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# Risk, Return and Risk Aversion: A Behavioral Rendition

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## **Abstract**

Behavioral finance and classical finance based on utility maximization appear to be mutually exclusive schools of thought. Despite the fundamental difference, we show that behavioral finance also has a linear relation between risk and return. This relation is obtained without the assumptions of market equilibrium, rational expectations, a specific utility function and the market portfolio. In the behavioral approach, the “pricing error” of CAPM is not an error. It is attributable to the higher-order moments of return. Empirical tests suggest that the relative risk aversion coefficient is positive and time-varying. Moreover, it correlates negatively with both volatility and return.

*Keywords:* Market Belief, Capital Asset Pricing, Risk, Risk Aversion

*JEL classification:* G12

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## I. Introduction

One of the central tenets of finance is that investors expect higher return for taking risk. They exchange some of their riskless securities for risky assets because they expect the total payoff in the long run to be optimal in terms of the risk-return trade-off. Since the publication of CAPM by Sharpe (1964), Lintner (1965) and Mossin (1966), the notions of mean-variance efficiency and systematic risk have become the basis for analyzing the trade-off. In the same vein, generalized versions of CAPM such as Merton (1973)'s ICAPM and Ross (1976)'s APT all predict a positive relation between risk and return.

Beyond the realm of rational expectations, Lakonishok, Shleifer, and Vishny (1994) argue that investors overreact to past earnings growth and overestimate future growth rates. The speculative bubble in the later half of 1990s is an example of mispricing. This example demonstrates that stock prices can markedly deviate from their fundamental values for several years. After all, economic agents are humans who err naturally. Odean (1998) reports that retail investors all too often sell away their star performers prematurely but hold on to under-performing stocks to avoid realizing capital losses, a phenomenon described by Tversky and Kahneman (1991) as loss aversion. Professional traders are also not immune from making mistakes. Indeed, the financial industry is rife with anecdotes of mistakes<sup>2</sup>, some of which are as dramatic and catastrophic as the downfalls of the LTCM and the Barings Bank. These market behaviors incomprehensible within the classical framework of equilibrium and utility optimization suggest that a new approach is needed to examine the relation between risk and return.

In this paper, we present a model based on market belief to describe the behavior of stock returns. The market belief represents a summary of all market participants' opinions. We do not take side on whether their views are rational or irrational. Our starting point is to acknowledge that various dynamics affecting price moves are too complex to be decomposed entirely within the philosophical framework of reductionism. The coupling among the state variables makes it intractable to isolate one factor totally from the influence of the others. We adopt an agnostic stand concerning investors' consumption-investment tradeoff and introduce market belief to

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<sup>2</sup>Ritter (1996) gives an educational account of his rise and fall in futures trading.

capture the aggregate behavior of market participants, who have the freedom to trade or abstain from trading.

The conclusion of our paper is this: Expected return is linearly related to risk. If we further assume investors are risk averse, then the alluded relation will have to be positive. This is an old result, but a key feature of how this result is derived lies in our model's required assumptions, which are parsimonious compared to extant asset pricing models. It is intriguing that the same linear relation between risk and return emerges from a model that relaxes the assumptions concerning market equilibrium, rational expectations, a specific utility function as well as the elusive market portfolio. On the other hand, neither do we need to assert that investors are overconfident and their trading decisions are psychologically biased<sup>3</sup>. But, to the extent that market belief is subjective and not based on maximization of some objective function, our approach may be labelled as behaviorist.

Although the parsimonious stance of our approach also has its drawback in being silent about the microeconomic dynamics governing the state variables, it enables empirical tests to be conducted and interpreted in a straightforward manner. Regardless of whether the portfolio is mean-variance efficient or whether market equilibrium holds, our tests provide overwhelming evidence of a positive relation between risk and return<sup>4</sup>. The tests are independent of the models for conditional variance, as this paper's approach does not require a GARCH-type treatment of risk. In addition, we uncover a negative relation between risk aversion and volatility, and suggest the combined effect of higher-order moments of return as a plausible cause of "pricing error."

The rest of the paper is as follows. Section II introduces our basic model based on the concept of market belief. Section III connects the parameter of the basic model with relative risk aversion. The proxy we use for market belief, as well as the related econometric specifications are discussed in Section IV. Section V documents our empirical analysis and results. Concluding

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<sup>3</sup>Daniela, Hirshleifer, and Teoh (2002) provide a comprehensive review of behaviorist-oriented literature. Campbell (2000)'s review and Cochrane (2001)'s exposition cover both standard and anomalous findings.

<sup>4</sup>The coefficient of this relation has been a subject of controversy due to conflicting findings in the empirical literature. For example, French, Schwert, and Stambaugh (1987) find a positive relation between risk and return while Campbell (1987) and Glosten, Jagannathan, and Runkle (1993) find the relation to be negative. Many other papers have also looked into this issue but the conclusions are divided.

remarks are made in Section VI.

## II. Basic Model

### A. Assumptions

Instead of pursuing the classic stochastic optimization approach, we consider a probabilistic framework with the following assumptions.

A1 : Market participants' beliefs and trading strategies are heterogeneous.

A2 : Market participants' beliefs and trading strategies are largely unpredictable.

A3 : The probability density function of return exists and is differentiable.

Be it active or side-lined, we define market participants as people who have an interest in the capital assets being traded in the markets. They include retail investors, fund managers, institutional trading desks, market makers, as well as company insiders, financial analysts and reporters. More often than not, the amount, quality and timeliness of information, as well as the capacity to process and to act upon the information vary significantly among various categories of market participants. Even for the same information, different analysts will have dissimilar beliefs regarding the fair value of an asset. Besides the heterogeneous composition, trading motivation, strategies and styles are also different. Some trades are driven by the needs for hedging, portfolio re-balancing and risk reduction. Others are purely speculative and in some instances, motivated by private information. Some investors prefer growth potential as opposed to value; some trade by momentum while others contrarian. In addition to various classes of trading strategies, investment horizons, holding power, portfolio management and cash flow constraints vary. Therefore, the first assumption is reasonably satisfied in the real world.

Although seldom explicitly stated, the second assumption underlies most if not all theories of finance. The second assumption is valid for heavily traded stocks such as those listed on the NYSE. These stocks are highly liquid and the breadth of ownership is wide. Even for illiquid stocks, the lack of liquidity depth makes even informed traders incapable of profiting from their

information optimally. In conjunction with the first assumption, the implication is that any single group of market participants has limited ability to manipulate the market and control the price. As a result, price levels and order flows are non-deterministic from the perspective of any single group of investors because the trading decisions of other market participants are largely unpredictable. Although the second assumption has the flavor of Efficient Market Hypothesis, we do not insist that security prices fully reflect all available information all the time.

Almost surely, all financial and econometric theories assume the probability density function of return exists. Our model is no exception. However, in contrast to most capital asset pricing theories that typically require the probability density function to be Gaussian, our model has no further assumption concerning its parametric form. Nonetheless, for technical reason, we need to assume that the probability density function is differentiable with respect to time and return.

In summary, the three assumptions underlying the basic model are reasonably fulfilled in the financial markets. Traditional notions such as market equilibrium, information efficiency, as well as optimal portfolio and consumption choice are not invoked. Furthermore, our framework does not require the assumption of a particular utility function to describe the preferences of representative investors.

