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# Efficient Child Care Subsidies†

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*We study the design of child care subsidies in an optimal welfare problem with heterogeneous private market productivities. The optimal subsidy schedule is qualitatively similar to the existing US scheme. Efficiency mandates a subsidy on formal child care costs, with higher subsidies paid to lower income earners and a kink as a function of child care expenditure. Marginal labor income tax rates are set lower than the labor wedges, with the potential to generate negative marginal tax rates. We calibrate our simple model to features of the US labor market and focus on single mothers with children aged below 6. The optimal program provides stronger participation but milder intensive margin incentives for low-income earners with subsidy rates starting very high and decreasing with income more steeply than those in the United States.* (*JEL* D82, H21, H24, J13, J16, J32)

The transition of mothers' role from traditional homemakers to potential breadwinners over the past decades indicates the increasing involvement of mothers as active members of the labor force. In parallel, policymakers are increasing their focus on child care subsidy programs. In the United States, programs such as the Dependent Care Tax Credit (DCTC) and the Child Care and Development Fund (CCDF) are benefiting from increased funding.[1] The CCDF was established as part of the landmark Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA), whose main goal was to increase employment and reduce welfare

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[1] Recent debates include the 2011 Obama Administration's proposal to double the DCTC for families earning below \$85,000 (Tax Policy Center 2010a). More than \$5.6 billion federal funds was allocated to the CCDF in 2017 (Office of Child Care 2017).

dependence among low-income single mothers (Grogger and Karoly 2005, Fang and Keane 2004). When introducing the PRWORA, the then-US president declared:

> *First, the new bill is strong on work. It provides $4 billion more for child care so that mothers can move from welfare to work, and protects their children by maintaining health and safety standards for day care.*
>
> Bill Clinton, 1996 Press Conference
> (White House Communications Agency, 1996)

Similarly, during the European Council meeting in Barcelona, it was agreed that:

> *Member States should remove disincentives to female labour force participation and strive [...] to provide childcare by 2010 to at least 90 percent of children between 3 years old and the mandatory school age and at least 33 percent of children under 3 years of age.*
>
> Point 32 in the Presidency Conclusions of the 2002 EC meeting
> (European Council, 2002)

Child care subsidies to promote labor supply is also a major argument in a recent OECD report (OECD 2016).

Even though there is a vast literature on the impact of child care subsidies on employment of mothers[2] and considerable policy debates on affordability of child care, none has so far looked at the optimal design of child care subsidies. We study the design of such subsidies within an optimal welfare framework, where heterogeneous agents have private information on labor market productivities. Agents have child care needs and allocate effort between the primary labor market and household child care activities.

We show that it is optimal to pay a *positive child care subsidy* on formal child care costs and that higher child care subsidies should be paid to lower income earners. We therefore add an efficiency reason to existing debates for providing child care subsidies to low-income earners and suggest that a *sliding scale child care subsidy scheme* would be an optimal way of promoting employment while achieving redistributional goals. Moreover, very much in line with the qualitative features of the existing scheme in the United States, *the optimal subsidy must be kinked* as a function of child care expenditure. An agent whose formal child care expenditure is lower than the kink point faces a positive subsidy while it is optimal to set a nonpositive subsidy for child care expenditure above the kink point.

The intuition for the three key qualitative features of the efficient child care subsidy schedule is as follows. Achieving redistributional goals (or raising tax revenues) require income transfers across productivity levels, which in turn may discourage formal labor supply. Since household child care increases the individual return from reducing labor hours, a *positive child care subsidy rate* is required to discourage household child care. Moreover, for each given skill type, the higher the labor income required, the lower the time available for household child care such that there is a lesser need to discourage such activities. Hence, the optimal child care subsidy rate *decreases with income*. Finally, the *kink feature of optimal subsidies* is needed to

[2] For example, Bainbridge, Meyers, and Waldfogel (2003); Blau (2003); Blau and Tekin (2007); Ho (2013, 2015); Kimmel (1995); Tekin (2005).

discourage both high- and low-skilled individuals from digressing from the optimal labor supply. Whereas a high productivity agent is typically tempted to generate a lower labor income and increase household child care, a low-skilled individual may be tempted to produce a higher labor income and reduce household child care. Both of these possibilities are detrimental to incentives and may hinder the achievement of public goals. Since low- and high-skilled individuals face different marginal returns from household child care, the optimal child care subsidy schedule is kinked such that the target rates are different before and after the kink.

By jointly designing child care subsidies and nonlinear income dependent child allowances, we show that the new policy tool cannot be replicated by a negative marginal tax rate based on earned income of low-skilled workers alone (such as, for example, the Earned Income Tax Credit).[3] Our implementation exercise generates an interesting discrepancy between the standard labor wedge (which is always positive in our model) and the marginal tax on earned income. In particular, the *optimal marginal taxes are set at lower rates than the labor wedges* due to the interaction with the sliding scale pattern of child care subsidies. While the sliding scale pattern directly counteracts non-local child care deviation incentives, it may have disincentive effects on labor supply. Marginal income tax rates are, therefore, lowered so as to adequately counterbalance such disincentives. This discrepancy is particularly relevant at low income levels and may potentially lead to negative marginal taxes on income.

This paper also provides some quantitative estimates of the optimal child care subsidy rates implied by our Mirrleesian framework with child care and labor supply margins. We calibrate our simple model to features of the US labor market and focus on single mothers with children aged below 6. According to US Census data, the number of children living with a single mother has nearly tripled between 1960 to 2016, with nearly one-quarter (17 million) of children currently living with a single mother. We focus on single mothers with young children because they tend to have high child care needs and are often targeted by generous transfer programs. Our study is, therefore, designed to focus on low- and middle-income earners. Focusing on single mothers within a heterogeneous agent framework enables us to identify key trade-offs involved in child care while abstracting from the practical complexity of modeling joint transfers in multi-adult households. The optimal program seems to provide stronger participation incentives but milder intensive margin incentives for low-income earners compared to the US scheme. In particular, the optimal child care subsidy rates start at very high levels and decrease with income more steeply than those in the current US scheme.

*Literature*.—Barnett (1993) argues that child care subsidies should be offered to mothers with young children to counteract the disincentive effects of the current tax system on labor supply. A similar principle emerges in the representative agent models of Kleven, Richter, and Sørensen (2000), which studies linear commodity taxation in

[3] Consider a high-skilled individual who faces a positive wage subsidy when earning low income but a positive marginal tax when earning high income. Such an individual would be tempted to work a few hours in the formal sector, enjoy the wage subsidy, and, in addition, engage in higher household child care so as to save on formal child care costs. Thus, child care subsidies have an independent role to play in our context.

presence of home production. We confirm this important principle.[4] However, we also find that the optimal pattern of child care subsidies across income groups do not mimic at all the shape of the labor income taxes, suggesting a richer role for such instrument.[5]

To implement the constrained efficient allocation, we allow the government to use formal child care subsidies to indirectly tax home activities, which would otherwise be detrimental for incentive compatibility. This is similar in spirit to the exercises performed in the New Dynamic Public Finance literature (Golosov and Tsyvinski 2006, Golosov et al. 2013, Kocherlakota 2010, Saez 2002b, Werning 2011), where both labor supply and saving wedges are considered. However, the child care margin is different from the saving margin studied in these works, both economically and technically. Crucially, due to the non-separability between labor supply and child care, the implementation of the second-best allocation in our model *requires a kink* in the subsidy schedule. Thanks to the additive separability assumption between consumption and leisure in these studies, savings can instead be taxed linearly.[6]

The introduction of child care relates our paper to the literature on income taxation in the presence of non-market activities (Beaudry, Blackorby, and Szalay 2009; Choné and Laroque 2011; Saez 2002a). This literature considers heterogeneous cost of labor market participation and has argued that it is optimal to subsidize low-income earners in the form of negative marginal income tax rates. We consider a different framework where mothers differ in labor market productivities but face the same hourly cost of formal child care. As in these works, our model involves a multidimensional choice problem.[7] Although we cannot adopt the standard "local approach," the model permits a sharp characterization of the optimal allocation by focusing on only the downward incentive constraints.

Also related to our paper is the literature in quantitative micro- and macroeconomics that aims at numerically computing welfare or labor supply gains from policy reforms, as opposed to characterizing the optimal child care subsidy and nonlinear transfer scheme as we do. Bick (2016) and Domeij and Klein (2013) find that child care subsidies may encourage labor supply of German mothers. Guner, Kaygusuz, and Ventura (2016) finds that an increase in child care subsidies in the US context may increase labor supply, especially along the extensive margin of participation. Blundell and Shephard (2012) estimates a structural labor supply model and focus on single mothers in the United Kingdom. Our work complements these studies in that it analyzes a richer (and hence more flexible) policy tool in an optimal welfare framework with heterogeneous private market productivities. Flexibility supported by rigorous economic principles may provide some valuable

[4] For a similar principle emerging in a different context, see Koehne and Sachs (2016).

[5] In fact, even in the existing US scheme, child care subsidies seem to follow a somewhat more complex pattern. For example, since the Earned Income Tax Credit scheme implies a negative income tax rate for low-income earners with young children, if child care subsidies were to merely mimic (counteract) the pattern of the marginal income taxes, child care costs should be taxed, not subsidized, for low-income earners.

[6] For the need of a kink in savings taxation in the presence of nonseparabilities, see Kocherlakota (2004).

[7] There are important differences in the framework considered, which imply different technical difficulties and require a different approach. In Beaudry, Blackorby, and Szalay (2009), the different activities are perfect substitutes, while in Choné and Laroque (2011) and Saez (2002a), agents face heterogeneous fixed costs of participation to the labor market. Our model contemplates two genuinely different intensive margins (work and child care). Our framework is more closely related to Besley and Coate (1995) (see also, Brett 1998), but the characteristic of our model does not allow for the (more standard) local-approach adopted in that paper. Instead, we follow a line of attack to the problem that is similar to that indicated by Matthews and Moore (1987).

insights into the assessment of complex schemes such as the existing one in the United States. Moreover, studying the efficient design of child care subsidies jointly with optimal child allowances allows us to understand how they have an independent role from income taxes.

We document the main components of child care subsidy programs in the United States in Section I. In Section II, we present our model of the household where mothers choose both labor supply in the primary market and household provided child care. Optimal policy and implementation results are presented in Sections III and IV, respectively. The calibration exercise and numerical results are presented in Section V. In Section VI, we discuss some extensions to our framework, while Section VII concludes.

## I. US Child-Related Subsidy Programs

This section describes the 2010 US tax and subsidy scheme with a particular focus on child care subsidies and child dependent allowances. We outline the main features of interest in two major child care (price-related) subsidy programs, the Dependent Care Tax Credit (DCTC) and the Child Care and Development Fund (CCDF). We then describe the child dependent tax exemptions and allowances that are available to families with children under the federal income tax scheme, the Earned Income Tax Credit (EITC), and the Temporary Assistance to Needy Families (TANF). Further details are reported in online Appendix B.3.

*Child Care Subsidies (DCTC and CCDF)*.—The DCTC is a non-refundable federal income tax credit program available to families with children aged under 13 and covers part of child care expenses. The CCDF is a block grant fund managed by states within certain federal guidelines. CCDF subsidies are available as vouchers or as part of direct purchase programs to families with children under 13 and with income below 85 percent of the state median income.

**Employment Requirements:** Both subsidy programs are conditional on parental employment. The DCTC is a tax credit available only to families who earn income and pay taxes while the CCDF is available to low-income families who are engaged in work-related activities.[8]

**Sliding Scale:** In both the DCTC and the CCDF, the child care subsidy rate declines as income increases.[9] The DCTC has a tax credit rate of 35 percent of child care expenses for families with annual gross income of less than $15,000. The tax credit rate declines by 1 percent for each $2,000 of additional income until it reaches a constant tax credit rate of 20 percent for families with annual gross income above $43,000. Whereas the federal recommended subsidy rate for the CCDF is 90 percent, only a certain proportion of eligible households receive the

[8] In 2010, 81 percent of families receiving CCDF were employed with the remaining families in training (Administration for Children and Families 2012).

[9] While there are differences across states in the generosity of the subsidy rates, in all states, the child care subsidy rates strictly follow a sliding scale pattern (Gabe, Lyke, and Spar 2001).

