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Mei Lin
*Singapore Management University, mlin@smu.edu.sg*

Ruhai Wu
*McMaster University*

Wen Zhou
*The University of Hong Kong*

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Platform Subsidy with Endogenous Network Effects∗

Mei Lin  Ruhai Wu  Wen Zhou†
Singapore Management University  McMaster University  The University of Hong Kong

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Abstract

This paper examines a monopoly platform’s strategy in offering subsidies based on a microfoundation of trades on the two-sided platform. A key feature is the endogenous network effects that arise from sellers’ and buyers’ entry and trading decisions. The model also captures both horizontal and vertical dimensions of product differentiation, which directly affect these decisions. The results illustrate that the platform substitutes differentiation in the horizontal dimension for that in the vertical dimension in its optimal decision. We show that it is never optimal for the platform to subsidize sellers in the absence of vertical differentiation. When products are vertically differentiated, the platform may subsidize sellers if the market is sufficiently liquid. Vertical differentiation and characteristics of a liquid market mitigate the negative same-side network effect among sellers and allow the platform to internalize higher gains to offset the subsidy. Regardless of vertical differentiation, buyer-side subsidy is possible given an illiquid market, in which seller competition is less severe. However, the degree of vertical differentiation works against the buyer-side subsidy. Our findings connect platform pricing strategies to market and user characteristics, which applies directly to managerial decisions and deepens theoretical understanding.

Keywords: Two-sided platforms, subsidy, variety, quality, network effects, vertical differentiation

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†Authors are listed in the alphabetical order: Mei Lin, mlin@smu.edu.sg, +65 6808 5284; Ruhai Wu, wuruhai@mcmaster.ca, 905-525-9140 x23048; Wen Zhou, wzhou@business.hku.hk, +852 3917 5665.
1 Introduction

Platform business models are becoming increasingly prevalent in the recent decade. They are characterized by an intermediary service offered to multiple distinct groups of users who derive value from the presence of other users in the same and/or different groups on the platform. The classic examples span a wide range including shopping malls which host stores and shoppers, credit cards which serve merchants and cardholders, newspaper or magazines that bring together advertisers and readers, and many more. Other instances of platforms enabled by information technology are also flourishing: Smartphone application markets with third-party applications for end users, computer operating systems that enable marketplaces of software for the OS users, online intermediaries such as Airbnb and HomeAway which provide listings of vacancies for those seeking lodging, and online markets that facilitate transactions between third-party sellers and buyers.

For many platform, transactions among users play a role in the platform’s decision problem. On these platforms, users’ benefits are generated based on their strategic decisions, such as sellers’ pricing choices and buyers’ purchasing choices. Their entry decisions depend on these benefits from transactions, which are then closely tied to the platform’s pricing decisions. The literature on two-sided platforms is prolific in exploring many aspects of the platform strategies; however, many studies take the interactions between platform users as given rather than as strategic choices. Even among the recent research that focuses on transactions on the platform, few studies fully endogenize the three tightly connected components: platform’s two-sided pricing problem, users’ entry decisions, and their transaction strategies.¹ Our study aims to address this gap.

The major element in this paper is the microfoundation of trades in the platform owner’s pricing problem. The microfoundation characterizes platform users’ strategic interactions and their equilibrium surplus from entering and trading on the platform. Such an approach has two powerful merits. First, it allows network effects to emerge endogenously through users’ decisions and analytically illustrates both cross-side and same-side network effects. Second, the microfoundation allows us to study how product characteristics and market conditions affect the platform owner’s pricing decisions. This novel framework connects users’ trade-level interactions with the platform’s pricing decisions. We discuss the two merits in more detail below.

Endogenizing network effects is important for cultivating deeper understanding of platform strategies. In the existing research, the positive cross-side network effect is often specified exogenously, such that each user on one side of the platform benefits from participation of users on the other side. Moreover, such specification is often linear and assumes away network effects within the same side. Whereas the general intuition driving such assumptions may stand in many examples of platforms, these assumptions set aside rich details of

¹See Section 2 for an extended discussion on related papers.
the transactions on the platform, which shape the network effects as well as interactions within each side. In an earlier debate, Liebowitz and Margolis (1994) also point out the limitations of assumptions of network effects. Although their concern lies in the concept of network *externality*, it nevertheless motivates a stronger theoretical foundation for network effects studied in the two-sided context. Hagiu (2009) is among the first to model endogenous network effects in the platform’s pricing decisions. His model features seller competition, which affects the surplus on both sides. We endogenize network effects by solving both buyers’ and sellers’ entry decisions and trading choices and explicitly characterizing the trading equilibrium. As a result, while confirming the positive cross-side network effects, we also show that they are not always linear as commonly assumed. Furthermore, the equilibrium generates a negative same-side network effect on the seller side, which is a characteristic of seller competition in Hagiu’s (2009) model.

The microfoundation also enables us to examine product variety and quality variation in the platform’s pricing problem. These dimensions of heterogeneity not only determine sellers’ pricing decisions and buyers’ purchasing decisions, they also impact the platform’s strategies. Through embedding the microfoundation in the platform’s decision problem, we tease out the economic mechanisms that connect user heterogeneity to the platform’s strategies. The managerial relevance of product variety and quality on a platform is ubiquitous. Amazon, eBay, and Taobao are known as online platforms for trading a rich diversity of products, spanning hundreds of categories and varying quality levels. Mobile application markets, such as Apple’s App Store and Google Play, have also grown to offer applications that have varying ratings and suit a multitude of purposes. In addition, some platforms have a global presence, such as Craigslist which has more than 700 local sites in 70 countries and Airbnb with an even wider coverage. While serving multiple regions, these platforms face varying quality heterogeneity as a result of economic, cultural, and political characteristics of different regions. Thus, it is inevitable for platform owners to strategize contingent on heterogeneity of products offered by their sellers.

Based on the microfoundation, we study how a monopoly platform’s two-sided pricing strategy depends on product quality heterogeneity and other market characteristics. More specifically, should the platform owner subsidize buyers or sellers? How does quality heterogeneity play into this decision? And how do other aspects of the market environment guide the platform’s strategy? We investigate these questions by analyzing the platform’s pricing problem when it collects entry fees from both sides. Endogenous entry on both sides not only depends on the entry fees but is also connected to the microfoundation that characterizes interactions between the two sides after entry. We model such interactions by introducing

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2 They raise methodological concerns regarding the assumption that “the benefits of an activity depend upon the number of participants” (p. 149) in describing network externality. The article’s main argument is that network *externality* carries more specific meanings than network *effect* and should be analyzed with more rigor.
quality heterogeneity to the circular city model to capture both horizontal and vertical product differentiation. Both cross-side and same-side network effects then emerge endogenously in sellers’ and buyers’ equilibrium trading decisions. Based on this equilibrium, the platform’s optimal strategy in offering subsidies is established. We identify the conditions that lead to subsidy on each side. We also illustrate the impact of quality heterogeneity on the optimal fees and profits.

We find that, when sellers’ products are more vertically differentiated, it is optimal for the platform to attract more sellers, which reduces horizontal differentiation. The intuition is the following. A more vertically differentiated market allows higher quality sellers to attract more buyers and charge higher prices; meanwhile, lower quality sellers lose some buyers and suffer a price cut. Overall, the gains outweigh the losses, the total profits among sellers increase with vertical differentiation. Similarly, the total surplus of buyers is also higher. Given that the platform internalizes its users’ surplus through entry fees, it gains from increased vertical differentiation. More importantly, this gain is sharpened if the platform admits more sellers, which reduces horizontal differentiation and therefore allows quality to play a more dominant role in competition. Thus, it is optimal for the platform to admit more sellers when the market is more vertically differentiated. This suggests that the two dimensions of differentiation are substitutes for the platform in equilibrium.

In terms of platform pricing, we show that it is never optimal for the platform to subsidize sellers absent vertical differentiation. When sellers have uniform quality, any additional entry by sellers necessarily intensifies seller competition and reduces each seller’s profit. Although part of such profit loss, such as that related to the price cut, would be recovered through the increased surplus of buyers, the other losses, those from the reduced market share, would not be compensated and would be absorbed by the platform. Intuitively, absence vertical differentiation, the seller-side competition is highly intense such that a marginal entry on the seller side always yields a negative net surplus for the platform, ruling out any incentive to subsidize the seller side.

