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Abstract The incomplete contracts literature often cites indescribable contingencies as a major obstacle to the creation of complete contracts. Using agents' minimum foresight concerning possible future *payoffs*, Maskin and Tirole (Rev Econ Stud 66:83–114, 1999) show that indescribability does not matter for contractual incompleteness as long as there is symmetric information at both the contracting stage and the trading stage. This is called the *irrelevance theorem*. The following generalization of the irrelevance theorem is shown here: indescribability does not matter even in the presence of asymmetric information at the trading stage, as long as there is symmetric information at the contracting stage. This is an important clarification because Kunimoto (Econ Lett 99:367–370, 2008) shows that indescribability can matter if there is *asymmetric* information at *both* stages. It is thus argued that asymmetric information at the

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contracting stage is necessary for indescribability to be important in the rational agents contracting model.

Keywords Asymmetric information · Bayesian implementation · Incentive compatibility · Incomplete contracts · Indescribability · Individual rationality · Irrelevance theorem

JEL Classification C72 · D78 · D82

1 Introduction

Almost everyone would agree that actual contracts appear quite incomplete. Since the early works of Grossman and Hart (1986) and Hart and Moore (1990), the incomplete contracts literature has successfully answered many organizational questions.[1] A crucial assumption for this literature is that the two parties cannot write an ex ante contract which specifies appropriate investment levels, and appropriate asset usage as a function of every possible contingency. To justify this apparent incompleteness, the literature often invokes the assumption that, at the contracting stage, agents are able to forecast their possible future *payoffs* (this is needed in order to be able to make rational investment choices before uncertainty is realized.), but state contingent (physical) *actions* cannot be described, i.e., are *indescribable*.[2] Maskin and Tirole (1999) (henceforth, MT) observe that using this agents' minimum foresight concerning the possible payoff contingencies, the parties can design a message game *at the trading stage* (ex post) that effectively describes all the trades that have not been described *at the contracting stage* (ex ante). They show (in their Theorem 1) that when the parties can use such a game and renegotiation is preventable, inability to foresee future contingencies *by itself* does not constrain contracting. MT also show (in their Theorem 3) that when the parties are risk-averse and unbounded transfers and lotteries can be used, renegotiation together with indescribability does not constraint the set of implementable outcomes. When the parties are risk-neutral, MT only establish (in their Theorem 4) that indescribability places no *additional* constraint on contracting, but do not rule out renegotiation-proofness constraints.[3]

The basic idea of Maskin and Tirole (1999) is: if agents have trouble describing *physical* contingencies, they can write contracts that *ex ante* specify only the possible *payoff* contingencies. Then, later on, when the state of the world is realized, they can fill in the physical details. It follows that the only serious complication is incentive compatibility: will it be in each agent's interest to specify these details truthfully? But

[1] See Tirole (1999) for an excellent survey of this literature.

[2] See footnote 4 in Hart and Moore (1990): "*...Note that there is no inconsistency in assuming, on the one hand, that date 0 contingent statements are infeasible and, on the other hand, that agents have perfect foresight about the consequences of this lack of feasibility ...*".

[3] In certain environments, Che and Hausch (1999), Hart and Moore (1999), and Segal (1999) show that optimal complete contracts take the form of "simple contracts". These works are considered *foundations of incomplete contracts*. To be consistent with MT's irrelevance theorems, these authors do not argue that indescribability *by itself* is an essential ingredient for contractual incompleteness.

implementation theory can be used to ensure that truthful specification occurs in equilibrium. I consider indescribability as simply a constraint that filling in the "physical" details is impossible at the contracting stage.[4]

This paper is primarily concerned with the question of under what conditions indescribability itself is important for contractual incompleteness. Kunimoto (2008) made some progress in this direction: indescribability can matter when there is *asymmetric* information at *both* the contracting and the trading stages, using the standard complete contracts model otherwise. More specifically, it is shown there that there is a set of implementable contracts that always induces the ex ante efficient investment and trade when the states are describable, while only the no-investment-no-trade contract can be implemented when the states are indescribable. The current paper clarifies the extent to which indescribability does not matter by extending MT's irrelevance theorem to the environments in which there is symmetric information at the contracting stage but asymmetric information at the trading stage (See Sect. 4).[5] In other words, I show that indescribability places no additional constraint on contracting by "rational" agents, but does not rule out *Bayesian implementability* constraints (See Sect. 3 and the Appendix). I therefore aruge that asymmetric information at the contracting stage can result in indescribability mattering for contractual incompleteness. For example, this asymmetric information might be viewed as the agents' previous experiences to the current situation.

