12-2016

Endogenous quality choice, signaling, and welfare

Gea M. LEE
Singapore Management University, gmlee@smu.edu.sg

Seung Han YOO

Endogenous Quality Choice, Signaling, and Welfare*

Gea M. Lee† Seung Han Yoo‡

December, 2016

Abstract

We consider a model in which each worker endogenously selects his own type through a private investment decision and selects a public signal in the labor market. Signaling then contributes to social welfare through its influence on the quality choice. We offer a rationale for the argument that there are too many high-type workers from a welfare perspective, identifying circumstances under which separating equilibrium generates too many high-type workers while having to use the incentive-compatible signal and treat high-type workers differently in the market. The inefficiency can then be reduced in pooling equilibrium.

Keywords and Phrases: Investment, Endogenous quality, Signaling, Welfare
JEL Classification Numbers: **D63, I21, J24**

---

*This is a substantially revised version of the paper "Job Market Signaling with Human Capital Investment" dated in 2013. We are grateful to Kyle Bagwell, Christine Ho, Hian Teck Hoon, Fali Huang, Keiichi Kawai, Takashi Kunimoto, Mark Machina, Andy Skrzypacz, Ross Starr, Joel Sobel, and Joel Watson for helpful discussions and comments. We also thank seminar participants at UCSD, SNU, Singapore Management, Sogang, Sungkyunkwan Universities, and Australasian Public Choice conference. Part of this work was done while Seung Han was visiting UCSD. The hospitality of UCSD's Department of Economics is gratefully acknowledged. Of course, all remaining errors are ours.

†School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903 (e-mail: `gmlee@smu.edu.sg`).

‡Department of Economics, Korea University, Seoul, Republic of Korea 136-701 (e-mail: `shyoo@korea.ac.kr`).

1 Introduction

We consider a situation in which a seller makes a private investment to endogenously determine the quality of the product that the seller provides in a competitive market. The consequent asymmetric information in the market in return causes a moral hazard problem in the investment stage: given a single price in the market, sellers have no incentive to make the investment that would upgrade the quality. In the setting, signaling contributes to social welfare through its influence on the quality choice. The no-investment problem can be solved by separating signal, because sellers can then be treated differently in the market, and by pooling signal as well, because sellers can then reduce signaling costs even when they are treated equally in the market.

In this paper, we formalize the setting using a labor-market model in which each worker endogenously selects his own type through a private investment decision and then selects a public signal in the market.[1] In the model, the worker decides whether to make the investment and become high type, or to remain as low type, by comparing the future benefit that will be manifested through signaling with the investment costs that are drawn from a prior distribution of workers' inborn cost types. An equilibrium consists of a proportion of workers who make the investment to be high type, and a signaling form. The proportion of high-type workers in the population is referred to as the *investment ratio*, and the signaling takes the form of either separating or pooling. We raise the question: can we rationally say that there are too many high-type workers from a welfare perspective? The answer is not obvious because the workers' selection of types is made in their interests. In separating equilibrium, some workers choose to become high type for their own benefits while causing no welfare loss to the remaining workers. Thus, with the use of separating equilibrium alone, we cannot argue that there are too many high-type workers, even when most of workers make the costly investment to be treated differently from a very small fraction of the remaining workers.

We show, however, that there exist circumstances under which separating equilibrium generates too many high-type workers from a welfare perspective. The use of pooling equilibrium is essential for the finding due to the accompanying feature: pooling signal affects all workers and entails a tension between the generation of high-type workers and the signaling costs of low-type workers. In particular, we use a subtle link between pooling and separating equilibrium to identify conditions under which we can make the following statements: (i) for any separating equilibrium, there exists a pooling equilibrium that approximates the separating equilibrium in

[1] Although we adopt a familiar labor-market model in this paper, our main theme can be generally extended for the setting in which an investment decision endogenously generates asymmetric information in the market, and asymmetric information in the market in return causes the moral hazard problem in the investment stage.

terms of the investment ratio and social welfare; (ii) this pooling equilibrium has overinvestment; and (iii) there exists an optimal pooling equilibrium that restricts the inefficiency of overinvestment. We find that three statements hold under a *single* condition: pooling signal reaches a "saturation point" such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. Under the condition, the tension observed in pooling signal implies that it is socially preferred to reduce signaling costs than to increase high-type workers. In this case, separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal and treat high-type workers differently in the market. The inefficiency of overinvestment can then be reduced in pooling equilibrium where workers use the same signal without having to be treated differently. On the other hand, there also exist circumstances under which pooling signal remains sufficiently effective in generating the investment ratio further. In this case, pooling equilibrium generates too few high-type workers, and the inefficiency of underinvestment can be reduced in separating equilibrium.

In practice, it is commonly argued in the mass media that there are too many college graduates typically based on limited job openings. It is, however, difficult to support the argument perhaps for two main reasons. First, despite limited job openings, high school graduates may choose to go to college for their own benefits. Indeed, there exists a significant wage gap between college-educated and high-school-only workers in real data.[2] Second, a fundamental question of whether and how the signal (college degree) contributes to human capital is rarely discussed or answered in the argument.[3] In regard to the specific issue, our model broadly indicates that, despite the significant wage gap, if the capacity for education to increase the aggregate human capital reaches a saturation point over the education level between high school and college, then it becomes reasonable to argue that signaling costs of college degree are too high, and there are too many college graduates, from a welfare perspective.

Our model is related to a few existing models. Fang (2001) contains an investment stage before workers select signaling, and highlights an economic role of "social culture" by showing

[2] The college wage premium substantially increased between 1980 and 2005 in the US, and it has been studied by a vast body of literature (see, for example, Taber (2001), Fang (2006), Goldin and Katz (2007a, 2007b), Walker and Zhu (2008) and Cunha, Karahan, and Soares (2011) among many others).

[3] Since the classical papers of Spence (1973, 1974), the "information-conveying" aspect of signaling has produced a large body of literature (see Kreps and Sobel (1994) and Riley (2001) for literature survey). The information-conveying aspect of education has also been empirically tested (Wolpin (1977), Riley (1979), Lang and Kropp (1986), Tyler, Murnane and Willett (2000), Bedard (2001)). For example, using a unique data set containing the General Educational Development (GED) test scores, Tyler, Murnane and Willett (2000) identify the signaling value of the GED, net of human capital effects. They observe that there are substantial signaling effects for young white dropouts, estimated at about 20% earnings gain after 5 years.

that there exists a separating equilibrium in which the seemingly irrelevant activity, social culture, becomes an endogenous signaling instrument for the workers who invested in skills. In his model, however, pooling equilibrium is inferior to separating equilibrium in which workers make the investment to be treated differently. Using a signaling setting in which the market (receiver) observes an informative grade in addition to the regular signal, Daley and Green (2014) show that some degree of pooling emerges in equilibria and that, if the market's prior belief that the sender is high type approaches one, then the equilibrium converges to the complete-information outcome, pooling with no costly signaling. In their discussion of the possibility that there is an ex ante privately-observed investment, they predict that the investment remains inefficiently low even in the presence of informative grades given that it takes additional resources to be treated differently as high type. In our model, the receiver's belief is endogenously supported only if signaling is large enough to support the belief. We find that there are important welfare implications that have not been observed by the existing information-conveying argument, showing that signaling may overly generate high-type workers.

Recent papers by Hermalin (2013) and Kawai (2014) consider a situation in which an investment in an asset made by a seller endogenously determines the value of the asset, and a potential buyer cannot observe the seller's investment decision made prior to trade. In those models, there is a key trade-off between the provision of ex ante incentive for investment and the achievement of ex post efficiency in trade: if trade is sure to happen, then the seller has no incentive to invest ex ante, and if no trade is anticipated, then the seller has incentive to invest for her own benefit. In equilibria, investment and trade occur both with a positive probability when the buyer cannot observe the seller's investment, or receive any signal of it. In particular, Hermalin (2013) observes that a holdup problem arises when the buyer has all the bargaining power and the problem may cause overinvestment. Our model also establishes the existence of overinvestment when an ex ante investment results in asymmetric information between sellers and buyers, but it allows that signaling is a natural option for sellers and that trade surely occurs in a competitive market.[4]

This paper is organized as follows. We introduce the model in Section 2, and provide the existence of separating and pooling equilibria in Section 3. In Section 4, we offer a rationale for the assertion that there may be too many, or too few, high-type workers from a welfare

[4] Goldlücke and Schmitz (2014) consider an ex ante investment as well, but in a different context where a seller can make an observable investment to improve his product specialized for a buyer, showing that a seller's signaling motive can alleviate the ex ante underinvestment (i.e., the hold-up problem). A key insight of their model is that if the seller has private information about the fraction of the ex post surplus that he can realize on his own, then his large investment can serve as signal of having the strong outside options that affect the buyer's take-it-or-leave-it offer.

perspective. We provide numerical examples in Section 5, and concluding remarks in Section 6. All the proofs are collected in an appendix.

