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Bargaining and competition revisited

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Abstract

We show the robustness of the Walrasian result obtained in models of bargaining in pairwise meetings. Restricting trade to take place only in pairs, most of the assumptions made in the literature are dispensed with. These include assumptions on preferences (differentiability, monotonicity, strict concavity, bounded curvature), on the set of agents (dispersed characteristics) or on the consumption set (allowing only divisible goods).

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1. Introduction

There are two theories of decentralized exchange. The Jevonsian tradition is based on pairwise interactions and it is explained in terms of exploiting gains from trade when the marginal rates of substitution of two agents differ. The Edgeworthian tradition, on the other hand, allows for groups of agents to interact and relies on the elimination of all coalitional recontracting possibilities. Modern presentations of both traditions are found in [3–5] and [2], respectively. These contributions describe decentralized matching and bargaining games whose sets of equilibrium outcomes coincide with the Walrasian allocations of an exchange economy.

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In this paper, we show that the Walrasian results found in [3,5] (see also [9]) are robust to the relaxation of many of the assumptions on which it rested.[2] Dagan et al. [2] already set out to make progress in this direction, but their approach was based on a procedure in which coalitions of any finite size perform the trades.[3] The current paper rests on even weaker assumptions than those used in [2], and it uses Gale's [3] pairwise meetings original procedure.

We argue that the only assumptions on which the decentralization result relies are continuity and local non-satiation of preferences and a condition to rule out problems at the boundary of the consumption set. As explained in [7], there are two key steps in the result: efficiency and budget balance. To obtain efficiency, our result is based exclusively on a separation argument—Lemma 6—, which in the general case exploits the convexifying effects of the continuum: pairwise efficiency implies Pareto efficiency (thanks to the boundary assumption). Budget balance is proved in Lemma 7 making an assumption on the equilibrium which turns out to be also necessary. The novelty of this assumption is that it uses deviations of agents that hold their initial endowments (and not only agents that are about to leave the market, as the earlier published proofs did).[4] For simplicity in the presentation, we write the model and proofs making more assumptions than the ones we really need, and we discuss in the last section how they can be dispensed with.

2. The economy

Time is discrete and is indexed by the non-negative integers. There is a continuum of agents in the market. Each agent is characterized by his initial bundle, and by his von Neumann–Morgenstern utility function. The consumption set for each agent is $\mathbb{R}^L_+$, i.e., we consider for now only infinitely divisible goods. Each agent chooses the period in which to consume, that is, to leave the market. We denote by D the event for an agent in which he never leaves the market.

For now, we shall make the assumption of a finite-type economy. That is, the agents initially present in the market are of a finite number "K" of types. The symbol K also denotes the set of types. All members of any given type k have the same

[2] These results and the current paper deal with the characterization question, i.e., the identification of conditions under which *all* equilibria of the game support Walrasian allocations. Conversely, Gale [4] and Dagan et al. [2]'s Theorem 2 identify weak conditions in order to construct equilibrium strategies behind each Walrasian allocation. In particular, although their model involves coalitional meetings, the strategies proposed in the proof of Dagan, Serrano and Volij's Theorem 2 prescribe trade only between pairs of agents, making them readily applicable in our model. Hence, we shall not be concerned with this question here.

[3] McLennan and Sonnenschein [8] also relaxed some of the assumptions made in [3]. However, their paper uses unlimited short sales, which is problematic, as argued in [1,2].

[4] Gale [5] also uses agents holding initial endowments for parts of its proof. However, his arguments continue to use a uniform bounded curvature assumption, while ours do not.

utility function

$$u_k : \mathbb{R}^L_+ \cup \{D\} \to \mathbb{R} \cup \{-\infty\}$$

and the same initial endowment $\omega_k \in \mathbb{R}^L_{++}$.

For each type k there is initially a measure n_k of agents in the market (with $\sum_{k=1}^K n_k = 1$).

