Dynamic Platform Pricing on Innovative Products

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December 28, 2013

Abstract

Many two-sided platforms offer innovative hardware products that improve in quality and enter the market sequentially. We analyze the impact of the decrease in the production cost on a monopoly platform owner’s dynamic two-sided pricing problem, in which buyers are strategic and exert a cross-side network effect to the seller side. Our findings show that a greater decrease in cost raises the optimal price of the low-quality product and allocates more buyer-side demand to the future market. Furthermore, such decrease in cost may also lead to a higher optimal price for the future higher-quality product, given a sufficiently significant quality improvement. Thus, a greater decrease in cost can enable the platform to position its product line to the high-end market. Compared to the base case in which the seller side is absent, we find that, in the two-sided model, the network effect intensifies the impact of cost decrease on the price of the low-quality product and makes the forward intertemporal demand shift on the buyer-side more pronounced. Moreover, the network effect propagates intertemporally, which may reverse the impact of cost decrease on the optimal price of the high-quality product compared to that in the base case, when the quality improvement is less significant. Our work underscores the importance of network effects in a dynamic platform pricing problem and highlights buyers’ strategic behavior, production cost, and quality improvements in two-sided pricing.

Keywords: Dynamic pricing, two-sided platforms, sequential innovation, strategic consumers
1 Introduction

Many platforms not only connect multiple groups of users, they also directly offer the platform hardware devices. Some platform owners have historically manufactured their devices in house. A well-known example is Apple with an array of electronic products, such as iPads, iPhones, and Macbooks, which serve consumers on one side and bring abroad application developers on the other side. Video game consoles have the similar characteristics, with Sony PlayStation, Microsoft XBox, and Nintendo Wii all being platform-owned hardware devices. Although for some platforms—such as Windows—the hardware market has been dominated by third-party PC manufacturers, it has become increasingly evident that platform owners are also tapping into the hardware market. In the tablet market, Windows Surface RT is among the latest Microsoft-made tablet running Windows applications. Google has introduced its own laptop Chromebook that allows consumers to use Chrome OS applications (Burns (2012)).

The two-sided business model with an additional hardware product offering is important in several respects: First, studies on information products, including those related to two-sided platforms, often consider marginal cost to be zero, which is not applicable to hardware production; Second, the platform hardware is targeted primarily toward the consumer side, which creates complex asymmetry among different sides of users; Third, whereas service provision is often subscription-based, the durability of hardware products emphasizes intertemporal considerations in the pricing model. This study addresses these points in the two-sided platform context.

Similar to other types of IT products, the platform devices advance through waves of technological innovation. These quality-improved products are often released to the market sequentially. The iPad, for example, has had a number of upgrades in terms of display, CPU, wireless capability, and other specifications; also, Windows Surface Pro has been widely anticipated since the release of the Surface RT. Such patterns in introducing platform-based products have a number of implications. Inevitably, the innovation trend sparks wide interest and online discussions regarding upcoming products. Consumers’ purchasing decisions are rarely made only based on the existing products; more often, purchasing decisions are forward-looking with anticipations for the pricing and quality of future products. Thus, consumers are strategic. In addition, sequential introductions of innovative products often involve dynamic pricing considerations by the platform owner. While
managing different versions of products, the platform owner tends to adopt pricing strategies that are based on the evolving state of the market by setting price at the launch of new products, rather than committing to a long-term pricing plan.

The platform market is driven by a multitude of forces, including the network effects across two sides of the platform, consumers’ strategic behaviors, and the dynamic nature of sequential innovation; these forces interact with one another in shaping the platform owner’s prices and profits. The cross-side network effect is a defining characteristic of two-sided platforms, where more users on one side help to attract users on the other side. In the example of iPad, a wider iPad adoption means more downloads and revenues for app developers in the App Store; and a larger network of app developers adds value to iPad users as well. These network effects lead to tradeoffs for the platform owner, who uses pricing as an instrument to balance such tradeoffs and optimize revenues from both sides. In particular, by cutting the price charged to one side, the platform might suffer some revenue loss on this side but gain a larger user base, which in turn increases the demand on the other side. In the context of dynamic pricing, such tradeoffs also have additional intertemporal significance. With forward-looking consumers, the platform may need to manage the two-sided tradeoffs differently than if consumers were myopic. It is important to evaluate whether to leverage consumers’ valuation for future consumption and the cross-side network effect to yield higher future revenues at some short-term loss. Furthermore, as the platform owner undertakes dynamic pricing strategies that take into account past consumptions, characteristics of the remaining market, and changes in production costs, the differences in the strengths of the network effects and product quality over time may be critical for setting optimal prices.

Platform owner’s pricing strategies are inevitably linked to changes in production costs, which often have a decreasing trend given rapid technological advancements. Firms can effectively cut costs for future production in many ways. Facing uncertain future costs, in many industries, firms engage in upfront negotiations with their suppliers to secure a lower procurement cost for the future. The automobile manufacturers follow different contracts to negotiate prices with suppliers of auto parts. In the IT industry, there is a wide speculation on ways Apple contracts with its factories on the cost of later generations of iPhones and iPads (Opam (2011)). Besides through contractual agreements, the decrease in production cost can also be achieve by learning-by-doing by the manufacturer and trust establishment in supply chain relationships. Anticipating a lower
future production cost offers opportunities for the platform owner to adjust prices dynamically for strategic allocations of intertemporal demand.

The main objective of this paper is to examine how decreasing production cost of platform-owned hardware impacts the platform owner’s dynamic two-sided pricing decisions. Our research highlights sequential introduction of quality-improving hardware products that the platform offers to the buyer side. As we address the platform owner’s pricing strategy that segments the hardware market intertemporally, we also consider the seller-side fee that the platform owner sets. Through the one-sided base case, we illustrate the difference in a platform owner’s strategy compared to that of a traditional hardware producer (without the seller-side market) driven by cross-side network effects between the two sides.

Our work differentiates with existing studies on two-sided platforms by considering innovative hardware products. Research work on two-sided pricing mechanisms and platform strategies has been tremendously fruitful and continues to expand rapidly. However, to the best of our knowledge, most insights thus far do not account for the platform’s role in introducing hardware devices, which is an essential element in many platform owners’ strategies as their business models evolve. Our study contributes to the literature by exploring this facet of platforms’ decision problem. We discuss the related works and contrast them with our study in Sections 2 and 7.

Our model captures sequential introduction of quality-improved products and dynamic pricing decisions of a monopoly platform. We derive the optimal price of the low-quality product on the buyer side and the fee on the seller side in the present and those of the high-quality product in the future. Furthermore, we analyze the impact of the decrease in cost (from the present to the future) on the optimal prices. To sharpen the intuitions about quality improvement, a baseline case without the seller side is illustrated. And then, we characterize the optimal results in the two-sided case involving both dynamic effects of sequential introduction and the cross-side network effect from the buyer side to the seller side. The comparison with the base case highlights the role of network effects in the platform’s strategies. We also numerically analyze the two-sided case with cross-side network effects between the two sides in both directions. The numerical results are consistent with those from the analytical model.

Absent the seller side, the base case focuses on the interactions between the platform and buyers. It shows that a greater decrease in cost leads to a higher optimal price for the low-quality
product which is introduced first. As decreasing cost makes the future market more profitable, a higher price for the low-quality product encourages some buyers to delay purchase to the future market and extracts more surplus from those who purchase early nevertheless. More interestingly, a greater decrease in cost can raise the optimal price for the high-quality product introduced later, when the quality improvement exceeds a certain threshold. The forward intertemporal demand shift populates the future market with more high-valuation buyers who can yield a sufficiently high margin to compensate for reduced demand due to the higher price. As a result, the product line is pushed toward the high-end market. However, if the quality improvement is below the threshold, the optimal price for the high-quality product will be lower given a greater decrease in cost.

The intuition established in the base case builds the foundation for understanding the two-sided problem, in which the platform sets prices on both buyer- and seller-side and needs to account for the cross-side network effect. Compared to the base case, a greater decrease in cost raises the buyer-side price of the low-quality product to a greater extent, which in turn causes a more pronounced forward shift of buyer-side demand. The reason lies in the network effect – buyers not only generate sales, they also attract sellers; thus, buyers become more valuable to the platform. By having more buyers in the future market, the platform also derives additional seller-side revenues. In this sense, a greater decrease in cost further enhances profitability in the future market because the platform can benefit from profits on both sides of the market. The cost decrease then generates a more pronounced impact on the buyer-side price, compared to the base case, for both the low-quality and high-quality products. Moreover, the network effect reduces sensitivity of the platform’s pricing strategy to quality improvement in the future market. Even given a minor quality improvement, the decreasing cost for the future market may still induce a higher optimal buyer-side price for the high-quality product, whereas this optimal price is reduced in the base case.

We further extend the two-sided model by adding the network effect from the seller side to the buyer side, so that buyers value both product quality and seller-side network size. Unfortunately, bi-directional network effects compromises analytical tractability of the model. Thus, we study it numerically; the findings are consistent with the analytical results. In fact, the new network effect does not alter the platform’s intertemporal incentives because it amplifies the increases in demands on both sides in the future market. As a result, at a greater cost decrease, the platform is still able to set a higher price for the low-quality product to allocate more buyers to the future market, and
such effects are greater compared to those in the base case.