2 Model

We explore a situation in which each worker determines his own quality of labor through a private investment decision, and this endogenous quality choice causes asymmetric information in the market. We consider signaling to be a natural option for workers, and reveal a notable feature of the model implied by the endogenous quality choice.

2.1 Endogenous quality choice and signaling

We consider a labor market that has a unit mass of workers. Each worker has an inborn investment cost level c, which is drawn from an absolutely continuous distribution function $G(c)$ with the support $[\underline{c}, \overline{c}]$, where $\overline{c} > \underline{c} \geq 0$. The level of c captures a composite cost depending on the individual worker's aggregate endowment such as intellect, health, maturity, initial wealth, and parental environment. The density $g \equiv G'$ is everywhere positive. Given the inborn cost, each worker makes an endogenous choice of his own type $q \in \{H, L\}$, and this choice is privately observed. The choice of $q = L$ needs no investment and the choice of $q = H$ needs an investment that incurs the inborn cost. A worker next selects a publicly observable signal $e \in \mathbb{R}_+$ in the labor market where two risk-neutral firms engage in a Bertrand-type competition with simultaneous wage offers. The worker earns wage $w \in \mathbb{R}_+$ if he is hired by one of the two firms and has zero utility from outside options. Thus, a worker who selects type q has the type-dependent payoff

$$\begin{cases} u_q(w, e) - c & \text{if} \quad q = H \\ u_q(w, e) & \text{if} \quad q = L, \end{cases}$$

where $u_q(w, e)$ is strictly increasing in w and strictly decreasing in e. A firm obtains the value $y_q \in \mathbb{R}_+$ when it employs a worker with type $q \in \{H, L\}$. The worker's investment improves the value, $y_H > y_L$.

The utility function $u_q(w, e)$ is continuous and includes the following standard assumptions. The Spence-Mirrlees property (SMP) holds in the function. Formally, an increase in signal is less costly for a high-type worker than for a low-type worker: if $e' > e$, then

$$u_H(w, e') - u_H(w, e) > u_L(w, e') - u_L(w, e). \tag{1}$$

In addition, we assume that $u_q(w, e)$ has no "cross effect" between q and w. In other words, the utility gain associated with wage increase is type-irrelevant: if $w' > w$, then

$$u_H(w', e) - u_H(w, e) = u_L(w', e) - u_L(w, e). \tag{2}$$

This assumption greatly simplifies our analysis and is satisfied for all separable utility functions, $u_q(w,e) = v(w) - c_q(e)$, for any increasing function $v(\cdot)$. For no signal $e = 0$, it is reasonable to assume that the level of utility is type-irrelevant:

$$u_H(w,0) = u_L(w,0). \tag{3}$$

This assumption and SMP imply $u_H(w,e) > u_L(w,e)$ for all $e > 0$.

The time line is described as follows:

Time 1. Nature chooses c.

Time 2. Each worker chooses type q.

Time 3. Each worker chooses signal e.

Time 4. The two firms simultaneously make wage offers.

Time 5. Each worker accepts the highest wage and produces. For indifferent offers, he randomly selects one firm.

The worker's investment strategy at time 2 is a mapping $Q : [\underline{c}, \overline{c}] \to \{H, L\}$, and the worker's signaling strategy at time 3 is a mapping $E : \{H, L\} \to \mathbb{R}_+$. A worker makes the investment decision by comparing the investment cost with the benefit that will be manifested through the signaling choice. As we present below, the equilibrium investment strategy Q takes the form of a "cutoff strategy" with a threshold cost type k such that workers who are endowed with cost type $c < k$ $(c > k)$ make the investment (no investment). An equilibrium is called an *interior* equilibrium if it has the threshold on an interior point of the support $[\underline{c}, \overline{c}]$, $k \in (\underline{c}, \overline{c})$, and an equilibrium is called a *boundary* equilibrium otherwise. The strategy of firm i at time 4 is a mapping $w_i : \mathbb{R}_+ \to \mathbb{R}_+$ for $i = 1, 2$. Each firm observes signal e and forms the (common posterior) belief $\mu(e)$, the probability of $q = H$. Given the Bertrand-type competition, in equilibrium, each firm's strategy satisfies $w(e) = w_i(e) = \mu(e)\, y_H + (1 - \mu(e))\, y_L$ for all i.

A strategy profile $\{(Q(c), E(q)), w(e)\}$ is a *perfect Bayesian equilibrium* if in each time line, the strategy of each player is the best response to the other players' strategies, and the belief is updated by the Bayes' rule where possible.[5]

[5] Formally, a set of strategies $\{(Q(c), E(q)), (w_i(e))_{i=1}^2\}$ and a belief function $\mu(e)$ constitute a perfect Bayesian equilibrium if

(i) $(Q(c), E(q))$ is optimal for the worker given $(w_i(e))_{i=1}^2$;

(ii) $\mu(e)$ is derived from $E(q)$ via the Bayes' rule where possible;

(iii) $(w_i(e))_{i=1}^2$ is a Nash equilibrium of the simultaneous move game in which both firms make wage offers to the worker knowing that $q = H$ with probability $\mu(e)$.

2.2 Separating and pooling equilibria

We characterize two interior equilibria, separating and pooling. We begin by analyzing the signaling stage at time 3. For separating equilibria, let $e_H \equiv E(H) \neq e_L \equiv E(L)$. The Bayes' rule entails that $\mu(e_H) = 1$ and $\mu(e_L) = 0$ on the equilibrium path; thus, y_H (y_L) becomes the wage for high-type (low-type) workers, and low-type workers maximize utility by selecting $e_L = 0$. For pooling equilibria, let $e \equiv E(H) = E(L)$. The Bayes' rule entails that $\mu(e) = \lambda$ on the equilibrium path, where λ denotes the proportion of high-type workers; thus, the expected value $\mathbb{E}^\lambda[y] = \lambda y_H + (1-\lambda) y_L$ becomes the wage for both types. The proportion of high-type workers, λ, is *endogenously* determined by the workers' investment decision.

Separating and pooling equilibria must satisfy incentive compatibility conditions:

$$u_H(y_H, e_H) \geq u_H(y_L, 0) \text{ and } u_L(y_L, 0) \geq u_L(y_H, e_H),$$

and

$$u_H(\mathbb{E}^\lambda[y], e) \geq u_H(y_L, 0) \text{ and } u_L(\mathbb{E}^\lambda[y], e) \geq u_L(y_L, 0).$$

A separating signal e_H must be in an interval, $e_H \in [\underline{e}_H, \overline{e}_H]$, where $\underline{e}_H$ and $\overline{e}_H$ are respectively defined by binding constraints,

$$u_H(y_H, \overline{e}_H) = u_H(y_L, 0) \text{ and } u_L(y_L, 0) = u_L(y_H, \underline{e}_H), \tag{4}$$

with $\overline{e}_H > \underline{e}_H > 0$ from the assumption $u_L(y_L, 0) = u_H(y_L, 0)$. A pooling signal e must be in an interval, $e \in [0, \overline{e}(\lambda)]$, where the upper bound $\overline{e}(\lambda)$ is defined by the binding constraint

$$u_L(\mathbb{E}^\lambda[y], \overline{e}(\lambda)) = u_L(y_L, 0). \tag{5}$$

There is no overlap in the use of signal in two equilibria, $\overline{e}(\lambda) < \underline{e}_H$, for all $\lambda < 1$.[6]

Consider next the investment stage. For separating equilibria, if a worker selects $q = H$, then he has utility $u_H(y_H, e_H) - c$, and if a worker selects $q = L$, then he has utility $u_L(y_L, 0)$. Hence, in the investment stage, an interior separating equilibrium has a threshold:

$$k_s = u_H(y_H, e_H) - u_L(y_L, 0). \tag{6}$$

For pooling equilibria, if a worker selects $q = H$, then he has utility $u_H(\mathbb{E}^\lambda[y], e) - c$, and if a worker selects $q = L$, then he has utility $u_L(\mathbb{E}^\lambda[y], e)$. Hence, in the investment stage, an interior pooling equilibrium has a threshold:

$$k_p = u_H(\mathbb{E}^\lambda[y], e) - u_L(\mathbb{E}^\lambda[y], e).$$

[6] For $\lambda < 1$, we have $\overline{e}(\lambda) < \underline{e}_H$ from $u_L(y_H, \underline{e}_H) = u_L(y_L, 0) = u_L(\mathbb{E}^\lambda[y], \overline{e}(\lambda))$ and $y_H > \mathbb{E}^\lambda[y]$.