With vertical differentiation, seller subsidy is possible if the market is sufficiently liquid. In the context of our model, a liquid market is characterized by several market conditions including low entry costs on both sides, a high average of product quality, buyers’ weak horizontal preferences, and a large potential buyer market. These characteristics lead to a larger network size on both sides of the platform in equilibrium. Lower entry costs and a larger potential buyer market both directly lead to more entry on both sides. Higher average quality and weaker horizontal preferences of buyers indirectly make the platform entry more attractive to both sides. As discussed previously, vertical differentiation leads to more sellers on the platform. More sellers imply reduced horizontal differentiation, which makes the effect of vertical differentiation more pronounced. The platform is then more likely to internalize additional net benefits created by vertical differentiation through admitting the marginal seller; therefore, seller-side subsidy can be optimal.
Regardless of whether vertical differentiation is present, the platform may subsidize buyers if the market is sufficiently illiquid. In a less liquid market, fewer sellers are present. The dampened competition yields higher profit gains for sellers, as additional buyers enter the platform. Given that the platform internalizes sellers’ profits, it is more inclined to offer buyer-side subsidy in an illiquid market. This finding, compared to that of the seller-side subsidy, highlights the asymmetry between the two sides. The degree of vertical differentiation expands the seller-side network size in equilibrium, which enhances market liquidity and reduces the platform’s incentive to subsidize buyers.

The remaining of the paper is organized as follows: In Section 2, we discuss the related literature. We describe the model setup in Section 3 and analyze the model in Section 4. And then, Section 5 explores the relationships of horizontal and vertical differentiation in equilibrium. Section 6 presents the platform’s optimal prices and its subsidization strategies. Finally, we conclude the paper in Section 7.

2 Related literature

One of the contributions of our paper is offering a new theory on platform subsidy. In the seminal works including Rochet and Tirole (2003), Armstrong (2006), and Parker and Van Alstyne (2005), a consistent insight is that the strength of network effect exerted by users on one side to the other side (i.e., the cross-side network effect) is a key determinant of the platform’s pricing strategies and that, by symmetric construction, the platform tends to subsidize the side that contributes a stronger network effect. Recent studies explore these network effects in more depth by incorporating same-side network effects and building microfoundations to model interactions on the platform. Hagiu (2009) endogenizes the cross-side network effects and features a negative same-side network effect among sellers. He finds that when consumers have stronger preferences for variety the platform relies more on the seller side for generating profits. Lin et al. (2011) find that the platform may subsidize buyers when their valuation for quality is more dispersed. The current paper has a different focus compared to these two papers, but all three papers show that mitigated seller competition—whether as an indirect result of a higher transportation cost \( t \) (or lower market liquidity) in the current paper, stronger consumer preferences for variety (Hagiu 2009), or more dispersed consumer valuation for quality (Lin et al. 2011)—may allow the platform to subsidize buyers or generate more revenues from the seller side. Wright (2012) re-examines the question of whether the fees set by payment card platforms are biased against retailers. He shows that because retailers account for cardholders’ benefits for using the card as compared to off-platform transaction, but not vice versa, the fee on retailers is too high as compared to the level that would maximize social welfare.

An important development in the recent literature is the focus on users interactions on the platform (Economides and Katsamakas 2006, Hagiu 2009, Belleflamme and Peitz 2010,
Casadesus-Masanell and Halaburda (2011, Lin et al. 2011, Halaburda and Piskorski 2013, Hagiu and Wright 2013 and others). Economides and Katsamakas (2006) model pricing decisions of application developers (i.e., sellers) on operating system platforms and compare industry profits and market shares of proprietary and open platforms. Casadesus-Masanell and Halaburda (2011) focus on interactions among buyers when selecting sellers’ applications and discuss the platform’s strategies in controlling the variety of applications. They show that the platform should limit such variety to facilitate coordination among buyers to select common applications for consumption complementarity. Halaburda and Piskorski (2013) consider the tradeoffs between choice and competition created by a large number of potential matches. They analyze platform users’ surplus based on a microfoundation and find that two platforms can coexist with the one limiting choices charging higher prices. Belleflamme and Peitz (2010) study the effect of intermediation on sellers’ investment incentives and highlight the role of singlehoming and the nature of investment. They find that overinvestment by sellers is possible for platforms with paid access. Gans (2012) demonstrates a unravelling problem when a seller sets the price after the platform sets the access fees. He finds that the platform cannot charge buyers a positive fee in equilibrium and has to profit from sellers through revenue sharing. Edelman and Wright (2013) explore the impact of platform price coherence (i.e., sellers are restricted to charge the same price on and off the platform) on consumer surplus and social welfare and illustrate the case of harmful intermediation.

Compared with the aforementioned papers, our paper offers at least three distinctive features: price competition, endogenous network effects, and heterogeneity of platform users. Our microfoundation characterizes price competition among sellers, which is absent in some studies (Belleflamme and Peitz 2010, Casadesus-Masanell and Halaburda 2011, Gans 2012, Halaburda and Piskorski 2013). Price competition is a natural way to capture interactions on marketplace-type platforms. It also enables us to model the two sides’ decisions in a fairly general setting. The equilibrium outcome produces surplus on each side of the platform, which connects to two sides’ entry decisions.

Our microfoundation endogenizes not only the transactions on the platform, but also the number of sellers based on their entry decisions. Many studies with endogenous entry take cross-side network effects as given (Rochet and Tirole 2003, Parker and Van Alstyne 2005, Armstrong 2006), while some other studies endogenize cross-side network effects but not entry. Both Edelman and Wright (2013) and Lin et al. (2011) model seller entry, but the number of sellers on the platform is limited to two. We allow multiple sellers to enter the platform, which enables network effects to arise from sellers’ entry and price competition. Economides and Katsamakas (2006) and Wu and Lin (2013) both consider competition among multiple sellers; however, the network size on the seller side is exogenous and fixed. In their studies, the focus is on the platform’s strategy facing the existing user base. Our interest lies in how the platform attracts users’ participation on both sides with consideration for equilibrium trades on the platform, allowing us to fully endogenize both cross-side and
same-side network effects.

Another novelty in our paper is to connect heterogeneities of sellers’ quality and buyers’ preferences to the platform’s strategies. In fact, the discussion on heterogeneity in the context of platforms is building new knowledge. Boudreau (2012) empirically shows that it is the heterogeneity of sellers that generates the variety of software they produce. Chao and Derdenger (2013) emphasize that consumer heterogeneity is a primary driver of the platform’s bundling strategies. Hagiu and Spulber (2013) focus on complementarity or substitutability between the content provided by the platform and the sellers’ products and derive insights into the platform’s strategies in its investment on the content and pricing. We highlight the impact of quality heterogeneity of sellers’ products on the platform’s strategies to subsidize users.

3 Model setup

Three types of players make decisions in this game: a platform, multiple potential sellers, and multiple potential buyers. The platform charges each seller and buyer an entry fee, denoted by \( R_s \) and \( R_b \), respectively. Sellers and buyers incur entry costs in addition to the entry fees. The total number of potential buyers is \( z \). They are heterogeneous in the entry cost \( c \) which follows a uniform distribution on [0, \( C \)]. We assume that sellers have the same entry cost, \( f \).\(^3\) Entry fees and entry costs differ in two aspects. First, entry fee is a transfer payment between seller/buyer and the platform, whereas entry cost is a deadweight loss, to the three parties combined. Second, the endogenous entry fees can be negative, which suggests a subsidy provided by the platform, whereas the exogenous entry costs are always positive.

On the platform, sellers and buyers trade products that are differentiated both horizontally and vertically. We introduce quality heterogeneity to Salop’s circle to model interactions between the two sides (Salop 1979). Horizontal differentiation is represented by sellers’ locations on the circle, where buyers’ horizontal preferences are uniformly distributed. We adopt the common assumption that each seller offers only one product (Hagiu 2009); thus, seller and product are conceptually equivalent. Vertical differentiation refers to quality heterogeneity among sellers. Each seller’s quality is uncertain before entry. Many reasons, such as the serendipitous nature of R&D and market environment, may lead to such uncertainties. Let sellers’ quality levels be independent and identically distributed with mean \( \mu \) and variance \( \sigma^2 \). To ensure that each seller on the platform has a positive market share, we assume that \( \sigma^2 \) is not too high.

\(^3\)We model heterogeneous buyer-side entry cost to ensure an interior solution of buyer entry scale. Such heterogeneity is unnecessary on the seller side because a seller’s expected profit decreases as more sellers enter the platform, which leads to an interior entry scale on the seller side even though their entry costs are identical. If sellers also differ in their entry costs, our major findings continue to hold.
Upon entry, the two sides trade with complete information. Since sellers typically need to choose product type before product development, we assume that sellers locate on the circle prior to their quality realization. Given that sellers face the identical quality distribution and have the same expected profit, they are located equidistantly on the circle upon entry, after which their quality levels are realized. Sellers compete in price, and buyers have unit demand. Buyer $j$ receives surplus $v_i - p_i - td_{ij}$ by purchasing from seller $i$ that has quality $v_i$, where $p_i$ is the price set by seller $i$, $d_{ij}$ is the distance between buyer $j$ and seller $i$, and $t$ is buyers’ unit transportation cost. The distance can be interpreted as the degree of misfit between the buyer’s horizontal preference and the seller’s product. We normalize sellers’ production cost to zero.