## **B. Dynamics**

Market participants and the dynamics of stock price exhibit far complex behaviors to be caged submissively within the paradigms of mean-variance efficiency and expected utility maximization. To come to grips with the complexity and heterogeneity in the market, we take the phenomenological approach of not a priori impose any scheme that postulates how market participants are behaving. Nebulous as the notion may be, it is convenient to use “market belief” to describe what market participants *think* the market or a particular stock is heading. Based on their respective opinions, some of the market participants may trade, while some may just think but do no act.

More concretely, suppose the prevailing outlook at time  $t$  is such that the probability of upward price move over a horizon  $h$  is  $p_+(t; h)$ . This probability is not directly observable. It

represents an aggregate and presumably subjective view of market participants who process the information available at time  $t$  and second-guess what others are thinking and going to act. As befitting the notion of probability, we write  $p_-(t; h) = 1 - p_+(t; h)$  as the probability of downward price move. The difference,  $p_+(t; h) - p_-(t; h)$ , is called the market belief, which is an unobservable summary statistic of the economy.

As a remark, the binomial option pricing model also has a similar probabilistic structure for the upward and downward moves in the underlying assets. Thus, it is not entirely obnoxious to consider the difference of these two probabilities as market belief, though in a different context with dissimilar motivation. Now, let  $\ell \geq 0$  be the ensemble average excess return when a particular stock or portfolio is deemed to move up in the future. For simplicity, we let  $-\ell$  be the average excess return when it is perceived that asset prices will move downwards.

We denote the excess return of a portfolio over a horizon  $h$  as  $r_h$ . It is a random variable. Taking all market participants' opinions into consideration, the conditional expected excess return  $E_t[r_h]$  is expressed as, after suppressing the arguments in  $p_+(\cdot)$  and  $p_-(\cdot)$ ,

$$E_t[r_h] = p_+ \ell + p_- (-\ell) = (p_+ - p_-) \ell. \quad (1)$$

We use the subscript  $t$  to indicate the expectation is conditional on the information set at time  $t$ . As anticipated, the conditional expected excess return is dependent on the market belief,  $p_+ - p_-$ . The conditional variance is  $V_t[r_h] \equiv E_t\left[\left(r_h - E_t[r_h]\right)^2\right] = E_t[r_h^2] - E_t[r_h]^2$ . Since  $E_t[r_h^2] = p_+ \ell^2 + p_- (-\ell)^2 = \ell^2$  and substituting in equation (1), we get

$$V_t[r_h] = (1 - (p_+ - p_-)^2) \ell^2.$$

With these quantities, the rates of conditional expected excess return and variance are defined accordingly as

$$\tilde{\mu} \equiv \frac{1}{h} E_t[r_h] = \frac{(p_+ - p_-) \ell}{h}, \quad (2)$$

$$\tilde{\nu} \equiv \frac{1}{h} V_t[r_h] = \frac{1 - (p_+ - p_-)^2}{h} \ell^2. \quad (3)$$

The probability density function of  $r_h$  is denoted as  $f(r_h, t)$ . If one were to consider a time scale  $h$  that is small compared to, say, several decades, and wonder what would happen to the probability density function at the next time step  $t + h$ , the following equation ensues (see, for example, Kwok (1999)).

$$f(r_h, t + h) = p_+ f(r_h - \ell, t) + p_- f(r_h + \ell, t).$$

This equation expresses the forward evolution of the probability density function as being dependent on the current time and the state of excess return, which can only be  $r_h - \ell$  or  $r_h + \ell$  with probability density  $f(r_h - \ell, t)$  and  $f(r_h + \ell, t)$  respectively. If the excess return were  $r_h - \ell$ , it would become  $r_h$  with probability density  $p_+ f(r_h - \ell, t)$  in the next time step  $t + h$ . If it were  $r_h + \ell$ , it would become  $r_h$  in the next time step with probability density  $p_- f(r_h + \ell, t)$ .

Performing Taylor's expansion up to the first order in  $h$  and second order in  $\ell$ , we obtain

$$\begin{aligned} f(r_h, t) + \frac{\partial f}{\partial t} h + O(h^2) &= p_+ \left( f(r_h, t) - \frac{\partial f}{\partial r_h} \ell + \frac{1}{2} \frac{\partial^2 f}{\partial r_h^2} \ell^2 \right) + p_- \left( f(r_h, t) + \frac{\partial f}{\partial r_h} \ell + \frac{1}{2} \frac{\partial^2 f}{\partial r_h^2} \ell^2 \right) \\ &+ O(\ell^3). \end{aligned}$$

After a re-arrangement of terms and using the identity  $p_+ + p_- = 1$ , it becomes

$$\frac{\partial f}{\partial t} = -\frac{\ell}{h}(p_+ - p_-) \frac{\partial f}{\partial r_h} + \frac{\ell^2}{2h} \frac{\partial^2 f}{\partial r_h^2} + O(h) + O\left(\frac{\ell^3}{h}\right). \quad (4)$$

To gain some insight from equation (4), we consider the hypothetical limits  $h \rightarrow 0$  and  $\ell \rightarrow 0$ . Equation (4) then reduces to a special form of the Fokker-Planck equation that describes the forward diffusion,

$$\frac{\partial f}{\partial t} = -\mu \frac{\partial f}{\partial r_h} + \frac{\nu}{2} \frac{\partial^2 f}{\partial r_h^2}.$$

In this equation,  $\mu$  is the drift rate and  $\nu$  the coefficient of diffusion:

$$\mu = \lim_{h, \ell \rightarrow 0} \frac{(p_+ - p_-) \ell}{h}; \quad (5)$$

$$\nu = \lim_{h, \ell \rightarrow 0} \frac{\ell^2}{h}. \quad (6)$$



It is interesting to note that while  $\mu$  is identical to  $\tilde{\mu}$  in equation (2),  $\nu$  is not quite the same as  $\tilde{\nu}$ . More alarming is the fact that  $\mu$  and  $\nu$  are not of the same order of magnitude. The drift rate  $\mu$  is linear in  $\ell$  whereas the diffusion coefficient  $\nu$  is quadratic. To yield finite values for both  $\mu$  and  $\nu$  when  $h, \ell \rightarrow 0$ , a consistency relation has to be introduced. Namely, the market belief  $p_+ - p_-$  has to satisfy

$$p_+ - p_- = \hat{\gamma}_0 + \tilde{\gamma}_1 \ell, \quad (7)$$

where  $\hat{\gamma}_0$  and coefficient  $\tilde{\gamma}_1$  are the parameters. In other words, up to a coefficient  $\tilde{\gamma}_1$ ,  $p_+ - p_-$  must be of the same order of magnitude as  $\ell$ , so that  $\mu$  is comparable with  $\nu$  in having the same leading order of  $\ell^2$ . Otherwise, the variance would be too small relative to the expected excess return, which is contrary to the reality of uncertainty. It also implies that  $\hat{\gamma}_0$  represents the sum of all the terms of order higher than one:

$$\hat{\gamma}_0 \equiv \sum_{j=2}^{\infty} c_j \ell^j,$$

where  $c_j$ 's are the corresponding coefficients. In a way, our preferential treatment of mean and variance at the expense of higher-order moments restricts the probability density function to a class of compact distributions elaborated in Chapter 12 of Ingersoll (1987).