We assume that there exists a continuous function $\phi_k : \mathbb{R}^L_+ \to \mathbb{R}$ that is strictly increasing and strictly concave on $\mathbb{R}^L_+$.[5] We also assume that $-\infty < \phi_k(x) < \phi_k(\omega_k)$ for every $x \in \partial\mathbb{R}^L_+$. Then,

$$u_k(x) = \begin{cases} \phi_k(x) & \text{if } x \in \mathbb{R}^L_+, \\ -\infty & \text{if } x = D. \end{cases}$$

Definition 1. An allocation is a K-tuple of bundles $(x_1, \ldots, x_K)$ for which $\sum_{k=1}^K n_k x_k = \sum_{k=1}^K n_k \omega_k$.

Definition 2. An allocation $(x_1, \ldots, x_K)$ is Walrasian if there exists a price vector $p \neq 0$ such that for all k the bundle x_k maximizes u_k over the budget set, $\{x \in \mathbb{R}^L_+ | px \leqslant p\omega_k\}$.

3. The game and the equilibrium concept

We study the model of Gale's [3,5], as described in [9]. We go over its details at present.

In every period each agent is matched with a partner with probability $\alpha \in (0, 1)$ (independent of all past events). The probability that any given agent is matched in any given period with an agent in a given set is proportional to the measure of that set in the market in that period. It follows from this specification of the matching technology that in every period there are agents who have never been matched. The finite-type economy, along with this matching technology, implies that even though agents leave the market as time passes, at any finite time a positive measure of every type remains.

Once a match is established, each party learns the type and current bundle of his opponent. With equal probability, one of them is selected to propose a vector z of goods, to be transferred to him from his opponent. Let a pair $\{i,j\}$ be matched and call i and j the proposer and responder, respectively.

We denote by x_i the proposer's original bundle when this pairwise meeting begins, and by x_j the responder's original bundle. Suppose that i's proposal is accepted. Then, we denote by $x_i + z$ the proposer's new bundle and by $x_j - z$ the responder's

[5] We shall comment on how to relax strict concavity and strict monotonicity, as well as the finite-type and divisible-good assumptions in our last section.

new bundle. We require that any proposal result in a net trade z satisfying the following feasibility condition, $x_i + z \in \mathbb{R}^L_+$ and $x_j - z \in \mathbb{R}^L_+$. After a proposal is made, the other party either accepts or rejects the offer.

The market exit rules are as follows. In the event an agent rejects an offer, he chooses whether or not to stay in the market. An agent who makes an offer, accepts an offer, or who is unmatched, must stay in the market until the next period: he may not exit.

There is no discounting. Therefore, an agent who exits obtains the utility of the bundle he holds at that time. An agent who never exits receives a utility of $-\infty$.

A strategy for an agent is a plan that prescribes his bargaining behavior for each period, for each bundle he currently holds, and for each type and current bundle of his opponent. An agent's bargaining behavior is specified by the offer to be made in case he is chosen to be the proposer and, for each possible offer, one of the actions "accept", "reject and stay", or "reject and exit".

An assumption that leads to this definition of a strategy is that each agent observes the index of the period, his current bundle, and the current bundle and type of his opponent, but no past events beyond his own personal history.[6]

Like Gale [3–5] and [9], we restrict attention to the case in which all agents of a given type use the same strategy. As trade occurs, the bundle held by each agent changes. Different agents of the same type, even though they use the same strategy, may execute different trades. Thus the number of different bundles held by agents may increase. However, the number of different bundles held by agents is finite at all times. Thus in any period the market is given by a finite list $(k_i, c_i, v_i)_{i=1,\dots,I_t}$, where v_i is the measure of agents who are still in the market in period t, currently hold the bundle c_i, and are of type k_i. We call such a list a *state of the market*. We say that an agent of type k who holds the bundle c is characterized by (k, c).

Associated with each K-tuple of strategies σ, one can define a state of the market $\rho(\sigma, t) = (k_i, c_i, v_i)_{i=1,\dots,I_t}$ in each period t. Although each agent faces uncertainty, the presence of a continuum of agents and the symmetry of strategies allow us to define ρ in a deterministic fashion (see [9, pp. 160–161]). For example, since in each period the probability that any given agent is matched is α, we take the fraction of agents with any given characteristic who are matched to be precisely α. The reader is referred to [9] for a description of how to obtain $\rho(\sigma, t+1)$ from $\rho(\sigma, t)$ given σ.

