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Indescribability and asymmetric information at the contracting stage

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Abstract

Maskin and Tirole [Maskin, E., Tirole, J., 1999. Unforeseen contingencies and incomplete contracts. Review of Economic Studies, 66, 83-114] show that indescribability does not matter for contractual incompleteness when there is symmetric information both at the contracting stage and at the trading stage. Following their setup, I show that with *asymmetric* information at both stages, indescribability can matter. © 2007 Elsevier B.V. All rights reserved.

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JEL classification: C72; D78; D82

1. Introduction

There is now a vast literature on incomplete contracts (see Tirole (1999) for a survey), which has successfully answered, among other things, the meaning of ownership and the nature and financial structure of the firm. In addressing these issues, the literature has focused only on *incomplete* contracts in the sense that even if the agents would like to add contingent clauses, they are prevented from doing so because states are too expensive to *describe* at the contracting stage. Maskin and Tirole (1999) and Maskin (2002), however, show that indescribability is not binding as a constraint in generating contractual incompleteness. Their results rely on two assumptions always invoked by the literature: (1) there is *symmetric* information both at the contracting stage and at the trading stage and (2) agents can probabilistically forecast their possible future *payoffs*.

The basic idea of Maskin and Tirole (1999) is given as follows: if agents have trouble describing *physical* contingencies, they can write contracts that ex ante specify only the possible payoff contingencies. Then, later on, when the state of

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the world is realized, they can fill in the physical details. It follows that the only serious complication is incentive compatibility: will it be in each agent's interest to specify these details truthfully? But implementation theory with symmetric information can be used to ensure that truthful specification occurs in equilibrium. Here, indescribability is simply considered a constraint that filling in the "physical" details is impossible before the trading stage. Following Maskin and Tirole's (1999) setup, I investigate under what conditions indescribability matters. I propose asymmetric information at the contracting stage as a reason why indescribability can matter.¹ When there is asymmetric information at the contracting stage, indescribability can simply be a constraint that the agents are unable to elicit their private information about physical details of the states at the contracting stage and they find it more costly to do so at the trading stage. With this in mind, this paper's main result shows by a simple model that there is a set of implementable contracts that always induce the ex ante efficient trade when the states are describable, while only the no-trade contract can be implemented when the states are indescribable.

¹ Tirole (1999) indeed speculated this possibility in Section 4.1, although my answer may not be what he had in mind.

2. Simple model

This paper will focus on a simple bilateral contracting model. $^{\rm 2}$

2.1. The underlying economy

Consider a bilateral contracting environment where a buyer (*B*) and a seller (*S*) can trade one (but no more than one) of two widgets indexed by ℓ and *r* that the seller will produce and the buyer will use to produce the final good.³ There are five dates in this contractual relationship: At date 0, some partial information about the state is revealed. At date 1, there is asymmetric information in which the buyer makes an unobservable investment. At date 3, an additional information about the state is realized. At date 4, the agents implement the trade specified by the contract.

Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ be the set of states of the world. The value of the buyer and the cost of the seller (in terms of money) that they assign to widget $k = \ell$, *r* in state ω are denoted $v_B^k(\omega)$ and $c_S^k(\omega)$, respectively, and suppose

$$v_B^{\ell}(\omega_1) = v_B^{\ell}(\omega_3) = 0, \quad v_B^{\ell}(\omega_2) = 15 \text{ and } v_B^{\ell}(\omega_4) = 0.$$

$$v_B^{r}(\omega_1) = v_B^{r}(\omega_3) = 0, \quad v_B^{r}(\omega_2) = 0 \text{ and } v_B^{r}(\omega_4) = 15.$$

$$c_S^{\ell}(\omega_1) = c_S^{\ell}(\omega_3) = 1, \quad c_S^{\ell}(\omega_2) = 10, \text{ and } c_S^{\ell}(\omega_4) = 10.$$

$$c_S^{r}(\omega_1) = c_S^{r}(\omega_3) = 1, \quad c_S^{r}(\omega_2) = 10, \text{ and } c_S^{r}(\omega_4) = 10.$$

2.2. Information structure

Each agent faces a different information structure, depending on whether the contracting stage (date 2) or the trading stage (date 4) is considered. Each agent's private information about physical details of the states is given as a partition of Ω .

