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Managing Seller Heterogeneity in a Competitive Marketplace

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Abstract

The growth of online marketplaces is accompanied by significant heterogeneity of the third-party sellers. The marketplace owner often applies policies that favor the sellers who offer higher values to buyers, which puts the lower-value sellers at an even greater disadvantage. This leads to the phenomenon of Matthew Effect. Our study focuses on a marketplace owner's policy in managing seller heterogeneity and analyzes Matthew Effect in a competitive market environment. By extending the circular city model, we analytically examine the price competition among a large number of sellers that differ both in variety and in their value offerings. We present the closed-form equilibrium solution for the sellers' pricing strategies, which illustrates the ripple effect that exerts competitive pressure from seller to seller at a diminishing magnitude. Furthermore, we show that, by providing each seller with the support that enhances its value offering proportionally, the marketplace owner further sharpens seller heterogeneity and creates Matthew Effect. This results in an increased profit for the marketplace owner and improved consumer surplus. The optimal level of such value-based support is dependent on the distribution of the sellers' values and the buyers' horizontal preferences. Our findings suggest that Matthew Effect improves social welfare by increasing the total profits of sellers and the marketplace owner, as well as the consumer surplus; moreover, it creates business opportunities for new sellers in the online marketplace.

Keywords: *Seller heterogeneity, variety, Matthew Effect, circular city model*.

"For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath."

-- Matthew 25:29, *The King James Bible*.

1. Introduction

Along with the exciting business opportunities that come with online marketplaces, the many sellers that emerge bring forth a high degree of heterogeneity. As Amazon welcomes on board more third-party sellers, product condition and shipping service further depend on the variability in the individual sellers' business knowledge and practice. Similarly, on Taobao, the C2C market of the Alibaba Group, some sellers are committed to building a reputable online business, while others may be simply selling for a hobby. Airbnb is a marketplace that enables sharing of unused living space between hosts and travelers; hosts' hospitality can also vary significantly. A traveler might encounter a longer delay in the response from an ad-hoc host who only participates occasionally, compared to that from a host who is routinely seeking renters for longer stays. Also, in mobile application markets, as the markets are open to both professional and amateur developers, not all third-party developers can create games that are addictively entertaining. In general, sellers are highly heterogeneous in the *value* they offer to buyers. Such value offering is analogous to the combination of sellers' product quality and service performance and is characterized by a set of specific attributes that are relevant to the marketplace.

The heterogeneity of sellers' value offerings (or "value" for short) plays a pivotal role in determining buyers' purchasing choices, sellers' price competition, and ultimately the marketplace owner's profit. It is well established theoretically and apparent in practice that the degree of sellers' value heterogeneity directly determines the buyers' purchasing decisions. Meanwhile, each seller strategically considers other sellers' characteristics relative to those of its own while setting price. In many marketplaces, the sellers with higher value offerings tend to capture a larger market than newer and weaker sellers, who are pressured to set lower prices. Thus, sellers' heterogeneous characteristics are important to strategic interactions among sellers and buyers in the marketplace and to the sellers' profits, part of which is then absorbed by the marketplace owner.

Aside from value heterogeneity, the sellers' variety further complicates the economic mechanisms in the marketplace. Variety refers to the number of horizontally differentiated sellers, who carry products of different attributes—such as color, style, functionality, and location—to target buyers of different tastes. For example, on a typical day, the eBay market displays more than 2 million postings for women's shoes alone, which can be sorted into different styles, brands, heel heights, sizes, and so on. Similarly, Airbnb listings show apartments located near downtown, country homes in tranquil neighborhoods, studios next to hiking trails, and other locations that suit renters' different needs. More variety could lead to an increasingly crowded marketplace with highly similar products. As a result, the substitution effect intensifies competition between sellers, which may lead to a more aggressive pricing strategy. On the other hand, with the increased similarity between sellers' products, their value offerings based on attributes such as service quality may become a more prominent factor in buyers' purchasing choices. Therefore, heterogeneity in sellers' values offerings and seller variety interact and jointly determine the sellers' and buyers' surplus.

Interestingly, marketplace owners manage seller heterogeneity by rewarding sellers based on their values offerings. Both Taobao and T-Mall, two of Alibaba Group's marketplaces, selectively offers promotional opportunities to higher-value sellers who can then further enhance their appeals to buyers. Airbnb dynamically ranks listings based on hosts' reviews, nights booked, response rate, and cancellations. In effect, the hosts that offer lower values to travelers get less exposure, which create competitive pressure on their pricing and profits. Moreover, T-Mall applies higher fees on lower-value sellers (Fletcher 2011), which puts these sellers at an even greater disadvantage relative to higher-value competitors. App markets, such as Apple's App Store and Google Play, promote applications that win the developer contests and attain high scores for the Top-Charts criteria.[1] Similarly, Facebook implemented a rule change in 2008 that allowed more engaging apps to further engage more users by allocating to them a higher number of notifications relative to the less engage apps (Claussen et al. 2013). A common characteristic among these value-based treatments is that they favor high-value sellers and, in turn, sharpen heterogeneity of sellers, which alters seller competition and may lead to profound economic implications for the sellers, the buyers, and the marketplace owner.

These value-based treatments enhance heterogeneity of the sellers and, thus, create *Matthew Effect*. Matthew Effect is a concept that originates from the *Gospel of Matthew* and describes the phenomenon of the "rich" receiving an additional advantage over those who are struggling and, in turn, getting "richer"; in other words, it is the effect of an unequal treatment that favors those who already have a superior status. Matthew

[1] Based on the industry observations and analysis, the ranking of App Store Top Charts is continuously adjusted and have factored in app rating, engagement and many other criteria aside from the number of downloads (Perez 2013).

Effect has been applied to sociological, economic, and other scientific contexts. It was first adopted by Merton (1968) to describe the misallocation of credit in the rewards systems for scientific work. It is interesting to explore whether, in online marketplaces, Matthew Effect that results from the value-based treatments also leads to "misallocation" of revenues and welfare among the users. These treatments shift the revenues between sellers of different values through the economic mechanism of trades; meanwhile, part of the total revenues is absorbed by the marketplace owner. It is unclear to which extent the marketplace owner might gain from creating Matthew Effect. It is also important to examine who gain and lose in this phenomenon in the presence of competitive forces. In this paper, we address this issue and, more specifically, the following questions:

(1) How does value heterogeneity among a variety of sellers affect their competition, pricing strategies, and profits?

(2) How does the Matthew Effect created through the marketplace owner's value-based treatment impact seller competition, the profits in the marketplace, and consumer surplus?

(3) What is the optimal level of the marketplace owner's value-based treatment for managing seller heterogeneity? How does this optimal level depend on sellers' and buyers' characteristics?

(3) How does the marketplace owner's value-based treatment affect the optimal seller variety in the marketplace?

We extend the circular city model (Salop 1979) to allow differentiated seller values among a large number of sellers who also differ horizontally. The closed-form solution for the equilibrium prices and profits of these sellers is obtained. We then derive the marketplace owner's profits based on the sellers' equilibrium profits and analyze a policy that offers each seller support by increasing its value offering proportionally. This value-based support leads to Matthew Effect, as higher-value sellers receive higher support than lower-value sellers, and value heterogeneity is sharpened. We then solve for the marketplace owner's optimal support level and examine its properties. Finally, we discuss the implication of Matthew Effect for the marketplace owner's profit, the sellers' profits, the consumer surplus, and the number of sellers in the market.