Definition 3. A market equilibrium is a particular type of perfect Bayesian equilibrium: it is a K-tuple σ^* of strategies, one for each type, and the "state of

[6] As explained in [3], thanks to the presence of a continuum of agents and since personal histories are private information, there is no loss of generality in making this Markov assumption on the strategies. Two agents with the same utility function and bundle in period t, but with different private histories, must receive in equilibrium the same expected utility, as otherwise one could profitably deviate by imitating the other. This shows that the value function $V_k(c, t)$ used in our proof is well defined as being independent of payoff irrelevant histories.

the market'' beliefs $\rho(\sigma^*, t)$ both on and off the equilibrium path for each time t, such that:

- For any trade z, bundles c and c', type k, and period t, the behavior prescribed by each agent's strategy from period t on is optimal, given that in period t the agent holds c and has either to make an offer or to respond to the offer made by his opponent, who is of type k and holds the bundle c', given the strategies of the other types, and given that the agent believes that the state of the market is $\rho(\sigma^*, t)$.

Each agent is an entity of measure 0 in the continuum and each of them has met only a finite number of agents in all the rounds up to round t. A fixed profile of strategies played by the continuum of agents determines the state of the market in round t with independence of the actions of such a set of measure 0. This is the rationale for the beliefs assumed, even after an agent observes deviations from the equilibrium strategies. It is also easy to see that, in a discretized version of the game, our equilibria are also sequential equilibria.

4. The theorem

Suppose that the market equilibrium strategy calls for agents characterized by (k, c) who are matched in period t with agents characterized by (k', c') to reject *some* offer z and leave the market. These agents are said to be *ready to leave the market* in period t.

Theorem 1. *At every market equilibrium, each agent of type $k \in K$ leaves the market with the bundle x_k with probability* 1, *where $(x_1, \ldots, x_K)$ is a Walrasian allocation.*

Proof. Consider a market equilibrium. All of our statements are relative to this equilibrium. All agents of type k who hold the bundle c at the beginning of period t (before their match has been determined) face the same probability distribution of future trading opportunities. Thus in equilibrium all such agents have the same expected utility, $V_k(c, t)$ (see footnote 5 again).

Lemma 1. $V_k(c, t) \geqslant u_k(c)\ \forall k,\ c,\ t.$

Proof. Suppose that an agent of type k who holds the bundle c in period t makes the zero trade offer whenever he is matched and is chosen to propose a trade, and reject every offer and leaves the market when he is matched and chosen to respond. Clearly, he is matched and chosen to respond to an offer in finite time with probability 1. □

Lemma 2. $V_k(c, t) \geqslant V_k(c, t+1)\ \forall k,\ c,\ t.$

Proof. By proposing the zero trade and rejecting any offer and staying in the market, any agent in the market in period t is sure of staying in the market until period $t+1$ with his current bundle. $\square$

Lemma 3. *For an agent of type k who holds the bundle c and is ready to leave the market in period t we have* $V_k(c, t+1) = u_k(c)$.

Proof. From Lemma 1, we have $V_k(c, t+1) \geqslant u_k(c)$. Suppose that $V_k(c, t+1) > u_k(c)$ and the circumstances that make the agent leave the market are realized. Then he would leave with the bundle c and obtain the utility of $u_k(c)$. However, he is better off by deviating and staying in the market until period $t+1$, contradicting his equilibrium strategy. $\square$

Lemma 4. *Suppose that an agent of type k holds the bundle c and is ready to leave the market in period t. Then it is optimal for him to accept any offer z (of a transfer from him to the proposer) for which* $u_k(c-z) > u_k(c)$.

Proof. If he accepts the offer, from Lemma 1, we have $V_k(c-z, t+1) \geqslant u_k(c-z)$, and therefore, $V_k(c-z, t+1) > u_k(c)$. If he rejects the offer, then we have, from Lemma 3, $V_k(c, t+1) = u_k(c)$. Combining this with the previous inequality, we obtain the result. $\square$

Lemma 5. *For each type* $k \in K$, *there exists a period* t^* *such that for every* $t \geqslant t^*$ *there is a positive measure of agents of type k who are ready to leave the market with the bundle* c_k *in period t.*

Proof. Suppose first that there is a set of agents of type k with positive measure who hold the bundle c_k and are ready to leave the market in period t^*. Given the matching technology, a positive measure of such agents have been unmatched in any future time after t^* and remain ready to leave the market by Lemmas 2 and 3.