Since the utility gain from wage increase is type-irrelevant by the assumption in (2),

$$u_H(\mathbb{E}^{\lambda}[y], e) - u_L(\mathbb{E}^{\lambda}[y], e) = u_H(0, e) - u_L(0, e),$$

a threshold k_p is given as

$$k_p = u_H(0, e) - u_L(0, e). \tag{7}$$

It follows from (6) that an increase in separating signal e_H discourages workers from becoming high type while causing no welfare loss to the workers who remain as low type. An increase in pooling signal e affects all workers, and it is immediate from (7) and SMP in (1) that an increase in e encourages workers to become high type.

Lemma 1 *The separating threshold k_s is a strictly decreasing function of $e_H \in [\underline{e}_H, \overline{e}_H]$, whereas the pooling threshold k_p is a strictly increasing function of $e \in [0, \overline{e}(\lambda)]$.*

We now define interior and boundary equilibria. An interior separating equilibrium is defined as a pair (k_s^*, e_H^*) that satisfies

$$k_s^* = u_H(y_H, e_H^*) - u_L(y_L, 0) \in (\underline{c}, \overline{c}) \text{ and } e_H^* \in [\underline{e}_H, \overline{e}_H]. \tag{8}$$

An interior pooling equilibrium is defined as a pair $\left(k_p^*, e^*\right)$ that satisfies

$$k_p^* = u_H(0, e^*) - u_L(0, e^*) \in (\underline{c}, \overline{c}) \text{ and } e^* \in [0, \overline{e}\left(G\left(k_p^*\right)\right)], \tag{9}$$

where the proportion of high-type workers in the population, $G(k_p^*)$, is endogenous.

In a boundary equilibrium, workers are treated equally by the same wage and thus they select no costly signal, $e = 0$. Given that workers are treated equally and select no signal, we have $u_H(w, 0) - u_L(w, 0) = 0$ from the assumption in (3) and find that workers have no incentive to make the investment. Thus, the boundary equilibrium with no investment can solely survive.

Lemma 2 *A unique boundary equilibrium (k_b^*, e_b^*) exists with $G(k_b^*) = e_b^* = 0$.*

In the model, there can be two types of interior equilibria, separating and pooling, and there is a unique boundary equilibrium. We henceforth restrict attention to non-trivial interior equilibria; in what follows, a separating (pooling) equilibrium refers to an interior separating (pooling) equilibrium.[7]

[7] In our analysis below, the boundary equilibrium with no investment is considered only under the ban on signaling.

2.3 Signaling as socially beneficial device

We now highlight a notable feature of the model: signaling contributes to social welfare through its influence on the workers' selection of types. The ban on signaling (no signaling) leads to a pooling equilibrium in which workers receive the same wage and thus select no signal, and given $u_H(w,0) - u_L(w,0) = 0$, workers make no investment. Therefore, the ban on signaling results in the boundary equilibrium with social welfare $u_L(y_L, 0)$. If the ban on signaling is lifted, then the no-investment problem can be solved. In a separating equilibrium with e_H, the workers with $c \in (\underline{c}, k_s)$ select high type to be treated differently with a higher wage y_H, $u_H(y_H, e_H) - c > u_L(y_L, 0)$, while the remaining workers have utility $u_L(y_L, 0)$. In a pooling equilibrium with e, although all workers are treated equally by the same wage, the workers with $c \in (\underline{c}, k_p)$ select high type to reduce signaling costs, $u_H\left(\mathbb{E}^\lambda[y], e\right) - u_L(\mathbb{E}^\lambda[y], e) = u_H(0, e) - u_L(0, e) > c$, while the remaining workers have utility $u_L(\mathbb{E}^\lambda[y], e) \geq u_L(y_L, 0)$.

Lemma 3 *In our model, any separating or pooling equilibrium generates a strictly higher welfare than the ban on signaling.*

We in turn show that signaling becomes socially wasteful with no influence on the workers' selection of types. To this end, we consider a benchmark model in which the proportion of high type is *exogenously* fixed at $\lambda \in (0, 1)$. In any pooling equilibrium with $e > 0$, the ban on signaling benefits all workers who receive the same wage $\mathbb{E}^\lambda[y]$ regardless of their types. In the separating equilibrium with $\underline{e}_H$, the ban on signal benefits low-type workers since $u_L(\mathbb{E}^\lambda[y], 0) > u_L(y_L, 0)$, and it benefits high-type workers only if λ is sufficiently large to satisfy $u_H(\mathbb{E}^\lambda[y], 0) > u_H(y_H, \underline{e}_H)$. As we show in the Appendix, the concavity of u_L ensures that the benefit of low type is greater than the loss of high type even for small λ.

Lemma 4 *In the benchmark model, (i) the ban on signaling generates a strictly higher welfare than any pooling equilibrium with $e > 0$; and (ii) if u_L is concave in w, then the ban on signaling generates a strictly higher welfare than any separating equilibrium.*

3 Existence of equilibria

In this section, we establish the existence of separating and pooling equilibria. We assume that u_q is differentiable in what follows.

We proceed to examine the separating equilibrium with the least costly signal $\underline{e}_H$ that satisfies Cho-Kreps' criterion (Cho and Kreps (1987)). Since the threshold k_s is strictly decreasing in

$e_H \in [\underline{e}_H, \overline{e}_H]$, the separating equilibrium with $\underline{e}_H$ has the highest level $\overline{k}_s$,

$$\overline{k}_s = u_H\left(y_H, \underline{e}_H\right) - u_L\left(y_L, 0\right) = u_H(y_H, \underline{e}_H) - u_L(y_H, \underline{e}_H),$$

where the second equality follows from the definition of $\underline{e}_H$, $u_L\left(y_L, 0\right) = u_L\left(y_H, \underline{e}_H\right)$ in (4). Given the assumption that the utility gain from wage increase is type-irrelevant, the threshold becomes

$$\overline{k}_s = u_H\left(0, \underline{e}_H\right) - u_L(0, \underline{e}_H). \tag{10}$$

We now adopt two useful notations. First, we define a function

$$\Delta\left(e\right) \equiv u_H\left(0, e\right) - u_L\left(0, e\right)$$

to capture how signal e determines the type-relevant gain that workers expect when making the investment. Notice that, for any pooling signal e and the separating signal $\underline{e}_H$, we can use the same function Δ to represent investment thresholds:[8]

$$k_p = \Delta(e) \text{ and } \overline{k}_s = \Delta(\underline{e}_H).$$

Second, we define a distribution function

$$D(e) \equiv G(\Delta\left(e\right))$$

to examine how signal e generates the proportion of high-type workers in the population that is hereafter referred to as the *investment ratio*. The function $D(e)$ is strictly increasing in pooling signal $e \in [0, \overline{e}\left(\lambda\right)]$ for all $D(e) \in (0, 1)$. The slope $D'\left(e\right)$ is sufficiently steep (flat) if an increase in pooling signal e is sufficiently effective (ineffective) in increasing the investment ratio further. For instance, the slope $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$ may be steep (flat) for $e \geq e^*$, if the population density $g(c)$ is high (low) for $c \geq \Delta(e^*)$, and (or) if the magnitude of $\Delta'(e)$ is large (small) for $e \geq e^*$.[9]

We next use those functions, $\Delta(e)$ and $D(e)$, and establish the existence of equilibria. There exists a separating equilibrium with $\underline{e}_H$ if and only if $\Delta(\underline{e}_H) \in (\underline{c}, \overline{c})$, or equivalently $D(\underline{e}_H) \in (0, 1)$. In the separating equilibrium, the signal $\underline{e}_H$ motivates the workers with cost types below $\overline{k}_s = \Delta(\underline{e}_H)$ to make the investment and results in the investment ratio $D(\underline{e}_H) = G(\Delta(\underline{e}_H))$. We also establish the existence of a pooling equilibrium using the correspondence

$$\{x \in [0, 1] : x = D\left(e\right) \text{ for } e \in [0, \overline{e}\left(\lambda\right)]\}. \tag{11}$$

[8] Note that Δ captures the type-relevant gain *net of income effect*. We also know that pooling signal cannot exceed the level $\underline{e}_H$ and that the separating signal $\underline{e}_H$ has the feature in (10). Thus, for any pooling signal and the separating signal $\underline{e}_H$, we can use the same function Δ.

[9] Reall that SMP implies $\Delta'(e) > 0$. In broad terms, the magnitude of $\Delta'(e)$ refers to the degree of SMP.

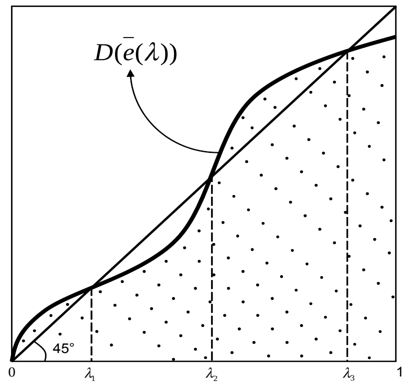

Figure 1: Two intervals of equilibrium proportions

The correspondence has the maximum value $D(\bar{e}(\lambda)) = G(\Delta(\bar{e}(\lambda)))$ for the highest pooling signal $\bar{e}(\lambda)$ given λ. The following proposition shows that there exists a pooling equilibrium with some $e \in [0, \bar{e}(\lambda)]$ if and only if the function $D(\bar{e}(\lambda))$ reaches the 45 degree line for some $\lambda \in (0,1)$. This existence condition means that, given $\lambda \in (0,1)$, the highest pooling signal $\bar{e}(\lambda)$ motivates the workers with cost types below $k_p = \Delta(\bar{e}(\lambda))$ to make the investment and results in the investment ratio $D(\bar{e}(\lambda))$ becoming at least as high as λ.