By imposing equation (7), the diffusion coefficient  $\nu$  becomes approximately the rate of conditional variance  $\tilde{\nu}$  expressed in equation (3), reinforcing the sensibility of our economic interpretation. Equation (7) is based upon the first two assumptions of heterogeneity and limited predictability of market belief. With market participants having diverse views and trading strategies, one could surmise that the market belief is seldom overwhelmingly one-sided or homogeneous. Moreover, by definition,  $|p_+ - p_-| \leq 1$ . This is in line with the fact that under normal circumstances, the average excess return is typically well within  $\pm 100\%$  over a short horizon. Large jumps associated with distressful events are rare and do not occur at the daily or monthly frequency. Accordingly, up to a coefficient  $\tilde{\gamma}_1$ , the market belief  $p_+ - p_-$  is of the same order as  $\ell$  most of the time.

Although the Fokker-Planck equation is known to be solvable, we use it only to illustrate the limiting case of equation (4) for interpreting the drift rate and the coefficient of diffusion. Its

Gaussian diffusion solution is not used in our model. In fact, we do not need to know the explicit functional form of  $f(r_h, t)$  that solves equation (4). More importantly, the holding period  $h$  and the average excess return  $\ell$  of market participants need not be infinitesimally small. So long as the first two terms on the right side of equation (4) are large relative to the higher-order terms, and in light of equations (2) and (3), we obtain the following:

$$\tilde{\mu} = \frac{p_+ - p_-}{h} \ell; \quad (8)$$

$$\tilde{\nu} = \frac{\ell^2}{h} + O(\ell^3). \quad (9)$$

Equation (8) relates the rate of conditional expected excess return  $\tilde{\mu}$  with the market belief. Up to the higher order of  $\ell^3$ , we interpret  $\tilde{\nu}$  in equation (9) as the rate of conditional variance. A characteristic worth highlighting is that the rate of conditional expected excess return  $\tilde{\mu}$  is not constrained to be positive. It depends on the sign of  $p_+ - p_-$ .

Applying equation (7), we obtain from equations (8) and (9) a linear relation between the rates of conditional mean  $\tilde{\mu}$  and variance  $\tilde{\nu}$  as follows:

$$\tilde{\mu} = \tilde{\gamma}_0 + \tilde{\gamma}_1 \tilde{\nu}, \quad (10)$$

where

$$\tilde{\gamma}_0 \equiv \hat{\gamma}_0 \ell/h = \sum_{j=3} c_j \ell^j / h.$$

What equation (10) suggests is that the rate of conditional expected excess return is a linear function of the rate of conditional variance. We have thus transformed the belief of market participants into an empirical asset pricing model in which  $\tilde{\gamma}_0$  and  $\tilde{\gamma}_1$  are two parameters to be estimated from the data. While  $\tilde{\gamma}_0$  accounts for the higher-order contributions, the coefficient  $\tilde{\gamma}_1$  measures the increment of risk premium per unit of conditional variance. Therefore, we have demonstrated that the conditional expected excess return is linearly related to risk as measured by the conditional variance.

From the CAPM's point of view,  $\tilde{\gamma}_0$  represents the "pricing error." In equilibrium, this pricing error should be zero. However, this sum of higher-order moments of return in the framework of

this paper is not required to vanish. It is the residual, not a pricing error. Its existence does not threaten the dominance of the conditional variance in equation (10).

### III. Model of Risk Aversion

In this section, we introduce additional assumptions to explore the possibility of interpreting  $\tilde{\gamma}_1$  as the relative risk aversion. We use the utility function  $u(S)$  to express market's overall view and preference of the price or index level  $S$ . The asset could be an individual stock, or an index portfolio. This aspect differs fundamentally from extant asset pricing models for which the utility is usually a specific function of wealth consisting of *all* assets including human capital. To begin with, we assume that

A4 : The utility function  $u(S)$  exists and is differentiable.

A5 : The utility function is strictly concave and for every integer  $k \geq 3$  and for all  $S$ ,

$$S^{k-1} \left| \frac{\frac{d^k u(S)}{dS^k}}{\frac{du(S)}{dS}} \right| \leq 1 \quad (11)$$

Utility function is fundamental in economics and finance. Justification for its postulated existence is hardly needed. Beyond its existence and differentiability, we do not need the assumption regarding the functional form of  $u(S)$ , although its differentials have to satisfy a technical condition expressed in equation (11). For reasons that will become evident later, this technical condition is to align our model of risk aversion with the probability density function of return being in the class of compact distributions discussed previously.

In general, higher  $S$  makes asset owners happy. But for many others, especially those who intend to acquire the asset, they are unhappy that the price has become too high. Conversely, asset owners will be unhappy when  $S$  is low while others who intend to buy, they prefer a lower  $S$  at the time of purchase. In any case, for a given  $S$ ,  $u(S)$  reflects the aggregate preference of market participants.

We define the relative risk function  $\omega(S)$  as

$$\omega(S) \equiv -\frac{S u''(S)}{2 u'(S)}. \quad (12)$$

Up to a factor of two,  $\omega(S)$  is no difference from the standard definition, although as remarked earlier, market participants' utility is a function of  $S$  instead of total wealth. If  $u(S)$  is strictly concave, then  $u'(S) \equiv du/dS > 0$  and  $u''(S) < 0$ , implying that  $\omega(S) > 0$ . It is tantamount to saying that market participants are risk averse. When facing choices with compatible returns, market participants tend to choose the more certain alternatives. If two portfolios have the same perceived risk, their expected returns should be comparable. On the other hand,  $\omega(S) \leq 0$  corresponds to market participants being indifferent to risk or having an affinity for risk taking. Through empirical analysis, we will be able to ascertain investors' attitude towards risk taking by the sign of  $\omega(S)$  and its statistical significance.

On the basis of their belief concerning the future price level relative to the current level  $S$ , and depending on their respective financial circumstances and degree of risk aversion, market participants can choose to trade or to abstain. Suppose market participants decide to trade in the market. Then they stand a chance  $p_+$  of gaining an amount equal to  $\ell S$  per unit of asset, and a probability of  $p_-$  losing by as much. Their expected utility of trading is

$$E_t[\text{Trading}] = p_+ u((1 + \ell)S) + p_- u((1 - \ell)S). \quad (13)$$

It represents the conditional expected utility of traders when the price moves up, or moves down with respect to the current price  $S$  by an amount  $\ell S$  in the future. Applying Taylor's theorem and in view of Assumption A5, the terms on the right side of equation (13) are

$$u(S) + \ell S u'(S) \left( (p_+ - p_-) + \frac{1}{2} \ell S \frac{u''(S)}{u'(S)} + O(\ell^2) \right). \quad (14)$$