Thus, it only remains to show the existence of such t^*. We argue by contradiction. Suppose that there exists a type k such that there is no positive measure of agents of type k ready to leave the market at any time $t < \infty$. In this case the expected utility of almost all agents of type k is $u_k(D) = -\infty$. On the other hand, given the matching technology, at any point in time there is a positive measure of agents of type k who hold ω_k, and given our assumption about the utility of the initial endowment, they can be sure of getting the utility of their initial bundle in finite time by proposing the zero trade whenever necessary and rejecting the offer and leaving the market as soon as they are chosen to be responders. $\square$

Lemma 6. *Consider any period t such that a positive measure of traders leaves the market in periods* $t' \leqslant t$. *Consider the different characteristics of traders* (k_i, c_i) *for* $i \in I_t$. *Let* $E_t \subseteq I_t$ *be the set of characteristics present up to period t for which a positive measure of agents has left the market. Suppose that for all* $i = (k_i, c_i) \in E_t$, *each member of the exiting set of agents of characteristics i leaves the market in period* t_i

with the bundle x_i. Then there is a vector $p \in \mathbb{R}^L_{++}$ that supports the upper contour set of u_i at x_i for every $i \in E_t$.

Proof. First, we define the following sets:

$$A_i^+ = \{z \in \mathbb{R}^L \mid u_i(x_i + z) > u_i(x_i)\}$$

and

$$A_i^- = \{z \in \mathbb{R}^L \mid u_i(x_i - z) > u_i(x_i)\}.$$

It is clear that for all $i \in I_t$, A_i^+ and A_i^- are convex sets. Second, if one defines $A_{-i}^+ = \sum_{j \neq i} A_j^+$ and $A_{-i}^- = \sum_{j \neq i} A_j^-$, we have that A_{-i}^+ and A_{-i}^- are also convex sets. Note that if $z \in A_i^+$, then for any $\beta \in (0,1)$, $\beta z \in A_i^+$, as follows from convexity and continuity of preferences. The same observation applies to A_{-i}^-.

Further, we show now that $A_i^+ \cap A_{-i}^- = \emptyset$ for all $i \in I_t$. Suppose, contrary to the claim, that there exist $i \in I_t$ and $z \in A_i^+ \cap A_{-i}^-$. Since $z \in A_{-i}^-$, there exist $(z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_{I_t})$ with $\sum_{j \neq i} z_j = z$ such that $u_j(x_j - z_j) > u_j(x_j)$ for all $j \neq i$. First, we shall construct a profitable deviation of any agent of characteristic i who is ready to leave the market with the bundle x_i. Using the observation at the end of last paragraph about A_i^+ and A_{-i}^-, we have $\beta z \in A_i^+$ and $\beta z \in A_{-i}^-$ for all $\beta \in (0,1)$. Notice that for every $i \in I_t$, $x_i \in \mathbb{R}^L_{++}$ by our boundary assumption. If we take β sufficiently small, we have that $x_i - \sum_{j \neq i} \beta z_j \in \mathbb{R}^L_{++}$. In other words, in no matter what order he executes the net trades $(\beta z_1, \ldots, \beta z_{i-1}, \beta z_{i+1}, \ldots, \beta z_{I_t})$, his bundle after each of these trades continues to be feasible. Consider the following deviation by an agent of characteristic i who is ready to leave the market with the bundle x_i:

- The first time that he is matched with an agent of characteristic $j \neq i$ who is ready to leave the market with the bundle x_j and if the agent of characteristic i is chosen to be the proposer, propose the trade βz_j.
- Reject any offer and leave the market when he is chosen to be the responder as soon as he achieves the bundle $x_i + \beta z$.
- Otherwise, propose the zero trade, reject every offer whenever necessary, and stay in the market.