The rest of the paper is organized as follows. In Sect. 2, I set up the benchmark case without indescribability. In Sect. 3, I give definitions and notations needed to address the case of indescribability. In Sect. 4, I establish the main result as the extended irrelevance theorem *á la* Maskin and Tirole (1999) in the environments where there is asymmetric information at the trading stage. Section 5 illustrates those conceptually complex notions introduced in Sect. 3 and the main result provided in Sect. 4 through an example. This section also illustrates most of concepts and the results provided in Maskin and Tirole (1999). Section 6 concludes.

2 The benchmark case without indescribability

2.1 Bilateral contracting

Consider a bilateral contracting environment in which two agents trade goods. There are four dates in this contractual relationship: At date 1, the agents sign a contract. This stage is called the *contracting* stage in this paper. Throughout this paper, I assume that there is *symmetric information* at the contracting stage.[6] At date 2, the agents make

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[4] See Maskin (2002) for an illustrative exposition of Maskin and Tirole (1999).

[5] In fact, Maskin and Tirole (1999) wrote in footnote 6 in their paper as follows: *We conjecture, however, that the irrelevance theorem extends the case of ex post asymmetric information in the sense that the ex ante describability or indescribability of states continues not to matter under our assumptions.* I will make this conjecture a formal result because it clarifies the extent to which indescribability does not matter, given the result of Kunimoto (2008).

[6] This is consistent with the incomplete contracts literature. Kunimoto (2008), on the other hand, assume that there is "asymmetric" information at the contracting stage.

non-contractible investments. At date 3, each agent only receives his own signal. This stage is called the *trading* stage, at which the agents could have "asymmetric" information. At date 4, the agents implement the trade specified by the contract.

Let $N = \{1, 2\}$ be the set of agents. For each agent $i \in N$, let $\mathcal{T}_i = \{\tau_i^1, \ldots, \tau_i^{k_i}\}$ be a finite set of agent i's types (signals). Let $\mathcal{T} \equiv \mathcal{T}_1 \times \mathcal{T}_2$ be the set of states of the world and $\mathcal{T}_j$ for $j \neq i$ denote the other agent j's type space from agent i's point of view. Note that *symmetric information* at the trading stage can be seen as a special case in which both agents have the same type space $\mathcal{T}_1 = \mathcal{T}_2$ and it is common knowledge that both agents always receive the same signal.

Each agent i makes a *non-contractible* investment $e_i \in E_i$. Each pair $e = (e_1, e_2) \in E_1 \times E_2 = E$ gives rise, stochastically, to a state of the world τ characterized by (1) an action set A^τ and (2) ex post payoff functions $u_i : A \times \mathcal{T} \to \mathbb{R}$. Here $A \equiv \bigcup_{\tau \in \mathcal{T}} A^\tau$. More formally, let $\tilde{\tau}$ be a random vector taking values in $\mathcal{T}$ with associated distribution p over $\mathcal{T}$ conditional on $e \in E$ where

$$p(\tau | e) = \text{Prob}\{\tilde{\tau} = \tau \ | e\}.$$

I assume that they have prior probabilistic beliefs about states conditional on the choice of investments and that they share a common prior expressed by the conditional probabilities $p(\tau | e)$.

A *complete contract* can be defined as a mapping $f : \mathcal{T} \to A$ with the property that $f(\tau) \in A^\tau$ for each $\tau \in \mathcal{T}$.

The contract f induces an investment "game" between the agents in which, given $e = (e_1, e_2)$, agent i's payoff at the contracting stage is

$$\sum_{\tau \in \mathcal{T}} p(\tau | e) u_i \left(f(\tau); \tau\right) - c_i(e_i),$$

where $c_i(e_i)$ is agent i's cost of investment.