Since there is no gain or loss for market participants who abstain from trading,  $p_+ u(S) + p_- u(S) = u(S)$ . Hence, the expected utility of market participants who do not trade is  $u(S)$ . The expected utility of trading  $E_t[\text{Trading}]$  and the expected utility of not trading,  $u(S)$ , are related

as follows:

$$E_t[\text{Trading}] = u(S) + C\Delta u. \quad (15)$$

In this equation,  $C \equiv C(S, \ell)$  is a function of the payoff  $\ell S$  and  $\Delta u$  is the change in utility. Borrowing Pratt (1964)'s concept of "risk premium," the adjustment term  $C\Delta u$  reconciles the difference<sup>5</sup> between the expected utility of trading and that of not trading. As an emphasis, there is no need to impose that market participants maximize their expected utility. Neither do we require the price  $S$  to be efficient. It may simply be an indicative price of a range of prices. In this sense, our model is independent of whether the market is in equilibrium. Worded differently, the supply and demand of asset in the market is such that  $S$  may or may not be the market clearing price at time  $t$ . As in the discussion of Assumption A2, trading friction in the market can impede informed traders to benefit from their privileged information. More specifically, if almost all market participants decide to abstain from trading<sup>6</sup>, it is not certain whether  $S$  represents the equilibrium price. It may be, or maybe not. In any case, the validity of our model of risk aversion does not depend on the equilibrium and efficiency properties of  $S$ .

After re-arranging and suppressing the argument of the utility function in equation (14), we obtain from equation (15) an expression for the market belief,

$$p_+ - p_- = \frac{C\Delta u}{\ell S u'} + \ell \left( -\frac{S u''}{2 u'} \right) + O(\ell^2). \quad (16)$$

Given that  $\ell$  is the ensemble average excess return, and typically less than one over a short horizon, we can express it as  $\Delta S/S$ . Therefore, the first term in equation (16) yields

$$C \frac{\Delta u}{\Delta S} \frac{1}{u'},$$

which is approximately  $C$ , as  $u' \approx \Delta u/\Delta S$ . Hence, equation (16) becomes

$$p_+ - p_- = C + O(\ell^2) + \omega(S) \ell. \quad (17)$$

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<sup>5</sup>In our context, this "risk premium" is not to be confused with the risk premium of capital asset pricing.

<sup>6</sup>A survey of the Trades and Quotes (TAQ) database suggests quite a number of securities do not have transactions every trading day even though market makers post quotes and stand ready to trade.

At this juncture, we need to make an assumption about  $C$ .

A6 : The disparity between the expected utility of trading and that of not trading is small.

Specifically, under Assumption A6,  $C$  is of the order  $\ell^2$  or higher. We can then identify equation (17) with equation (7), which results in  $\tilde{\gamma}_0 = C + O(\ell^2)$  and

$$\tilde{\gamma}_1 = \omega(S). \quad (18)$$

In view of this result, equation (10) with  $\tilde{\gamma}_1$  being the relative risk aversion function is essentially the same as ICAPM under certain conditions enunciated by Merton (1980) and Campbell (1987). However, the context in which the relation arises is fundamentally different. The econometrics for CAPM, ICAPM and other asset pricing models necessitate a joint test of their resulting specifications and market equilibrium. As discussed in the literature, there are several major difficulties associated with the joint test, including the selection of a proxy for the market portfolio. In contrast, since our model requires neither the assumption of market equilibrium nor the existence of the market portfolio, empirical tests based on equation (10) are conceptually simpler and straightforward.

From a different angle, our model offers a fresh perspective on the myriad empirical tests of CAPM and its variants. Although market indexes are commonly used as proxies for the market portfolio, our model suggests that Roll (1977)'s critique for not using the true market portfolio can be circumvented because the assumptions concerning the portfolio of all assets and market equilibrium are not needed in our approach.

Another major difference from the CAPM and ICAPM is that  $\tilde{\gamma}_0$  need not vanish. Collectively,  $\tilde{\gamma}_0$  corresponds to other risk elements not captured by the conditional variance. In our model, the combined higher-order effect  $\tilde{\gamma}_0$  may become significant. It corresponds to the "pricing error" or anomalous return of extant asset pricing models. However, in our framework, there is nothing anomalous about  $\tilde{\gamma}_0$ . It arises whenever the higher-order contributions are not as negligible.

In view of equation (18), it is evident that the relative risk aversion  $\tilde{\gamma}_1$  is necessarily stochastic. Being a function of the price level  $S$ , the  $\tilde{\gamma}_1$  estimate based on historical data will likely be

different from the ex ante  $\tilde{\gamma}_1$ . On the other hand, if the unknown functional form of the utility function remains unchanged and if the price level is relatively stable, our model suggests that the relative risk aversion will not change much. The opposite case of changing price level and utility function, which is more likely in reality, will result in time-varying risk aversion.

In sum, we gain an economic insight that the aggregate belief of market participants plays a crucial role in determining the relation between risk and return. Although it turns out to have the same linear form as the ICAPM, our probability-based approach is parsimonious and realistic in the assumptions. There is no need to assume a particular utility function, whether the equilibrium is attained and whether rational expectations are at work. Moreover, the relation is applicable to both individual stocks as well as index portfolios. The “pricing error” of classical models is allowed in our model.

## IV. A Proxy for Market Belief

This paper is essential theoretical. However, to add an empirical flavor to the theory, this section discusses some tests of asset pricing models to motivate our simpler econometric approach that is independent of how the time-series dynamics of conditional variance is modelled. Since the concept of market belief as discussed in this paper is relatively new, the main intent is to demonstrate that there is a proxy for market belief that yields economically reasonable results.

To begin with, we note that in most tests of existing capital asset pricing models, the behavior of unobservable risk represented by  $\tilde{\nu}$  is dependent on the econometric model used. Since the seminal papers of Engle (1982) and Bollerslev (1986), GARCH models are found to be particularly appropriate for capturing the persistent behavior of conditional variance<sup>7</sup>. But when used in the tests of asset pricing models, Baillie and DeGennaro (1990), Nelson (1991) and Glosten, Jagannathan, and Runkle (1993) document that the coefficient of the linear relation between

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<sup>7</sup>The GARCH literature has grown dramatically, as can be seen from the reviews by Bollerslev, Chou, and Kroner (1992) as well as Bera and Higgins (1993). Though it is also beyond the scope of this paper to provide a thorough survey of the research involving GARCH or its variants for modelling conditional heteroskedasticity in the context of testing CAPM or ICAPM, related papers include Engle, Lilien, and Robbins (1987), Chou (1988), Campbell and Hentschel (1992), Whitelaw (1994), Theodossiou and Lee (1995), Braun, Nelson, and Sunier (1995), Chiang and Doong (2001), in addition to Chou, Engle, and Kane (1992).

conditional return and variance is null and even negative, contradicting its interpretation as the measure for relative risk aversion in ICAPM. Brandt and Kang (2002) model the conditional mean and volatility as a latent vector autoregressive process and find the relation to be negative as well.

One of the plausible explanations for these conflicting conclusions could well be that the true dynamics of the second-order conditional moment is subtle and difficult to model. In addition to volatility clustering, another regular feature of conditional variance is the asymmetry in the positive and negative price moves. The contemporaneous return and conditional volatility is *negatively* correlated. This asymmetric effect may dilute and even overwhelm the positivity of the risk aversion estimate.