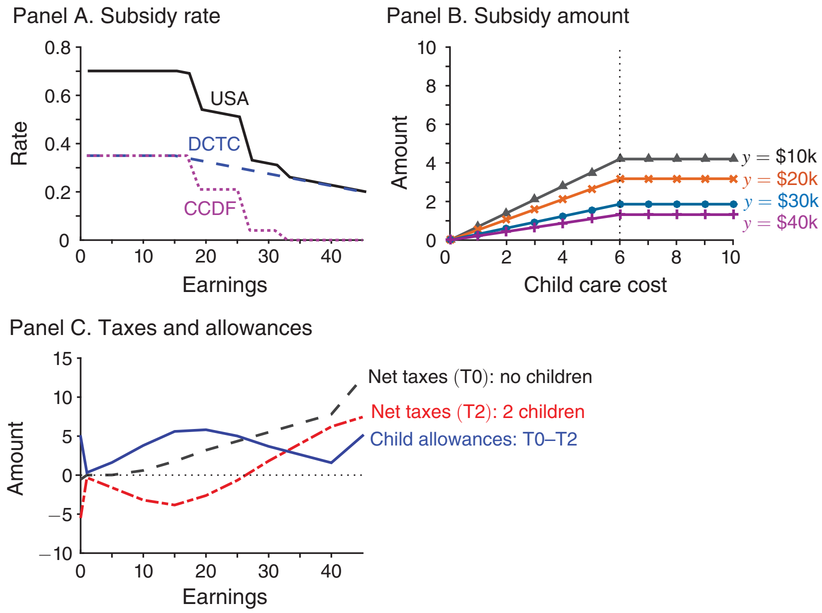


FIGURE 1. 2010 US TAX AND SUBSIDY SCHEDULES ($THOUSANDS)

*Notes:* Panel A reports child care subsidy rates under DCTC and CCDF, and the consolidated rates (solid line) as a function of gross family income. Panel B reports the amounts of child care subsidies received as a function of total formal child care costs and by family income ( $y$) for a family with two children aged below 13. Panel C depicts the amounts of net income taxes payable as a function of gross family income for a single person with 0 and 2 children. The net income taxes include TANF, federal and social security taxes, and EITC. The difference between net income taxes for a single person without and with children are represented by the solid line, and are interpreted as the child allowances that a parent is eligible for under the US welfare system.

subsidy: 39 percent, 24 percent, and 5 percent of households living, respectively, below, between 101 percent and 150 percent, and above 150 percent of the poverty threshold (US Department of Health and Human Services 2009). Panel A of Figure 1 illustrates the average child care subsidy rates under the DCTC and the CCDF according to family income.[10]

**Decreasing Coverage:** The coverage rate decreases with total expenditure on child care. The DCTC has a cap on child care expenditure of $3,000 for families with one child and $6,000 for families with two children. As of 2010, the CCDF maximum reimbursement rates ranged from $280 per week (Puerto Rico) to $1,465 per month (New York) for an infant in full-time formal child care (Minton et al. 2012). In addition, 17 states had a cap on the number of hours of formal child care use, ranging from 45 hours per week (Michigan) to 20 hours per day (Montana). Panel B of Figure 1 illustrates the amount of child care subsidy that a family with two eligible children would receive under the DCTC and CCDF. We illustrate the scheme for families with two children as our sample of interest, single mothers with

[10] States using CCDF funding are also required to have co-payments from the family that increase with family income. We do not take into account the state wide variations in co-payments in our analysis and focus on the average subsidy rates at the federal level. Following the allocation rates described above, Figure 1 is drawn by imputing an average CCDF subsidy rate of 35.1 percent, 21.6 percent, and 4.5 percent to households with income below, between 101 percent and 150 percent, and above 150 percent of the poverty threshold, respectively.

children aged below 6, have two children on average (see Section V for details). Consistent with the rates reported in panel A, the slope of the subsidy amount schedule decreases with family income before the cap of $6,000.

*Child Allowances* (*Tax Exemptions*, *EITC*, *and TANF*).—In addition to subsidies on the cost of formal child care, parents are also eligible for relatively generous child dependent allowances that are conditional on the presence of children in the household. Under the federal income tax scheme, taxable income is based on earnings minus standard deductions of $5,700 for a single childless person and $8,400 for a single parent, minus exemptions of $3,650 for each taxpayer and dependent. Both childless individuals and parents are subject to social security (SS) taxes set at 7.65 percent of earnings.

Working families are eligible for the EITC, which is a refundable tax credit and follows a "trapezoid" pattern.[11] Parents are also eligible for TANF, which is a cash assistance program for families with children aged below 18. In 2010, nearly 80 percent of TANF recipients were unemployed while a family with two children received on average $412 of TANF benefits per month (US Department of Health and Human Services 2011).

Panel C of Figure 1 illustrates the net income taxes payable by a single childless person and by a single parent with two children, computed as federal income and SS taxes minus EITC benefits for the employed, and minus TANF and additional benefits for the unemployed (see Section V for details). The demographic dependent child allowances are computed as the difference between net taxes of a childless individual and net taxes of a single parent with two children. The figure illustrates at least three qualitative properties of the US tax and transfer system. First, child allowances are equivalent to nonlinear income transfers. Second, the increasing pattern of the dashed line indicates that childless households always face a positive marginal tax on income. Third, the child allowances paid to mothers with children below 6 imply a negative marginal income tax, as indicated by the decreasing segment of the dash-dotted line, for earnings below $15,000.[12]

## II. Model

From the richness of the US child related transfer and subsidy program, a few normative questions emerge. Is it economically sensible to pay a positive child care subsidy to working mothers? Can the same margin be accounted for with properly designed taxes and transfers on labor income? Should the child care subsidy rate depend on earned income? If yes, should marginal taxes for working mothers be adjusted relative to those levied on childless households? And should the child care subsidy rate depend on total child care cost? In particular, should there be a cap above which the subsidy rate is zero?

[11] For a single childless person, EITC benefits are phased in at a rate of 7.65 percent up to a maximum of $457 in benefits. Families with children benefit from much more generous EITC benefits. For example, for a single parent with two children, EITC benefits are phased in at a rate of 40 percent up to a maximum of $5,036 in benefits.

[12] While we focus on the federal income tax, some states also impose state income taxes with rates ranging from 0 percent to 11 percent. Low-income parents would still benefit from a negative marginal tax rate even if we were to take into account the highest marginal tax rate of 11 percent (Tax Policy Center 2010b).

In order to address these questions, a flexible economic model is needed, where rich patterns of income taxes and child care subsidies can be studied. The framework presented in this section, introduces the possibility of engaging in household child care in an optimal welfare problem à la Mirrlees in a centralized economy. This relatively simple model captures some of the key trade-offs faced by working mothers. We address the optimal design of a nonlinear transfer and subsidy scheme that implements the optimum in a decentralized economy in Section IV.

*Agents and Technologies*.—Consider an economy with a continuum of agents who are heterogeneous in market productivities $z$. We consider discrete levels of market productivity, with $z_1 = 0$ being the minimum and $z_N > 0$ the maximum, that is, $z \in Z := \{z_1, \ldots, z_i, \ldots, z_N\}$. Agents of type $z_i$ constitute a fraction $\pi(z_i) > 0$ of the population, with $\sum_{i=1}^{N} \pi(z_i) = 1$. We interpret agents with $z_1 = 0$ as agents who are subject to adverse labor market conditions (the involuntarily unemployed or unlucky), thereby rendering their market productivity zero.

Agents can allocate effort to market work or to household child care. An agent who devotes $l \geq 0$ units of effort on the market produces $y = zl$ of consumption goods. Each agent has child care needs that are normalized to 1 unit of effort, and devote effort level $h \geq 0$ toward them. The remaining amount of child care is covered by purchasing child care from the formal child care market at cost $\omega$ per unit.[13] We assume that $z_N > \omega > 0$.[14]

Agents' utility function is additive in consumption $c$ and effort cost $v(e)$:

$$c - v(e),$$

where $e = l + h$ is total effort and $c$ represents household consumption net of formal child care cost $f := \omega(1-h)$.[15]

We note that the theoretical results that follow are generalizable to the case where formal child care is an imperfect substitute for household care and we can also allow for endogenous formal child care quality. The assumption of quasilinear utility is also made for analytical tractability as in Besley and Coate (1995) and Kleven, Kreiner, and Saez (2009). The extensions to the model are discussed in Section VI. The case with log preferences is also solved numerically in Section V.

ASSUMPTION 1: *The cost function is strictly increasing and strictly convex:* $v'(e) > 0$ *and* $v''(e) > 0$ *for all e. In addition, assume that* $v'(0) = 0$.

[13] We interpret child care needs as the amount of child care time that can be substituted for paid care during a normal working week. In other words, while $h = 0$ implies that full-time formal child care is employed, it does not necessarily imply that mothers never look after their children. For example, mothers could still be taking care of their children during evenings after work.

[14] Whenever either one of the inequalities is not satisfied, our framework specifies into a standard Mirrlees optimal tax model. First, as can be seen by analogy to the proof of Proposition 1(iii), when $\omega = 0$ then $h(z) = 0$ for all $z$. In addition, from Proposition 2(i), if $z_N \leq \omega$ then all agents will either be pooled into unemployment or engage in full-time household child.

[15] Child care wedges are present in Proposition 2(v) due to the non-separabilities between labor and child care. An alternative utility specification, delivering the same result, could explicitly treat leisure as a good: $c + v(T - e)$, where $T$ is the total endowment of time and $e$ is effort.

*Laissez-Faire Equilibrium.*—Suppose that agents face no taxes nor subsidies and there are no insurance markets. They solve

$$\max_{l \geq 0, h \geq 0} zl - \omega(1-h)^{+} - v(l+h),$$

where $(1-h)^{+} := \max\{0, 1-h\}$. In the laissez-faire equilibrium, high productivity agents specialize into employment while low productivity agents provide household child care. If $z > \omega$, they optimally choose $h = 0$ and $l > 0$. These high productivity agents consume $c = zl - \omega$ and labor supply solves $z = v'(l)$. When agents have $z < \omega$, they all choose $h > 0$. Low productivity agents with employment opportunities $(0 < z < \omega)$ may also work after all child care needs have been taken care of, that is, if $h = 1$. Since household child care does not depend on labor market productivities, all unemployed agents engage in the same level of household child care and enjoy the same consumption. On the other hand, among employed agents, both earnings and consumption increase in $z$.

*Government and Information.*—Consider a government who aims at maximizing social welfare. The government does not observe market productivities. The government, however, knows the probability distribution of the different types of agents among the population. The government cannot observe labor supply while it can observe output from the labor market (i.e., earnings, $y$), and the total cost of formal child care purchased by each agent $(f)$. Since $f = \omega(1-h)$, household child care $(h)$ is verifiable (while leisure is not observable). For the purpose of the present application, we endow the government with the amount $M$ of resources to be shared among agents. We interpret $M$ as resources allocated to the group of agents we are interested in (i.e., single mothers with young children), which are obtained from general taxation or other sources that are not studied in this paper. By the revelation principle, we can restrict ourselves to direct mechanisms defined over $Z$.

DEFINITION 1: *An allocation consists of consumption functions $c : Z \to \mathbb{R}$, market production functions $y : Z \to \mathbb{R}_{+}$, and household-provided child care functions $h : Z \to \mathbb{R}_{+}$, for all types. Let $\Omega$ be the set of such allocations.*

The government also has to satisfy the budget constraint, which can be written as

$$\sum_{i=1}^{N} \pi(z_i) c(z_i) + \omega \leq \sum_{i=1}^{N} \pi(z_i)\left[y(z_i) + \omega h(z_i)\right] + M. \tag{1}$$

Modeling the problem as though the government takes all production and assigns consumption and child care, is equivalent to imposing a net tax of $T(z) := y(z) - \omega(1 - h(z)) - c(z)$ on each agent of type $z$. Constraint (1) is equivalent to $\sum_i \pi(z_i) T(z_i) + M \geq 0$.

The government faces the standard trade-off between redistributing resources and preserving work incentives. In the laissez-faire allocation, utility increases in $z$ among employed agents and the unemployed get the lowest utility level. Should the government provide too generous redistribution toward low $z$ types, high $z$ types would be tempted to mimic low $z$ types by decreasing effort.

*Constrained Efficient Allocation* (*Second-Best*).— Since each agent has private information on market productivity, the government faces a set of incentive compatibility constraints. The incentive constraints guarantee the truthful revelation of agents' type $z$. Agents will only reveal their true type if government policy is such that utility from telling the truth is higher than utility from pretending to be a different type.

DEFINITION 2: *A reporting strategy is a mapping* $\sigma : Z \to Z$. *By the revelation principle, the planner aims at implementing the truth-telling strategy*, $\sigma^*$, *where* $\sigma^*(z) = z, \forall z \in Z$.

With private information, government allocation has the same domain as above but is based on agents' declarations $\sigma$. The definition of an allocation must be reinterpreted accordingly, but still follows Definition 1.

Let

$$V(\sigma|z) \coloneqq c(\sigma) - v\left(\frac{y(\sigma)}{z} + h(\sigma)\right)$$

be the utility that agent of type $z$ obtains by pretending to be of type $\sigma$. The government must guarantee that the agent prefers the truth-telling strategy to any other strategy. Truth-telling requires that for all $z \in Z$,

$$V(z|z) \geq V(\sigma|z), \quad \forall \sigma \in Z. \tag{2}$$

A key question in the design of an efficient welfare program is how to optimally trade off redistribution for effort incentives. The objective of the government is to maximize welfare:

$$W(c, y, h; \phi) = \sum_i \pi(z_i)\phi(z_i)\left[c(z_i) - v\left(\frac{y(z_i)}{z_i} + h(z_i)\right)\right], \tag{3}$$

where the function $\phi : Z \to \mathbb{R}_+$ defines the social weighting given by the authorities to the different agents' classes $z \in Z$.

DEFINITION 3: *A second-best allocation is a solution to the maximization of the objective* (3) *over* $(c, y, h) \in \Omega$ *subject to the budget constraint* (1) *and the incentive constraints* (2).

## III. The Optimal Allocation

In this section, we characterize the constrained efficient (second-best) allocation. In a standard Mirrlees problem with unidimensional choice of effort, it is customary to use a "local approach" (i.e., solve the *relaxed problem* that only imposes local incentive compatibility constraints). Under the standard assumption that preferences satisfy the "single-crossing property of indifference curve maps" (i.e., the marginal rate of substitutions between the choices $y$ and $c$ are monotone in agent's type $z$), the solution derived from the relaxed problem coincides with the solution to the global problem. In addition, a robust result in the standard optimal taxation model is that one can focus on (local) downward incentive constraints and hence always obtain downward distortions, that is, positive labor wedges.

Our model involves a multidimensional choice of effort (work and child care). The monotonicity of marginal rates of substitution between any pair of choices does not imply the "single crossing property of indifference curve maps." The most typically adopted approach in the literature on multidimensional choice is to still use a local approach and look for conditions that guarantee that the solution to the relaxed problem delivers a uniformly monotone allocation.[16] Unfortunately, in our framework, uniform monotonicity of the optimal allocations cannot easily be guaranteed a priori. We will hence follow a non-local approach.[17] We look for conditions that guarantee what Matthews and Moore (1987) refers to as *double crossing*. This, in turn, allows us to only focus on downward incentive constraints as shown in Lemma 1. Assumption 2 guarantees that the utility levels generated by any two allocations, $\left(\bar{c}, \bar{y}, \bar{h}\right)$ and $\left(\hat{c}, \hat{y}, \hat{h}\right)$, cross no more than twice in the $z$ space (see Lemma 2 and Figure A1 in the online Appendix).

ASSUMPTION 2: *Let $e > 0$. The ratio $v''(e)/v'(e)$ is decreasing in e.*

Standard cost functions such as the quadratic, the constant Frisch elasticity: $v(e) = \left(1/\theta\right)\left(e^{1+\gamma}/(1+\gamma)\right)$, $\theta, \gamma > 0$, and the exponential cost functions, satisfy this assumption.

