{\beta}_D^u$ such that $\overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$ if $\beta < \overline{\beta}_D^u$, and $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$ otherwise. We also observe $\overline{\beta}_D^u$ increases in α_F due to increasing difference between the total production cost with each technology, and increases in B_2 due to increasing $\overline{\beta}_D^0$. These observations have important implications for the impact of β on the optimal technology choice as analyzed in §5.3.

In summary, accounting for the budget constraints is important in avoiding technology mis-specification. More importantly, the capacity intensity of each technology, the pooling value of the operating budget with dedicated technology and the level of financial flexibility in the production stage have a pronounced effect on the direction of the mis-specification.

5.2 The Impact of A Tighter Capital Budget

In this section, we investigate the impact of the capital budget B_1 on the optimal technology choice. We first analyze the case with identical capacity intensities:

Proposition 7
$$(\alpha_F = \alpha_D) \frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} = 0$$
 if $0 < \beta \leq \overline{\beta}_D^0$ or $\beta = 1$. $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} > 0$ if $\max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right) \leq \beta < 1$.

In other words, we can analytically perform sensitivity analysis for $0 < \beta \leq \overline{\beta}_D^0$ and $\beta \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$. In between, we resort to numerical experiments. Panel a of Figure 5 summarizes the generic pattern observed in all our numerical instances.

There is a capital budget threshold $B_1^u(\beta)$ such that when $B_1 > B_1^u(\beta)$, the optimal capacity investment level with either technology is not constrained by B_1 , and thus, B_1 has no impact. As β increases, because the optimal capacity level with each technology increases, a higher B_1 becomes necessary for an unconstrained budget, and thus, $B_1^u(\beta)$ increases. As proven in Proposition 7, when β is sufficiently low, the $B_1 > B_1^u(\beta)$ region subsumes the entire B_1 range; and when β is sufficiently high, this region ceases to exist. When $B_1 < B_1^u(\beta)$, B_1 matters. In this region, in the absence of the capital budget constraint, the total capacity investment cost is higher with flexible technology.¹⁰ Therefore, as B_1 decreases, this technology is more negatively affected, and is adopted for a smaller unit investment cost range.

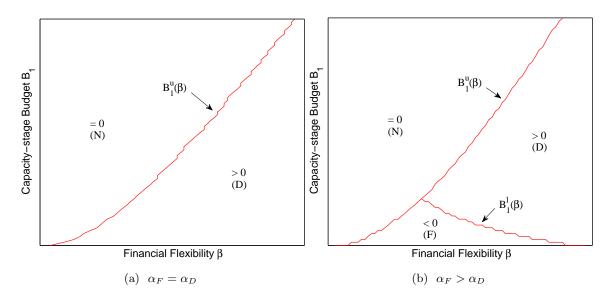


Figure 5: The impact of the capital budget B_1 on $\overline{\eta}_F(\eta_D)$ for a given (β, B_1) : "= 0", "> 0", and "< 0" denote the sign of $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1}$. $\{N, F, D\}$ document the technology that is adopted for a larger unit investment cost range, and thus, is less negatively affected by a tighter B_1 . N means neither technology is affected.

¹⁰To understand this, consider sufficiently high β . In the absence of the B_1 constraint, following Corollaries 1 and 2 in §4, the optimal capacity investment is characterized by the β -flexible level with both technologies. When $\eta_F = \eta_F^u(\eta_D)$, with identical capacity intensities, the total capacity investment cost is the same with each technology. Because $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$, as follows from Proposition 5, and the total capacity investment cost with technology T decreases in η_T , it is higher with flexible technology when $\eta_F = \overline{\eta}_F(\eta_D)$.

A parallel result with Proposition 7 can be obtained with minor modifications when flexible technology has a larger capacity intensity:

Proposition 8 ($\alpha_F > \alpha_D$) $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} = 0$ if $0 < \beta \leq \overline{\beta}_D^0$. There exists $\overline{\beta}_F^3 \in [\overline{\beta}_D^2, 1)$ such that $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} > 0$ if $\overline{\beta}_D^3 \leq \beta \leq 1$.

In other words, we can analytically perform sensitivity analysis for $0 < \beta \leq \overline{\beta}_D^0$ and $\beta \geq \overline{\beta}_D^3$. In between, we resort to numerical experiments. Panel b of Figure 5 summarizes the generic pattern observed in all our numerical instances. Unlike Panel a, in the $B_1 < B_1^u(\beta)$ region, we observe a sub-region $B_1 < B_1^l(\beta)$ in which $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} < 0$. The intuition is as follows:

With a sufficiently high β , the total capacity investment cost is higher with flexible technology. Therefore, dedicated technology is less negatively affected by a tighter B_1 (the D region in Panel b). As β decreases, the optimal capacity investment level decreases with each technology. Because the optimal capacity level decreases to a larger extent with flexible technology, the total capacity investment cost is lower with this technology.¹¹ Therefore, flexible technology is less negatively affected by a tighter B_1 (the F region in Panel b). In our numerical experiments, we observe that $\overline{\beta}_D^u$ decreases in B_1 in the F region of Panel b, and thus, $B_1^l(\beta)$ also decreases in β .

In summary, as depicted in Figure 5, the dominant regime is one where dedicated technology is adopted for a larger unit investment cost range, and thus, is the best response to the tightening of the capital budget. The reason is that dedicated technology has lower total capacity investment cost. This finding is reversed when flexible technology has a strictly larger capacity intensity, the capital budget is severely constraining and the financial flexibility is moderate. In this case, the operating budget considerations become critical: Because the total capacity investment level is less sensitive to changes in financial flexibility with dedicated technology, this technology has a higher total capacity investment cost. Therefore, flexible technology is the best response to the tightening of the capital budget. Managerially, these results are important because they imply that the optimal technology adopted should differ depending on the severity of the capital and operating budget constraints. Thus, indiscriminately adopting the same technology as financial constraints get tighter can be a detrimental strategy.

¹¹When the optimal capacity investment is characterized by the β -flexible level with both technologies, for sufficiently low β , the total capacity investment cost is lower with flexible technology when $\eta_F = \eta_F^u(\eta_D)$. Moreover, as follows from §5.1, $\overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$ for sufficiently low $\beta < \overline{\beta}_D^u$. Because the total capacity investment cost with technology T decreases in η_T , it is lower with flexible technology when $\eta_F = \overline{\eta}_F(\eta_D)$.

