3-2019

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Citation

HO, Christine. Optimal social insurance with informal child care. (2019). *The B.E. Journal of Economic Analysis & Policy*. 1-49.

Available at: https://ink.library.smu.edu.sg/soe_research/2067

Optimal Social Insurance with Informal Child Care*

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January 2017

Abstract

This study investigates whether the possibility of engaging in household child care may exacerbate the incentives of parents and grandparents to falsely claim disability benefits. I present an efficient implementation case for subsidizing formal child care costs of the disabled and propose an implementation of the optimal scheme that consists of capped formal day care subsidies, non-linear income taxation and asset-testing. I calibrate a multi-generational family model with persistence in privately observed shocks to match key features of the US labor and child care markets, and find that day care subsidies may lead to sizeable cost savings.

JEL: H21, H24, H31, J14, J22
Keywords: social insurance, day care subsidies, multi-member family

*I would like to thank James Banks, David Blau, Samuel Belinski, Richard Blundell, Tomoki Fujii, Nicolas Jacquet, Claus Kreiner, Guy Laroque, Kathleen McGarry, Costas Meghir, Nicola Pavoni, Ian Preston, Thomas Sargent, Conny Wunsch, seminar attendees at University College London, Sciences Po., Boston College, University of Essex, University of St. Gallen, Academia Sinica, Singapore Management University, ESWC, PET Conference, Optimal Taxation and Public Policies in Macroeconomics Workshop University of Macau, EEA-ESEM Conference, RES Conference, IAREP Workshop, and LSE Public Economics presentation for insightful comments and suggestions. All mistakes remain my own.

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Disability insurance and child related tax benefits are important components of government policy in the United States. Social Security disability benefit payments totalled \$135 billion and made up 17.5% of Social Security benefits in 2013 (US Budget, FY2014, Table 13.1). There were 10.1 million recipients of Social Security Disability Insurance (SSDI) benefits and 9.2 million recipients of Social Security Income (SSI) benefits (SSA, 2013). Child related tax benefits, such as the Earned Income Tax Credit (EITC) and dependent tax exemptions, are also relatively large in scope and magnitude. Families' tax savings from those two programs amounted to \$104 billion in 2013 (Maag, 2013). The EITC benefited 26 million working families and made an estimated 13.1 million children less poor (CBPP, 2014).

In contrast, US price related child care subsidy programs that are linked to formal child care costs are relatively small. Families' tax savings from the Child and Dependent Care Tax Credit (CDCTC) totalled \$4 billion in 2013 (Maag, 2013) while the budget request for the Child Care and Development Fund (CCDF) was \$6 billion to support 1.4 million children in 2015 (DHHS, 2014). Meanwhile, child care subsidies tend to be relatively large in Scandinavia (Guner et al., 2014; Havnes and Mogstad, 2011), whereas recent policy debates in Europe expressed a desire to move towards universal day care. The European Union stated as policy goal "to provide childcare by 2010 to at least 90% of children between 3 years old and the mandatory school age" to encourage labor force participation of mothers (European Council, 2002).

This study presents an efficient implementation case for subsidizing formal child care costs of the disabled and proposes an implementation of the optimal scheme that consists of capped formal day care subsidies, combined with non-linear income taxation and asset-testing. The framework is a dynamic Mirrleesian one, where the government seeks to provide social insurance to multi-member households whose adult members, such as parents and grandparents, are subject to privately observed disability shocks. Healthy household members may allocate their time between working on the primary labor market and household child care activities. I employ a recursive formulation with history dependence in privately observed shocks, where the government minimizes expected costs subject to delivering a given level of promised utility to each household and subject to preserving the work incentives of healthy members.

In an optimal social insurance framework with private disability shocks, disability benefits cannot be too generous or else, healthy individuals may be tempted to mimic the disabled by not working and claim the benefits.[1] With household child care activities, healthy household members who mimic the disabled by not working, may not only claim disability benefits, but also save on formal child care costs by looking after the children themselves. In addition, household child care interact with households' incentives to engage in sub-optimal savings. The possibility of engaging in household

[1]Social Security defines disability as the inability to engage in substantial gainful activity due to an physical or psychological impairment. Those include back pain and mental illness which are hard to verify. Among SSDI beneficiaries, 27% received the benefits on the grounds of musculoskeletal diseases and 35% on the grounds of mental disorders (SSA, 2012). The controlling evidentiary weight is placed on an applicants' own health practitioner, thereby making the screening process easier for applicants (Autor and Duggan, 2006; Hu et al., 2001). The literature has documented evidence of moral hazard in long-term disability insurance (Autor et al., 2014; Gruber, 2000; Haveman et al., 1991; Low and Pistaferri, 2015; Maestas et al., 2013).

child care therefore exacerbates the incentives to mimic the disabled. Implementation of the constrained optimal allocations in a decentralized economy thus needs to account for such joint deviation incentives. The use of child care subsidies helps to directly counterbalance households' child care deviation incentives by making such activities unattractive relative to formal child care.

In the disability insurance context studied in this paper, the case for such subsidies stem from the implementation exercise as opposed to an optimal wedges argument traditionally employed in the optimal income tax literature. In particular, the case for child care subsidies does not involve optimally distorting child care choices in the optimal scheme. Rather, the relevant "wedges" are those of would be mimickers, who have an incentive to jointly deviate on their labor supply and child care in a decentralized economy. This paper shows that child care subsidies help implement the optimal scheme, where it is *as though* the government could control household members' child care, in a decentralized economy where the government may not actually monitor household members' child care.

I propose an implementation of the constrained optimum with capped formal day care subsidies, non-linear income taxation and asset-testing in a decentralized economy. The subsidized day care is equivalent to a full price subsidy on formal child care costs and may be provided through free public day care up to a cap at the optimal level of formal child care use. Such scheme implements the constrained optimum for single (grand)parent households and for multi-member households with high earnings capacity, *as though* the government could control household members' child care, when in fact, it precludes from such monitoring needs. In the context of multi-member households, formal day care subsidies may not only help counter the incentive issues of mothers but also the incentive issues of related family members.

The incentives to mimic the disabled are more perverse for multi-member households with at least one healthy member with low earnings capacity. The household child care incentives of such members interact with those of mimickers. Even though day care subsidies ensure that the optimal level of formal child care is used, members may still engage in sub-optimal individual levels of household child care. In the absence of monitorable household child care, a costlier constrained optimum may be implemented for such households, that is, one equivalent to the solution to the optimal program with hidden household child care activities.

The quantitative exercise incorporates demographic heterogeneity across households in terms of presence of parents and grandparents, marital status, gender, and education, and number and age of children. The multi-generational family model is calibrated to match key features of the US labor and child care markets given the current tax and benefit system. The exercise seeks to quantify the government's potential cost savings from using a policy tool (day care subsidies) that directly smoothens the exacerbation of incentives caused by household child care. Such cost savings stem from single (grand)parent and multi-member households with high earnings capacity. The benchmark model is the constrained optimal one where the government may indirectly control child care activities through subsidized day care whereas the comparison model is one where the government may not control child care activities.

I find that the proposed implementation with subsidized day care may lead to average cost savings of 0.05% to 3.31%, relative to the case that would deliver the same level of welfare to each household but where the government may not use day care subsidies. The higher the relative exacerbation of incentives, the higher the cost savings from subsidized day care. I find higher cost savings for single parent households and for multi-generational households with both a single parent and a single grandparent present. Average cost savings ranged between 0.23% and 2.2% for single mothers with higher cost savings among less educated mothers. Cost savings for single fathers ranged between 0.1% and 1.23%, for two parent households between 0.05% and 0.8%, and for households with a grandparent present between 0.2% and 3.31%. Such cost savings increase with formal child care costs since the incentives to shirk are further exacerbated.[2]

The verticalization of families due to greater life expectancy, and the increasing prevalence of multi-generational households with children aged below 18 makes it important for policy to take intergenerational linkages into consideration (Bengtson, 2001; Ho, 2015a). From US Census, approximately 7.5 million (10%) children lived with a grandparent in 2010. Meanwhile, 21.1% of pre-primary school aged children with a working mother benefited from grandparent provided child care averaging 23 hours per week (Laughlin, 2013). Data from the Health and Retirement Study indicates that 44% of grandmothers receiving Social Security disability benefits provided at least 2 hours of grandchild care per week while 30% of such grandmothers provided at least 20 hours of grandchild care per week. Grandparent provided child care has been found to increase labor supply of mothers (Cardia and Ng, 2003; Compton and Pollak, 2014; Maurer-Fazio et al., 2011) while child care needs have been found to influence labor supply of grandparents (Marcotte and Wang, 2007; Rupert and Zanella, 2014).

This paper is related to the optimal social insurance literature and to the optimal tax literature with multi-dimensional choice (Beaudry, P., Blackorby, C., and Szalay, 2009; Besley and Coate, 1995; Choné and Laroque, 2011; Diamond and Mirrlees, 1978; Kleven et al., 2000; Rothschild and Scheuer, 2016). Albanesi and Sleet (2006) propose a recursive labor and wealth tax system that implements a dynamic Mirrlees optimum, where an agent experiences identically and independently distributed private skills shocks. Golosov and Tsyvinski (2006) make a case for asset-testing in a dynamic model of social insurance where an agent is subject to privately observed absorbing disability shocks. Álvarez-Parra and Sánchez (2009) design an optimal unemployment insurance scheme in the presence of a hidden labor market.

The paper is also related to the literature on child care subsidies. Proponents of child care subsidies have argued for such subsidies on the grounds of encouraging mothers to become self-sufficient, in order to counteract the disincentive effects of income taxation, and to promote higher quality child care (Barnett, 1993; Blau, 2003; Currie, 2001; Domeij and Klein, 2012; Heckman and Cunha, 2010). There is general agreement in the literature that child care subsidies are positively associated with higher labor supply of mothers (Bick, 2015; Guner et al., 2014; Havnes and Mogstad, 2014). Child care subsidies also lead to a substitution from relative care to formal child care (Havnes and Mogstad, 2011; Tekin, 2007), and to a possible increase in the labor supply of coresident

[2]Pareto improvement may be achieved through redistribution of the cost savings as lump sum transfers. I do not seek to model redistribution across the different family types but rather focus on quantifying the efficiency gains from subsidized day care for each household structure.

grandmothers (Ho, 2015b). This suggests that day care subsidies may lead to welfare gains by increasing the labor supply of both parents and grandparents.

This paper studies a dynamic Mirrleesian framework in which multi-member households may engage in primary labor market and household child care activities. I argue for the use of an additional policy tool, day care subsidies, that directly circumvents the child care incentives of mimickers. Such subsidies may not be replicated by non-linear income taxation or disability benefits: Whereas decreasing income taxes and disability benefits may discourage would be mimickers, such a scheme would be costlier than one that directly circumvents the relevant child care incentives. This study provides a novel justification for child care subsidies that stem from an efficient implementation argument, that is, deterring parents and grandparents from "shirking".

Section I. describes presents the recursive formulation of the government problem in a centralized economy. In Section II., I present a case for subsidizing formal child care costs of the disabled and propose an implementation of the constrained optimum in a decentralized economy, comprising of capped formal day care subsidies, asset-testing and non-linear income taxation. Section III. describes the quantitative exercise and Section IV. presents numerical results from simulations of the optimal policy as well as from counterfactual policies without day care subsidies. Section V. concludes.

I. Model

In this section, I present the centralized government problem where it is as though the government decides on the allocations of consumption, household child care and labor supply. In Section II., I discuss an implementation of the constrained optimal allocations in a decentralized economy where households make their own choices. In a decentralized set up, agents may have incentives to deviate from the optimal allocations, for example, by jointly mimicking the disabled and engaging in sub-optimal household child care activities. I argue that the use of child care subsidies may help counterbalance such deviation incentives in the implementation exercise.

The centralized set up is a dynamic Mirrleesian model of social insurance where household members are subject to privately observed absorbing disability shocks. Healthy household members can allocate their time between work on the labor market and household child care activities whereas disabled household members can neither work on the labor market nor at home. Households can meet child care needs through healthy members' household child care or by purchasing formal child care at an exogenously given price. The first incurs a disutility of effort cost and the second incurs a budget cost. The framework is one where the government seeks to minimize the expected costs of social insurance subject to delivering a given level of promised utility to the household and subject to preserving work incentives.

A. *Agents*

Agents in the model are a continuum of ex-ante identical family households. I use the terms family and household interchangeably throughout the text. Let I be the total number of adults in the household and denote the adult members of the family by the index $i \in \{1,2,...,I\}$. For example, $i=1$ could represent the father and $i=2$ the mother. I consider a finite horizon time frame with T discrete periods, $t=0,1,...,T$, during which the household's adult composition is fixed and child care needs may be relevant to the family. The family household is a decision unit in the model.[3]

Child care needs and time allocation In each period, households have child care needs of $n_t \geq 0$ that evolve deterministically over time. n_t may be interpreted as the required time that children need to spend in child care, either through household child care or through formal child care. Healthy adults can devote time (effort) to the labor market $l_t^i \geq 0$, or to household child care $h_t^i \geq 0$. When a healthy adult works on the labor market, the latter earns $y_t^i = w_t^i l_t^i$, where $w_t^i \geq 0$ is wage per unit of labor. The remaining child care needs not covered by household child care (i.e., the difference between child care needs and the sum of household child care supplied by all family members: $n_t - \sum_i h_t^i$) is purchased by households from the formal child care market at price $p_t > 0$ per unit. w_t^i and p_t are exogenous and evolve deterministically over time.

Preferences Family preferences are separable across periods and the future is discounted at rate $\beta \in (0,1)$. Within period family preferences are separable in family consumption $c_t > 0$ and effort of each adult member $e_t^i = l_t^i + h_t^i$. Intra period family preferences are given by:

$$u(c_t) - \sum_i v_i\left(e_t^i\right),$$

where u is a concave function of consumption with $u' > 0$, $u'' < 0$ and $\forall i$, v_i is a convex function of effort with $v_i' \geq 0$, $v_i'' \geq 0$. Assume that $v_i(0) = v_i'(0) = 0$.

Disability states Adults are subject to absorbing disability shocks, that is, once an adult is disabled, the disability is permanent. Disabled adults can neither work on the market nor at home so that $e_t^i = 0$. Such assumptions are based on the fact that Social Security pays disability benefits only for long-term total disability and less than 0.5% of SSDI and SSI beneficiaries leave the disability rolls and return to work (42 U.S.C. 1320b-19, The Public Health and Welfare).[4]

[3] There is evidence that child care subsidies targeting young mothers may affect the labor supply of coresident grandmothers and that public assistance crowds out family support (Abbott et al., 2013; Ho, 2015b; Rosenzweig and Wolpin, 2007; Schoeni, 2002). For the sake of tractability, I assume that the family household is a decision unit (Cardia and Ng, 2003). The efficiency case for subsidizing formal child care costs would hold irrespective, as long as the possibility of engaging into household child care exacerbates mimicking incentives. I also focus on a finite horizon model as opposed to an infinite horizon overlapping generations (OLG) model so as to model policy based on the current generation only. In a dynamic optimal social insurance OLG model, efficient allocations are tied to the behavior of previous generations so that the standard immizeration result would hold (Atkeson and Lucas, 1992; Ho, 2013), which may be a normatively unappealing policy implication (Sleet and Yeltekin, 2005).

[4] The case for day care subsidies would still hold if disability shocks were not absorbing or if agents were subject to multiple partial disability shocks or if the disabled could engage in household child care activities (Ho, 2013).

In any period, a family may have from zero to all members disabled. Let S_t denote the set of relevant disability states that a family may be subject to in period t and denote the state of family disability by the index $s_t \in S_t$, which captures the number and the identity of disabled members. Note that by the absorbing nature of disability shocks, the relevant state set in the current period, S_t, would depend of the previous period's state set, S_{t-1}. In particular, if one family member is disabled in the previous period, then the same member must be disabled in the current period. Let $\pi_t(s_t, s_{t-1})$ be the conditional probability that the family is in state s_t in the current period given state s_{t-1} in the previous period. Normalize $\sum_{s_t \in S_t} \pi_t(s_t, s_{t-1}) = 1$.

B. Government

The government is risk-neutral and provides social insurance to families in the least costly way possible, while guaranteeing the family an expected utility level of at least V. The initial promised utility, V, can be interpreted as an exogenously determined parameter capturing the generosity level of the welfare system. The government discounts future costs at rate $\beta = \frac{1}{1+r}$, where r is the exogenous interest rate.

Information structure The government knows the distribution of disability shocks but actual disability shocks are private information to the household. The government may verify household assets A_t, formal child care costs $f_t = p_t\left(n_t - \sum_i h_t^i\right)$ and household members' labor supply l_t^i.

C. Government Problem

By the revelation principle, one can focus on a direct mechanism where in each period, households declare their disability state $s_t \in S_t$. The government then specifies allocations according to the declared state. The optimal allocations may be solved recursively as the government minimizing expected costs of social insurance subject to promise keeping, threat keeping, and incentive compatibility constraints.

Allocations The centralized set up is as though the government takes all production and assets from agents, and decides on the allocations of consumption, household child care and labor supply of agents. In particular, in each period, for a declaration of state s_t, the government specifies for all household members, labor supply $l_t^i(s_t)$ and household child care $h_t^i(s_t)$, and allocates transfers of $b_t(s_t)$ to households. Households then incur formal child care costs of $f_t(s_t) = p_t\left(n_t - \sum_i h_t^i(s_t)\right)$ and consume $c_t(s_t) = b_t(s_t) - p_t\left(n_t - \sum_i h_t^i(s_t)\right)$. The government also delivers continuation utility $V_{t+1}(s_t)$ to households that truthfully report s_t and threatened continuation utilities $\tilde{V}_{t+1}(s_t, \tilde{s}_t), \forall \tilde{s}_t \in S_t$ to households that are in state $\tilde{s}_t$ but falsely report s_t.

Promise keeping constraint The continuation utility allocated in the current period becomes the expected discounted utility delivered to truthful agents the following period. The promise keeping constraint in period t is given by:

$$\sum_{s_t \in S_t} \pi_t(s_t, s_{t-1}) \left[u(c_t(s_t)) - \sum_i v_i\left(h_t^i(s_t) + l_t^i(s_t)\right) + \beta V_{t+1}(s_t) \right] = V_t(s_{t-1}), \qquad (1)$$

where $V_t(s_{t-1})$ is the promised utility for agents who truthfully declared state s_{t-1} in the previous period. $V_t(s_{t-1}) = V$ in the first period and $V_{t+1}(s_t) = 0$ in the last period.