I say that the pair (e^*, f) is *feasible* if, given a complete contract f, the unique equilibrium of the investment "game" consists of each agent i selecting $e_i = e_i^*$:

1. e^* constitutes a Nash equilibrium:

$$\sum_{\tau \in \mathcal{T}} p(\tau | e^*) u_i(f(\tau); \tau) - c_i(e_i^*) \geq \sum_{\tau \in \mathcal{T}} p\left(\tau | e_i, e_j^*\right) u_i(f(\tau); \tau) - c_i(e_i)$$

   for all $i \in N$ and all $e_i \in E_i$, and
2. there is no other equilibrium

Denote by $e^*(f)$ the unique Nash equilibrium of the investment game associated with a complete contract f. Let

$$A^\tau = X^\tau \times Y,$$

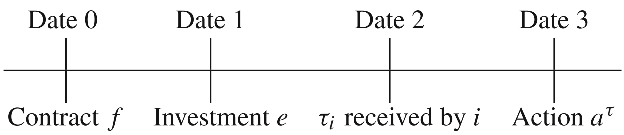
where X^τ is state-dependent and Y is state-independent. Assume for simplicity that X^τ is finite for any $\tau \in \mathcal{T}$. For concreteness, I shall assume that the choice of $y \in Y$

specifies the distribution of money. That is, $y = (y_1, y_2)$, where y_i is agent i's allotment of money. The set of feasible allocations Y is given as follows:

$$
\begin{aligned}
&Y_i \equiv \mathbb{R} \quad \text{for all } i \in N \text{(unbounded transfer is possible)}, \quad \text{and} \\
&Y = \{(y_1, y_2) \in Y_1 \times Y_2 \mid y_1 + y_2 \leq 0\}.
\end{aligned}
$$

I shall suppose that $u_i(\cdot; \tau)$ depends on x and y_i only, and that for all $\tau \in \mathcal{T}$ and all $x \in X^\tau$, $u_i(x, y_i; \tau)$ is strictly increasing and continuous in y_i in the feasible set Y_i. In what follows, I focus on the set of complete contracts whose induced investment game possesses the unique Nash equilibrium.

To sum up, the timing of the events is portrayed below:



2.2 Welfare-Neutral complete contracts

This paper will be particularly concerned with complete contracts that are *welfare-neutral*. The *interim payoff* of agent i generated by a complete contract f when his type is τ_i is given:

$$
U_i(f|\tau_i) \equiv \sum_{\tilde{\tau} \in \mathcal{T}} p\big(\tilde{\tau}|\ \tilde{\tau}_i = \tau_i, e^*(f)\big) u_i\left(f(\tilde{\tau}); \tilde{\tau}\right) - c_i\big(e_i^*(f)\big).
$$

Let X^* be the set of all mappings $\tilde{x}(\cdot)$ with the property that $\tilde{x}(\tau) \in X^\tau$ for any $\tau \in \mathcal{T}$. Two types τ_i and τ_i' are said to be *equivalent* ($\tau_i \sim \tau_i'$) if there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that, for each $\tilde{x} \in X^*$ and each $y_i \in Y_i$,

$$
U_i(\tilde{x}, y_i|\tau_i) = \alpha U_i\big(\tilde{x}, y_i|\tau_i'\big) + \beta.
$$

That is, two types are equivalent if the interim cardinal preferences over all complete contracts are the same. This paper requires that a complete contract always result in the same interim payoffs between these equivalent types. (After all, it is only payoffs that matter).

A complete contract $f : \mathcal{T} \to A$ is *welfare-neutral* if, for every $i \in N$, whenever $\tau_i \sim \tau_i'$, there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that

$$
U_i(f|\tau_i) = \alpha U_i\big(f|\tau_i'\big) + \beta.
$$

Welfare-neutrality is not an innocuous assumption but fairly weak one in the standard incomplete contracts environment.

3 The case of indescribability

3.1 Indescribable states

I would like to know what can be achieved when the states are indescribable. If parties are able to perform dynamic programming, then the very least they can formulate a probability distribution over the *possible interim payoffs*.[7] To represent possible interim payoffs, let

$$V_i : Z \times Y_i \to \mathbb{R},$$

be a "number-based" interim payoff function for agent i, where Z is an abstract set, which will be endowed with enumerability. The only difference between V_i and an ordinary interim payoff function $U_i(\cdot, y_i|\tau_i)$ is that, in the former, the domain of complete contracts has been replaced by this set of Z. The set Z is a dummy index reflecting the fact that agents cannot forecast physical actions. This paper deviates from the literature by allowing the agents to have asymmetric information at the trading stage. On the other hand, this paper follows the literature by assuming that whether or not a given action is feasible can be verified at the trading stage. In other words, for any state $\tau \in \mathcal{T}$, the set of actions at state τ, X^τ should be part of each player i's description of the state τ_i: For any $\tau \in \mathcal{T}$, I essentially assume

$$X^\tau = \bigcup_{\tau' \in \mathcal{T} \text{ s.t. } \tau'_1 = \tau_1} X^{\tau'} = \bigcup_{\tau' \in \mathcal{T} \text{ s.t. } \tau'_2 = \tau_2} X^{\tau'}.$$