Proposition 1 *(i) There exists a separating equilibrium with $\underline{e}_H$ if and only if $\Delta(\underline{e}_H) \in (\underline{c}, \bar{c})$, or equivalently $D(\underline{e}_H) \in (0,1)$.*

(ii) There exists a pooling equilibrium if and only if $D(\bar{e}(\lambda)) \geq \lambda$ for some $\lambda \in (0,1)$.

Figure 1 depicts the case with two sets of equilibrium proportions, $[0, \lambda_1]$ and $[\lambda_2, \lambda_3]$, where the dotted area below the curve $D(\bar{e}(\lambda))$ represents the correspondence in (11). Since $u_L(\mathbb{E}^\lambda[y], \bar{e}(\lambda)) = u_L(y_L, 0)$ where $\mathbb{E}^\lambda[y] = y_L + \lambda(y_H - y_L)$, the highest signal $\bar{e}(\lambda)$ is strictly increasing for all $\lambda \in (0,1)$ with boundary values, $\bar{e}(0) = 0$ and $\bar{e}(1) = \underline{e}_H$. Thus, using the same function Δ for interior and boundary values, we can find that $\Delta(\bar{e}(\lambda))$ is strictly increasing for all $\lambda \in (0,1)$ with $\Delta(\bar{e}(0)) = 0$ and

$$\Delta(\bar{e}(1)) = \Delta(\underline{e}_H) = \bar{k}_s,$$

and that $D(\bar{e}(\lambda))$ is strictly increasing for all $D(\bar{e}(\lambda)) \in (0,1)$ with the vertical intercept

$$D(\bar{e}(1)) = D(\underline{e}_H) = G(\bar{k}_s).$$

If the wage gap, $y_H - y_L$, becomes larger given y_L, then $D(\bar{e}(\lambda))$ shifts up since $\bar{e}(\lambda)$ increases given $\lambda > 0$. The function shifts more if the gain from making the investment, $\Delta(e)$, is larger.

In the following proposition, we impose a condition on the slope of $D(\overline{e}(\lambda))$: the slope is sufficiently small such that a pooling equilibrium exists and generates the investment ratio that approaches $D(\overline{e}(1))$. If $\Delta(\overline{e}(1)) > \overline{c}$, then the condition immediately holds: if $\Delta(\overline{e}(1)) > \overline{c}$, then $D(\overline{e}(\lambda))$ is perfectly flat on the top, $D(\overline{e}(\lambda)) = 1$ on $[\lambda', 1]$ for some $\lambda' \in (0,1)$, and thus a pooling equilibrium exists and generates the investment ratio that approaches 1. As we confirm in Section 5, the condition on the slope of $D(\overline{e}(\lambda))$ plays a key role in our justification for the assertion that there are too many high-type workers for a welfare perspective.

Proposition 2 *(i) If $\Delta\left(\overline{e}\left(1\right)\right) > \overline{c}$, then there exists a pooling equilibrium with λ sufficiently close to* 1.

(ii) Suppose $\underline{c} < \Delta\left(\overline{e}\left(1\right)\right) \leq \overline{c}$. If there exists a sufficiently small $\lambda' > 0$ such that $dD\left(\overline{e}\left(\lambda\right)\right)/d\lambda$ is sufficiently small on $\left[\lambda', 1\right]$, then there exists a pooling equilibrium. In addition, if $dD\left(\overline{e}\left(\lambda\right)\right)/d\lambda$ converges to zero, the investment ratio in the pooling equilibrium converges to $D\left(\overline{e}\left(1\right)\right)$.

Notice that the slope of $D(\overline{e}(\lambda))$ depends on the slope $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$. The condition on the slope of $D(\overline{e}(\lambda))$, stated in Proposition 2 (ii), is likely to hold if the wage gap, $y_H - y_L$, is sufficiently large given y_L so that $D(\overline{e}(\lambda))$ is above 45 degree line for some λ, and an increase in pooling signal e is ineffective in increasing the investment ratio so that the slope $D'(e)$ is sufficiently flat above a certain level.

4 Too many (too few) high-type workers

In this section, we offer a theoretical foundation of the argument that there are too many, or too few, high-type workers from a welfare perspective. For this purpose, we say that the government uses the regulation ($\underline{E}$) (the regulation ($\overline{E}$)) if it imposes a lower bound $\underline{E}$ (an upper bound $\overline{E}$) on the use of signal. We then associate the regulation ($\underline{E}$) (the regulation ($\overline{E}$)) with circumstances under which the government promotes (restricts) the generation of high-type workers to maximize social welfare.

4.1 Regulation

We here show that the government can use the regulation ($\underline{E}$) or ($\overline{E}$) to support a particular equilibrium as a unique equilibrium satisfying the Cho-Kreps' intuitive criterion. Suppose that the government uses the regulation ($\underline{E}$) such that a pair $(k_s^*, e_H^*) = (u_H(y_H, \underline{E}) - u_L(y_L, 0), \underline{E})$ satisfies (8) and thus is a separating equilibrium. Then the pair becomes a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion, since there is no signaling below $\underline{E}$ to which a type H worker can deviate, and there exists no pooling equilibrium given that pooling signal e must satisfy $e < \underline{e}_H \leq \underline{E} = e_H^*$.[10] Suppose next that the government uses the regulation $(\overline{E})$ such that a pair $(k_p^*, e^*) = (u_H(0, \overline{E}) - u_L(0, \overline{E}), \overline{E})$ satisfies (9) and thus is a pooling equilibrium. Then the government can support the pair as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion, since there is no signaling above $\overline{E}$ to which a type H worker can deviate, and there exists no separating equilibrium given that separating signal e_H must satisfy $e_H > \overline{e}(\lambda^*) \geq e^* = \overline{E}$. The following lemma reports this finding.

Lemma 5 *For any separating (pooling) equilibrium, the regulation ($\underline{E}$) (the regulation ($\overline{E}$)) can support the separating (pooling) equilibrium as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.*

The government can also affect the workers' selection of signal and investment through its tax policy. Formally, for any separating (pooling) equilibrium, there is a tax policy under which the separating (pooling) equilibrium satisfies the Cho-Kreps' intuitive criterion. A key idea is that the government can impose a tax on the signal above e^* (below e_H^*) if it implements a pooling (separating) equilibrium with e^* (e_H^*), and the level of tax is determined to prevent the potential deviation by high-type workers.[11] To deliver our main findings simply, we focus on the regulation ($\underline{E}$) or ($\overline{E}$) in this paper.

4.2 Tension in pooling

We now express social welfare in terms of investment while observing that workers have surplus and employers earn zero profits in the competitive market. A separating equilibrium generates the social welfare:

$$\begin{aligned} U_s(k_s) &= \int_{\underline{c}}^{k_s} [u_H(y_H, e_H) - c] dG(c) + \int_{k_s}^{\overline{c}} u_L(y_L, 0)\, dG(c) \\ &= u_L(y_L, 0) + \int_{\underline{c}}^{k_s} [k_s - c] dG(c), \end{aligned}$$

where the second equality follows from $k_s = u_H(y_H, e_H) - u_L(y_L, 0)$. The social welfare consists of two parts: the utility $u_L(y_L, 0)$ that is secured for all workers and the surplus of investment

[10] If $\Delta(\overline{e}(1)) \geq \overline{c}$, or equivalently $D(\overline{e}(1)) = 1$, then the separating equilibrium with $e_H^* = \underline{e}_H$ does not exist, but a separating equilibrium with $e_H^* \in (\underline{e}_H, \overline{e}_H]$ may exist. Our finding implies that if a separating equilibrium with $e_H^* \in (\underline{e}_H, \overline{e}_H]$ exists, then the regulation ($\underline{E}$) can support it as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.