An analytical derivation of the constrained efficient allocation also requires an assumption on the social weighting function $\phi(\cdot)$.

ASSUMPTION 3: *Let $\mathbb{E}[\phi] := \sum_{i=1}^{N} \pi(z_i)\phi(z_i)$. We have $\phi(z_1) \geq \mathbb{E}[\phi]$. Moreover, for $j \geq 3$, the weight $\phi(z_j)$ is lower than the average social welfare weight: $\phi(z_j) \leq \mathbb{E}[\phi]$.*

Note that Assumption 3 is satisfied by the Utilitarian welfare function with equal weights $\phi(z_i) \equiv 1$ on all agents. In this case, however, the allocation would display no trade-off between efficiency and redistribution. At the other extreme, the conditions of Assumption 3 are satisfied by the Rawlsian welfare function: $W^R(c,e) := \min_i\left\{c(z_i) - v\left(e(z_i)\right)\right\}$. As we will see below, incentive compatibility implies that $c(z_i) - v\left(e(z_i)\right)$ increases with $i$, and hence, the Rawlsian

[16] This is what Matthews and Moore (1987) refers to as "attribute ordering." For example, since both the marginal rates of substitution between $(-c)$ and $y$, and between $(-c)$ and $h$ decrease with $z$, if $y$ and $h$ were either *both* monotone increasing or both monotone decreasing in $z$, the allocation would satisfy the single crossing property for the agent's problem and hence local incentive constraints would imply global incentive compatibility (see Lemma 0 in Matthews and Moore 1987 and Section 7.3 in Fudenberg and Tirole 1991).

[17] Besley and Coate (1995, Section VII) solves a model similar to ours using a local approach and assuming monotonicity of the marginal rates of substitution. Crucially, they also assume that $\omega = 0$ and $z_1 > 0$. This implies that all agents are optimally required to choose $h = 0$. Their model, hence, reduces to a version of the standard Mirrlees framework where the monotonicity of the marginal rates of substitution implies single crossing of the indifference curve maps.

criterion implies $\phi(z_1) > 0$ and $\phi(z_i) = 0$ for $i > 1$. The Rawlsian criterion can be seen as the limit case for the following class of welfare objectives:

$$\widehat{W}(c, e; \rho) := \left( \sum_{i=1}^{N} \pi(z_i) \left[ c_i - v(e_i) \right]^{\rho} \right)^{\frac{1}{\rho}},$$

for $\rho \to -\infty$. Intuitively, for $\rho$ finite but sufficiently low, the implied Pareto weights satisfy Assumption 3. Although it allows for non-monotone $\phi$s, Assumption 3 is satisfied whenever the government has a sufficiently strong desire for redistribution at the bottom.[18]

LEMMA 1 (DIC Approach): *Under Assumptions 1, 2, and 3, any solution to the second-best problem where only downward incentive constraints (DIC) are imposed, that is, when the set of conditions* (2) *is relaxed to be* $\sigma \leq z$, *delivers an optimal allocation. In addition, the "local" downward incentive constraints (LDIC) can be imposed as equalities. Finally, if the upward incentive constraint (UIC) is binding for two types* $z_j < z_k$, *then it is optimal for all agents with type* $z_i : z_j \leq z_i \leq z_k$ *to receive the same allocation (i.e., bunching).*

PROOF:

See online Appendix A.

Lemma 1 states that the solution from the relaxed second-best problem, where the government maximizes the objective (3) subject to the budget constraint (1) and only the DIC in (2), delivers a solution to the original problem. Given the relaxed problem with DIC only, we show that the LDIC must be satisfied with equality. This crucially relies on the fact that preferences satisfy the double crossing property. Should LDIC between type $z_{i+1}$ and type $z_i$ be slack, then the double crossing property implies that the non-local DIC for preventing type $z_{i+1}$ from mimicking lower types will also be slack. It would, therefore, be possible to improve welfare at no additional cost and without violating incentives, by redistributing from type $z_{i+1}$ to all other types. Under Assumption 3, such redistribution will weakly improve welfare. It is then straightforward to show that when the LDIC bind, the UIC will also be satisfied.[19]

Thereafter, we indicate the allocation obtained using Lemma 1 as "the optimal allocation," and denote it by adding an asterisk as superscript.

PROPOSITION 1 (Minimal Properties): *Under Assumptions 1, 2, and 3, we have:*

(*i*) *The "net surplus"* $y^*(z) + \omega h^*(z) - c^*(z)$ *is non-decreasing in* $z$.

[18] The requirement that $\phi(z_1) \geq \mathbb{E}[\phi]$ guarantees a well-defined problem and can be replaced by a participation constraint. The function $\phi$ is typically assumed to be non-increasing so that $\phi(z_1) \geq \mathbb{E}[\phi]$ is automatically satisfied.

[19] Since $z_1 = 0$, the UIC between $z_1$ and $z_2$ is either straightforwardly satisfied (if $y^*(z_2) > 0$) or it implies bunching (whenever $y(z_2) = 0$). We hence do not need to show a binding LDIC for $z_2$.

$(ii)$ *Utility of agents in equilibrium* $V^*(z|z)$ *is non-decreasing in* $z$, *and strictly increasing between any two levels* $z_{i+1} > z_i$ *when* $y^*(z_i) > 0$.

$(iii)$ *For all* $z$, $h^*(z) \leq 1$.

PROOF:

See online Appendix A.

Points (i) and (ii) in Proposition 1 summarize a general principle. Obtaining a larger net surplus from high types is the sole reason why the government is ready to trade off redistribution and screen agents instead of pooling them. Point (iii) states that providing household child care beyond child care needs would be costly in terms of effort without yielding any additional return. This implies that providing $h > 1$ does not help satisfy the incentive constraints. This is because consumption is a superior instrument to achieve separation between types.

PROPOSITION 2 (Characterization): *Under Assumptions 1, 2, and 3, we have:*

$(i)$ *Unemployment: Recall that* $z_1 = 0$. *We have* $y^*(z_1) = 0$ *and* $h^*(z_1) > 0$, *where*

$$1 - \frac{1}{\omega} v'\big(h^*(z_1)\big) \geq 0, \tag{4}$$

*with equality whenever* $v'(1) \geq \omega$. *If* $v'(1) \leq \omega$, *then* $h^*(z_1) = 1$. *In addition, for all* $z$ *such that* $y^*(z) = 0$, *type* $z$ *gets the same allocation as type* $z_1$.

$(ii)$ *Low productivity: Let* $z \leq \omega$. *We have* $h^*(z) > 0$, *and if* $y^*(z) > 0$, *then* $h^*(z) = 1$.

$(iii)$ *Segmentation: If* $y^*(z) > 0$, *then* $y^*(z') > 0$ *for all* $z' > z$. *And hence, if* $y^*(z) = 0$, *then* $y^*(z') = 0$ *for all* $z' < z$.

$(iv)$ *Monotonicity: Let* $z' > z$ *for which we have no bunching. If* $h^*(z') \leq h^*(z)$, *then* $y^*(z') > y^*(z)$; *and, equivalently: if* $y^*(z') \leq y^*(z)$, *then* $h^*(z') > h^*(z)$.

$(v)$ *Wedges for the employed: Let* $z_i$ *be such that* $y^*(z_i) > 0$. *Then labor wedges are non-negative:*

$$1 - \frac{1}{z_i} v'\big(e^*(z_i)\big) \geq 0. \tag{5}$$

*If, in addition,* $h^*(z_i) > 0$, *then the child care wedges are also non-negative:*

$$1 - \frac{1}{\omega} v'\big(e^*(z_i)\big) \geq 0. \tag{6}$$

*Both wedges are strictly positive whenever* $\phi(z_{i+1}) < \mathbb{E}[\phi]$.
*For* $i = N$, *the labor wedge is zero and* $h^*(z_N) = 0$.

PROOF:

See online Appendix A.

The intuition for result (i) is simple. When $y(z) = 0$, market productivity does not matter so that all agents receive the same allocation: we have pooling among the unemployed. Result (ii) states that low market productivity types may provide positive labor supply only when all child care needs have been met. Statement (iii) delivers a minimal monotonicity condition: if an agent is employed, then more productive agents will also be employed. Statement (iv) concludes the monotonicity properties of the allocation. Wedges in (v) are direct consequences of the fact that only DIC matter in our model.

The wedge (6) implies that it is optimal to distort $h^*(z_i)$ downward. Intuitively, the key limits to redistribution is constituted by the discouraging effect of taxes on formal labor. Household-provided child care constitutes a productive way to employ spare time by shirkers. Subsidizing formal child care reduces the return of household-provided child care hence enhancing redistribution possibilities, or equivalently, allowing to achieve the same redistributional goal with a lower efficiency cost.

A more formal argument for why it is optimal to reduce household child care can be made by considering the following variational exercise. Recall from Lemma 1 that only the downward incentive constraints matter. Suppose that we have an optimal allocation $(c^*, y^*, h^*)$ such that for some $z_i < z_N$, we have $h^*(z_i) > 0$ and (6) is satisfied with equality:

$$\omega = v'\left(\frac{y^*(z_i)}{z_i} + h^*(z_i)\right).$$

Suppose now that the government decreases $h^*(z_i)$ by $\epsilon$ and decreases $c^*(z_i)$ by $\omega\epsilon$ such that the government budget constraint (1) is still satisfied. Indicate the new level of consumption and household child care as $c^\varepsilon$ and $h^\varepsilon$, respectively. Since (6) is satisfied with equality, for $\varepsilon$ small, a true telling agent $z_i$ will also get the same utility level as before:

$$c^\varepsilon - v\left(\frac{y^*(z_i)}{z_i} + h^\varepsilon\right) = c^*(z_i) - v\left(\frac{y^*(z_i)}{z_i} + h^*(z_i)\right).$$

In geometrical terms, such perturbation corresponds to a small movement along agent $z_i$'s indifference curve. Consider now an agent with productivity $z' > z_i$ who contemplates mimicking agent $z_i$. Since $y^*(z_i)/z' < y^*(z_i)/z_i$, the strict convexity of $v(\cdot)$ implies that

$$c^\varepsilon - v\left(\frac{y^*(z_i)}{z'} + h^\varepsilon\right) < c^*(z_i) - v\left(\frac{y^*(z_i)}{z'} + h^*(z_i)\right).$$

In other terms, agent $z'$ will now find mimicking $z_i$ less attractive, thereby relaxing the DIC and allowing for higher redistribution.[20]

## IV. The Shape of Efficient Child Care Subsidies

As described in Section I, the existing child care subsidy scheme is rather complex. First, it involves only a partial coverage of formal child care costs. Second, the coverage is nonlinear: the subsidy has a formal child care expenditure cap above which the subsidy rate is reduced to zero. Third, the subsidy rate decreases with labor income. We are interested in understanding whether such features are optimal.

In this section, we propose a nonlinear transfer and subsidy scheme that implements the constrained efficient allocation in a decentralized economy. We note that while Assumptions 2 and 3 are sufficient conditions that allow us to analytically characterize the optimal allocations, we do not need to impose those assumptions for our implementation exercise. The proposed implementation is more general and prevents both upward and downward deviations in the global problem.

### A. *Child Care Wedges and Joint Deviations*

As indicated in (6), point (v) of Proposition 2, it is optimal for the marginal rate of substitution between consumption and child care to be lower than the return to child care (in consumption terms) for certain agents. Such discrepancies are known as *wedges* in public finance. If agents could freely choose child care (that is not necessarily socially optimal), wedges would be eliminated. A typical way to preserve a wedge is to use a tax policy. In our case, a positive subsidy on formal child care would reduce the privately perceived return to household child care and generate a wedge qualitatively similar to (6). In our framework, however, the relationship between the wedge and the optimal subsidy on child care is not so straightforward. Instead, we show that the optimal subsidy must be kinked as a function of the level of formal child care cost, very much in line with the qualitative features of the existing scheme in the United States. An agent whose expenditure on formal child care is lower than the kink point faces a subsidy while it is optimal to set the subsidy to zero (or even to perhaps impose a positive tax) for formal child care cost above the kink point.

The reason why the connection between wedges and taxes breaks down in our framework is as follows. The wedge (6) is calculated by figuring out the shadow return to child care of an agent who produces the socially optimal quantities as a function of her skills. Setting the subsidy equal to this wedge eliminates the agent's desire to provide suboptimal child care *when she produces the socially optimal quantities associated with her z type*. However, in a market economy with taxes, an agent might find it optimal to adopt a *joint deviation* of producing a different amount and adjusting the level of child care provided.

[20] More precisely, since the DIC of all types above $z_i$ are relaxed, the government can decrease the consumption of such higher types, generate budget savings, and redistribute them uniformly across all types to improve total welfare.

An optimal tax and subsidy schedule has to be designed so as to deter such joint deviations.

In order to more formally grasp the economic forces shaping child care subsidies in our framework, consider the "local" wedge as in (6):

$$WE(z_i|z_i) \ := \ 1 - \frac{1}{\omega}v'\left(\frac{y^*(z_i)}{z_i} + h^*(z_i)\right).$$

Let $h^*(z_i) < 1$. Suppose that the government is able to induce agent $z_i$ to produce $y^*(z_i)$. Hence, $WE(z_i|z_i) \geq 0$ represents a necessary condition for the agent to choose $h^*(z_i)$.