The remaining of the paper is organized as follows. We discuss the related literature in Section 2. Section 3 introduces the model. In Section 4, we focus on the base case without considering the seller side problem to illustrate the fundamental intertemporal forces exerted by product quality and costs. In Section 5, we solve the full model and contrast the results with those in the base case. To account for bi-directional network effects, we conduct numerical studies in Section 6 and show that the insights obtained in Section 5 persist. Section 7 concludes the paper.

## 2 Related Literature

Our work is closely related to three streams of literature: two-sided platforms, sequential innovation, and strategic customers. To the best of our knowledge, our paper is among the first to consider the dynamic pricing problem of a two-sided platform that sequentially introduces innovative hardware devices in the presence of strategic consumers. Moreover, we connect thoughts from these bodies of literature to gain further understanding on problems across these domains.

The literature on two-sided platforms explores the platform’s pricing problem taking into consideration network effects, user multi-homing, platform governance, and innovation. The earlier works include Rochet and Tirole (2002), Rochet and Tirole (2003), Caillaud and Jullien (2003), Parker and Alstyne (2005), and Armstrong (2006). Continuing from this line of literature, more recent studies examine the innovation issues on platforms. Lin et al. (2011) study the innovation race among sellers of a two-sided market. By analyzing innovation incentives and price competition among sellers, they find the platform’s optimal two-sided pricing strategy. They show that the seller-side fee may have a positive impact on sellers’ innovation incentives, while the buyer-side fee slows down the innovation race. Boudreau (2012) conducts an empirical study on the effect of the number of applications on software variety. He finds that an increase in the number of application producers leads to an overall reduction in innovation incentives, which creates a tension with the positive network effects assumed by many studies of two-sided markets. Hagiu (2009) accounts for the effect of consumers’ preference for variety. He examines the effect of such variety on the platform’s pricing strategies and discusses how the seller-side pricing structure influences sellers’ innovation incentives. These studies focus on innovation that drives the products offered by sell-
ers to buyers, whereas we devote our attention to the the platform’s strategies in managing the innovative hardware market through two-sided pricing.

Although studies on two-sided pricing models have been commonly based on static settings to derive crisp insights and maintain analytical tractability, recently a growing body of research work has begun to explore dynamic strategies in the platform context. Hagiu (2006) investigates price commitment by a platform, where one side of the platform arrives before the other side. He finds that the platform can attract the early-arrival side without committing to a low price for the late-arrival side. Also allowing the consumer side to arrive first, Bhargava et al. (2013) examine the platform’s product line expansion strategy with uncertainty on developer-side participation. They find the dependencies of the expansion strategy on the fixed cost for expansion and uncertainty on developer participation. Lin et al. (2011) study sellers’ dynamic innovation race to create products for the platform market and find implications on the platform’s pricing decisions. Rather than focusing on sellers’ dynamics, our work considers the platform’s sequential decisions. Zhu and Iansiti (2012) consider forward-looking consumers and focus on a platform’s entry problem in competition with an incumbent, with constant quality. Through both analytical modeling and empirical validation, they find that, when both the network effect and consumers’ valuation for future applications are sufficiently low, a platform entrant may capture its market with quality advantage. Whereas Zhu and Iansiti (2012) do not model the platform owner’s prices, Dou et al. (2012) analytically study a platform’s pricing decision on the buyer side. By comparing strategic buyers with myopic buyers, Dou et al. (2012) identify that the two types of buyers exhibit different behaviors only when the platform owner operates a license model or a limited-time freemium model with a positive switching cost.

To contrast with the existing studies on the platform owner’s dynamic strategies, we analyze the platform’s pricing strategies on both the buyer and seller sides. Our findings echo those in the related papers by also illustrating the importance of product quality as well as the platform owner’s and buyers’ strategic considerations. Furthermore, we emphasize other factors such as cross-side network effects within each period and their intertemporal impacts. Unique to our model, the platform owner offers multiple products introduced over a period of time, whereas the current literature typically accounts for a single product/service offered by the platform. Other distinctive features in our paper include endogenous two-sided pricing decisions, quality improvements, and
the decreasing production cost.

Network effects and other key elements of two-sided platforms have been considered in a variety of contexts. Gilbert and Jonnalagedda (2011) anchor on the concept of “contingent product,” which is the product that is required to consume a durable good (e.g., ink is the contingent product of printer). They evaluate the lock-in strategy with consideration for strategic consumers and find that the firm’s ability to commit to shutting down the production of the durable good plays an important role. Our model aims to address a two-sided problem and accounts for bi-directional network effects between the two sides. Bhargava and Choudhary (2004) study versioning strategies of a platform (“infomediary” in their paper) that provides matching services for the two sides with the option of value-added services. They find that it is optimal for the platform to offer two versions of matching services, those with and without the value-added services, and the versioning incentives are stronger compared to a traditional seller as a result of network effects. Cheng et al. (2011) evaluate net neutrality policies by studying the the broadband service provider as a platform, which charges a fee to consumers and possibly also a price to the content provider side. By modeling two-sided pricing, they find that abolishing net neutrality benefits the broadband service provider while taxing the content providers; the change to consumer surplus further depends on relative capabilities of the content providers in generating revenues. Guo et al. (2013) further examine the net neutrality problem by considering the broadband service provider’s options to also discriminate the consumer side. Their findings emphasize the importance of the platform making strategic decisions on both the content provider side and the consumer side simultaneously. Hao et al. (2013) focus on mobile advertising platforms and examine different strategies of the platform owner in pricing ads and those of application developers in publishing ads. Chou et al. (2012) incorporate a new element of supply chain operational costs into a two-sided pricing problem. Whereas the conventional theory on platform subsidies may hold, in some cases the platform extracts surplus on both sides to offset the supply chain costs.

Our paper is also related to the literature on sequential innovation. Our model builds on those in Dhebar (1994) and Kornish (2001), which examine the problem of a durable-goods monopolist selling low-quality and high-quality products in the first and second period, respectively. They examine whether there exists an equilibrium pricing strategy when the pace of quality improvement varies. Dhebar (1994) concludes that rapid quality improvement is not desirable even with the
option of upgrading the low-quality products, whereas Kornish (2001) shows that any large quality improvement could be optimal under different parameter settings without offering the special upgrading pricing in the second period. Our work considers a platform producing quality-improving hardware devices, for which upgrades are not commonly feasible; thus, we focus on other issues such as two-sided pricing. Bhattacharya et al. (2003) investigate how to optimally introduce high technology products with an option of holding the low quality products until the high quality products launch. They show that introducing low quality products before high quality products may be still preferred. For topics on sequential innovation, Ramachandran and Krishnan (2008) provide a detailed review.

A key component in most dynamic pricing models is strategic consumer behavior, that is, consumers are forward-looking and may delay their purchases to maximize their utilities over time. Researchers are often interested in how a monopolist optimally prices a single product over time. Stokey (1979) and Bulow (1982) show that a monopolist is forced to price at the marginal cost; Besanko and Winston (1990) prove that the optimal price decreases over time due to consumers’ strategic behavior. Levin et al. (2010) analytically illustrate that, for a monopolist offering a perishable product, accounting for consumers’ strategic behaviors is critical for obtaining maximum revenues. By taking into account capacity constraints/inventory, the literature in operations management shows that markdown or markup could be optimal (see Su (2007) and Aviv and Pazgal (2008)). Liu and Zhang (2012) extend the model of Besanko and Winston (1990) to a duopoly market with vertically differentiated products and obtain a similar optimal pricing strategy. Our work emphasizes the role of production cost in the firm’s and consumers’ decisions. We show that, as future production cost decreases, consumers’ strategic behaviors make possible for the firm to raise the price of the low-quality product in the first period and to raise or lower the price of the high-quality product in the second period, depending on the extent of quality improvement.

3 The Model

Consider a monopoly two-sided platform owner that facilitates transactions between two groups of users through an exclusive hardware product, which is offered solely by the platform owner to its buyer side. This product improves in quality sequentially: A low-quality version is available in the
first period, followed by a high-quality in the second period. Suppose these products are durable goods, so that buyers purchase either quality of product but not both. Furthermore, unlike in the case of software products, buyers cannot simply update the low-quality hardware to obtain the high-quality version.

Let \( q_i \) and \( p_i \) denote the quality and the selling price of product \( i \) respectively, where \( i = L, H \). \( q_i \) is exogenously given. Following the common assumption (Netessine and Taylor (2007)), let the production cost of hardware be a convex function in quality with \( 0 < \beta_1 < 1 \) denoting the costliness of quality in period 1. Suppose technology becomes less costly over time, the unit costs of the low- and high-quality products are \( \beta_1 q^2_L \) and \( (\beta_1 - \delta)q^2_H \) respectively, where \( \delta \) is the reduction in cost from period 1 to period 2 and \( \beta_1 > \delta > 0 \).