The equilibrium of seller competition shows a ripple effect that analytically captures the competition dynamics among a variety of heterogeneous sellers: Changes to any seller's price propagate to its immediate neighbors, then to their neighbors, and so on at a rapidly decreasing magnitude. This suggests that strategizing with only the first- and second-degree neighbors is of primary importance. Furthermore, the average equilibrium price does not depend on sellers' value because the ripple effect distributes value effects across all sellers' prices, keeping the sum of the prices constant.

Based on a percentage royalty revenue model, the equilibrium analysis then leads to the study of the marketplace owner's value-based support policy and Matthew Effect. We show that the marketplace owner's expected profit is independent of the average of sellers' values, which implies that a uniform treatment to all sellers is ineffective. The change to each seller's value is offset by competition between the neighboring sellers – the degree of value heterogeneity remains unchanged. The value-based support, which leads to Matthew Effect, indeed results in a higher total profit among sellers and, therefore, a higher profit for the marketplace owner. The increase in value variation shifts more competitive advantage to the higher-value sellers, resulting in a more imbalanced competition. The gains of these sellers outweigh the loss of lower-value sellers; thus, the total profit increases.

Moreover, we find that the marketplace owner needs to calibrate the optimal level of the value-based support based on both sellers' and buyers' characteristics. A higher average of sellers' values makes the support more costly without affecting the marketplace owner's profits; thus, a higher average value reduces the optimal support level. Conversely, a higher value variance means that value heterogeneity is more responsive to the support, and the optimal support level is higher. Buyers' transportation cost reflects the degree of horizontal differentiation among sellers. A higher transportation cost mitigates neighboring sellers' competition, which in turn reduces the effectiveness of the support and lowers the optimal support level.

We also examine the impact of Matthew Effect on the buyers' surplus and the optimal seller variety. We find that the value-based support leads to a higher expected consumer surplus. While buyers consume higher values offered by the sellers under the support, the increase in price is suppressed by the sellers' competition. As a result, the buyers are overall better off under Matthew Effect. Moreover, we show that the seller variety that maximizes the marketplace owner's profit increases, when the marketplace owner offers the value-based support. In the absence of value heterogeneity, an increase in seller variety intensifies competition and reduces the marketplace owner's profits; however, with heterogeneous values, the Matthew Effect created by the value-based support enhances value heterogeneity and make the positive effect of variety more pronounced. Therefore, the marketplace owner benefits from bringing more sellers into the market, which generates new opportunities for online selling.

The equilibrium characterization of users' transactions in our work offers an important contribution to the literature of spatial competition with heterogeneity. A few recent studies also incorporate heterogeneity in the circular city model (Alderighi and Piga 2012, Syverson 2004, Vogel 2008). To the best of our knowledge, we are the first to derive the closed-form equilibrium solution in the circular city setting with seller heterogeneity and price propagation between sellers. The availability of explicit solution forms removes the reliance on numerical approximation—such as that in Alderighi and Piga (2012)—and permits further analytical studies. Specifically, the equilibrium solution indicates the degrees of the ripple effect between sellers of different values at different locations, which is fundamental to understanding competition among a large number of heterogeneous sellers. This takes a significant step forward from the traditional frameworks that are often limited to only two sellers and leads to a rigorous analysis of management strategies for a vibrant marketplace.

The findings in this paper lead to a number of managerial insights. First, in a large marketplace, sellers can primarily compete directly and indirectly against the few that offer the most similar products. The interactions with the remaining competitors have insignificant impact on the profits. Second, a marketplace policy that shifts the overall seller value is ineffective in a saturated market; on the other hand, the value-based policy that many marketplace owners implement indeed results in higher profits for the higher-value sellers as well as for the marketplace owner. Third, the marketplace owner should also adjust the level of its policy according to the changes in the average of sellers' value and the degrees of their value heterogeneity and horizontal differentiation. Fourth, the Matthew Effect created from the value-based support policy not only benefits the higher-value sellers and the marketplace owner, but also makes the buyers better off and creates new online selling opportunities.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature and highlight the contribution of our work. Section 3 presents the model setup. In Section 4, we characterize and analyze the equilibrium of competing sellers in the marketplace. Based on these results, in Section 5, we examine the marketplace owner's value-based support and discuss the Matthew Effect that emerges in such a policy. Finally, Section 6 concludes the paper.

2. Literature

Our study is related to the literature on horizontal differentiation and localized competition (Hotelling 1929, Salop 1979). Among the many applications of the circular city model (Alderighi and Piga 2012, Syverson 2004, Vogel 2008), Alderighi and Piga (2012) is most directly related to our work. Unlike Syverson (2004) and Vogel (2008), both which do not consider price propagation between sellers, Alderighi and Piga (2012) capture the ripple effect of the sellers' pricing decisions that propagate around the circle highlighted in our model. Based on numerical solution of equilibrium prices, Alderighi and Piga (2012) generalize the properties of the coefficients and derive implications for geographic concentration of downstream retailers in relation to their upstream wholesalers. Our work makes two major contributions: First, the heterogeneity that we introduce to the circular city model lies in firms' value offerings, which affects both sellers' strategies and buyers' valuations, whereas firms' heterogeneity in efficiency studied by Alderighi and Piga (2012) is not directly relevant to buyers' decisions. Thus, value heterogeneity in our work introduces new complexity that is not accounted for previously. More importantly, we obtain the closed-form, analytical solution of the equilibrium prices, market shares and profits, which enables us to further analyze dependencies of the equilibrium on quality heterogeneity.

Economides (1993) also accounts for firms' quality in a circular city model. He compares two setups in which firms make entry (location), quality, and pricing decisions. He derives symmetric equilibrium among the firms in each scenario and finds that their precommitment on quality leads to a greater variety but lower quality. Even though quality variation is considered in Economides (1993), quality heterogeneity is not present in equilibrium. Our study tackles the problem of characterizing the equilibrium among horizontally differentiated sellers who also have heterogeneous quality levels. The equilibrium characterization also provides the foundation for analyzing other dimensions of heterogeneity among a large number of sellers.

Competition among a large number of sellers generates on-going research interests in various electronic markets. Motivated by security software's unique market structure of many vendors with low market shares, Dey et al. (2012) study the underlying economic forces exerted by the negative network effect associated with indirect attacks. By modeling the competition between many security software vendors, they provide a theoretical explanation for the high equilibrium prices and low market coverage. Xu et al. (2011) analyze oligopolistic price competition between multiple firms in the online search environment, where the firms have different ranks in consumers' search ordering. They find that, in equilibrium, the firms have asymmetric mixed-strategy for setting prices for both exogenous and endogenous consumer search strategies. Chen and Stallaert (2014) study behavioral targeting in online advertising and allow advertisers to target different users. They show how the heterogeneity in both advertisers' values and users' preferences affects the publisher's and advertisers' benefits from behavioral targeting. Our work also explores heterogeneity among a multitude of sellers and their competitive pricing strategies, in the context of online marketplaces. Our findings contribute to this stream of knowledge by capturing both value heterogeneity and seller variety and by evaluating the marketplace owner's policy in managing seller heterogeneity.