5.3 The Impact of Financial Flexibility in the Production Stage

In this section, we investigate the impact of the financial flexibility β in the production stage on the optimal technology choice. With identical capacity intensities, this impact depends on the pooling value of the operating budget with dedicated technology:

Proposition 9 $(\alpha_F = \alpha_D) \frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} = 0 \text{ if } 0 < \beta \leq \overline{\beta}_D^0. \frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} > 0 \text{ if } \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right) \leq \beta < 1.$

In other words, we can analytically perform sensitivity analysis for $\beta \leq \overline{\beta}_D^0$ and $\beta \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$. In between, we resort to numerical experiments. Figure 6 summarizes the generic pattern observed in all our numerical instances. The impact of B_2 on our results is also highlighted as it parameterizes the severity of the budget constraint.

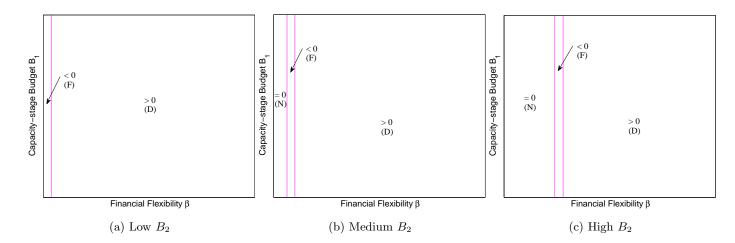


Figure 6: The impact of the financial flexibility β on $\overline{\eta}_F(\eta_D)$ for a given (β, B_1) and B_2 with $\alpha_F = \alpha_D$: "= 0", "> 0" and "< 0" denote the sign of $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta}$. $\{N, F, D\}$ document the technology that is adopted for a larger unit investment cost range, and thus, is less negatively affected by a lower β . N means neither technology is affected.

When β is sufficiently low, following Corollaries 1 and 2 in §4, neither technology is impacted by β . Therefore, as shown in Proposition 9, β has no impact on the optimal technology choice (the *N* regions in Figure 6).¹² In the remaining β range, β matters. When β is moderately low, as discussed at the end of §4.2, the pooling value of the operating budget with dedicated technology increases in β , and thus, flexible technology is adopted for a smaller unit investment cost range (the F regions in Figure 6). When β is moderately high, the optimal capacity is given by the β -flexible level with both technologies, and the total production cost is higher with flexible technology. Therefore, this technology benefits

¹²Because $\overline{\beta}_D^0$ decreases in B_2 , the N region in Figure 6 shrinks as B_2 decreases.

more from a higher β , and is adopted for a larger unit investment cost range. When β is sufficiently high, following Corollaries 1 and 2, the capital budget is constraining with both technologies. With identical capacity intensities, the total production cost is the same with each technology. As β increases, as discussed at the end of §4.2, because the pooling value of the operating budget with dedicated technology decreases, flexible technology is adopted for a larger unit investment cost range, as shown in Proposition 9.

When flexible technology has a larger capacity intensity, the relative production costs with each technology plays a key role for the impact of β on the optimal technology choice.

Proposition 10 $(\alpha_F > \alpha_D)$ $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} = 0$ if $0 < \beta \leq \overline{\beta}_D^0$. For $\beta \in \left[\max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right), 1 \right)$, there exist a unique $\overline{\alpha}_F > \alpha_D$ such that $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} > 0$ if $\alpha_F < \overline{\alpha}_F$, and $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} < 0$ otherwise.

In other words, we can analytically perform sensitivity analysis in the same β range with Proposition 9. For the remaining β range, we resort to numerical experiments. Figure 7 summarizes the generic pattern observed in all our numerical instances.

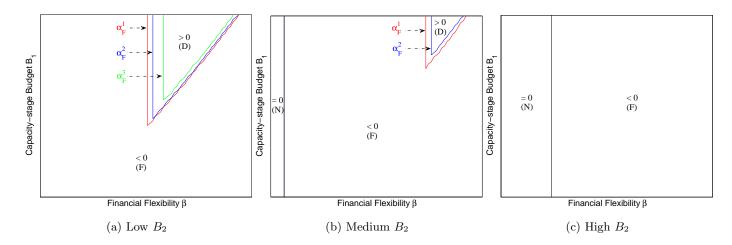


Figure 7: The impact of the financial flexibility β on $\overline{\eta}_F(\eta_D)$ for a given (β, B_1) and B_2 with $\alpha_F^j = \alpha_D + l^j$ where $l^j \in \{\frac{1-\alpha_D}{8}, \frac{1-\alpha_D}{4}, \frac{1-\alpha_D}{2}\}$: "= 0", "> 0" and "< 0" denote the sign of $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta}$. $\{N, F, D\}$ document the technology that is adopted for a larger unit investment cost range, and thus, is less negatively affected by a lower β .

In comparison with the $\alpha_F = \alpha_D$ case, the majority of the *D* regions in Figure 6 are replaced with the *F* regions in Figure 7. This pattern can be explained by two observations:

1) When β is sufficiently high such that the capital budget is constraining with both technologies, the impact of β on the optimal technology choice crucially depends on α_F . In this case, the total production cost with technology $T \in \{D, F\}$ is given by $\frac{1-\alpha_T}{\alpha_T}B_1$. Because $\alpha_F > \alpha_D$, the total production cost is higher with dedicated technology, and thus, all else equal, this technology benefits more from a higher β . However, following Corollary 3 in §4.2, a higher β also decreases the pooling value of the operating budget with dedicated technology. As shown in Proposition 10, when α_F is smaller than a threshold $\overline{\alpha}_F$, the pooling effect dominates, and as β increases, flexible technology is adopted for a larger unit investment cost range. Otherwise, the higher total production cost effect dominates, and dedicated technology is adopted for a larger unit investment cost range. In our numerical setting, we observe $\alpha_F > \overline{\alpha}_F$, and thus, as β increases, dedicated technology is adopted for a larger unit investment cost range, as depicted in Figure 7.

2) When β is moderately high such that the capital budget is not constraining, the impact of β on the optimal technology choice crucially depends on the ordering between the budget-constrained and budget-unconstrained investment cost thresholds. For low β values within this range, as follows from §5.1, $\overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$ for $\beta < \overline{\beta}_D^u$. In this case, the total production cost is higher with dedicated technology, and thus, this technology benefits more from a higher β , and is adopted for a larger unit investment cost range. For larger β values within this range, $\beta > \overline{\beta}_D^u$, $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$. In this case, the total production cost is higher with flexible technology, and thus, this technology benefits more from a higher β , and is adopted for a larger unit investment cost range (the *D* regions in Figure 7). Because $\overline{\beta}_D^u$ increases in B_2 and α_F (as discussed in §5.1), the *D* region shrinks as B_2 or α_F increases.