In a framework where privately observed shocks are assumed to be independent over time, the promised continuation utility would be the only state variable that we need to keep track of (Albanesi and Sleet, 2006; Atkeson and Lucas, 1992). With time-independent shocks, expected utility of agents would be common knowledge in every period. In other words, expected utility in the current period would be the same for all agents who declared s_{t-1} in the previous period, irrespective of whether the agents were truthful or not, since agents have the same distribution of privately observed shocks.

Threat keeping constraints When shocks are history dependent, such as with absorbing disability shocks, expected utility of agents would not be common knowledge in every period.[5] One therefore also needs to keep track of the history of disability shocks, and of additional state variables, $\tilde{V}_{t+1}(s_t, \tilde{s}_t)$ for all $\tilde{s}_t$ pretending to be s_t. $\tilde{V}_{t+1}(s_t, \tilde{s}_t)$ can be interpreted as the threatened continuation utility of agents in state $\tilde{s}_t$ pretending to be in state s_t (Fernandes and Phelan, 2000). The threatened continuation utilities allocated in the current period then becomes the expected discounted utilities that the government needs to deliver to previously untruthful agents via the following period's threat keeping constraints.

In addition, in a multi-member context, mimicking households may still privately choose a state declaration s_t that would maximize their current expected discounted utility, even if they are in a different state $\tilde{s}_t$ in the current period. The threat keeping constraints therefore capture the different probability distributions of previous period's mimickers, as well as their current private optimizing behavior. The constraints keep track of potential multi-period deviations.[6]

[5] To see this, consider two agents who declared to be in state s_{t-1} in the previous period. The first agent was truthful while the second agent was untruthful and actually in state $\tilde{s}_{t-1} \neq s_{t-1}$. Then, in the current period, the expected utility based on the distribution of disability shocks would be conditional on s_{t-1} for the truthful agent but conditional on $\tilde{s}_{t-1}$ for the mimicker.

[6] To shed more light on the interpretation of such constraints in a multi-member context, consider a two parent household. In period $t-1$, the family is in state $\tilde{s}_{t-1}$ (both parents are healthy) but declare to be in state s_{t-1} (the mother falsely reports disability). The threat keeping constraint ensures that the family gets the threatened utility $\tilde{V}_t(s_{t-1}, \tilde{s}_{t-1})$ in period t, which takes into account the probability distribution of health shocks and the private optimizing behavior. Consider two scenarios in period t. In Scenario A, the parents are still healthy (state $\tilde{s}_t$). The family still reports disability for the mother due to its absorbing nature but the family may choose to report whether the father is disabled or not. The family therefore chooses a report s_t to maximize utility and get continuation utility $\tilde{V}_{t+1}(s_t, \tilde{s}_t)$, which will become next period's threatened utility. In Scenario B, the parents are disabled (state $\tilde{s}_t$). The family truthfully reports $s_t = \tilde{s}_t$ and get continuation utility $\tilde{V}_{t+1}(s_t, \tilde{s}_t) = V_{t+1}(s_t)$, which will become next period's promised utility.

The threat keeping constraints in period t, $\forall \tilde{s}_{t-1} \in S_{t-1}$:

$$\sum_{\tilde{s}_t \in S_t} \pi_t\left(\tilde{s}_t, \tilde{s}_{t-1}\right) \max_{s_t} \left[u\left(c_t\left(s_t\right)\right) - \sum_i v_i\left(h_t^i\left(s_t\right) + l_t^i\left(s_t\right)\right) + \beta \tilde{V}_{t+1}\left(s_t, \tilde{s}_t\right)\right] = \tilde{V}_t\left(s_{t-1}, \tilde{s}_{t-1}\right), \tag{2}$$

where $\tilde{V}_t\left(s_{t-1}, \tilde{s}_{t-1}\right)$ is the threatened utility of agents who were in state $\tilde{s}_{t-1}$ in the previous period but declared to be in state s_{t-1}.

Incentive compatibility constraints To incentivize truthful declarations, the expected discounted utility from truth-telling needs to be no less than the expected discounted utility from mimicking. The incentive compatibility constraints in period t, $\forall s_t, \tilde{s}_t \in S_t$:

$$\begin{aligned} & u\left(c_t\left(s_t\right)\right) - \sum_i v_i\left(h_t^i\left(s_t\right) + l_t^i\left(s_t\right)\right) + \beta V_{t+1}\left(s_t\right) \\ \geq\ & u\left(c_t\left(\tilde{s}_t\right)\right) - \sum_i v_i\left(h_t^i\left(\tilde{s}_t\right) + l_t^i\left(\tilde{s}_t\right)\right) + \beta \tilde{V}_{t+1}\left(\tilde{s}_t, s_t\right). \end{aligned} \tag{3}$$

Honest households get allocated continuation utility $V_{t+1}\left(s_t\right)$, to be delivered through next period's promise keeping constraint while dishonest households get allocated the continuation utility $\tilde{V}_{t+1}\left(\tilde{s}_t, s_t\right)$, to be delivered through next period's threat keeping constraint for a household that was in state s_t but declared to be in state $\tilde{s}_t$.[7]

Constrained optimization problem Let $\mathbf{\tilde{V}_{t+1}}\left(\mathbf{s_t}, \mathbf{\tilde{s}_t}\right)$ be a vector of threatened continuation utilities capturing $\tilde{V}_{t+1}\left(s_t, \tilde{s}_t\right)$, for all $\tilde{s}_t \in S_t$. In every period t, for agents who were in state $s_{t-1} \in S_{t-1}$ in the previous period, the government chooses allocations that minimize expected costs:

$$G_t\left(V_t\left(s_{t-1}\right), \mathbf{\tilde{V}_t}\left(\mathbf{s_{t-1}}, \mathbf{\tilde{s}_{t-1}}\right)\right) = \underset{c_t, l_t, h_t, V_{t+1}, \tilde{V}_{t+1}}{Min} \tag{4}$$

$$\sum_{s_t \in S_t} \pi_t\left(s_t, s_{t-1}\right) \left[c_t\left(s_t\right) + p_t\left(n_t - \sum_i h_t^i\left(s_t\right)\right) - \sum_i w_t^i l_t^i\left(s_t\right) + \beta G_{t+1}\left(V_{t+1}\left(s_t\right), \mathbf{\tilde{V}_{t+1}}\left(\mathbf{s_t}, \mathbf{\tilde{s}_t}\right)\right)\right],$$

subject to promise keeping constraint (1), threat keeping constraints (2), and incentive compatibility constraints (3).

Every period, the government specifies for all household members i, labor supply $l_t^i\left(s_t\right)$ and household child care $h_t^i\left(s_t\right)$, and allocates transfers of $b_t\left(s_t\right)$ to households, which is equivalent to the government choosing consumption of agents since $c_t\left(s_t\right) = b_t\left(s_t\right) - p_t\left(n_t - \sum_i h_t^i\left(s_t\right)\right)$. The government also allocates continuation utility $V_{t+1}\left(s_t\right)$ and threatened continuation utilities $\tilde{V}_{t+1}\left(s_t, \tilde{s}_t\right), \forall \tilde{s}_t \in S_t$, to households for each declaration of state $s_t \in S_t$. In the initial period, $V_t\left(s_{t-1}\right) = V$, the initial promised utility. In the last period, $\forall \tilde{s}_t \in S_t$, $V_{t+1}\left(s_t\right) = \tilde{V}_{t+1}\left(s_t, \tilde{s}_t\right) = 0$, and $G_{t+1}\left(V_{t+1}\left(s_t\right), \mathbf{\tilde{V}_{t+1}}\left(\mathbf{s_t}, \mathbf{\tilde{s}_t}\right)\right) = 0$.

[7] Note that by the nature of disability, only abled family members may mimic the disabled and not vice versa so that only a subset of states are relevant for mimicking purposes. Household members who claim to be disabled must also have $v_i\left(h_t^i\left(\tilde{s}_t\right) + l_t^i\left(\tilde{s}_t\right)\right) = 0$.

D. *Constrained Optimal Allocations*

LEMMA 1. Let $(c_t^*(s_t), l_t^*(s_t), h_t^*(s_t))$ solve the government problem (4).

(i) Production efficiency. Healthy members with $w_t^i \geq p_t$ devote all of their effort to the labor market: $l_t^{i*}(s_t) > 0$ and $h_t^{i*}(s_t) = 0$.

Healthy members with $w_t^i < p_t$ provide household child care: $l_t^{i*}(s_t) \geq 0$ and $h_t^{i*}(s_t) > 0$; $l_t^{i*}(s_t) = 0$ when $\sum_i h_t^{i*}(s_t) < n_t$.

(ii) No distortion of effort. For healthy members with $w_t^i \geq p_t$, the consumption-labor and consumption-child care margins are respectively given by:

$$u'(c_t^*(s_t))\, w_t^i = v_i'\left(e_t^{i*}(s_t)\right) \text{ and } u'(c_t^*(s_t))\, p_t \leq v_i'\left(e_t^{i*}(s_t)\right),$$

with the latter satisfied with strict inequality for $w_t^i > p_t$.

For healthy members with $w_t^i < p_t$, the consumption-labor and consumption-child care margins are respectively given by:

$$u'(c_t^*(s_t))\, w_t^i < v_i'\left(e_t^{i*}(s_t)\right) \text{ and } u'(c_t^*(s_t))\, p_t = v_i'\left(e_t^{i*}(s_t)\right),$$

when $\sum_i h_t^{i*}(s_t) < n_t$.

(iii) Intertemporal savings wedge. For each period $t < T$, the inverse Euler equation holds and there is an inter-temporal wedge between current and future marginal utilities of consumption:

$$u'(c_t^*(s_t)) < \sum_{s_{t+1} \in S_{t+1}} \pi_{t+1}(s_{t+1}, s_t)\, u'\left(c_{t+1}^*(s_{t+1})\right).$$

Proof: See Appendix A.

The constrained optimal allocations described in Lemma 1 are fairly intuitive. It is optimal for healthy household members whose wages are higher than the cost of formal child care to work on the labor market, and for healthy household members whose wages are lower than the cost of formal child care to engage in household child care and work on the labor market only after all child care needs have been met. The consumption-labor and consumption-child care margins of healthy household members are not distorted, and the inverse Euler equation holds.

II. Implementation

The efficiency argument for subsidizing the formal child care costs of the disabled stem from the implementation of the constrained optimum in a decentralized economy. I discuss how households would behave if they could deviate from the optimal allocations, and what policy tools can provide agents with the right incentives to counteract such deviations incentives. In particular, I show that if agents could deviate from the socially optimal level of household child care, then the incentives of healthy household members to mimic the disabled would be exacerbated since they can then claim disability benefits and also save on formal child care costs. In addition, household child care incentives interact with households' incentives to engage in sub-optimal savings, resulting in triple deviation incentives to (i) mimic the disabled, (ii) engage in sub-optimal savings, and (iii) engage in sub-optimal household child care.

I next propose an implementation of the constrained optimum in a decentralized economy through the use of capped formal day care subsidies, asset-testing and non-linear income taxation. The subsidized day care is equivalent to a full price subsidy on formal child care costs and may be provided through free public day care capped at the optimal level of formal child care use. Such scheme implements the constrained optimum for single (grand)parent households and for multi-member households with high earnings capacity, *as though* the government could control household members' child care, when in fact, it precludes from such monitoring needs. The incentives to mimic the disabled are more perverse for multi-member households with at least one healthy member with low earnings capacity. A costlier constrained optimum, equivalent to the solution to the optimal program with hidden household child care activities, may be implemented for such households.

A. *Private Deviation Incentives*

Child Care Deviation Incentives

Suppose that household members could deviate from the optimal level of household child care. Households take as given the allocations of transfers $b_t^*(s_t)$, and labor supply $l_t^{i*}(s_t), \forall i$, specified by the government from the constrained optimal problem (4). The household then chooses household child care to maximize expected utility.

Illustration with $T = 1$ and $I = 1$ Consider the private problem of an agent who is healthy (state s) but declares to be disabled (state $\tilde{s}$). Let $\tilde{c}(\tilde{s}, s)$ and $\tilde{h}(\tilde{s}, s)$ respectively denote consumption and household child care of an agent in state s claiming to be in state $\tilde{s}$. The agent solves:

$$\underset{\tilde{c}(\tilde{s},s),\tilde{h}(\tilde{s},s)}{Max} u(\tilde{c}(\tilde{s}, s)) - v\left(\tilde{h}(\tilde{s}, s)\right),$$

subject to the budget constraint: $\tilde{c}(\tilde{s}, s) + p\left(n - \tilde{h}(\tilde{s}, s)\right) = b^*(\tilde{s})$. $b^*(\tilde{s})$ are the constrained optimal transfers allocated to a disabled agent. The transfers are equal to the constrained optimal levels of consumption and cost of formal child care: $b^*(\tilde{s}) = c^*(\tilde{s}) + p(n - h^*(\tilde{s})) = c^*(\tilde{s}) + pn$ (since $h^*(\tilde{s}) = 0$ for a disabled agent).

The private consumption-child care margin of the agent is given by:

$$u'(\tilde{c}(\tilde{s}, s))\, p = v'\left(\tilde{h}(\tilde{s}, s)\right).$$

A healthy agent will engage in household child care when mimicking the disabled. By engaging in household child care, the agent can save on formal child care costs and enjoy higher consumption of: $\tilde{c}(\tilde{s},s) = c^*(\tilde{s}) + p\tilde{h}(\tilde{s},s)$. In particular, consumption increases by the value of household child care. Thus, the maximized utility of a mimicker is greater than the constrained optimal one: $u(\tilde{c}(\tilde{s},s)) - v\left(\tilde{h}(\tilde{s},s)\right) > u(c^*(\tilde{s}))$, where the right hand side of the inequality takes into account the fact that effort cost $v(0) = 0$. Now, in the case of one agent, the incentive constraint preventing a healthy agent from mimicking a disabled agent is binding at the constrained optimum:

$$u(c^*(s)) - v(l^*(s) + h^*(s)) = u(c^*(\tilde{s})). \tag{5}$$

It follows that the incentive constraint (5) is violated if a mimicker may engage in sub-optimal household child care activities.

Implications for subsidizing formal child care The private incentives to mimic the disabled are exacerbated in the presence of household child care activities, which make the incentive constraints harder to satisfy. Child care subsidies on formal child care costs may therefore help smoothen the exacerbation of incentives by making household child care less attractive to mimickers.

Consider the illustration example above with $T = 1$ and $I = 1$. A full subsidy on formal child care costs incurred by disabled agents, coupled with an equivalent decrease in the transfers provided to the disabled, would ensure that agents do not engage in any household child care when mimicking the disabled. In particular, consider a disability transfer scheme that subsidizes the price of formal child care at a rate of $\tau = 1$ and that transfers $b(\tilde{s}) = b^*(\tilde{s}) - \tau pn$ to the disabled.

A truly disabled agent uses full time formal child care and therefore gets total subsidies of τpn whereas disability transfers decrease by the full subsidy amount. The budget constraint of a truly disabled agent is therefore given by:

$$\underbrace{c(\tilde{s})}_{\text{Consumption}} + \underbrace{pn}_{\text{Formal child care cost}} - \underbrace{\tau pn}_{\text{Child care subsidies}} = \underbrace{b(\tilde{s})}_{\text{Disability transfers}}.$$

It follows that the disabled agent's consumption is still the same as in the constrained optimum: $c(\tilde{s}) = b(\tilde{s}) = b^*(\tilde{s}) - \tau pn = c^*(\tilde{s}) + pn - \tau pn = c^*(\tilde{s})$. The truly disabled agent therefore gets the same utility as in the constrained optimum: $u(c^*(\tilde{s}))$.

Now, a mimicker also gets a full subsidy rate $\tau = 1$ on formal child care such that the subsidized price is: $(1-\tau)p = 0$. The mimicker will therefore not engage in any household child care. Consumption of the mimicker is equal to the constrained optimal one, $\tilde{c}(\tilde{s},s) = c^*(\tilde{s})$, and utility is the same as in the constrained optimum, $u(c^*(\tilde{s}))$. It follows that the incentive compatibility constraint (5) is satisfied under such a scheme.

The use of a full price subsidy on formal child care costs discourages the mimicker from engaging in sub-optimal household child care activities when mimicking the disabled. In the meantime, the decrease in disability transfers by the amount of total child care subsidies paid, helps maintain the cost of the program to the government. Thus, rather than providing disability transfers as a lump sum payment $b^*(\tilde{s})$, providing child care subsidies at a rate of $\tau = 1$ to the disabled and equivalently lower disability transfers $b(\tilde{s})$ help implement the constrained optimal allocations.

An alternative scheme is to provide subsidized day care to both healthy and disabled agents up to the optimal level of formal day care use, and decrease the transfers provided to agents by the cost of formal day care. In particular, disabled agents benefit from free formal day care up to $f^*(\tilde{s}) = pn$ and get reduced transfers of $b(\tilde{s}) = b^*(\tilde{s}) - f^*(\tilde{s})$. This is equivalent to the scheme with a full subsidy on the price of formal child care for the disabled. Healthy agents also benefit from free day care up to $f^*(s) = p(n - h^*(s))$ and get reduced transfers of $b(s) = b^*(s) - f^*(s)$. Healthy agents still engage in the optimal levels of labor supply and household child care. Consumption of healthy agents are also at the optimal level and the incentive compatibility constraint (5) holds.

Triple Deviation Incentives

The incentives of agents to deviate from the optimal level of savings are well-known (Golosov and Tsyvinski, 2006; Kocherlakota, 2010) and not delved in detail here. The incentives to engage in sub-optimal savings interact with the incentives to engage in sub-optimal household child care and increase the utility that households may get when mimicking the disabled, thereby resulting in triple deviation incentives.