[11] The proof for this result can be provided upon request.

that is available only for the workers with cost types below k_s. Integrating by parts, we can rewrite $U_s(k_s)$ as

$$U_s(k_s) = u_L(y_L, 0) + \int_{\underline{c}}^{k_s} G(c)dc. \tag{12}$$

A pooling equilibrium has the social welfare:

$$U_p(k_p) = \int_{\underline{c}}^{k_p} [u_H(\mathbb{E}^\lambda[y], e) - c]dG(c) + \int_{k_p}^{\overline{c}} u_L(\mathbb{E}^\lambda[y], e)dG(c).$$

Using $k_p = u_H(\mathbb{E}^\lambda[y], e) - u_L(\mathbb{E}^\lambda[y], e)$ and integration by parts, we find that the social welfare consists of the utility $u_L(\mathbb{E}^\lambda[y], e)$ that is secured for all workers and the surplus of investment that is available only for the workers with cost types below k_p:

$$U_p(k_p) = u_L(\mathbb{E}^\lambda[y], e) + \int_{\underline{c}}^{k_p} G(c)dc, \text{ where } \lambda = G(k_p) \text{ and } e = \Delta^{-1}(k_p). \tag{13}$$

Having $U_s(k_s)$ and $U_p(k_p)$, we can analyze the relationship between social welfare and investment. In a separating equilibrium, an increase in the workers' investment unambiguously raises the welfare $U_s(k_s)$. Since k_s is strictly decreasing in $e_H \in [\underline{e}_H, \overline{e}_H]$, the welfare highest at $U_s(\overline{k}_s)$ when $e_H = \underline{e}_H$. In a pooling equilibrium, by contrast, an increase in k_p has a trade-off: an increase in k_p raises the expected wage and the surplus of investment, but it increases signaling costs of workers who remain as low type,

$$U_p'(k_p) = \frac{\partial U_p}{\partial k_p} + \frac{\partial U_p}{\partial e}\frac{de}{dk_p} = \left(\frac{\partial u_L}{\partial w} g(k_p)(y_H - y_L) + G(k_p)\right) + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)}, \tag{14}$$

where $1/\Delta'(e)$ follows from the inverse function $e = \Delta^{-1}(k_p)$. Thus, pooling signal affects all workers and entails a tension between the generation of high-type workers and the signaling costs of low-type workers. To relate this feature to findings in the following subsection, we here identify the conditions on $g(k_p)$ and $\Delta'(e)$ under which $U_p(k_p)$ is strictly decreasing in k_p. Notice that the conditions remain valid for any wage gap, $y_H - y_L$.

Lemma 6 *In a pooling equilibrium, if $g(k_p)$ or $\Delta'(e)$ is sufficiently small (large) at $e = \Delta^{-1}(k_p)$, then $U_p(k_p)$ is strictly decreasing (increasing) in k_p.*

4.3 Distortions in investment

We finally ask the main question: can we rationally say that there are too many high-type workers from a welfare perspective? In a separating equilibrium, the workers with lower cost types make the investment to receive a higher wage while causing no welfare loss to the remaining workers. Indeed, the social welfare $U_s(k_s)$ strictly increases in k_s. Therefore, with the use of separating equilibrium alone, it is impossible to assert that there are too many high-type workers, even when most of workers make the costly investment to be treated differently from a very small fraction of the remaining workers.

We find, however, that there exist circumstances under which there are too many high-type workers. The use of pooling equilibrium is essential for this finding due to the accompanying tension between the generation of high-type workers and the signaling costs of low-type workers. We begin by recalling that the separating equilibrium with $\underline{e}_H$ has the social welfare:

$$U_s(\overline{k}_s) = u_L(y_L, 0) + \int_{\underline{c}}^{\overline{k}_s} G(c)dc.$$

From (12) and (13), we also find the difference in the social welfare:

$$U_p(k_p) - U_s(\overline{k}_s) = u_L(\mathbb{E}^{\lambda}[y], e) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p} G(c)dc, \tag{15}$$

where $u_L(\mathbb{E}^{\lambda}[y], e) - u_L(y_L, 0) \geq 0$ with equality only if $e = \overline{e}(\lambda)$ from (4).

Suppose now that $\Delta(\overline{e}(1)) \in (\underline{c}, \overline{c})$, or equivalently, $D(\overline{e}(1)) \in (0, 1)$. Then there exists the separating equilibrium with $\underline{e}_H$ that has the the investment ratio $G(\overline{k}_s) = D(\overline{e}(1))$. Denote this investment ratio by $\overline{\lambda} \equiv G(\overline{k}_s) = D(\overline{e}(1))$. We next impose a condition on the slope of $D(e)$: the slope $D'(e)$ is sufficiently flat for $e \geq e^* \equiv \overline{e}(\lambda^*)$ such that a fixed point $\lambda^* = D(\overline{e}(\lambda^*))$ approximates the investment ratio $\overline{\lambda} = D(\overline{e}(1)) \in (0, 1)$. Defining the threshold k_p^* by $G(k_p^*) = \lambda^*$, we have

$$U_p(k_p^*) - U_s(\overline{k}_s) = u_L(\mathbb{E}^{\lambda^*}[y], e^*) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p^*} G(c)dc = \int_{\overline{k}_s}^{k_p^*} G(c)dc < 0. \tag{16}$$

The condition on the slope $D'(e)$ leads to two points. First, k_p^* approaches $\overline{k}_s$ and thus $U_p(k_p^*)$ approaches $U_s(\overline{k}_s)$. Second, the pooling equilibrium with e^* has overinvestment, since the conditions on $g(k_p)$ and $\Delta'(e)$ reported in Lemma 6 imply that the social welfare $U_p(k_p)$ is strictly decreasing in k_p when $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$ is sufficiently small. Thus, the condition on $D'(e)$ means that there is k_p^{**} such that $k_p^{**} < k_p^*$ and $U_p(k_p^{**})$ is greater than U_s for any separating equilibrium. If $\Delta(\overline{e}(1)) = \overline{c}$, then a similar result can be obtained. Lastly, if $\Delta(\overline{e}(1)) > \overline{c}$, even without the condition on $D'(e)$, there is a flat interval of $D(\overline{e}(\lambda))$ on the top, $D(\overline{e}(\lambda)) = 1$ on $[\lambda', 1]$, and the same conclusion follows. Once a superior form of signaling is identified, the social planner can implement the optimal policy based on Lemma 5.

Proposition 3 *(i) If $\Delta(\overline{e}(1)) > \overline{c}$, then the regulation $(\overline{E})$ maximizes social welfare.*

(ii) Suppose $\underline{c} < \Delta(\overline{e}(1)) \leq \overline{c}$. Given $D(\overline{e}(1)) > 0$, if there exists a sufficiently small $\lambda' > 0$ such that the slope $D'(e)$ is sufficiently small on $\{e : D(e) = \lambda,\ \lambda \in [\lambda', 1]\}$, then the regulation $(\overline{E})$ maximizes social welfare.

In summary, in the parameter range where $\underline{c} < \Delta(\overline{e}(1)) \leq \overline{c}$, due to the restriction on the slope $D'(e)$, we can make the following statements:[12] (i) Proposition 2 ensures that, for any separating equilibrium, there exists a pooling equilibrium that approximates the separating equilibrium in terms of the investment ratio and social welfare; (ii) Lemma 6 implies that this pooling equilibrium has overinvestment; and (iii) it follows from Proposition 1 and Lemma 6 that there exists an optimal pooling equilibrium that restricts the inefficiency of overinvestment. Therefore, there exist circumstances under which there are too many high-type workers from a welfare perspective.[13] Intuitively, the condition on $D'(e)$ corresponds to a situation in which pooling signal has a saturation point such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. Under the condition, the tension observed in pooling signal implies that it is socially preferred to reduce signaling costs than to increase high-type workers. In this case, separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal and treat high-type workers differently in the market. The inefficiency of overinvestment can then be reduced in pooling equilibrium where workers use the same signal without having to be treated differently.

We can also identify circumstances under which there are too few high-type workers from a welfare perspective. If the slope of $D(e)$ is sufficiently steep for some range, then it is uncertain that a pooling equilibrium exists, and even when a pooling equilibrium exists, it may generate too few high-type workers. To formalize this argument, suppose $\Delta(\overline{e}(1)) \in (\underline{c}, \overline{c})$. Given $G(\overline{k}_s) = D(\overline{e}(1)) \in (0, 1)$, if λ^* that satisfies $\lambda^* = D(\overline{e}(\lambda^*))$ is sufficiently smaller than $D(\overline{e}(1))$, then the term $u_L(\mathbb{E}^{\lambda^*}[y], e^*)$ in (16) approaches $u_L(y_L, 0)$, but k_p^* does not approach $\overline{k}_s$. Then $U_s(\overline{k}_s)$ is greater than U_p for any potential pooling equilibrium. This condition corresponds to a situation in which pooling signal remains sufficiently effective in generating the investment ratio further.[14] In this case, there are too few high-type workers in pooling equilibrium, and the inefficiency of

[12] If $\Delta(\overline{e}(1)) > \overline{c}$, then a pooling equilibrium exists with its investment ratio approaching $D(\overline{e}(1))$ that cannot be achieved by any separating equilibrium. Thus, we exclude this parameter range when arguing that the inefficiency of overinvestment can be reduced in pooling equilibrium.