Our study is also related to two-sided platforms, as marketplaces act as platforms that serve sellers and buyers. Our work is concerned with the management of platform users and their transactions within the platform, whereas the platform literature has focused primarily on user entry into the platform, which then links to the network effects and two-sided fees (Rochet and Tirole 2003, Parker and Van Alstyne 2005, Armstrong 2006). The recent research efforts in this literature are actually moving toward the heterogeneity of users. Boudreau (2011) points out that, in software development, generating variety depends on "the heterogeneity and diversity of producers, rather than just the added numbers of producers per se" (p. 1). He empirically shows that the effect of introducing additional software producers on innovation depends on the heterogeneity of their software programs. Also, Ceccagnoli et al. (2012) find that differences in independent software vendors' (ISVs') capabilities create heterogeneity in their performance improvements.

Several other papers in the platforms literature also consider seller heterogeneity by modeling the transactions between the sellers and buyers. Hagiu (2009) and Lin et al.

(2011) examine buyer preferences and seller competition in the pricing decisions of a two-sided platform. Whereas in Hagiu (2009) many homogeneous sellers compete on a platform, Lin et al. (2011) model two sellers of differentiated quality. Thus, the former provides a richer discussion regarding seller variety, and the latter on sellers' quality levels. Different from these two studies, our work allows many sellers that differ in terms of both variety and value offerings to compete on a platform. A challenge in this setting may reside in analyzing the price competition between more than two sellers with different attributes. Our equilibrium characterization overcomes this obstacle and provides a theoretical foundation for the related questions.

3. Model Setup

In a marketplace, transactions take place between multiple sellers with heterogeneous value offerings and buyers with heterogeneous tastes. Following Salop (1979), a continuum of buyers is distributed uniformly on a circle of unit circumference. Their locations represent their tastes. Denote by n the number of sellers in the marketplace. n represents the degree of seller variety, as variety is generally proportional to the number of sellers (Boudreau 2011).

Whereas in Salop (1979) the variety of sellers have a uniform value, our model allows these sellers to be heterogeneous in their value offerings to the buyers. Sellers' values include the set of specifically attributes that are relevant to the marketplace (as defined in Section 1). For example, in the case of Taobao.com, a seller's value may be an aggregate measure of its product condition, promptness in responding to potential buyers' enquiries, delivery service, product guarantee, experience as a Taobao seller, and any other additional offerings to buyers. Denote seller i's value by v_i. Let the sellers' values follow a distribution with the cumulative distribution function $F(v)$.

Following the common assumption in the applications of the circular city model, sellers are located equidistantly on the circle (Chen and Stallaert 2014). Without the loss of generality, let seller i be located at $\frac{i}{n}$ and offer value $v_i > 0$ at price p_i, where $i \in \{0,1,\dots,n-1\}$. Note that seller $n-1$ is adjacent to seller 0 on the circle. Each buyer purchases from one seller and incurs a transportation cost due to the difference between this seller and that of an ideal match. The transportation cost is linear in the distance between the seller and the buyer at the rate t. Thus, buyer x's utility from purchasing seller i is $v_i - p_i - t \cdot \left|\frac{i}{n} - x\right|$.

The marketplace owner offers support to sellers based on their values. For instance, Taobao often offers promotional opportunities to sellers with higher value offerings in terms of relevant attributes discussed previously. These sellers are then able to reach more buyers, gain additional selling tactics, and further improve their values. Similarly, on Airbnb, the hosts are presented to renters based on their responses to potential renters, cleanliness, hospitality, cancelation rate, and any other characteristics of the property areas. Airbnb's ranking based on the combined metric of hosts' values provides higher-value hosts with better opportunities to showcase their properties and other value attributes. In short, higher-value sellers gain more from the value-based support compared to the lower-value sellers – this phenomenon exhibits Matthew Effect. Mathematically, let the marketplace owner's decision be the proportional level of support, α, based on each seller's original value. Thus, under this policy, seller i effectively offers an additional value of αv_i. The marketplace owner incurs a variable cost from the support: $c(\alpha v_i)^2$; it is commonly assumed that achieving a higher value is increasingly costly.

The marketplace owner collects a percentage royalty, γ, from each seller's profit and chooses the level of support based on the total royalty collected under the support less the cost. We let the marketplace owner's decision be based on the distribution of sellers' values, as it is often more feasible to utilize the aggregate characteristics of the users in a large marketplace. The timing of the game is the following: The marketplace owner sets the level of the support; the support then takes effect on each seller's value; with complete information, the sellers set prices simultaneously while the buyers make purchasing decisions; finally, both the sellers' and the marketplace owner's profits are realized.

4. Competing Sellers and Equilibrium

By backward induction, we first analyze the buyers' purchasing decisions and the sellers' equilibrium prices and profits in the marketplace. Based on these equilibrium results, we examine the marketplace owner's policy problem in Section 5.

Along the circumference of the circle, each seller's price and quality directly affect its neighboring sellers' decisions; the same effect is then carried to these neighboring sellers' neighboring sellers. The effect propagates around the circle, generating a *ripple effect*. To mathematically model this continuous competitive effect around the circle, we extend the range of i, such that $i \in \mathbb{Z}$, where seller i and seller $i \pm n$ are the same entity. For example, seller 0 and seller n refer to the same seller, so do sellers 1, $n + 1$, and $2n + 1$, and so on.

Following the convention in the literature of spatial competition (Eaton and Lipsey 1978, Syverson 2004, Alderighi and Piga 2012), we examine the equilibrium in which all sellers are locally competitive in the market; in other words, no seller's value is too low to attract any buyer. Condition 1 is sufficient for ruling out such cases:

Condition 1. *For any* $i \in \mathbb{Z}$, $.|v_i - v_{i+1}| < \frac{t}{n}$.

The buyer located at x between sellers i and $i+1$ is indifferent between the two sellers if

$$v_i - p_i - t\left(x - \frac{i}{n}\right) = v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n} - x\right).$$

From here, we identify the marginal buyer between sellers i and $i+1$:

$$x = \frac{1}{2t}(v_i - v_{i+1} - p_i + p_{i+1}) + \frac{i}{n} + \frac{1}{2n}.$$

Thus, seller i's demand is expressed as,

$$q_i = \frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n}. \tag{1}$$

Its profit is $\pi_i = (1-\gamma)p_i q_i$, which is concave in p_i. The n sellers' optimal prices are given by the following n first order conditions (FOCs):

$$p_i = \frac{t}{2n} + \frac{1}{4}(2v_i - v_{i+1} - v_{i-1} + p_{i+1} + p_{i-1}), \forall i \in \{0, 1, 2, \dots, n-1\}. \tag{2}$$

Proposition 1 (Marketplace Equilibrium). *There exists a unique Nash equilibrium, in which seller* $\boldsymbol{i}$*'s price is*

$$p_i^* = \frac{t}{n} + v_i - \sum_{d=0}^{n-1} b_d v_{i-d}, \forall i \in \{0,1, \dots n-1\}, \tag{3}$$

where $b_d = \frac{\delta^{n-d}+\delta^d}{\sqrt{3}(\delta^n-1)} > 0, \delta = 2+\sqrt{3}$. *Its equilibrium demand is* $q_i^* = \frac{1}{n} + \frac{v_i}{t} - \frac{1}{t}\sum_{d=0}^{n-1} b_d v_{i-d}$, *and its equilibrium profit is*

$$\pi_i = (1-\gamma)\frac{{p_i^*}^2}{t}. \quad (4)$$

Each seller's equilibrium price, demand, and profit are increasing in its own value and decreasing in the value of the other sellers.