The game unfolds in three stages. In stage I, the platform sets $R_s$ and $R_b$. In stage II, sellers and buyers simultaneously make entry decisions by paying their respective entry fees and incurring entry costs. Upon entry, sellers are located equidistantly, and buyers are assigned uniformly on the circle. In stage III, sellers’ quality levels are realized and become public information, based on which the two sides trade.

4 Analysis

In this section, we solve the subgame-perfect Nash equilibrium by backward induction, assuming rational expectations.

4.1 Stage III: Price competition

We first analyze the equilibrium trading decisions, taking the number of sellers and buyers who enter the platform, $n_s$ and $n_b$, as given. We focus on the equilibrium in which market is fully covered, which implies that sellers are sufficiently competitive, ruling out the case of local monopolies. The competitive case more closely resembles the vibrant platform markets that motivate this study. Consider a circle of unit circumference. Without loss of generality, let the location of the $i$th seller be $\frac{i}{n_s}$ for $i = 0, 1, \ldots, n - 1$. A buyer who is located between sellers $i$ and $i + 1$ at distance $x$ from seller $i$ is indifferent between buying from either seller if $v_i - p_i - t(x - \frac{i}{n_s}) = v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n_s} - x\right)$. To ensure that all sellers obtain a positive market share, or that no buyer prefer to purchase from sellers other than the two most nearby sellers, we suppose the quality variance $\sigma^2$ does not exceed a certain level, an assumption consistent with the condition for localized competition in Alderighi and Piga (2012). The location of the marginal buyer between the two sellers is $x^*_{i,i+1} = \frac{(v_i - p_i) - (v_{i+1} - p_{i+1})}{2t} + \frac{1}{n_s} + \frac{1}{2n_s}$. Similarly, the location of the marginal buyer between sellers $i$ and $i - 1$ is $x^*_{i-1,i} = \frac{(v_{i-1} - p_{i-1}) - (v_i - p_i)}{2t} + \frac{i-1}{n_s} + \frac{1}{2n_s}$. Then seller $i$’s market share is

$$q_i = n_b\left(\frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n_s}\right).$$
Given that seller \( i \)'s revenue is \( \pi_i = q_ip_i \), the first-order condition (FOC) with respect to \( p_i \) gives,

\[
p_i = \frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{4} + \frac{t}{2ns}.
\]

From here, we have \( q_i = \frac{ns}{t}p_i \) and \( \pi_i = \frac{ns}{t}p_i^2 \).

The optimal price of seller \( i \) depends on the prices charged by its two neighboring sellers, \( i - 1 \) and \( i + 1 \). In equilibrium, the prices of all \( ns \) sellers must be solved simultaneously. Wu and Lin (2013) provides a full analysis of the equilibrium solution, based on which the equilibrium price for seller \( i \) is

\[
p_i^* = \frac{t}{ns} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j},
\]

where \( b_j = \frac{\delta^{n_s-j} + \delta^j}{\sqrt{3} (\delta^{n_s-1})} \) and \( \delta = 2 + \sqrt{3} \).

4.2 Stage II: Entry

Seller \( i \)'s expected profit from entering the platform is \( E(\pi_i) - Rs - f \), where \( E(\pi_i) \) is its expected revenue, with the expectation taken on product quality of all \( ns \) sellers. Since all sellers are ex ante identical, their expected revenues are the same: \( E(\pi_i) = E(\pi) \).

**Lemma 1** A seller’s expected revenue is:

\[
E(\pi) = \frac{n_b}{t} \left[ \frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s) \right],
\]

where \( g_s(n_s) = n_s \left( 1 - \frac{4}{3\sqrt{3}} \frac{\delta^{n_s+1}}{\delta^{n_s-1}} + \frac{2n_s}{3} \frac{\delta^{n_s}}{(\delta^{n_s-1})^2} \right) > 0 \). The following network effects emerge:

1. A positive cross-side network effect (+CSNE) from buyers to sellers

2. A negative same-side network effect (-SSNE) among sellers.

**Proof.** Available by request to authors. ■

The network effects that are commonly assumed in most related studies turn up endogenously in our model. Cross-side network effect is defined by that more participation on one side increases the surplus of those participating on the other side. Here, \( E(\pi) \) is proportional to the number of buyers on the platform, \( n_b \), which suggests a positive cross-side network effect exerted by the buyer side to the seller side; this is consistent with the common assumption. More interestingly, the equilibrium generates a negative same-side network effect; that is, a seller’s expected surplus is decreasing in the number of sellers, \( ns \). Same-side network
is rarely studied in the literature of two-sided platforms. One exception is Hagiu (2009), in which a seller’s surplus is assumed to decrease in the number of participating sellers. The difference in our model is that we derive this feature endogenously by explicitly modeling sellers’ price competition.

If buyer $j$ enters the platform, her expected surplus is $E(u) - R_b - c_j$, where $c_j$ is her entry cost, and $E(u)$ is her expected utility from trading on the platform, with the expectation taken on her location and the product quality of the seller from whom she makes the purchase.

**Lemma 2** A buyer’s expected utility from entering the platform is:

$$E(u) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s),$$

where $g_b(n_s) \equiv n_s \left( \frac{1}{6\sqrt{3} \delta^{n_s+1}} - \frac{n_s}{3^{n_s} (\delta^{n_s+1})^2} \right) > 0$. The expected utility shows a positive cross-side network effect (+CSNE) from sellers to buyers; however, the same-side network effect is absent among buyers.

**Proof.** Available by request to authors.

Network effects are also manifested in the buyer-side surplus. $E(u)$ increases with $n_s$ indicating, again, a positive cross-side network effect in the reverse direction of that in Lemma 1. The manifestation of positive cross-side network effects in both directions echoes the standard network effect assumptions. However, endogenizing network effects here shows that the marginal effect is not always constant, contrary to the assumption in many existing models. Meanwhile, $E(u)$ is independent of $n_b$, which rules out any same-side network effect among buyers. Although in some examples—such as networked video games and group-buying—a positive same-side network effect might apply, this effect is likely to only further sharpen the economic forces in the current model.

Furthermore, Lemmas 1 and 2 reveal that quality heterogeneity benefits both sides. Taking the number of buyers and sellers as given, a higher degree of quality heterogeneity mitigates sellers’ horizontal competition, which leads to a higher revenue. Buyers’ expected surplus also increases, as those purchasing from higer-quality sellers realize a gain that offsets the loss incurred by those purchasing from lower-quality sellers. Building on these observation, Sections 5 and 6 explore the effects of quality heterogeneity in more depth and from the platform’s perspective.

A seller enters the platform as long as its expected profit is non-negative. Therefore, the equilibrium number of the sellers on the platform, $n_s$, must satisfy the free-entry condition, $E(\pi) - R_s - f = 0$, or equivalently,

$$R_s = E(\pi) - f.$$
A buyer with entry cost $c_j$ enters the platform if and only if $E(u) - R_b - c_j \geq 0$, or equivalently $c_j \leq E(u) - R_b$. Given the uniform distribution of $c$, the equilibrium number of buyer entering the platform is $n_b = \frac{z}{C} (E(u) - R_b)$, or

$$n_b = \frac{z}{C} \left[ \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s) - R_b \right]. \quad (5)$$

### 4.3 Stage I: Platform pricing

The platform’s optimization problem is,

$$\max_{R_s, R_b} \Pi(R_s, R_b) = n_s R_s + n_b R_b. \quad (6)$$

By substituting Eq. (4) into the profit function, we have $\Pi = n_b \left[ R_b + \frac{R_b}{n_b} E(\pi) \right] - n_s f$. Given the one-to-one mapping between $R_s$ and $n_s$, it is equivalent to allow the platform to choose $n_s$ instead of $R_s$. After substituting Eq. (5) and the expression for $E(\pi)$, the optimization problem becomes:

$$\max_{n_s, R_b} \Pi = \frac{z}{C} \left\{ \mu - \frac{9t}{4n_s} + \frac{\sigma^2}{t} (g_b(n_s) - g_s(n_s)) \right\} \left\{ R_b + \frac{t}{n_s} + \frac{\sigma^2}{t} g_s(n_s) \right\} - n_s f. \quad (7)$$

By taking the FOC with respect to $R_b$, we have,

$$R_b = \frac{1}{2} \left[ \mu - \frac{9t}{4n_s} + \frac{\sigma^2}{t} (g_b(n_s) - g_s(n_s)) \right].$$

Given this choice, the platform solves

$$\max_{n_s} \Pi(n_s) = \frac{z}{4C} \left[ \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} g(n_s) \right]^2 - n_s f, \quad (7)$$

where $g(n_s) \equiv g_b(n_s) + g_s(n_s) = n_s \left( 1 - \frac{7}{6\sqrt{3} \beta^{n_s} + 1} + \frac{n_s}{3} \left( \delta^{n_s} - 1 \right)^2 \right)$. Suppose the second-order condition is satisfied, the following first-order condition will then define a unique, interior equilibrium solution, $n_s^*$:

$$\Pi'(n_s) = \frac{z}{2C} \left[ \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} g(n_s) \right] \left[ \frac{t}{4n_s^2} + \frac{\sigma^2}{t} g'(n_s) \right] - f = 0. \quad (8)$$

The remaining variables are expressed as functions of $n_s^*$ shown in Table 1.