Setting marginal income tax rates equal to the labor wedges (5) and marginal child care subsidy rates equal to the child care wedges $WE(z_i|z_i)$, however, will not be enough to implement the constrained optimum. This is because those who tell the truth about their type are not the only ones who would want to increase $h$. In fact, higher types who declare to be of a type $\sigma = z_i$ will have even greater incentives to overprovide $h$ (while also engaging in suboptimal market work). In particular, consider agent $z_{i+1}$ declaring to be of type $z_i$. The "joint deviation wedge" for this agent is given by

$$WE(z_i|z_{i+1}) \ := \ 1 - \frac{1}{\omega}v'\left(\frac{y^*(z_i)}{z_{i+1}} + h^*(z_i)\right).$$

Clearly $WE(z_i|z_{i+1}) > WE(z_i|z_i)$, that is, agents of type $z_{i+1} > z_i$ face a joint deviation child care wedge that is larger than the child care wedge for a true-telling agent of type $z_i$. In other words, if we were to set the child care subsidy rate to $WE(z_i|z_i)$, then agent $z_{i+1}$ pretending to be of type $z_i$ and producing the recommended level of income $y^*(z_i)$ for this declaration, finds it optimal to increase $h$ beyond $h^*(z_i)$. This is problematic since, as shown in Lemma 1, the LDIC is binding at the optimal allocation. This implies that, whenever the child care subsidy rate is set equal to $WE(z_i|z_i)$, agent $z_{i+1}$ finds it strictly more advantageous to declare $\sigma = z_i$, produce $y^*(z_i)$ and choose $h > h^*(z_i)$ compared to declaring the truth (and choosing the recommended values $\left(y^*(z_{i+1}), h^*(z_{i+1})\right)$ for his type). These complications are even stronger when non-local DIC are binding, a non-pathological feature of the optimal allocation in our multidimensional choice setting. For the purpose of implementing a second-best allocation, it is therefore important to consider the possibility of joint deviations in declaring a different type $\sigma$ and engaging in a non-optimal level of $h$.

*A Graphical Representation of the Optimal Child Care Subsidy Schedule.*— The rational behind the qualitative shape of the efficient subsidy scheme can be seen graphically as follows. Recall that $V^*(\sigma|z)$ is the value for agent $z$ of declaring $\sigma$ according to the constrained efficient allocation:

$$V^*(\sigma|z) \ := \ c^*(\sigma) - v\left(\frac{y^*(\sigma)}{z} + h^*(\sigma)\right),$$

where $\left(c^*(\sigma), y^*(\sigma), h^*(\sigma)\right)$ are the constrained optimal allocations associated with type $\sigma$. Second-best optimal net taxes are given by

$$T^*(\sigma) = y^*(\sigma) - c^*(\sigma) - \omega\left(1 - h^*(\sigma)\right).$$

Suppose now that agents can privately choose which type to declare, $\sigma \in Z$, as well as household provided child care. Taking the second-best optimal $y^*(\sigma)$ and $T^*(\sigma)$ as given, an agent $z$ therefore chooses $\sigma$ and $h$ so as to maximize her private utility:

$$\max_{\sigma,h} \underbrace{y^*(\sigma) - T^*(\sigma) - \omega(1-h)^+}_{c} - v\left(\frac{y^*(\sigma)}{z} + h\right). \tag{7}$$

If each agent who reports $\sigma$ engages in the constrained efficient level of household child care associated with type $\sigma$ (i.e., $h = h^*(\sigma)$), then incentive compatibility would imply that all agents would reveal their true type. A necessary condition for this to happen is that the agent faces a subsidy that solves her first-order condition with respect to household child care at $h^*(\sigma)$. We would thus require a subsidy rate equal to the *joint deviation child care wedge* at $h = h^*(\sigma)$. Let $s(\sigma|z)$ be such a rate:

$$s(\sigma|z) = WE(\sigma|z) := 1 - \frac{1}{\omega}v'\left(\frac{y^*(\sigma)}{z} + h^*(\sigma)\right).$$

Hence, we have

$$\left(1 - s(\sigma|z)\right)\omega - v'\left(\frac{y^*(\sigma)}{z} + h^*(\sigma)\right) = 0.$$

We illustrate the private problem (7) of an agent of type $z$ declaring to be of type $\sigma$ in panel A of Figure 2. In the absence of child care subsidies, the slope of the budget constraint, $c = y^*(\sigma) - T^*(\sigma) - \omega(1-h)$, is equal to the cost of formal child care $\omega$. The agent engages in household child care $h(\sigma|z) \in (0,1)$ given by the tangency point between the agent's indifference curve and budget constraint at point A. To implement the constrained optimum, we need to induce any agent who declares $\sigma$ to choose the constrained optimal household child care, $h^*(\sigma)$. A child care subsidy rate set equal to the joint deviation wedge of the agent at $h^*(\sigma)$ ensures that the slope of the budget constraint becomes $\left(1 - s(\sigma|z)\right)\omega$. Agent $z$ declaring $\sigma$ will therefore choose $h^*(\sigma)$ at point B.

This hypothetical subsidy scheme is, however, infeasible since the subsidy rates are dependent on the true type $z$ of the agent, which is nonobservable. We therefore need to design a subsidy scheme that does not rely on observing $z$. Suppose that, as in panel A of Figure 2, in the absence of child care subsidies, an agent $z$ reporting $\sigma$ has incentive to engage in $h > h^*(\sigma)$. Such deviation, would be discouraged by setting the subsidy rate equal to the joint deviation wedge of highest type $z_N$:

$$WE(\sigma|z_N) = 1 - \frac{1}{\omega}v'\left(\frac{y^*(\sigma)}{z_N} + h^*(\sigma)\right).$$

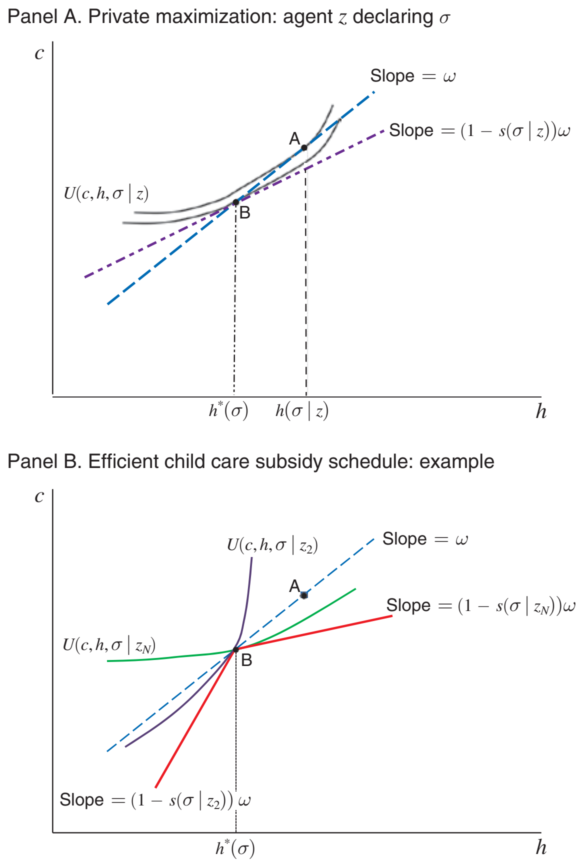


FIGURE 2. THE SHAPE OF CHILD CARE SUBSIDIES

*Notes:* $U(c,h,\sigma|z)$ corresponds to the objective function in (7). Panel A: In the absence of child care subsidies, agent $z$ declaring $\sigma$ engages in household child care level $h(\sigma|z)$, given by the tangency point between the agent's indifference curve and the agent's budget constraint at point A. A hypothetical child care subsidy rate set equal to the joint deviation wedge of the agent at $h^*(\sigma)$ would ensure that an agent $z$ declaring $\sigma$ will choose $h^*(\sigma)$ at point B. Such hypothetical subsidy rate is infeasible as it would depend on the true type $z$, which is not observed. Panel B: A subsidy rate that is set equal to the maximum joint deviation wedge $s(\sigma|z_N) = WE(\sigma|z_N)$ when $h \geq h^*(\sigma)$ and to the minimum joint deviation wedge $s(\sigma|z_2) = WE(\sigma|z_2)$ when $h < h^*(\sigma)$, ensures that any agent declaring to be of type $\sigma$ chooses the optimal level of household child care $h^*(\sigma)$. An example of such a scheme is depicted by the solid line budget constraint with a kink at $h^*(\sigma)$.

Since $WE(\sigma|z_N) \geq WE(\sigma|z)$ for all $z$, no $z$ declaring $\sigma$ would ever choose $h$ above $h^*(\sigma)$. Symmetrically, setting a subsidy rate equal to $WE(\sigma|z_2)$ guarantees that each agent $z$ reporting $\sigma$ has an incentive to choose $h \leq h^*(\sigma)$. Such a scheme is illustrated by the solid line kinked budget constraint in panel B of Figure 2. The scheme displays a kink point at $h^*(\sigma)$. At point B in panel B, the steeper segment of the kinked budget constraint is tangent to the indifference curve for agent $z_2$ while the flatter segment of the kinked budget constraint is tangent to the indifference curve for agent $z_N$. Since the indifference curve of any $z$ reporting $\sigma$ would lie in between the indifference curves associated with $z_2$ and $z_N$

at the kink point, any agent reporting $\sigma$ would choose $h^*(\sigma)$. This principle is used in Proposition 3, where we also show that $z_2$ can be replaced by the productivity level of the highest unemployed type whenever the later is higher than $z_2$.

To sum up, a kink in the optimal child care subsidy schedule arises because of the presence of upward and downward deviation incentives in our model. As long as there are more than two types with $z > 0$, some low productivity agents may choose to mimic higher types and provide lower household child care than optimal for this level of income. Symmetrically, some high productivity agents may choose to mimic lower types and provide higher household child care than optimal. Due to the non-separability between labor supply and child care, *the marginal rate of substitution between consumption and child care at the same income level*, *differs across types.* Thus, for a given $y$, to incentivize true-telling agents to choose the optimal level of formal child care, and at the same time discourage both higher and lower types from mimicking the true-teller, different subsidy rates are offered according to whether formal child care is higher than or lower than the optimal level for each given income bracket.[21]

## B. *Implementation*

We first discuss an implementation that relies on direct mechanism and subsequently map our proposed implementation using a version of the taxation principle.

Recall that for any real number $x$, we adopt the notation $x^+ := \max\{0,x\}$ and $x^- := \min\{x,0\}$. Let $Z_0^* := \{z \in Z \mid y^*(z) = 0\}$ denote the set of types pooled into unemployment, and $\bar{z}_0 := \max\{z_2, \max Z_0^*\}$ the highest type between $z_2$ and the unemployed.

PROPOSITION 3: *Let $f^*(\sigma) := \omega(1 - h^*(\sigma))$ be the optimal formal child care cost associated with the optimal $h^*(\sigma)$. The following subsidy rates and transfers implement the constrained optimum.*

(*i*) *For employed agents*, *we have*

$$\text{if } \sigma \notin Z_0^*, \quad \text{then } s(\sigma,f) = \begin{cases} \left(1 - \frac{1}{\omega} v'\left(\frac{y^*(\sigma)}{z_N} + h^*(\sigma)\right)\right)^+, & \text{if } f \le f^*(\sigma); \\ \left(1 - \frac{1}{\omega} v'\left(\frac{y^*(\sigma)}{\bar{z}_0} + h^*(\sigma)\right)\right)^-, & \text{if } f > f^*(\sigma). \end{cases}$$

[21] Werning (2011) studies nonlinear capital taxation and argues that there is no need for a kinked tax system in the presence of continuous types. In his context, saving deviations can only occur ex ante (i.e., before agents know their type) and kinks may only occur at finitely many points (when the agent is indifferent between two or more reports). From an ex ante perspective, the finitely many kinks get an expected probability weight of 0 in a continuous type framework. There are two main reasons why Werning's (2011) argument does not apply in our framework and kinks will be more pervasive even with a continuum of types. First and foremost, child care deviations occur at the interim stage (i.e., after agents know their type) and hence no ex ante integration is possible. Second, our multidimensional choice model fails to admit a local approach. In particular, since the true-telling constraints may also bind for non-adjacent types, the considerations made above typically apply to the case with a continuum of types as well.

(*ii*) *For unemployed agents, the subsidy rate is zero: if* $\sigma \in Z_0^*$, *then* $s(\sigma, f) = 0, \forall f$.

(*iii*) *For all* $\sigma \in Z$, *the optimal transfer scheme is set as follows:*

$$T(\sigma) = y^*(\sigma) - c^*(\sigma) - f^*(\sigma) + s(\sigma, f^*(\sigma))f^*(\sigma);$$

*where* $c^*(\cdot)$ *and* $y^*(\cdot)$ *are the consumption and income functions of the second-best allocation.*

PROOF:

See online Appendix A.

The identification of the type $\bar{z}_0$ permits one to minimize the subsidy rates in the second segment of the subsidy schedule while analytically guaranteeing the implementation of the second best. The operators $x^+$ and $x^-$ have a similar aim. They imply that the child care subsidy rate is set to zero whenever such zero rate is "analytically sufficient" to implement the second best. We further note that for employed agents, if $f^*(\sigma) = \omega$, then only the subsidy rates associated with the first segment $f \leq f^*(\sigma)$ are relevant. Similarly, if $f^*(\sigma) = 0$, then only the subsidy rates associated with the second segment $f > f^*(\sigma)$ are relevant.

As described above, child care subsidies in statement (i) ensure that each agent declaring $\sigma$ chooses the optimal level of formal child care cost associated with $\sigma$, $f^*(\sigma)$, no matter what her true type is. In particular, the first [second] segment ensures that agents will not want to overprovide [underprovide] household child care, analogous to the principle illustrated in Figure 2. From statement (iii), income transfers are then adjusted to yield the same consumption to agents as in the constrained optimum: $c^*(\sigma) = y^*(\sigma) - T(\sigma) - (1 - s(\sigma, f^*(\sigma))f^*(\sigma)$. Since agents earn the same and receive the same consumption levels as in the second-best optimum, such allocations are incentive compatible and also satisfy the government budget constraint.

Statement (ii) deals with child care subsidies offered to the unemployed. Since market productivities are irrelevant for them, they are all the same and there are no incentives problem among them. There is thus no need to subsidize child care of the unemployed.

The implementation is straightforward in the sense that we do not need to compute who deviates where and by how much, that is, we do not need to compute all joint deviation wedges: the proposed kinked subsidy schedule, based on the joint deviation wedges associated with $z_N$ and $\bar{z}_0$, ensures that no agent would deviate from the optimal child care level. The child care subsidies are conditional on formal child care cost being verifiable.[22]

The optimal subsidy rates and transfers schedule incorporates features that match the qualitative ones of the US system, that is, a cap on formal child care costs and

[22] One might consider alternative implementation schemes that may include, for example, day care vouchers equivalent to the subsidy rate up to the optimal level of formal child care and extreme income taxes for households with formal child care expenses above the level mandated by the optimal program.

subsidy rates that decrease with earnings for formal child care costs below the cap. We propose such a scheme using a variation of the taxation principle below.