Proposition 1 characterizes the closed-form equilibrium solution of the circular city model with heterogeneous values. To the best of our knowledge, no prior studies provide such an equilibrium characterization. Among closely related works, most models abstract out seller heterogeneity when studying the sellers' pricing decisions (Syverson 2004; Vogel 2008); one exception is Alderighi and Piga (2012), which shows the uniqueness of equilibrium when multiple firms with heterogeneous production costs compete in a circular city setting; however, they do not characterize the equilibrium and, instead, use numerical approximations to analyze the properties of the equilibrium. The significance of the closed-form solution in our work goes beyond mathematical elegance. As analytically shown below, it empowers us to rigorously illustrate the impact of value heterogeneity on sellers' equilibrium strategies and profits, and on the marketplace owner's policy decisions.

Proposition 1 offers several insights. First, intuitively, each seller's equilibrium price, demand, and profit are increasing in its own value. When a seller improves its value, ceteris paribus, it attracts additional buyers located further away in terms of taste. The value superiority compensates for a certain degree of mismatch, enabling the seller to raise its price and gain a higher profit. Equivalently, when a seller's value is reduced, it is not only pressured to cut price, its market share also shrinks as some buyers switch to the neighboring sellers. Given that both the market share and price of a seller are proportional to its own value (Proposition 1), the value has a quadratic effect on sellers' equilibrium profits.

Second, any seller's equilibrium price, demand, and profit are not only negatively affected by its neighboring competitors' values, but also by those of more remote sellers. The ripple effect is the force that connects all sellers: When a seller increases its value offering, the direct competition between that seller and its neighbors induces the neighboring sellers to cut price; then these sellers' neighboring sellers are also pressured to cut prices. This price cutting strategy propagates around the circle riding on the ripple effect and hits every seller.

The magnitude of the value effect depends on sellers' relative locations. In particular, a seller experience a more pronounced negative impact from the value improvement of another seller that offers a more similar product (i.e., located more nearby). Eq. (3) shows that $b_d < b_{d'}$ if $\min\{d, n-d\} < \min\{d', n-d'\}$, which implies that the impact of value diminishes from seller to seller. Moreover, notice that the indirect, negative impact of a seller' value to the remote sellers diminishes quickly as it passes through sellers. The impacts of a seller's value to the prices of its immediate neighbors, second-degree (neighbor's) neighbors, and third-degree (neighbors' neighbors') neighbors converge to $\frac{1}{\sqrt{3}(2+\sqrt{3})}$, $\frac{1}{\sqrt{3}(2+\sqrt{3})^2}$, and $\frac{1}{\sqrt{3}(2+\sqrt{3})^3}$, respectively (i.e., 0.155, 0.041, and 0.011, respectively), as n increases.[2] Therefore, in a marketplace with many sellers, it is adequate for each seller to strategize against its first- and second-degree competitors.

The ripple effect illustrates the competitive forces that are absent from both the circular city model without value heterogeneity and a duopoly competition. Our equilibrium characterization explicitly identifies the strengths of cross-seller value effect with the coefficient b_d for different sellers. In contrast, without value heterogeneity, the circular city model does not allow any seller to hold an advantage over its competitors. Seller heterogeneity is often of interest, if not elemental, in marketplaces. Duopoly models allow richer analysis of seller differentiation; however, the seller base of size two also considerably restricts in-depth discussions of marketplaces where a large number of diverse sellers are present. Our work makes a contribution by examining the effects of seller heterogeneity on the competition where a high number of sellers make strategic decisions.

Proposition 2 (Average Price and Quality). *The average equilibrium price of sellers,* $\bar{\boldsymbol{p}} = \frac{\boldsymbol{t}}{\boldsymbol{n}}$*, is independent of the value offering.*

Although the value offering affects individual sellers' pricing strategies, it exerts no impact on the average price among sellers. Sellers' price adjustments that respond to changes in the value offering follow a zero-sum game. As shown in Proposition 1, an increase in one seller's value raises its own price and reduces the price of all other sellers. The ripple effect distributes the shift in one seller's price to remaining sellers and keeps the average price constant. Expectedly, the average price decreases in the number of

[2] In fact, the convergence is sufficient fast such that, even at $n = 10$, the impact on the third-degree neighbor is miniscule.

sellers on the platform. With more sellers in the marketplace, their products are more similar; the intensified competition then leads to an overall price cut.

5. Marketplace Owner's Problem

In this section, we examine the marketplace owner's problem of setting the optimal level of the value-based support that generates Matthew Effect among the sellers. In particular, the marketplace owner maximizes the total royalty collected from the transactions with consideration for the costs incurred from the support. First, by substituting in sellers' equilibrium profits from Proposition 1, we derive the marketplace owner's expected profit based on the distribution of the sellers' values.

Lemma 1 *The marketplace owner's expected profit is:*

$$E(\Pi^*) = \frac{\gamma t}{n} + \frac{\gamma n}{t} \cdot Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right). \quad (5)$$

Notice that the marketplace owner's profit is independent of the expected value of sellers' value offerings. This suggests that a policy that benefits all sellers equally generates no additional surplus, positive or negative, to sellers or to the marketplace owner. As discussed in Lemma 1, any shifts in the expected value dissipate in seller competition, and only the relative—not absolute—values of neighboring sellers determine their profits. The marketplace owner's royalty, which is a fraction of sellers' total profits, is then also independent of any uniform change to all sellers' values.

As the marketplace owner offers the value-based support that enhances the sellers' values proportionally, the expected profit under the support is:

$$E\left(\Pi_p^*\right) = \frac{\gamma t}{n} + \frac{\gamma n}{t} \cdot (1+\alpha)^2 Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right)$$

$$- E\left(\sum_{i=0}^{n-1} c(\alpha v_i)^2\right)$$

$$= \frac{\gamma t}{n} + \frac{\gamma n}{t} \cdot (1+\alpha)^2 Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right)$$

$$- nc\alpha^2\left(Var(v) + E^2(v)\right). \tag{6}$$

Therefore, the increase in the marketplace owner's expected profit is:

$$E\left(\Pi_p^*\right) - E(\Pi^*)$$

$$= \frac{\gamma n}{t} Var(v)(\alpha^2 - 2\alpha)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right)$$

$$- nc\alpha^2\left(Var(v) + E^2(v)\right). \tag{7}$$

Proposition 3 (Optimal Support Level) *Offering a common support to all sellers equally does not impact the marketplace owner's expected profit. But, the marketplace is better off providing value-based support to its sellers. The optimal level is:*

$$\alpha^* = \frac{1}{\frac{ct}{\gamma}\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right)\left(1 + \frac{E(v)^2}{Var(v)}\right) - 1}. \tag{8}$$

The value-based support to sellers increases the marketplace owner's profit by enhancing value heterogeneity among the sellers. Recall that Proposition 1 shows an increase in a seller's value leads to higher price and demand for this seller. The value-based support allows the higher-value sellers to lift price and attract more buyers; meanwhile, as the lower-value sellers become more disadvantaged, they are pressured to cut price while losing market shares. In other words, by rewarding higher-value sellers with an advantage, the competition among sellers becomes more imbalanced. The gains of the high-value sellers outweigh the loss of the low-value sellers, as the former fulfill more transactions than the latter. Therefore, the total profits among sellers, hence the total royalty that the marketplace owner collects is higher.