---

6 As long as $\sigma^2$ is not too large, the platform’s objective function is well behaved. We can show that $g(n_s)$ is increasing and concave in $n_s$ with $g(1) = 0$. If $\sigma^2 = 0$, $\Pi(n_s) = \frac{z}{4C} \left( \mu - \frac{t}{m} \right)^2 - n_s f$ is concave in $n_s$. Therefore, when $\sigma^2$ is sufficiently small, the second-order condition can always be satisfied.
Table 1: Endogenous variables as functions of $n_s$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sellers $n^*_s$</td>
<td>$n^<em>_s = \frac{z}{2c} \left( \mu - \frac{t}{4n^</em>_s} + \frac{\sigma^2}{T} g(n^*_s) \right)$</td>
</tr>
<tr>
<td>Number of buyers $n^*_b$</td>
<td>$n^<em>_b = \frac{z}{2c n^</em>_s} \left( \frac{1}{n^<em>_s} + \frac{\sigma^2}{T} g_s(n^</em>_s) \right) \left( \mu - \frac{t}{4n^<em>_s} + \frac{\sigma^2}{T} g(n^</em>_s) \right) - f$</td>
</tr>
<tr>
<td>Seller-side fee $R^*_s$</td>
<td>$R^<em>_s = \frac{z}{2cn^</em>_s} \left( \frac{1}{n^<em>_s} + \frac{\sigma^2}{T} g_s(n^</em>_s) \right) \left( \mu - \frac{t}{4n^<em>_s} + \frac{\sigma^2}{T} g(n^</em>_s) \right) - f$</td>
</tr>
<tr>
<td>Buyer-side fee $R^*_b$</td>
<td>$R^<em>_b = \frac{1}{2} \left( \mu - \frac{9t}{4n^</em>_s} + \frac{\sigma^2}{T} (g_b(n^<em>_s) - g_s(n^</em>_s)) \right)$</td>
</tr>
<tr>
<td>Platform’s profit $\Pi^*$</td>
<td>$\Pi^* = \frac{z}{4c} \left( \mu - \frac{t}{4n^<em>_s} + \frac{\sigma^2}{T} g(n^</em>_s) \right)^2 - n^*_s f$</td>
</tr>
</tbody>
</table>

5 The two dimensions of differentiation

This model simultaneously captures vertical and horizontal differentiation. The degree of vertical differentiation is represented by the variance of quality, $\sigma^2$; the degree of horizontal differentiation is defined by the equilibrium number of sellers on the platform, $n^*_s$. A higher $n^*_s$ implies a shorter distance between neighboring sellers, or a lower degree of horizontal differentiation. Furthermore, the degree of vertical differentiation is exogenously determined, whereas the degree of horizontal differentiation is endogenously chosen by the platform. The following proposition depicts the relationship between the two dimensions of differentiation in equilibrium:

**Proposition 1** $\frac{dn^*_s}{d\sigma^2} > 0$, that is, when sellers differ more vertically, in equilibrium the platform admits more sellers, which reduces horizontal differentiation.

**Proof.** The first-order condition to the optimization problem (7) can be written as $\Pi'(n^*_s(\sigma^2), \sigma^2) \equiv 0$. Take total differentiation with respect to $\sigma^2$ to get $\frac{\partial^2 \Pi}{\partial n^*_s^2} \frac{dn^*_s}{d\sigma^2} + \frac{\partial^2 \Pi}{\partial n^*_s \partial \sigma^2} = 0$. By the second-order condition, $\frac{\partial^2 \Pi}{\partial n^*_s \partial \sigma^2} = \Pi''(n^*_s) < 0$. Then $\frac{dn^*_s}{d\sigma^2}$ has the same sign as $\frac{\partial^2 \Pi}{\partial n^*_s \partial \sigma^2}$. Given the expression of $\Pi'(n^*_s)$ as in (8), we have $\frac{\partial^2 \Pi}{\partial n^*_s \partial \sigma^2} > 0$ as $g(n^*_s) > 0$ and $g'(n^*_s) > 0$. As a result, $\frac{dn^*_s}{d\sigma^2} > 0$. □

We explain Proposition 1 in the following two steps. First, quality heterogeneity among sellers exerts a positive impact on both sellers’ profits and buyers’ surplus. Second, this positive impact raises the marginal benefit of seller entry, which the platform internalizes.
Thus, as the degree of vertical differentiation increases, it is optimal for the platform to tradeoff some horizontal differentiation by admitting more sellers. We further elaborate the two points below.

First consider a seller’s expected profit given an increase in quality variance through a mean-preserving spread.\(^7\) The quality levels of above-average sellers increase with the spread, leading to higher prices for these sellers. Meanwhile, such quality improvements attract more buyers despite the price increase, so above-average sellers’ market shares also expand. Thus, above-average sellers gain from both higher prices and larger market shares.\(^8\)

On the other hand, below-average sellers’ quality levels deteriorate in the mean-preserving spread. In equilibrium, their prices are lower, market shares shrink, and profits are lost. Since the number of transactions made at a lower price is reduced, while those made at a higher price is expanded, the losses of below-average sellers are then outweighed by the gains among the above-average sellers. Therefore, on average, a seller’s expected profit increases with quality variance (i.e., \(g_s(n_s) > 0\)). In brief, a higher quality heterogeneity allows the higher-quality sellers to benefit at the expense of lower-quality sellers; the former dominates the latter because the higher-quality sellers hold an advantage in market shares. As a result, the expected net effect of a higher quality variance is positive for sellers.

A higher quality variance also alters some buyers’ choices and leads to a higher expected surplus for buyers (i.e., \(g_b(n_s) > 0\)). A buyer purchasing from an above-average seller realizes a higher surplus in the mean-preserving spread from the improved quality, despite the price increase. Similarly, a buyer purchasing from a below-average seller suffers a loss in surplus as a result of the spread, despite the price cut. The mean-preserving spread then may induce some of these buyers to switch to another seller to mitigate their losses. Following the seller-side intuition, given that the above-average sellers hold larger market shares to begin with, more buyers realize a gain; furthermore, even some of the buyers who suffer a loss have the opportunity to reduce the loss by switching. Therefore, on average, a buyer’s expected surplus is higher as quality variance among sellers increases.

Let us now consider the second force: Admitting more sellers intensifies the positive impact of quality variance on both sides (i.e., \(g'_s(n_s) > 0\) and \(g'_b(n_s) > 0\), as a result of buyers’ equilibrium purchasing decisions. As the number of sellers grows, sellers are located more densely, and the degree of horizontal differentiation is reduced. Seller quality then becomes a more important factor in buyers’ purchasing decisions. Thus, a buyer is less captive to a particular seller and is more inclined to switch to a different seller facing a higher quality variance. This sharpens the shifts in sellers’ market shares and, in turn, magnifies the effects on their profits that are internalized by the platform. Thus,

\(^7\)We use mean-preserving spread for the convenience of illustration. The intuition generally applies for other changes in distribution as quality variance increases while keeping the mean constant.

\(^8\)Both effects are clear from the equilibrium solution (e.g., Eq. (1)). These effects are examined in more detail in Wu and Lin (2013).
admitting an additional seller generates higher marginal benefit for the platform as quality variance increases. It is then optimal for the platform to reduce horizontal differentiation by admitting more sellers.

Proposition 1 suggests that vertical and horizontal differentiations are substitutes in equilibrium. Understanding quality heterogeneity on the seller side helps the platform owner to more efficiently plan for its capacity, such as space allocation for a shopping mall, technical resource scaling for serving end users, IT staffing for managing application developers, and so on. Furthermore, the relationship between the two dimensions of differentiation guides the platform’s decision in managing sellers’ entry scale. In some cases, platform owners exercise limited or no control over the degree of quality heterogeneity on the seller side. For example, as Airbnb expands to serve over 33,000 cities in 192 countries, differences in income distribution, culture, and government regulations across these regions may create variations in quality of hosts. Hosts in certain regions might be more consistent in terms of lodging space quality and hospitality than those in other regions. Our finding implies that the platform owner may be better off tailoring the network size of its hosts in different regions to the degree of quality variation based on regional characteristics. In other cases, the platform has access to various instruments to influence the outcomes of sellers’ quality. Smartphone platform owners such as Apple and Google employ various contests and even directly screen applications to craft the distribution of application qualities. Our finding then offers insights into how platforms might consider balancing the number of applications while controlling for quality heterogeneity.