Denote by Ψ_i^c a partition of Ω of agent i (i=B,S) at the contracting stage, where ψ_i^c is a generic element of Ψ_i^c which is agent i's *type* at the contracting stage. Similarly, denote by Ψ_i^t a partition of Ω of agent i at the trading stage, where ψ_i^t is a generic element of Ψ_i^c which is agent i's type at the trading stage. Define $\Psi^c \equiv \Psi_B^c \times \Psi_S^c$ and $\Psi^t \equiv \Psi_B^t \times \Psi_S^t$ where $\psi^c(\psi^t)$ is a generic element of $\Psi^c(\Psi^t)$.

The information structure at the contracting stage is summarized below:

$$\begin{split} \Psi_B^{\mathrm{c}} &= \{\omega_1, \omega_2, \omega_3, \omega_4\} \\ \Psi_S^{\mathrm{c}} &= \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}. \end{split}$$

At the contracting stage, the buyer is completely uninformed of the state, while the seller knows exactly which widget should be traded: widget ℓ should be traded when the seller's type is $\{\omega_1, \omega_2\}$, while widget *r* should be traded when the seller's type is $\{\omega_3, \omega_4\}$.⁴ The literature has instead assumed that no information is revealed to anyone at the contracting stage, i.e., $\Psi_B^c = \Psi_S^c = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. This is called symmetric information at the contracting stage. The information structure at the trading stage is summarized as follows:

$$\begin{aligned} \Psi_B^{t} &= \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\} \\ \Psi_S^{t} &= \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}\}. \end{aligned}$$

At the trading stage, the seller is completely informed of the state, while the buyer only knows either $\{\omega_1, \omega_2\}$ or $\{\omega_3, \omega_4\}$, i.e., which widget should be traded.

2.3. Complete contracts

The set of feasible allocations A^{ω} at state $\omega \in \Omega$ is defined as follows:

$$A^{\omega} = X^{\omega} \times Y^{\omega}$$

where X^{ω} and Y^{ω} will be defined momentarily. X^{ω} is defined as:

$$X^{\omega} = \left\{ (x_{\ell}(\omega), x_r(\omega)) \in \{0, 1\}^2 | x_{\ell}(\omega) + x_r(\omega) = 0 \text{ or } 1 \right\}$$

where for each $k = \ell$, *r* and each $\omega \in \Omega$, $x_k(\omega) = 0$ stands for the case where widget *k* is not produced at state ω and $x_k(\omega) = 1$ stands for the case where widget k is produced and delivered to the buyer at state ω . Furthermore, $x_{\ell}(\omega) + x_r(\omega) = 0$ means that no widget is produced at state ω and $x_{\ell}(\omega) + x_r(\omega) = 1$ respects the constraint that only one type of widgets can be produced and traded at state ω . Y^{ω} is defined as:

$$Y^{\omega} = \{y(\omega) \in \mathbb{R}_+\}$$

where $y(\omega)$ denotes the monetary transfer from the buyer to the seller at state ω . Define $A \equiv \bigcup_{\omega \in \Omega} A^{\omega}$. A *complete contract* is a function

$$\xi : \Omega \to A$$
 such that $\xi(\omega) \in A^{\omega} \forall \omega \in \Omega$.

Note that any complete contract is required to be deterministic. This turns out to be a non-trivial restriction on the set of complete contracts. With the set of notations provided above, I can define each agent's state-dependent ex post utility corresponding to a complete contract $\xi(\cdot) = (x_{\ell}(\cdot), x_r(\cdot), y(\cdot))$ such that for each $\omega \in \Omega$,

$$u_B(\xi(\omega);\omega) = \begin{cases} 15 - y(\omega) & \text{if either } \omega = \omega_2 \text{ and } x_\ell(\omega_2) = 1 \\ & \text{or } \omega = \omega_4 \text{ and } x_r(\omega_4) = 1 \\ -y(\omega) & \text{otherwise} \end{cases}$$

 $^{^2\,}$ This paper's model is quite consistent with the general model of Kunimoto (2006) and Maskin and Tirole (1999).