Another plausible cause is that the relative risk aversion may be stochastic. To examine the stochastic nature of risk aversion, Chou, Engle, and Kane (1992) generalize the ARCH-in-mean model of Engle, Lilien, and Robbins (1987) by postulating the following system:

$$\tilde{\mu}_t = a_t + b_t \tilde{\nu}_t + e_t, \quad (19)$$

$$b_t = b_{t-1} + \epsilon_t, \quad (20)$$

$$\tilde{\nu}_t = c_0 + c_1 \eta_{t-1}^2 + c_2 \tilde{\nu}_{t-1}, \quad (21)$$

where the GARCH surprise variable is defined as  $\eta_t = \tilde{\mu}_t - E_{t-1}[\tilde{\mu}_t]$ . Equation (19) is an econometric specification for the linear relation between the conditional excess return and conditional variance derived in Section II. The errors  $e_t$  and  $\epsilon_t$  are assumed to be uncorrelated Gaussians with zero means. Their variances are  $\tilde{\nu}_t$  and  $\sigma_b^2$  respectively. The variance  $\sigma_b^2$  accounts for the stochastic nature of  $b_t$ , which is a state variable of the economy. If  $\sigma_b^2$  were small, the system would reduce to the ARCH-in-mean specification with fixed  $b_t$ . Chou, Engle, and Kane (1992) find that their  $b_t$  estimates vary significantly over the sample period from 1926 through 1987. Although at the 5% level, the point estimates for  $b_t$  range from significantly negative, weakly negative, null, weakly positive to positive over their sample period as seen from their Figure 3, they conclude that  $b_t$  is positive.

Essentially, our probability-based model provides an alternative *raison d'être* for the time-



varying GARCH-in-mean model with equation (19) being the main object of investigation. As discussed in Section III,  $b_t$  for the theoretical  $\tilde{\gamma}_1$  is free to be time-varying. However, it should be positive, as equations (17) and (18) endow an economic meaning to  $\tilde{\gamma}_1$  as the relative risk aversion coefficient.

Since we have highlighted the importance of market belief and how it enters into asset pricing, we shall focus on finding a proxy for it. Suppose on a particular stock market and in a particular month, there are  $N$  securities listed on the exchange. In the unlikely event that all the  $N$  securities have positive excess returns, the belief of market participants must have been very positive. Conversely, if the prices of these  $N$  securities all decline, the market belief must have been very negative.

In general, it is more likely that some securities will advance, some decline, and the rest remain unchanged. If more (less) securities have advanced than declined,  $B_t \equiv p_+ - p_-$  will be positive (negative). Since  $B_t$  is bounded,  $-1 \leq B_t \leq 1$ , we consider the difference between the numbers of advances and declines divided by  $N$  as an indicator of market belief at time  $t$ . Denoting the i.i.d. disturbance as  $\epsilon_t$  and the monthly excess return of the security market as  $R_t$ , we specify

$$B_t = \gamma_0 + \gamma_1 R_t + \epsilon_t \quad (22)$$

for equation (7), or equation (17). In our empirical tests documented in the next section,  $t$  is measured in the unit of a month.

Unlike the GARCH-in-mean specification, equation (22) is a linear regression. The problem of finding a correct econometric model for the conditional variance is circumvented. Though it is not a direct test of equation (10), it provides a necessary validation examination for the probability-based approach to asset pricing. Furthermore, from the viewpoint of estimation, equation (22) is much simpler compared to the GARCH-in-mean specification.

## V. Data and Estimation

This section documents a set of tests motivated by the model. First and foremost, we test whether  $\gamma_1$  is positive and significant. We are also interested in ascertaining the extent of higher-order effects encapsulated in  $\gamma_0$ . In addition, we look into the relation between relative risk aversion  $\gamma_1$  and return volatility, as intuitively one would expect risk aversion to be negatively correlated with volatility.

We obtain from the CRSP database monthly returns for each security issue traded on the NYSE. To be included in the sample, the security issue must have positive number of shares outstanding and the market capitalization must be positive. The sample has 4,449 NYSE securities in the period from January 1926 through December 2002. The total number of securities in the CRSP database increases from only 268 in January 1926 to 2,875 in October 1998. Since then, the total number declines to 2,541 securities in December 2002. Over the entire sample period, the average monthly total number is 1,240. For each month, we count the numbers of NYSE securities that register positive, zero and negative returns. These numbers are then employed to compute  $B_t$  to yield a time series of 924 observations. The distribution of these 924 observations are shown in Figure 1. As evident from the histogram, the mass of the distribution is centered around zero with a slight skew, implying that the market belief is rarely one-sided.

We conduct augmented Dickey-Fuller tests to ascertain whether the time series of monthly market belief has unit root from 1 to 50 lags. Since the absolute values of the  $t$ -statistics are at least four times larger than the 1% critical value, the test results do not support the null hypothesis of a unit root in all of the specifications with no deterministic term, a constant term, as well as a constant and a trend. Based on these results, there is reason to believe that monthly market belief is stationary over the sample period.

The monthly returns of price-weighted DJIA are collected from Reuters. For the value-weighted S&P 500, the monthly returns are extracted from the CRSP database. We use the one-month T-bill rates as the riskless returns. Figure 2 plots the time series of monthly DJIA excess returns and market belief. Visually, the two time series do not appear to have much resemblance. For instance, during the Great Depression in the 1930s, monthly excess return ex-

hibits rather high volatility whereas the corresponding period in the time series of market belief does not differ much from the other sample period. Nonetheless, market crashes and sharp rises appear to have coinciding spikes. In Table I, we provide the descriptive statistics for the excess returns of DJIA and S&P 500, along with the market belief. These three monthly time series have mild autocorrelation but strong positive cross-correlation. The autocorrelation of 0.0825 in the first lag of market belief is not inconsistent with Assumption A2, as this assumption does not insist that market belief is totally random. We also record the fact that the non-contemporaneous correlations between the excess returns of these two indexes with the market belief are insignificant.

For these two indexes, linear regression analysis based on equation (22) over the entire sample period suggests that their  $\gamma_1$  estimates are statistically significant while  $\gamma_0$  estimates are not. We have employed Newey-West's correction for serial correlation and heteroskedasticity with sufficient number of lags to compute the standard errors. The relative risk aversion coefficient denoted by  $\gamma_1$  is estimated to be 6.73 with a  $t$ -statistic of 7.56 for the DJIA, and 6.67 with a  $t$ -statistic of 6.79 using the S&P 500. The  $\gamma_0$  estimates are 0.0098 and -0.0028 respectively. Both  $\gamma_0$  estimates are, however, not statistically significant. Durbin-Watson tests do not reject the hypothesis that the residuals are uncorrelated. From the value of the adjusted  $R^2$ , we infer that 66.4% of the variation in the market belief is explained by the DJIA excess return<sup>8</sup>. The goodness-of-fit improves slightly to 68.3% when the excess return of S&P 500 is used as the regressor. In light of these results, we conclude that market participants are indeed risk averse.