From Lemma 5, there is a positive measure of agents of each characteristic $j \neq i$ who are ready to leave the market with the bundle x_j in every period. In addition, from Lemma 4, it is optimal for each such agent of each characteristic $j \neq i$ to accept the offer βz_j. Given the matching technology, any agent of characteristic i who is ready to leave the market with the bundle x_i can eventually meet as many agents of every characteristic $j \neq i$ who are ready to leave the market with the bundle x_j as he wishes. Thus, with probability 1, he can achieve the utility $u_i(x_i + \beta z) > u_i(x_i)$ given the belief that agents of other characteristics follow their equilibrium strategies, which is a contradiction. Therefore, we have established that $A_i^+ \cap A_{-i}^- = \emptyset$.

Consider now the strict upper contour set at the bundle x_i for each characteristic i:

$$B_i(x_i) = \{y_i \in \mathbb{R}^L_{++} | u_i(y_i) > u_i(x_i)\},$$

and their sum

$$B(x) = \sum_{i \in I_t} B_i(x_i), \quad \text{where } x = \sum_{i \in I_t} x_i.$$

For the given x, define the set

$$\{x\} = \left\{ x = \sum_{i \in I_t} y_i \text{ for some } y_i \in \mathbb{R}^L_{++} \right\}.$$

Both $B(x)$ and $\{x\}$ are convex sets.

Furthermore, we show now that $B(x) \cap \{x\} = \emptyset$. Suppose, contrary to the claim, that their intersection is non-empty. That is, there exist $(y_1, \ldots, y_{I_t})$ with $\sum_{i \in I_t} y_i = x$ such that $u_i(y_i) > u_i(x_i)$ for all i. Let $y_i = x_i + z$ and $y_j = x_j - z_j$ for all $j \neq i$. Since $\sum_{i \in I_t} y_i = x$, we have $z = \sum_{j \neq i} z_j$. Then this contradicts $A_i^+ \cap A_{-i}^- = \emptyset$.

Therefore, by the separating hyperplane theorem, there exists $p \in \mathbb{R}^L$ and a constant r such that $px \leqslant r$ and such that for every $y \in B(x)$, $py \geqslant r$. Let $\varepsilon > 0$ and denote by $\varepsilon^L \in \mathbb{R}^L_{++}$ a vector where all its components are ε. By strict monotonicity of preferences, $x + |I_t|\varepsilon^L \in B(x)$. Taking a sequence of ε's converging to 0, we obtain that $r = px$.

Finally, consider an arbitrary $y_i \in B_i(x_i)$. Clearly, we have that $y_i + \sum_{j \neq i} x_j + |I_t - 1|\varepsilon^L \in B(x)$, and therefore, $p[y_i + \sum_{j \neq i} x_j + |I_t - 1|\varepsilon^L] \geqslant px$, or $py_i + p|I_t - 1|\varepsilon^L \geqslant px_i$. And again taking a sequence of $\varepsilon \to 0$, we obtain that $py_i \geqslant px_i$. Since the utility functions are continuous, we have that for every z_i such that $u_i(z_i) \geqslant u_i(x_i)$, $pz_i \geqslant px_i$, as we wanted to show. Of course, the fact that $p \in \mathbb{R}^L_{++}$ comes from strict monotonicity, now that we know that p supports the upper contour sets at x_i for every characteristic i. $\square$

Lemma 7. *Let p, x and the sets A_i^- be as defined in Lemma 6 and its proof. Let $I = \bigcup_t I_t$ be the set of all characteristics present in equilibrium, and let $A^- = \sum_{i \in I} A_i^-$. Let $z_i^* = x_i - \omega_i$ be characteristic i's net trade in equilibrium. Assume that for every characteristic i for which $pz_i^* < 0$, there exists $\theta_i \in \mathbb{R}^L$ small enough, $u_i(x_i + \theta_i) > u_i(x_i)$, for which there exists $\beta > 0$ small enough such that $\beta\theta_i \in A^-$.*

Then, the market equilibrium is payoff equivalent to the Walrasian equilibrium (p, x).