Indeed, the Matthew Effect generated by the value-based support is beneficial for the marketplace owner and the higher-value sellers. The equilibrium analysis of the sellers' pricing strategies and the buyers' purchasing choices rigorously illustrates the economic mechanism behind the Matthew Effect. By creating advantages for higher-value sellers, the marketplace owner is able to reduce the less profitable transactions, increase the number of more profitable ones, and further raise the profits of the latter. This practice is nevertheless controversial, especially when the disparity in the treatment to sellers is too extreme and a large number of lower-value sellers are unable to handle the competitive pressure (He 2011). Thus, we further discuss the optimal level of the value-based support in terms of characteristics of the sellers and buyers.

Proposition 4 *For a higher average of sellers' value offerings, the marketplace owner's optimal support level is lower; however, for a higher variance of sellers' value offering, the marketplace owner's optimal support level is higher.*

Both the average and the variance of sellers' values impact the extent of the marketplace owner's support. A higher average value offering reduces the optimal support level because it makes the support more costly at the same level without affecting the total profits. The optimal level is, therefore, adjusted downward to offset the cost increase. On the other hand, value heterogeneity incentivizes a higher support level. The reason is that more heterogeneous value offerings are more sensitive to the value-based support. The marketplace owner can then sharpen value heterogeneity more cost-effectively.

Proposition 4 points out that, because the support is not costless, the marketplace owner should calibrate the extent of this effort based on the characteristics of its sellers. The expected seller value offering and value heterogeneity play starkly different roles on the marketplace owner's decisions. A marketplace with more lower-value sellers or a wider range of seller value can offer more generous support. As sellers change over time, it is necessary that the marketplace owner re-evaluates its support offering.

Proposition 5 *For a higher buyer transportation cost, the marketplace owner's optimal support level is lower.*

When sellers are more differentiated in terms of variety, the marketplace owner offers a lower support level. Eq. (7) indicates that the transportation cost, t, dampens the marketplace owner's gains from the support to sellers. The reason is that, if the sellers are more differentiated in terms of variety, any changes to their value offerings will have a lower impact on the neighboring sellers. In other words, the marketplace owner's support becomes less effective. Therefore, the optimal level is lower with more differentiation in seller variety. This seems to suggest that differentiations in terms of variety and value heterogeneity are complementary in their effects on the marketplace owner's profits.

In addition to a value-based support discussed above, it is interesting to note that the marketplace can also achieve profit gains by undermining lower-value sellers. Given that the marketplace owner yields higher profits with more value variation among sellers, enhancing such heterogeneity at the lower end has the same effect. If lower-value sellers have further reduced value offerings, the competition also becomes more imbalanced, which allows higher-value sellers to reap more profits and results in higher profits for the marketplace owner. This finding coincides with T-Mall's policy that imposes a heavier burden on lower-value sellers through increased security deposits (Fletcher 2011). This strategy is intriguing because the lower-value sellers' service is impaired due to the tightened financial constraint, and the impairment due the policy revision makes them even less capable than the higher-value sellers in obtaining the deposit refund.

Other Implications of Value-Based Support

We have established thus far that the Matthew Effect created through the value-based support leads to an increase in the total profits in the marketplace and, therefore, benefits the marketplace owner. However, as the increases in sellers' values are accompanied by higher prices, it is not obvious how the value-based support impacts the welfare of the buyers. We study the buyers' tradeoffs between receiving higher value offerings from the sellers and paying higher prices and derive the following result.

Proposition 6 (Consumer Surplus) *The value-based support leads to a higher expected consumer surplus.*

Under the value-based support, some buyers purchase from the same seller they would choose absent the support, while other buyers choose a different seller as a result of the shifts in sellers' values. For the first group of buyers, the additional benefit from the sellers' value increase is greater than the increase in price. Even though the higher-value sellers are able to raise their prices, the competition from the other sellers keeps the extent of price lift below the extent of value increase. Therefore, the value-based support raises the surplus of the first group of buyers. Meanwhile, the second group of buyers "switch" to the higher-value sellers under the value-based support. (In other words, they would choose a lower-value seller absent the value-based support.) Their surplus is also improved because, by “switching”, they are effectively making a choice that yields a higher surplus than transacting with the seller of choice absent the support. Thus, all the buyers obtain a higher surplus under the value-based support.

By creating the Matthew Effect, the marketplace owner not only generate more profit overall, it also raises the buyers’ payoffs; thus, the Matthew Effect improves the social welfare. Aside from the lower-value sellers who are disadvantaged, Matthew Effect brings benefit to all the other players in the marketplace. Clearly, the phenomenon of Matthew Effect in the online marketplaces is economically desirable, not only from the marketplace owner’s perspective. However, it may still be worthwhile to further evaluate this biased treatment against the weaker sellers.

The value-based support policy for the management of seller value heterogeneity has additional implications for the seller variety. The marketplace owner’s profit function identifies an optimal seller variety. By examining Eq. (5), we can see that as the number of seller increases, leading to greater variety, the impact on the marketplace owner’s profit is mixed. The first term of Eq. (5) alone shows a negative effect of seller variety on the marketplace owner’s profit. This suggests that, absent value heterogeneity, an increase in seller variety leads to greater similarity among sellers, which intensifies competition and reduces each seller's profit. The second term of Eq. (5) shows a positive effect of variety that is related to sellers’ value heterogeneity. The difference in sellers’ value offerings mitigates the competition intensity from the more crowded marketplace and allows the sellers to leverage their value offerings in setting prices.

Proposition 7 (Optimal Variety) *As the variance of sellers' value offerings increases, the optimal seller variety also increases.*

As the value-based support provided by the marketplace owner alters the sellers' competition, the seller variety that leads to the highest profit for the marketplace owner becomes greater. The positive effect of variety on the marketplace owner's profit is more pronounced given a higher degree of value heterogeneity (Eq. (5)). An increase in seller variety leads to more similarity among sellers horizontally; thus, sellers' value offerings play a more prominent role in the buyers' and competing sellers' decisions. It is then intuitive to see that, raising sellers' value heterogeneity allows a greater optimal level of variety, as the negative (competition) effect can be offset by the strengthened positive heterogeneity effect.