In summary, with identical capacity intensities, as depicted in Figure 6, the dominant regime is one where dedicated technology is the best response to lower financial flexibility. This finding is reversed when the financial flexibility is sufficiently low. These results are driven by the impact of financial flexibility on the pooling value of the operating budget with dedicated technology: Lower financial flexibility increases this pooling value unless the financial flexibility is sufficiently low. When flexible technology has a larger capacity intensity, the total production cost is lower with this technology, and thus, all else equal, this technology is less negatively impacted by lower financial flexibility. When the capacity intensity of flexible technology is sufficiently large, this effect outweighs the increasing pooling value of the operating budget with dedicated technology: As depicted in Figure 7, the dominant regime is one where flexible technology is the best response to lower financial flexibility. Our results show that when flexible technology has larger capacity intensity, financial flexibility and flexible technology are substitutes unless the capital budget is moderately constraining and the financial flexibility is also moderate; otherwise they are complements.

6 Conclusion

This paper contributes to the operations management literature on stochastic capacity and technology investment in multi-product firms by analyzing the impact of financial constraints on the flexible versus dedicated technology choice. The majority of papers in this literature (often implicitly) assume that operational investments are made with abundant financial resources. Yet operations managers often rely on limited budgets both in capacity investment and production. This is the first paper that studies how the capital budget constraint and the operating budget uncertainty jointly shape the flexible-versus-dedicated technology choice and the optimal capacity investment with each technology.

We identify that flexibility in how to allocate the (common) operating budget based on the demand realizations plays a key role on the optimal technology choice. As explained at the beginning of §5.1, this operating budget flexibility brings dedicated technology closer to flexible technology in terms of the overall resource network's flexibility. Therefore, the technology choice is impacted by to what extent this flexibility is beneficial with dedicated technology (the pooling value of the operating budget with dedicated technology). We uncover that because this pooling value is ignored, not accounting for financial constraints may result in the more frequent adoption of flexible technology than warranted when both technologies have the same capacity intensity (the ratio of unit capacity cost to total unit capacity and production cost). When flexible technology has a higher capacity intensity, the same technology mis-specification is also observed with high financial flexibility in the production stage. Otherwise, not accounting for financial constraints results in the more frequent adoption of dedicated technology than warranted. The reason is that flexible technology has lower total production cost than that of dedicated technology in this case, and not accounting for the possibility of an operating budget constraint ignores this benefit.

We observe that one type of technology is not a panacea for tighter financial constraints - not only the severity but also the stage of these financial constraints are important drivers of the right technology choice. Consider the case where both technologies have similar production costs,¹³ and thus, the capacity intensity is strictly larger with flexible technology. Our results demonstrate that dedicated technology is the best response to the tightening of the capital budget and should be adopted for a larger unit investment cost range unless this budget is severely constraining and the financial flexibility in the production stage is moderate. Dedicated technology is the best response to lower financial flexibility and should be adopted for a larger unit investment cost range only when the capital budget

¹³In practice, this is relevant, for example, with highly automated technologies (Fine and Freund 1990).

is moderately constraining and the financial flexibility is also moderate. These results underline the importance of considering the production costs rather than only capacity costs, which have been the main focus of the extant literature.

Another important implication of our results is on the link between operational flexibility and operating leverage (the ratio of total capacity cost to the total expected production cost). In a *budget-unconstrained environment*, because capacity intensity is higher with flexible technology, intuitively, operating leverage is higher with this technology in comparison with dedicated technology. Is operating leverage higher with flexible technology in a *budget-constrained environment*? Interestingly, our results imply that the operating leverage can be lower with flexible technology in this environment, in particular, this is case when the financial flexibility is moderately low. This is because other factors such as the pooling value of the operating budget with dedicated technology and the financial flexibility level in the production stage become effective in a budget-constrained environment.

The capacity intensity of a technology is affected by its automation level. When the highly automated technology requires a higher capacity cost but a lower labor cost than the less automated technology, the former has a higher capacity intensity. Thus, our results underline the importance of considering financial constraints when deciding the automation level of the production technology. The capacity intensity of a technology may also be affected by the location of the production plant. This is because labor costs, which can constitute a big part of production costs, may vary with respect to the plant location. Therefore, our results underline the need for firms to take a holistic view of the technology adoption together in the presence of financial constraints.

Consider a business unit that relies on funds from its parent company. The financial flexibility of this unit is closely linked to the product portfolio of its parent company. For example, business units producing a premium product are more likely to be allocated sufficient budgets to cover their operating costs, thus, they have a higher financial flexibility in the production stage. The financial flexibility of the business unit may also vary based on the diversification level of the product portfolio of its parent company. If the products are highly diversified, one may argue that their production costs are not strongly positively correlated. In this case, if the focal business unit requires additional financing due to an unexpected increase in the operating cost, the headquarters can provide additional financing by reallocating funds from the other business units. Because the financial flexibility is critical in the optimal technology choice, there is value in coordinating the technology

investment and the product portfolio decision.

Our modeling of the operating budget uncertainty captures the resource constraints in the production environment due to the financial problems of the firm. In practice, such a constraint can also be a result of the financial problems of the firm's suppliers. In particular, a financially troubled supplier may not be able to deliver the required volume of components for production. For example, as discussed in Nussel and Sherefkin (2008), Chrysler, the auto manufacturer, temporarily closed four assembly plants and canceled a shift at a fifth plant as a result of component shortages after Plastech, its supplier of different types of trim (such as door panels, floor consoles and engine covers), encountered financial problems. Babich (2010) discusses other examples in the automotive industry. The resource constraint in the production environment due to a supplier's financial problem can be captured by reinterpreting the operating budget as the available volume of a common component for both products, and the operating cost as the procurement cost. In particular, β represents the likelihood of sufficient delivery volume from the supplier regardless of the capacity investment level, B_2 denotes the total value of components received, and y_T denotes the procurement cost of each component with technology $T \in \{D, F\}$. Therefore, $\frac{B_2}{y_T}$ denotes the component volume available for production which captures the severity of the component shortage. Because the procurement cost is the same for each technology, flexible technology has a strictly larger capacity intensity. In this setting, our results, as summarized in Figure 7, demonstrate that technology adopted in anticipation of a higher component shortage possibility crucially depends on the severity of this shortage: Dedicated technology should be adopted unless the component shortage is expected to be severe. Otherwise, flexible technology can be the weapon of choice, in particular, this is the case when the shortage possibility is moderately high.