Illustration with $T = 2$ and $I = 1$ Consider the problem of an agent who is healthy (state s_0) in period $t = 0$ and may be healthy (state s_1) or disabled (state $\tilde{s}_1$) in period $t = 1$. Suppose that the agent always claims to be disabled in the second period. The agent takes as given the allocations of transfers $b_t^*(s_t) = c_t^*(s_t) + p_t(n_t - h_t^*(s_t))$ and labor supply $l_t^*(s_t)$ specified by the government $\forall t, s_t$ from the constrained optimal problem (4). The agent then privately chooses consumption, household child care and assets $A_1(s_0) \in \mathbb{R}$ to maximize expected utility:

$$\underset{c,h,A}{Max} \qquad u(c_0(s_0)) - v(l_0^*(s_0) + h_0(s_0))$$

$$+\beta\left\{\pi_1(s_1,s_0)\left[u(\tilde{c}_1(\tilde{s}_1,s_1)) - v\left(\tilde{h}_1(\tilde{s}_1,s_1)\right)\right] + \pi_1(\tilde{s}_1,s_0)\left[u(c_1(\tilde{s}_1))\right]\right\},$$

subject to the budget constraints in period 0, in period 1 if healthy and in period 1 if disabled, respectively:

$$c_0(s_0) + p_0(n_0 - h_0(s_0)) + A_1(s_0) = b_0^*(s_0),$$
$$\tilde{c}_1(\tilde{s}_1,s_1) + p_1\left(n_1 - \tilde{h}_1(\tilde{s}_1,s_1)\right) = b_1^*(\tilde{s}_1) + (1+r)A_1(s_0),$$
$$c_1(\tilde{s}_1) + p_1 n_1 = b_1^*(\tilde{s}_1) + (1+r)A_1(s_0).$$

The agent's private Euler equation is given by:

$$u'(c_0(s_0)) = \pi_1(s_1,s_0)u'(\tilde{c}_1(\tilde{s}_1,s_1)) + \pi_1(\tilde{s}_1,s_0)u'(c_1(\tilde{s}_1)).$$

This is in contrast to the socially optimal intertemporal wedge from Lemma 1(iii). Households have an incentive to underconsume and oversave in the first period relative to the social optimum: $c_0(s_0) < c_0^*(s_0)$ and $A_1(s_0) > 0$. Note that the transfer scheme considered implies zero household savings at the optimum. One may consider alternative transfer and non-zero savings schemes that deliver the same net present value of transfers to households. For example, $b_0^*(s_0) = c_0^*(s_0) + p_0(n_0 - h_0^*(s_0)) + A_1^*(s_0)$ and $b_1^*(s_1) = c_1^*(s_1) + p_1(n_1 - h_1^*(s_1)) - (1+r)A_1^*(s_0)$ for $s_1 \in S_1$ and $A_1^*(s_0) \in \mathbb{R}$. The incentive to underconsume and oversave in the first period would still be present due to the socially optimal inter-temporal wedge: $c_0(s_0) < c_0^*(s_0)$ and $A_1(s_0) > A_1^*(s_0)$.

The incentives to oversave interact with the incentive to overprovide household child care in the first period. In particular, if $c_0(s_0) < c_0^*(s_0)$, then the private marginal benefit from household child care is greater than the socially optimal one:

$$u'(c_0(s_0))\,p_0 - v'(l_0^*(s_0) + h_0(s_0)) > u'(c_0^*(s_0))\,p_0 - v'(l_0^*(s_0) + h_0^*(s_0)),$$

where the right hand side of the inequality corresponds to the optimal consumption-child care margin from Lemma 1(ii). Households therefore have an incentive to increase household child care $h_0(s_0)$ beyond the optimal level $h_0^*(s_0)$. In addition, a falsely disabled agent would want to engage in household child care activities as explained above. The agent therefore oversaves in the first period and engages in higher than optimal household child care activities in both periods.

With triple deviation incentives, the agent gets higher utility when (i) mimicking the disabled, (ii) oversaving and (iii) overproviding household child care. Implementation of the constrained optimal allocations in a decentralized economy thus require policy tools that would preclude agents from engaging in sub-optimal levels of savings and household child care.

B. *Implementation of the Constrained Optimal Allocations*

I now show that a combination of capped formal day care subsidies, non-linear income taxation and asset-testing implement the constrained optimum in a decentralized economy for single adult households and for multi-member households with high earnings capacity. Note that there is a direct mapping between earnings $y_t^i = w_t^i l_t^i \; \forall i$, formal child care costs f_t and disability claims s_t. The policy tools may therefore be modelled as functions of earnings and formal child care costs. For the ease of exposition, I stick to the direct mechanism notation.

Decentralized Household Problem Consider a household in state s_t. When claiming state $\tilde{s}_t$, the household needs to provide labor supply $l_t^{i*}(\tilde{s}_t) \; \forall i$ and face the associated policy scheme. The household problem can be solved in two steps: (i) For each possible state claim $\tilde{s}_t \in S_t$, the household chooses consumption $\tilde{c}_t(\tilde{s}_t, s_t)$, household child care $\tilde{h}_t^i(\tilde{s}_t, s_t) \; \forall i$, and assets $\tilde{A}_{t+1}(\tilde{s}_t, s_t)$ that maximize utility, and (ii) the household claims the state that yields the highest utility.

$$\tilde{U}_{s_t}\left(\tilde{s}_{t-1}, s_{t-1}\right) = \underset{\tilde{s}_t \in S_t}{Max} \left\{ \underset{\tilde{c}_t(\tilde{s}_t,s_t),\tilde{h}^i_t(\tilde{s}_t,s_t),\tilde{A}_{t+1}(\tilde{s}_t,s_t)}{Max} \right.$$

$$\left. u\left(\tilde{c}_t\left(\tilde{s}_t, s_t\right)\right) - \sum_i v_i\left(l^{i*}_t\left(\tilde{s}_t\right) + \tilde{h}^i_t\left(\tilde{s}_t, s_t\right)\right) + \beta \sum_{s_{t+1} \in S_{t+1}} \pi_{t+1}\left(s_{t+1}, s_t\right) \tilde{U}_{s_{t+1}}\left(\tilde{s}_t, s_t\right) \right\},$$

subject to the household budget constraints $\forall \tilde{s}_t \in S_t$

$$\tilde{c}_t\left(\tilde{s}_t, s_t\right) + I_t\left(p_t, n_t, \sum_i \tilde{h}^i_t\left(\tilde{s}_t, s_t\right), f^*_t\left(s_t\right)\right) + \tilde{A}_{t+1}\left(\tilde{s}_t, s_t\right)$$

$$= \sum_i w^i_t l^{i*}_t\left(\tilde{s}_t\right) - T_t\left(\tilde{s}_t\right) + (1+r)\tilde{A}_t\left(\tilde{s}_{t-1}, s_{t-1}\right).$$

$I_t\left(p_t, n_t, \sum_i \tilde{h}^i_t\left(\tilde{s}_t, s_t\right), f^*_t\left(s_t\right)\right)$ represents subsidized formal child care costs and $T_t\left(\tilde{s}_t\right)$ are net income taxes.

A policy scheme implements the constrained optimum if household choices coincide with the optimal allocations $\left(c^*_t\left(s_t\right), l^*_t\left(s_t\right), h^*_t\left(s_t\right)\right)$, $\forall t, s_t$. Proposition 2 outlines a simplistic scheme with free capped day care provision and asset limits set at zero. Alternative day care subsidies and non-zero asset limits are then discussed.

PROPOSITION 1. The following scheme implements the constrained optimum from the government problem (4) for single adult households and for multi-member households with $w^i_t \geq p_t \ \forall i$.

(i) Subsidized formal day care Households benefit from free day care capped at the optimal level of formal child care, $\forall t, s_t$:

$$f^*_t\left(s_t\right) = p\left(n_t - \sum_i h^{i*}_t\left(s_t\right)\right).$$

(ii) Non-linear income taxes and asset-testing The government imposes net taxes and asset-testing, $\forall t, s_t$:

$$T_t\left(s_t\right) = \begin{cases} \sum_i w^i_t l^{i*}_t\left(s_t\right) - c^*_t\left(s_t\right) & \text{if} \quad A_t\left(s_{t-1}\right) \leq 0 \\ \sum_i w^i_t l^{i*}_t\left(s_t\right) - c^*_t\left(s_t\right) + (1+r)A_t\left(s_{t-1}\right) & \text{if} \quad A_t\left(s_{t-1}\right) > 0 \end{cases}.$$

Proof: See Appendix B.

The implementation may be summarized as follows. Subsidized formal day care ensures that households engage in the optimal level of household child care while asset-testing ensures that households engage in the optimal level of savings. Once households engage in the optimal levels of child care and savings, the non-linear income taxes ensure that households get the optimal level of consumption. It follows that the promise keeping (1), threat keeping (2) and incentive compatibility (3) constraints hold, thereby delivering social insurance in the least costly way possible.

Day care subsidies Subsidized day care capped at the optimal level of formal child care makes household child care beyond the constrained optimal level relatively unattractive to households. Subsidized day care may be implemented through direct reimbursements to day care centers up to $f_t^*(s_t)$ or equivalently, through a full price subsidy on formal child care at a rate of $\tau = 1$ and with the total subsidy capped at $f_t^*(s_t)$. If follows that household child care beyond the constrained optimal level would not help households save on formal child care costs, while it would be costly in terms of effort. Households would therefore be discouraged from engaging in non-optimal household child care activities. In other words, day care subsidies ensure that households choose the optimal level of formal child care, which in turn, ensures that individual household members engage in the optimal levels of individual household child care.

Non-zero asset limits One may construct indeterminate combinations of non-linear income taxes and non-zero asset limits that deliver the same net present value of net taxes to agents. In particular, given that day care subsidies ensure that households engage in the optimal level of household child care, any combination of asset limits $\bar{A}_t(s_{t-1}) \in \mathbb{R}$ and income taxes $T_t(s_t)$ may be defined recursively from the household budget constraint $\forall t, s_t$:

$$c_t^*(s_t) + \bar{A}_{t+1}(s_t) = \sum_i w_t^i l_t^{i*} - T_t(s_t) + (1+r)\bar{A}_t(s_{t-1}),$$

where $\bar{A}_{t+1}(s_t) = 0$ in the last period $t = T$.

The non-linear income tax and asset-testing policy schedule is then given by:

$$T_t(s_t) = \begin{cases} T_t^0(s_t) & \text{if} \quad A_t(s_{t-1}) \leq \bar{A}_t(s_{t-1}) \\ T_t^0(s_t) + (1+r)A_t(s_{t-1}) & \text{if} \quad A_t(s_{t-1}) > \bar{A}_t(s_{t-1}) \end{cases},$$

where $T_t^0(s_t) = \sum_i w_t^i l_t^{i*}(s_t) + (1+r)\bar{A}_t(s_{t-1}) - \bar{A}_{t+1}(s_t) - c_t^*(s_t)$.

C. *Implementation of the Constrained Optimum with Hidden Actions*

If formal day care subsidies cannot be used as a policy tool, then the implementable optimum would be equivalent to the solution to the government problem with hidden household child care, which would be costlier. Such hidden activities are analogous to hidden actions models in the optimal social insurance literature (Álvarez-Parra and Sánchez, 2009; Golosov and Tsyvinski, 2007; Pavoni and Violante, 2007).

The government problem with hidden household child care is similar to problem (4) but with additional child care constraints. In particular, the government minimizes expected costs subject to delivering the initial promised utility V to households and subject to preserving work incentives. In addition, the government needs to explicitly take into account private child care incentives $\forall \tilde{s}_t, s_t \in S_t$:

$$\mathbf{\tilde{h}_t}\left(\mathbf{\tilde{s}_t}, \mathbf{s_t}\right) = \underset{\tilde{h}_t^i(\tilde{s}_t, s_t)}{arg\ max}\, u\left(c_t\left(\tilde{s}_t\right) + p_t\left(\sum_i \tilde{h}_t^i\left(\tilde{s}_t, s_t\right) - \sum_i h_t^i\left(\tilde{s}_t\right)\right)\right) - \sum_i v_i\left(l_t^i\left(\tilde{s}_t\right) + \tilde{h}_t^i\left(\tilde{s}_t, s_t\right)\right),$$

where $\mathbf{\tilde{h}_t}\left(\mathbf{\tilde{s}_t}, \mathbf{s_t}\right)$ is the vector of household child care $\tilde{h}_t^i\left(\tilde{s}_t, s_t\right)$ provided by members in a family in state s that declares to be in state $\tilde{s}$. $c_t\left(\tilde{s}_t\right)$, $h_t^i\left(\tilde{s}_t\right)$ and $l_t^i\left(\tilde{s}_t\right)$ are respectively, consumption, household child care and labor supply of a family that is truly in state s.

Appendix C. shows that the qualitative features of Lemma 1 still hold in the government problem with hidden household child care. On the other hand, the government now accounts for the fact that some of those who claim to be disabled may engage in household child care. In particular, the consumption-child care margin of mimickers is not distorted (while previously in problem (4), the consumption-child care margin of mimickers was distorted relative to the private optimum).

The constrained optimum with hidden household child care would be costlier relative to that of problem (4). Consider again the illustration with $T = 1$ and $I = 1$, where an agent is healthy (state s) but declares to be disabled (state $\tilde{s}$). By enabling the government to indirectly control household child care, formal day care subsidies help implement the constrained optimal allocations analogous to problem (4). In other words, incentive constraint (5) is satisfied with equality:

$$u\left(c^*\left(s\right)\right) - v\left(l^*\left(s\right) + h^*\left(s\right)\right) = u\left(c^*\left(\tilde{s}\right)\right).$$

In the absence of day care subsidies, this incentive constraint would be violated as mimickers would engage in sub-optimal household child care activities:

$$u\left(c^*\left(s\right)\right) - v\left(l^*\left(s\right) + h^*\left(s\right)\right) < \underset{\tilde{h}(\tilde{s},s)}{Max}\, u\left(c^*\left(\tilde{s}\right) + p\tilde{h}\left(\tilde{s}, s\right)\right) - v\left(\tilde{h}\left(\tilde{s}, s\right)\right).$$

It follows that a different constrained optimum (with hidden household child care) that creates a larger spread between utilities of the abled and of the disabled would preserve work incentives. As preferences are subject to diminishing marginal utility, it would be costlier to the government to deliver the initial promised utility V.

Cost savings from day care subsidies In the absence of day care subsidies, implementation of the constrained optimum equivalent to the solution to the government problem with hidden household child care may be done through the use of non-linear income taxation and asset-testing, defined recursively from the household budget constraint $\forall t, s_t$. Conversely, the use of day care subsidies help implement the constrained optimal allocations equivalent to the solution to problem (4) for single (grand)parent households and for multi-member households with high earnings capacity. Thus, by enabling the government to indirectly control household child care, day care subsidies help the government experience some cost savings for such households. In Section III., I quantify such cost savings for different household structures.

Multi-member households with low earnings capacity The incentives to mimic the disabled are more perverse in the presence of a healthy member with low earnings capacity, $w_t^i < p_t$. The child care incentives of such members interact with those of falsely disabled members. Thus, even if the household uses the optimal level of formal child care, household members may still save on total effort cost by reallocating household child care across mimickers and healthy members with $w_t^i < p_t$, which exacerbates the incentives to falsely claim disability (see Appendix D. for an illustration). In this case, the constrained optimum equivalent to the solution to the government problem with hidden household child care may be implemented. Such households will not be relevant to our quantitative analysis.

III. Quantitative Analysis

The quantitative analysis allows for observed demographic heterogeneity across households in terms of structure (presence of parents and grandparents), adult characteristics (marital status, gender, and education) and child characteristics (number and age of children). I denote a particular family household type by the index k which conveys all the relevant information on household composition. The index i still denotes a particular adult member. Each household type may be of different sizes or face different life cycle profiles of child care needs. Adult household members may have different effort costs, life cycle probabilities of being disabled, and wage profiles.

The qualitative results presented in Section II. are applicable to any family type. In the optimal social insurance program, the government minimizes expected costs subject to delivering a given promised utility to each household type and subject to preserving work incentives. The model may therefore be solved separately for each family type. The quantitative analysis aims at (i) quantifying the optimal allocations and (ii) quantifying the efficiency gains from subsidized day care. Such gains are measured as the cost savings from implementing the constrained optimal allocation with subsidized day care, as opposed to implementing a different allocation without subsidized day care. The benchmark model is the constrained optimal one where the government may control household child care (through subsidized day care) whereas the comparison model is one where the government may not control household child care. The cost savings are computed for each household type and may be redistributed to households as lump sum transfers to improve welfare under a normative welfare criterion that is not delved into in this paper.

A. Parameters

Family preferences are given by:

$$ln\left(c_{kt}\right) - \sum_i \alpha_k^i \frac{\left(e_{kt}^i\right)^{1+\gamma}}{1+\gamma}.$$

The felicity of consumption is logarithmic.[8] α_k^i is an effort cost parameter and γ is the reciprocal of the Frisch elasticity of labor supply.

Parameters to be calibrated are the discount factor β, preference parameters $\left\{\gamma, \alpha_k^i\right\}$, the life cycle probabilities of being disabled $\hat{\pi}_{kt}^i$, the life cycle profiles of wages w_{kt}^i, child care needs n_{kt} and price of formal child care p_{kt}. I calibrate initial promised utility V_k for each family type according to the US welfare system: Social Security and Federal Taxes, Earned Income Tax Credit (EITC), Child and Dependent Care Tax Credit (CDCTC), Child Care and Development Fund (CCDF), Social Security Disability Insurance (SSDI), and Supplemental Security Income (SSI).

B. Demographics

Household adult structure The adult structure of each household is assumed to be constant over the time frame $t = 0, ..., T$, which corresponds to the finite life cycle of a multi-generational household during which child care needs may be relevant. The demographic composition of households is designed to match the composition of US households from the Current Population Survey (CPS) years 2005 to 2014.

I define an adult to be part of the parent or grandparent generation based on age and irrespective of the presence of children. Those aged 25 to 49 are part of the parent generation and those aged 50 to 74 are part of the grandparent generation. To keep the terminology simple, I refer to an adult in the parent generation as "father" or "mother". Similarly, an adult in the grandparent generation is referred to as "grandfather" or "grandmother". I consider parent households with adults aged only 25 to 49, grandparent households with adults aged only 50 to 74, as well as intergenerational households with adults aged 25 to 74. Adults may be single or married.[9]

I consider a 5 period model where each period t corresponds to a 5 year time interval. I base the life cycle of a multi-generational household on the age of the mother when she is present in the household so that $t = 0, 1, 2, 3, 4$ correspond to a household with a mother aged respectively 25-29, 30-34, 35-39, 40-44, 45-49. When there is no mother present but a grandmother is present, I base the life cycle of the household on the age of the grandmother so that $t = 0, 1, 2, 3, 4$ correspond to a household with a grandmother aged respectively 50-54, 55-59, 60-64, 65-69, 70-74. Finally, when there are neither mother nor grandmother in the household, I use the age of the father.

I do not consider households with structures that make up less than 2% of the sample. There are thus 7 possible structures: 3 parent households (single mother, single father, two parents), 2 grandparent households (single grandmother, two grandparents), and 2 intergenerational households (single grandmother and single father, single grandfather and single mother). The household structures are represented in Table 1. Parent households make up 61% of the sample while grandparent households make up 28% of the sample, and 11% of households are intergenerational.

[8] The logarithmic function is normalized by adding 1 to its argument so that $lim_{c\to 0} u(c) = 0$.