Viewed this way, a state τ can alternatively be expressed as $(X^\tau, V^\tau, \varphi^\tau)$, where:

1. X^τ is the (physical) action space compatible with state τ, where $|X^\tau| = k^\tau$;
2. $V^\tau = (V_1^{\tau_1}, V_2^{\tau_2})$, where $V_i^{\tau_i} : K^\tau \times Y_i \to \mathbb{R}$ constitutes agent i's number-based interim payoff function and $K^\tau = \{1, \ldots, k^\tau\}$;
3. $\varphi^\tau : K^\tau \to X^\tau$ is a mapping from numbers into the action space. That is, associated with $y_i \in Y_i$, $\varphi^\tau(k)$ implements the interim payoff vector $V_i^{\tau_i}(k, y_i)$ in type τ_i: $U_i(\varphi^\tau(k), y_i|\tau_i) = V_i^{\tau_i}(k, y_i)$ for all $y_i \in Y_i$ and $i \in N$. The function φ^τ can be interpreted as a *deciphering key*.

Here $|\cdot|$ stands for the cardinality of a set. I let $\mathcal{V} = \mathcal{V}_1 \times \mathcal{V}_2$ denote the set of possible *interim* number-based payoff functions $V = (V_1, V_2)$ where $V_1 \in \mathcal{V}_1$ and $V_2 \in \mathcal{V}_2$.

When I say that agents can perform dynamic programming in the case of indescribable states, I mean that they can formulate prior belief $\hat{p}(V|e)$ about how states affect interim payoffs. Beliefs $\hat{p}$ are consistent with beliefs in the describable states model if, for all $V \in \mathcal{V}$ and all $e \in E$,

[7] Maskin and Tirole (1999) instead look at the possible *ex post* payoffs because they focus on symmetric information at the trading stage.

$$\hat{p}(V|e) = \sum_{\tau \in \mathcal{T}_V} p(\tau|e),$$

where $\mathcal{T}_V = \{\tau \in \mathcal{T} \,|V^\tau \equiv U^\tau \circ \varphi = V$ for some deciphering key $\varphi\}$.

3.2 Number-Based contracts

When actions are no longer describable in advance, parties cannot pre-specify the action they would like to implement in each state. However, they can in principle pre-specify the interim *utilities* they would like to implement. I call

$$\hat{f} : \mathcal{V} \to Z \times Y,$$

a *number-based contract*, where for any number-based interim payoff functions V, $\hat{f}(V)$ specifies an integer $z \in Z$ and a describable action $y_i \in Y_i$ for each $i \in N$ such that $(V_1(z, y_1), V_2(z, y_2))$ are the corresponding interim utilities to be implemented.

A number-based contract $\hat{f}$ is said to be *welfare-neutral* if, for each $i \in N$, whenever $\tau_i \sim \tau_i'$, there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that

$$V_i^{\tau_i}\left(k^\tau, y_i^{\tau_i}\right) = \alpha V_i^{\tau_i'}\left(k^{\tau'}, y_i^{\tau_i'}\right) + \beta$$

where $\hat{f}(V_1^{\tau_1}, V_2^{\tau_2}) = (k^\tau, y_1^{\tau_1}, y_2^{\tau_2})$ and $\hat{f}(V_1^{\tau_1'}, V_2^{\tau_2'}) = (k^{\tau'}, y_1^{\tau_1'}, y_2^{\tau_2'})$.

The number-based contract $\hat{f}$ *corresponds* to the complete contract $f : \mathcal{T} \to A$, if for all $i \in N$, all $V_i \in \mathcal{V}_i$, and for all $\tau_i \in \mathcal{T}_i$ such that $V_i^{\tau_i} = V_i$,

$$V_i\left(\hat{f}\right) = U_i(f|\tau_i).$$

It is routine to verify that a complete contract f is welfare-neutral if and only if there exists a welfare-neutral number-based contract corresponding to f.