6 Platform fees

In this section, we turn to the platform’s pricing strategies. In particular, we look for the conditions under which the platform subsidizes the entry of either side. We proceed by first characterizing the pricing strategy absent vertical differentiation (i.e., $\sigma^2 = 0$); and then, we study how a positive $\sigma^2$ affects the platform’s subsidization strategies.

6.1 No vertical differentiation

When $\sigma^2 = 0$, the equilibrium expressions are greatly simplified. The FOC, Eq. (8) is reduced to the following:

$$\Pi'(n_s) = \frac{zt}{8Cn_s^2} \left( \mu - \frac{t}{4n_s} \right) - f = 0. \quad (9)$$

Proposition 2 Without vertical differentiation, sellers are never subsidized; buyers are subsidized ($R_b^* < 0$) if and only if

$$\frac{Ctf}{z} > \frac{16}{729} \mu^3. \quad (10)$$
Therefore, the platform is more likely to subsidize buyers if there are fewer potential buyers \((z \text{ is small})\), the product quality is low \((\mu \text{ is small})\), buyers’ or sellers’ entry is more costly \((C \text{ or } f \text{ is large})\), or buyers have strong horizontal preferences \((t \text{ is large})\).

**Proof.** The FOC (9) implies that \(\frac{zt}{C(n_s)^2} \left( \frac{1}{4n_s^4} \right) = 8f\), so the seller-side fee (from Table 1) \(R_s^* = \frac{zt}{2C(n_s^2)^2} \left( \frac{1}{4n_s^4} - f \right) = 3f > 0\). For the buyer-side fee, \(R_b^* = \frac{1}{2} \left( \frac{1}{4n_s^4} - \frac{9}{4f} \right) < 0\) if and only if \(n_s^* < \frac{9}{4f}\). Because \(\Pi''(n_s) < 0\), \(n_s^* < \frac{9}{4f}\) if and only if \(0 = \Pi'(n_s^*) > \Pi'(\frac{9}{4f}) = \frac{16}{729} \varepsilon n_s^2 - f\), which leads to the condition stated in the proposition.

In this base case without quality heterogeneity, the platform’s pricing strategies for the two sides are in sharp contrast. Whereas it can be optimal for the platform to pay for buyers’ participation, such subsidy is never optimal on the seller side. We can explore the driving forces of subsidy by examining the benefit of admitting the marginal seller or buyer. To more clearly illustrate the intuition, let us re-consider the mathematical formulation of the platform’s optimization problem, Eq. (6). We now let the platform choose \(n_b\) and \(n_s\) simultaneously, instead of the previous approach of choosing \(R_b\) and \(n_s\) sequentially in Section 4.3. Note that the two approaches are mathematically equivalent. Given \(n_b\) and \(n_s\) in the objective function of Eq. (6), the two sides’ entry conditions then determine their respective entry fees: \(R_s = E(\pi) - f = \frac{t}{n^2} n_b - f\), and \(R_b = E(u) - C z n_b = \mu - \frac{9}{4n_s^4} - C z n_b\). The FOCs with respect to \(n_s\) and \(n_b\) are listed below.

\[
\begin{align*}
\frac{\partial \Pi}{\partial n_s} &= n_b \frac{\partial R_s}{\partial n_s} + n_s \frac{\partial R_b}{\partial n_s} + R_s^* = 0, \quad (11) \\
\frac{\partial \Pi}{\partial n_b} &= n_b \frac{\partial R_b}{\partial n_b} + n_s \frac{\partial R_s}{\partial n_b} + R_b^* = 0. \quad (12)
\end{align*}
\]

Let us first consider the FOC for subsidizing the seller side (Eq. (11)); the rationale for the buyer-side subsidy follows the same mechanism. The optimality of the seller-side subsidy (i.e., \(R_s^* < 0\)) requires that, holding the number of buyers fixed, the marginal seller admitted creates a positive net benefit for the platform (i.e., \(n_s \frac{\partial R_s}{\partial n_s} + n_b \frac{\partial R_b}{\partial n_s} > 0\)) excluding the subsidy offered to the marginal seller; mathematically, the FOC must hold. The platform’s revenues from the fees\(^{10}\) collected on the two sides shift as the additional seller enters. The platform must reduce the seller-side fee to admit an additional seller, which incurs a loss on the total fee collected from the existing sellers (i.e., \(n_s \frac{\partial R_s}{\partial n_s} < 0\)) — we call this the same-side loss (SSL). Meanwhile, having more sellers exerts a positive network effect on the buyer side, allowing the platform to charge a higher buyer-side fee and realize a gain, that is \(n_b \frac{\partial R_b}{\partial n_b} > 0\), which we refer to as the cross-side gain (CSG). The loss on the seller side, SSL, and the

\(^{9}\)Notice that the numerical value in Condition (10), \(\frac{16}{729}\), comes from a general feature of the circular city model. It is not an outcome of any specific assumptions.

\(^{10}\)We use the term *fee* in the general sense to include subsidy, which is a negative “fee.”
gain on the buyer side, CSG, sum up to the net benefit the platform obtains from the base networks of buyers and sellers. A positive net benefit implies that subsidy is optimal on the side where entry occurs. Table 2 lists the SSL and CSG for an additional entry on either side.

Table 2: Breakdown of the Net Benefit from an Additional Entry (No Vertical Differentiation)

<table>
<thead>
<tr>
<th>Side of Entry</th>
<th>Same-Side Loss</th>
<th>Cross-Side Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller side</td>
<td>$n_s \frac{\partial R_s}{\partial n_s} = -\frac{2n_s}{n_s^2}$</td>
<td>$n_b \frac{\partial R_b}{\partial n_b} = \frac{5n_b}{4n_s^2}$</td>
</tr>
<tr>
<td>Buyer side</td>
<td>$n_b \frac{\partial R_b}{\partial n_b} = -\frac{Cn_b}{z}$</td>
<td>$n_s \frac{\partial R_s}{\partial n_s} = \frac{1}{n_s}$</td>
</tr>
</tbody>
</table>

On the seller side, admitting an additional seller does not yield any positive net benefit for the platform. To see this, suppose the platform admits a new seller while keeping the buyer-side network unchanged, fees on both sides must then be adjusted to satisfy their respective free entry conditions. Without vertical differentiation, an increase in the number of sellers necessarily reduces the equilibrium price as a result of intensified competition; this loss is recovered on the buyer side because buyers pay lower prices and obtain higher surplus. Each seller suffers a second loss: Its market share shrinks to accommodate for the new seller. Only part of this loss is recovered on the buyer side: Buyers’ horizontal preferences are better served, hence transportation costs are reduced. However, the remaining part of sellers’ loss is a pure loss. As the platform internalizes shifts in surplus on the two sides through optimizing entry fees, it incurs a net loss from admitting the new seller. Therefore, it is never an optimal strategy for the platform to subsidize seller entry.

The negative same-side effect that applies to sellers is absent on the buyer side (Lemmas 1 and 2). When the platform expands the buyer-side network, buyers’ surplus is not affected. On the other hand, sellers’ demand increases, which leads to higher profits. The platform internalizes such benefits through the entry fees. Under the condition specified in Proposition 2, the marginal buyer that the platform admits generates a positive net benefit, rendering subsidizing the buyer side an optimal strategy.

The condition that governs buyer-side subsidy is satisfied when entry costs on both sides and buyers’ transportation cost are high, the potential buyer market is small, and/or the expected quality of sellers is low. Higher costs incurred by buyers and a lower expected

---

Note that this is not affected when $\sigma^2 = 0$. Sellers’ expected profit becomes $E(\pi) = \frac{1}{n_s^2} n_s$, which decreases with the number of sellers $n_s$, whereas buyers’ expected surplus becomes $E(u) = \mu - \frac{5\tau}{4n_s^2}$, which is independent of $n_b$. 

16
quality both reduce buyers’ surplus and discourage their participation on the platform. Given a smaller number of buyers, the platform’s loss on the buyer side from offering subsidies is mitigated; as shown in Table 2, the magnitude of SSL for the additional buyer-side entry is lower as \( n_b \) decreases. Meanwhile, a higher entry cost for sellers and a smaller potential buyer market imply a lower expected profit for sellers, which reduces seller-side participation. With fewer sellers present, the platform internalizes a higher CSG on the seller side from the marginal buyer; the reason is that having more sellers would otherwise intensify competition, which dissipates the value created by the buyer side (in Table 2, \( \frac{1}{n_s} \) decreases in \( n_s \)). Briefly, the marginal loss from admitting an additional buyer is mitigated by a smaller buyer-side network, and the gain is augmented by a smaller seller-side network. Therefore, for smaller networks on both sides, or a more illiquid market, the platform obtains a higher marginal net benefit from admitting the additional buyers, which is conducive to buyer-side subsidy.