Turning to the investigation of the time-varying nature of risk aversion<sup>9</sup>, we perform the same regression but using subsamples comprising of 120 monthly observations each. The subsample is rolled over by a year, yielding 68 rolling estimates for the relative risk aversion coefficient. We find that all the  $\gamma_1$  estimates are statistically significant. Figure 3 plots the DJIA's  $\gamma_1$  estimate for each subsample along with the standard deviation  $\sigma$  of subsample's monthly excess

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<sup>8</sup>Despite having only 30 stocks as components, it is intriguing that the DJIA Index and the price moves of all securities traded on the NYSE have a large commonality. When the prices of most DJIA's component firms rise (fall), many other securities' price levels tend to advance (decline) as well. The intra-day casualty between individual security price moves and changes in the DJIA's index level may be a subject of interest for further research.

<sup>9</sup>More recently, Rosenberg and Engle (2002) find that their empirical risk aversion moves in the opposite direction of business cycle. Brandt and Wang (2003) show that aggregate risk aversion varies in response to news about inflation and relates to business conditions.

return for gauging the level of return volatility. The starting year of the subsample is indicated on the horizontal axis. It appears that during the second World War and the few years following the end of the war, investors were much more risk averse. The peak occurs in the subsample spanning the period from January 1943 to December 1952. Since then, it seems that the risk aversion coefficient is on the downward trend. Interestingly, as evident in Figure 3, the relation between  $\gamma_1$  and the standard deviation  $\sigma$  of monthly excess return is negative. When the risk aversion rises, the standard deviation falls, and vice versa. The same pattern (not plotted) is also observed when the monthly excess return of S&P 500 is used instead. To quantify the extent of correlation, we consider the change in  $\gamma_1$  estimates and the change in  $\sigma$  values because by construction these time series of rolling estimates are unit-root non-stationary. Respectively, the correlation coefficients are found to be -0.73 and -0.77, which are statistically significant. On the other hand, the non-contemporaneous correlations between these first-order differenced time series are not significant.

The contemporaneous negative relation may be understood qualitatively as follows. When market participants are less risk averse, their trading tends to be speculative in nature. More speculative trades have a tendency to generate larger fluctuation. On the other hand, when investors are more risk averse, they tend to be more conservative and do not engage as much in speculative transactions. As a result, prices do not fluctuate as much. Hence, when investors are less (more) risk averse, the market will likely to be more (less) volatile. As a further remark, there was a stock market bubble from the mid-1920s till 1929 when the market crashed. Volatility was unusually high in this period. Furthermore, the large increments in  $\gamma_1$  from 1932 to 1933 and 1933 to 1934 coincide with the Great Depression and the looming second War World. The large jump in the risk aversion from 1938 to 1939 and 1940 to 1941 may also be attributed to the start of War World II and the cold war that followed. During these periods of economic uncertainty, investors might have become more risk averse and conservative in their trading strategies. Correspondingly, market volatility declined. Another observation concerns the recent decline in the risk aversion coefficient since 1990, which coincides with the beginning of the stock market bubble associated with the dotcom fever.

Table II provides further evidence of positive risk aversion coefficients. Each subsample now

spans two decades and the sample period is rolled over by two years. In line with our model’s prediction,  $\gamma_1$  estimates are positive and statistically significant while most  $\gamma_0$  estimates are usually not. Out of the 29 subsample periods, seven in the case of DJIA and eight in the case of S&P 500 have statistically significant  $\gamma_0$  estimates. They are also economically significant. In contrast to CAPM, our model does not preclude the possibility of a “pricing error” manifested by the statistically significant  $\gamma_0$  estimate. With  $\gamma_0$  being a collection of higher order terms, it is tempting to speculate that the combined effect of third-order skewness and fourth-order kurtosis might be one of the culprits behind these non-vanishing  $\gamma_0$  estimates, which could further be traced to the disparity between the expected utility of trading and that of abstaining. Nonetheless, over longer sample period as reported earlier, these pricing errors disappear.

In contrast to most existing tests, our results documented in TableII have higher statistical power. All the adjusted  $R^2$  values are larger than 64%. More importantly, the  $t$ -statistics for the  $\gamma_1$  estimates are all larger than 6 even after using Newey-West’s procedure to correct for any potential serial correlation and heteroskedasticity in the residual. Our risk aversion coefficients are in the range of 5 to 13 during our sample period. Taking into account the factor of two in equation (12), the range that corresponds to the standard definition of relative risk aversion is 10 to 26. In magnitude, this range is fairly compatible with the estimates by, for example, Ait-Sahalia and Lo (2000), Jackwerth (2000), Guo and Whitelaw (2003), as well as Bliss and Panigirtzoglou (2004), though it is larger than the estimates by Brandt and Wang (2003).

## VI. Conclusion

We show that a simple model of market belief based upon parsimonious but realistic assumptions gives rise to a linear relation between risk and return. Despite the fundamental difference in the approach, this relation is essentially the same as that of the ICAPM. Thus, it appears that the relation between risk and return is universal, being independent of the approach taken to derive it. In other words, if we consider the space of all possible relations between risk and return, the linear relation is the fixed point in this space where at least our behaviorist approach and classical finance based on utility maximization converge.

But, in contrast to extant capital asset pricing models, which apply strictly to the elusive market portfolio and under the condition of market equilibrium, our model is applicable to a wider variety of portfolios including individual stocks, regardless of whether the equilibrium is attained. Rescinding the theoretical market portfolio of classical finance provides a basis for using any widely adopted index such as the S&P 500 or the DJIA as a market benchmark. In addition, there is no need to assume a particular utility function. More importantly, our model can accommodate the “pricing error,” which is attributable to the higher-order moments of return in light of this paper’s approach.

For empirical analysis, we use the monthly returns of all securities traded on the NYSE. The sample period is from January 1926 through December 2002 and the sample size is 4,449. As a proxy for market belief, the difference between the numbers of advancing and declining securities divided by the total number of securities traded on the NYSE is used. Our empirical study provides overwhelming evidence that the risk aversion coefficient is statistically significant and positive. We also find that market participants’ risk aversion is time-varying and negatively correlated with volatility. This negative relation appears to coincide with major historical events in the 20th century. In magnitude, the estimated relative risk aversion coefficients are compatible with the estimates in the literature. All these economically reasonable results support the theory and the proxy used in this paper.

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**Table I**  
**Descriptive Statistics**

This table lists the descriptive statistics for the time series of monthly DJIA excess returns, S&P 500 excess returns and the proxy for market belief. The sample period is January 1926 through December 2002. Monthly return series of 4,449 NYSE securities are obtained from the CRSP database. These series are used to compute the monthly market belief as the ratio of the difference between the monthly numbers of advancing and declining NYSE securities over the total number of securities traded on the NYSE in that month. To compute the excess returns of the two indexes, one-month T-bill rates from the CRSP database are used. For the autocorrelation coefficients of lags 1 to 4, we indicate those estimates that are significant at the 5% level in bold.