This result suggests that the marketplace owner should take into account indirect outcome of the Matthew Effect, as it implements the value-based policies. The change in sellers' competition dynamics implies that the marketplace owner can welcome more sellers. The increased the seller base will sharpen the benefits that sellers' value heterogeneity creates for the marketplace owners, the higher-value sellers, as well as the buyers. Therefore, Matthew Effect helps to create more opportunities for online selling. Its benefits extend beyond those on the existing sellers and buyers in the marketplace.

6. Conclusion

This paper analyzes the competition among a large number of heterogeneous sellers and examines the marketplace owner's value-based policy in managing seller heterogeneity. We model the pricing strategies of a variety of sellers with differentiated value offerings and find the ripple effect that drives interdependencies of sellers' pricing strategies. Based on the strategic interactions between the sellers and buyers, we study the marketplace owner's value-based support policy. We show that offering the support that increases each seller's value proportionally leads to an increased profit for the marketplace owner. The optimal support level depends on the average and variance of the sellers' values – which alter the cost-effectiveness of the support – as well as on the buyers' transportation cost. Furthermore, the value-based support also improves consumer welfare and leads to higher optimal seller variety. This suggests that Matthew Effect may lead to higher social welfare and create new business opportunities for online sellers.

Several future research directions can be further explored. Information asymmetry in a large, competitive marketplace may not be well understood. As online buyers may face uncertainty regarding the sellers' value offerings, introducing uncertainties about seller heterogeneity may be a meaningful direction. A follow-up work could introduce the marketplace owner's pricing problem for the entry of sellers and buyers to connect more closely to the theories of two-sided platform. Our equilibrium characterization prepares the foundation for such an analysis.

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Appendix

Proofs

Proof of Proposition 1

From Eq. ((**2**), we derive $p_{i+1} = 4p_i - p_{i-1} - \frac{2t}{n} - (2v_i - v_{i+1} - v_{i-1})$

Or, $p_i = 4p_{i-1} - p_{i-2} - \frac{2t}{n} - (2v_{i-1} - v_i - v_{i-2})$. (9)

Eq. ((9) can be written as a set of *n* FOCs:

$$p_i - (2-\sqrt{3})p_{i-1} = (2+\sqrt{3})\left(p_{i-1} - (2-\sqrt{3})p_{i-2}\right) - \frac{2t}{n} - (2v_{i-1} - v_i - v_{i-2}) \quad (10)$$

$$(2+\sqrt{3})\left(p_{i-1} - (2-\sqrt{3})p_{i-2}\right) = (2+\sqrt{3})[(2+\sqrt{3})\left(p_{i-2} - (2-\sqrt{3})p_{i-3}\right) - \frac{2t}{n} - (2v_{i-2} - v_{i-1} - v_{i-3})] \quad (11)$$

…

$$(2+\sqrt{3})^{n-1}\left(p_{i+1} - (2-\sqrt{3})p_i\right) = (2+\sqrt{3})^{n-1}[(2+\sqrt{3})\left(p_i - (2-\sqrt{3})p_{i-1}\right) - \frac{2t}{n} - (2v_i - v_{i+1} - v_{i-1})] \quad (12)$$

Summing up the *n* FOCs, we have

$$\left[1 - (2+\sqrt{3})^n\right](p_i - (2-\sqrt{3})p_{i-1}) = -\frac{2t}{n}\sum_{k=0}^{n-1}(2+\sqrt{3})^k - \sum_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-1} - v_{i-k} - v_{i-k-2})$$

Or,

$$p_i - (2-\sqrt{3})p_{i-1}$$
$$= \frac{1}{1+\sqrt{3}} \cdot \frac{2t}{n}$$
$$- \frac{1}{\left[1-(2+\sqrt{3})^n\right]} \sum_{k=0}^{n-1} (2+\sqrt{3})^k (2\, v_{i-k-1} - v_{i-k}$$
$$- v_{i-k-2}) \tag{13}$$

From here, we derive the following *n* equations:

$$p_i = (2-\sqrt{3})p_{i-1} + \frac{1}{1+\sqrt{3}} \cdot \frac{2t}{n} - \frac{1}{\left[1-(2+\sqrt{3})^n\right]} \sum_{k=0}^{n-1} (2+\sqrt{3})^k (2\, v_{i-k-1}$$
$$- v_{i-k} - v_{i-k-2}) \tag{14}$$

$$(2-\sqrt{3})p_{i-1} = (2-\sqrt{3})\Bigg[(2-\sqrt{3})p_{i-2} + \frac{1}{1+\sqrt{3}} \cdot \frac{2t}{n}$$
$$- \frac{1}{\left[1-(2+\sqrt{3})^n\right]} \sum_{k=0}^{n-1} (2+\sqrt{3})^k (2\, v_{i-k-2} - v_{i-k-1} - v_{i-k-3})\Bigg] \tag{15}$$

…

$$\left(2-\sqrt{3}\,\right)^{n-1} p_{i+1}$$

$$= \left(2-\sqrt{3}\,\right)^{n-1} \Bigg[\left(2-\sqrt{3}\,\right) p_i + \frac{1}{1+\sqrt{3}} \cdot \frac{2t}{n}$$

$$- \frac{1}{\left[1-\left(2+\sqrt{3}\right)^n\right]} \sum_{k=0}^{n-1} \left(2+\sqrt{3}\,\right)^k \left(2\, v_{i-k} - v_{i-k+1} - v_{i-k-1}\right) \Bigg] \tag{16}$$

Sum up these *n* equations, we have

$$\left[1-\left(2-\sqrt{3}\right)^n\right] p_i = \frac{1}{1+\sqrt{3}} \cdot \frac{2t}{n} \textstyle\sum_{l=0}^{n-1} \left(2-\sqrt{3}\,\right)^l - \sum_{l=0}^{n-1} \left(2-\sqrt{3}\,\right)^l \frac{1}{\left[1-\left(2+\sqrt{3}\right)^n\right]} \sum_{k=0}^{n-1} \left(2+\right.$$

$$\left.\sqrt{3}\,\right)^k \left(2\, v_{i-k-l-1} - v_{i-k-l} - v_{i-k-l-2}\right)$$

Therefore, seller i's equilibrium price is

$$p_i = \frac{t}{n} - \frac{1}{\left[1-\left(2+\sqrt{3}\right)^n\right]\left[1-\left(2-\sqrt{3}\right)^n\right]} \sum_{l=0}^{n-1} \sum_{k=0}^{n-1} \left(2-\sqrt{3}\,\right)^l \left(2 + \sqrt{3}\,\right)^k \left(2\, v_{i-k-l-1} - v_{i-k-l} - v_{i-k-l-2}\right) \tag{17}$$