3.3 Mechanisms

As long as the complete contract f is welfare-neutral, my task is to ensure that the set of conditions under which the interim utilities prescribed by the corresponding number-based contract $\hat{f}$ actually get implemented. There are two difficulties for this: (1) the true number-based interim payoff functions V^τ are private information and (2) the true deciphering key φ^τ is not verifiable. Recall that V^τ and φ^τ are introduced in Sect. 3.1. Accordingly, $\hat{f}$ must be implemented indirectly. This is where I make use of implementation theory.

Let $\Gamma = (M, g)$ be a mechanism. Here $M = M_1 \times M_2$ refers to the set of message profile $m = (m_1, m_2)$. The outcome function $g : M \to A$ assigns to each message profile m an alternative $g(m) \in A$. Given an arbitrary prior p, a mechanism induces a Bayesian game $\Gamma(p)$ in which each agent's type is his signal, and after observing his signal, agent i selects a message from the set M_i. A strategy in $\Gamma(p)$ for agent i is a rule $\sigma_i : \mathcal{T}_i \to M_i$. A strategy profile $\sigma = (\sigma_1, \sigma_2)$ lists a strategy for each agent.

The interim payoff of agent i generated by a Bayesian game $\Gamma(p)$ when a strategy profile σ is played and his type is τ_i is given:

$$U_i(g \circ \sigma | \tau_i) \equiv \sum_{\tilde{\tau}_j \in \mathcal{T}_j} p(\tilde{\tau}_j, \tau_i | e^*(g \circ \sigma)) u_i(g(\sigma(\tilde{\tau}_j, \tau_i)); \tilde{\tau}_j, \tau_i) - c_i(e_i^*(g \circ \sigma)).$$

A strategy profile σ is said to be a *Bayesian (Nash) equilibrium* of the game $\Gamma(p)$ if for each agent i, state τ, and message m_i,

$$U_i(g \circ \sigma | \tau_i) \geq U_i(g \circ (m_i, \sigma_j) | \tau_i).$$

A mechanism $\Gamma = (M, g)$ is said to *implement* a complete contract f in Bayesian equilibrium if, for all $\tau \in \mathcal{T}$, the following two conditions hold: (1) there is a Bayesian equilibrium whose outcome coincides with $f(\tau)$ and (2) for every Bayesian equilibrium σ, we have $g(\sigma(\tau)) = f(\tau)$. A complete contract f is said to be *Bayesian implementable* if there exists a mechanism Γ that implements f in Bayesian equilibrium. If a complete contract f is welfare-neutral, it is not difficult to show that f is Bayesian implementable if and only if every Bayesian equilibrium interim *utilities* (not physical outcomes) in each state is the same as those prescribed by f. Thus, I say that a number-based contract $\hat{f}$ is Bayesian implementable if it corresponds to a welfare-neutral complete contract f that is Bayesian implementable.[8]

4 To the extent indescribability does not matter

To state the extended irrelevance theorem, I propose the following assumption.

Assumption A 1. There exist a describable alternative $x^0 \in \bigcap_{\tau \in \mathcal{T}} X^\tau$ and transfer $y^0 \in Y$.

2. For any two complete contracts f and f' and any $i \in N$, there exists $y_i \in \mathbb{R}$ such that

$$U_i(f' | \tau_i) > U_i\left(f_X, f_{Y_i} - y_i | \tau_i\right) \quad \text{for all } \tau_i \in \mathcal{T}_i,$$

where $f_X : \mathcal{T} \to X$ and $f_{Y_i} : \mathcal{T} \to Y_i$.

Assumption A1 implies that there is always the no-trade outcome as a describable option. Assumption A2 requires that given any two complete contracts, there exists (potentially large) monetary transfer which compensates agents from a switch from one contract to the other. These assumptions are basically satisfied in the incomplete contracting literature.

[8] When physical outcomes cannot be described in advance, the best we can do is to ensure that equilibrium *utilities* are the same as those prescribed by the contract.