Relaxing the assumptions made on the modeling of the budget constraints gives rise to a number of interesting areas for future research. First, there is our simplifying assumption of a two-point characterization of the operating budget uncertainty. Second, we focus on budget constraints that are unrelated to demand prospects. When the possibility of an operating budget constraint increases with lower demand prospects (because of tighter credit terms), the correlation between the operating budget and demand uncertainty should be considered. Third, we assume exogenous capital budget and operating budget uncertainty. Consider a business unit that relies on budgets allocated by its parent company. The parent company allocates these budgets by considering its own capital availability and the investment opportunities from all business units. Studying this allocation decision in an equilibrium setting should prove to be an interesting problem for future research.

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A Appendix

Throughout the appendix, we use $M_F \doteq \mathbb{E}\left[\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}\right].$

Name	Description		
c_T	Unit capacity investment cost of technology T		
y_T	Unit production cost of technology T		
$\alpha_T = \frac{c_T}{c_T + y_T}$	Capacity intensity of technology T		
$\eta_T = c_T + y_T$	Unit (aggregate) investment cost of technology T		
B_1	Capital budget		
B_2	Operating budget realization		
β	Probability of ample operating budget		
$b\in(-\infty,-1)$	Constant price elasticity of demand		
$\mu_{\xi} = \mathbb{E}[\xi_1] = \mathbb{E}[\xi_2]$	Expected demand		
ρ	Demand correlation		
Q_T^{i*}	Optimal production volume for product $i \in \{1, 2\}$ with technology T		
K_T^*	Optimal capacity investment level with technology T		
K_T^u	Budget-unconstrained capacity investment level with technology T		
K_T^{β}	β -flexible capacity investment level with technology T		
K_D^P	Pooling-optimal capacity investment level with dedicated technology		
Ψ_T^*	Optimal expected profit with technology T when the operating budget is common		
$\widehat{\Psi}_D^*$	Optimal expected profit with dedicated technology when the operating budget is product-specific		
$\Delta^P = \Psi^*_D - \widehat{\Psi}^*_D$	Pooling value of the (common) operating budget with dedicated technology		
$\eta_F^u(\eta_D)$	Flexible investment cost threshold in a budget-unconstrained environment		
$\overline{\eta}_F(\eta_D)$	Flexible investment cost threshold in a budget-constrained environment		

Table 1: Summary of Notation

Proof of Proposition 1: For $K_F \in \left[0, \frac{B_2}{y_F}\right]$, $\frac{\partial \Psi_F}{\partial K_F} = -(c_F + y_F) + M_F \left(1 + \frac{1}{b}\right) K_F^{\frac{1}{b}}$, and for $K_F \in \left(\frac{B_2}{y_F}, \frac{B_1}{c_F}\right]$, $\frac{\partial \Psi_F}{\partial K_F} = -c_F + \beta \left[M_F \left(1 + \frac{1}{b}\right) K_F^{\frac{1}{b}} - y_F\right]$. Because b < -1, $\frac{\partial^2 \Psi_F}{\partial K_F^2} < 0$ in each K_F range, and thus, Ψ_F is piecewise concave in K_F with a kink at $K_F = \frac{B_2}{y_F}$. Moreover, $\left.\frac{\partial \Psi_F}{\partial K_F}\right|_{0^+} \to \infty$. $\left.\frac{\partial \Psi_F}{\partial K_F}\right|_{\left(\frac{B_2}{y_F}\right)^-} \ge \left.\frac{\partial \Psi_F}{\partial K_F}\right|_{\left(\frac{B_2}{y_F}\right)^+}$ is equivalent to $B_2 \le y_F \left(\frac{\left(1 + \frac{1}{b}\right)M_F}{y_F}\right)^{-b}$, and $\left.\frac{\partial \Psi_F}{\partial K_F}\right|_{\left(\frac{B_2}{y_F}\right)^+} \le 0$ is equivalent to $B_2 \ge y_F K_F^{\beta}$, where $K_F^{\beta} = \left(\frac{\left(1 + \frac{1}{b}\right)M_F}{\frac{c_F}{\beta} + y_F}\right)^{-b}$. When $B_{2} \geq y_{F}K_{F}^{\beta}, \text{ i.e. } \frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{\left(\frac{B_{2}}{y_{F}}\right)^{+}} \leq 0, \text{ because } \frac{B_{1}}{c_{F}} > \frac{B_{2}}{y_{F}}, K_{F}^{*} = \min\left(K_{F}^{u}, \frac{B_{2}}{y_{F}}\right) \text{ where } K_{F}^{u} = \left(\frac{\left(1+\frac{1}{b}\right)M_{F}}{c_{F}+y_{F}}\right)^{-b} \text{ is the unique solution to } \frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{K_{F}^{u}} = 0 \text{ for } K_{F} \leq \frac{B_{2}}{y_{F}}. \text{ When } B_{2} < y_{F}K_{F}^{\beta}, \text{ i.e.}$ $\frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{\left(\frac{B_{2}}{y_{F}}\right)^{+}} > 0, \text{ because } K_{F}^{\beta} < \left(\frac{\left(1+\frac{1}{b}\right)M_{F}}{y_{F}}\right)^{-b}, \frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{\left(\frac{B_{2}}{y_{F}}\right)^{-}} > \frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{\left(\frac{B_{2}}{y_{F}}\right)^{+}}, \text{ and thus, } \Psi_{F} \text{ is increasing in } K_{F} \text{ for } K_{F} \leq \frac{B_{2}}{y_{F}}. \text{ Therefore, } K_{F}^{*} = \min\left(K_{F}^{\beta}, \frac{B_{1}}{c_{F}}\right), \text{ where } K_{F}^{\beta} \text{ is the unique solution to } \frac{\partial \Psi_{F}}{\partial K_{F}}\Big|_{K_{F}^{\beta}} = 0 \text{ for } K_{F} > \frac{B_{2}}{y_{F}}. \blacksquare$