To be able to describe the subsidy rates and transfer scheme as only a function of income, we need an additional monotonicity assumption. We abuse in notation and indicate by $f(y)$ the formal child care level associated with income $y$. For all values of $y$ such that there is a $\sigma_y : y = y^*(\sigma_y)$, we associate $f(y) = f^*(\sigma_y)$. Unfortunately, such mapping does not deliver a well-defined function whenever the optimal allocation associates multiple values of $f$ to one $y$. We will hence assume a well-defined function $f(\cdot)$.

ASSUMPTION 4: *Let* $\mathcal{Y} = \{y \in \mathbb{R}_+ \mid \exists z \in Z : y = y^*(z)\}$ *be the set of equilibrium income values, and for all* $y \in \mathcal{Y}$ *define* $f(y) := f^*(\sigma_y)$. *Assume that* $f(\cdot)$ *is single valued.*

As we will see in Section VB, $f$ turns out to be non-decreasing in $y$ in all of our numerical simulations. For practical purposes, we can extend the domain of $f(\cdot)$ to $\mathbb{R}_+$ so as to obtain a weakly monotone (piecewise constant) function. For $y \geq 0,\ y \notin \mathcal{Y}$, we set $f(y) = f(m(y))$ where $m(y) := \max\{\hat{y} \in \mathcal{Y} \mid \hat{y} \leq y\}$. The consumption function is analogously constructed: $c(y) = c(m(y))$, where for all $y \in \mathcal{Y}$, $c(y) = c^*(\sigma_y)$.

PROPOSITION 4: *Under Assumption* 4, *there is a* $\overline{T} \in \mathbb{R}$ *such that the following subsidy rates and transfers implement the constrained optimum.*

(*i*) *For employed agents* (*who earn* $y > 0$), *we have*

$$s(y,f) = \begin{cases} \left(1 - \frac{1}{\omega} v'\left(\frac{y}{z_N} + 1 - \frac{f(y)}{\omega}\right)\right)^+, & \text{if } f \leq f(y); \\ \left(1 - \frac{1}{\omega} v'\left(\frac{y}{\bar{z}_0} + 1 - \frac{f(y)}{\omega}\right)\right)^-, & \text{if } f > f(y); \end{cases}$$

*if* $y \in \mathcal{Y}$, *then* $T(y) = y - c(y) - f(y) + s(y, f(y)) f(y)$; *otherwise*, $T(y) = \overline{T}$.

(*ii*) *For unemployed agents* (*with* $y = 0$), *the second-best allocation is implemented by having*

$$s(0, f) \equiv 0, \quad \text{and} \quad T(0) = -c(0) - f(0).$$

PROOF:

See online Appendix A.

Note that when $f(y) = \omega$, the child care subsidies for the employed with $f < f(y)$ follow a *sliding scale* pattern, that is, they decrease with labor earnings. To see this, suppose that productivity type $z_N$ produces a lower output than the optimal one associated with her type (i.e., she is mimicking a low-income earner). In this case, type $z_N$ has an incentive to engage in higher than optimal household child care. The lower the mimicked output, the lower the marginal cost of providing an extra hour of child care within the household and hence, the higher the non-local child care deviation incentives for type $z_N$. Thus, higher child care subsidies are paid to lower income earners to counterbalance such incentives.

In our implementation, labor wedges and marginal taxes on income do not coincide. Using the private first-order condition of the agent with respect to $y$, evaluated at the agent's optimal formal child care cost choice $f(y)$, we obtain

$$T'(y) = 1 - \frac{1}{z} v'\left(\frac{y}{z} + 1 - \frac{f(y)}{\omega}\right) + s'_y(y, f(y)) f(y). \tag{8}$$

When $s'_y(y,f) \leq 0$, since $f(y) \geq 0$, our implementation implies a marginal income tax that is no greater than the labor wedge.[23] This observation might contribute to the debate over the optimality of imposing a negative marginal income tax rate on low-income earners. The debate has focused on the possibility of having *negative labor wedges* whenever there is a strong desire to redistribute toward low-skilled individuals (Choné and Laroque 2011; Saez 2002a). As we saw in Section I (see panel C of Figure 1), only low-income working mothers face negative marginal taxes in the US system. Meanwhile, they also face child care subsidies that decrease with earnings. Equation (8) indicates that *when child care subsidies follows a sliding scale*, *optimal negative marginal taxes can be compatible with positive labor wedges.*

*Child Allowances*.—In order to implement the second-best allocation in a way that is compatible with the current US income tax schedule, we let $T^a(y)$ denote net taxes faced by a childless individual earning $y$ in the actual US tax and benefit system (the corresponding schedule is the dashed line T0 in panel C of Figure 1). As described in Section I, the existing child allowances $A^a(\cdot)$ include child related federal income tax exemptions and EITC if employed, and TANF benefits if unemployed (this schedule corresponds to the solid line in panel C).

We take the general income tax scheme for the childless $T^a(y)$ as given and keep it fixed. *The optimal child care subsidy rates* $s(y,f)$ *from Proposition* 4, *together with the optimal child allowances*, $A(y)$, *implement the constrained optimum*, where child allowances are defined as

$$A(y) := T^a(y) - T(y),$$

and $T(y)$ are the total optimal transfers from Proposition 4.[24] It is straightforward to see that since child care subsidy rates are the same as in Proposition 4, agents would engage the optimal level of formal child care. In addition, since $T(y) = T^a(y) - A(y)$, Proposition 4 implies that consumption and utility would also be the same as under the second best.

[23] Note that the function $s(y,f)$ is not differentiable in $y$ at $f = f(y)$. For all practical purposes, we can focus on the initial segment of the subsidy rate schedule, where cost of formal child care is below $f(y)$.

[24] Note that this does not imply redistribution from the childless to parents but rather that one can keep the current income tax for the childless fixed and partially reform the system for parents via child allowances.

## V. Quantitative Analysis

In this section, we present an illustrative simulation exercise based on a calibrated version of our framework. We focus on single mothers with at least one child aged below 6 and calibrate our model to match features of the US labor market. We then simulate our optimal policy results and compute the optimal child care subsidies and child allowances in the context studied in this paper. We also numerically analyze the optimal program allowing for income effects. This section delivers at least three useful pieces of information that complement the theoretical analysis performed so far. First, by bringing our model to the data, we provide an explicit real-world interpretation of the key variables of our model. Second, we show a few typical characteristics of the optimal allocation (such as the monotonicity of consumption and income) that cannot be shown analytically. Third, we give a sense of the order of magnitude of welfare gains related to the labor supply margin when child care considerations are taken into account within our Mirrleesian framework.

### A. *Summary of the Calibration Exercise*

We consider the following preferences for the household:

$$U(c,e) = \frac{c^{1-\alpha} - \kappa}{1-\alpha} - \frac{1}{\theta}\frac{e^{1+\gamma}}{1+\gamma},$$

where $\alpha$ represents the coefficient of relative risk aversion (CRRA), and $1/\gamma$ represents the wage elasticity of labor supply. In the baseline specification we consider quasilinear preferences (with $\alpha = \kappa = 0$). We also display the results for the case with log-consumption preferences (i.e., $\alpha = \kappa = 1$) as in Guner, Kaygusuz, and Ventura (2016).[25]

We fix $\alpha$ and calibrate the remaining parameters, which are: the preference parameters $\gamma$ and $\theta$, the probability of facing adverse labor market conditions $\pi(0)$, the distribution of market productivities $\pi(z)$ when $z > 0$, the child care needs normalized to one unit of effort (which corresponds to choosing a normalization for effort $e$), the cost of formal child care $\omega$, and the amount of net resources allocated to single mothers *M* under the current US system. Table 1 summarizes the parameter values and relevant moments used for the baseline calibration. Below, we summarize the rationales behind our parametric choices; an extensive description of the calibration process is reported in online Appendix B.

Following the literature on wage elasticity among women (Heckman and Macurdy 1980 and Blundell, Meghir, and Neves 1993), we set $\gamma = 1$ corresponding to an elasticity of 1. We also conduct sensitivity analysis by considering a more conservative elasticity of 0.5, corresponding to preference parameter $\gamma = 2$ (Chetty et al. 2011).

We specify the probability that agents suffer from adverse labor market conditions ($z_1 = 0$) as the proportion of involuntarily unemployed single mothers with children below 6 in the 2010 Current Population Survey (CPS), which is 11 percent.

[25] We also considered CRRA specification with $\alpha = 2$ (Bick 2016; Domeij and Klein 2013), which yielded similar qualitative results with minor quantitative differences.

Table 1—Baseline Parameterization

| Parameter | Value | Moments to match | Source |
|---|---|---|---|
| $\gamma$ | 1 | Wage elasticity of labor supply 1 | Heckman and Macurdy (1980)<br>Blundell, Meghir, and Neves (1993) |
| $\theta$ | See Table B3 | Average hours of work | CPS 2010 |
| $\pi(0)$ | 11 percent | Proportion involuntarily unemployed | CPS 2010 |
| $\pi(z)$ | See Figure B1 | Empirical distribution of wages | CPS 2010 |
| $e = 1$ | 24 hours | Hours of non-family child care per week | Rosenbaum and Ruhm (2007)<br>Laughlin (2010) |
| $\omega$ | \$5.10 | Average hourly child care cost | Child Care Aware of America (2012) |
| $M$ | See Table B3 | Net transfers to single mothers | Federal and SS Tax, EITC, DCTC, CCDF, TANF |

We interpret market productivity types $z > 0$ as hourly wages when agents are not voluntarily unemployed. Wages of non-working mothers are imputed using two-step selection correction methods à la Heckman. In the numerical exercise, we discretize the wage space into 50 wage centiles ranging between \$2.40 to \$32.21 such that we have 2 percent of mothers within each centile.

Child care needs are interpreted as the amount of child care time that can be substituted for paid care.[26] Given a normal working week of 40 hours and family provided care (from grandparents and other relatives) of 16 hours per week (Rosenbaum and Ruhm 2007; Laughlin 2010), mothers need to make alternative child care arrangements for the remaining 24 hours per week. We thus set one unit of effort as equal to 24 hours per week.[27] Average hourly cost of formal child care is calibrated as $\omega = \$5.10$ based on the 2010 US average cost (Child Care Aware of America 2012). As sensitivity check, we also consider a higher $\omega = \$6.40$, corresponding to \$10,000 a year for a child in full-time day care.

We calibrate the parameter $\theta$ such that, given the 2010 US tax and benefit system and the selection corrected empirical distribution of wages, the average hours of work predicted from an agent's private optimization problem match the average hours of work of single mothers with children aged below 6 in the CPS. Since the US welfare system tends to be generous toward single mothers, we also calibrate the amount of net transfers, $M$, already allocated to them in the US budget. In other words, we take as given the current generosity of the United States toward single mothers. Online Appendix Table B3 reports the calibrated values of $\theta$ and $M$ for the different parametric specifications of the model.

*Model Validity*.— We made several assumptions while calibrating our model. The main assumptions include the use of the wage distribution to model the distribution of market productivity and our model implication that people with employment

[26] The literature tends to incorporate paid child care as a fixed cost of work or by assuming that one hour of work requires one hour of paid child care (Blundell and Shephard 2012; Domeij and Klein 2013; Guner, Kaygusuz, and Ventura 2016). Our child care modeling is closer to Bick (2016) which also allows for the possibility of unpaid household child care as a genuinely separate margin from labor supply.

[27] The online Appendix also considers a specification with one unit of effort set equal to 34 hours per week, corresponding to a 50-hour working week minus 16 hours of family care.

opportunities and wages above the reservation wage would work. As an external validity check, we compute the employment elasticity implied by our model based on the proportion of women who would leave employment as a result of a 2 percent increase in the cost of formal child care. According to our calibrated baseline model, the employment elasticity with respect to the cost of child care is $-0.89$, which lies within the average range of child care price elasticities estimated in the literature.[28]

*Social Welfare Criterion*.—In line with the literature, we consider a concave and increasing social welfare function (Kleven, Kreiner, and Saez 2009; Piketty and Saez 2013), analogous to the one discussed in Section III:

$$\widehat{W}(c,e;\rho) \;:=\; \left(\sum_{i=1}^{N}\pi(z_i)\left[U(c_i,e_i)\right]^{\rho}\right)^{\frac{1}{\rho}}.$$

For our benchmark case with quasilinear utility, we consider a moderate desire for redistribution by assuming a logarithmic welfare function ($\rho = 0$). By taking the derivative with respect to $c$ at the optimal allocation, we recover the (here endogenous) social welfare weights: $\phi^*(z_i) := \frac{1}{V^*(z_i|z_i)} = \frac{1}{c^*(z_i) - \frac{1}{\theta}\frac{e^*(z_i)^{1+\gamma}}{1+\gamma}}$.

Although the function $\phi^*$ may not necessarily satisfy the sufficient condition stated in Assumption 3, a simple corollary to Lemma 1 suggests the following algorithm. Compute the optimal allocation of the relaxed problem with only (local and non-local) DICs. If the LDICs are satisfied with equality, all UICs are also satisfied and all properties of Lemma 1 hold. The numerical algorithm is described in online Appendix B.5.

We also consider the purely utilitarian social welfare criterion ($\rho = 1$) with log-consumption preferences.[29] This criterion maps into our main specification with quasilinear preferences when the (endogenous) social welfare weights are set to $\phi^*(z) := c^*(z)^{-\alpha}$. Moreover, we consider *Pareto improving reforms*, where we solve the government problem as before except that we impose the additional constraints that each agent type gets at least as much utility as under the actual US tax and benefit system.

## B. *Results*

*Constrained Optimal Allocations*.— We start with a quick look at the optimal allocation. The solid lines in Figure 3 illustrate results from the baseline case with quasilinear preferences and logarithmic social welfare. In the figure, we also report results for a higher cost of formal child care, lower labor supply elasticity, and log-consumption preferences with utilitarian social welfare. When performing the sensitivity analyses, we recalibrate $\theta$ and $M$, while keeping the other parameters at the baseline level.