### 6.2 Vertical differentiation

We now consider the general case with vertical differentiation among sellers. Given \( \sigma^2 > 0 \), \( g_s(n_s) \) and \( g_b(n_s) \) take more complex forms. For analytical tractability, we use approximation to simplify these functions while still capture the main forces in the model. When \( n_s \) is not too small, \( \frac{\sigma^2}{n_s} \) is easily well above 10. Thus, approximating \( \frac{n_s \sigma^2}{(\sigma^2 - 1)^2} \) at 0, which requires \( n_s \) to be sufficiently high, is consistent with the actual equilibrium. For the remaining analysis, let \( \delta_1 = 1 - \frac{7}{6\sqrt{3}} \), \( \delta_2 = 1 - \frac{4}{3\sqrt{3}} \), and \( \delta_3 = 1 - \frac{\sqrt{3}}{2} = \frac{1}{23} \). The platform’s optimization problem (7) is then simplified to:

\[
\max_{n_s} \Pi(n_s) = \frac{z}{4C} \left( \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} \delta_1 n_s \right)^2 - n_s f,
\]

Our numerical study without applying approximation shows that the equilibrium \( n_s \) is easily well above 10. Thus, approximating \( \frac{\sigma^2}{(\sigma^2 - 1)^2} \) at 0 and \( \frac{n_s \sigma^2}{(\sigma^2 - 1)^2} \) at 0, which requires \( n_s \) to be sufficiently high, is consistent with the actual equilibrium. For the remaining analysis, let \( \delta_1 = 1 - \frac{7}{6\sqrt{3}} \), \( \delta_2 = 1 - \frac{4}{3\sqrt{3}} \), and \( \delta_3 = 1 - \frac{\sqrt{3}}{2} = \frac{1}{23} \). The platform’s optimization problem (7) is then simplified to:

\[
\max_{n_s} \Pi(n_s) = \frac{z}{4C} \left( \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} \delta_1 n_s \right)^2 - n_s f,
\]

Table 3: Numerical Examples

<table>
<thead>
<tr>
<th>( n_s )</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\sigma^2}{n_s} )</td>
<td>1.00277</td>
<td>1.00074</td>
<td>1.00020</td>
<td>1.00005</td>
<td>1.00001</td>
<td>1.00000</td>
<td>1.00000</td>
</tr>
<tr>
<td>( \frac{n_s \sigma^2}{(\sigma^2 - 1)^2} )</td>
<td>0.00693</td>
<td>0.00222</td>
<td>0.00069</td>
<td>0.00021</td>
<td>0.00006</td>
<td>0.00002</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
for which the FOC is,
\[
\Pi'(n_s) = \frac{z}{2C} \left( \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} \delta_1 n_s \right) \left( \frac{t}{4n_s^2} + \frac{\sigma^2}{t} \delta_1 \right) - f = 0. \tag{14}
\]
Note that \( \Pi''(n_s) = \frac{z}{2C} \frac{3t^4 + 16t^2 \delta_4 n_s^2 - 8 \mu t n_s}{16t^2 n_s^4} \). For any given \( n_s \), we can have \( \Pi''(n_s) < 0 \) as long as \( \mu \) is sufficiently large and/or \( \sigma^2 \) is sufficiently small. As long as all feasible \( n_s \) are finite, the second-order condition is satisfied, and the FOC is both necessary and sufficient to define a unique solution of \( n_s^* \). The endogenous variables are expressed as functions of \( n_s \) in the table below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sellers</td>
<td>( n_s^* )</td>
</tr>
<tr>
<td>Number of buyers</td>
<td>( n_b^* = \frac{2C}{z} \left( \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} \delta_1 n_s \right) )</td>
</tr>
<tr>
<td>Seller-side fee</td>
<td>( R_s^* = \frac{z}{2C} \left( \frac{t}{n_s^2} + \frac{\sigma^2}{t} \delta_2 \right) \left( \frac{t}{4n_s^2} + \frac{\sigma^2}{t} \delta_1 \right) - f )</td>
</tr>
<tr>
<td>Buyer-side fee</td>
<td>( R_b^* = \frac{1}{2} \left( \mu - \frac{9t}{4n_s} - \frac{\sigma^2}{t} \delta_3 n_s \right) )</td>
</tr>
<tr>
<td>Platform’s profit</td>
<td>( \Pi^* = \frac{z}{4C} \left( \mu - \frac{t}{4n_s} + \frac{\sigma^2}{t} \delta_1 n_s \right)^2 - n_s f )</td>
</tr>
</tbody>
</table>

Let \( \delta_5 \equiv \frac{1}{2} - \frac{5}{27} \sqrt{3} > 0 \) and \( \delta_6 \equiv \frac{\sqrt{3}}{324} (353 - 189 \sqrt{3}) > 0 \):

**Proposition 3** When sellers are vertically differentiated, they are subsidized \( (R_s^* < 0) \) if and only if
\[
\frac{Ctf}{z} < \sigma^2(\delta_5 \mu + \delta_6 \sigma). \tag{15}
\]

Therefore, the platform is more likely to subsidize sellers when entry on either side is less costly \( (C \ or \ f \ is \ lower) \), horizontal preferences are weaker \( (t \ is \ smaller) \), more potential buyers are present \( (z \ is \ larger) \), the expected quality is higher \( (\mu \ is \ larger) \) or sellers differ more in quality \( (\sigma^2 \ is \ large) \).

**Proof.** From the first-order condition (14), at the optimal \( n_s^* \), \( f = n_b^* \left( \frac{t}{4n_s^2} + \frac{\sigma^2}{t} \delta_1 \right) \). Plug this into the expression of \( R_s^* \) to get \( R_s^* = n_s^* \left( \left( \frac{t}{n_s^2} + \frac{\sigma^2}{t} \delta_2 \right) - \left( \frac{t}{4n_s^2} + \frac{\sigma^2}{t} \delta_1 \right) \right) \). Then \( R_s^* < 0 \).
if and only if \( n^* s > \frac{\delta t}{\sigma} \), where \( \delta = \frac{\sqrt{243}}{4} \). Because \( \Pi''(n_s) < 0 \), \( n^* s > \frac{\delta t}{\sigma} \) if and only if \( 0 = \Pi'(n^*_s) < \Pi'(\frac{\delta t}{\sigma}) \), which leads to (15).

Proposition 3 suggests that, when sellers are vertically differentiated, the platform may obtain a positive net benefit from admitting the marginal seller, which contrasts with the base case (without vertical differentiation) where the platform always incurs a loss from the entry of the marginal seller. Recall that the net benefit is the sum of the same-side loss (SSL) and the cross-side gain (CSG). With vertical differentiation, the SSL from the additional seller-side entry is unchanged, under approximation. Even without approximation, the effect that arises from vertical differentiation, in the new term with \( \sigma^2 \), is only minuscule. The reason for this negligible impact of quality heterogeneity on the SSL is related to the ripple effect in sellers’ competition. Sellers’ equilibrium price (Eq. (1)) shows that the effect of a seller’s quality on other sellers is the strongest for the immediate neighbors, and such effect diminishes like ripples as it traverses to the neighbors’ neighbors, and so on.\(^\text{12}\) In fact, the magnitude of the effect is reduced to nearly zero at the fourth-degree neighbor on either side of a seller; in other words, the effect of quality is concentrated on only six nearest neighboring sellers. Therefore, admitting an additional seller, given that \( n_b \) is larger than six, does not generate significant impact that is new with the presence of quality heterogeneity.

On the other hand, the CSG from admitting the additional seller, generated on the buyer side, is altered when sellers are vertically differentiated. The CSG is now \( \frac{5tn^*_b}{4n^*_s} + \frac{\sigma^2}{6\sqrt{\delta t}} \), in which the second term is the additional gain from quality heterogeneity. The degree of quality heterogeneity sharpens the CSG from the additional seller-side entry, because buyers are more likely to switch to a higher-quality seller after a new seller enters. As sellers locate closer together after the new entry, buyers’ horizontal matching is improved; thus, quality becomes a more important factor, which creates opportunity for some buyers to switch to a higher quality seller. A higher variance of quality implies that higher-quality sellers’ quality levels are further enhanced; thus, buyers are more likely to change purchasing decisions, which leads to a greater increase in their expected surplus and a higher CSG for the platform. The unchanged SSL and increased CSG lead to an overall higher net benefit for the platform; therefore, the presence of vertical differentiation creates a possibility for seller-side subsidy.