Proof of Corollary 1: Because $B_1 < c_F K_F^u$ and $B_2 < y_F K_F^u$, $K_F^* = \min\left(\frac{B_1}{c_F}, \max\left(\frac{B_2}{y_F}, K_F^\beta\right)\right)$. $\frac{\partial K_F^\beta}{\partial \beta} > 0$, $K_F^\beta \to \infty$ for $\beta = 0$, and $K_F^\beta = K_F^u$ for $\beta = 1$. For part i, Let $\overline{\beta}_F^0 \doteq \left(1 + \left(\frac{c_F + y_F}{c_F}\right) \left(\left(\frac{y_F K_F^u}{B_2}\right)^{-\frac{1}{b}} - 1\right)\right)^{-1} \in (0, 1)$ denote the unique solution to $K_F^\beta(\beta) = \frac{B_2}{y_F}$. When $\beta \leq \overline{\beta}_F^0$, $\Psi_F^* = -(c_F + y_F)K_F^* + M_F K_F^{*(1+\frac{1}{b})}$, where $K_F^* = \frac{B_2}{y_F}$, and thus, $\frac{\partial \Psi_F^*}{\partial \beta} = 0$. When $\beta > \overline{\beta}_F^0$, $K_F^* = \min\left(K_F^\beta, \frac{B_1}{c_F}\right)$, and $\frac{\partial \Psi_F^*}{\partial \beta} = \frac{\partial \Psi_F(K_F)}{\partial \beta}\Big|_{K_F^*}$, where $\Psi_F(K_F) = -c_F K_F + \beta\left[M_F K_F^{(1+\frac{1}{b})} - y_F K_D\right] + (1-\beta)\left[M_F\left(\frac{B_2}{y_F}\right)^{(1+\frac{1}{b})} - B_2\right]$. We define $H(K_F) \doteq \frac{\partial \Psi_F(K_F)}{\partial \beta} = M_F\left(K_F^{(1+\frac{1}{b})} - \left(\frac{B_2}{y_F}\right)^{(1+\frac{1}{b})}\right) - y_F\left(K_F - \frac{B_2}{y_F}\right)$, where $H\left(\frac{B_2}{y_F}\right) = 0$. It is easy to establish that $\frac{\partial H(K_F)}{\partial K_F} > 0$ is equivalent to $K_F < \left(\frac{(1+\frac{1}{b})M_F}{y_F}\right)^{-b}$. Since $K_F^* < K_F^u < \left(\frac{(1+\frac{1}{b})M_F}{y_F}\right)^{-b}$, $H(K_F^*) > 0$. an thus, $\frac{\partial \Psi_F^*}{\partial \beta} > 0$. For part ii, let $\overline{\beta}_F^1 \doteq \left(1 + \left(\frac{c_F + y_F}{c_F}\right)\left(\left(\frac{c_F K_F^u}{B_1}\right)^{-\frac{1}{b}} - 1\right)\right)^{-1}$ denote the unique solution to $K_F^\beta(\beta) = \frac{B_1}{c_F}$ such that $K_F^* = \max\left(\frac{B_2}{y_F}, K_F^\beta\right) < \frac{B_1}{c_F}$ for $\beta \leq \overline{\beta}_F^1$, and $K_F^* = \frac{B_1}{c_F}$ otherwise. Because $\frac{B_1}{c_F} > \frac{B_2}{y_F}$ and $B_1 < c_F K_F^u$, $\overline{\beta}_F^1 \in (\overline{\beta}_F^0, 1)$. When $\beta \leq \overline{\beta}_F^1, \frac{\partial \Psi_F}{\partial B_1} = 0$. When $\beta > \overline{\beta}_F^1, \Psi_F^* = \Psi_F\left(\frac{B_1}{c_F}\right)$, and $\frac{\partial \Psi_F^*}{\partial B_1} > 0$ because $\frac{B_1}{c_F} < (K_F^u <)\left(\frac{(1+\frac{1}{b})M_F}{y_F}\right)^{-b}$.

Proof of Proposition 2: The proof is omitted.

Proof of Proposition 3: The proof is similar to Proposition 1, and is omitted. **Proof of Proposition 4**: For $K_D > \frac{B_2}{2y_D}$, from (4), we obtain $\frac{\partial^2 \Psi_D}{\partial K_D^2} < 0$, and thus, Ψ_D is piecewise concave in K_D . When $\frac{\partial \Psi_D}{\partial K_D}\Big|_{\left(\frac{B_2}{2y_D}\right)^+} \le 0$, which is equivalent to $B_2 \ge 2y_D\hat{K}_D$, where $\hat{K}_D = \left(\frac{(1+\frac{1}{b})(\mu_{\xi}-(1-\beta)\mathbb{E}[\min(\tilde{\xi}_1,\tilde{\xi}_2)])}{c_D+\beta y_D}\right)^{-b}$, Ψ_D is unimodal in K_D , and $K_D^* = \min\left(K_D^u, \frac{B_2}{2y_D}\right)$. Here, $K_D^u = \left(\frac{(1+\frac{1}{b})\mu_{\xi}}{c_D+y_D}\right)^{-b}$ is the unique solution to $\frac{\partial \Psi_D}{\partial K_D}\Big|_{K_D^u} = 0$ for $K_D \le \frac{B_2}{2y_D}$. When $\frac{\partial \Psi_D^P}{\partial K_D}\Big|_{\left(\frac{B_2}{2y_D}\right)^+} > 0$, i.e. $B_2 < 2y_D\hat{K}_D$, because $\hat{K}_D < K_D^u$ following from the assumption $\frac{c_D}{c_D+y_D} > 1 - \frac{\mathbb{E}[\min(\tilde{\xi}_1,\tilde{\xi}_2)]}{\mu_{\xi}}$, $B_2 < 2y_DK_D^u$, i.e. $\frac{\partial \Psi_D}{\partial K_D}\Big|_{\left(\frac{B_2}{2y_D}\right)^-} > 0$. Therefore, Ψ_D is unimodal in K_D , and $K_D^* = \min\left(K_D^P, \frac{B_1}{2c_D}\right)$, where $K_D^P > \frac{B_2}{2y_D}$ is the unique solution to $\frac{\partial \Psi_D}{\partial K_D}\Big|_{K_D^P} = 0$ with $\frac{\partial \Psi_D}{\partial K_D} = -2c_D + \beta \left[\left(1 + \frac{1}{b}\right) 2\mu_{\xi}K_D^{\frac{1}{b}} - 2y_D\right] + (1 - \beta) \int_{\Omega_D^C} \left(1 + \frac{1}{b}\right) \left[\max(\xi_1, \xi_2)K_D^{\frac{1}{b}} - \min(\xi_1, \xi_2) \left(\frac{B_2}{y_D} - K_D\right)^{\frac{1}{b}}\right] f(\xi_1, \xi_2)d\xi_1d\xi_2$. When $K_D > \frac{B_2}{y_D}$, $\Omega_D^C = \emptyset$. Therefore, when $B_2 < y_D K_D^\beta$, $K_D^P = K_D^\beta = \left(\frac{(1 + \frac{1}{b})\mu_{\xi}}{\frac{C}{\beta} + y_D}\right)^{-b}$. **Proof of Corollary 2:** The proof is similar to the proof of Corollary 1, and is omitted. For $B_2 \in (2y_D \hat{K}_D(\beta)\Big|_{\beta=0}, 2y_D K_D^u)$, $\overline{\beta}_D^0$ is the unique solution to $B_2 = 2y_D \hat{K}_D(\beta)$, whereas for $B_2 \leq 2y_D \hat{K}_D(\beta)\Big|_{\beta=0}$, $\overline{\beta}_D^0 = 0$. $\widehat{\beta}_D^0$ is the unique solution to $B_2 = y_D K_D^\beta$. **Proof of Corollary 3:** The proof is omitted.