In addition to setting the buyer-side hardware price, the platform also charges a fixed fee on the seller side in each period, denoted by \( s_t \), where \( t = 1, 2 \). Sellers’ participation on the platform depends on the buyer-side network size and the seller-side fee, following a linear demand function (Banker et al. (1998)). Let the seller-side demand decrease in \( s_t \) at the rate of \( k \), as a result of the quantity effect. Thus, for a buyer-side network of size \( \Theta \), the seller-side demand function is \( v_t \Theta - ks_t \), where \( v_t \) is the marginal network effect exerted by the buyer-side in period \( t \).

Consider a continuum of buyers of density \( \mu \). Buyers are heterogeneous in their valuation (or willingness-to-pay) for quality. A buyer with valuation \( \theta \) receives utility \( \theta q_i - p_i \) from product \( i \) for \( i = L, H \). Without loss of generality, let a buyer’s utility for not joining the platform (by not purchasing the hardware product) be zero. A buyer purchases either the low- or high-quality product (in period 1 or 2, respectively) that gives him a higher utility, provided that the utility is positive. Furthermore, the firm cannot identify the \( \theta \) value for each buyer, but it observes that \( \theta \) is uniformly distributed over \([0, 1]\). We assume all agents have rational expectations about the future as they maximize payoffs.

In deriving analytical results, we focus on the case where the seller-side demand is increasing in the buyer-side network size, and the buyer-side demand is determined by their preferences for quality. This simplification allows the model to remain tractable as we examine the platform’s dynamic, two-sided pricing strategy. It is applicable to platforms that have established a critical mass of sellers, in which case buyers’ purchasing decisions are primarily based on the characteristics of the hardware product. To also incorporate the network effect from the seller side to the buyer
side, we perform numerical analysis with bi-directional network effects in Section 6 and show that the findings are qualitatively consistent with the analytical results in this section.

We follow the assumption adopted in the literature that innovation is not “too rapid,” such that quality only improves in absolute terms and not in present-value terms (Liu and Zhang (2012). Mathematically, this implies that $q_L > \alpha q_H$, where $\alpha$ the common per-period discount factor.) Violation of this condition rules out the subgame-perfect equilibrium for sequential product introduction (see Dhebar (1994)), implying that the optimal pricing strategy may lead to consumer regret. As a result, buyers with higher valuations are early adopters who purchase the low-quality product in period 1, whereas those with lower valuations may purchase the high-quality product in period 2. Let $\theta_L$ denote the valuation of the customer who is indifferent between buying the low-quality product in period 1 and the high-quality product in period 2, and $\theta_H$ denotes the valuation of the customer who is indifferent between buying the high-quality product and nothing.

The timeline of the events is the following: In period 1, only the low-quality product is available. The platform sets prices charged on both sides, $p_L$ and $s_1$; and both the buyer- and seller-side demands are realized. In period 2, the high-quality product replaces the low-quality product. Based on the current state of the market, the platform sets $p_H$ and $s_2$; and again, the demands on both sides are realized. We solve for the subgame-perfect equilibrium in this dynamic game, such that $p_H$, $s_2$, and buyers’ purchasing decisions are all best responses at the start of period 2, given valuations of buyers in the remaining market. In period 1, all players make forward-looking decisions anticipating such subgame-perfect future strategies. This contrasts with models of committed pricing, in which the platform makes a static decision for both periods upfront without further optimizing at the start of period 2.

The platform’s profit functions in two periods are:

$$\Pi_2(p_H, s_2) = (\theta_L - \frac{p_H}{q_H}) (p_H - (\beta_1 - \delta)q_H^2) \mu + s_2 \left[ v_2(\theta_L - \frac{p_H}{q_H}) - ks_2 \right],$$

$$\Pi_1(p_L, s_1) = (1 - \theta_L)(p_L - \beta_1 q_L^2)\mu + s_1 [v_1(1 - \theta_L) - ks_1] + \alpha \Pi_2(\theta_L).$$

In the remaining of the paper, we normalize $\mu$ to 1, and $v_1, v_2$ and $k$ are scaled by $\frac{1}{\mu}$ accordingly.
4 One-Sided Base Case

We begin by solving the optimal prices in a simpler model, in which the seller side is absent, to more clearly illustrate the dynamics on the buyer side. In the one-sided model, we set \( s_t = 0 \) for both periods. The platform’s profit functions are reduced to the following:

\[
\Pi_2(p_H) = \left( \theta_L - \frac{p_H}{q_H} \right) (p_H - (\beta_1 - \delta)q_H^2),
\]
\[
\Pi_1(p_L) = (1 - \theta_L)(p_L - \beta_1q_L^2) + \alpha \Pi_2(\theta_L).
\]

We derive the optimal price in each period (see Appendix B.1), based on which comparative statics yield the effect of cost decrease on the platform’s strategies and buyers’ purchasing decisions.

**Proposition 1.** A greater decrease in cost induces the platform to set a higher price for the low-quality product in period 1, which leads to a demand shift from period 1 to period 2.

Decreasing cost increases profitability of the high-quality product; thus, the platform is better off with an expanded future market. The optimal price in period 1 is then higher, which not only extracts more surplus from the buyers who are early adopters but also creates a larger market in period 2. Facing a higher price in period 1, only buyers with the highest valuation purchase early, which allows the platform to price-discriminate more aggressively. More buyers are then willing to delay purchase – the potential market is larger in period 2. The platform’s pricing strategy in period 2 must align with buyers’ rational expectations such that those who delay purchase are indeed better off by giving up consumption in period 1.

The quality gap across periods is a key factor in the effect of decreasing cost on the price in period 2. In Figure 1, only the area above the 45-degree line is feasible to ensure \( q_L < q_H \). Region I is also not feasible because \( q_L > \alpha q_H \). The quality gap widens from Region III to Region II, leading to the following proposition.

**Proposition 2.** 2a. For a major quality improvement such that \( \alpha q_H < q_L \leq \frac{5}{4} \alpha q_H \) (i.e., in Region II), a greater decrease in cost leads to a higher optimal price in period 2 (for the high-quality products). Meanwhile, the total demand across the two periods decreases.

2b. For a minor quality improvement such that \( \frac{5}{4} \alpha q_H < q_L < q_H \) (i.e., in Region III), a greater decrease in cost leads to a lower optimal price in period 2 (for the high-quality products). Meanwhile,
the total demand of the two periods increases.

The platform can leverage a greater cost decrease either to position its product line to the high-end market or to increase the total market share. In the former strategy, the platform sets a higher price for the high-quality product in period 2 (as well as that of the low-quality product in period 1 as shown in Proposition 1). This result contradicts the conventional wisdom that lower costs lead to lower prices, which holds in a static setting. In a dynamic setting, the price increase in period 1 creates a forward demand shift that allocates a segment of buyers with high valuations to period 2; a sufficiently substantial quality improvement then allows the platform to exploit these buyers at a higher price in period 2. Even though the total market share is reduced, the price increases in both periods induce buyers to time their purchases such that the platform obtains a higher margin from all purchases.

Decreasing cost leads to a lower price for the high-quality product in period 2 (and a higher price for the low-quality product in period 1 as shown in Proposition 1) only when the quality improvement in period 2 is not significant. In this case, the platform cannot extract sufficient surplus from the high-valuation buyers through a higher price; thus, instead, the optimal strategy is to set a lower price to capture a wider market.
The extent of cost decrease has several strategic implications for the platform. A more drastic cost decrease may be a powerful leverage to charge higher prices in the present and in the future (Propositions 1 and 2a), as long as a major quality improvement for newer versions of the product is anticipated. Through these pricing strategies, the platform segments the market in favor of the future market, which is made more profitable due to the cost decrease. In the spirit of second-degree price discrimination, the platform not only exploits buyers with sufficiently high valuations to purchase early, it may also be able to extract more surplus from those who switch to delay purchase. The outcome of such strategic price adjustments is the exclusion of buyers with lower valuations. As a result, a greater decrease in cost may allow the platform to profit more from fewer buyers.

5 Two-Sided Pricing

In this section, we turn to the full model. The two-sidedness significantly alters and complicates the platform’s pricing problem. The platform not only faces a dynamic problem in setting the price on the buyer-side, it also prices the seller-side fee; the cross-side network externality is an important factor in the platform’s optimal two-sided pricing decision.

Lemma 1 characterizes the optimal buyer-side price, the valuation of the buyer who is indifferent between purchasing the high-quality product and consuming nothing, and the optimal seller-side fee in period 2. Define \( W \equiv \frac{v^2}{2kq_H} \).

Lemma 1. In the period 2 subgame, the optimal price of the high-quality product, the buyer with the lowest valuation, and the optimal seller-side fee in this period are given by:

\[
\begin{align*}
p^*_H & = \frac{\theta_L q_H + (\beta_1 - \delta)q_H^2 - \theta_L v^2}{2 - W} \quad (1) \\
\theta^*_H & = \frac{\theta_L + (\beta_1 - \delta)q_H - \theta_L W}{2 - W} \quad (2) \\
s^*_2 & = \frac{v^2}{2k} \left( \theta_L - \frac{p^*_H}{q_H} \right) . \quad (3)
\end{align*}
\]

Let us first inspect the optimal buyer-side price in period 2 with and without the seller side (Eq. (1) and Eq. (8)). The comparison illustrates forces created by the network effect that are independent of intertemporal issues. The expressions of \( p^*_H \) in the two cases show that, with the
seller-side, both the denominator and the numerator of $p^*_H$ are reduced. These two differences exert forces in opposite directions.