The condition for seller-side subsidy includes less costly entry on either side, weaker horizontal preferences for buyers, more potential buyers, and a higher average quality, which jointly describe a highly liquid platform market with a larger network size on both sides. Given a large number of sellers, the platform’s CSG from admitting an additional seller is further amplified. In such a crowded market, sellers located more narrowly, which makes the effect of quality more pronounced. Thus, buyers are less captive to any particular seller due to its location advantage, because horizontal preferences are well served in this crowded

\(^\text{12}\)For a detailed discussion on the ripple effect see Wu and Lin (2013).
market; instead, buyers are more inclined to switch to a seller who offers a higher quality. In brief, the value created by sellers’ quality heterogeneity is enhanced by the intensity of their horizontal competition, allowing the platform to derive a higher net benefit from the marginal seller and provide subsidy to the seller side.

We now turn to the buyer-side subsidy. Let 
\[
\delta_7 = \frac{4(35+12\sqrt{3})}{729}, \quad \delta_8 = \frac{28+\sqrt{3}}{26}, \quad \text{and} \quad \delta_9 = \frac{29+4\sqrt{3}}{61},
\]
and we have the following result:

**Proposition 4** When sellers are vertically differentiated, the platform subsidizes the buyer side \(R_b^* < 0\) if and only if
\[
\frac{Ctf_z}{\bar{z}} > \delta_7 (\mu + \theta) (\mu - \delta_8 \theta)(\mu - \delta_9 \theta),
\]
(16)

where \(\theta = \sqrt{\mu^2 - 9\delta_3 \sigma^2}\).

**Proof.** Given the expression of \(R_b^*(n_s)\), \(R_b^* < 0\) if and only if \(n_s < \frac{q}{2(\mu + \theta)} \equiv \bar{n}\), where \(\theta = \sqrt{\mu^2 - 9\delta_3 \sigma^2}\). Because \(\Pi''(n_s) < 0\), \(n_s^* < \bar{n}\) if and only if \(0 = \Pi'(n_s^*) > \Pi'(\bar{n})\), which is reduced to (16). It can be shown that the right hand side is increasing in both \(\mu\) and \(\sigma^2\), so the condition is less likely to be satisfied when \(\sigma^2\) is larger.

Proposition 4 specifies the condition for the buyer-side subsidy when sellers are vertically differentiated. Note that Condition (16) at \(\sigma^2 = 0\) is identical to Condition (10) from Proposition 2. Also note that the right hand side of Condition (16) is increasing in both \(\mu\) and \(\sigma^2\), meaning that the condition is less likely to be satisfied when \(\sigma^2\) is higher. Other parameters have same effect on the condition as those in Proposition 2.

The qualitative interpretation for Proposition 4 is consistent with that in the base case (Proposition 2, Condition (10)): With quality heterogeneity among sellers, an illiquid platform market is still more conducive to buyer-side subsidy. However, the degree of quality heterogeneity tightens Condition (16), working against the buyer-side subsidy. Based on Proposition 1, a higher degree of quality heterogeneity leads to a larger seller-side network size in equilibrium. The buyer-side network size also becomes larger as a result of the positive cross-side network effect exerted by the expanded seller side. As the platform reduces the buyer-side fee to admit the additional buyer, the SSL increases with the buyer-side network. Furthermore, given a larger seller-side network, the CSG from admitting the additional buyer is reduced due to the intensity of seller competition. Thus, at a higher quality heterogeneity, the net benefit the platform obtains is lower, making the buyer-side subsidy less likely.

The driving forces for seller-side and buyer-side subsidies, as illustrated in their respective conditions, are clearly reversed. Whereas characteristics of a liquid platform market more likely leads to a subsidy for sellers, the buyer-side subsidy relies on a less liquid platform market; quality heterogeneity works favorably for the seller-side subsidy, while constraining the condition for the buyer-side subsidy. Such asymmetry is curious because existing
discussions related to two-sided pricing are often generalizable to either side. For example, a well-known insight from earlier works (Parker and Van Alstyne 2005, Armstrong 2006) is that the platform is more inclined to subsidize the side that exerts a stronger network effect onto the other side. Without contradicting previous findings, we identify the stark asymmetry by fully characterizing the trades between the two sides. By allowing network effects to emerge from strategic interactions endogenously, we show that such asymmetry arises from the negative same-side network effect that is unique to the seller side because of competition. On the other hand, the buyers often do not exhibit such rivalry among themselves. Therefore, the two sides create different dynamics in the net benefit the platform internalizes from admitting the marginal seller/buyer as detailed in our findings.

In practice, platforms on which seller and buyer sides trade are ubiquitous, including online and offline marketplaces of a wide variety of goods or services, software platforms, gaming platforms, and many more. Our findings provide a new angle of explaining subsidies on these platforms. Buyer-side subsidy is commonly observed, especially when the networks are not large. For example, Microsoft subsidized Xbox by offering the console at a very low price. Although the strategy was designed in part to compete with other console makers, not having high liquidity in the market of players and game developers allowed Microsoft to recover the consumer-side subsidy from the game developer side. Also, it is well known that public transportation systems, such as the Mass Transit Railway (MTR) in Hong Kong, often subsidize the passenger side. While collecting low fares from passengers, MTR profits from renting out limited retail spaces located closely around the exits and within the stations. Without a fierce competition, the differentiation among the stores allows stores to generate profit from the volume of passenger traffic that MTR brings through subsidization; the profits are ultimately absorbed by MTR.

The evidence for seller subsidy is more difficult to observe. Our findings suggest that it can be optimal for the platform if the networks on both sides are large or the quality heterogeneity on the seller side is high. In contrast to game consoles’ buyer-side subsidy, computer operating systems often subsidize the sellers (software developers). Microsoft Windows is known to subsidize developers by offering free or low-cost software development kit (SDK), while charging high prices on the user side. Part of the reason may be that the quality of console games is not as variable as quality of software programs, which require higher expertise on development as well as design knowledge related to specific fields that the software serves. Industry fairs also come close to illustrating this case, including wedding expos, computer fairs, and many others. Some of these fairs, especially those that are large in scale and held in central locations (e.g., metropolitan areas) tend to charge an entrance fee on the buyer side and attract a large crowd nevertheless. Because of easy access for attendees and low setup costs for vendors, both sides incur low entry costs to participate. Although

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13Many platforms also do not fall under this umbrella, such as the dating platform, which facilitates matching rather than trade.
a rental fee might still be collected on the seller side, the organizer may profit substantially on the buyer side to subsidize some of the sellers’ costs.

6.3 Impact of $\sigma^2$

We find that the platform should reduce the seller fee, $R^*_s$, and raise the buyer fee, $R^*_b$, as quality heterogeneity, $\sigma^2$, increases.\(^{14}\) It would seem intuitive to reason that a higher quality heterogeneity suggests mitigated competition, which may increase sellers’ profit and allow the platform to charge a higher seller fee. Our finding on the seller fee show the opposite because competition intensity also depends on the number of sellers; in fact, increased quality heterogeneity attracts more entry on the seller side (Proposition 1), which offsets mitigated competition. The increased variety of products with a wider range of quality creates value for the buyer side, allowing the platform to set a higher fee for buyers. When taking into account both direct and indirect effects of increased quality heterogeneity on the two sides, the platform’s optimal strategy is to lower the seller-side fee. The contrast between the platform’s pricing strategies on the two sides echoes the “seasaw principle” discussed in Rochet and Tirole (2006).

These results are consistent with our findings on platform subsidies from Section 6. Proposition 3 shows that the platform is more likely to subsidize sellers at a higher degree of quality heterogeneity. Here, as quality heterogeneity increases, the platform reduces the seller fee, implying that seller subsidy becomes more likely. Whereas Proposition 3 states the conditions under which the optimal fees are positive/negative, here we focus more on the properties of the optimal fees regardless of whether they are subsidized. An additional implication is that the extent of the subsidy that the platform offers to sellers is greater when sellers are more heterogeneous in quality. Furthermore, as Proposition 4 shows, quality heterogeneity among sellers works against buyer subsidy; here we see that the platform either offers a smaller subsidy or charges a higher fee to buyers when quality varies more greatly.

The comparative statics of the optimal fees provide a basis for how the platform can adjust its pricing when market condition, such as quality heterogeneity of sellers, changes. For example, if smartphone application developers succeed in a particular technology innovation and release games of superior visual experience or applications that more seamlessly integrate with the operating system, the newly introduced applications effectively widen quality heterogeneity in the market. Facing increased quality heterogeneity enabled by developer innovation, the platform may consider reducing the fee on the developer side while raising the price charged on the user side. The same strategy may be implemented in reverse. For instance, Airbnb may find it necessary to exclude hosts below a certain quality to avoid legal disputes, and the reduced quality heterogeneity would suggest a higher fee for the hosts.