[9] In the CPS, a household is identified by the household number and current address of residence. Family members within a household are identified using the family identication number.

Table 1: Household Structure

Household	Adults	Mother	Father	Grandma	Grandpa	Prop.
Parent	Single mother	✔				0.13
	Single father		✔			0.05
	Two parents	✔	✔			0.43
Grandparent	Grandmother			✔		0.04
	Grandparents			✔	✔	0.24
Intergenerational	Grandmother & Father		✔	✔		0.04
	Grandfather & Mother	✔			✔	0.07

Note: Proportions computed from CPS data. Parents are aged 25 to 49 and grandparents 50 to 74.

Household child characteristics Child arrival rates are exogenously given in the model. The interpretation of child arrival is inclusive of births of own children and arrival of grandchildren into the household. I assume that children may arrive only in the first three periods, corresponding to when adults in the parent generation are aged 25 to 39. The arrival rates of children vary by household structure and time period, and are calibrated according to the proportion of households with children aged below 5 in the CPS.

For parent households, I limit the total number of children to 3 and the maximum number of children arriving in one period is limited to 2. This yields a maximum of 17 profiles of child arrivals for each parent household. For grandparent households and intergenerational households with a single father, I limit the total number of children to 1. This implies a maximum of 4 profiles of child arrivals for those households. For intergenerational households with a single mother, I limit the total number of children to 2 and the maximum number of children arriving in one period is also 2. This yields a maximum of 9 profiles of child arrivals for such intergenerational households.[10]

I report the proportion of households with children aged below 5 in Table 2. As can be seen from the Table, parent households and intergenerational households with mothers have the highest child arrival rates. On the other hand, grandparent households and intergenerational households with single fathers had the lowest child arrival rates. I compute the proportion of households facing each child arrival profile using the information in Table 2. For example, the proportion of single mothers who have one child in every period is computed as $a_0 \times a_1 \times a_2$ where a_t is the proportion of single mothers with one child aged below 5 in period t. I normalize the sum of the proportions to 1 for each household structure.

Child care needs and price of formal child care I define child care needs n_{kt} as the portion of the working week during which child care is required. A normal working week is 40 hours which is normalized to 1 unit of time. I assume that children require full time child care only in the first period of their life. Since an adult can look after several children at the same time, household child care needs are based on age of the

[10]Less than 6% of parent households had more than 3 children or had more than two 5 year old children at any given point in time. Less than 6% of grandparent households and intergenerational households with a single father (mother) had more than 1 child (2 children).

Table 2: Proportion with Children Aged Below 5

Household	Adults	# Kids < 5	t = 0	t = 1	t = 2	# Profiles
Parent	Single mother	1 kid	0.45	0.35	0.25	17
		2 kids	0.18	0.09	0.05	
	Single father	1 kid	0.37	0.37	0.25	17
		2 kids	0.17	0.11	0.06	
	Two parents	1 kid	0.40	0.39	0.31	17
		2 kids	0.21	0.21	0.13	
Grandparent	Grandmother	1 kid	0.07	0.07	0.05	4
	Grandparents	1 kid	0.02	0.01	0.01	4
Intergenerational	Grandmother & Father	1 kid	0.07	0.07	0.05	4
	Grandfather & Mother	1 kid	0.24	0.25	0.19	9
		2 kids	0.08	0.08	0.06	

Note: Proportions computed from CPS data.

youngest child and is 1 unit of time if the latter is a newborn.

Price of formal child care p_{kt} depends on the number of newborns in the household. I calibrate p_{kt} according to data from Child Care Aware of America (2014) fact sheet, which is an annual report on child care costs based on statistics from state Child Care Resource and Referral agencies and from the latest market rate surveys. Among families that use formal child care, infants and toddlers aged 4 were in either center-based care or family child care homes.[11]

The calibration of hourly cost of formal child care for a child aged 0-5 is done as follows. For each state, I first compute the average annual costs that a child would incur in center-based care and in family child care home assuming that the child faces the infant cost for 2 years and the toddler cost for 3 years. I then pro-rate the center-based and family child care home costs according to the proportion of space attributed to each facility type. In the next step, given 50 working weeks of 40 hours each a year, I compute the hourly cost of child care in each state. Finally, I pro-rate this cost by the proportion of children aged less than 4 in each state and convert to 2010 dollars using the Bureau of Labor Statistics (BLS) Consumer Price Index (CPI) calculator. This yields an hourly child care cost of \$3.88 per child. In sensitivity analysis, I recalibrate the model using a higher cost of formal child care of \$5 per child, which corresponds to annual formal child care cost of \$10,000 for a child in full time day care.[12]

Disability rates The life cycle probabilities of being disabled $\hat{\pi}^i_{kt}$ vary according to gender, marital status and age group, and household adult composition. I assume that the probability of being disabled is independent across household members and across households. The probability of being disabled is based on the CPS question "Does ...

[11] Family care homes are typically licensed facilities that provide paid formal child care to a small group of children. I classify this arrangement as formal child care to avoid confusion with the informal household child care provided by parents and grandparents in the model.

[12] Average hourly cost of formal child care ranged between \$2.47 (Mississipi) and \$9.46 (Columbia).

have a health problem or a disability which prevents work or which limits the kind or amount of work?”.[13]

Table 3: Proportion Disabled

Household	Adults	t				
		0	1	2	3	4
Parent	Single mother	0.05	0.07	0.09	0.10	0.12
	Single father	0.02	0.04	0.06	0.09	0.12
	Married mother	0.02	0.02	0.03	0.04	0.05
	Married father	0.03	0.03	0.03	0.03	0.04
Grandparent	Single grandmother	0.15	0.19	0.23	0.23	0.23
	Married grandmother	0.08	0.10	0.13	0.13	0.14
	Married grandfather	0.07	0.11	0.15	0.15	0.16
Intergenerational	Single grandmother	0.10	0.16	0.23	0.23	0.23
	Single father	0.09	0.12	0.15	0.15	0.15
	Single grandfather	0.07	0.11	0.18	0.18	0.19
	Single mother	0.06	0.06	0.06	0.06	0.06

Note: Proportions computed from CPS data.

I report the proportion of disabled individuals by household adult composition in Table 3. As can be seen from the Table, older individuals have higher probabilities of being disabled compared to younger individuals of the same gender and marital status. On the other hand, single parents and grandparents were more likely to be disabled compared to their married counterparts of the same gender and age group. I determine the transition probabilities of households being in a given state π_{kt} based on the proportions in Table 3. For example, a single mother has probability $\left(1-\hat{\pi}^i_{k0}\right)$ of being healthy in period 0. For subsequent periods, she has conditional probability $\frac{\left(1-\hat{\pi}^i_{kt}\right)}{\left(1-\hat{\pi}^i_{kt-1}\right)}$ of being healthy in period t given that she was healthy in period $t-1$.

Wages I calibrate hourly wages according to the wage profiles of workers in the CPS. I first divide gross earnings by hours of work to get hourly wages. Wages are then adjusted to 2010 dollars using the BLS CPI calculator. 2% of the sample of workers had earnings or hours information missing and 1.8% had wages of more than \$100 per hour, which I drop from the sample. I allow wages to vary according to gender, marital status, age group, household adult composition and education, and take the average across each category. I allow for two education levels: high school or less, and more than high school education. The wage profiles of household members are reported in Appendix Table A1. On average, gross wages for the non-retired are higher than the cost of formal child care, $w^i_{kt} > p_{kt}$.

[13] Since disability is absorbing in the model, I also posit that the proportion of disabled individuals cannot decrease over time. In particular, if the proportion disabled is lower for a older age group than for a younger age group, I assume that the proportion disabled is the same as that of the younger age group. This was relevant for single grandmothers in the last two periods.

C. US Tax and Benefit System

Social Security and Federal taxes Social Security taxes are calculated as 6.2% of the first $106,800 earnings (SSA, 2010). Taxable income is computed as gross earnings minus exemptions and deductions. Deductions are $5,700 for singles, $8,400 for household heads, and $11,400 for married couples. Each individual and dependent also gets personal exemptions of $3,650. Federal income tax brackets are given in Appendix Table A2.

EITC The EITC is a refundable tax credit designed for lower income working families. The phase-in rate, maximum credit, phase-out rate and income limits depend on the number of children aged below 18 in the household. The income limits also depend on a tax payers filing status (i.e., single, head of household or married). The EITC schedule is given Appendix Table A2.

CDCTC The CDCTC is a non-refundable tax credit program available to working families with children under 13. The CDCTC has a tax credit rate of 20% to 35% of child care expenses up to a cap of $3k for families with one child and $6k for families with two or more children (Tax Policy Center, 2010). The 35% credit rate applies to families with annual gross income of less than $15k, and declines by 1% for each $2k of additional income until it reaches a constant tax credit rate of 20% for families with annual gross income above $43k.

CCDF The CCDF is a block grant fund managed by states within certain federal guidelines. CCDF subsidies are available as vouchers or as part of direct purchase programs to working families with children under 13 and with income below 85% of the state median income. I set the CCDF rate to 90% which is the recommended subsidy rate under Federal guidelines although there are variations across states. I take into account the fact that only a certain proportion of eligible households received the CCDF subsidy: 39%, 24%, and 5% of potentially eligible children living in households respectively, below the poverty threshold, between 101 to 150% of the poverty threshold, and above 150% of the poverty threshold but below the CCDF eligibility threshold of 85% of state median income (DHHS, 2014). US median household income was $51,144 in 2010. The poverty thresholds for different family sizes are given in Appendix Table A2.

SSDI To be eligible for disability benefits, one must have worked for at least 5 out of the 10 most recent years with the benefits being permanent thereafter. SSDI benefits are based on the age at which one becomes disabled and Average Indexed Monthly Earnings (AIME). SSDI benefits are automatically converted to retirement benefits when the recipient is past the retirement age of 65. I assume that if a person is disabled, that person is disabled at the start of the period and the relevant AIME is a summary of earnings from the previous periods. I compute SSDI benefits as follows:

$$SSDI^i_{kt} = \begin{cases} 0.9AIME^i_{kt-1} & \text{if} \quad AIME^i_{kt-1} \in [0, d_1] \\ 0.9d_1 + 0.32\left(AIME^i_{kt-1} - d_1\right) & \text{if} \quad AIME^i_{kt-1} \in [d_1, d_2] \\ 0.9d_1 + 0.32\left(d_2 - d_1\right) + 0.15\left(AIME^i_{kt-1} - d_2\right) & \text{if} \quad AIME^i_{kt-1} > d_2 \end{cases},$$

where d_1 and d_2 are bend points. In 2010, d_1 was equal to \$761 and d_2 was equal to \$4,586 (SSA, 2014). I use the following formula to approximate AIME:

$$AIME^i_{kt} = \frac{1}{2}\left(AIME^i_{kt-1} + min\left\{ssbase_t, y^i_{kt}\right\}\right),$$

where $ssbase_t$ is the Social Security base wage of \$106,800. I assume that parents are not eligible for SSDI in period $t = 0$ while grandparents are eligible to claim disability benefits in the first period.[14] The relevant AIME if a grandparent is disabled in the first period is approximated from average earnings of individuals aged 45-49 with the same gender, marital status, and education.

SSI SSI is a means-tested program that provides benefits to low income individuals aged above 65 and to the disabled. The definition of disability is the same as under SSDI although there are no contribution requirements under SSI. It is possible to receive both SSI and SSDI if income is sufficiently low.[15] To be eligible for SSI, countable resources need to be less than \$2k for an individual and \$3k for a couple (Morton, 2014). I use household assets as the measure of resources. SSI benefits are reduced one-for-one for income. SSI benefits are approximiated as follows:

$$SSI^i_{kt} = max\left\{0, \bar{SSI}^i_{kt} - SSDI^i_{kt}\right\},$$

where $\bar{SSI}^i_{kt}$ is the maximum SSI benefits. In 2010, the maximum monthly benefits available to a single individual and to a couple were respectively, \$674 and \$1,011.

D. *Preference Parameters*

The 5 year period discount factor is set at $\beta = 0.95$ corresponding to an annual interest rate of $r = 1\%$. The Frisch elasticity of labor supply is set to 0.5 which corresponds to $\gamma = 2$ (Chetty et al., 2011; Domeij and Klein, 2012; Pistaferri, 2003). The effort cost parameter α^i_k varies by gender, marital status, age group (25-49 and 50-74), and household adult structure. α^i_k is internally calibrated to match average weekly labor hours of working adults without children aged below 18 in the household in the CPS.

[14] Less than 3% of SSDI recipients were aged 25-29. Meanwhile, 75% of the working age population are eligible for SSDI benefits with 70% of recipients being aged above 50. 53% of SSDI beneficiaries are male and 47% are female. When SSI is taken into account, 90% of the working age population are insured against disability (SSA, 2011a).

[15] 85% of SSI recipients received the benefits based on disability in 2010 and 34% of SSI recipients also received Social Security benefits (SSA, 2011b).

The calibration of α_k^i is done as follows. First, define a grid of possible values over α. Then, for each household structure, find the labor supply predicted by the model for healthy workers, $l_k^i(\alpha)$, where α is a vector of grid points associated with the effort cost parameters of all household members. I solve each household's utility maximization problem by taking into account the US Social Security and Federal Taxes and EITC. I then minimize the sum of squares of the distance between labor supply predicted by the model and average weekly labor hours from the CPS, $\hat{l_k^i}$:

$$\alpha_k = argmin \sum_i \left\{ E\left[l_k^i(\alpha)\right] - \hat{l_k^i} \right\}^2.$$

The average labor hours and calibrated effort cost parameters are reported in Table 4. As can be seen from the Table, the weekly labor hours predicted by the model are matched very closely to average labor hours in the CPS.[16]

Table 4: Effort Cost Parameter

Household	Adults	Hours		α
		Data	Model	
Parent	Single mother	39.86	39.98	0.58
	Single father	41.54	41.56	0.53
	Married mother	39.63	38.30	0.39
	Married father	43.67	44.20	0.32
Grandparent	Single grandmother	37.27	37.22	0.72
	Married grandmother	37.04	39.15	0.32
	Married grandfather	42.37	43.96	0.34
Intergenerational	Single grandmother	38.73	38.72	0.40
	Single father	41.48	41.34	0.30
	Single grandfather	43.69	43.74	0.33
	Single mother	39.24	39.23	0.35

Note: α calibrated to match average weekly hours of work of working adults in the CPS averaged over time periods.

I report the life cycle profiles of labor supply averaged over all child compositions in Appendix Figures A1 to A3. As can be seen from Figure A1, the model replicates the life cycle profile of labor supply for working parents very closely. The life cycle profiles of adult members in households with a grandparent are also closely replicated although slightly overestimated especially for married grandmothers in Figure A2 and single grandfathers in Figure A3.

E. *Initial Promised Utility*

The initial promised utility is calibrated for each household type according to their adult composition and child composition profiles. V_k is set equal to the expected utility of the

[16] Recall from Lemma 1(ii) that the consumption-labor supply margin of healthy workers is not distorted.

household under the US tax and benefit system according to household members' actual health and disability status. In particular, V_k reflects the "desired" welfare level of each family type under the current system taking into account Social Security and Federal Taxes, EITC, DCTC, CCDF, SSDI and SSI.

F. *Computation*

The government problem (4) is solved by backward induction for each of the 208 household types. First define a grid over promised utility V. Starting from the final period, for each grid point, find the allocations that minimize expected costs while satisfying the promise keeping and incentive compatibility constraints, and find the threatened utilities $\hat{V}$ that can be delivered through the threat keeping constraints. In the penultimate period, repeat the procedure taking into account the fact that the continuation utilities for truthful and untruthful agents will become respectively, the promised and threatened utilities in the final period. The procedure is repeated until the first period. Given the calibrated initial promised utility V, the optimal allocations for each possible disability history can then be computed.

IV. Numerical Results

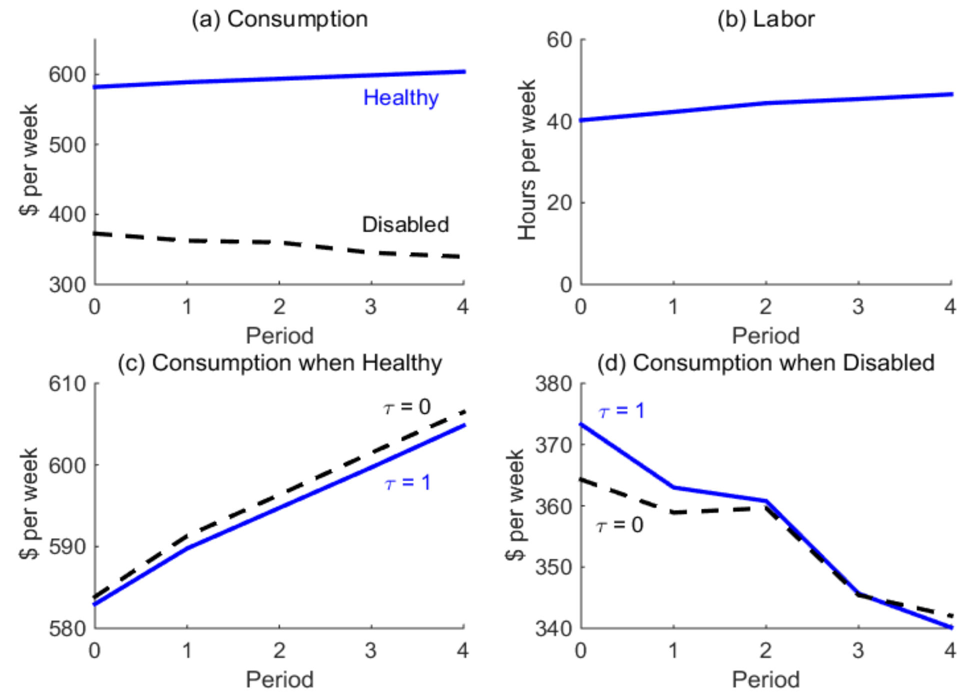
This section characterizes the constrained optimal allocations. Those allocations are then compared to the case where day care subsidies are not available, thereby allowing agents to deviate on child care in a decentralized economy. Finally, I compute the cost savings from the optimal scheme with subsidized day care relative to an alternative scheme without subsidized day care.

A. *Optimal Allocations*

Figures 1, 2 and 3 report the constrained optimal allocations for respectively single mothers, married parents and single grandmothers, averaged over all child arrival profiles and education groups. The constrained optimal allocations for the remaining 4 household adult structures are reported in Appendix Figures A4 to A7. The optimal allocations as implemented by the scheme described in Proposition 1, are illustrated in Panels (a) and (b) of the Figures.