By re-indexing the *v*'s, we can write Eq. ((17) as:

$$p_i^* = \frac{t}{n} - \frac{1}{\left[1-\left(2+\sqrt{3}\right)^n\right]\left[1-\left(2-\sqrt{3}\right)^n\right]} \sum_{m=0}^{2n} b_m v_{i-m} \tag{18}$$

where $b_0 = -1$;

$$b_m = -\frac{1}{\sqrt{3}}\left(\left(2+\sqrt{3}\right)^m - \left(2-\sqrt{3}\right)^m\right) \quad \text{for } m = 1,2,\ldots,n-1;$$

$$b_n = \frac{\sqrt{3}-1}{\sqrt{3}}\left(2+\sqrt{3}\right)^n + \frac{\sqrt{3}+1}{\sqrt{3}}\left(2-\sqrt{3}\right)^n$$

$$b_m = -\frac{1}{\sqrt{3}}\left(\left(2+\sqrt{3}\right)^{2n-m} - \left(2-\sqrt{3}\right)^{2n-m}\right) \quad \text{for } m = n+1, n+2, \dots, 2n-1;$$

$$b_{2n} = -1$$

Note that v_i, v_{i-n}, v_{i-2n} refer to the same seller, and that for $m = 1,2,\dots,n-1$, v_{i-m} and v_{i-m-n} refer to the same seller. Therefore, combine the parameters of the same v_i, for $i = 0,1,\dots n-1$, we have

$$p_i^* = \frac{t}{n} + v_i - \sum_{d=0}^{n-1} b_d v_{i-d} \tag{19}$$

where, $b_d = \frac{\left(2+\sqrt{3}\right)^{n-d} + \left(2+\sqrt{3}\right)^{d}}{\sqrt{3}\left(\left(2+\sqrt{3}\right)^{n}-1\right)} > 0$. From Eq. ((**2**), we derive the equilibrium demand $q_i = \frac{p_i}{t} = \frac{1}{n} + \frac{v_i}{t} - \frac{1}{t}\sum_{d=0}^{n-1} b_d v_{i-d}$. Thus, Seller i's equilibrium profit is $\pi_i^* = (1-\gamma)p_i^* q_i^* = (1-\gamma)\frac{p_i^{*2}}{t}$. □

Proof of Proposition 2

The result follows immediately from $\overline{p} = \frac{t}{n}$. □

Proof of Lemma 1

The marketplace owner's expected profit is the expectation of the aggregate of sellers' profits multiplied by the fraction γ:

$$E(\Pi^*) = \frac{\gamma}{t} E\left[\sum_{i=0}^{n-1} p_i^{*2}\right] = \frac{n\gamma}{t} E\left[\left(\frac{t}{n} + v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right]$$

$$= \frac{n\gamma}{t} E\left[\frac{t^2}{n^2} + \frac{2t}{n}\left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right]$$

$$= \frac{\gamma t}{n} + \frac{n\gamma}{t} E\left[\frac{2t}{n}\left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right].$$

It can be shown that $\sum_{d=0}^{n-1} b_d = 1$, based on which we further simplify:

$$E\left[\frac{2t}{n}\left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right]$$

$$= \frac{2t}{n} E(v)\left(1 - \sum\nolimits_{d=0}^{n-1} b_d\right)$$

$$+ E\left(v_i^2(1-b_0)^2 - 2(1-b_0)v_i \sum_{d=1}^{n-1} b_d v_{i-d} + \left(\sum_{d=1}^{n-1} b_d v_{i-d}\right)^2\right)$$

$$= (1-b_0)^2 E(v^2) - 2(1-b_0)\sum_{d=1}^{n-1} b_d E^2(v) + E\left(\sum_{d=1}^{n-1} b_d v_{i-d}\right)^2$$

$$= (1-b_0)^2 E(v^2) - 2(1-b_0)E^2(v)$$

$$+ \sum_{d=1}^{n-1} b_d^2 E(v^2) + \sum\nolimits_{j,k=1;j\neq k}^{n-1} b_j b_k E^2(v)$$

$$= \left((1-b_0)^2 + \sum\nolimits_{d=1}^{n-1} b_d^2\right)[Var(v) + E^2(v)] - 2(1-b_0)E^2(v)$$

$$+ \sum\nolimits_{j,k=1;j\neq k}^{n-1} b_j b_k E^2(v)$$

$$= Var(v)\left[(1-b_0)^2 + \sum\nolimits_{d=1}^{n-1} b_d^2\right]$$

$$+ E^2(v)\left[(1-b_0)^2 - 2(1-b_0)^2 + \left(\sum\nolimits_{d=1}^{n-1} b_d^2\right)^2\right]$$

$$= Var(v)\left[(1-b_0)^2 + \sum\nolimits_{d=1}^{n-1} b_d^2\right]$$

$$= \ Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right).$$

Therefore, $E(\Pi^*) = \frac{\gamma t}{n} + \frac{\gamma n}{t} \cdot Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n+1}{\delta^n-1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n-1)^2}\right)$. □

Proof of Proposition 3

The platform is interested in solving α to maximize the gains from offering quality support minus the costs incurred:

$$\frac{\gamma n}{t} Var(v)(\alpha^2 + 2\alpha)\left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3} \frac{\delta^n}{(\delta^n - 1)^2}\right) - nc\alpha^2\big(Var(v) + E^2(v)\big).$$

The FOC with respect to α is:

$$2n\alpha \left[\frac{\gamma}{t} Var(v)\left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3} \frac{\delta^n}{(\delta^n - 1)^2}\right) - c\big(Var(v) + E^2(v)\big)\right]$$
$$+ \frac{2\gamma n}{t} Var(v)\left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3} \frac{\delta^n}{(\delta^n - 1)^2}\right) = 0.$$

Suppose the second-order condition is satisfied, such that $c > \frac{\frac{\gamma}{t} Var(v)\left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3} \frac{\delta^n}{(\delta^n - 1)^2}\right)}{Var(v) + E^2(v)}$, we have an internal solution $\alpha^* > 0$. □

Proof of Proposition 4

It is straightforward to show that Eq. (8) decreases in $E(v)$ and increases in $Var(v)$. □

Proof of Proposition 5

It is straightforward to show that Eq. (8) decreases in t. □

Proof of Proposition 6

Define the buyer who is indifferent between the sellers located at 0 and $\frac{1}{n}$ as $x^* = \frac{1}{2t}(p_1 - p_0 - v_1 + v_0) + \frac{1}{2n}$, such that for a buyer located at x between 0 and $\frac{1}{n}$, if $0 \leq x < x^*$, the buyer buys from the seller located at 0; if $x^* \leq x \leq \frac{1}{n}$, she buys from the seller at $\frac{1}{n}$. The expected utility for the buyers located between 0 and 1 is:

$$u\big|_{\left[0,\frac{1}{n}\right]} = \int_0^{x^*}(v_0 - p_0 - tx)dx + \int_{x^*}^{\frac{1}{n}}\left(v_1 - p_1 - t\left(\frac{1}{n} - x\right)\right)dx$$

$$= (v_0 - p_0)x^* - \frac{t}{2}x^{*2} + \left(\frac{1}{n} - x^*\right)\left(v_1 - p_1 - \frac{t}{n}\right) + \frac{t}{2}\left(\frac{1}{n^2} - x^{*2}\right)$$

$$= -tx^{*2} + x^*\left(v_0 - p_0 - v_1 + p_1 + \frac{t}{n}\right) + \frac{1}{n}(v_1 - p_1) - \frac{t}{2n^2}$$

$$= -t\left(\frac{1}{2t}(p_1 - p_0 - v_1 + v_0) + \frac{1}{2n}\right)^2 + \left(\frac{1}{2t}(p_1 - p_0 - v_1 + v_0) + \frac{1}{2n}\right)\left(v_0 - p_0 - v_1 + p_1 + \frac{t}{n}\right) + \frac{1}{n}(v_1 - p_1) - \frac{t}{2n^2}$$

Define $a \equiv p_1 - p_0 - v_1 + v_0$, then, $u\big|_{\left[0,\frac{1}{n}\right]} = -t\left(\frac{1}{2t}a + \frac{1}{2n}\right)^2 + \left(\frac{1}{2t}a + \frac{1}{2n}\right)\left(a + \frac{t}{n}\right) + \frac{1}{n}(v_1 - p_1) - \frac{t}{2n^2}$.