Proof. For each characteristic $i = (k, c) \in I$ present in the market, define the following set:

$$\Gamma_i = \{z \in \mathbb{R}^L \mid u_i(\omega_i + z) > u_i(x_i)\}.$$

We shall show now that for every $i \in I$, $pz_i^* \geqslant 0$, that is, $px_i \geqslant p\omega_i$. We argue by contradiction. Suppose there exists characteristic i such that $pz_i^* < 0$. Consider θ_i as in the statement of the lemma. We have that $z_i = z_i^* + \theta_i \in \Gamma_i$. Since θ_i is small enough, we obtain $pz_i < 0$ by continuity. Further, by our assumption, we have that there exists $\beta > 0$ small enough such that $\beta z_i \in A^-$.

Since $\beta z_i \in A^-$, there exist $(\beta z_1', \ldots, \beta z_I')$ with $\sum_{j \in I} \beta z_j' = \beta z_i$ such that $u_j(x_j - \beta z_j') > u_j(x_j)$ for all $j \in I$. Since A^- is convex and the closure of it contains $0 \in \mathbb{R}^L$, we can choose β arbitrarily small. Recall our assumption that $\omega_i \in \mathbb{R}^L_{++}$. Hence, if one takes a sufficiently small β, we have that $\omega_i - \sum_{j \in I} \beta z_j' \in \mathbb{R}^L_{++}$. Take the smallest integer $N \in \mathbb{N}$ such that $\beta \geqslant 1/N$.

Consider the following deviation by an agent of characteristic i. Instead of following his equilibrium strategy, he will use this other, starting at the beginning of the game when he holds his initial bundle ω_i:

- The first N times that he is matched with an agent of characteristic j who is ready to leave the market with the bundle x_j and if the agent of characteristic i is chosen to be the proposer, offer the trade $(1/N)z_j'$.
- Reject any offer and leave the market when he is chosen to be the responder as soon as he finishes trading N times with each $j \in I$ ready to leave the market, as prescribed above.
- Otherwise, propose the zero trade, reject every offer whenever necessary, and stay in the market.

Following this strategy, in no matter what order he executes the net trades $((1/N)z_1', \ldots, (1/N)z_I')$, his bundle after each of these trades continues to be feasible. From Lemma 5, there is a positive measure of agents of each characteristic j who are ready to leave the market with the bundle x_j in every period. In addition, from Lemma 4, it is optimal for each such agent of characteristic j to accept the offer $(1/N)z_j'$. Given the matching technology, any agent of characteristic i who holds his initial bundle ω_i can eventually meet as many agents of every characteristic j who are ready to leave the market with the bundle x_j as he wishes. Thus, with probability 1, he can achieve the utility $u_i(\omega_i + z_i) > u_i(x_i)$ given the belief that other agents follow their equilibrium strategies, which is a contradiction.

Therefore, we have established that for all characteristics i present in the market $px_i \geqslant p\omega_i$. In addition, recall that p supports the indifference surface at x_i. That is, we have $V_i(\omega_i, t) \geqslant \max_{x \in \mathbb{R}^L_+} \{u_i(x) \mid px \leqslant p\omega_i\}$. By Lemma 2, we have $V_i(\omega_i, 0) \geqslant \max_{x \in \mathbb{R}^L_+} \{u_i(x) \mid px \leqslant p\omega_i\}$.

By efficiency of the Walrasian allocations, these inequalities must be equalities. That is,

$$V_i(\omega_i, 0) = \max_{x \in \mathbb{R}^L_+} \{u_i(x) \mid px \leqslant p\omega_i\}. \qquad \square$$

Lemma 8. *If the utility functions are strictly concave, every agent of type $k \in K$ leaves the market in finite time with a bundle x_k such that $(x_k)_{k \in K}$ is a Walrasian allocation.*

Proof. By strict concavity of utility functions, the market equilibrium outcome is a degenerate lottery, so each agent of type $k \in K$ receives in equilibrium the bundle x_k, the unique maximizer of his utility over the equilibrium budget set through ω_k. $\square$

This concludes the proof of the theorem.

5. Extensions

The theorem proved in this paper can be extended in several important directions.