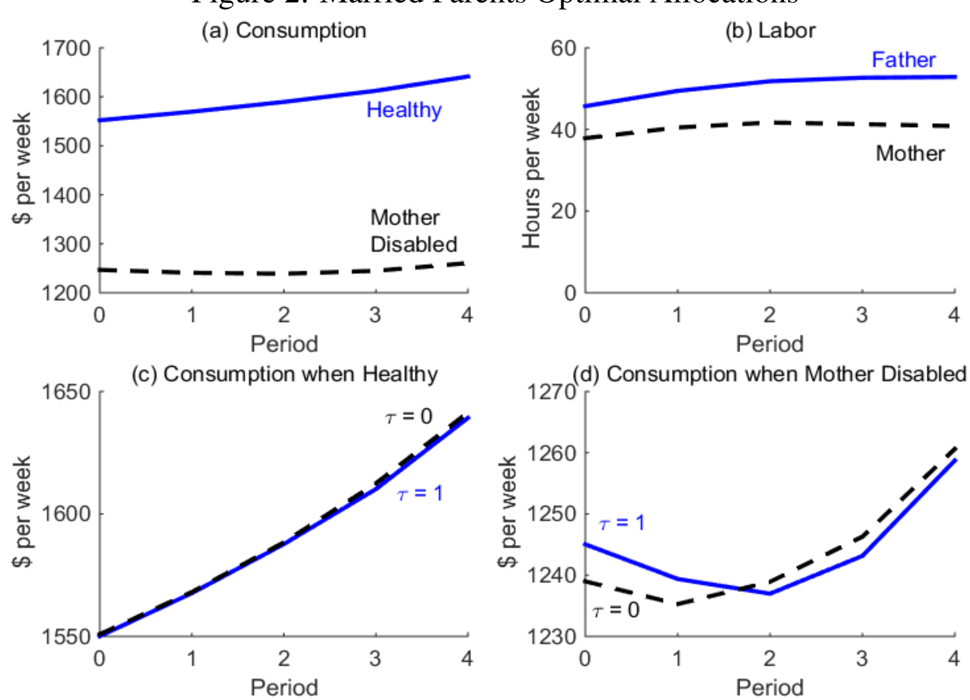
The solid lines in Panels (a) illustrate the consumption allocated to each household when all adult members of the household remain healthy in all periods. A common feature to note across all Figures, is that the consumption profile is non-decreasing over time. In particular, consumption is strictly increasing in households with a parent present. Similarly, consumption is strictly increasing in grandparent only households, until the retirement period 3, after which consumption is constant. The increasing consumption profiles are in line with the government providing dynamic work incentives to agents who remain healthy and work. In particular, agents who remain healthy are rewarded with higher consumption and future utilities, and therefore higher consumption in the future, so as to preserve work incentives.

Figure 1: Single Mothers Optimal Allocations



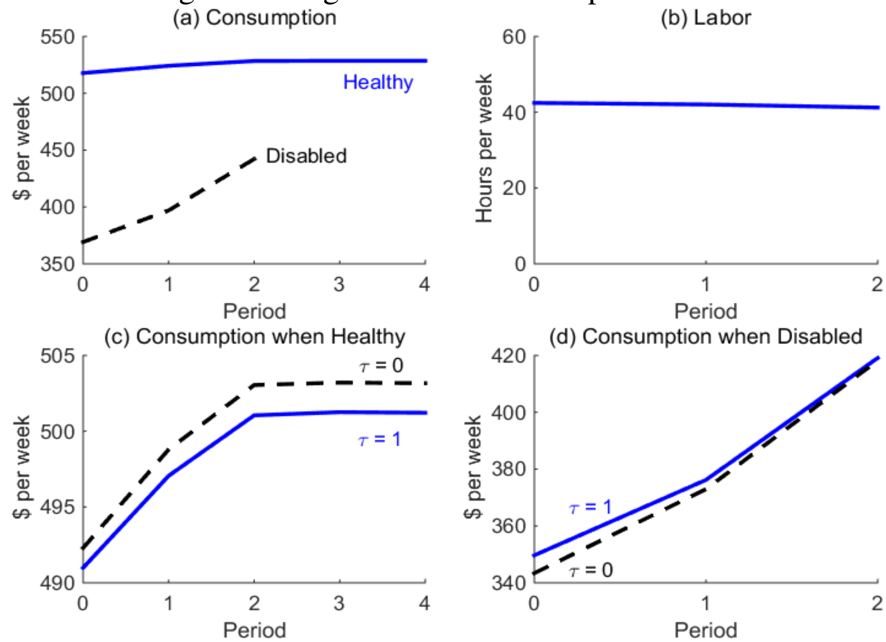
Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.

Figure 2: Married Parents Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.

Figure 3: Single Grandmothers Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children. Grandparents are retired in periods 3 and 4. Single grandmothers have zero probability of becoming disabled in period 3 and 4 if they were previously healthy.

The dashed lines of Panels (a) illustrate consumption allocated to a household when one household member becomes disabled in period 0, 1, 2, 3 or 4. In two parents households, the disabled member is the mother.[17] Consumption of the disabled is influenced by (i) life-cycle wage profiles and (ii) dynamic incentives. On one hand, it may be efficient to have lower consumption for the disabled when agents are more productive (i.e., have higher wages). This is so as to discourage healthy agents from mimicking the disabled. On the other hand, preservation of dynamic work incentives imply allocating higher future utilities to working agents, and therefore higher consumption when an agent becomes disabled later in life.

As can be seen from Appendix Table A1, parents have increasing wage profiles, except college educated married mothers whose wage declines in the last two periods. Conversely, grandparents have decreasing wage profiles. The life-cycle wage profile effect described above seems to dominate for those with rising wage profiles: The consumption profile of disabled single mothers is decreasing in Figure 1, which is in line with providing lower consumption to the disabled so as to preserve the work incentives of still abled and increasingly productive agents. Conversely, the dynamic incentives effect described above seems to prevail for those with declining wage profiles: The consumption profile of disabled single grandmothers is increasing in Figure 3, which is in line with providing higher future utilities to agents who remain abled, and therefore

[17]For two member households, I illustrate the more frequent situation where only the mother or the grandparent becomes disabled. This corresponds to the grandmother in Figure A5, and the grandparent in Figures A6 and A7.

higher consumption when an agent becomes disabled later in life.[18]

There are also two forces influencing optimal labor market effort dynamics when all household members are healthy: (i) life-cycle wage profiles and (ii) dynamic incentives. While it may be efficient for agents to work more as they get more productive, the increasing consumption profiles when healthy and the consumption-labor margin in Lemma 1(ii) imply that it may also be efficient for agents to work less in future periods. Once again, the first effect seems to dominate the second effect for those with rising wage profiles: Single mothers have increasing labor supply profiles in Figures 1, in line with encouraging more productive agents to work more. Conversely, the second effect seems to prevail for those with declining wage profiles: Single grandmothers have decreasing labor supply profiles in Figure 3, in line with the provision of dynamic incentives.

B. *Optimal Allocations without Day Care Subsidies*

I now compute the optimal allocations in the case where the government may not use day care subsidies, but may use non-linear income taxation and asset-testing. Thus, the constrained optimal allocations from problem (4) cannot be implemented in a decentralized economy as agents may now engage in non socially optimal household child care activities and the government may not use day care subsidies to counteract such private incentives. The benchmark model with subsidized day care is equivalent to the solution to the government problem (4) and the comparison model without subsidized day care is equivalent to the solution to the government problem with hidden household child care.

The optimal allocations with ($\tau = 1$) and without ($\tau = 0$) subsidized day care are illustrated by the solid and dashed lines in Panels (c) and (d) of Figures 1 to 3 and in Appendix Figures A4 to A7. I report the averaged profiles across households with children. Optimal labor market effort profiles (not illustrated) in the case without subsidized day care are similar and sometimes slightly lower compared to the case with subsidized day care. The main difference stems from the fact that consumption allocated to households where all members are healthy is higher compared to the case with subsidized day care, as can be seen from Panels (c). Conversely, consumption allocated to households when one member becomes disabled is lower in earlier periods compared to the case with subsidized day care, as can be seen from Panels (d).

The intuition behind this result relates to the fact that child care needs are relevant in the first three periods $t = 0, 1, 2$. The incentives to mimic the disabled so as to engage in private household child care activities are therefore exacerbated in earlier periods. Since the government cannot use subsidized day care to counterbalance such child care incentives, the only way to incentivize would be mimickers to be honest is by rewarding the healthy through higher consumption and penalizing the disabled through lower consumption. In later periods, when child care needs are less relevant, consumption of the disabled are also higher, in line with the dynamic incentives associated with providing higher future utility to those who were working in earlier periods. The gaps in

[18]Note that consumption is constant once all members are disabled or retired since all uncertainty has been resolved.

consumptions with and without subsidized day care are wider for households with more children arriving when parents are aged 30-39 and grandparents aged 55-64. For one member households, consumption of the disabled without subsidized day care is up to \$61 per week lower compared to consumption of the disabled with subsidized day care. The corresponding figure for two member households is \$117 per week.

C. Cost Savings from Subsidized Day Care

I compute the cost savings associated with the case where the government may use subsidized day care compared to the case where the government may not use subsidized day care. The costs savings are computed such that both cases deliver the same initial promised utility, V_k, to households of type k. I take the difference between expected costs from both cases and compute the percentage cost savings relative to expected costs in the case where the government may not use subsidized day care. The difference in costs stems from the fact that with subsidized day care, the government may directly smoothen the exacerbated incentives of would be mimickers by targeting their private child care incentives. Conversely, without subsidized day care, households may engage in non-optimal household child care activities when mimicking the disabled. The higher the relative incentives to engage in non-optimal household child care, the more exacerbated the incentives of would be mimickers, and therefore the higher the relative cost savings from using subsidized day care as a policy tool.

The average cost savings are computed for each household type and range from 0.05% to 3.31%, with higher cost savings for single parent households and intergenerational households with both a parent and a grandparent present.[19] I report the average cost savings by education group and by total number of children in Table 5. As can be seen from the Table, single mothers with high school education or less have relatively higher cost savings compared to single mothers with college education. Cost savings for the former range between 1.65% and 2.2% and for the latter between 0.23% and 1.56%. A similar pattern is observed for single fathers, whose cost savings range between 0.1% and 1.23%, and with higher savings for families with more children. Cost savings for two parent households range between 0.05% and 0.8% and for grandparent households between 0.20% and 0.36%. Intergenerational households have cost savings ranging between 0.24% and 3.31%, with higher cost savings for families with college educated grandmothers and fathers, and for families with a grandfather, a mother, and two children in the household.

In sensitivity analysis, I recalibrate the initial promised utilities and recompute the associated cost savings for each family type under a higher hourly cost of formal child care of \$5 per child, which corresponds to annual formal child care cost of \$10,000 for a child in full time day care. As can be seen from Table 6, costs savings are rela-

[19] The cost savings may be interpreted as the relative information costs associated with private household child care incentives. I also computed the information costs associated with privately observed disability shocks by taking the difference between expected costs for the full information case and expected costs under the constrained optimum with subsidized day care. The information costs ranged between 0.44% to 59.35% with higher cost savings associated with intergenerational households with both a grandparent and a parent present.

tively higher for most household types. Cost savings range from 0.06% to 2.93% for parent households, 0.27% to 0.68% for grandparent households, and 0.4% to 13.7% for intergenerational households. The more expensive formal child care is, the higher the incentives to mimic the disabled so as to save on formal child care. The role of subsidized day care in counterbalancing those incentives therefore becomes even more important, thereby leading to higher costs savings.

Table 5: Cost Savings

Household	Adults	High School			College		
		1 kid	2 kids	3 kids	1 kid	2 kids	3 kids
Parent	Single mother	2.20%	1.88%	1.65%	0.23%	0.54%	1.56%
	Single father	0.18%	0.57%	1.23%	0.10%	0.14%	0.41%
	Two parents	0.11%	0.29%	0.80%	0.05%	0.07%	0.15%
Grandparent	Grandmother	0.27%	-	-	0.34%	-	-
	Grandparents	0.20%	-	-	0.36%	-	-
Intergenerational	Grandmother & Father	0.59%	-	-	2.59%	-	-
	Grandfather & Mother	0.27%	2.03%	-	0.24%	3.31%	-

Note: Cost savings are averaged over child arrival profiles. When there are two household members, I report cost savings for cases where both members have the same level of education.

Table 6: Sensitivity Analysis on Cost Savings

Household	Adults	High School			College		
		1 kid	2 kids	3 kids	1 kid	2 kids	3 kids
Parent	Single mother	1.77%	2.23%	2.93%	0.34%	1.01%	2.22%
	Single father	0.26%	0.67%	1.68%	0.15%	0.32%	0.53%
	Two parents	0.11%	0.25%	0.66%	0.06%	0.33%	0.45%
Grandparent	Grandmother	0.27%	-	-	0.45%	-	-
	Grandparents	0.68%	-	-	0.47%	-	-
Intergenerational	Grandmother & Father	0.62%	-	-	13.7%	-	-
	Grandfather & Mother	0.63%	1.18%	-	0.40%	3.80%	-

Note: Cost savings with hourly cost of formal child care of $5 per child, corresponding to $10,000 per year for a child in full time day care.

V. Conclusion and Discussion

This study proposes an implementation of optimal social insurance when disability shocks are private information in a multi-member and multi-choice framework. The possibility of engaging in household child care activities exacerbates the incentives to mimic the disabled, since in addition to receiving disability benefits, the household may save on formal child care costs. Subsidized day care capped at the optimal level of formal child care helps counteract such incentives for all members of the family. At the same time, non-linear income taxation and asset-testing prevent households from oversaving in earlier periods and therefore decrease the private incentives to mimic the disabled in later periods. Calibrating the model to match key features of the US labor and child care markets, I find that the use of subsidized formal day care may lead to sizeable cost savings, with higher cost savings for single mothers and for intergenerational households with both a parent and a grandparent present.

The model presented in this study is applicable to alternative home production needs such as elder care. Indeed, as long as home production needs are important enough to exacerbate the incentives to mimic the disabled and save on the cost of formal activities, subsidizing the formal activities that are substitutable to home production with a cap at the socially efficient level may be efficient. Such subsidies may be used in conjuction with income taxation and asset-testing so as to preserve the work incentives of healthy family members.

The case for day care subsidies would still hold even if disability shocks were not absorbing or in a model with multiple health shocks that results in varying degrees of disabilities. The case for day care subsidies would also hold for more general effort cost functions that allow for differing costs of effort from labor market and child care activities, or in a model with privately observed heterogenous labor market productivities (Ho and Pavoni 2016). While the optimal mix of primary labor market and household child care effort would still depend on the relative wage and cost of formal child care, as long as the opportunity to engage in non-optimal household child care activities exacerbates the incentives to mimic the disabled, there would be a role for day care subsidies to counterbalance such incentives.

While child care needs are defined as occurring due to the arrival of a child aged below 5 into the household, I note that child care needs may be broader in definition. For instance, school age children may also have after school care needs. In addition, it is possible that some multi-generational family members, such as grandparents, provide child care to their non-resident grandchildren. Such child care needs may also contribute to the exacerbation of incentives of healthy family members to mimic the disabled. In this case, the computed cost savings would provide a lower bound on the potential cost savings from subsidized day care. Conversely, it is unclear how cost savings would change if one were to allow for endogenous child care needs. Bick (2015) argues that higher child care subsidies financed with higher income taxes may have opposing effects on fertility such that child care subsidies may not influence fertility.

The model may be extended to incorporate heterogenous child care quality. The extension is straightforward if quality is reflected in child care costs. As shown in sensitivity analysis, higher child care costs may result in higher cost savings from subsidized day care. The model would be more complicated if quality choice were not observable by the government and if higher quality child care help improve the human capital of future generations. Bastani et al. (2013) explore the desirability of a refundable tax credit, tax deductability, and public provision of child care in a model with two agent's types, where the main focus is on motivating parents to choose higher quality child care. Cornelissen et al. (2015) find that while children from disadvantaged background were less likely to enrol in formal child care, they were also more likely to have higher gains in terms of school readiness and health outcomes. In addition to the cost savings from preventing parents and grandparents from "shirking", subsidized day care may therefore help encourage higher quality choices which may also result in higher human capital gains for children. I leave such interesting considerations for future research.

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A Appendix

A. *Proof of Lemma 1*

To keep the notation simple, I drop the constrained optimal subscript, *, and loosely use the notation $\tilde{s} > s$ to indicate that agents in state $\tilde{s}$ may mimic agents in state s. Let $\lambda_t(s_{t-1})$ denote the Lagrange multiplier associated with the promise keeping constraint (1) for agents who truthfully declared to be in state s_{t-1} in the previous period, $\tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1})$ denote the Lagrange mutipliers associated with the threat keeping constraints (2) for agents who declared to be in state s_{t-1} in the previous period when they were actually in state $\tilde{s}_{t-1} \in S_{t-1}$, and $\eta_t(s_t, \tilde{s}_t)$ denote the Lagrange multipliers associated with the incentive compatibility constraints (3) of agents who declare to be in state s_t when they are actually in state $\tilde{s}_t \in S_t$.

The first order conditions for each period $0 < t < T$ from problem (4) are:

$$
\begin{array}{rcrcl}
c_t(s_t) & : & \pi_t(s_t, s_{t-1}) - \left[\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) + \sum_{\tilde{s}_t > s_t} \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right)\right] u'(c_t(s_t)) & = & 0, \\
l_t^i(s_t) & : & \pi_t(s_t, s_{t-1}) w_t^i - \left[\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) + \sum_{\tilde{s}_t > s_t} \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right)\right] v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right) & \leq & 0, \\
h_t^i(s_t) & : & \pi_t(s_t, s_{t-1}) p_t - \left[\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) + \sum_{\tilde{s}_t > s_t} \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right)\right] v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right) & \leq & 0, \\
V_{t+1}(s_t) & : & -\pi_t(s_t, s_{t-1}) G'_{V_{t+1}(s_t)}\left(V_{t+1}(s_t), \mathbf{\tilde{V}_{t+1}(s_t, \tilde{s}_t)}\right) + \zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) & = & 0, \\
\tilde{V}_{t+1}(s_t, \tilde{s}_t) & : & -\pi_t(s_t, s_{t-1}) G'_{\tilde{V}_{t+1}(s_t, \tilde{s}_t)}\left(V_{t+1}(s_t), \mathbf{\tilde{V}_{t+1}(s_t, \tilde{s}_t)}\right) + \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right) & = & 0,
\end{array}
$$

where

$$
\begin{aligned}
\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) &= \lambda_t(s_{t-1}) \pi_t(s_t, s_{t-1}) + \sum_{\tilde{s}_{t-1} > s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}) \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) I\{\tilde{s}_t = s_t\} + \sum_{\tilde{s}_t < s_t} \eta_t(\tilde{s}_t, s_t), \\
\tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right) &= \sum_{\tilde{s}_{t-1} > s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}) \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) I\{\tilde{s}_t > s_t\} - \eta_t(s_t, \tilde{s}_t).
\end{aligned}
$$

$I\{\tilde{s}_t > s_t\}$ is an indicator function taking a value of 1 if it is privately optimal for those in state $\tilde{s}_t > s_t$ to declare to be in state s_t, and a value of 0 otherwise. $I\{\tilde{s}_t = s_t\}$ is an indicator function taking a value of 1 if an agent who was previously untruthful happens to be in state s_t in the current period, and a value of 0 otherwise. By incentive compatibility, such agents will find it optimal to be truthful and declare state s_t. They therefore get continuation utility $\tilde{V}_{t+1}(s_t, s_t) = V_{t+1}(s_t)$.