[28] Employment elasticities with respect to cost of child care for US single mothers with children aged below 6 range from $-0.5$ (Connelly 1992) to $-1.29$ (Connelly and Kimmel 2003).

[29] Further specifications of preferences and social welfare criteria are considered in the online Appendix.

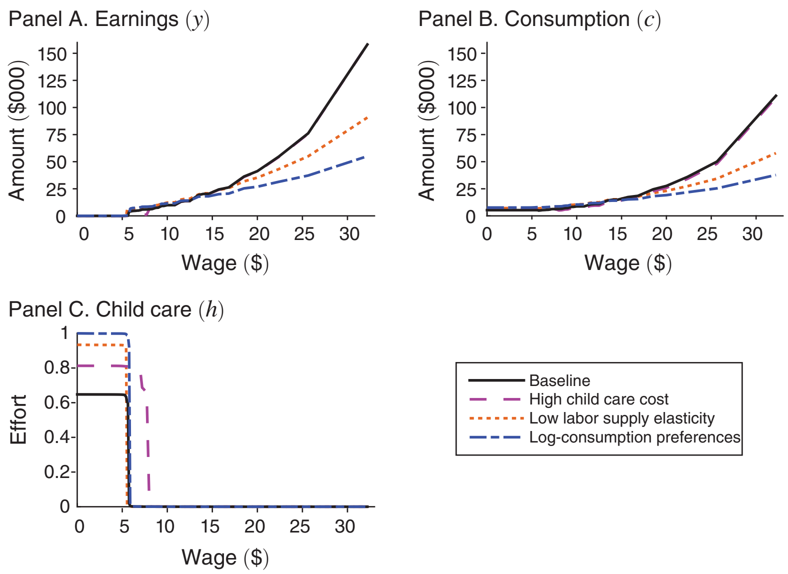


FIGURE 3. THE OPTIMAL ALLOCATION

*Notes:* The figure reports the optimal allocations under different specifications of our model. We display the baseline case with quasilinear preferences ($\alpha = 0$) and logarithmic social welfare ($\rho = 0$). We also present the specifications with high child care cost ($\omega =$ \$6.4), with low labor supply elasticity ($\gamma = 2$), and with log-consumption preferences ($\alpha = 1$) and the utilitarian social welfare ($\rho = 1$), with the remaining parameters set at the baseline level.

In all specifications, optimal earnings $(y)$ and consumption $(c)$ increase with market productivity $(z)$, as can be seen from panels A and B. Compared to the baseline, the flatter earnings profile in the specification with low labor supply elasticity indicates that it is harder to incentivize labor supply. It is also harder to incentivize the labor supply of higher productivity agents compared to lower productivity agents in the presence of income effects, as can be seen from the log-consumption specification.

As expected, unemployed mothers are pooled with the same consumption and household child care within a given specification, as can be seen from panel C. Working mothers tend not to engage in household child care $(h = 0)$, while a greater proportion of mothers tend to engage in household child care in the specification with high child care cost as can be seen from panel C.[30]

*Comparison with the United States*.—In Figure 4, we compare the optimal allocation with the one obtained under the US tax and benefit system and compute the implied welfare gains. For conciseness, we focus on the optimal allocations obtained from the baseline specification and the Pareto improving reform. The graphs in panels A and B are truncated at wage \$22.5 for emphasis (we omit the two highest wage

[30] Across all productivity levels, for the specifications reported in Figure 3, we have only four productivity types, $z =$ \$7.21 – \$7.71, in the specification with high formal child care cost ($\omega = 6.40$), where mothers work on the labor market and also provide household child care.

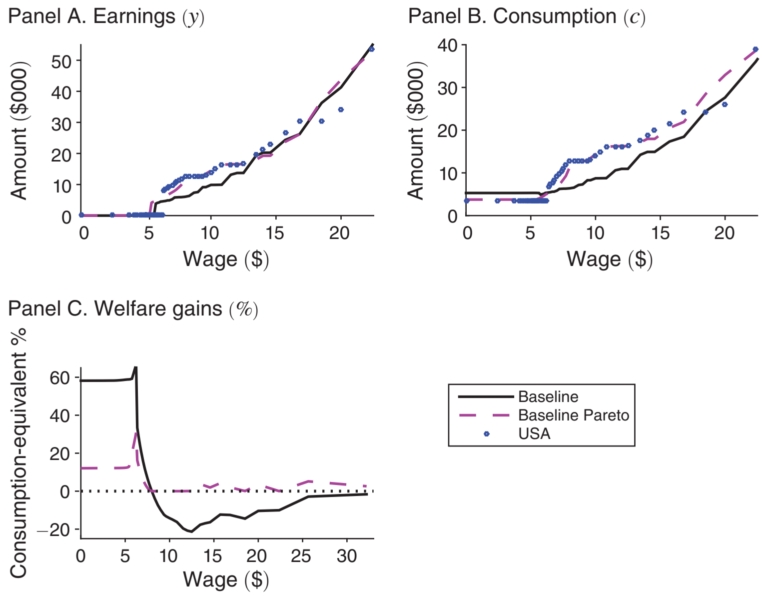


Figure 4. Optimal versus United States under 2010 US Tax and Benefit System

*Notes:* The solid lines represent the optimal allocations while the dotted lines represent the Pareto improving allocations, that is the optimal allocations with the additional constraints that every agent's welfare is at least as high as under the US status quo. The circles represent the simulated allocation obtained from solving the agent problem given the US status quo tax and subsidy scheme for mothers with two children below 13. The graphs in panels A and B are truncated at wage \$22.5 for emphasis (we omit the two highest wage bins of \$25.64 and \$32.21), while in panel C we display welfare gains for the whole range of productivities.

bins of \$25.64 and \$32.21). In the online Appendix, we report the welfare gains for other preference specifications and welfare criteria (see Table B5 and Figure B2).

The solid lines in panels A and B of Figure 4 illustrate the optimal allocations obtained from the baseline specification, while the dashed lines represent the optimal allocations from a Pareto improving reform. The circles represent the imputed allocations implied by the actual US tax and benefit system.[31] Panel C illustrates the welfare gains relative to the US status quo. The welfare gains for each $z_i$ are measured as the relative increase in consumption, $k(z_i)$, that generates the same welfare level as under target optimum: $U(c(z_i)(1+k(z_i)),e(z_i)) = U(c^*(z_i),e^*(z_i))$.

From panel A, the greater proportion of wage bins with zero earnings in the US case compared to the baseline and Pareto improving cases, indicate that in the optimal scheme, *a higher proportion of mothers work relative to the US system*. The incentive issues at hand suggest that properly designed child care subsidies may encourage labor supply, especially among low productivity types near the extensive margin of participation. We also see that the intensive margin incentives from

[31] More precisely, to get the allocation implied by the US system, we simulate the choice of single mothers taking into account federal and SS taxes, EITC, TANF, DCTC, and CCDF as per the agent's problem (B1) in the online Appendix.

the optimal allocations are milder for low productivity types but stronger for high productivity types.

From panel C, the optimal scheme generates higher welfare gains for low productivity mothers. This is because such mothers tend to have relatively low consumption under the US system as can be seen from panel B, while our social welfare criterion puts a greater weight on them. Furthermore, there are some sharp increases in welfare gains for low productivity mothers near the extensive margin of participation. Similar patterns emerge from the Pareto reform, although the welfare gains are toned down because the Pareto improving constraints limit the scope for redistribution from the high to the low types.

*Child Care Subsidies and Child Related Transfers*.—The optimal child care subsidy rates $s(y, f(y))$, optimal allowances $A(y)$, and net income taxes $T^a(y)$ are illustrated in Figure 5. For conciseness, we once again focus on the baseline specification and Pareto improving reform. In the bottom panels we report the full graphs while the top panels are zoomed for the range of earnings below \$42,000. Panel E illustrates the net income taxes faced by single childless individuals under the US tax system (i.e., Federal and SS Taxes, EITC, and unemployment benefits). The US net income taxes are computed as in Section I. As explained in the last part of Section IVB, we can keep actual net taxes faced by a single childless individual, $T^a(y)$, and find the corresponding optimal child allowances, $A(y) = T^a(y) - T(y)$ that deliver the second-best optimal allocation.

Panel A of Figure 5 reports the optimal child care subsidy rates for employed mothers according to the implementation in Proposition 4 and the US child care subsidy rates (i.e., DCTC and CCDF). We report the optimal subsidy rates corresponding to the first segment of the schedule, when $f \leq f(y)$:[32]

$$s(y,f) = \left(1 - \frac{v'\left(\frac{y}{z_N} + 1 - \frac{f(y)}{\omega}\right)}{\omega u'(c(y))}\right)^+ . \tag{9}$$

The optimal subsidy rates for the baseline specification start from 80 percent and decrease toward zero for earnings of \$30,000 or above. Our optimal subsidy scheme displays a similar qualitative feature to that of the United States: child care subsidy rates decline with earnings. Conversely, *the optimal subsidy rates decrease more steeply than the US ones*. This qualitative feature is robust across welfare criteria and parametric specifications (see online Appendix Table B6).[33]

[32] As noted in footnote 30, we have only 4 cases with $y > 0$ and $0 < f(y) < \omega$. The optimal subsidy rate associated with the second part of the kink when $f > f(y)$ was zero in all 4 cases; this shape is representable by a "cap" (at the optimal level of formal child care) such as the existing one in the United States. For all other cases, $f(y) = \omega$ so that only the rate in the first segment is relevant.

[33] Table B6 reports the optimal child care subsidy rates for the different specifications of the model as well as sensitivity analyses on the wage bins. The subsidy rates are quantitatively variable across specifications, with generally higher optimal subsidy rates for the specifications with high formal child care cost and with more inelastic labor supply, where the (non-local) child care deviation incentives may be higher. The subsidy schedule decreases even more steeply in the log-consumption preference specification. This is intuitive from equation (9): since consumption increases with earnings, the marginal utility of consumption in the denominator decreases and the subsidy rates thus decrease faster relative to the quasilinear utility case.

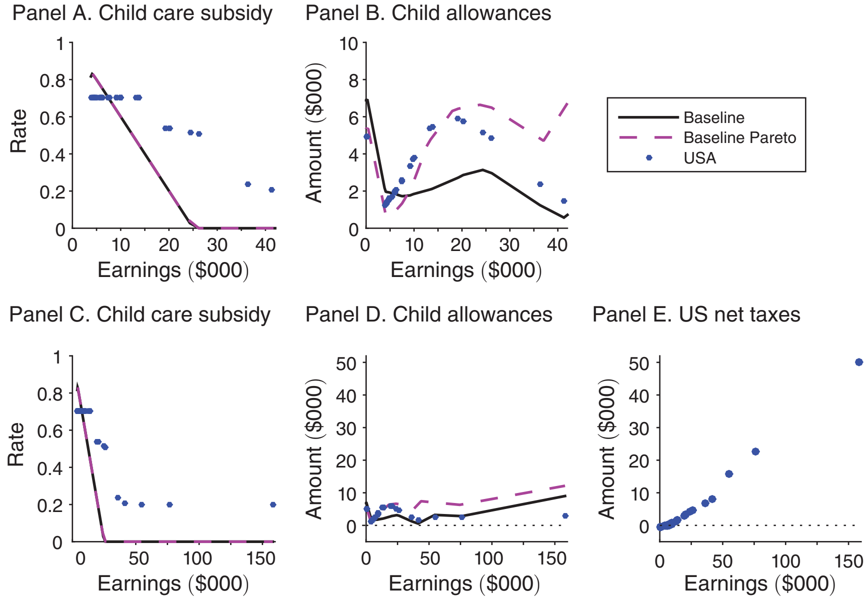


Figure 5. Second-Best Subsidies and Child Allowances versus Actual US System

*Notes:* The solid lines represent the optimal allocations, the dotted lines represent the Pareto improving allocations, and the circles denote US transfers. The lines in panel A depict the optimal child care subsidy rates, $s(y)$, while the circles denote the combined DCTC and CCDF subsidy rates. The lines in panel B depict the optimal child allowances, $A(y)$, while the circles capture income tax exemptions, EITC and TANF benefits faced by a single parent in the United States relative to a childless individual. Panels C and D report the full graphs for child care subsidy rates and child allowances. Panel E depicts the net income taxes, $T^a(y)$, faced by a single childless individual under the US tax system. The top panels are zoomed for the range of earnings below $42,000.

Panel B illustrates the optimal child allowances and the US child allowances (i.e., federal income tax exemptions, EITC, and TANF as computed in Section I). As argued above, the optimal program provides stronger participation incentives compared to the US scheme. The intensive margin incentives seem to be milder [stronger] for those earning below [above] the US median income.[34] In particular, for those earning below $20,000, the child care subsidy schedule is steeper while child care allowances tend to be flatter than those of the US scheme, especially at lower intermediate levels of earnings. Conversely, for those earning above $20,000, the child care subsidy schedule tends to be flatter while child allowances decrease less steeply relative to those in the US scheme.

Finally, note that in the case of the Pareto improving reform, the pattern of child allowances for low-income mothers closely mimic those of the United States. Recall that in this case, the optimal child care subsidy rates decrease more steeply than those in the United States and that participation is always enhanced in the optimal program (as can be seen from Figure 4). This suggests that a properly designed child care subsidy schedule may be an important key to incentivizing the labor supply of mothers.

[34] In 2010, the median income for a single mother was $24,370 (US Census, Table F-10: https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-families.html).