The interim payoff of agent i attainable from the no trade outcome (x^0, y^0) when his type is τ_i is given:

$$U_i\left(x^0, y_i^0|\tau_i\right) \equiv \sum_{\tilde{\tau}_j \in T_j} p\left(\tilde{\tau}_j, \tau_i|e^*(x^0, y^0)\right)u_i\left(x^0, y_i^0; \tilde{\tau}_j, \tau_i\right) - c_i\left(e_i^*(x^0, y^0)\right).$$

I shall define f to be individually rational at the trading stage.[9]

Definition 1 f is (interim) *individually rational* if

$$U_i(f|\tau_i) \geq U_i\left(x^0, y_i^0|\tau_i\right) \quad \text{for all } \tau_i \in \mathcal{T}_i \text{ and } i \in N.$$

I extend MT's irrelevance theorem to the environments in which there is symmetric information at the contracting stage but asymmetric information at the trading stage. This extension is important because Kunimoto (2008) shows that the irrelevance theorem is no longer valid when there is asymmetric information at both the contracting stage and the trading stage.[10]

Theorem 1 (Extended irrelevance theorem) *Suppose Assumption A holds. If the number-based contract $\hat{f}$ corresponds to a complete contract f that is welfare-neutral, Bayesian implementable, and individually rational, then $\hat{f}$ can be implemented in subgame perfect equilibrium even when the states are indescribable at the contracting stage.*

Proof of Theorem 1 The proof builds on standard implementation theory. In the implementation literature, the objective is to construct mechanisms that elicit agents' payoffs when the action space is known in advance but payoffs are neither known *ex ante* nor verifiable *ex post*. We show that such mechanisms can be extended to the case in which the action space cannot be forecast. Specifically, after the state τ is realized, we have agent 1 announce what he claims are (1) the realized action set, denoted X and (2) a mechanism Γ^X that, given X, implements the number-based contract $\hat{f}$ (the existence of an implementing mechanism is assured in the Appendix). We then allow agent 2 to challenge either announcement. Because we assume (as does the incomplete contracts literature) that courts can verify *at the trading stage* whether or not any given action is feasible (See Sect. 3.1), it is easy for agent 2 to challenge successfully agent 1 if he had omitted a feasible action from X or included an infeasible one. Moreover, assuming that X is the true action space, agent 2 can mechanically prove when a mechanism Γ^X fails to implement $\hat{f}$ by simply exhibiting a suboptimal equilibrium (which also constitutes a successful challenge). The incentive for agent 2 to challenge successfully is that she is then rewarded with the opportunity to receive a potentially large monetary transfer from agent 1 to agent 2 (this deters agent 1 from announcing a false action

[9] When there is symmetric information at the trading stage, individual rationality reduces to ex post individual rationality which is needed for MT's irrelevance theorem.

[10] In the light of Theorem 1, Maskin and Tirole (1999) show that when there is symmetric information at the trading stage, Bayesian implementability is a vacuous constraint.

space or non-implementing mechanism). If agent 2 does not challenge—which will occur only if agent 1's claim is true—the mechanism Γ^X is then played.

Note first that, since $u_i(\cdot)$ is strictly increasing in money and $\mathcal{T}_i$ is finite, there exists $y^{00} \in Y$ such that

$$U_i\left(x^0, y_i^{00}|\tau_i\right) < U_i\left(x^0, y_i^0|\tau_i\right) \quad \text{for all } \tau_i \in \mathcal{T}_i \text{ and all } i \in N.$$

Given the action space X, a mechanism $\Gamma^X = (M_1 \times M_2, g^X)$ is said to be *successful* provided that, for all possible pairs of number-based interim payoff functions $V \in \mathcal{V}$ for which $|X| = |Z|$ and all possible deciphering keys $\varphi : Z \to X$, the unique Bayesian equilibrium interim payoffs of the game $\Gamma^X(p)$ when agents have interim utility functions $(V_1(\varphi^{-1}(\cdot), \cdot), V_2(\varphi^{-1}(\cdot), \cdot))$ are $V(\hat{f})$, where $\varphi^{-1} : X \to Z$.