1. Strict monotonicity can be weakened to local non-satiation, since it is used only to find near-by strictly preferred bundles in the separation argument of Lemma 6 and in the use of the first welfare theorem in Lemma 7. This allows to extend the theory to "economic bads" if one considers the consumption set to be $\mathbb{R}^L$, to avoid problems with the boundary assumption on preferences.

2. Strict concavity, used only in Lemma 8, can be replaced with the assumption of aggregate risk aversion introduced in [2]. This assumption takes advantage of the convexification effects in the continuum. As explained in that paper, aggregate risk aversion can be derived from assumptions on individual preferences, by requiring a weak form of concavity on the quasiconcave covers of utility functions.

3. There is another sense in which the convexifying effects of the continuum help relaxing assumptions. Although we have assumed convexity of individual preference relations for the proof of Lemma 6, all is really needed is the convexity of the sum of preferred sets to make the separation argument go through. Assuming only that, one can adapt the proof of Proposition 1 in [8] to prove the existence of a unique separating price. That proof relies on differentiability, but it can be extended as follows. Using the notation in their proof, if at periods t and τ, there are two different supporting prices p_t and p_τ, we can have two cases: (a) if p_t intersects the relevant preferred set at period τ or p_τ intersects that at period t, one can use the same deviation proposed in McLennan and Sonnenschein's proof to find a contradiction; and (b) if that does not happen, both prices support both relevant preferred sets, and then we might as well choose one of them as our separating price.

4. Lemma 7 rests on an assumption made directly on the equilibrium. A sufficient condition for it is that there is at least one type with differentiable preferences because then the set A^- is smooth at 0. This minimal presence of differentiability is not necessary, though we have constructed examples with all preferences being non-differentiable in which all market equilibria have the Walrasian property. We choose to state the assumption as in Lemma 7 because it is a necessary condition for the theorem to hold. That is, using the notation found in the statement of Lemma 7, suppose the assumption were violated: this means that there would exist an equilibrium for which there exists a characteristic i with $pz_i^* < 0$. It follows then that the equilibrium outcome cannot be Walrasian, since at least characteristic i does not end up consuming his Walrasian bundle. (This argument shows that there is a large number of conditions that are necessary for the theorem; essentially, any statement with a preamble of the form "for every characteristic i such that $pz_i^* < 0$." The

advantage of the assumption of Lemma 7 is that, apart from necessary, it is also sufficient.)

5. The "finite type" assumption has been made only for expositional reasons. Alternatively, one could work with the condition of dispersed characteristics, as in [3]. Increasing the diversity of types in the population can only help the arguments of the proof, as long as there is a positive probability of meeting agents with a bundle in an open ball of a given bundle.

6. The theory extends also to indivisible goods, thereby reconciling the result in the limit with the limit theorem of Gale [6]. To do this, one should convexify the consumption set and work with the quasiconcave covers of utility functions, as done in [2]. Our key argument is based on separation and this can be attained following similar steps to the ones in our proof.

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References

[1] N. Dagan, R. Serrano, O. Volij, Comment on McLennan and Sonnenschein sequential bargaining as a noncooperative foundation for Walrasian equilibrium, Econometrica 66 (1998) 1231–1233.

[2] N. Dagan, R. Serrano, O. Volij, Bargaining, coalitions, and competition, Econom. Theory 15 (2000) 279–296.

[3] D. Gale, Bargaining and competition, part I: characterization, Econometrica 54 (1986) 785–806.

[4] D. Gale, Bargaining and competition, part II: existence, Econometrica 54 (1986) 807–818.

[5] D. Gale, A simple characterization of bargaining equilibrium in a large market without the assumption of dispersed characteristics, Working Paper 86-05, CARESS, University of Pennsylvania, 1986.

[6] D. Gale, Limit theorems for markets with sequential bargaining, J. Econom. Theory 43 (1987) 20–54.

[7] D. Gale, Strategic Foundations of General Equilibrium—Dynamic Matching and Bargaining Games, Cambridge University Press, Cambridge, 2000.

[8] A. McLennan, H. Sonnenschein, Sequential bargaining as a noncooperative foundation for Walrasian equilibrium, Econometrica 59 (1991) 1395–1424.

[9] M.J. Osborne, A. Rubinstein, Bargaining and Markets, Academic Press, San Diego, 1990.