Proof of Proposition 5: We use the following result throughout the proof:

Lemma A.1
$$2\eta_D K_D^u = \eta_F K_F^u|_{\eta_F^u(\eta_D)}$$
 where $\eta_F^u(\eta_D) = \eta_D \left[\frac{M_F}{2^{-\frac{1}{b}}\mu_{\xi}}\right]^{\frac{o}{b+1}}$

Following Corollary 3, $\Psi_D^* = \widehat{\Psi}_D^* + \triangle^P$, where \triangle^P denotes the pooling value of the (common) operating budget and $\widehat{\Psi}_D^*$ denotes the optimal expected profit when the budget is product-specific with the dedicated technology. It is easy to establish that $\Psi_F^*|_{\eta_F^u(\eta_D)} = \widehat{\Psi}_D^*$, and thus, $\overline{\eta}_F(\eta_D) = \eta_F^u(\eta_D)$ when $\triangle^P = 0$. Because $\frac{\partial \Psi_F^*}{\partial \eta_F} < 0$, when $\triangle^P > 0$, $\overline{\eta}_F(\eta_D) < \eta_F^u(\eta_D)$. Following Corollary 3, $\triangle^P = 0$ if $\beta \leq \overline{\beta}_D^0$ or $\beta = 1$, and $\triangle^P > 0$ otherwise.

$$\begin{split} & \operatorname{Proof} \operatorname{of} \operatorname{Proposition} 6: \ \operatorname{Let} L(\beta) \doteq \Psi_F^*(\beta)|_{\eta_F^u(\eta_D)} - \Psi_D^*(\beta), \text{ where } \eta_F^u(\eta_D) = \eta_D \left[\frac{M_F}{2^{-\frac{1}{b}}\mu_g}\right]^{\frac{b}{b+1}}.\\ & \operatorname{Because} \frac{\partial \Psi_F^*}{\partial \eta_F} < 0, \text{ for a given } \beta, \ \overline{\eta}_F(\eta_D) \gtrless \eta_F^u(\eta_D) \text{ is equivalent to } L(\beta) \gtrless 0. \text{ When } \\ & 0 < \beta \leq \overline{\beta}_D^0, \text{ following Corollary 2 and Proposition 4, } K_D^* = \frac{B_2}{2(1-\alpha_D)\eta_D} \text{ and } B_2 \geq 2(1-\alpha_D)\eta_D \widehat{K}_D. \text{ Because } \widehat{K}_D > K_D^\beta, B_2 > 2\eta_D K_D^u \frac{(1-\alpha_D)}{\left(\frac{\alpha}{\beta}+1-\alpha_D\right)^{-b}}. \text{ Let } G(\alpha) \doteq \frac{(1-\alpha)}{\left(\frac{\alpha}{\beta}+1-\alpha\right)^{-b}}.\\ & \operatorname{Because} \beta < \frac{\frac{b+1}{b+1}-\alpha}{1-\alpha} \text{ for } \beta \leq 1, \ \frac{G(\alpha)}{\alpha} < 0, \text{ and } G(\alpha_F) \leq G(\alpha_D) \text{ for } \alpha_F \geq \alpha_D. \text{ Therefore, } B_2 > 2\eta_D K_D^u G(\alpha_F) = \eta_F K_F^u|_{\eta_F^u(\eta_D)} G(\alpha_F), \text{ where the equality follows from Lemma A.1, and, from Proposition 1, <math>K_F^*|_{\eta_F^u(\eta_D)} = \min\left(K_F^u|_{\eta_F^u(\eta_D)}, \frac{B_2}{(1-\alpha_F)\eta_F^u(\eta_D)}\right). \text{ When } \\ & K_F^*|_{\eta_F^u(\eta_D)} = \frac{B_2}{(1-\alpha_F)\eta_F^u(\eta_D)}, \ L(\beta) = 2^{-\frac{1}{b}}\mu_\xi\left(\frac{B_2}{\eta_D}\right)^{(1+\frac{1}{b})}\left((1-\alpha_F)^{-(1+\frac{1}{b})} - (1-\alpha_D)^{-(1+\frac{1}{b})}\right) - \\ & B_2\left(\frac{1}{1-\alpha_F} - \frac{1}{1-\alpha_D}\right). \text{ It is sufficient to establish } L(\beta) > 0 \text{ in this case. Let } S(\alpha_F) \text{ denote } L(\beta) \text{ for a given } \alpha_F. \ \frac{\partial S(\alpha_F)}{\partial \alpha_F} = \frac{B_2}{(1-\alpha_F)^2}\left(\left(\frac{2(1-\alpha_F)\eta_D K_D^u}{B_2}\right)^{-b} - 1\right) > 0, \text{ because } B_2 < \\ & 2(1-\alpha_D)\eta_D K_D^u \text{ and } \alpha_F > \alpha_D. \text{ Because } S(\alpha_D) = 0, \ S(\alpha_F) > 0 \text{ for } \alpha_F > \alpha_D, \text{ and } K_D^* = \\ & \frac{B_1}{2\alpha_D\eta_D}. \text{ Let } H(\alpha) \doteq \frac{\alpha}{\left(\frac{\alpha}{\beta}+1-\alpha\right)^{-b}}. \text{ For } \beta \geq \left(1+\frac{1}{-(b+1)\alpha_D}\right)^{-1}, \ \frac{H(\alpha)}{\alpha} \geq 0, \text{ and thus, } H(\alpha_F) > \\ & H(\alpha_D). \text{ Therefore, } B_1 < 2\eta_D K_D^u H(\alpha_F) = \eta_F K_F^u|_{\eta_F^u(\eta_D)} H(\alpha_F) \text{ and, following Proposition} \end{aligned}$$