(i) Healthy members with $w_t^i \geq p_t$. From examining the first order conditions with respect to $l_t^i(s_t)$ and $h_t^i(s_t)$, one can rule out cases with both $l_t^i(s_t) = 0$ and $h_t^i(s_t) = 0$, when agents are healthy and $\pi_t(s_t, s_{t-1}) > 0$, since $v_i'(0) = 0$ and $w_t^i \geq p_t > 0$. I now show that $l_t^i(s_t) > 0$ and $h_t^i(s_t) = 0$. Suppose to the contrary that $l_t^i(s_t) = 0$ and $h_t^i(s_t) > 0$. Then, it would be possible to decrease $h_t^i(s_t)$ by $\varepsilon > 0$ and increase $l_t^i(s_t)$ by ε such that the total effort of member i is the same. The promise keeping, threat keeping and incentive compatibility constraints are still satisfied, while the government's expected costs decrease by $\pi_t(s_t, s_{t-1})\left(w_t^i - p_t\right)\varepsilon \geq 0$. Thus, $l_t^i(s_t) = 0$ and $h_t^i(s_t) > 0$ cannot be optimal. The same argument applies for cases where $l_t^i(s_t) > 0$ and $h_t^i(s_t) > 0$. It must therefore be that $l_t^i(s_t) > 0$ and $h_t^i(s_t) = 0$ for healthy members with $w_t^i \geq p_t$.

Healthy members with $w_t^i < p_t$. If $h_t^i(s_t)$ is an interior solution, then the first order condition with respect to $h_t^i(s_t)$ is satisfied with equality whereas the first order condition with respect to $l_t^i(s_t)$ is satisfied with strict inequality since $w_t^i < p_t$. Thus, $h_t^i(s_t) > 0$ and $l_t^i(s_t) = 0$. Suppose to the contrary that $h_t^i(s_t) = 0$ and $l_t^i(s_t) > 0$. Then, it would be possible to increase $h_t^i(s_t)$ by $\varepsilon > 0$ and decrease $l_t^i(s_t)$ by ε such that the total effort of member i is the same. The promise keeping, threat keeping and incentive compatibility constraints are still satisfied, while the government's expected costs decrease by $\pi_t(s_t, s_{t-1})\left(p_t - w_t^i\right)\varepsilon > 0$. Thus, $h_t^i(s_t) = 0$ and $l_t^i(s_t) > 0$ cannot be optimal. It must therefore be that $h_t^i(s_t) > 0$ and $l_t^i(s_t) = 0$ for healthy members with $w_t^i < p_t$, as long as all child care needs have not yet been met. Healthy household members with $w_t^i < p_t$ may work on the labor market only when all child care needs have been met through household child care since there are no cost savings from reallocating effort from labor to child care.

(ii) Healthy members with $w_t^i \geq p_t$. From **(i)**, $l_t^i(s_t) > 0$ and $h_t^i(s_t) = 0$. The first order condition with respect to $l_t^i(s_t)$ is satisfied with equality while the first order condition with respect to $h_t^i(s_t)$ is satisfied with inequality (strict inequality when $w_t^i > p_t$). Using the first order conditions with respect to $c_t(s_t)$, $l_t^i(s_t)$, $h_t^i(s_t)$ and rearranging, one gets:

$$u'(c_t(s_t))\,w_t^i = v_i'\left(e_t^i(s_t)\right) \text{ and } u'(c_t(s_t))\,p_t \leq v_i'\left(e_t^i(s_t)\right).$$

Healthy members with $w_t^i < p_t$. From **(i)**, the first order condition with respect to $h_t^i(s_t)$ is satisfied with equality as long as all child care needs have not yet been met (i.e., interior solution), while the first order condition with respect to $l_t^i(s_t)$ is satisfied with strict inequality. Using the first order conditions with respect to $c_t(s_t)$, $l_t^i(s_t)$, $h_t^i(s_t)$, and rearranging, one gets:

$$u'(c_t(s_t))\,w_t^i < v_i'\left(e_t^i(s_t)\right) \text{ and } u'(c_t(s_t))\,p_t = v_i'\left(e_t^i(s_t)\right).$$

(iii) Adding the first order conditions with respect to $V_{t+1}(s_t)$ and $V_{t+1}(s_t, \tilde{s}_t), \forall \tilde{s}_t > s_t$, and taking into account the fact that $I\{\tilde{s}_t = s_t\} + I\{\tilde{s}_t > s_t\} = I\{\tilde{s}_t \geq s_t\}$, one gets:

$$\pi_t(s_t, s_{t-1})\left[G'_{V_{t+1}(s_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right) + \sum_{\tilde{s}_t > s_t} G'_{\tilde{V}_{t+1}(s_t,\tilde{s}_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right)\right] = \psi_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right).$$

Using the first order condition with respect to $c_t(s_t)$, one gets:

$$\frac{1}{u'(c_t(s_t))} = G'_{V_{t+1}(s_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right) + \sum_{\tilde{s}_t > s_t} G'_{\tilde{V}_{t+1}(s_t,\tilde{s}_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right). \quad \text{(A1)}$$

Now, adding the first order conditions with respect to $c_t(s_t)$ for all $s_t \in S_t$, and taking into account the fact that $\sum_{s_t \in S_t} \sum_{\tilde{s}_t \geq s_t} \pi_t(\tilde{s}_t, \tilde{s}_{t-1})\, I\{\tilde{s}_t \geq s_t\} = 1$, one gets:

$$\sum_{s_t \in S_t} \frac{\pi_t(s_t, s_{t-1})}{u'(c_t(s_t))} = \lambda_t(s_{t-1}) + \sum_{\tilde{s}_{t-1} > s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}).$$

One has an anologous expression for the following period:

$$\sum_{s_{t+1} \in S_{t+1}} \frac{\pi_{t+1}(s_{t+1}, s_t)}{u'(c_{t+1}(s_{t+1}))} = \lambda_{t+1}(s_t) + \sum_{\tilde{s}_t > s_t} \tilde{\lambda}_{t+1}(s_t, \tilde{s}_t).$$

From the interpretation of the Lagrange multipliers, one has:

$$\sum_{s_{t+1} \in S_{t+1}} \frac{\pi_{t+1}(s_{t+1}, s_t)}{u'(c_{t+1}(s_{t+1}))} = G'_{V_{t+1}(s_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right) + \sum_{\tilde{s}_t > s_t} G'_{\tilde{V}_{t+1}(s_t,\tilde{s}_t)}\left(V_{t+1}(s_t), \tilde{\mathbf{V}}_{\mathbf{t+1}}(\mathbf{s_t}, \tilde{\mathbf{s}}_\mathbf{t})\right). \quad \text{(A2)}$$

The inverse Euler equation follows from equations (A1) and (A2):

$$\frac{1}{u'(c_t(s_t))} = \sum_{s_{t+1} \in S_{t+1}} \frac{\pi_{t+1}(s_{t+1}, s_t)}{u'(c_{t+1}(s_{t+1}))}.$$

Applying Jensen's inequality to the inverse Euler equation, one then gets the inter-temporal wedge between current and future marginal utilities of consumption:

$$u'(c_t(s_t)) < \sum_{s_{t+1} \in S_{t+1}} \pi_{t+1}(s_{t+1}, s_t)\, u'(c_{t+1}(s_{t+1})).$$

B. *Proof of Proposition 1*

Every period, households take the policy scheme as given and choose a state claim $\tilde{s}_t \in S_t$, consumption, household child care, and assets that maximize utility. From the decentralized household problem, the function $\mathbf{I}_t = p_t\left(\sum_i h_t^{i*}(\tilde{s}_t) - \sum_i \tilde{h}_t^i(\tilde{s}_t, s_t)\right)$ when $\sum_i h_t^{i*}(\tilde{s}_t) - \sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) > 0$, and $\mathbf{I}_t = 0$ otherwise. The cost of formal child care faced by the household is positive only if total household child care is lower than the optimal level (i.e., the cost of formal child care is beyond the free day care cap). Income taxes $T_t(\tilde{s}_t) = \sum_i w_t^i l_t^{i*}(\tilde{s}_t) - c_t^*(\tilde{s}_t)$ if $\tilde{A}_t(\tilde{s}_{t-1}, s_{t-1}) \leq 0$, and $T_t(\tilde{s}_t) = \sum_i w_t^i l_t^{i*}(\tilde{s}_t) - c_t^*(\tilde{s}_t) + (1+r)\tilde{A}_t(\tilde{s}_{t-1}, s_{t-1})$ otherwise.

Claim 1 Households have no incentives to engage in total household child care that is higher than the optimal level. Suppose to the contrary that households choose $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) > \sum_i h_t^{i*}(\tilde{s}_t)$. Then, households do not get any formal child care cost savings (in fact, they give up part of the free day care for which they are eligible) but need to exert costly effort on household child care. By reducing the effort of any member by $\varepsilon > 0$, the household can save on effort cost and benefit from free day care worth $p_t\varepsilon$. Thus, $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) > \sum_i h_t^{i*}(\tilde{s}_t)$ cannot be optimal.

Claim 2 Households claiming a state in which optimal free day care is full time will engage in the optimal level of household child care. This applies to households consisting of a combination of healthy members with high earnings capacity $w_t^i \geq p_t$ and disabled members only. Such households benefit from full time free day care $f_t^*(\tilde{s}_t) = p_t n_t$. Household members have no incentives to engage in higher than optimal household child care, $\tilde{h}_t^i(\tilde{s}_t, s_t) > h_t^{i*}(\tilde{s}_t) = 0$, as effort is costly and there is no scope to save on formal child care costs. In addition, $\tilde{h}_t^i(\tilde{s}_t, s_t)$ cannot be below $h_t^{i*}(\tilde{s}_t) = 0$ for all members. Household members will therefore engage in the constrained optimal level of household child care: $h_t^{i*}(\tilde{s}_t) = 0$ for all i.

I now get to the gist of the proof and show that households will accumulate zero assets and engage in the optimal level of household child care. The decentralized choices coincide with the constrained optimal allocations and are therefore incentive compatible.

Step 1 Households accummulate non-negative assets. The proof is done by contradiction using backward induction.

Last period ($t = T$) Suppose that assets carried over to the last period are negative: $\tilde{A}_T(\tilde{s}_{T-1}, s_{T-1}) < 0$. The last period's budget constraint is given by:

$$\tilde{c}_T(\tilde{s}_T, s_T) + \mathbf{I}_T = c_T^*(\tilde{s}_T) + (1+r)\tilde{A}_T(\tilde{s}_{T-1}, s_{T-1}).$$

It must therefore be that $\tilde{c}_T(\tilde{s}_T, s_T) < c_T^*(\tilde{s}_T)$ since $\tilde{A}_T(\tilde{s}_{T-1}, s_{T-1}) < 0$ and $\mathbf{I}_T \geq 0$.

For such household choice to be optimal, it must also be that the private Euler equation holds:

$$u'(\tilde{c}_{T-1}(\tilde{s}_{T-1}, s_{T-1})) = \sum_{s_T \in S_T} \pi_T(s_T, s_{T-1})\, u'(\tilde{c}_T(\tilde{s}_T, s_T)).$$

Now, from Lemma 1(iii), there is an inter-temporal wedge at the optimal consumption levels:

$$u'\left(c^*_{T-1}\left(\tilde{s}_{T-1}\right)\right) < \sum_{s_T \in S_T} \pi_T\left(s_T, s_{T-1}\right) u'\left(c^*_T\left(\tilde{s}_T\right)\right).$$

From the private Euler equation, the inter-temporal wedge, the fact that $\tilde{c}_T\left(\tilde{s}_T, s_T\right) < c^*_T\left(\tilde{s}_T\right)$ and from concavity of u, it must therefore be that $\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < c^*_{T-1}\left(\tilde{s}_{T-1}\right)$.

Penultimate period ($t = T-1$) From Claim 1, households do not have an incentive to overprovide total household child care: $\sum_i \tilde{h}^i_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) \leq \sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$. I now show that households do not have an incentive to underprovide total household child care: $\sum_i \tilde{h}^i_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) \geq \sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$. Suppose to the contrary that $\sum_i \tilde{h}^i_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < \sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$. From Lemma 1(ii), the consumption-child care margins at the optimal allocations are given by:

$$u'\left(c^*_T\left(\tilde{s}_T\right)\right) p_T - v'_i\left(l^{i*}_T\left(\tilde{s}_T\right) + h^{i*}_T\left(\tilde{s}_T\right)\right) \leq 0,$$

with equality when $w^i_{T-1} < p_{T-1}$ and $\sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right) < n_{T-1}$. It follows that if $\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < c^*_{T-1}\left(\tilde{s}_{T-1}\right)$, then

$$\begin{aligned} & u'\left(\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right)\right) p_{T-1} - v'_i\left(l^{i*}_{T-1}\left(\tilde{s}_{T-1}\right) + h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)\right) \\ & > u'\left(c^*_{T-1}\left(\tilde{s}_{T-1}\right)\right) p_{T-1} - v'_i\left(l^{i*}_{T-1}\left(\tilde{s}_{T-1}\right) + h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)\right). \end{aligned}$$

Since the marginal gain from household child care is higher, household members have greater incentives to increase household child care beyond the optimal level $h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$. Thus, it cannot be that $\sum_i \tilde{h}^i_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < \sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$. The household must therefore engage in the optimal level of total household child care $\sum_i \tilde{h}^i_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) = \sum_i h^{i*}_{T-1}\left(\tilde{s}_{T-1}\right)$.

The penultimate period's budget constraint is then given by:

$$\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) + \tilde{A}_T\left(\tilde{s}_{T-1}, s_{T-1}\right) = c^*_{T-1}\left(\tilde{s}_{T-1}\right) + (1+r)\tilde{A}_{T-1}\left(\tilde{s}_{T-2}, s_{T-2}\right).$$

Since $\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < c^*_{T-1}\left(\tilde{s}_{T-1}\right)$ and $\tilde{A}_T\left(\tilde{s}_{T-1}, s_{T-1}\right) < 0$, it must be that $\tilde{A}_{T-1}\left(\tilde{s}_{T-2}, s_{T-2}\right) < 0$.

For such household choice to be optimal, it must also be that the private Euler equation holds:

$$u'\left(\tilde{c}_{T-2}\left(\tilde{s}_{T-2}, s_{T-2}\right)\right) = \sum_{s_{T-1} \in S_{T-1}} \pi_{T-1}\left(s_{T-1}, s_{T-2}\right) u'\left(\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right)\right).$$

Now, from Lemma 1(iii), there is an inter-temporal wedge at the optimal consumption levels:

$$u'\left(c^*_{T-2}\left(\tilde{s}_{T-2}\right)\right) < \sum_{s_{T-1} \in S_{T-1}} \pi_{T-1}\left(s_{T-1}, s_{T-2}\right) u'\left(c^*_{T-1}\left(\tilde{s}_{T-1}\right)\right).$$

From the private Euler equation, the inter-temporal wedge, the fact that $\tilde{c}_{T-1}\left(\tilde{s}_{T-1}, s_{T-1}\right) < c^*_{T-1}\left(\tilde{s}_{T-1}\right)$ and from concavity of u, it must therefore be that $\tilde{c}_{T-2}\left(\tilde{s}_{T-2}, s_{T-2}\right) < c^*_{T-2}\left(\tilde{s}_{T-2}\right)$.

First period ($t = 0$) By following the same line of proof, it must be that $\tilde{A}_1\left(\tilde{s}_0, s_0\right) < 0$ from the second period's budget constraint, and that $\tilde{c}_0\left(\tilde{s}_0, s_0\right) < c^*_0\left(\tilde{s}_0\right)$ from the Euler equations between the first and second periods. In addition, $\sum_i \tilde{h}^i_0\left(\tilde{s}_0, s_0\right) = \sum_i h^{i*}_0\left(\tilde{s}_0\right)$ from the household's consumption-child care margins in the first period. The first period's household budget constraint is given by:

$$\tilde{c}_0\left(\tilde{s}_0, s_0\right) + \tilde{A}_1\left(\tilde{s}_0, s_0\right) = c^*_0\left(\tilde{s}_0\right).$$

Since $\tilde{c}_0(\tilde{s}_0, s_0) < c_0^*(\tilde{s}_0)$, it must be that $\tilde{A}_1(\tilde{s}_0, s_0) > 0$, which is a contradiction. Thus, it must be that households accummulate non-negative assets.

Step 2 Households accummulate zero assets.

From Step 1, the household accummulates non-negative assets. The household budget constraint for all $t < T$ may therefore be rewritten as:

$$\tilde{c}_t(\tilde{s}_t, s_t) + \mathbf{I}_t + \tilde{A}_{t+1}(\tilde{s}_t, s_t) = c_t^*(\tilde{s}_t).$$

From Claim 1, households do not have an incentive to overprovide total household child care: $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) \leq \sum_i h_t^{i*}(\tilde{s}_t)$. I now show that households do not have an incentive to underprovide total household child care: $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) \geq \sum_i h_t^{i*}(\tilde{s}_t)$. Suppose to the contrary that $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) < \sum_i h_t^{i*}(\tilde{s}_t)$. From Lemma 1(ii), the consumption-child care margins at the optimal allocations are given by:

$$u'(c_t^*(\tilde{s}_t))\, p_t - v_i'\left(l_t^{i*}(\tilde{s}_t) + h_t^{i*}(\tilde{s}_t)\right) \leq 0,$$

with equality when $w_t^i < p_t$ and $\sum_i h_t^{i*}(\tilde{s}_t) < n_t$. It follows that if $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) < \sum_i h_t^{i*}(\tilde{s}_t)$, then $\tilde{c}_t(\tilde{s}_t, s_t) < c_t^*(\tilde{s}_t)$ from the budget constraint since $\tilde{A}_{t+1}(\tilde{s}_t, s_t) \geq 0$. We therefore have

$$u'(\tilde{c}_t(\tilde{s}_t, s_t))\, p_t - v_i'\left(l_t^{i*}(\tilde{s}_t) + h_t^{i*}(\tilde{s}_t)\right) > u'(c_t^*(\tilde{s}_t))\, p_t - v_i'\left(l_t^{i*}(\tilde{s}_t) + h_t^{i*}(\tilde{s}_t)\right).$$

Since the marginal gain from household child care is higher, household members have greater incentives to increase household child care beyond the optimal level $h_t^{i*}(\tilde{s}_t)$. Thus, it cannot be that $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) < \sum_i h_t^{i*}(\tilde{s}_t)$. The household must therefore engage in the optimal level of total household child care: $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) = \sum_i h_t^{i*}(\tilde{s}_t)$.