Based on the distribution of the sellers' values, we can derive the following:

$$E(p_i^2) = E\left[\left(\frac{t}{n} + \frac{\sum_{d=0}^{n-1} b_d v_{i-d}}{\left((2+\sqrt{3})^n - 1\right)\left(1-(2-\sqrt{3})^n\right)}\right)^2\right] = \frac{t^2}{n^2} + \frac{E\left[\left(\sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right]}{\left((2+\sqrt{3})^n - 1\right)\left(1-(2-\sqrt{3})^n\right)}$$

$$\because \left(\sum_{d=0}^{n-1} b_d v_{i-d}\right)^2 = \left(\sum_{d=0}^{n-1} b_d v_{i-d} - E(v)\sum_{d=0}^{n-1} b_d\right)^2 = \left(\sum_{d=0}^{n-1} b_d (v_{i-d} - E(v))\right)^2$$

$$= \sum_{d=0}^{n-1} b_d^2 (v_{i-d} - E(v))^2 + 2\sum_{j\neq k} b_j b_k \left(v_{i-j} - E(v)\right)\left(v_{i-k} - E(v)\right)$$

$$\therefore E\left[\left(\sum_{d=0}^{n-1} b_d v_{i-d}\right)^2\right] = E\left[\sum_{d=0}^{n-1} b_d^2 \left(v_{i-d} - E(v)\right)^2 + 2\sum_{j\neq k} b_j b_k \left(v_{i-j} - E(v)\right)\left(v_{i-k} - E(v)\right)\right]$$

$$= \sum_{d=0}^{n-1} b_d^2 E\left[\left(v_{i-d} - E(v)\right)^2\right] = Var(v)\sum_{d=0}^{n-1} b_d^2$$

Therefore, $E(p_i^2) = \frac{t^2}{n^2} + \frac{\sum_{d=0}^{n-1} b_d^2}{\left((2+\sqrt{3})^n - 1\right)^2\left(1-(2-\sqrt{3})^n\right)^2} Var(v)$,

$E(p_i p_{i+1}) = \frac{t^2}{n^2} + \frac{\sum_{d=0}^{n-1} b_d b_{d+1}}{\left((2+\sqrt{3})^n - 1\right)^2 \left(1-(2-\sqrt{3})^n\right)^2} Var(v),$

$E(a) = 0$, and

$E(a^2) = E[(p_1 - p_0 - v_1 + v_0)^2] = E[(p_1 - p_0)^2] + E[(v_1 - v_0)^2] = 2E(p_i^2) -$

$2E(p_i p_{i+1}) + 2Var(v) = 2Var(v)\left(1 + \frac{\sum_{d=0}^{n-1} b_d (b_d - b_{d+1})}{\left((2+\sqrt{3})^n - 1\right)^2 \left(1-(2-\sqrt{3})^n\right)^2}\right).$

The total consumer surplus is:

$E(u|_x) = n \cdot E\left[u|_{\left[0,\frac{1}{n}\right]}\right] = n \cdot E\left[-t\left(\frac{1}{2t}a + \frac{1}{2n}\right)^2 + \left(\frac{1}{2t}a + \frac{1}{2n}\right)\left(a + \frac{t}{n}\right) + \frac{1}{n}(v_1 - p_1) -\right.$

$\left.\frac{t}{2n^2}\right]$

$= n\left[\frac{1}{4t}E(a^2) - \frac{5}{4}\frac{t}{n^2} + \frac{1}{n}E(v)\right] = \frac{n}{2t}\ Var(v)\left(1 + \frac{\sum_{d=0}^{n-1} b_d (b_d - b_{d+1})}{\left((2+\sqrt{3})^n - 1\right)^2 \left(1-(2-\sqrt{3})^n\right)^2}\right) - \frac{5}{4}\frac{t}{n} +$

$E(v)$

Notice that $1 + \frac{\sum_{d=0}^{n-1} b_d (b_d - b_{d+1})}{\left((2+\sqrt{3})^n - 1\right)^2 \left(1-(2-\sqrt{3})^n\right)^2} > 0$, therefore, the total consumer surplus increases in the variance of the sellers' values. □

Proof of Proposition 7

From Lemma 1, the optimal variety n^* must satisfy the following two conditions:

F.O.C. w.r.t. n

$$H(n) \equiv \frac{\partial E\left(\Pi^*\right)}{\partial n}$$

$$= -\frac{\gamma t}{n^2}$$

$$+ \frac{\gamma Var(v)}{t}\Bigg[1 - \frac{4}{3\sqrt{3}}\left(1 + \frac{2}{\delta^n - 1}\right) + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}$$

$$+ n\left(\frac{4}{3\sqrt{3}}\frac{2\delta^n ln\delta}{(\delta^n - 1)^2} + \frac{2}{3}\frac{\delta^n}{(\delta^n - 1)^2} - \frac{2n}{3}\frac{\delta^n + 1}{(\delta^n - 1)^3}\delta^n ln\delta\right)\Bigg]$$

$$= -\frac{\gamma t}{n^2}$$

$$+ \frac{\gamma Var(v)}{t}\Bigg[1 - \frac{4}{3\sqrt{3}}\left(1 + \frac{2}{\delta^n - 1}\right)$$

$$+ \frac{4n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\left(1 + \frac{2ln\delta}{\sqrt{3}} - \frac{nln\delta}{2} - \frac{nln\delta}{\delta^n - 1}\right)\Bigg] = 0.$$

S.O.C. w.r.t. n $\quad \frac{\partial^2 E(\Pi^*)}{\partial n^2} = \frac{\partial H}{\partial n} < 0$

$$\frac{\partial H}{\partial Var(v)} = 1 - \frac{4}{3\sqrt{3}}\left(1 + \frac{2}{\delta^n - 1}\right) + \frac{4n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\left(1 + \frac{2ln\delta}{\sqrt{3}} - \frac{nln\delta}{2} - \frac{nln\delta}{\delta^n - 1}\right).$$

It can easily be shown that $\frac{\partial H}{\partial Var(v)} > 0$ when $n \in \{1,2,3,4\}$; Furthermore, $\frac{\partial H}{\partial Var(v)}$ is monotonically increasing when $n > 4$. Therefore, $\frac{\partial H}{\partial Var(v)} > 0$.

Hence, $\frac{\partial n^*}{\partial Var(v)} = -\frac{\frac{\partial H}{\partial Var(v)}}{\frac{\partial H}{\partial n}}$. In other words, the variance of sellers' value offerings increases the optimal seller variety. □