1, $K_F^*|_{\eta_F^u(\eta_D)} = \frac{B_1}{\alpha_F \eta_F^u(\eta_D)}$. We obtain $L(\beta) = \beta \triangle_1 + (1-\beta) \triangle_2$, where

$$\Delta_1 \doteq -B_1 \left(\frac{1}{\alpha_F} - \frac{1}{\alpha_D} \right) + 2^{-\frac{1}{b}} \mu_{\xi} \left(\frac{B_1}{\eta_D} \right)^{(1+\frac{1}{b})} \left(\alpha_F^{-(1+\frac{1}{b})} - \alpha_D^{-(1+\frac{1}{b})} \right),$$

$$\Delta_2 \doteq \left(\frac{B_2}{\eta_D} \right)^{(1+\frac{1}{b})} \left(2^{-\frac{1}{b}} \mu_{\xi} (1-\alpha_F)^{-(1+\frac{1}{b})} - M_F (1-\alpha_D)^{-(1+\frac{1}{b})} \right).$$

Let $S_1(\alpha_F)$ denote \triangle_1 for a given α_F . Because $B_1 < 2\alpha_D\eta_D K_D^u$ by assumption, $B_1 < 2\alpha_F\eta_D K_D^u$, and thus, $\frac{\partial S_1(\alpha_F)}{\partial \alpha_F} < 0$. Because $S_1(\alpha_D) = 0$, $\triangle_1 < 0$ for $\alpha_F > \alpha_D$. Let $S_2(\alpha_F)$ denote \triangle_2 for a given α_F . $\frac{\partial S_2(\alpha_F)}{\partial \alpha_F} > 0$, and $S_2(\alpha_D) < 0$ because $M_F > 2^{-\frac{1}{b}} \mu_\xi$ for $\rho < 1$. Therefore, there exists a unique $\hat{\alpha}_F > \alpha_D$, which is the solution to $S_2(\hat{\alpha}_F) = 0$ such that when $\alpha_F < \hat{\alpha}_F$, $\triangle_2 < 0$, and $\triangle_2 > 0$ otherwise. The sign of $L(\beta)$ is given by the sign of $\triangle_1 + (\frac{1}{\beta} - 1)\triangle_2$. For $\alpha_F \le \hat{\alpha}_F$, $\triangle_1 < 0$ and $\triangle_2 \le 0$. Therefore, $L(\beta) < 0$ for $\beta \ge \overline{\beta}_D^2 \doteq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$. For $\alpha_F > \hat{\alpha}_F$, $\triangle_1 < 0$ and $\triangle_2 > 0$. Because L(1) < 0 and \triangle_2 is finite, there exists $\overline{\beta}_D^2 \in \left[\max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right), 1\right)$ such that $L(\beta) < 0$ for $\beta \ge \overline{\beta}_D^2$.

Proof of Proposition 7: When $0 < \beta \leq \overline{\beta}_D^0$ or $\beta = 1$, following Proposition 5, $\overline{\eta}_F(\eta_D) = \eta_F^u(\eta_D)$, and thus, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} = 0$. For the rest of the proof, we use the following result:

Lemma A.2 Consider $\alpha_F \geq \alpha_D$. When $\beta \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$, for a given η_D , the flexible threshold is given by $\overline{\eta}_F(\eta_D) = \eta_F^u(\eta_D) Z^{\left(\frac{b}{b+1}\right)}$, where

$$Z \doteq \frac{\left(\frac{B_1}{\alpha_F}\right)^{\left(1+\frac{1}{b}\right)} + \left(\frac{1}{\beta} - 1\right) \left(\frac{B_2}{1-\alpha_F}\right)^{\left(1+\frac{1}{b}\right)}}{\left(\frac{B_1}{\alpha_D}\right)^{\left(1+\frac{1}{b}\right)} + \frac{M_F}{2^{-\frac{1}{b}}\mu_{\xi}} \left(\frac{1}{\beta} - 1\right) \left(\frac{B_2}{1-\alpha_D}\right)^{\left(1+\frac{1}{b}\right)} + B_1 \left(\frac{1}{\alpha_F} - \frac{1}{\alpha_D}\right) \frac{\eta_D^{\left(1+\frac{1}{b}\right)}}{2^{-\frac{1}{b}}\mu_{\xi}}.$$
(5)

From Lemma A.2, $sgn\left(\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1}\right) = sgn\left(\frac{\partial Z}{\partial B_1}\right)$, which, for $\alpha_F = \alpha_D = \alpha$, is given by the sign of $\frac{\left(M_F - 2^{-\frac{1}{b}}\mu_{\xi}\right)}{2^{-\frac{1}{b}}\mu_{\xi}}\left(\frac{1}{\beta} - 1\right)\left(\frac{B_2}{1-\alpha}\right)^{\left(1+\frac{1}{b}\right)} > 0$ (because $M_F > 2^{-\frac{1}{b}}\mu_{\xi}$ for $\rho < 1$).

Proof of Proposition 8: When $\beta \leq \overline{\beta}_D^0$, as we established in the proof of Proposition 5, $K_D^* = \frac{B_2}{2(1-\alpha_D)\eta_D}$ and $K_F^*|_{\eta_F^u(\eta_D)} = \min\left(K_F^u|_{\eta_F^u(\eta_D)}, \frac{B_2}{(1-\alpha_F)\eta_F^u(\eta_D)}\right)$. Because $\overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$ (as follows from Proposition 6) and $\frac{\partial \eta_F K_F^u}{\partial \eta_F} < 0$, $B_1 > \alpha_F \eta_F K_F^*|_{\eta_F^u(\eta_D)} > \alpha_F \eta_F K_F^*|_{\overline{\eta}_F(\eta_D)}$. Therefore, following Corollaries 1 and 2, neither technology is B_1 constrained at the technology cost pair $(\eta_D, \overline{\eta}_F(\eta_D))$, and thus, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} = 0$. When $\beta \geq \overline{\beta}_D^2$, following Proposition 6, $\beta > \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right)$. Therefore, $\overline{\eta}_F(\eta_D)$ is as given in Lemma A.2. It follows that $sgn\left(\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1}\right) = sgn\left(\frac{\partial Z}{\partial B_1}\right)$, where Z is as defined in