*Marginal Tax Rates*.—Online Appendix Table B7 reports the marginal income tax rates corresponding to equation (8), which takes into account the labor wedges and the marginal child care subsidies. For $f(y) = \omega$, the expression specifies to

$$T'(y) = 1 - \frac{v'\left(\frac{y}{z}\right)}{zu'(c(y))} + s'_y(y,\omega)\omega.$$

The labor supply wedges are always positive while the child care subsidies decrease with income, thereby leading to marginal tax rates that are lower than the labor wedges. *Despite the steeply decreasing subsidy rates for child care costs*, *optimal marginal tax rates tend to be positive at all levels of earnings*. Thus, even though our child care framework qualitatively allows for the possibility of negative marginal income taxes, our quantitative exercise does not indicate an explicit need for such taxes in most specifications.[35]

## VI. Extensions

As argued in the introduction, promoting mothers' labor supply is a major argument for child care subsidies. Whereas our parsimonious framework highlights some key ingredients in the design of the shape of child care subsidies, there are some important considerations that are not captured by our framework. In this section, we discuss some extensions that may be relevant for policy, and which can be directly analyzed within our framework.

First, we allow for income effects. As shown in Section V, income effects may modify the quantitative results but have no qualitative implications. Below, we discuss conditions under which our analysis fully extends to the case with income effects.

Second, we consider child care quality and human capital externalities. Economists and policymakers also argue that formal child care quality may serve the purpose of improving child outcomes, especially for children from poor socioeconomic backgrounds (Blau and Currie 2006; Cascio and Schanzenbach 2013; Cornelissen et al. 2018; Havnes and Mogstad 2015). To gain intuition on how such considerations might interact with our results, we solve our model allowing for the quality choice margin of externally acquired child care and imperfect substitutability between formal and household child care. In addition, we allow for positive externalities in children's human capital, accounting for a potential failure in the child care market. The key assumption is that the child human capital production function does not depend on unobservable market productivity. We show how all considerations made above for child care subsidies can carry over to this extension.

Lastly, we briefly discuss married mothers. Even without the child care margin, multi-agent households constitute a challenge in the Mirrlees optimal tax literature (Frankel 2014; Kleven, Kreiner, and Saez 2009; Laroque and Pavoni 2017). In our context, the problem becomes one of multidimensional screening and

[35] A couple of exceptions occur in the Pareto improving specifications, where we have negative marginal tax rates for earnings of $10,000, as can be seen from online Appendix Table B7.

multidimensional choice that would involve jointly designing couple taxation and child care subsidies. Below, we solve a special case where the primary earner's labor supply is fixed, which permits a full and direct application of our analytical results to married mothers such that all qualitative results hold.[36]

### A. *Income Effects*

The key analytical modification in the presence of a concave felicity of consumption function relates to the conditions required to show that the LDICs bind. In particular, if consumption is monotone in $z$, as is the case in our simulations from Section V, then we can show that the LDIC bind under the same Assumptions 1–3 as before. Thus, all proofs and characterizations results of Lemma 1 and Propositions 1–2 would carry over to the model with income effects. Assumption 3 would need to be strengthened if one cannot guarantee the monotonicity of consumption. For example, one can show that the LDIC bind under all concave utility functions assuming a Rawlsian social welfare function. A less restrictive assumption could allow for positive welfare weights on the unemployed and the lowest type employed. The efficiency case for child care subsidies would still hold for more general nonseparable preference specifications although we may possibly lose the sliding-scale pattern. Our simulation results seem to indicate that the sliding scale pattern tends to survive to this extension though.

### B. *Endogenous Child Quality and Human Capital Externalities*

We first introduce endogenous child care quality and allow for imperfect substitutability between formal and household child care. Let child human capital be given by $k = g\big(h, q(1-h)\big)$, where $g$ is concave and $q$ denotes formal child care quality. Formal child care costs are given by $f = \omega p(q)(1-h)$, where $p(\cdot)$ is an increasing and convex function of $q$. The government can observe both $f$ and $q$. Since $\omega$ and the function $p$ are known, the government effectively controls $h$, and hence $k$.

Let us define the utility of household of type $z_i$ declaring to be of type $z_j$ under the allocation $\left\{c(z_i), y(z_i), q(z_i), h(z_i)\right\}_{i=1}^{N}$ as

$$V(z_j|z_i) := x(z_j) - v\left(\frac{y(z_j)}{z_i} + h(z_j)\right),$$

where $x(z_j) := c(z_j) + g\Big(h(z_j), q(z_j)\big(1 - h(z_j)\big)\Big) - \omega p\big(q(z_j)\big)\big(1 - h(z_j)\big)^{+}$. This formulation permits the same analysis of the incentive constraints as in the main model. Under Assumptions 1–3, preferences display the double crossing property and the LDIC bind. This enables the characterization of the optimal allocation by

[36] For the sake of tractability, the literature on couples taxation has previously assumed discrete labor supply choices or fully assortative mating. To the best of our knowledge, the literature has yet to analytically solve an optimal multidimensional screening tax problem with continuous labor supply choices for both persons in a couple. Introducing child care choices on top of this would be quite a challenge.

only imposing the LDIC, which is a key result of the main model. The government problem hence becomes

$$\max_{(V,x,k,y,h,q)} \sum_i \pi(z_i)\phi(z_i)V(z_i|z_i), \tag{10}$$

subject to

$$\sum_{i=1}^{N} \pi(z_i)\Big[y(z_i) - x(z_i) + k(z_i) - \omega p\big(q(z_i)\big)\big(1 - h(z_i)\big)^{+}\Big] \geq M,$$

$$k(z_i) = g\Big(h(z_i), q(z_i)\big(1 - h(z_i)\big)\Big),$$

$$V(z_i|z_i) \geq V(z_i|z_j), \qquad \forall j < i, \quad i = 2,3,\ldots,N,$$

where $V$ and $x$ are as defined above and $k(z_i) := g\Big(h(z_i), q(z_i)\big(1 - h(z_i)\big)\Big)$. The resource constraint is written as though the planner internalized all child human capital production. This is possible because the private market productivity $z$ does not enter into the child human capital production function. This also implies that child care quality should not be distorted away from the private optimum, which solves $\omega p'\big(q^*(z_i)\big) = \partial k/\partial q = g_2'\Big(h^*(z_i), q^*(z_i)\big(1 - h^*(z_i)\big)\Big)$.

Following very similar lines of proof to the main model, we can show the following. Lemma 1 and Proposition 1 (ii) and (iii) hold as is, while the expression of the "net surplus" in (i) is replaced with $y^*(z_i) - x^*(z_i) + g\Big(h^*(z_i), q^*(z_i)\big(1 - h^*(z_i)\big)\Big) - \omega p\big(q^*(z_i)\big)\big(1 - h^*(z_i)\big)$. Analogous results to Proposition 2 hold as well. Whereas results (iii), (iv), and the labor wedges (5) in (v) hold exactly as in the proposition, results (i) and the child care wedges (6) in (v) hold with $\omega$ replaced by the new net return to household child care:

$$\omega p(q) + g_1'\big(h, q(1-h)\big) - q g_2'\big(h, q(1-h)\big),$$

where $g_1'$ and $g_2'$ indicate the derivatives of $g$ with respect to the first and second arguments, respectively. Result (ii) will be typically lost for $g$ strictly concave but if $g$ is linear, then $q^*$ is independent of $z$ and (ii) holds with condition $z \leq \omega$ replaced by $z \leq \omega p(q^*) + g_1' - q^* g_2'$.

Once the qualitative properties of the wedges have been determined, the implementation results of Propositions 3 and 4 carry over, with $\omega$ replaced by the new net return to $h$. A kink in the child care subsidy schedule is still necessary since the private marginal cost of household child care, $v'\big((y/z) + h\big)$, differs across $z$ types, for the same $(y, h)$.

*Human capital externalities* can be introduced by modifying the government objective as follows (while keeping all constraints as described in problem (10)):

$$\max_{(V,x,k,y,h,q)} \sum_i \pi(z_i)\phi(z_i)V(z_i|z_i) + \sum_i \pi(z_i)\zeta(z_i)k(z_i).$$

In the above, $\zeta(z_i) \geq 0$ is decreasing in $z_i$. This specification includes both positive externalities generated by child human capital and the potential desire to redistribute child human capital from low- to high-income individuals. The optimality conditions for $x$ and $y$ in this extension are identical to the case just discussed. The optimality condition for $q$ simply indicates that the optimal quality of formal child care is higher than that chosen privately by the household as long as $h^*(z) < 1$. Human capital externalities are the only reason for distorting $q$ and such considerations do not affect the labor wedge.

*The presence of human capital externalities, however*, *may affect the wedge on h.* On one hand, since human capital externalities do not interact with the incentive constraints, the incentive component brings $h$ below the privately optimal quantity as discussed above, thereby calling for a subsidy on formal child care. On the other hand, the externality effect calls for a larger child human capital, $k$. Note that the marginal effect of $h$ on $k$ equals $\partial k^*(z^i)/\partial h = g_1'\left(h^*(z), q^*(z)\left(1 - h^*(z)\right)\right) - q^*(z)\, g_2'\left(h^*(z), q^*(z)\left(1 - h^*(z)\right)\right)$. If $\partial k^*(z^i)/\partial h > 0$, the downward distortion on $h$ is mitigated since household child care is more productive in generating child human capital. Conversely, if $\partial k^*(z^i)/\partial h < 0$, the downward distortion on $h$ is reinforced since formal child care is more productive. For simplicity, suppose that $g$ is linear, such that $g_1' = g_2' = 1$. Then, the sign of $\partial k^*(z^i)/\partial h$ only depends on $q^*(z_i)$. Under the assumption that $\zeta(z_i)$ decreases with $z_i$, $q^*(z_i)$ would decrease in $z_i$.[37] In this case, high market productivity mothers tend to be subsidized less than low productivity types (or may be even taxed), thereby reinforcing the sliding scale pattern of child care subsidies.

The presence of a child quality margin requires a new policy instrument for implementing the constrained efficient allocation. One possibility could be licensing, where child care centers are subject to health and safety inspections.[38] In this paper we do not have the ambition to provide a full implementation for the allocation with both child care time and quality margins. Bastani, Blomquist, and Micheletto (2017) explores the desirability of a refundable tax credit, tax deductability, and public provision of child care, where they focus on motivating parents to choose higher quality child care. We believe that child quality considerations are a very relevant and complex topic, that deserves a separate investigation.

### C. *Married Mothers*

As discussed above (and in footnote 36), even without considering the child care margin, the literature on optimal couple taxation relies on strong assumptions to maintain tractability. In this section, we propose a model of the household that

[37] When $g_1' = g_2' = 1$, the optimality condition for $q^*(z_i)$ becomes $\omega p'(q^*(z_i)) = 1 + \zeta(z_i)$. The result is hence implied by $p(\cdot)$ convex.

[38] For instance, the 2014 Child Care and Development Block Grant Act provides a new emphasis on the importance of providing high-quality child care and established annual monitoring for CCDF providers as well as a pre-licensure inspection for licensed CCDF providers. It also established comprehensive background checks for child care staff members of all licensed and CCDF-eligible providers (Matthews et al. 2015; US Department of Health and Human Services 2016).

permits the same qualitative characterization of the allocation and wedges as in our main model.

Let utility of a household under an allocation $\{c, y, h^1, h^2\}$ be given by

$$c - v\left(\frac{y}{z} + h^1\right) - v\left(\bar{l} + h^2\right),$$

where $h^1$ denotes primary earner's child care and $h^2$ denotes secondary earner's child care.

The key simplifying assumption is that *the primary earner works for a fixed amount* of time regardless of his productivity. The assumption of fixed hour occupations is perhaps less strong for highly regulated labor markets such as those in most European countries compared to those in the United States. We also assume that, although the government can observe total household expenditure on formal child care, it is unable to detect which partner provides household child care. We now show that the government problem can be analyzed using similar instruments as before and generates the same qualitative conclusions on formal child care subsidies.

For each $y/z$ and $h$, we can define the following function:

$$\hat{B}\left(\frac{y}{z}, h\right) := \min_{h^1, h^2} v\left(\frac{y}{z} + h^1\right) + v\left(\bar{l} + h^2\right),$$

subject to

$$h^1 \geq 0, \quad h^2 \geq 0; \quad h^1 + h^2 = h.$$

For ease of exposition, assume that the primary earner's labor supply is always higher than that of the secondary earner: $\bar{l} \geq y(z_i)/z_i$ for all $i$. In this case, $h^1 > 0$ unless $h = 0$ and hence,[39]

$$\hat{B}\left(\frac{y}{z}, h\right) = B\left(\frac{y}{z} + h\right) = \begin{cases} v\left(\frac{y}{z} + h\right) + v\left(\bar{l}\right), & \text{if } \frac{y}{z} + h \leq \bar{l}; \\ 2v\left(\frac{1}{2}\left(\frac{y}{z} + h + \bar{l}\right)\right), & \text{if } \frac{y}{z} + h > \bar{l}. \end{cases}$$

The function $B$ is continuous and differentiable in $(y/z) + h$. Importantly, $B$ inherits the double crossing property as long as $v$ satisfies Assumption 1. We can now define the new utility of household of type $z_i$ declaring $z_j$ as

$$V(z_j | z_i) := c(z_j) - B\left(\frac{y(z_j)}{z_i} + h(z_j)\right),$$

[39] The case where $\bar{l} < y(z_i)/z_i$ is analogous and would imply that $h^2 > 0$ whenever $h > 0$.

where $h(z_j)$ indicates *total household child care* mandated to a household of type $z_j$.[40] The government solves a similar problem to the baseline model:

$$\max_{(c,y,h)} \sum_i \pi(z_i)\phi(z_i)V(z_i|z_i),$$

subject to

$$\sum_i \pi(z_i)\left[y(z_i) + \bar{z}_i\bar{l} - \omega + \omega h(z_i) - c(z_i)\right] \geq M,$$

$$V(z_i|z_i) \geq V(z_i|z_j), \qquad \forall j < i, \quad i = 2,3,\ldots,N,$$

with $V$ as defined above and $\bar{z}_i := \sum_k \pi^k(z_i)\, z_k$ representing the average productivity of the primary earners married to secondary earners with productivity $z_i$. Note that the component $\bar{z}_i\bar{l}$ simply adds a constant to the government budget. The analysis of the incentive compatibility constraints and the first-order conditions is the same as in the main model, so all results in Lemma 1 and Propositions 1–2 hold. The implementation still involves a kink in the child care subsidy schedule, and a weaker form of sliding scale pattern holds as a function of household income as long as the society displays positive assortative mating.[41]

## VII. Conclusion

We provide an efficiency case for child care subsidies in an optimal heterogeneous agent welfare design problem. We show that optimal child care subsidy rates follow a sliding scale and that the coverage rates should contemplate a kink. These features are in line with the qualitative features of the existing US scheme. Although child care subsidies incentivize higher work participation, the sliding scale pattern may have disincentive effects on labor supply. To counterbalance such disincentives, marginal labor income taxes are set at lower rates than the labor wedges.