[13] The inefficiency of overinvestment may be reduced in pooling equilibrium, $k_p < \overline{k}_s$ and $U_p(k_p) > U_s(\overline{k}_s)$, even when the slope of the function $D(\overline{e}(\lambda))$ is moderately small.

[14] If $\Delta(\overline{e}(1)) = \overline{c}$, then the separating equilibrium with $\underline{e}_H$ does not exist, but the regulation $(\underline{E})$ can support a separating equilibrium with $e_H > \underline{e}_H$ that approximates the investment ratio $D(\overline{e}(1))$. Thus, the result in Proposition 4 holds when $\Delta(\overline{e}(1)) = \overline{c}$.

underinvestment can be reduced in separating equilibrium. The following proposition reports this finding.

Proposition 4 *Suppose* $\underline{c} < \Delta(\overline{e}(1)) \leq \overline{c}$*. Given* $D(\overline{e}(1)) > 0$*, if the maximum* λ^* *that satisfies* $\lambda^* = D(\overline{e}(\lambda^*))$ *is sufficiently smaller than* $D(\overline{e}(1))$*, then the regulation* ($\underline{E}$) *maximizes social welfare.*

5 Numerical examples

In this section, we use numerical analysis and report circumstances under which the inefficiency of overinvestment can be reduced in pooling equilibrium, $k_p < \overline{k}_s$ and $U_p(k_p) > U_s(\overline{k}_s)$.

We use the utility function, $u_q(w, e) = w - c_q(e)$ for $q \in \{L, H\}$, where $c_L(e) = e^2$ and $c_H(e) = ae^2$ for $a \in (0, 1)$. Then, $k_p = \Delta(e) = u_H(0, e) - u_L(0, e) = (1 - a)e^2$. From $u_L(\mathbb{E}^\lambda[y], \overline{e}(\lambda)) = u_L(y_L, 0)$, we have $\overline{e}(\lambda) = \sqrt{B\lambda}$, where B denotes the wage gap, $B \equiv y_H - y_L$. From $u_L(y_L, 0) = u_L(y_H, \underline{e}_H)$, we find $\underline{e}_H = \sqrt{B} = \overline{e}(1)$ and $\overline{k}_s = \Delta(\underline{e}_H) = (1-a)B$. We consider an exponential CDF:

$$G(c; \tau) = \frac{1 - e^{-\tau c}}{1 - e^{-\tau}}, \; c \in [0, 1] \text{ and } \tau > 0.$$

We then have

$$D(e) = G(\Delta(e); \tau) = \frac{1 - e^{-\tau(1-a)e^2}}{1 - e^{-\tau}} \text{ and } D(\overline{e}(\lambda)) = \frac{1 - e^{-\tau(1-a)B\lambda}}{1 - e^{-\tau}}.$$

The welfare comparison between the two signaling forms in (15) becomes

$$U_p(k_p) - U_s(\overline{k}_s) = B\left(\frac{1 - e^{-\tau k_p}}{1 - e^{-\tau}}\right) - \frac{k_p}{1 - a} + \left(\frac{k_p - \overline{k}_s + e^{-\tau k_p} - e^{-\tau \overline{k}_s}}{1 - e^{-\tau}}\right).$$

For a fixed $\overline{k}_s = (1 - a)B = 0.6$, Proposition 3 indicates that, if $(1 - a)$ is sufficiently small, or if the exponential parameter τ is sufficiently large, then there exists a pooling equilibrium that is superior to any feasible separating. For different parameters, we identify $D(\overline{e}(1)) = G(\overline{k}_s; \tau)$, fixed points $\lambda^* = D(\overline{e}(\lambda^*))$, and thresholds k_p^* corresponding to $\lambda^* = G(k_p^*; \tau)$. Table 1 summarizes the outcomes.

Table 1. Fixed point values						
$(1-a)$	B	$\overline{k}_s$	τ	$D(\overline{e}\,(1))$	λ^*	k_p^*
0.6	1	0.6	3	0.8784	0.8055	0.5458
0.3	2	0.6	3	0.8784	0.8055	0.5458
0.2	3	0.6	3	0.8784	0.8055	0.5458
0.3	2	0.6	2	0.8082	0.5797	0.4334
0.3	2	0.6	3	0.8784	0.8055	0.5458
0.3	2	0.6	4	0.9262	0.9016	0.5798

For $e = \overline{e}(\lambda^*)$, $u_L(\mathbb{E}^{\lambda^*}[y], e) - u_L(y_L, 0) = 0$ in (15) and $\lambda^* = G\left(k_p^*; \tau\right) < G\left(\overline{k}_s; \tau\right) = D(\overline{e}\,(1))$. Thus, for $k_p = k_p^*$, we have

$$B\left(\frac{1-e^{-\tau k_p^*}}{1-e^{-\tau}}\right) - \frac{k_p^*}{1-a} = 0, \text{ and } U_p\left(k_p^*\right) - U_s\left(\overline{k}_s\right) < 0.$$

However, for $(1-a)$ sufficiently small, there exist pooling equilibria with $k_p < k_p^*$ and $U_p\left(k_p\right) - U_s(\overline{k}_s) > 0$. Table 2 reports this result.

Table 2. Change in $(1-a)$					
$(1-a)$	B	$\overline{k}_s$	τ	$U_p(k_p^*) - U_s(\overline{k}_s)$	$U_p(k_p) - U_s(\overline{k}_s)$ for $k_p = 0.3$
0.6	1	0.6	3	-0.1087	-0.1066
0.3	2	0.6	3	-0.1707	0.0180
0.2	3	0.6	3	-0.2327	0.1425

Figure 2 illustrates the outcomes when $(1-a)$ decreases (i.e., $\Delta'(e) = 2(1-a)e$ decreases) while holding $\overline{k}_s = (1-a)\,B$ and τ fixed. The function $D(\overline{e}\,(\lambda))$ then remains the same, but the differential $U_p(k_p) - U_s(\overline{k}_s)$ shifts up on $[0, k_p^*]$ and results in $U_p(k_p) - U_s(\overline{k}_s) > 0$. Table 3 reports that for the exponential parameter τ sufficiently small, there exist pooling equilibria with $k_p < k_p^*$ and $U_p(k_p) - U_s(\overline{k}_s) > 0$.

Table 3. Changes in the exponential parameter					
$(1-a)$	B	$\overline{k}_s$	τ	$U_p(k_p^*) - U_s(\overline{k}_s)$	$U_p(k_p) - U_s(\overline{k}_s)$ for $k_p = 0.3$
0.3	2	0.6	2	-0.2276	-0.1602
0.3	2	0.6	3	-0.1707	0.0180
0.3	2	0.6	4	-0.1143	0.1717

Figure 3 illustrates the outcomes when τ increases. An increase in τ shifts $G(\Delta(e); \tau)$ such that $D(\overline{e}\,(\lambda))$ shifts up with a flatter slope for larger λ, and the differential $U_p(k_p) - U_s(\overline{k}_s)$ shifts up

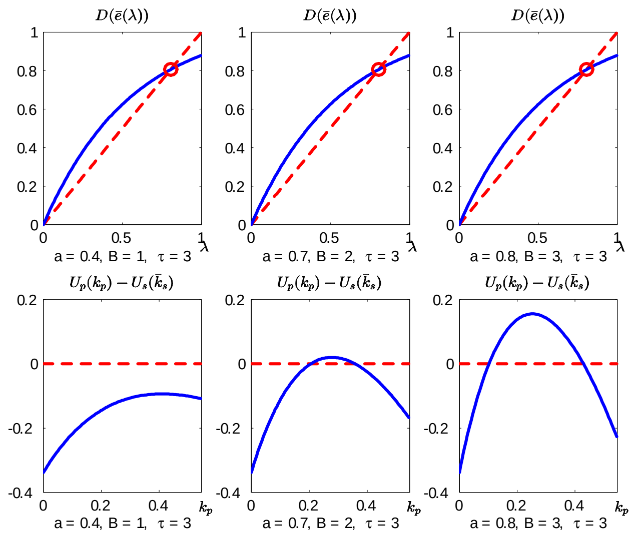

Figure 2: When $(1-a)$ decreases

on $[0, k_p^*]$. As a result, there exist pooling equilibria with $k_p < k_p^*$ such that $U_p(k_p) - U_s(\bar{k}_s) > 0$.