(5). This sign is given by the sign of $N(\beta) = \left(\frac{1}{\beta} - 1\right) \Delta_1 + \Delta_2$, where

$$\Delta_1 \doteq \frac{M_F}{2^{-\frac{1}{b}}\mu_{\xi}} \left(\frac{B_2}{\alpha_F(1-\alpha_D)}\right)^{(1+\frac{1}{b})} - \left(\frac{B_2}{\alpha_D(1-\alpha_F)}\right)^{(1+\frac{1}{b})} \left[1 - \left(1 - \frac{\alpha_D}{\alpha_F}\right) \left(\frac{B_1}{2\alpha_D\eta_D K_D^u}\right)^{-\frac{1}{b}}\right]$$

$$\Delta_2 \doteq \frac{B_1}{-(b+1)2^{-\frac{1}{b}}\mu_{\xi}} \left(\frac{\eta_D}{\alpha_F}\right)^{(1+\frac{1}{b})} \left(\frac{1}{\alpha_D} - \frac{1}{\alpha_F}\right).$$

Because $\alpha_F > \alpha_D$, $\Delta_2 > 0$. If $\Delta_1 \ge 0$ then $N(\beta) > 0$ for $\beta \ge \overline{\beta}_D^2$, and thus, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} > 0$. If $\Delta_1 < 0$, because N(1) > 0 and Δ_1 is finite, there exists $\overline{\beta}_D^3 \in \left[\overline{\beta}_D^2, 1\right)$ such that $N(\beta) > 0$ for $\overline{\beta}_D^3 \le \beta \le 1$, and thus, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial B_1} > 0$.

Proof of Proposition 9: When $0 < \beta \leq \overline{\beta}_D^0$, following Proposition 5, $\overline{\eta}_F(\eta_D) = \eta_F^u(\eta_D)$, and thus, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} = 0$. When $\beta \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right), \overline{\eta}_F(\eta_D)$ is as given in Lemma A.2, and $sgn\left(\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta}\right) = sgn\left(\frac{\partial Z}{\partial \beta}\right)$, where Z is as defined in (5). For $\alpha_F = \alpha_D = \alpha$, $sgn\left(\frac{\partial Z}{\partial \beta}\right)$ is given by $\frac{M_F}{2^{-\frac{1}{b}\mu_{\xi}}} \left(\frac{B_1}{\alpha}\right)^{(1+\frac{1}{b})} > 0$.

Proof of Proposition 10: When $\beta \leq \overline{\beta}_D^0$, as we established in the proof of Proposition 5, $K_D^* = \frac{B_2}{2(1-\alpha_D)\eta_D}$ and $K_F^*|_{\eta_F^u(\eta_D)} = \min\left(K_F^u|_{\eta_F^u(\eta_D)}, \frac{B_2}{(1-\alpha_F)\eta_F^u(\eta_D)}\right)$. Because $\overline{\eta}_F(\eta_D) > \eta_F^u(\eta_D)$ (as follows from Proposition 6) and $\frac{\partial \eta_F K_F^u}{\partial \eta_F} < 0$, $K_F^*|_{\overline{\eta}_F(\eta_D)} = \min\left(K_F^u|_{\overline{\eta}_F(\eta_D)}, \frac{B_2}{(1-\alpha_F)\overline{\eta}_F(\eta_D)}\right)$. Therefore, $\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta} = 0$. When $\beta \geq \max\left(\widehat{\beta}_D^0, \overline{\beta}_D^1, \left(1 + \frac{1}{-(b+1)\alpha_D}\right)^{-1}\right), \overline{\eta}_F(\eta_D)$ is as given in Lemma A.2, and $sgn\left(\frac{\partial \overline{\eta}_F(\eta_D)}{\partial \beta}\right) = sgn\left(\frac{\partial Z}{\partial \beta}\right)$, where Z is as defined in (5). After some algebra, we obtain that the sign of $\frac{\partial Z}{\partial \beta}$ is given by

$$\Delta_1 \doteq \frac{M_F}{2^{-\frac{1}{b}}\mu_{\xi}} \left(\frac{B_1\left(\frac{1}{\alpha_F} - 1\right)}{(1 - \alpha_D)} \right)^{(1 + \frac{1}{b})} - \left(\frac{B_1}{\alpha_D}\right)^{(1 + \frac{1}{b})} + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{1}{\alpha_D} - \frac{1}{\alpha_F}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{1}{\alpha_D} - \frac{1}{\alpha_D}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{1}{\alpha_D} - \frac{1}{\alpha_D}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{1}{\alpha_D} - \frac{1}{\alpha_D}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) + \eta_D^{(1 + \frac{1}{b})} \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) + \eta_D^{(1 + \frac{1}{b}}\mu_{\xi}^{(1 + \frac{1}{b})}\right) \left(\frac{B_1}{2^{-\frac{1}{b}}\mu_{\xi}}\right) + \eta_D^{(1 + \frac{1}{b}}\mu_{\xi}^{(1 + \frac{1}{b}}\mu_{\xi}}\right) + \eta_D^{(1 + \frac{1}{b}}\mu_{\xi}^{(1 + \frac{1}{b})}\mu_{\xi}^{(1 + \frac{1}{b}}\mu_{\xi}}\right)$$

Let $S(\alpha_F)$ denote Δ_1 for a given α_F . Using $M_F \geq 2^{-\frac{1}{b}}\mu_{\xi}$, we obtain $\frac{\partial S(\alpha_F)}{\partial \alpha_F} < 0$ because $B_1 < 2\alpha_D\eta_D K_D^u$ by assumption, $\alpha_F > \alpha_D$, $\alpha_D < 1$ and b < -1. It is easy to establish $S(\alpha_D) > 0$ and S(1) < 0. Because $\frac{\partial S(\alpha_F)}{\partial \alpha_F} < 0$, there exists a unique $\overline{\alpha}_F \in (\alpha_D, 1)$, which is the solution to $S(\overline{\alpha}_F) = 0$, such that $S(\alpha_F) > 0$ if $\alpha_F < \overline{\alpha}_F$ and $S(\alpha_F) < 0$ otherwise.

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