Our simulation exercise suggests that the optimal program may provide stronger work participation incentives but milder intensive margin incentives compared to the US scheme, especially for lower income earners. In all specifications considered, optimal child care subsidy rates decrease with income more steeply than those in the current US scheme. The simulations further suggest that there may be scope for nontrivial Pareto improving welfare gains from properly designed child care subsidies.

A main achievement of this paper is to formulate a flexible model of the design of child care subsidies and to derive a number of properties of the optimal scheme. This might serve to unify a body of literature and to suggest some new results. Despite the complexity of the resulting screening problem, the solution found is remarkably simple and can be explained intuitively. The theory that emerges has a non-local nature. We also discuss how our qualitative results are robust to the presence of

[40] In the terminology of the paper, the government recommends formal child care $f(z_j) = \omega\left(1 - h(z_j)\right)$.

[41] Along a given kink segment, the optimal child care subsidy rates vary with income generated by the secondary earner alone, $y(z_i)$. If $\bar{z}_i$ increases with $i$, then child care subsidies will *on average* decrease with household income.

income effects, endogenous child quality, human capital externalities, and married mothers, under some conditions.

The model has a number of limitations. First, the analysis assumes that individuals differ only with respect to their labor market productivities. Whereas this may not seem particularly realistic, we do not believe that this nullifies the value of our analysis. The considerations that we have uncovered are likely to be important in more general analyses. Second, our assumptions about how the labor market operates are somewhat restrictive. For example, we have abstracted from general equilibrium effects. We do, however, share this limitation with most of the literature on optimal income taxation.

We have also abstracted from dynamic considerations in our model. Following standard arguments, it can be shown that if market productivity does not change over time, then the optimal dynamic allocation is a repetition of the static one characterized in this paper (Baron and Besanko 1984; Fudenberg and Tirole 1991). With strictly concave preferences in consumption and stochastic $z$, matters become more complicated and the taxation of savings becomes relevant for redistribution (Ábrahám, Koehne, and Pavoni 2016; Ho 2019; Kocherlakota 2010). Finally, taking into account the parents' potential human capital accumulation from encouraging participation may also be very valuable. For example, Blundell et al. (2016) finds that single mothers with basic education earn little returns to experience while those with higher education have significant returns to experience. The fact that the potential gains from incentivizing participation are unequal across skill groups might have nontrivial implications for the optimal pattern of child care subsidies. We leave such interesting considerations for future research.

## REFERENCES

**Ábrahám, Árpád, Sebastian Koehne, and Nicola Pavoni.** 2016. "Optimal Income Taxation When Asset Taxation Is Limited." *Journal of Public Economics* 136: 14–29.

**Administration for Children and Families.** 2012. *Child Care and Development Fund: Report to Congress FY 2008–2011*. Washington, DC: Administration for Children and Families.

**Bainbridge, Jay, Marcia K. Meyers, and Jane Waldfogel.** 2003. "Child Care Policy Reform and the Employment of Single Mothers." *Social Science Quarterly* 84 (4): 771–91.

**Barnett, W. Stephen.** 1993. "New Wine in Old Bottles: Increasing the Coherence of Early Childhood Care and Education Policy." *Early Childhood Research Quarterly* 8 (4): 519–58.

**Baron, David P., and David Besanko.** 1984. "Regulation and Information in a Continuing Relationship." *Information Economics and Policy* 1 (3): 267–302.

**Bastani, Spencer, Sören Blomquist, and Luca Micheletto.** 2017. "Child Care Subsidies, Quality, and Optimal Income Taxation." CESifo Working Paper 6533.

**Beaudry, Paul, Charles Blackorby, and Dezsö Szalay.** 2009. "Taxes and Employment Subsidies in Optimal Redistribution Programs." *American Economic Review* 99 (1): 216–42.

**Besley, Timothy, and Stephen Coate.** 1995. "The Design of Income Maintenance Programmes." *Review of Economic Studies* 62 (2): 187–221.

**Bick, Alexander.** 2016. "The Quantitative Role of Child Care for Female Labor Force Participation and Fertility." *Journal of the European Economic Association* 14 (3): 639–68.

**Blau, David M.** 2003. "Child Care Subsidy Programs." In *Means-Tested Transfer Programs in the United States*, edited by Robert A. Moffitt, 443–516. Chicago: University of Chicago Press.

**Blau, David, and Janet Currie.** 2006. "Pre-School, Day Care, and After-School Care: Who's Minding the Kids?" In *Handbook of the Economics of Education*, Vol. 2, edited by Eric Hanushek and Finis Welch, 1163–1278. Amsterdam: Elsevier.

**Blau, David, and Erdal Tekin.** 2007. "The Determinants and Consequences of Child Care Subsidies for Single Mothers in the USA." *Journal of Population Economics* 20 (4): 719–41.

**Blundell, Richard, Monica Costa Dias, Costas Meghir, and Jonathan Shaw.** 2016. "Female Labor Supply, Human Capital and Welfare Reform." *Econometrica* 84 (5): 1705–53.

**Blundell, Richard, Costas Meghir, and Pedro Neves.** 1993. "Labour Supply and Intertemporal Substitution." *Journal of Econometrics* 59 (1–2): 137–60.
**Blundell, Richard, and Andrew Shephard.** 2012. "Employment, Hours of Work and the Optimal Taxation of Low-Income Families." *Review of Economic Studies* 79 (2): 481–510.
**Brett, Craig.** 1998. "Who Should Be on Workfare? The Use of Work as Part of an Optimal Tax Mix Requirements." *Oxford Economic Papers* 50 (4): 607–22.
**Cascio, Elizabeth U., and Diane Whitmore Schanzenbach.** 2013. "The Impacts of Expanding Access to High-Quality Preschool Education." *Brookings Papers on Economic Activity* (Fall): 127–92.
**Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber.** 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review* 101 (3): 471–75.
**Child Care Aware of America.** 2012. *Parents and the High Cost of Child Care.* Arlington, VA: Child Care Aware of America.
**Choné, Philippe, and Guy Laroque.** 2011. "Optimal Taxation in the Extensive Model." *Journal of Economic Theory* 146 (2): 425–53.
**Connelly, Rachel.** 1992. "The Effect of Child Care Costs on Married Women's Labor Force Participation." *Review of Economics and Statistics* 74 (1): 83–90.
**Connelly, Rachel, and Jean Kimmel.** 2003. "Marital Status and Full–Time/Part–Time Work Status in Child Care Choices." *Applied Economics* 35 (7): 761–77.
**Cornelissen, Thomas, Christian Dustmann, Anna Raute, and Uta Schönberg.** 2018. "Who Benefits from Universal Child Care? Estimating Marginal Returns to Early Child Care Attendance." *Journal of Political Economy* 126 (6): 2356–409.
**Domeij, David, and Paul Klein.** 2013. "Should Day Care Be Subsidized?" *Review of Economic Studies* 80 (2): 568–95.
**European Council.** 2002. *Presidency Conclusions.* Brussels: European Council.
**Fang, Hanming, and Michael P. Keane.** 2004. "Assessing the Impact of Welfare Reform on Single Mothers." *Brookings Papers on Economic Activity* (1): 1–95.
**Frankel, Alexander.** 2014. "Taxation of Couples under Assortative Mating." *American Economic Journal: Economic Policy* 6 (3): 155–77.
**Fudenberg, Drew, and Jean Tirole.** 1991. *Game Theory.* Cambridge, MA: MIT Press.
**Gabe, Thomas, Bob Lyke, and Karen Spar.** 2001. *Child Care Subsidies: Federal Grants and Tax Benefits for Working Families.* Washington, DC: Congressional Research Service, the Library of Congress.
**Golosov, Mikhail, Maxim Troshkin, Aleh Tsyvinski, and Matthew Weinzierl.** 2013. "Preference Heterogeneity and Optimal Capital Income Taxation." *Journal of Public Economics* 97 (1): 160–75.
**Golosov, Mikhail, and Aleh Tsyvinski.** 2006. "Designing Optimal Disability Insurance: A Case for Asset Testing." *Journal of Political Economy* 114 (2): 257–79.
**Grogger, Jeffrey, and Lynn A. Karoly.** 2005. *Welfare Reform: Effects of a Decade of Change.* Cambridge, MA: Harvard University Press.
**Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura.** 2016. "Child-Related Transfers, Household Labor Supply and Welfare." Human Capital and Economic Opportunity Working Paper 2017-001.
**Havnes, Tarjei, and Magne Mogstad.** 2015. "Is Universal Child Care Leveling the Playing Field?" *Journal of Public Economics* 127: 100–14.
**Heckman, James J., and Thomas E. Macurdy.** 1980. "Life Cycle Model of Female Labour Supply." *Review of Economic Studies* 47 (1): 47–74.
**Ho, Christine.** 2013. "Welfare Reform and At-Risk Mothers' Labour Supply." *Economic Papers* 32 (2): 249–57.
**Ho, Christine.** 2015. "Welfare-to-Work Reform and Intergenerational Support: Grandmothers' Response to the 1996 PRWORA." *Journal of Marriage and Family* 77 (2): 407–23.
**Ho, Christine.** 2019. "Optimal Social Insurance with Informal Child Care." *B.E. Journal of Economic Analysis & Policy* 19 (2): Article 24.
**Kimmel, Jean.** 1995. "The Effectiveness of Child-Care Subsidies in Encouraging the Welfare-to-Work Transition of Low-Income Single Mothers." *American Economic Review* 85 (2): 271–75.
**Kleven, Henrik Jacobsen, Claus Thustrup Kreiner, and Emmanuel Saez.** 2009. "The Optimal Income Taxation of Couples." *Econometrica* 77 (2): 537–60.
**Kleven, Henrik Jacobsen, Wolfram F. Richter, and Peter Birch Sørensen.** 2000. "Optimal Taxation with Household Production." *Oxford Economic Papers* 52 (3): 584–94.
**Kocherlakota, Narayana R.** 2004. "Wedges and Taxes." *American Economic Review* 94 (2): 109–13.
**Kocherlakota, Narayana R.** 2010. *The New Dynamic Public Finance.* Princeton, NJ: Princeton University Press.

**Koehne, Sebastian, and Dominik Sachs.** 2016. "Pareto-Efficient Tax Breaks." CESifo Working Paper 6147.

**Laroque, Guy, and Nicola Pavoni.** 2017. "Optimal Taxation in Occupational Choice Models: An Application to the Work Decisions of Couples." IFS Working Paper W17/07.

**Laughlin, Lynda.** 2010. *Who's Minding the Kids? Child Care Arrangements.* Washington, DC: US Department of Commerce.

**Matthews, Hannah, Karen Schulman, Julie Vogtman, Christine Johnson-Staub, and Helen Blank.** 2015. *Implementing the Child Care and Development Block Grant Reauthorization: A Guide for States.* Washington, DC: National Women's Law enter.

**Matthews, Steven, and John Moore.** 1987. "Monopoly Provision of Quality and Warranties: An Exploration in the Theory of Multidimensional Screening." *Econometrica* 55 (2): 441–67.

**Minton, Sarah, Christin Durham, Erika Huber, and Linda Giannarelli.** 2012. *The CCDF Policies Database Book of Tables: Key Cross-State Variations in CCDF Policies as of October 1, 2011.* Washington, DC: Office of Planning, Research and Evaluation.

**Office of Child Care.** 2017. "FY 2017 CCDF Allocations (Including Redistributed Funds)." Office of Child Care. https://www.acf.hhs.gov/occ/resource/fy-2017-ccdf-allocations-including-redistributed-funds (accessed September 26, 2019).

**Organisation for Economic Co-operation and Development (OECD).** 2016. *Who Uses Childcare? Background Brief on Inequalities in the Use of Formal Early Childhood Education and Care (ECEC) among Very Young Children.* Paris: OECD.

**Piketty, Thomas, and Emmanuel Saez.** 2013. "Optimal Labor Income Taxation." In *Handbook of Public Economics,* Vol. 5, edited by Alan J. Auerbach et al., 391–474. Amsterdam: Elsevier.

**Rosenbaum, Dan T., and Christopher J. Ruhm.** 2007. "Family Expenditures on Child Care." *B.E. Journal of Economic Analysis & Policy* 7 (1): Article 52.

**Saez, Emmanuel.** 2002a. "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses." *Quarterly Journal of Economics* 117 (3): 1039–73.

**Saez, Emmanuel.** 2002b. "The Desirability of Commodity Taxation under Nonlinear Income Taxation and Heterogeneous Tastes." *Journal of Public Economics* 83 (2): 217–30.

**Tax Policy Center.** 2010a. "Quick Facts: Child and Dependent Care Tax Credit."

**Tax Policy Center.** 2010b. "State Individual Income Taxes 2010." Tax Policy Center. https://www.taxpolicycenter.org/statistics/state-individual-income-tax-rates-2000-2019 (accessed September 26, 2019).

**Tekin, Erdal.** 2005. "Child Care Subsidy Receipt, Employment, and Child Care Choices of Single Mothers." *Economics Letters* 89 (1): 1–6.

**US Department of Health and Human Services.** 2009. "Child Care Eligibility and Receipt for Fiscal Year 2009." ASPE Issue Brief.

**US Department of Health and Human Services.** 2011. *Characteristics and Financial Circumstances of TANF Recipients, Fiscal Year 2010.* Washington, DC: US Department of Health and Human Services.

**US Department of Health and Human Services.** 2016. *Overview of 2016 Child Care and Development Fund Final Rule.* Washington, DC: US Department of Health and Human Services.

**Werning, Iván.** 2011. "Nonlinear Capital Taxation." Unpublished.

**White House Communications Agency.** 1996. "President Clinton's Press Statement on Welfare Reform."