First, in the two-sided case, the platform’s total profit is less sensitive to the buyer-side price. A lower buyer-side price not only stimulates the buyer-side demand, it also strengthens the network effect. Moreover, such network effect intensifies at an increasing rate because of the expanding buyer-side demand. As a result, in the two-sided case, if the platform is to set a lower buyer-side price, it can do so at a higher rate than in the base case to simultaneously take advantage of the seller-side profit gain. The similar intuition applies if the platform is to set a higher buyer-side price. This price increase weakens the network effect and at a decreasing rate. Overall, the total profits from the two sides are less responsive to the buyer-side price, compared with the case when the platform derives revenues solely from the buyers.

The intuition of the second force is straightforward. The seller-side introduces an additional incentive for the platform to obtain more buyers because of the benefits generated on the seller-side. This force simply shifts the buyer-side price downward from that in the base case. The net effect of the two forces then determines the difference of the buyer-side price in the two-sided case compared to that in the base case.

We can observe from Equations (1) and (3) that the platform optimally adjusts the buyer-side price and the seller-side fee in opposite directions. An increase in the buyer-side price diminishes the network effect on the seller-side and reduces the value of the platform to sellers. Thus, the optimal seller-side fee decreases. The platform’s optimal seller-side profit is also reduced.

Next we find the optimal results in period 1 and examine their properties.  

**Lemma 2.** In period 1, the optimal price for the low-quality product, the valuation of the buyer who is indifferent between purchasing the high-quality product and low-quality product, and the optimal seller-side fee are given by:

$$p^*_L = \frac{(2-W)q_L - \alpha q_H}{(2-W)q_L - \alpha q_H + (2-W)\beta_1 q^2_L - \frac{v^2}{2k}(2-W)} - \frac{\theta^*_L}{(2-W)q_L - 3\alpha q_H - \frac{v^2}{2k}(2-W)^2}$$

$$\theta^*_L = \frac{(2-W)q_L - \alpha q_H + (2-W)\beta_1 q^2_L - 2\alpha(\beta_1 - \delta)q^2_H - \frac{v^2}{2k}(2-W)}{2(2-W)q_L - 3\alpha q_H - \frac{v^2}{2k}(2-W)}$$

$$s^*_1 = \frac{v_1}{2k}(1 - \theta^*_L)$$
Comparing $p^*_L$ in the cases with and without the seller side, the differences are similar to those for $p^*_H$ – both the numerator and the denominator are reduced when the seller-side is introduced. The two opposing forces in period 2 also apply to the buyer-side price here. Aside from the same-period, cross-side tradeoffs, the platform’s pricing strategy in period 1 also depends on how the platform balances profits on the two sides in period 2. Notice that, in Eq. (5), the presence of the seller-side in period 2 affects buyers’ intertemporal purchasing decisions. Thus, introducing the seller-side complicates the platform’s strategies due to the interactions between prices on the two sides as well as the dynamic effects. The combination of these forces generates the net difference in the price in period 1 compared to the case without the seller side.

**Proposition 3.** A greater decrease in cost shifts demand from period 1 to period 2 and leads to a higher optimal buyer-side price (for the low-quality product) in period 1.

Proposition 1 from the base case still applies here. Considerations for the seller-side does not alter the qualitative effect of decreasing cost on the platform’s pricing strategies in period 1: A greater decrease in cost increases profitability in period 2 and allocates more buyer-side demand to the future market. It is optimal for the platform to encourage buyers to delay purchase to the more profitable period by setting a higher buyer-side price in period 1. This also allows the platform to extract more surplus from those who remain in period 1.

The difference in the two-sided model compared to the base case is that the impact of decreasing cost on the buyer-side price in period 1 not only triggers an intertemporal demand shift but also alters the strengths of the network effects in both periods. The strength of the network effect is transferred in the direction of the intertemporal demand shift; in other words, as more buyer-side demand is allocated to the future, the platform gains a stronger network effect later at a loss in the present time. This does not impact the platform’s pricing strategy qualitatively because the platform balances cross-period tradeoffs in network effects through seller-side pricing as well. We identify the intertemporal seller-side tradeoffs in Proposition 4 and further contrast the two-sided case with the base case in the following propositions.

**Proposition 4.** A greater decrease in cost leads to higher optimal fee and profit on the seller side in period 2, but lower optimal fee and profit on the seller side in period 1.

Decreasing cost not only incentivizes the platform to shift the buyer-side demand to the future
market, it also sharpens the difference in the seller-side profitability by weighing toward the future. The forward shift of the buyer-side demand transfers the strength of the network externality from period 1 to period 2. Thus, the platform is able to set a higher seller-side fee in period 2, based on its offering of an expanded buyer-side network. As a result, the platform gains from decreasing cost from both the sales of products on the buyer side and the revenues generated from the fees on the seller side. The additional benefits from the seller side in period 2 may strengthen the leverage that the platform obtains from decreasing cost compared to the base case, as shown in Propositions 5 and 6.

Proposition 5. In the two-sided model, both the intertemporal buyer-side demand shift and the buyer-side price increase for the low-quality product due to decreasing cost are more pronounced than in the base case.

In the two-sided model, the impact of decreasing cost on the buyer-side price in period 1 is stronger than in the base case. A higher buyer-side price in period 1 allows the platform not only to expand the buyer-side market in period 2 (same as in the base case) but also to profit further on the seller side in period 2 through the intensified buyer-to-seller network effect, as shown in Proposition 4. The latter (seller-side profits in period 2) increases the incentives for the former (creating more buyer-side demand in period 2); as a result, the platform’s optimal strategy is to set a higher buyer-side price in period 1 further than in the base case. An important implication is that, the network effect empowers the platform to more aggressively price discriminate and more effectively segment the market intertemporally, as a result of more sharply decreasing cost. As more focus is shifted to period 2, the platform only captures the most eager-to-buy consumers with the low-quality product and is able charge them a high premium for early adoption.

Proposition 6. 6a. For a major quality improvement (i.e., in Region II), a greater decrease in cost leads to a higher buyer-side price for the high-quality product. Furthermore, this impact is stronger in the two-sided case than in the base case. The total demand across the two periods decreases.

6b. For a minor quality improvement (i.e., in Region III), a greater decrease in cost nevertheless leads to a higher buyer-side price for the high-quality product, if the network effect in period 1, $v_1$, is sufficiently strong. Under certain conditions of the network effects in the two periods, the opposite may hold.
Consistent with the base case, in the two-sided model, given a substantial quality improvement a greater decrease in cost also enables the platform to set a higher price of the high-quality product. Because the demand shift forward is more pronounced here than in the base case, the future market is populated with more buyers with high valuations than in the base case. This alone allows the platform to set a higher buyer-side price in period 2 more aggressively than in the base case. Furthermore, two-sided pricing allows the platform to adjust the seller-side price to offset any loss on the buyer-side due to the price increase; thus, compared to the base case, the platform can again be more aggressive in setting higher buyer-side price based on the seller-side leverage (recall the discussion on Lemma 1 that the platform’s total profit is less sensitive to changes in the buyer-side price in the two-sided case than in the base case).

Proposition 6b further suggests that the cross-side network effect leads to a higher buyer-side price in period 2 even when it is not possible in the base case, that is, when the quality improvement is minor. Given such quality level, in the base case, a greater cost decrease leads to a lower price for the high-quality product (Proposition 2b); however, a higher price is possible in a two-sided model, depending on the strength of the network effect in period 1. Proposition 2a establishes that the network effect in period 1 drives up the impact of cost decrease on the buyer-side price. Thus, a sufficiently strong network effect can produce an impact on price that allocates considerably more buyers to period 2. The willingness-to-pay of these buyers allows the platform to extract more surplus, through a higher price. In effect, the network effect in period 1 creates an externality to the buyer-side demand and valuation in the next period; this intertemporal externality leads to a different price response in the two-sided model. Absent such a network effect, the impact of decreasing cost may not allocate sufficient high-valuation buyers to period 2 to allow a higher buyer-side price, for which the degree of quality improvement becomes more critical.

The network effect in period 2 plays a different role. When this network effect is sufficiently strong, greater decreases in cost may lead to a lower price for the high-quality product. This is more intuitive than the effect of the network effect in period 1: As the buyer-side demand attracts more sellers, the platform is inclined to obtain a larger buyer-side network, which leads to a lower buyer-side price. As a result, the optimal seller-side fee and profit are higher.

Our findings show that network effects have major implications for platforms’ pricing strategies both within the time period they arise and intertemporally. Whereas studying the base case helps
to disentangle the quality and cost effects from the otherwise complex mechanisms in the two-sided model, comparing the results in the two cases underscores that the two-sided features indeed yield new insights in evaluating dynamic pricing strategies. The presence of network effects intensifies the sensitivity of prices to the decrease in cost. Moreover, network effects can have dynamic impacts that propagate over the time horizon, leading to different strategies than those that are optimal in a one-sided pricing model. Thus, these rich intertemporal elements are meaningful, if not vital, to understanding a dynamic platform problem.