	DJIA Excess Return	S & P 500 Excess Return	Market Belief
Mean	0.0012	0.0031	0.0179
Standard Deviation	0.0553	0.0566	0.4567
Minimum	-0.3672	-0.2998	-0.9964
Maximum	0.3054	0.4212	0.9805
Skewness	-0.7418	0.3987	-0.2117
Kurtosis	9.668	12.280	2.265
Lag 1	<b>0.0689</b>	<b>0.0800</b>	<b>0.0825</b>
Lag 2	-0.0067	-0.0241	0.0217
Lag 3	<b>-0.0852</b>	<b>-0.1071</b>	-0.0344
Lag 4	0.0338	0.0165	0.0087
DJIA	1	0.9584	0.8152
S&P 500		1	0.8265
Market Belief			1

**Table II**  
**Time-Series Regression Results**

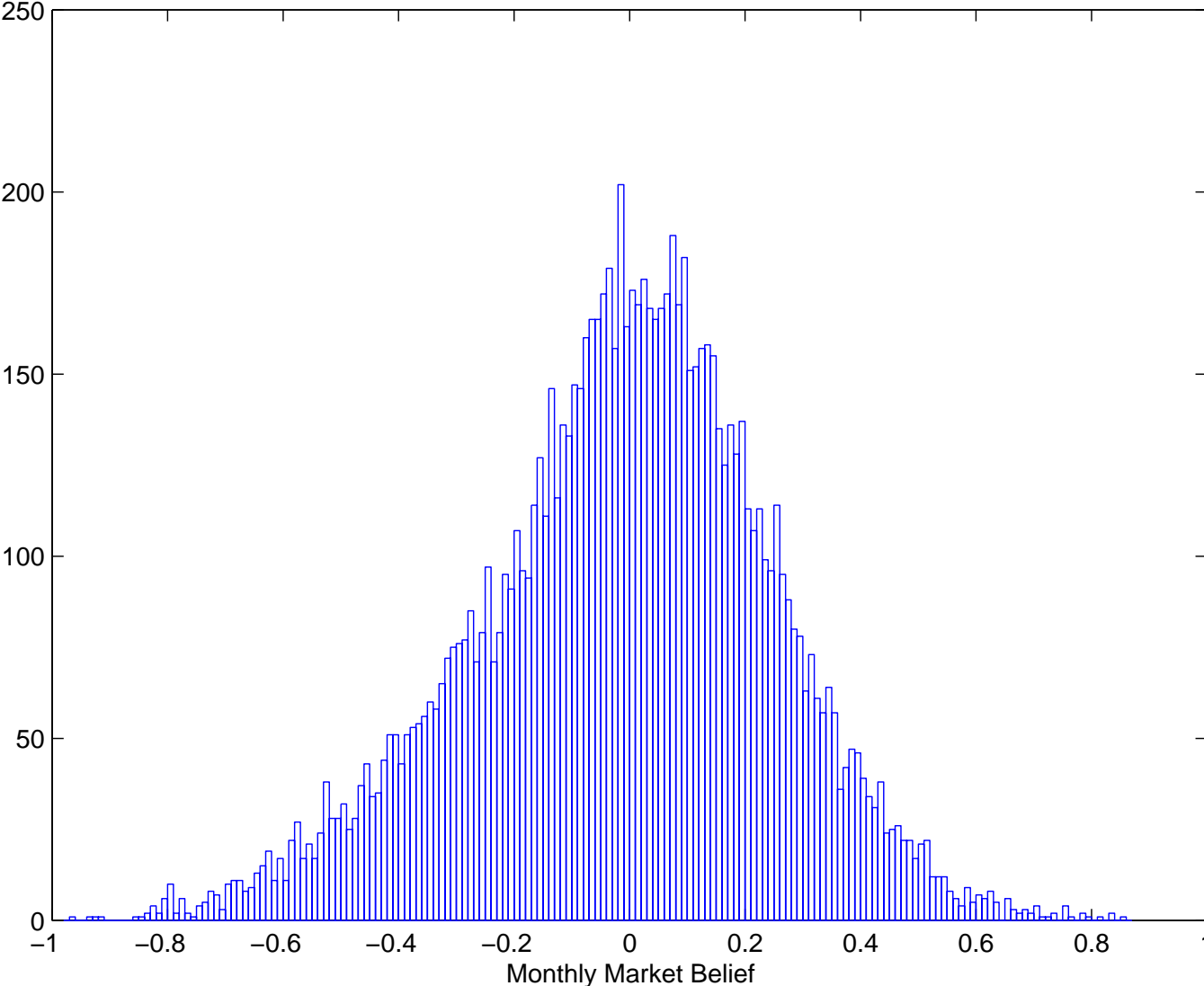
This table presents the results of regressing the monthly market belief, denoted as  $B_t$ , on the monthly excess return  $R_t$ . The one-month T-bill rate is used as the riskless rate. The ordinary least squares specification is

$$B_t = \gamma_0 + \gamma_1 R_t + \epsilon_t.$$

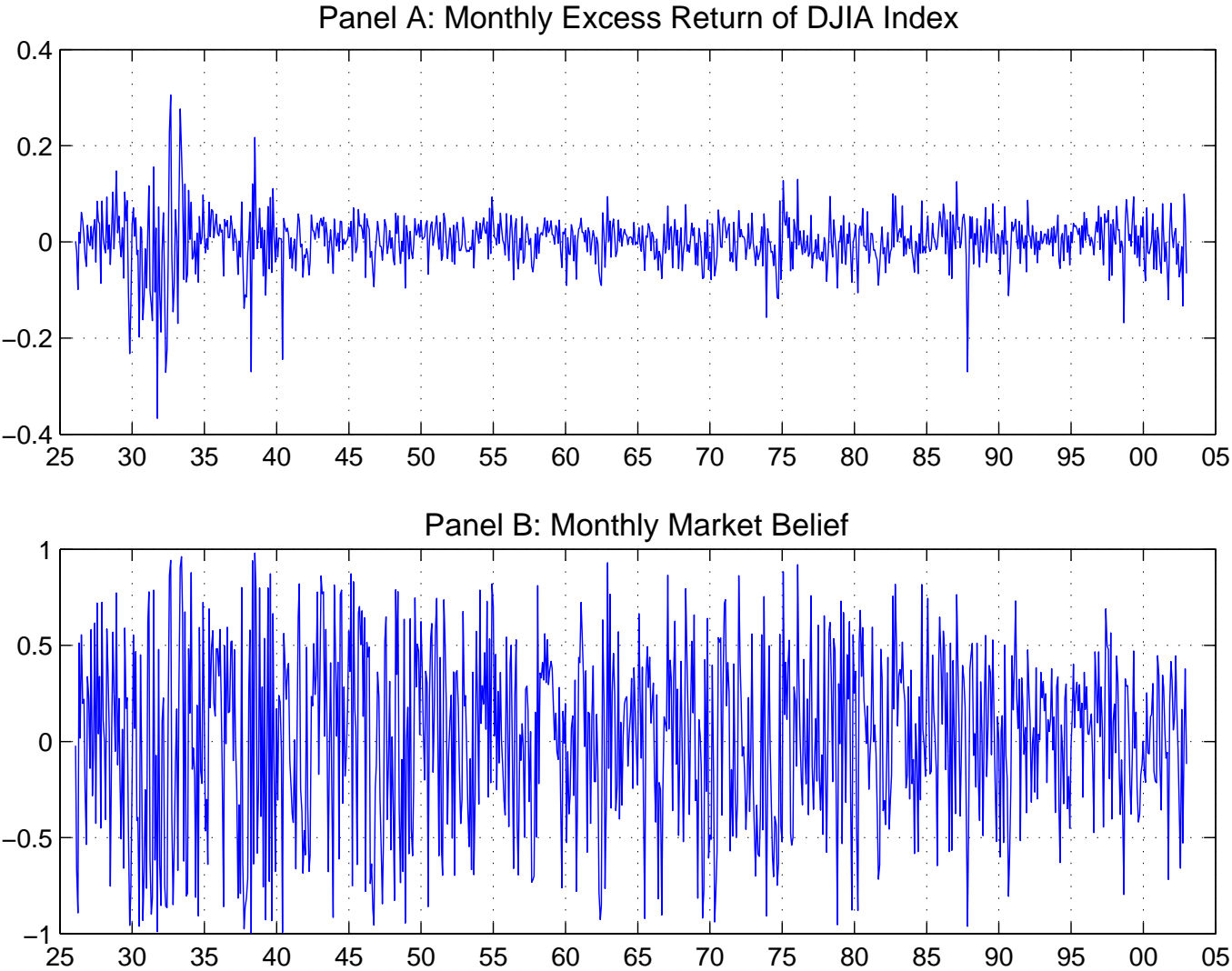
Each rolling subsample period spans 20 years. The first column states the starting years in January to the ending years in December of the subsample periods. Designated by  $\gamma_0$  and  $\gamma_1$ , the two columns contain the parameter estimates with their  $t$  statistics in the following two columns designated by  $t(\gamma_0)$  and  $t(\gamma_1)$ , respectively. The column designated by  $R^2$  tabulates the adjusted  $R^2$  in percent. Not all the  $\gamma_0$  estimates are statistically significant at the 5% level. Those that are significant are indicated in bold. In contrast, all the  $\gamma_1$  estimates are statistically significant. We have employed Newey-West's formula to correct the standard errors for serial correlation and heteroskedasticity.