³ This environment is very similar to the one of Hart and Moore (1999) and Segal (1999), although I do not discuss complexity of the environment at all.

⁴ At the contracting stage, only the seller knows which widget should be traded in this paper (asymmetric information), while both agents commonly believe that each widget is equally likely to be the right widget in Hart and Moore (1999) and Segal (1999) (symmetric information).

$$u_{S}(\xi(\omega);\omega) = \begin{cases} y(\omega) - 10 & \text{if } x_{\ell}(\omega) + x_{r}(\omega) = 1 \text{ and} \\ & \text{either } \omega = \omega_{2} \text{ or } \omega_{4} \\ y(\omega) & \text{if } x_{\ell}(\omega) + x_{r}(\omega) = 0 \\ y(\omega) - 1 & \text{otherwise} \end{cases}$$

2.4. Investment

At date 0, Nature determines agents' type at the contracting stage by the probability measure μ over Ψ^c with the following property:

$$\mu(\{\omega_1,\omega_2\}) = 1/2$$
 and $\mu(\{\omega_3,\omega_4\}) = 1/2$.

With private information at the contracting stage, only the buyer makes an investment β in human capital which increases the probability that the "right" widget entails high value. There are only two levels of investment: either $\beta=1$ (investment) or $\beta=0$ (no investment). The cost of investment $c(\beta)$ (in terms of money) is given as

$$c(\beta) = \begin{cases} 1 \text{ if } \beta = 1\\ 0 \text{ if } \beta = 0. \end{cases}$$

It is a common knowledge that the likelihood and the way these states of the world depend upon private information at the contracting stage and the buyer's investment:

$$\begin{aligned} &P(\omega_1 | \{\omega_1, \omega_2\}, \beta = 1) = P(\omega_3 | \{\omega_3, \omega_4\}, \beta = 1) = 1/3 \\ &P(\omega_2 | \{\omega_1, \omega_2\}, \beta = 1) = P(\omega_4 | \{\omega_3, \omega_4\}, \beta = 1) = 2/3 \\ &P(\omega_1 | \{\omega_1, \omega_2\}, \beta = 0) = P(\omega_3 | \{\omega_3, \omega_4\}, \beta = 0) = 1/2 \\ &P(\omega_2 | \{\omega_1, \omega_2\}, \beta = 0) = P(\omega_4 | \{\omega_3, \omega_4\}, \beta = 0) = 1/2 \end{aligned}$$

Here I read " $P(\omega_1|\{\omega_1,\omega_2\}, \beta=1)$ " as the likelihood that ω_1 is realized conditional upon the seller's type being $\{\omega_1,\omega_2\}$ and the buyer's positive investment. The above specification implies that the buyer's investment is relationship-specific, in particular, pays off only if the buyer receives the "right" widget. I say that the pair (β^*,ξ) is feasible if, given a complete contract ξ ,

$$\beta^* \in \arg \max_{\beta} \sum_{\psi^{\mathsf{c}} \in \Psi^{\mathsf{c}}} \mu(\psi^{\mathsf{c}}) \sum_{\omega \in \psi^{\mathsf{c}}} P(\omega | \psi^{\mathsf{c}}, \beta) u_B(\xi(\omega); \omega) - c(\beta).$$

Finally, the schematic diagram for the timing of the events is summarized below:



3. Under what conditions does indescribability matter?

I will characterize a set of implementable complete contracts as the one satisfying *incentive compatibility* (IC) and *individual rationality* (IR) at the relevant timing, either at the contracting stage or at the trading stage. A complete contract satisfies IC if, for each agent, telling the truth is a best strategy provided that all other tell the truth. A complete contract satisfies IR if, the resulting expected utility of each agent is at least as the same as the one corresponding to the no-trade outcome.

3.1. When Ω is indescribable at the contracting stage

When Ω is indescribable, agents are simply unable to elicit information about Ψ^c . At the trading stage in which Ω is describable, however, agents are able to elicit information about Ψ^t . Thus, in this environment, I restrict attention to the set of complete contracts which satisfy IC and IR at the trading stage.