6 Two-Sided Model with Bi-Directional Network Effects

In this section, we consider an additional network effect in the direction from the seller side to the buyer side. Let $\gamma_t$ denote each buyer’s marginal utility for an additional seller on the other side of the platform; that is, $\gamma_t$ measures the strength of the network effect of the seller-side on the buyer-side in period $t$. The utility function of a buyer with $\theta$ in period $t$ is then $\theta q_t + \gamma_t(v_t \Theta_t - ks_t)q_t - p_t$, where $q_t = q_L, p_t = p_L$ for $t = 1$, $q_t = q_H, p_t = p_H$ for $t = 2$, and $\Theta_t$ is the buyer-side network size determined by all buyers’ preferences based on this utility function. Hence, $\theta_H$ and $\theta_L$ satisfy the following conditions:

\[
(\theta_H + \gamma_2(v_2(\theta_L - \theta_H) - ks_2))q_H - p_H = 0,
\]
\[
(\theta_L + \gamma_1(v_1(1 - \theta_L) - ks_1))q_L - p_L = \alpha((\theta_L + \gamma_2(v_2(\theta_L - \theta_H) - ks_2))q_H - p_H).
\]

The platform’s profit in the two periods are:

\[
\Pi_2(p_H, s_2) = (\theta_L - \theta_H)(p_H - (\beta_1 - \delta)q_H^2)\mu + s_2(v_2(\theta_L - \theta_H) - ks_2),
\]
\[
\Pi_1(p_L, s_1) = (1 - \theta_L)(p_L - \beta_1 q_L^2)\mu + s_1(v_1(1 - \theta_L) - ks_1) + \alpha \Pi_2.
\]

Without loss of generality, $\mu$ is normalized to 1, and $v_1, v_2$ and $k$ are scaled by $\frac{1}{\mu}$. Following the method in Section 5, we obtain the expressions of $p_H$ and $s_2$ in terms of $\theta_L$ using the first-order conditions (FOCs) of $\Pi_2$ with respect to $p_H$ and $s_2$. However, model complexity does not permit derivation of the expressions for $p_L$ and $s_1$. Therefore, we use numerical analysis to compute these values and examine the effect of future cost reductions on the platform’s and buyers’ decisions.
We test a wide range of numerical values that satisfy the conditions to ensure concavity of the profit function and bounds of buyer valuations. The degree of intertemporal quality improvement is an important determinant for the effect of cost reduction on the price of high-quality product. Thus, we consider different combinations of \( \delta \) and \( \alpha \) while fixing other parameters: \( q_L, q_H, \beta_1, v_1, v_2, k, \gamma_1 \), and \( \gamma_2 \). We verify the robustness of the qualitative results based on different feasible values of these parameters that satisfy conditions specified in Section 5. In accordance with these conditions, we vary \( \delta \) values in the range from 0.01 to 0.15.

The numerical results show that the price of the low-quality product increases and the buyer demand of low-quality product decreases as \( \delta \) increases, which is consistent with Propositions 3. Figure 2 shows the result for \( \alpha = 0.55 \). When \( \delta \) increases, \( \theta_L \) increases, indicating that the buyer-side demand is shifted from period 1 to period 2.

Figure 3 illustrates the results for the price and profit on the seller-side in period 1 as \( \delta \) increases. Since more buyers are inclined to delay their purchase, the network effects enable the platform to set a higher seller-side fee in period 2 but a lower fee in period 1, which coincides with Proposition 4. As a result, the profit on the seller-side is lower in period 1 and higher in period 2.

Figure 4 compares the cases of one-sided and two-sided business models in terms of the changes in the price of the low-quality product (\( \Delta p_L \)) and the demand in period 1 (\( \Delta \theta_L \)), as the cost in period 2 decreases. Again, the findings revealed are consistent with Proposition 5: The two-sided business model provides the platform additional benefits of setting a higher price of the low-quality product.
product and shifting more buyers from the low-quality product market to the high-quality product market. With the seller-side, more buyers tend to purchase the high-quality product for a higher surplus; and the platform enjoys a higher profit generated on the seller-side due to the network effects and, hence, sets a higher price for the low-quality product.

Figure 3: Optimal Fee and Profit on Seller-Side in Period 1

Following Proposition 6, here cost reductions in period 2 lead to a higher price of the high-quality product due to the demand shift from period 1 to period 2 if the higher quality product improves the quality substantially (Region II). Moreover, the rate of increase is higher with the consideration of the seller-side. When the quality improvement is less substantial (in Region III), the firm may still set a higher price for the high-quality product, whereas it sets a lower price when

Figure 4: Demand and Price of Low-Quality Product for One-sided and Two-sided Models

Following Proposition 6, here cost reductions in period 2 lead to a higher price of the high-quality product due to the demand shift from period 1 to period 2 if the higher quality product improves the quality substantially (Region II). Moreover, the rate of increase is higher with the consideration of the seller-side. When the quality improvement is less substantial (in Region III), the firm may still set a higher price for the high-quality product, whereas it sets a lower price when
the network effects are not present, i.e., in the base case. Figure 5 demonstrates the results for Region II ($\alpha = 0.65$) and Region III ($\alpha = 0.55$).

![Figure 5: Price of High-Quality Product in Region II and III](image)

Although considering the network externalities in both directions significantly complicates the model, we obtain the same insights as those derived analytically based on unidirectional cross-side network effect in Section 5. The intuitions offered by the analytical model can be extended to understand the consistency in findings. The addition of the seller-to-buyer network effect does not conflict with the platform’s intertemporal incentives. In Section 5, we find that in the two-sided case the platform induces more buyers to delay purchase by setting a higher price compared to the one-sided case, because buyers in period 2 can indirectly generate additional profits for the platform by attracting more sellers. Here, when sellers also have this effect on the buyer-side demand, the platform still relies more on period 2 for profits relative to period 1. In fact, shifting buyers to period 2 not only stimulates the seller-side demand, the effect also feeds back on the buyer-side and induces more buyers to purchase in period 2. Whereas qualitative insights are consistent whether the network effect is unidirectional or bidirectional, there may be quantitative differences depending...
on the strength of network effects.

7 Discussion and Conclusion

In this paper, we examine a monopolist’s dynamic pricing decisions facing decreasing production costs, when it introduces quality-improving products sequentially. We compare strategies of a firm that only sells to buyers with those of a platform that markets to both buyers and sellers, where cross-side network effects are present. Contrary to the conventional wisdom, we find that future cost reductions enable the firm to set a higher price of the current product and as well as that of the future product that has a sufficiently high quality. Thus, future cost reductions shift the demand forward and enable high-end positioning of the product line. Furthermore, whereas the one-sided and two-sided business models allow the firm to adopt the similar strategy, in the two-sided case the network effects offer the platform a stronger leverage to set a higher price because the network effects further stimulate the demand and profits in the future market.

This study underscores important factors, in addition to network effect, to be considered by platform owners that are entering the hardware market. Driven by innovative technologies, platform owners’ pricing decisions not only depend on the strength of network effects between different groups of users, successful two-sided pricing strategies also account for intricate interplays between network effects and other hardware-relevant factors. In the case of Apple, securing future cost discounts through negotiation with hardware factories upfront has enabled the company to optimally segment the market across multiple releases through strategic pricing. Anticipating future cost reductions, Apple paces consumers’ adoption of different versions of iPad to gradually achieve remarkable market saturation. A steep price of the first iPad (relative to other brands of tablets) achieves both the positioning of the product as a premium tablet and sufficient market demand for more attractive, later versions. In the meantime, the cross-side network effects spur the growth of the App Store, which currently features over 500,000 applications. In the emerging trend of platform-owned hardware devices, Google, Microsoft and other major players also face dynamic decisions for pricing hardware for their types of platforms.

Our findings also offer new theoretical insights into cannibalization in an intertemporal context. When high-quality product cannot be offered before the low-quality product, we can leverage
future cost reductions to mitigate the intertemporal cannibalization problem. The firm’s strategy to set a higher price in response to a greater decrease in future cost suggests that overestimating future costs may lead to product underpricing. Anticipating a higher future cost drives the firm to capture more demand early on by cutting the price. A suboptimally low price of the product introduced early cannibalizes the demand for the future product, which in turn leads to further underpricing. This is because buyers with high valuation opt for the underpriced product early on, and the buyers remaining in the market have low willingness to pay when the higher-quality product becomes available. Therefore, without accurately anticipating future costs, the firm could face a severe intertemporal cannibalization problem, which incurs profit losses due to underpricing.

By comparing the one-sided and two-sided cases, our results suggest that it is even more important for a firm with a two-sided business model to reduce and accurately estimate future costs. Here the potential cost of intertemporal cannibalization is exacerbated because the buyer-side demand in the future market has an additional effect of attracting more sellers; the cross-side network effect feeds back on the buyer-side demand, expanding the market significantly and yielding profit gains for the platform. As the platform balances its intertemporal tradeoffs, the presence of network effects further emphasizes the more profitable market, inducing the platform to allocate more demand to the market where cost is lower. Therefore, a two-sided platform enjoys a greater advantage by anticipating future costs and holds more market power to position its current and future products to the high-end market.