Subsample Period	DJIA					S&P				
	$\gamma_0$	$\gamma_1$	$t(\gamma_0)$	$t(\gamma_1)$	$R^2$	$\gamma_0$	$\gamma_1$	$t(\gamma_0)$	$t(\gamma_1)$	$R^2$
1926–1945	0.007	5.61	0.3	11.2	68.5	-0.015	5.43	-0.7	8.2	69.5
1928–1947	-0.007	5.68	-0.3	10.7	67.2	-0.026	5.50	-1.2	8.1	69.4
1930–1949	0.000	5.97	0.0	9.3	66.1	-0.019	5.66	-0.9	7.5	68.2
1932–1951	-0.005	6.60	-0.2	7.5	64.7	-0.014	5.80	-0.7	6.3	65.0
1934–1953	-0.012	9.04	-0.6	8.2	71.7	-0.017	8.51	-1.0	10.2	77.0
1936–1955	-0.006	9.04	-0.3	7.9	70.9	-0.017	8.51	-0.9	9.6	75.0
1938–1957	-0.009	9.10	-0.4	7.5	70.5	-0.020	8.60	-1.0	8.7	74.1
1940–1959	-0.008	10.82	-0.4	8.6	74.5	-0.017	10.31	-0.9	10.1	78.3
1942–1961	-0.024	12.15	-1.8	22.8	79.3	<b>-0.039</b>	11.52	-3.2	27.0	82.9
1944–1963	<b>-0.033</b>	11.85	-2.4	24.5	80.4	<b>-0.045</b>	11.40	-3.6	27.5	83.2
1946–1965	<b>-0.035</b>	11.59	-2.7	25.0	80.5	<b>-0.044</b>	11.20	-3.5	27.4	82.6
1948–1967	-0.015	11.20	-1.1	26.0	80.0	<b>-0.032</b>	11.02	-2.5	27.5	82.2
1950–1969	-0.006	10.78	-0.4	27.4	78.6	<b>-0.028</b>	10.99	-2.2	26.7	81.6
1952–1971	-0.002	10.64	-0.2	30.4	78.5	-0.022	10.74	-1.7	28.4	82.6
1954–1973	-0.004	10.24	-0.3	22.1	77.8	-0.024	10.66	-1.7	27.1	82.1
1956–1975	-0.007	9.58	-0.4	16.3	75.6	-0.020	9.79	-1.3	12.3	78.9
1958–1977	0.010	9.40	0.6	16.5	73.6	-0.006	9.72	-0.4	12.1	77.4
1960–1979	0.022	9.36	1.3	17.4	73.7	0.000	9.75	0.0	12.8	78.2
1962–1981	0.033	9.27	1.8	18.1	73.6	0.011	9.43	0.7	13.3	76.8
1964–1983	<b>0.042</b>	9.04	2.5	18.2	72.6	0.024	9.13	1.5	13.5	75.6
1966–1985	<b>0.041</b>	9.00	2.4	19.2	73.8	0.019	9.06	1.3	14.4	77.3
1968–1987	0.032	7.92	1.8	11.5	70.7	0.018	8.26	1.1	13.2	75.9
1970–1989	0.031	7.72	1.8	11.7	70.9	0.019	8.05	1.3	13.7	76.1
1972–1991	0.031	7.48	1.8	12.4	71.9	0.018	7.78	1.2	14.4	75.8
1974–1993	<b>0.045</b>	7.37	3.0	12.1	71.7	<b>0.034</b>	7.60	2.8	14.6	76.6
1976–1995	<b>0.047</b>	7.44	3.1	10.8	71.9	<b>0.035</b>	7.96	2.9	13.8	78.3
1978–1997	<b>0.042</b>	7.27	2.8	11.0	72.2	<b>0.031</b>	7.79	2.6	14.2	78.7
1980–1999	0.016	6.67	1.0	11.6	69.2	0.007	7.07	0.5	14.7	74.4
1982–2001	0.019	6.20	1.2	12.5	68.6	0.016	6.53	1.0	14.2	71.8

**Figure 1. Histogram of Market Belief.** Market belief is defined as the aggregate view of market participants. The ratio of the difference between the monthly numbers of advancing and declining NYSE securities over the total number of securities traded on the NYSE is used as a cross-sectional proxy for the monthly market belief.



**Figure 2. Monthly Excess Return of DJIA Index and Monthly Market Belief.** The sample period is from January 1926 to December 2002, a total of 924 observations over 77 years. Monthly DJIA Index returns are gathered from Reuters and the one-month T-bill rates are obtained from the CRSP database to compute the monthly excess returns. Market belief is defined as the aggregate view of market participants. The ratio of the difference between the monthly numbers of advancing and declining NYSE securities over the total number of securities traded on the NYSE is used as a cross-sectional proxy for the monthly market belief. The horizontal axis marks the calendar year in two-digit format.



**Figure 3. Historical Risk Aversion Estimates for DJIA Index and Standard Deviations of Monthly Excess Return.** For each ten-year subsample, a risk aversion estimate is obtained from time-series regression. Rolling over the subsample period by 12 months, we obtain a total of 68 relative risk aversion estimates, which are plotted with ‘o.’ The horizontal axis marks the January of each subsample’s starting year. We also compute the annualized standard deviation of monthly excess return for each subsample. The computed series is plotted with ‘x’ and expressed in percent. These two time series appear to move in the opposite direction.