The household budget constraint then becomes:

$$\tilde{c}_t(\tilde{s}_t, s_t) + \tilde{A}_{t+1}(\tilde{s}_t, s_t) = c_t^*(\tilde{s}_t).$$

This implies that $\tilde{c}_t(\tilde{s}_t, s_t) < c_t^*(\tilde{s}_t)$ whenever $\tilde{A}_{t+1}(\tilde{s}_t, s_t) > 0$ and $\tilde{c}_t(\tilde{s}_t, s_t) = c_t^*(\tilde{s}_t)$ whenever $\tilde{A}_{t+1}(\tilde{s}_t, s_t) = 0$. Thus, $\tilde{A}_{t+1}(\tilde{s}_t, s_t) > 0$ cannot be optimal as the household gets a lower stream of consumption compared to the case when $\tilde{A}_{t+1}(\tilde{s}_t, s_t) = 0$. The household will therefore choose to accummulate zero assets.

Step 3 Households consume the optimal level of consumption.

From Steps 1 and 2, the household accummulates zero assets: $\tilde{A}_t(\tilde{s}_{t-1}, s_{t-1}) = 0$. The household budget constraint may therefore be rewritten as:

$$\tilde{c}_t(\tilde{s}_t, s_t) = c_t^*(\tilde{s}_t).$$

Households thus consume the optimal level of consumption.

Step 4 Individual household members engage in the optimal level of household child care.

From Step 2, households engage in the optimal level of total household child care: $\sum_i \tilde{h}_t^i(\tilde{s}_t, s_t) = \sum_i h_t^{i*}(\tilde{s}_t)$. In addition, from Claim 2, individual members in households claiming a state in which optimal free day care is full time will engage in the optimal level of household child care: $\tilde{h}_t^i(\tilde{s}_t, s_t) = h_t^{i*}(\tilde{s}_t) = 0, \forall i$. Now consider a household that claims a state in which optimal free day care is less than full time: $f_t^*(\tilde{s}_t) = p_t\left(n_t - \sum_i h_t^{i*}(\tilde{s}_t)\right)$. This applies to healthy single parent or single grandparent households with low earnings capacity $w_t^i < p_t$. From Lemma 1(i), it is optimal for such members to engage in strictly positive household child care $h_t^{i*}(\tilde{s}_t) > 0$. As there is only one adult member, individual household child care is equal to total household child care, which is optimally implemented through the optimal free day care.

Step 5 The decentralized allocations are incentive compatible.

Steps 1 to 4 show that for any state claim, household choices coincide with the optimal allocations. It follows that the promise keeping (1), threat-keeping (2) and incentive compatibility (3) constraints of the government problem (4) hold. In other words, the household will choose to claim its true state. Thus, a scheme with subsidized day care, non-linear income taxation and asset-testing implements the constrained optimal allocations $(c_t^*(s_t), l_t^*(s_t) h_t^*(s_t))$, $\forall t, s_t$.

C. Government Problem with Hidden Household Child Care

I follow the first-order approach (Rogerson, 1985) and impose the private first order conditions of agents with respect to household child care as additional constraints. Recall that $c_t(s_t) = b_t(s_t) - p_t\left(n_t - \sum_i h_t^i(s_t)\right)$ and $\tilde{c}_t(s_t, \tilde{s}_t) = b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)$. In every period t, for agents who were in state $s_{t-1} \in S_{t-1}$ in the previous period, the government chooses allocations to minimize expected costs:

$$G_t\left(V_t(s_{t-1}), \mathbf{\tilde{V}_t(s_{t-1}, \tilde{s}_{t-1})}\right) = \underset{b_t, l_t, h_t, \tilde{h}_t, V_{t+1}, \tilde{V}_{t+1}}{Min}$$

$$\sum_{s_t \in S_t} \pi_t(s_t, s_{t-1}) \left[b_t(s_t) - \sum_i w_t^i l_t^i(s_t) + \beta G_{t+1}\left(V_{t+1}(s_t), \mathbf{\tilde{V}_{t+1}(s_t, \tilde{s}_t)}\right) \right],$$

subject to the promise keeping constraint

$$\sum_{s_t \in S_t} \pi_t(s_t, s_{t-1}) \left[u\left(b_t(s_t) - p_t\left(n_t - \sum_i h_t^i(s_t)\right)\right) - \sum_i v_i\left(h_t^i(s_t) + l_t^i(s_t)\right) + \beta V_{t+1}(s_t) \right] = V_t(s_{t-1}),$$

Threat keeping constraints $\forall \tilde{s}_{t-1} \in S_{t-1}$:

$$\sum_{\tilde{s}_t \in S_t} \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) \max_{s_t} \left[u\left(b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right) \right.$$

$$\left. - \sum_i v_i\left(\tilde{h}_t^i(s_t, \tilde{s}_t) + l_t^i(s_t)\right) + \beta \tilde{V}_{t+1}(s_t, \tilde{s}_t) \right] = \tilde{V}_t(s_{t-1}, \tilde{s}_{t-1}),$$

Incentive compatibility constraints $\forall s_t, \tilde{s}_t \in S_t$:

$$u\left(b_t(s_t) - p_t\left(n_t - \sum_i h_t^i(s_t)\right)\right) - \sum_i v_i\left(h_t^i(s_t) + l_t^i(s_t)\right) + \beta V_{t+1}(s_t)$$

$$\geq u\left(b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(\tilde{s}_t, s_t)\right)\right) - \sum_i v_i\left(\tilde{h}_t^i(\tilde{s}_t, s_t) + l_t^i(\tilde{s}_t)\right) + \beta \tilde{V}_{t+1}(\tilde{s}_t, s_t),$$

Child care constraints $\forall i, \forall s_t, \tilde{s}_t \in S_t$:

$$u'\left(b_t(s_t) - p_t\left(n_t - \sum_i h_t^i(s_t)\right)\right) p_t - v_i'\left(l_t^i(s_t) + h_t^i(s_t)\right) \leq 0.$$

$$u'\left(b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right) p_t - v_i'\left(l_t^i(s_t) + \tilde{h}_t^i(s_t, \tilde{s}_t)\right) \leq 0.$$

Let $\lambda_t(s_{t-1})$ denote the Lagrange multiplier associated with the promise keeping constraint for agents who truthfully declared to be in state s_{t-1} in the previous period, $\tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1})$ denote the Lagrange mutipliers associated with the threat keeping constraints for agents who declared to be in state s_{t-1} in the previous period when they were actually in state $\tilde{s}_{t-1} \in S_{t-1}$, $\eta_t(s_t, \tilde{s}_t)$ denote the Lagrange multipliers associated with the incentive compatibility constraints of agents who declare to be in state s_t when they are actually in state $\tilde{s}_t \in S_t$, $\phi_t^i(s_t)$ denote the Lagrange multipliers associated with the child care constraints of agents who are honestly in state s_t, and $\tilde{\phi}_t^i(s_t, \tilde{s}_t)$ denote the Lagrange multipliers associated with the child care constraints of agents who declare to be in state s_t when they are actually in state $\tilde{s}_t \in S_t$.

Let $I\{\tilde{s}_t > s_t\}$ is an indicator function taking a value of 1 if it is privately optimal for those in state $\tilde{s}_t > s_t$ to declare to be in state s_t, and a value of 0 otherwise. $I\{\tilde{s}_t = s_t\}$ is an indicator function taking a value of 1 if an agent who was previously untruthful happens to be in state s_t in the current period, and a value of 0 otherwise. By incentive compatibility, such agents will find it optimal to be truthful and declare state s_t. They therefore get household child care $\tilde{h}_t^i(s_t, \tilde{s}_t) = h_t^i(s_t)$ and continuation utility $\tilde{V}_{t+1}(s_t, s_t) = V_{t+1}(s_t)$.

Claim 3 The Lagrange multipliers associated with the child care constraints are zero. The first order condition with respect to $\tilde{h}_t^i(s_t, \tilde{s}_t)$ is given by:

$$\begin{aligned} &\left[\sum_{\tilde{s}_{t-1}>s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}) \sum_{\tilde{s}_t>s_t} \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) I\{\tilde{s}_t > s_t\} - \sum_{\tilde{s}_t>s_t} \eta_t(s_t, \tilde{s}_t)\right] \left[f.\tilde{o}.c.(s_t, \tilde{s}_t)\right] \\ &- \sum_{j\neq i} \sum_{\tilde{s}_t>s_t} \tilde{\phi}_t^j(s_t, \tilde{s}_t) u''\left(b_t(s_t) - p_t\left(n_t - \textstyle\sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right) p_t^2 - \sum_{\tilde{s}_t>s_t} \tilde{\phi}_t^i(s_t, \tilde{s}_t)\left[s.\tilde{o}.c.(s_t, \tilde{s}_t)\right] \quad \leq \quad 0, \end{aligned}$$

where $f.\tilde{o}.c.(s_t, \tilde{s}_t)$ and $s.\tilde{o}.c.(s_t, \tilde{s}_t)$ are respectively, the private first and second order conditions with respect to child care: $f.\tilde{o}.c.(s_t, \tilde{s}_t) = u'\left(b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right) p_t - v_i'\left(l_t^i(s_t) + \tilde{h}_t^i(s_t, \tilde{s}_t)\right)$ and $s.\tilde{o}.c.(s_t, \tilde{s}_t) = u''\left(b_t(s_t) - p_t\left(n_t - \sum_i \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right) p_t^2 - v_i''\left(l_t^i(s_t) + \tilde{h}_t^i(s_t, \tilde{s}_t)\right)$. From Khun-Tucker conditions, $\tilde{\phi}_t^i(s_t, \tilde{s}_t) \geq 0$. In particular, if $\tilde{h}_t^i(s_t, \tilde{s}_t) = 0$, then the child care constraint is non-binding and $\tilde{\phi}_t^i(s_t, \tilde{s}_t) = 0$. If $\tilde{h}_t^i(s_t, \tilde{s}_t) > 0$, then the child care constraint is binding and $\tilde{\phi}_t^i(s_t, \tilde{s}_t) \geq 0$. Private optimality then implies that $f.\tilde{o}.c.(s_t, \tilde{s}_t) = 0$ and $s.\tilde{o}.c.(s_t, \tilde{s}_t) < 0$. Since utility is concave, it must therefore be that $\tilde{\phi}_t^i(s_t, \tilde{s}_t) = 0\ \forall i, \forall s_t, \tilde{s}_t \in S_t$. A similar line of proof shows that $\phi_t^i(s_t) = 0\ \forall i, \forall s_t \in S_t$.

The government's first order conditions are then given by:

$$\begin{aligned} b_t(s_t) \quad &: \quad \pi_t(s_t, s_{t-1}) - \left[\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) + \sum_{\tilde{s}_t>s_t} \tilde{\zeta}_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\right] u'(c_t(s_t)) = 0, \\ l_t^i(s_t) \quad &: \quad \pi_t(s_t, s_{t-1}) w_t^i - \zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right) \\ &\qquad + \sum_{\tilde{s}_t>s_t} \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right) v_i'\left(\tilde{h}_t^i(s_t, \tilde{s}_t) + l_t^i(s_t)\right) \leq 0, \\ h_t^i(s_t) \quad &: \quad \zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(c_t(s_t)) p_t - v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right)\right] \leq 0, \\ \tilde{h}_t^i(s_t, \tilde{s}_t) \quad &: \quad \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(\tilde{c}_t(s_t, \tilde{s}_t)) p_t - v_i'\left(\tilde{h}_t^i(s_t, \tilde{s}_t) + l_t^i(s_t)\right)\right] \leq 0, \\ V_{t+1}(s_t) \quad &: \quad -\pi_t(s_t, s_{t-1}) G'_{V_{t+1}(s_t)}\left(V_{t+1}(s_t), \mathbf{\tilde{V}_{t+1}}(\mathbf{s_t}, \mathbf{\tilde{s}_t})\right) + \zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) = 0, \\ \tilde{V}_{t+1}(s_t, \tilde{s}_t) \quad &: \quad -\pi_t(s_t, s_{t-1}) G'_{\tilde{V}_{t+1}(s_t, \tilde{s}_t)}\left(V_{t+1}(s_t), \mathbf{\tilde{V}_{t+1}}(\mathbf{s_t}, \mathbf{\tilde{s}_t})\right) + \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right) = 0, \end{aligned}$$

where

$$\begin{aligned} \zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right) &= \lambda_t(s_{t-1}) \pi_t(s_t, s_{t-1}) + \sum_{\tilde{s}_{t-1}>s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}) \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) I\{\tilde{s}_t = s_t\} + \sum_{\tilde{s}_t<s_t} \eta_t(\tilde{s}_t, s_t), \\ \tilde{\zeta}_t\left(\pi_t, \tilde{\lambda}_t, \eta_t\right) &= \sum_{\tilde{s}_{t-1}>s_{t-1}} \tilde{\lambda}_t(s_{t-1}, \tilde{s}_{t-1}) \pi_t(\tilde{s}_t, \tilde{s}_{t-1}) I\{\tilde{s}_t > s_t\} - \eta_t(s_t, \tilde{s}_t). \end{aligned}$$

Claim 4 The qualitative features of Lemma 1 hold. (i) Using the first order conditions with respect to $b_t(s_t)$ and $l_t^i(s_t)$, and summing across the first order conditions with respect to $h_t^i(s_t)$ and $\tilde{h}_t^i(s_t, \tilde{s}_t)$ $\forall \tilde{s}_t > s_t$, we have respectively:

$$\begin{aligned} &\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(c_t(s_t))w_t - v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right)\right] \\ &+ \sum_{\tilde{s}_t > s_t} \tilde{\zeta}_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(\tilde{c}_t(s_t, \tilde{s}_t))w_t - v_i'\left(l_t^i(s_t) + \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right] \leq 0, \end{aligned} \tag{A3}$$

$$\begin{aligned} &\zeta_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(c_t(s_t))p_t - v_i'\left(h_t^i(s_t) + l_t^i(s_t)\right)\right] \\ &+ \sum_{\tilde{s}_t > s_t} \tilde{\zeta}_t\left(\pi_t, \lambda_t, \tilde{\lambda}_t, \eta_t\right)\left[u'(\tilde{c}_t(s_t, \tilde{s}_t))p_t - v_i'\left(l_t^i(s_t) + \tilde{h}_t^i(s_t, \tilde{s}_t)\right)\right] \leq 0. \end{aligned} \tag{A4}$$

When $w_t^i \geq p_t$ and (A3) is satisfied with equality, (A4) will be satisfied with strict inequality. It must therefore be that $l_t^i(s_t) > 0$ and $h_t^i(s_t) = 0$ for healthy members with $w_t^i \geq p_t$. By the same line of thought, $h_t^i(s_t) > 0$ and $l_t^i(s_t) = 0$ for healthy members with $w_t^i < p_t$, as long as all child care needs have not yet been met. Similar lines of proof as in Appendix A. may then be used to show that (ii) and (iii) also hold.

D. Multi-Member Households with Low Earnings Capacity

Consider an illustration with $T = 1$ and $I = 2$. Suppose that a household consisting of two healthy members (state s) claims that member $i = 1$ is disabled. Suppose that $w^2 < p$. I now show that the exacerbation of incentives to mimic the disabled still exists despite the household benefiting from free day care of $f^*(s) = p\left(n - h^{2*}(s)\right)$. This is because the falsely disabled member $i = 1$ has incentives to engage in household child care activities, which would enable the healthy member $i = 2$ to spend less effort on household child care. Let $\varepsilon \geq 0$ be the increase (decrease) in household child care of member $i = 1$ ($i = 2$). The mimicker household solves:

$$\underset{\varepsilon}{Max}\, u(c(\tilde{s}, s)) - v_1(\varepsilon) - v_2\left(l^{2*}(\tilde{s}) + h^{2*}(\tilde{s}) - \varepsilon\right),$$

subject to the budget constraint: $\tilde{c}(\tilde{s}, s) = w^2 l^{2*}(\tilde{s}) - T(\tilde{s})$, where income tax $T(\tilde{s}) = w^2 l^{2*}(\tilde{s}) - c^*(\tilde{s})$. Thus, $\tilde{c}(\tilde{s}, s) = c^*(\tilde{s})$. The private first order condition of the household is given by:

$$-v_1'(\varepsilon) + v_2'\left(l^{2*}(\tilde{s}) + h^{2*}(\tilde{s}) - \varepsilon\right) \leq 0,$$

with strict inequality when $\varepsilon = 0$. But then, since $v_1'(0) = 0$ and $l^{2*}(\tilde{s}) + h^{2*}(\tilde{s}) > 0$ from Lemma 1(i), we cannot have strict inequality. It must therefore be that $\varepsilon > 0$.[20] It follows that the utility that the mimicker household gets in deviation is higher than the constrained optimal one, which still exacerbates the incentive constraint.

[20] Note that the result would hold even if $v_1'(0) > 0$, provided that the effort cost functions are convex enough. In other words, as long as the incremental effort cost of household member $i = 1$ is lower than that of household member $i = 2$, it would be privately optimal to have $\varepsilon > 0$.