Furthermore, our work pushes forward the literature on two-sided platforms by accounting for important new elements such as strategic consumers and production costs. Related to the notion of dynamic platform pricing with sequential entry of the two sides (Hagiu 2006, Bhargava et al. 2012), we allow the consumer side to self-select into early- and late-adopters of sequentially innovative platform products. Thus, rather than focusing on the dynamics of the two sides of users, we are interested in the platform’s dynamic acquisition of the consumer side market through quality-improving products while balancing its profits on the other side. More recently, costs in the context of two-sided pricing is receiving increasing attention. Bhargava et al. (2012) emphasizes the role of fixed costs for product expansion in the platform’s versioning strategies, while Chou et al. (2012) point out that supply chain operational costs may alter the conventional understanding on platform subsidization strategies. In our work, the production cost of platform products enters
the context of quality innovation. We show that its variability impacts the platform’s dynamic prices and strategies in intertemporal market segmentation.

A few limitations exist in the current paper and point to several directions for future research. First, we assume that the firm discontinues the low-quality product when the high-quality product is introduced. In practice, the low-quality product may continue to be on market. Appendix B.2 formulates an extension of the model from Section 4 and shows that our findings still hold when the low-quality product remains in period 2. However, studying this scenario in a two-sided model requires additional assumptions about compatibility issues on both sides of the platform for co-existing qualities. The topic of this extension is beyond the scope of this work. Another interesting extension is to consider variations in buyers’ purchasing behavior beside the typical “unit demand” for durable goods. One possibility is for some buyers to demand multiple products of different qualities. A related direction is that in the second-hand market, which emerges through many online channels for many types of products. Existing users then have the option of upgrading their current device by selling their products and then buying a newer and better version.

Appendix

A Proofs

Proof of Proposition 1 In period 1, the demand for the low-quality product is \(1 - \theta^*_L\). Based on Equations (9) and (10)

\[
\frac{\partial (1 - \theta^*_L)}{\partial \delta} = \frac{-2q_H^2 \alpha}{4q_L - 3\alpha q_H} < 0.
\]

The demand for the high-quality product in period 2 is \(\theta^*_L - \theta^*_H\).

\[
\frac{\partial (\theta^*_L - \theta^*_H)}{\partial \delta} = \frac{q_H^2 \alpha}{4q_L - 3\alpha q_H} > 0 \quad \text{and} \quad \frac{\partial p^*_L}{\partial \delta} = \frac{\alpha^2 q_H^3}{8q_L - 6\alpha q_H} > 0
\]

\(\square\)

Proof of Proposition 2 In Region II

\[
\frac{\partial p^*_H}{\partial \delta} = \frac{2q_H^3 \alpha}{8q_L - 6\alpha q_H} - \frac{q_H^2}{2} = \frac{q_H^2}{2} \left[ -\frac{4q_L}{4q_L - 3\alpha q_H} + 5\alpha q_H \right] \geq 0.
\]

The total demand is \(1 - \theta^*_H\), and

\[
\frac{\partial (1 - \theta^*_H)}{\partial \delta} = -\frac{\partial (\theta^*_H)}{\partial \delta} = -q_H \frac{\partial p^*_H}{\partial \delta} \leq 0.
\]
In Region III,
\[ \frac{\partial p^*_H}{\partial \delta} = \frac{q^*_H}{2} \left[ -4q_L + 5\alpha q_H \right] < 0 \text{ and } \frac{\partial (1 - \theta^*_H)}{\partial \delta} = -q_H \frac{\partial p^*_H}{\partial \delta} > 0. \]

\[ \boxed{} \]

**Proof of Lemma 1** The expected profit function is
\[ \Pi_2(p_H, s_2) = (\theta_L - \frac{p_H}{q_H})(p_H - (\theta_1 - \delta)q_H^2) + s_2 \left[ v^2_2(\theta_L - \frac{p_H}{q_H}) - ks_2 \right] \]

By taking the FOC w.r.t. \( s_2 \), we get \( s^*_2 = \frac{v^2_2}{4k}(\theta_L - \frac{p_H}{q_H}) \). Substituting \( s^*_2 \) into the profit function yields,
\[ \Pi_2(p_H, s_2) = (\theta_L - \frac{p_H}{q_H})(p_H - (\theta_1 - \delta)q_H^2) + \frac{v^2_2}{4k}(\theta_L - \frac{p_H}{q_H})^2. \]

The FOC wrt to \( p_H \) gives us
\[ p^*_H = \frac{\theta_Lq_H + (\beta_1 - \delta)q_H^2 - \theta_L\frac{v^2_2}{4k}}{2 - W}. \]

The SOC wrt to \( p_H \) requires the concavity condition: \( 2 - W > 0 \).

\[ \boxed{} \]

**Proof of Lemma 2** Plugging Eq. (1) for \( p^*_H \) into the profit function, we have
\[ \Pi^*_2(\theta_L) = \frac{(\theta_L - (\beta_1 - \delta)q_H)^2}{2 - W} \left[ q_H - \frac{v^2_2}{4k} \right] \]

Notice that to ensure positive demand and marginal profit, the following conditions must be satisfied:
\[ \theta_L - (\beta_1 - \delta)q_H > 0 \text{ and } q_H - \frac{v^2_2}{2k} > 0. \]

Furthermore, we derive the following to be substituted into the FOC of \( \Pi_1 \).
\[ \frac{\partial \Pi_2}{\partial \theta_L} = \frac{2(q_H - \frac{v^2_2}{4k})(\theta_L - (\beta_1 - \delta)q_H)}{2 - W} = \frac{q_H}{2 - W}(\theta_L - (\beta_1 - \delta)q_H) \]

The indifferent buyer \( \theta^*_L \) must satisfy \( 2 - W \left( \theta^*_Lq_L - p_L \right) = \alpha \left[ \theta^*_Lq_H - (\beta_1 - \delta)q_H^2 \right] \).
\[ \theta^*_L = \frac{(2 - W)p_L - \alpha(\beta_1 - \delta)q_H^2}{(2 - W)q_L - \alpha q_H}, \]

where \( (2 - W)q_L - \alpha q_H > 0. \)

The platform’s profit function in period 1 is
\[ \Pi_1(p_L, s_1) = (1 - \theta^*_L)(p_L - \beta_1q^*_L) + s_1 \left[ v_1(1 - \theta^*_L) - ks_1 \right] + \alpha \Pi^*_2(\theta^*_L). \]

The FOC wrt \( s_1 \) gives \( s^*_1 = \frac{v^2_1}{4k}(1 - \theta^*_L) \). We can rewrite the profit in period 1 as,
\[ \Pi_1(p_L) = (1 - \theta^*_L)(p_L - \beta_1q^*_L) + \frac{v^2_1}{4k}(1 - \theta^*_L)^2 + \alpha \Pi^*_2(\theta^*_L) \]
By taking the FOC wrt $p_L$, we get
\[
2(2-W)p_L = (2-W)\beta_1q_L^* + (2-W)q_L - \alpha q_H + \alpha(\beta_1 - \delta)q_H^2 - \frac{v_1^2}{2k}(1 - \theta_L^*)(2-W) + \alpha q_H(\theta_L^* - (\beta_1 - \delta)q_H).
\]

Therefore,
\[
p_L^* = \frac{(2-W)q_L - \alpha q_H}{(2-W)[2(2-W)q_L - 3\alpha q_H] - \frac{v_1^2}{2k}(2-W)^2} \left[ (2-W)q_L - \alpha q_H + (2-W)\beta_1q_L^* - \frac{v_1^2}{2k}(2-W) \right] - \frac{v_1^2}{2k}(2-W)\alpha(\beta_1 - \delta)q_H^2
\]
\[
= \frac{(2-W)q_L - \alpha q_H}{(2-W)[2(2-W)q_L - 3\alpha q_H] - \frac{v_1^2}{2k}(2-W)^2} \left[ -\frac{v_1^2}{2k}(2-W)\alpha(\beta_1 - \delta)q_H^2 \right]
\]
\[
= \frac{(2-W)q_L - \alpha q_H}{(2-W)[2(2-W)q_L - 3\alpha q_H] - \frac{v_1^2}{2k}(2-W)^2} \left[ -\frac{v_1^2}{2k}(2-W)\alpha(\beta_1 - \delta)q_H^2 \right]
\]

To ensure concavity of the profit function, we have the following condition:
\[
(2-W)[2(2-W)q_L - 3\alpha q_H] - \frac{v_1^2}{2k}(2-W)^2 > 0,
\]
which implies that $q_L - \frac{v_1^2}{2k} > 0$. Using $(2-W)q_L - \alpha q_H)\theta_L^* = (2-W)p_L^* - \alpha(\beta_1 - \delta)q_H^2$, we get
\[
\theta_L^* = \frac{(2-W)q_L - \alpha q_H + (2-W)\beta_1q_L^* - 2\alpha(\beta_1 - \delta)q_H^2 - \frac{v_1^2}{2k}(2-W)}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W)}
\]