6 Conclusions

In this paper, we examine a situation in which each worker endogenously determines the quality of labor through a private investment decision, and the consequent asymmetric information in the market in return causes a moral hazard problem in the investment stage. We consider a model in which signaling is a natural option for workers and socially beneficial due to its influence on the workers' investment. We offer a theoretical foundation for the argument that there are too many high-type workers from a welfare perspective. We identify circumstances under which pooling signal reaches a saturation point such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. In this case, it is socially preferred to reduce signaling costs than to increase high-type workers, and separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal to treat high-type workers differently in the market. The inefficiency of overinvestment can be reduced only in pooling equilibrium where workers use the same signal without having to be treated differently. We also identify circumstances under which pooling signal remains sufficiently effective in generating the investment ratio further. In this case, pooling equilibrium

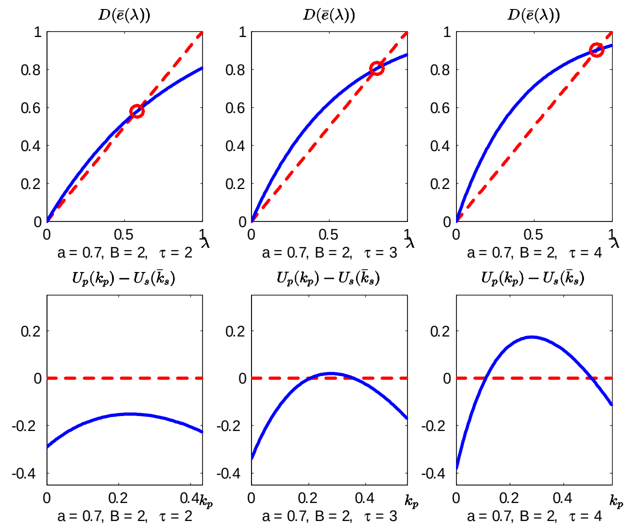

Figure 3: When τ increases

generates too few high-type workers, and the inefficiency of underinvestment can be reduced in separating equilibrium.

Our findings are based on a model that has fairly standard features. Thus, the main theme of our model can be generally extended for the setting in which an investment decision endogenously generates asymmetric information about the quality of products in the market, and this asymmetric information in the market in return causes a moral hazard problem in the investment stage.

7 Appendix

Proof of Lemma 4. In the benchmark model, a separating equilibrium (e_L, e_H) generates the social welfare:

$$\lambda u_H (y_H, e_H) + (1-\lambda) u_L (y_L, 0) .$$

Since $u_H (y_H, e_H)$ is strictly decreasing in $e_H \in [\underline{e}_H, \bar{e}_H]$, the least costly signaling for type H, $\underline{e}_H$, generates the highest social welfare in the separating equilibrium:

$$U_s = \lambda u_H (y_H, \underline{e}_H) + (1-\lambda) u_L (y_L, 0) .$$

A pooling equilibrium, $e_H = e_L = e$, generates the social welfare:

$$U_p = \lambda u_H \left(\mathbb{E}^{\lambda} [y] , e \right) + (1-\lambda) u_L \left(\mathbb{E}^{\lambda} [y] , e \right) .$$

In comparison, the ban on signaling leads to the same wage $\mathbb{E}^{\lambda}[y]$ and generates the social welfare:

$$U_0 = \lambda u_H\left(\mathbb{E}^{\lambda}[y], 0\right) + (1-\lambda)u_L\left(\mathbb{E}^{\lambda}[y], 0\right).$$

For a separating equilibrium, since $u_H(y_H, 0) > u_H(y_H, \underline{e}_H)$, we have

$$\lambda u_H(y_H, 0) + (1-\lambda)u_L(y_L, 0) > U_s.$$

Thus, to verify the result $U_0 > U_s$, it suffices to show that

$$\lambda u_H\left(\mathbb{E}^{\lambda}[y], 0\right) + (1-\lambda)u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - [\lambda u_H(y_H, 0) + (1-\lambda)u_L(y_L, 0)] \geq 0.$$

The LHS of this inequality becomes

$$\begin{aligned}
&\lambda[u_H\left(\mathbb{E}^{\lambda}[y], 0\right) - u_H(y_H, 0)] + (1-\lambda)[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_L, 0)] \\
&= \lambda[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_H, 0)] + (1-\lambda)[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_L, 0)] \\
&= u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - [\lambda u_L(y_H, 0) + (1-\lambda)u_L(y_L, 0)] \geq 0.
\end{aligned}$$

The first equality follows from the assumption that the utility gain from any wage increase is type-irrelevant, and the last inequality is given by concavity of u_L in w. For a pooling equilibrium, for any $e > 0$, it is immediate that $U_0 > U_p$. ■

Proof of Proposition 1. Suppose first that there exists $\lambda \in (0,1)$ such that $D(\overline{e}(\lambda)) \geq \lambda$. Define a correspondence $\Psi : [0,1] \rightrightarrows [0,1]$ using (9) such that

$$\Psi(\lambda) \equiv \{x \in [0,1] : x = D(e) \text{ for } e \in [0, \overline{e}(\lambda)]\}.$$

Thus, an equilibrium fraction of type H, λ^*, is a fixed point of Ψ, $\lambda^* \in \Psi(\lambda^*)$. Since $D(e) \in (0,1)$ is an increasing function of e, the correspondence can be rewritten as $\Psi(\lambda) = [0, D(\overline{e}(\lambda))]$, and the condition implies the existence of $\lambda^* \in (0,1)$ such that $\lambda^* \in \Psi(\lambda^*)$ and $\left(k_p^*, e^*\right)$ is derived from $G\left(k_p^*\right) = D(e^*) = \lambda^*$. Suppose next that there exists a pooling equilibrium and $D(\overline{e}(\lambda)) < \lambda$ for all $\lambda \in (0,1)$. Then only a boundary pooling equilibrium with $\lambda = 0$ or $\lambda = 1$ exists, which causes a contradiction. ■

Proof of Proposition 2. *(i)* Suppose $\Delta(\overline{e}(1)) > \overline{c}$. Since $\overline{e}(\lambda)$ is an strictly increasing function of $\lambda \in (0,1)$, there exists a unique $\lambda' < 1$ such that $\Delta\left(\overline{e}(\lambda')\right) = \overline{c}$. This implies that $D(\overline{e}(\lambda)) = 1$ and $D(\overline{e}(\lambda)) > \lambda$ for $\lambda \in [\lambda', 1)$, and Proposition 1 implies the existence of a pooling equilibrium with λ sufficiently close to 1.

(ii) If $D(\bar{e}(1)) \in (0,1)$ and there exists a sufficiently small $\lambda' > 0$ such that $dD(\bar{e}(\lambda))/d\lambda > 0$ is sufficiently small on $[\lambda', 1]$, then there exists $\lambda^* \in [\lambda', 1)$ such that $D(\bar{e}(\lambda^*)) = \lambda^*$ with λ^* sufficiently close to $D(\bar{e}(1))$. If $D(\bar{e}(1)) = 1$, $dD(\bar{e}(\lambda))/d\lambda < 1$ at $\lambda = 1$ is sufficient to have a pooling equilibrium with λ sufficiently close to 1. ■

Proof of Lemma 5. Suppose that the government uses the regulation ($\underline{E}$) such that a pair $(u_H(y_H, \underline{E}) - u_L(y_L, 0), \underline{E})$ satisfies (8) and is a separating equilibrium. If there is a separating equilibrium with $e_H > \underline{E}$, then the equilibrium cannot satisfy the intuitive criterion: a high-type worker can attain a higher payoff by deviating from the separating equilibrium, and a low-type worker cannot imitate the action of the high-type worker. For any deviation $e' \in (\underline{E}, e_H)$, we find that

$$u_H(y_H, e') > u_H(y_H, e_H) \text{ and } u_L(y_L, 0) > u_L(y_H, e').$$

The former inequality follows from $e' < e_H$, and the latter is from $u_L(y_L, 0) = u_H(y_H, \underline{e}_H) > u_L(y_H, e')$ for $e' > \underline{E} \geq \underline{e}_H$.