Suppose that there is always trade. After ψ^{t} is realized, efficiency consideration requires that the "right" widget be traded in the "right" states: $x_{\ell}(\omega_2) = 1$ and $x_r(\omega_4) = 1$. Since the seller perfectly identifies the underlying state, IR for the seller implies that its price in states ω_2 or ω_4 must be at least 10. IC for the seller immediately implies that it must be sold at the same price in all states. However, any such complete contract ξ would violate IR for the buyer which requires that, for any ω , $v(\omega) \le 9$ when $\beta^*=1$ and $y(\omega) \le 7.5$ when $\beta^*=0$. In either case, a sequence of arguments then follows: (1) IR for the seller implies that $x_k(\omega)=0$ for each k and $\omega=\omega_2, \omega_4$; (2) IR for the buyer implies that $y(\omega)=0$ for any ω ; and (3) IC for the seller implies that $x_k(\omega) = 0$ for any k and ω . In sum, the only complete contracts satisfying IC and IR are the no-trade contract: $x_k(\omega)=0$ and $v(\omega)=0$ for each k, ω . As a result, it is optimal for the buyer to make no investment.

3.2. When Ω is describable at the contracting stage

The previous analysis already shows that if the agents elicit information at the trading stage, only the no-trade contract can be executed. When Ω is describable, however, the agents are able to elicit information about Ψ^c at the contracting stage.

Therefore, I explore the possibility that the agents elicit information only at the contracting stage, i.e., the set of complete contracts satisfying IC and IR *at the contracting stage*.

Let ξ be a complete contract. Since $\omega_1(\omega_3)$ is indistinguishable from $\omega_2(\omega_3)$ at the contracting stage, I must have

$$\xi(\omega_1) = \xi(\omega_2)$$
 and $\xi(\omega_3) = \xi(\omega_4)$.

IC for the seller implies that the widget must be sold at the same price. Thus, ξ has the following property:

$$y(\omega_1) = y(\omega_3) = y(\omega_2) = y(\omega_4)$$

Provided that there is always the ex ante efficient trade, for any $\omega \in \Omega$, I let

$$\xi(\omega) = (x_{\ell}(\omega), x_{r}(\omega), y(\omega))$$

=
$$\begin{cases} (1,0,y) \text{ if } \omega = \omega_{1} \text{ or } \omega_{2} \\ (1,0,y) \text{ if } \omega = \omega_{3} \text{ or } \omega_{4} \end{cases}$$
(1)

Suppose that the buyer's investment is $\beta^* = 1$. IR for the seller implies that the expected cost of the widget is at least as

great as 7. On the other hand, IR for the buyer implies that the expected value of the widget is at most as great as 9. Hence, IR for both agents can be summarized into the following condition

$$7 \le y \le 9. \tag{2}$$

Any complete contract ξ with Eqs. (1) and (2) satisfies IC and IR at the contracting stage provided the buyer makes an investment.⁵ Finally, I want to show that $(\beta^*,\xi)=(1,\xi)$ is feasible. Define $U_B(\xi|\beta)$ to be the expected utility of the buyer at the contracting stage corresponding to ξ given β . For any $y \ge 0$, I have

$$U_B(\xi|\beta = 1) = 15 \times \frac{2}{3} - y - 1 = 9 - y > 7.5 - y$$
$$= 15 \times \frac{1}{2} - y = U_B(\xi|\beta = 0).$$

Thus, given ξ , it is optimal for the buyer to make an investment.

4. Concluding remarks

This paper asks the following question: "under what conditions does indescribability matter?" Asymmetric information at the contracting stage is the answer I promote in this paper. In particular, I show by a simple model that there is a set of implementable contracts that always induce the ex ante efficient trade when the states are describable, while only the no-trade contract can be implemented when the states are indescribable. When there is asymmetric information at the contracting stage, indescribability *by itself* significantly reduces the set of implementable contracts.

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⁵ Besides, x is welfare neutral in the sense of Kunimoto (2006) which, in turn, is an interim payoff generalization of Maskin and Tirole (1999)'s counterpart.