To ensure $\theta_L^* \leq \beta_1 - \delta \leq 1$, we need the following condition.
\[
(\beta_1 - \delta)q_H \leq \frac{(2-W)q_L - \alpha q_H + (2-W)\beta_1q_L^* - 2\alpha(\beta_1 - \delta)q_H^2 - \frac{v_1^2}{2k}(2-W)}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W)} \leq 1
\]

\[\square\]

Proof of Proposition 3

\[
\frac{\partial \theta_L^*}{\partial \delta} = \frac{2\alpha q_H^2}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W)} > 0.
\]

Since the demand in period 1 is $1 - \theta_L^*$, it decreases as $\delta$ increases.
\[
\frac{\partial p_L^*}{\partial \delta} = \frac{q_H^2 \left[ \alpha^2 q_H + \frac{v_1^2}{2k}(2-W) \right]}{(2-W)[2(2-W)q_L - 3\alpha q_H] - \frac{v_1^2}{2k}(2-W)^2} > 0.
\]

\[\square\]

Proof of Proposition 4

\[
\frac{\partial s_1^*}{\partial \delta} = \frac{v_1}{2k} \left[ -\frac{\partial \theta_L^*}{\partial \delta} \right] < 0,
\]
\[
s_2^* = \frac{v_2}{2k} \left[ \theta_L^* - \frac{p_H^*}{q_H} \right] = \frac{v_2}{2k} \left[ \theta_L^* - (\beta_1 - \delta)q_H \right] \quad \text{and} \quad \frac{\partial s_2^*}{\partial \delta} = \frac{v_2}{2k(2-W)} \left[ \frac{\partial \theta_L^*}{\partial \delta} - q_H \right] > 0.
\]
Since the seller-side profits is $k s_i^2$ in period $i$, $i \in \{1, 2\}$; the results for the seller-side profits follow immediately.

**Proof of Proposition 5**

$$\frac{\partial \theta_L^*}{\partial \delta} = \frac{2\alpha q_H^2}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k} (2-W)},$$

which is greater in magnitude compared to $\frac{\partial \theta_L^*}{\partial \delta}$ in the one-sided case because the numerators are equal and $2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k} (2-W) < 4q_L - 3\alpha q_H$.

To compare the intensity of the price increase on $p_L^*$ in one-sided and two-sided cases, we find the difference of the magnitude of $\frac{\partial p_L^*}{\partial \delta}$ in the two cases. Let $\omega$ denote the difference of $\frac{\partial p_L^*}{\partial \delta}$ in the two cases and denote by $D$ the product of the denominators of $\frac{\partial p_L^*}{\partial \delta}$ in the two cases.

$$D \cdot \omega = \left[ \alpha^2 q_H + \frac{v_1^2 \alpha}{2k} (2-W) \right] \cdot (8q_L - 6\alpha q_H) - \left[ \alpha^2 q_H \right] \cdot (2-W) \left[ 2(2-W)q_L - 3\alpha q_H \right] - \frac{v_1^2}{2k} (2-W)^2$$

$$= \left[ \alpha^2 q_H + \frac{v_1^2 \alpha}{2k} (2-W) \right] \cdot (8q_L - 6\alpha q_H) + \left[ \alpha^2 q_H \right] \cdot [2W(2-W)q_L] + \left[ \alpha^2 q_H \right] \cdot \frac{v_1^2}{2k} (2-W)^2$$

$$> 0$$

**Proof of Proposition 6**

$$\frac{\partial p_H^*}{\partial \delta} = \left( p_H - \frac{v_1^2}{2k} \right) \frac{\partial \theta_L^*}{\partial \delta} - \frac{q_H^2}{2-W} = -\frac{2\alpha q_H - \frac{v_1^2}{2k}}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k} (2-W) - 1}$$

We first need to show that $\frac{\partial p_H^*}{\partial \delta} \geq 0$. To do that, we combine the numerator of the terms in the parentheses:

$$-4q_L + 5\alpha q_H - 2\alpha \frac{v_1^2}{2k} + 2Wq_L + \frac{v_1^2}{2k} (2-W) = (1-W)(-4q_L + 5\alpha q_H) + W(-2q_L + 3\alpha q_H) + \frac{v_1^2}{2k} (2-W)$$

where $4q_L - 5\alpha q_H \leq 0$ in Region II; thus, $2q_L - 2.5\alpha q_H \leq 0$, implying that $2q_L - 3\alpha q_H < 0$. Also, notice that $1-W > 0$ because $q_H - v_2^2 > 0$. Therefore, $\frac{\partial p_H^*}{\partial \delta} > 0$.

To compare with the one-sided case, recall that in the one-sided case:

$$\frac{\partial p_H^*}{\partial \delta} = \frac{q_H^2}{2} \left[ \frac{2\alpha q_H - \frac{v_1^2}{2k}}{4q_L - 3\alpha q_H} - 1 \right]$$

If $\frac{2\alpha(q_H - v_1^2/2k)}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W)} > \frac{2\alpha q_H}{4q_L - 3\alpha q_H}$, then the increase in $p_H^*$ is more pronounced in the two-sided case than in the one-sided case. After multiplying the product of the numerators
\[
\left(2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W) \right) \cdot [4q_L - 3\alpha q_H]\) on both sides, LHS - RHS is the following:
\[
2\alpha(q_H - \frac{v_1^2}{2k}) \cdot [4q_L - 3\alpha q_H] - 2\alpha q_H \cdot \left[2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W) \right]
\]
\[
= 2\alpha q_H [4q_L - 3\alpha q_H] - 2\alpha \frac{v_1^2}{2k} [4q_L - 3\alpha q_H] - 2\alpha q_H [4q_L - 3\alpha q_H] + 2\alpha q_H \left[2Wq_L + \frac{v_1^2}{2k}(2-W) \right]
\]
\[
= 2\alpha q_H \frac{v_1^2}{2k}(2-W) + 2\alpha Wq_H [-2q_L + 3\alpha q_H]
\]
In Region II, \(4q_L \leq 5\alpha q_H\). Thus, \([-2q_L + 3\alpha q_H] > 0\), implying that \(LHS - RHS > 0\). Therefore, the increase in \(p^*_H\) is more pronounced in the two-sided case than in the one-sided case.

\[
\frac{\partial p^*_H}{\partial \delta} = \frac{q_H^2}{2-W} \left[ \frac{2\alpha(q_H - \frac{v_1^2}{2k})}{2(2-W)q_L - 3\alpha q_H - \frac{v_1^2}{2k}(2-W)} - 1 \right]
\]
\[
= \frac{q_H^2}{2-W} \left[ \frac{(2-W)(-2q_L + 2.5\alpha q_H) + 2Wq_H + \frac{v_1^2}{2k}(2-W)}{4q_L - 3\alpha q_H - 2Wq_L - \frac{v_1^2}{2k}(2-W)} \right]
\]
where \(-2q_L + 2.5\alpha q_H < 0\) in Region III and the numerator is increasing in \(v_1\). Therefore, for \(v_1\) sufficiently high, \(\frac{\partial p^*_H}{\partial \delta} \geq 0\).

\[\]
Proof The firm’s profit function is:

$$\Pi_2(p_H) = (\theta_L - \theta_H)(p_H - (\beta_1 - \delta)q_H^2) = (\theta_L - \frac{p_H}{q_H})(p_H - (\beta_1 - \delta)q_H^2).$$

The first-order condition gives $p_H^* = \frac{\theta_Lq_H + (\beta_1 - \delta)q_H^2}{2}$ and $\theta_H^* = \frac{\theta_H + (\beta_1 - \delta)q_H}{2}$. □

Now we examine the firm’s pricing decision for the low-quality product in period 1. Buyers time their purchases by comparing their utilities of consuming the products offered in the two periods. Let $\theta_L^*$ denote the valuation of the buyer who is indifferent between purchasing in the two periods under the optimality condition. Thus, buyers with valuation $\theta \geq \theta_L^*$ purchase in period 1. Given $\theta_L^* q_L - p_L = \alpha(\theta_L^* q_H - p_H^*(\theta_L^*))$, the indifferent buyer is defined by the following expression:

$$\theta_L^* = \frac{2p_L - \alpha(\beta_1 - \delta)q_H^2}{2q_L - \alpha q_H},$$

where $2q_L - \alpha q_H$ is positive.

A number of factors directly influence the value of $\theta_L^*$. Ceteris paribus, a lower product cost in period 2 would induce buyers to delay purchase; also, a lower quality or a higher price in period 1 would have the similar effect. Using the first-order condition (FOC) of the firm’s expected profit function in period 1, we can obtain the optimal price for the low-quality products and the optimal indifference buyer $\theta_L^*$ as shown in Lemma 4.