Suppose next that the government uses the regulation ($\overline{E}$) such that a pair $(u_H(0, \overline{E}) - u_L(0, \overline{E}), \overline{E})$ satisfies (9) and is a pooling equilibrium. Let $\widehat{E}$ satisfying $u_L(w, \widehat{E}) = u_L(y_H, \overline{E})$, where $w \equiv \theta y_H + (1-\theta) y_L$ is the pooling's wage. Then, for each $e \in [\widehat{E}, \overline{E}]$, we have $u_L(w, e) \leq u_L(w, \widehat{E}) = u_L(y_H, \overline{E}) \leq u_L(y_H, e')$ for all $e' \leq \overline{E}$. Hence, such e satisfies the criterion, since there is no $e' \leq \overline{E}$ such that $u_L(w, e) > u_L(y_H, e')$. Now, we show that there exists $\overline{E}$ such that any $e \in [\underline{E}, \widehat{E})$ does not satisfy the criterion. Choose $\underline{E}$ satisfying $u_H(w, \underline{E}) < u_H(y_H, \overline{E})$. Suppose that there is a pooling equilibrium with such e. Then, $u_L(w, e) > u_L(w, \widehat{E}) = u_L(y_H, \overline{E})$, and $u_H(w, e) \leq u_H(w, \underline{E}) < u_H(y_H, \overline{E})$. A type H worker can attain a higher payoff by deviating from the pooling equilibrium to $\overline{E}$, and a type L worker cannot imitate the action of the type H worker. Hence, $[\underline{E}, \widehat{E}) \cup \{\overline{E}\}$ yields a unique equilibrium. ■

Proof of Lemma 6. The result is immediate for a sufficiently small $\Delta'(e) > 0$. We thus focus on the condition on $g(k_p)$. Let $g(k_p) = 0$. Then, given $\Delta'(e) = \partial u_H/\partial e - \partial u_L/\partial e$,

$$U_p'(k_p) = G(k_p) + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)} < 1 + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)} = \frac{1}{\Delta'(e)} \left(\Delta'(e) + \frac{\partial u_L}{\partial e} \right) = \frac{1}{\Delta'(e)} \frac{\partial u_H}{\partial e} < 0.$$

Hence, for a sufficiently small $g(k_p) > 0$, $U_p'(k_p) < 0$. ■

Proof of Proposition 3. *(i)* $\Delta(\bar{e}(1)) > \bar{c}$. From Proposition 1, there does not exist a separating equilibrium with $\underline{e}_H$, but from Proposition 2 (i), there exists a pooling equilibrium with λ sufficiently close to 1. The pooling's signal level e corresponding to the threshold $\bar{c}$ is given as $\overline{c} = \Delta(e)$, and $\Delta(\overline{e}(1)) > \overline{c} = \Delta(e)$ implies $\overline{e}(1) > e$. Recall the definition of $\overline{e}(\lambda)$ such that $u_L(\mathbb{E}^{\lambda}[y], \overline{e}(\lambda)) = u_L(y_L, 0)$ from (5). By $\overline{e}(1) > e$, $u_L(\mathbb{E}^1[y], e) > u_L(\mathbb{E}^1[y], \overline{e}(1)) = u_L(y_L, 0)$. Then,

$$\begin{aligned} U_p(\overline{c}) - U_s(\overline{c}) &= u_L(\mathbb{E}^1[y], e) - u_L(y_L, 0) + \int_{\overline{c}}^{\overline{c}} G(c)dc \\ &= u_L(\mathbb{E}^1[y], e) - u_L(y_L, 0) > 0, \end{aligned}$$

and $U_s(\overline{c}) > U_s$ for any feasible separating's social welfare U_s. Hence, there exists k_p sufficiently close to $\overline{c}$ such that $U_p(k_p) > U_s$ for any feasible separating's social welfare U_s. The result follows from Lemma 5.

(ii) $\underline{c} < \Delta(\overline{e}(1)) \leq \overline{c}$. We divide the proof into two cases.

Case 1. $\Delta(\overline{e}(1)) = \overline{c}$. From Proposition 1, there does not exist a separating equilibrium with $\underline{e}_H$. If $D'(e)$ is sufficiently small at e with $D(e) = 1$, from Proposition 2 (ii), there exists a pooling equilibrium with λ sufficiently close to 1. The pooling's signal level e corresponding to the threshold $\overline{c}$ is given as $\overline{c} = \Delta(e)$, and $\Delta(\overline{e}(1)) = \overline{c} = \Delta(e)$ implies $\overline{e}(1) = e$. Then,

$$\begin{aligned} U_p(\overline{c}) - U_s(\overline{c}) &= u_L(\mathbb{E}^1[y], e) - u_L(y_L, 0) + \int_{\overline{c}}^{\overline{c}} G(c)dc \\ &= u_L(\mathbb{E}^1[y], e) - u_L(y_L, 0) = 0, \end{aligned}$$

and $U_s(\overline{c}) > U_s$ for any feasible separating's social welfare U_s. If $D'(e)$ is sufficiently small at e with $D(e) = 1$, from Lemma 6, $U_p(k_p)$ is strictly decreasing at $\overline{c}$. Hence, there exists k_p sufficiently close to $\overline{c}$ such that $U_p(k_p) > U_s$ for any feasible separating's social welfare U_s. The result follows from Lemma 5.

Case 2. $\underline{c} < \Delta(\overline{e}(1)) < \overline{c}$. From Proposition 1, there exists a separating equilibrium with $\underline{e}_H$, which has a threshold $\overline{k}_s = \Delta(\overline{e}(1))$. Denote the separating's human capital accumulation by $\overline{\lambda} \equiv G(\overline{k}_s)$. Now, if $D'(e)$ is sufficiently small on $\{e : D(e) = \lambda, \lambda \in [\lambda', 1]\}$, from Proposition 2 (ii), there exists a pooling equilibrium. In particular, choose a fixed point λ^* sufficiently close to $\overline{\lambda}$ such that $D(\overline{e}(\lambda^*)) = \lambda^*$. Denote k_p^* satisfying $G(k_p^*) = \lambda^*$. It follows from $D(\overline{e}(\lambda^*)) = \lambda^* = D(e)$ that $\overline{e}(\lambda^*) = e$, and $u_L(\mathbb{E}^{\lambda^*}[y], e) = u_L(\mathbb{E}^{\lambda^*}[y], \overline{e}(\lambda^*)) = u_L(y_L, 0)$. Then,

$$\begin{aligned} U_p(k_p^*) - U_s(\overline{k}_s) &= u_L(\mathbb{E}^{\lambda^*}[y], e) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p^*} G(c)dc \\ &= \int_{\overline{k}_s}^{k_p^*} G(c)dc < 0. \end{aligned}$$

However, for a fixed $\overline{k}_s$, as $D'(e) \to 0$ for all e satisfying $D(e) \geq \lambda^*$, so $k_p^* \to \overline{k}_s$, which leads to $U_p(k_p^*) - U_s(\overline{k}_s) \to 0$. In addition, for a fixed $\overline{k}_s$, as $D'(e) \to 0$ for all e satisfying $D(e) \geq \lambda^*$, from Lemma 6, $U_p(k_p)$ is strictly decreasing at k_p^*. Hence, there exists k_p sufficiently close to k_p^* such that $U_p(k_p) > U_s$ for any feasible separating's social welfare U_s. The result follows from Lemma 5. ■

References

Bedard, K. (2001), Human capital versus signaling models: university access and high school dropouts, Journal of Political Economy 109, 749-775.

Cho, I.-K. and Kreps, D. M. (1987), Signaling games and stable equilibria, Quarterly Journal of Economics 102, 179-222.

Cunha, F. Karahan, F. and Soares, I. (2011), Returns to skills and the college premium, Journal of Money, Credit and Banking 43, 39-86.

Daley, B. and Green, B. (2014), Market signaling with grades, Journal of Economic Theory 151, 114-145.

Fang, H. (2001). Social culture and economic performance, American Economic Review 91, 924-937.

Fang, H. (2006). Disentangling the college wage premium: Estimating a model with endogenous education choices, International Economic Review 47, 1151-1185.

Goldin, C. and Katz, L. F. (2007a), The race between education and technology: The evolution of U.S. educational wage differentials, 1890 to 2005, NBER Working Paper 12984.

Goldin, C. and Katz, L. F. (2007b), Long-run changes in the wage structure: Narrowing, widening, polarizing, Brookings Papers on Economic Activity 2: 135–65.

Goldlücke, S. and Schmitz, P. (2014), Investments as signals of outside options, Journal of Economic Theory 150, 683-703.

Hermalin, B. E. (2013), Unobserved investment, endogenous quality, and trade, RAND Journal of Economics 44, 33-55.

Kawai, K. (2014), Dynamic market for lemons with endogenous quality choice by the seller, Games and Economic Behavior 84, 152-162.

Kreps, D. and Sobel, J. (1994), Signaling. In Handbook of Game Theory, vol. 2, ed. R.J. Aumann and S. Hart. Amsterdam: North-Holland.

Lang, K. and Kropp, D. (1986), Human capital versus sorting: the effects of compulsory attendance laws, Quarterly Journal of Economics 101, 609-624.

Riley, J.G. (1979), Testing the educational screening hypothesis, Journal of Political Economy 87, s227-s252.

Riley, J.G. (2001), Silver signals: twenty-five years of screening and signaling, Journal of Economic Literature 39, 432-478.

Spence, A. M. (1973), Job market signaling, Quarterly Journal of Economics 87, 355-374.

Spence, A. M. (1974), Market Signaling. Harvard University Press, Cambridge, MA.

Taber, C. R. (2001), The rising college premium in the eighties: Return to college or return to unobserved ability? Review of Economic Studies 68, 665-691.

Tyler, J. H. Murnane, R. J. and Willett, J. B. (2000), Estimating the labor market signaling value of the GED, Quarterly Journal of Economics 115, 431-468.

Walker, I. and Zhu, Y. (2008), The college wage premium and the expansion of higher education in the UK, The Scandinavian Journal of Economics 110, 695-709.

Wolpin, K.I. (1977), Education and screening, American Economic Review 67, 949-958.