A natural question that follows is, how does quality heterogeneity affect the platform’s

\(^{14}\)The proof of $\frac{\partial R^*_s}{\partial \sigma^2} < 0$ is relegated to the appendix. $\frac{\partial R^*_b}{\partial \sigma^2} > 0$ can be illustrated numerically.
revenues? It can be easily proven that the platform’s total revenue increases as sellers become more heterogeneous in quality. Thus, by reducing the seller fee and raising the buyer fee, the platform can boost its total revenue in response to a higher degree of quality heterogeneity. At optimal fees, the platform can benefit from including more types of sellers to create a more diverse marketplace. This would obviously advise against mechanisms to screen out lower quality sellers for the sole reason of improving overall seller quality. Understandably, some platform might still exclude sellers that may lead to other undesirable costs (e.g., review applications for security problems). Our result cautions the platforms to examine their screening criteria so as to avoid unintended narrowing of quality heterogeneity among sellers.

We also find that as the degree of quality heterogeneity increases, the platform’s revenue source shifts away from the seller side and toward the buyer side; that is, \( \frac{n^* R_s}{n^* R_b} \), decreases with \( \sigma^2 \). This follows intuitively from our previous comparative statics, as the platform focuses less on charging sellers and more on charging buyers at higher quality heterogeneity. The implications are meaningful regarding the design of platform business model and fee structure. Understanding the quality heterogeneity of its sellers, the platform owner can more effectively allocate its marketing and operating resources for a seller-focus or buyer-focus strategy.

7 Discussion and conclusion

This paper examines a monopoly platform’s two-sided pricing problem, using an approach that endogenizes network effects through the microfoundation of trades between sellers and buyers. We capture both horizontal and vertical dimensions of differentiation among competing sellers and derive the platform’s optimal strategies related to these types of seller differentiation. We find that the two dimensions of differentiations are substitutes for the platform, who reduces horizontal differentiation by admitting more sellers as the degree of vertical differentiation increases. Furthermore, vertical differentiation plays a key role in the platform’s subsidization strategy: Seller-side subsidy is only possible in the presence of vertical differentiation. On the buyer-side, the platform may provide subsidy when the market is illiquid; however, vertical differentiation on the seller side reduces the platform’s incentive to subsidize buyers.

A few studies in the literature of two-sided platform also discuss seller-side competition. Armstrong (2006) hypothesizes on the platform’s strategies related to intraplatform competition, that is, competition among the participating sellers. Using a stylized example, he illustrates that the platform may allow such competition on the seller side if it can charge the buyer side a higher fee. However, without being able to charge the buyer side, the platform can only profit by ruling out seller competition. Hagiu (2009) rigorously confirms this insight through a model of platform pricing that also captures the negative effect of competition
on sellers. His intuition further incorporates substitutability between sellers and buyers’ preferences for variety. Specifically, when buyers have stronger preferences for variety, the platform profits more on the seller side, for which competition is mitigated as a result of reduced substitutability (Hagiu 2009). Our results echo this general insight: A platform derives more revenues on the seller side (i.e., is more likely to subsidize buyers) when the market is illiquid. Seller competition is dampened in such a market because fewer sellers enter the platform in equilibrium.

Our work differs significantly from Hagiu (2009) in both focus and approach. Whereas he compares the total rents the platform extracts from the two sides, we examine the conditions under which the platform charges a negative fee on each side. We analyze the optimal entry fees, which lead to managerial insights specific to platform pricing. We do not, however, aim to study platform competition or sellers’ innovation incentives, which are important in his work. Another important difference in our work is that, rather than assuming a negative effect from seller-side competition, we derive sellers’ surplus by characterizing their equilibrium competitive strategies. The motivation is not to confirm that the competition effect is indeed negative, which is apparent. The merit of the approach lies in connecting the platform’s decisions with richer characteristics of sellers and buyers, which may indirectly impact seller-side competition. In particular, we show that a greater quality heterogeneity is more conducive to seller-side subsidy and discourages buyer-side subsidy. This seems contradictory to the existing findings in Armstrong (2006) and Hagiu (2009) because, if greater quality heterogeneity mitigates sellers’ horizontal competition, the insight from Armstrong (2006) and Hagiu (2009) would predict that the seller-side fee is likely to increase and that the buyer side may be subsidized. However, the effect of quality heterogeneity on seller-side competition is ambiguous. As Proposition 1 shows, although a higher quality heterogeneity increases sellers’ expected profit ceteris paribus, in equilibrium the platform internalizes this gain by admitting more sellers, which intensifies their horizontal competition. Therefore, it is paramount to embed the effect of quality heterogeneity in the platform’s decisions, because directly applying existing theories—which are valid—would otherwise be misguided.

In fact, our analysis on the same-side loss (SSL) and cross-side gain (CSG) echoes the spirit of Armstrong (2006) and Hagiu (2009). For both Propositions 2 and 4, we explain that a less liquid platform market is conducive to buyer-side subsidy because the platform’s CSG from admitting an additional buyer would otherwise suffer with more sellers on board. The interactions of the cross-side and same-side network effects illustrated here is consistent with the idea that intensified competition from more sellers reduces the platform’s incentive to subsidize buyers.

We suggest several future research directions that are not addressed in the current work. One natural extension is to study competing platforms with a microfoundation for transactions on each platform. Competing platforms may differ in their technologies and designs so that platforms are asymmetric in terms of horizontal differentiation and quality hetereoge-
ity of products traded within their own markets. The asymmetry can yield new insights into how platforms set entry fees depending on both its own and its competitor’s market characteristics. Also, platforms may be able to strategically inform buyers of sellers’ products. For example, platforms’ investments in technologies that improve buyers’ shopping interface, provide recommendations, or offer powerful search functionalities are instrumental in facilitating certain transactions. Thus, endogenizing the related variables can be an interesting direction.

References


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Appendix: Other proofs

In reference to Footnote 14, the proof of $\frac{\partial R^*_n}{\partial \sigma^2} < 0$:

**Proof.** From Eq. (8), we can write $\mu - \frac{t}{4n_s} \frac{\sigma^2}{t} g(n_s) = \frac{2fc}{\frac{t}{4n_s} + \frac{\sigma^2}{t} g'(n_s)}$. Thus, we can express $R^*_n$ as the following:

$$R^*_n = \frac{f}{n_s} \left[ \frac{t}{4n_s} + \frac{\sigma^2}{t} g'(n_s) \right] - f \left[ \frac{4f}{n_s} + \frac{4\sigma^2}{n_s} g(n_s) \right] = f \left[ \frac{3A + 4B(\frac{g(n_s)}{n_s} - g'(n_s))}{A + 4Bg'(n_s)} \right],$$

where $A \equiv \frac{t}{n_s}$ and $B \equiv \frac{\sigma^2}{t}$.

$$\frac{\partial R^*_n}{\partial A} = \frac{f}{[A + 4Bg'(n_s)]^2} \left[ 3(A + 4Bg'(n_s)) - \left( 3A + 4B \left( \frac{g(n_s)}{n_s} - g'(n_s) \right) \right) \right] = \frac{4f B}{[A + 4Bg'(n_s)]^2} \left[ 4g'(n_s) - \frac{g(n_s)}{n_s} \right].$$
\[
\frac{\partial R_s}{\partial B} = f \left\{ \frac{4}{[A + 4Bg'(n_s)]^2} \left[ 4 \left( \frac{g_s(n_s)}{n_s} - g'(n_s) \right) (A + 4Bg'(n_s)) \right. \right.
\]
\[\left. \left. - 4g'(n_s) \left( 3A + 4B \left( \frac{g_s(n_s)}{n_s} - g'(n_s) \right) \right) \right] \right\}
\]
\[= \frac{4fA}{[A + 4Bg'(n_s)]^2} \left[ \frac{g_s(n_s)}{n_s} - 4g'(n_s) \right].
\]

Define \( D \equiv \frac{4f}{[A + 4Bg'(n_s)]^2} \left[ \frac{g_s(n_s)}{n_s} - 4g'(n_s) \right] \).

\[
\frac{\partial R_s}{\partial \sigma^2} = \frac{\partial R_s}{\partial A} \cdot \frac{\partial A}{\partial \sigma^2} + \frac{\partial R_s}{\partial B} \cdot \frac{\partial B}{\partial \sigma^2}
\]
\[= -DB \cdot \frac{-2t}{n_s^3} \cdot \frac{\partial n_s}{\partial \sigma^2} + \frac{DA}{t}
\]
\[= D \left( \frac{2tB}{n_s^3} \cdot \frac{\partial n_s}{\partial \sigma^2} + \frac{A}{t} \right).
\]

Notice that \( \frac{\partial n_s}{\partial \sigma^2} > 0 \) and \( D < 0 \); therefore, \( \frac{\partial R_s}{\partial \sigma^2} < 0. \)