**Lemma 4.** In period 1, the optimal price for the low-quality product and the valuation of a buyer who is indifferent between purchasing the high-quality product and low-quality product are given by

$$p_L^* = \frac{(2q_L - \alpha q_H + 2\beta_1 q_L^2)(2q_L - \alpha q_H) - \alpha^2(\beta_1 - \delta)q_H^3}{8q_L - 6\alpha q_H},$$

$$\theta_L^* = \frac{2q_L - \alpha q_H + 2\beta_1 q_L^2 - 2\alpha(\beta_1 - \delta)q_H^2}{4q_L - 3\alpha q_H}.$$  

(11)  

(12)

Proof Taking into account $\theta_L^*$, the firm’s expected profit in period 1 is

$$\Pi_1(p_L) = (1 - \theta_L^*(p_L))(p_L - \beta_1 q_L^2) + \frac{\alpha q_H}{4}(\theta_L^*(p_L) - (\beta_1 - \delta)q_H)^2$$
The FOC w.r.t. $p_L$ is

$$(2q_L - \alpha q_H - 4p_L + 2\beta_1 q_L^2 + \alpha(\beta_1 - \delta)q_H^2)(2q_L - \alpha q_H) + \alpha q_H(2p_L - \alpha(\beta_1 - \delta)q_H^2) - \alpha(\beta_1 - \delta)q_H^2 - \alpha(\beta_1 - \delta)q_H^2(2q_L - \alpha q_H) = 0$$

From our assumption of moderate innovation with $q_L > \alpha q_H$, we have $-8q_L + 6\alpha q_H < 0$ and the SOC is $\frac{\partial^2 \Pi_1}{\partial p_L^2} = \frac{-8q_L + 6\alpha q_H}{(2q_L - \alpha q_H)^2} < 0$. We can obtain the optimal price as shown in Eq. (11). By substituting $p_L^*$ into Eq. (10), we get expression for $\theta_L^*$ as shown in Eq. (12).

Notice that $\theta_L$ must satisfy $\theta_H \leq \theta_L \leq 1$, therefore we need the following conditions.

$$\begin{align*}
(\beta_1 - \delta)q_H &\leq \frac{2q_L - \alpha q_H + 2\beta_1 q_L^2 - 2\alpha(\beta_1 - \delta)q_H^2}{4q_L - 3\alpha q_H} \leq 1 
\end{align*}$$

(B.2) Coexistence of High- and Low-Quality Products in Period 2

The firm’s objective is to find $p_H^*$ and $p_L^*$ such that $\Pi^* = max_{p_H,p_L} \Pi(p_H,p_L)$. Since there is a one-to-one correspondence between $(p_H,p_L)$ and $(\theta_H,\theta_L)$, the firm’s profit function in the second period can be written as the following:

$$\Pi_2(\theta_H,\theta_L) = (\theta_L - \theta_H)(p_H - (\beta_1 - \delta)q_H^2) + (\theta_H - \theta_L)(p_L - (\beta_1 - \delta)q_L^2)$$

The firm’s maximization problem is

$$\begin{align*}
max_{\theta_H,\theta_L} \Pi_2(\theta_H,\theta_L) \\
s.t. \ 0 \leq \theta_L \leq \theta_H \leq 1
\end{align*}$$
Based on the FOCs, we derive:

\[
\theta^*_{2L} = \frac{\theta_L + (\beta_1 - \delta)q_L}{2}, \quad \theta^*_{H} = \frac{\theta_L + (\beta_1 - \delta)(q_H + q_L)}{2},
\]

\[
p^*_{2L} = \frac{\theta_L q_L + (\beta_1 - \delta)q^2_L}{2}, \quad \text{and} \quad p^*_H = \frac{\theta_L q_H + (\beta_1 - \delta)q^2_H}{2}.
\]

Notice that \(p^*_H\) remains unchanged compared to that in Appendix B.1; as a result, \(\theta^*_L\), the indifference buyer between purchasing in the two periods, and \(p^*_L\) are also identical to those in the main model – Proposition 1 holds for this setting.

An decrease in the costliness of quality provides more cost savings for a product of a higher quality; thus, the lower cost in period 2 raises profitability of the high-quality product more significantly than that of the low-quality product. It is intuitive that, within period 2, the optimal strategy is to induce a demand shift from the low-quality product to the high-quality product. The intertemporal demand shift, however, further depends on the platform’s optimal strategy in setting the price of the low-quality product in period 1.

**Proposition 7.**

7a. For a major quality improvement (i.e., in region II of Figure 6), a greater decrease in cost leads to a higher optimal price for both the low-quality and high-quality products in period 2. As a result, the total demand over two periods decreases.

7b. For a minor quality improvement (i.e., in region IV of Figure 6), a greater decrease in cost leads to a lower optimal price for both the low-quality and high-quality products in period 2. As a result, the total demand over two periods increases.

7c. When the quality improvement is moderate (i.e., in region III of Figure 6), a greater decrease in cost leads to a lower (higher) optimal price for the high-quality (low-quality) product in period 2. As a result, the total demand over two periods decreases.

**Proof.** We first define two indicator functions of quality ratio \(q \equiv \frac{q_L}{q_H}\). These indicator functions will be used to illustrate comparative statics results of the prices of the high- and low-quality products with respect to \(\delta\).

\[
f_1(q) = 5\alpha - 4q = \bar{f}(q) + 2\alpha
\]

\[
f_2(q) = 2\alpha + 3\alpha q - 4q^2 = q\bar{f}(q) + 2\alpha
\]
where \( f(q) = 3\alpha - 4q \). The values of \( f_1(\cdot) \) and \( f_2(\cdot) \) depend on \( q \) as described in Table 1.

The \((q_L, q_H)\)-space can be divided into four regions shown in Figure 6. Notice that regions I & II are identical to those from Figure 1, whereas region III is further split into two regions.

**Table 1: Regions of the \((q_L, q_H)\)-Space**

<table>
<thead>
<tr>
<th>Region</th>
<th>Condition</th>
<th>( f_1(q) )</th>
<th>( f_2(q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td>( q &lt; \alpha )</td>
<td>Not Feasible</td>
<td></td>
</tr>
<tr>
<td>Region II</td>
<td>( \alpha &lt; q \leq \frac{5\alpha}{4} )</td>
<td>( f_1(q) \geq 0, f_2(q) &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>Region III</td>
<td>( \frac{5\alpha}{4} &lt; q \leq b )</td>
<td>( f_1(q) &lt; 0, f_2(q) \geq 0 )</td>
<td></td>
</tr>
<tr>
<td>Region IV</td>
<td>( q &gt; b )</td>
<td>( f_1(q) &lt; 0, f_2(q) &lt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Notice that \( f_1(\cdot) \) is a monotonic and decreasing function. Given that \( f_1(\frac{5\alpha}{4}) = 0 \), in region II \( f_1(q) \geq 0 \). Moreover, in region II \( \tilde{f}(q) < 0 \). Since \( q < 1 \), then \( f_1(q) < f_2(q) \) for all feasible values of \( q \) in region II. Thus, \( f_2(q) > 0 \).

In regions III & IV, \( f_1(q) < 0 \). Also notice that \( f_2(\cdot) \) is decreasing when \( q > \frac{3\alpha}{8} \). \( f_2(b) = 0 \) then implies that \( f_2(q) \geq 0 \) for \( \frac{5\alpha}{4} < q \leq b \) (region III). For \( q > b \) (region IV), \( f_2(q) < 0 \).

7a. In region II, we have \( f_1(q) \geq 0, f_2(q) > 0 \), so
\[ \frac{\partial p_H^*}{\partial \delta} = \frac{q_H^3}{4q_L - 3\alpha q_H} \quad \text{and} \quad \frac{\partial p_{2L}^*}{\partial \delta} = -\frac{q_L^2}{2} + \frac{q_H^2q_L\alpha}{4q_L - 3\alpha q_H} \quad \geq 0 \]

The total demand is \( 1 - \theta_{2L}^* \), and \( \frac{\partial (1 - \theta_{2L}^*)}{\partial \delta} = -q_L \frac{\partial p_{2L}^*}{\partial \delta} < 0 \)

7b. In region IV, \( f_1(q) < 0, f_2(q) < 0, \) so

\[ \frac{\partial p_H^*}{\partial \delta} = \frac{q_H^3}{2} \left[ \frac{f_1(q)}{4q_L - 3\alpha q_H} \right] < 0 \quad \text{and} \quad \frac{\partial p_{2L}^*}{\partial \delta} = \frac{q_L q_H^2}{8q_L - 6\alpha q_H} f_2(q) < 0, \]

\[ \frac{\partial (1 - \theta_{2L}^*)}{\partial \delta} = -q_L \frac{\partial p_{2L}^*}{\partial \delta} > 0. \]

7c. In region III, \( f_1(q) < 0, f_2(q) \geq 0, \) so

\[ \frac{\partial p_H^*}{\partial \delta} = \frac{q_H^3}{2} \left[ \frac{f_1(q)}{4q_L - 3\alpha q_H} \right] < 0 \quad \text{and} \quad \frac{\partial p_{2L}^*}{\partial \delta} = \frac{q_L q_H^2}{8q_L - 6\alpha q_H} f_2(q) \geq 0, \]

\[ \frac{\partial (1 - \theta_{2L}^*)}{\partial \delta} = -q_L \frac{\partial p_{2L}^*}{\partial \delta} < 0. \]

\[ \square \]

References


Hao, L., H. Guo and R. Easley (2013), A mobile platform’s monetizing strategy for advertising under agency pricing for app sales. Working paper, University of Notre Dame.