Table A1: Hourly Wages

Household	Adults	Education	t 0	1	2	3	4
Parent	Single mother	High School	12.04	12.86	13.85	14.11	14.65
		College	15.80	18.13	20.59	22.14	23.74
	Single father	High School	15.47	16.37	18.30	19.03	19.39
		College	20.13	23.28	25.49	27.48	28.78
	Married mother	High School	13.04	14.00	14.95	15.16	15.23
		College	19.94	23.71	25.42	25.01	24.71
	Married father	High School	16.78	18.78	20.30	21.15	21.76
		College	23.67	28.81	32.38	33.94	34.54
Grandparent	Single grandmother	High School	14.84	14.47	14.03	-	-
		College	24.26	24.22	23.33	-	-
	Married grandmother	High School	15.89	16.22	15.62	-	-
		College	25.22	25.10	24.30	-	-
	Married grandfather	High School	22.39	21.87	21.16	-	-
		College	34.40	33.24	32.23	-	-
Intergenerational	Single grandmother	High School	15.22	15.04	13.77	-	-
		College	24.19	22.95	22.91	-	-
	Single father	High School	13.47	14.78	16.03	18.04	20.99
		College	17.85	19.78	24.49	26.71	32.26
	Single grandfather	High School	21.73	21.05	20.94	-	-
		College	33.77	33.11	33.07	-	-
	Single mother	High School	11.99	12.54	13.98	14.93	15.61
		College	16.85	18.72	23.55	23.99	25.06

Note: Hourly wage in 2010 dollars computed by dividing gross earnings by hours of work from CPS data. I take the mean across adults for each household structure and education level. Grandparents are retired in periods 3 and 4.

Table A2: 2010 US Tax and Benefit System

Federal Income Tax Rates [a]

Tax rate	Taxable income		
	Single	Head	Married
10%	Less than $8,375	Less than $11,950	Less than $16,750
15%	$8,375 - $34,000	$11,950 - $45,550	$16,750 - $68,000
25%	$34,000 - $82,400	$45,550 - $117,650	$68,000 - $137,300
28%	$82,400 - $171,850	$117,650 - $190,550	$137,300 - $209,250
33%	$171,850 - $373,650	$190,550 - $373,650	$209,250 - $373,650
35%	$373,650 and above	$373,650 and above	$373,650 and above

EITC [b]

# Children below 18	All Filing Status			Single and Head		Married	
	Phase-in rate	Maximum Credit	Phase-out rate	Phase-out Income	Income Limit	Phase-out Income	Income Limit
0	7.65%	$457	7.65%	$7,480	$13,460	$12,480	$18,470
1	34%	$3,050	15.98%	$16,450	$35,535	$21,460	$40,545
2	40%	$5,036	21.06%	$16,450	$40,363	$21,460	$45,373
3 or more	45%	$5,666	21.06%	$16,450	$43,352	$21,460	$48,362

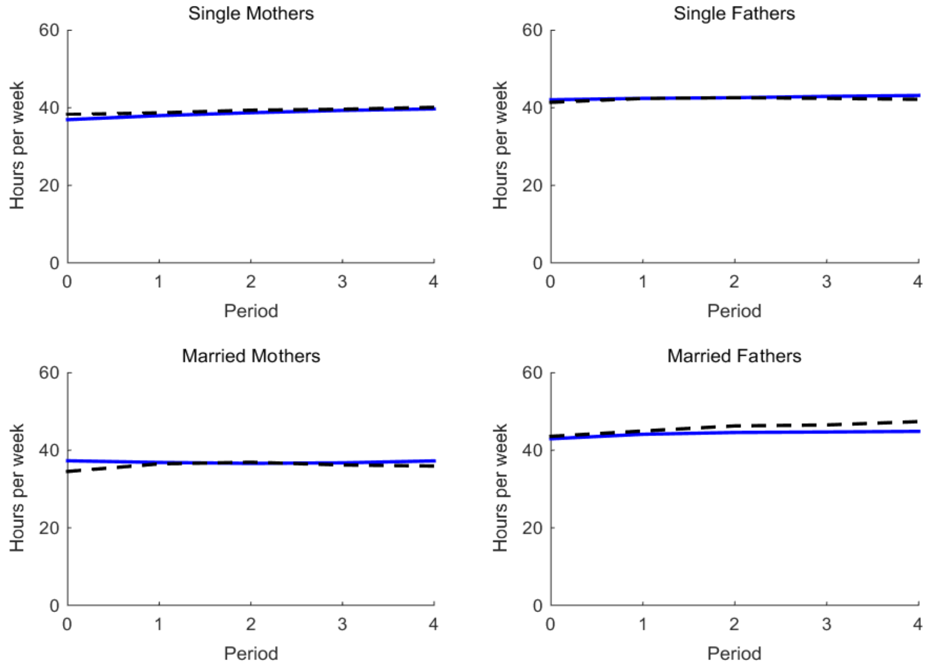
Poverty Thresholds [c]

Size of family unit	No. of Children below 18		
	1	2	3
Two people	$15,030		
Three people	$17,552	$17,568	
Four people	$22,859	$22,113	$22,190
Five people	$27,518	$26,675	$26,023

Sources: a. http://www.moneychimp.com. b. Historical Earned Income Tax Credit Parameters, Tax Policy Center. Phase-out income for married filing jointly status computed by author based on phase-out rate and income limit. c. U.S. Census Bureau.

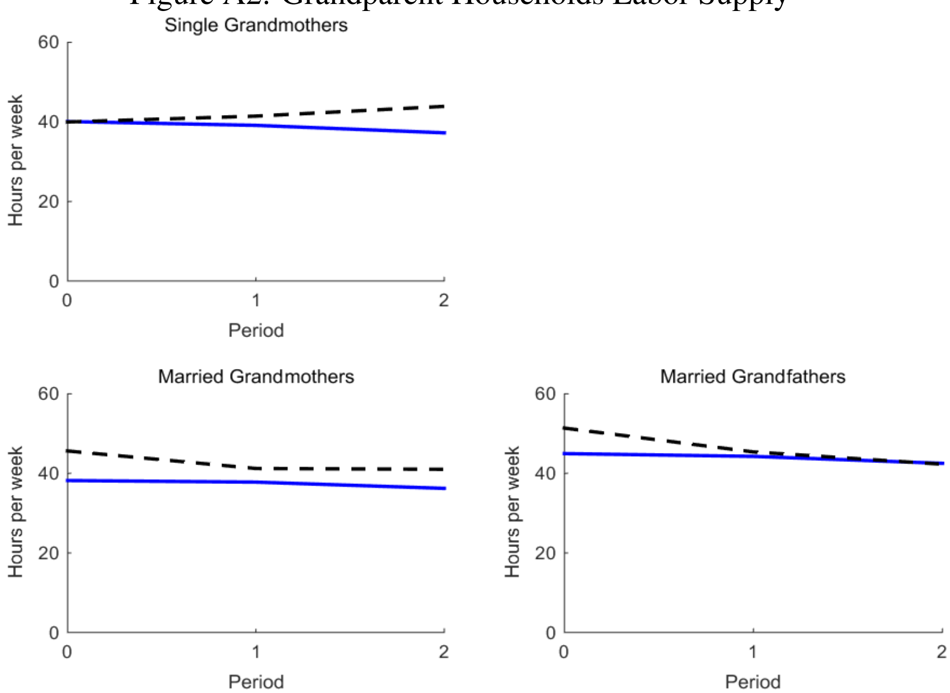
Filing status Federal income tax brackets depend on a tax payers filing status. I assume that households with a single parent or grandparent file taxes under the single status when there are no children present in the household and file under the head of household status when there are children below 18 present. Married households, on the other hand, file jointly for taxes irrespective of presence of children. For intergenerational households with children aged below 18, I assume that the grandparent files as head of household while the parent files under the single status. If the grandparent is disabled or retired, then the parent files as the head of household. To qualify as head of household, one must be unmarried, provide for more than half of housing expenses, and have a qualifying dependent who may be a descendant aged below 18 or a disabled relative of any age (Inland Revenue Service).

Figure A1: Parent Households Labor Supply



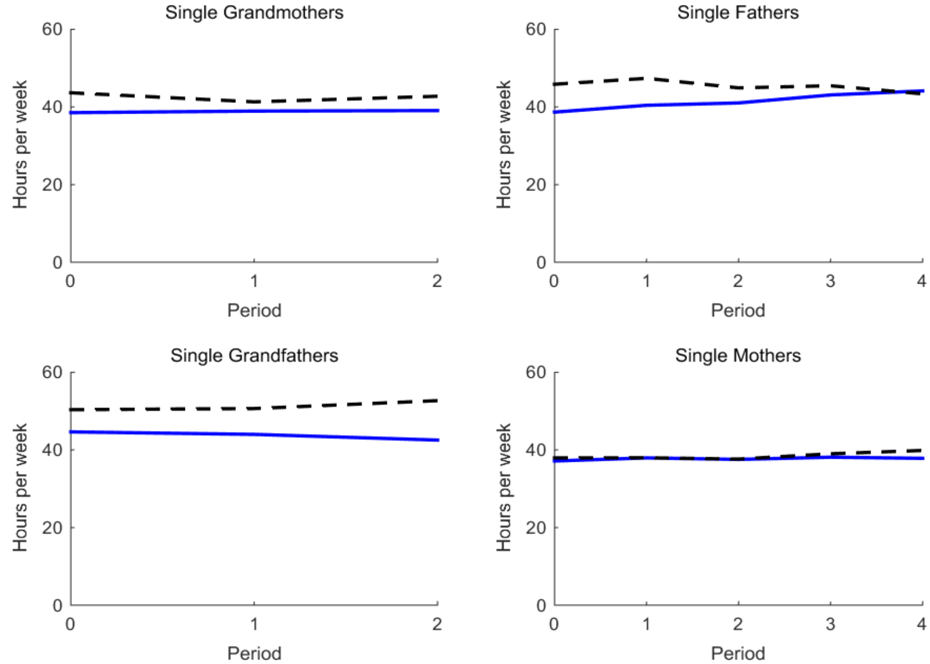
Note: Solid lines represent average hours from CPS and dashed lines represent average hours from model.

Figure A2: Grandparent Households Labor Supply



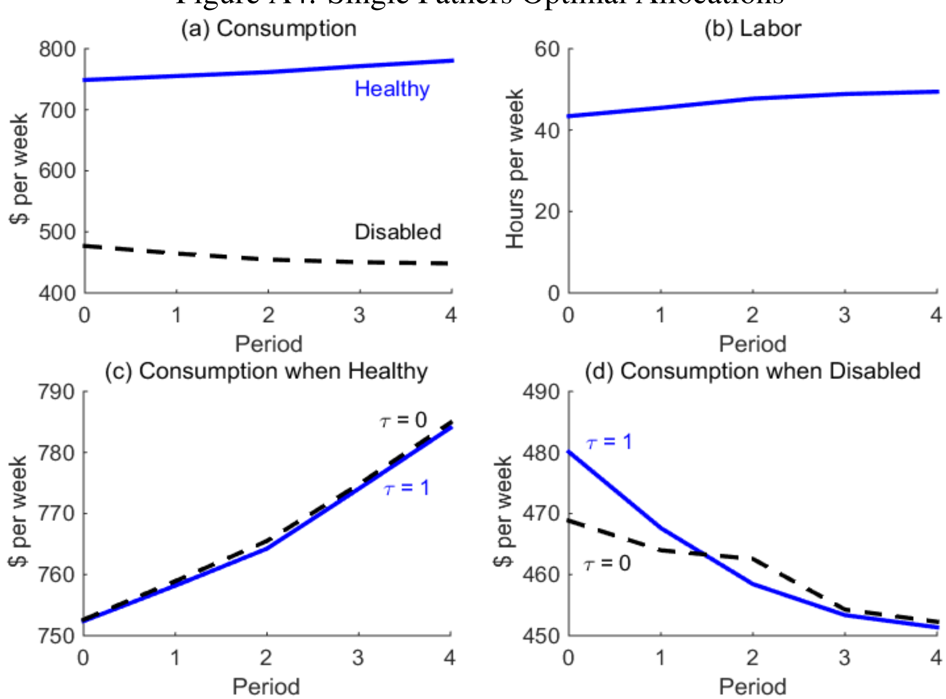
Note: Solid lines represent average hours from CPS and dashed lines represent average hours from model.

Figure A3: Intergenerational Households Labor Supply



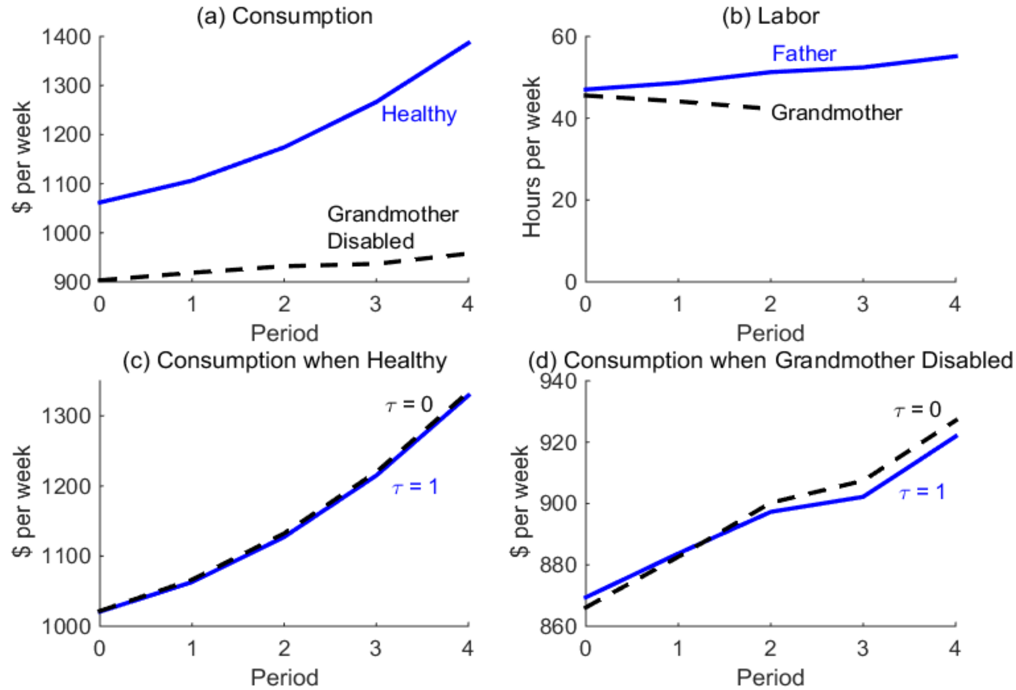
Note: Solid lines represent average hours from CPS and dashed lines represent average hours from model.

Figure A4: Single Fathers Optimal Allocations



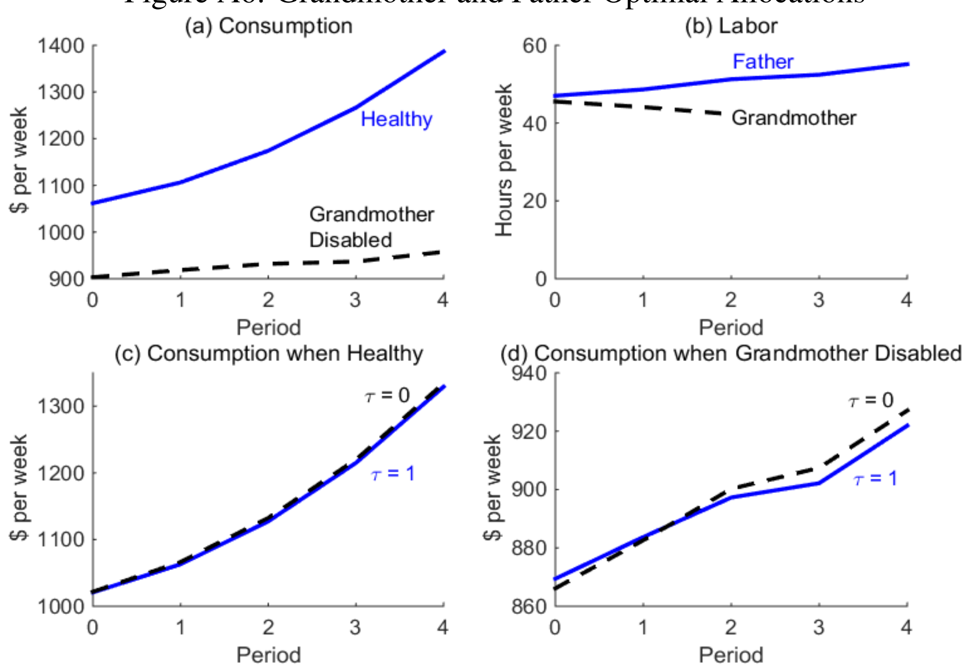
Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.

Figure A5: Married Grandparents Optimal Allocations



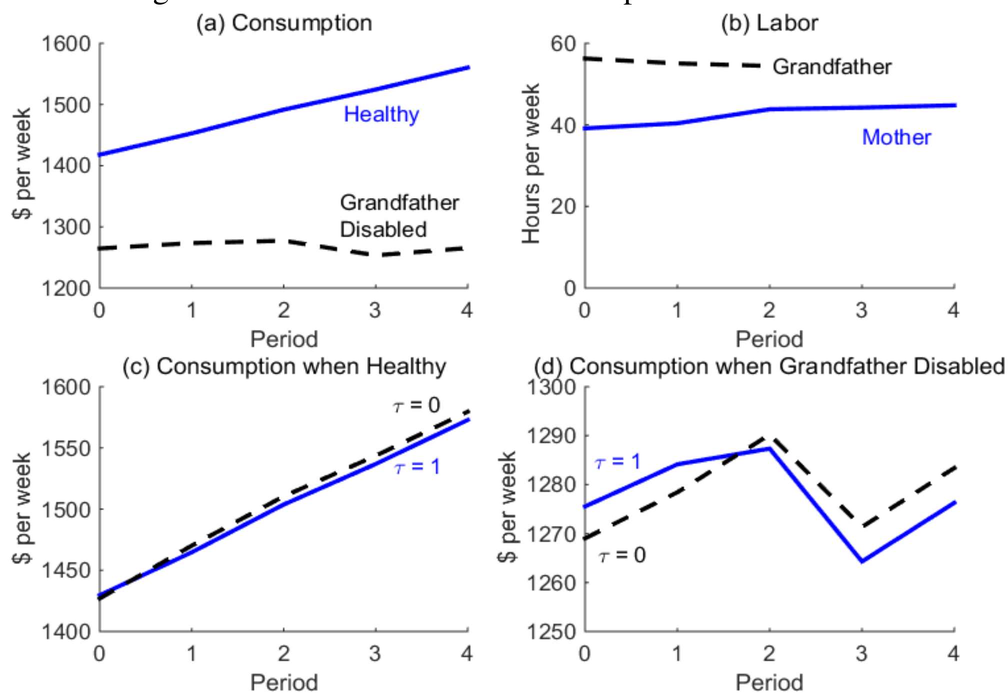
Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.

Figure A6: Grandmother and Father Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.

Figure A7: Grandfather and Mother Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with $(\tau = 1)$ and without $(\tau = 0)$ formal day care subsidies, averaged among households with children.