For the time being, suppose that, for all X for which there exists $\tau \in \mathcal{T}$ with $X^\tau = X$, there exists a successful mechanism. Then the parties can sign a contract that stipulates that, once the state is realized:

1. agent 1 proposes a mechanism $\Gamma^X = (M, g^X)$;
2. agent 2 can accept the proposal or challenge it;
3. if agent 2 challenges, then she demonstrates that the proposed mechanism Γ^X is not successful;
4. if agent 2 succeeds with the challenge, then she can name any money transfer (y_1, y_2) with $y_1 + y_2 = 0$. After this monetary transfer is made, the mechanism Γ^X is played; if she fails with the challenge, the outcome (x^0, y^{00}) is implemented; in either case, the execution of the contract ends at this point;
5. if agent 2 accepts agent 1's proposal, then the mechanism Γ^X is played and the action that this leads to is implemented.

We claim that, for each $\tau \in T$, the unique equilibrium interim payoffs of this contract are $(V_1^{\tau_1}(\hat{f}), V_2^{\tau_2}(\hat{f}))$. To see this, note that if agent 1 proposes a corresponding successful mechanism $\Gamma^X = (M, g^X)$ and if agent 2 accepts, then, by definition of "successful mechanism," the resulting continuation equilibrium interim payoffs are indeed $(V_1^{\tau_1}(\hat{f}), V_2^{\tau_2}(\hat{f}))$. Moreover, it is uniquely optimal for agent 2 to accept this proposal, since any challenge would fail and therefore result in outcome (x^0, y^{00}), which from individual rationality of the corresponding f, is worse than $V_2^{\tau_2}(\hat{f})$. It remains only to show that in equilibrium agent 1 must make such a proposal and agent 2 always has an incentive to challenge agent 1's "unsuccessful" mechanism. Observe that if agent 1 did not make such a proposal, then agent 2 could challenge successfully (either by showing that X is not the true feasible action set or by exhibiting number-based interim payoff functions $(V_1, V_2) \in \mathcal{V}$, a deciphering key φ, and a Bayesian equilibrium of $\Gamma^X(p)$ when agents have interim utility functions $(V_1(\varphi^{-1}(\cdot), \cdot), V_2(\varphi^{-1}(\cdot), \cdot))$ such that agents' Bayesian equilibrium interim payoffs are not $V(\hat{f})$. Moreover, by doing so, Assumption A2 guarantees that agent 2 could get a strictly higher interim payoff than $V_2^{\tau_2}(\hat{f})$. Thus, agent 2 always has an incentive to challenge agent 1's "unsuccessful" mechanism. On the other hand, since utility function is strictly increasing in money, agent 1 then would get an interim payoff strictly less than $V_1^{\tau_1}(\hat{f})$, a suboptimal outcome. Hence, agent 1 has an incentive to propose a "successful" mechanism.

To complete the proof, we must show that, for all $\tau \in \mathcal{T}$, there exists a successful mechanism Γ^{X^τ}. To do this, our job is reduced to checking the sufficient conditions for Bayesian implementation (Mookherjee and Reichelstein 1990) are satisfied. We relegate this part of the proof to the Appendix. □

5 Illustration

I would like to illustrate the concepts introduced in Sect. 3 and the main result in Sect. 4 through an example. Suppose that a buyer (B) and a seller (S) can trade a single indivisible good that the seller will produce and the buyer will consume. Let $\mathcal{T} = \{\tau_\ell, \tau_h\}$ be the set of types of the seller. I assume that the seller is informed of the state at the trading stage, while the buyer keeps remaining uninformed. Here I can ignore the buyer's type because he is completely uninformed. The value to the buyer and the cost to the seller (in terms of money) that they assign to the good in state τ are denoted $v_B(\tau)$ and $c_S(\tau)$, respectively, and suppose

$$v_B(\tau_\ell) = 0 \quad \text{and} \quad v_B(\tau_h) = 15$$
$$c_S(\tau_\ell) = 1 \quad \text{and} \quad c_S(\tau_h) = 10.$$

The set of feasible allocations A^τ at state $\tau \in \mathcal{T}$ is defined as follows:

$$A^\tau = X^\tau \times Y^\tau$$

where X^τ and Y^τ will be defined momentarily. X^τ is defined as:

$$X^\tau = \{x(\tau) \in \{0, 1\}\}$$

where for each $\tau = \tau_\ell, \tau_h, x(\tau) = 0$ stands for the case where the good is not produced at state τ and $x(\tau) = 1$ stands for the case where the good is produced and delivered to the buyer at state τ. Y^τ is defined as:

$$Y^\tau = \{y(\tau) | \; y(\tau) \in \{0, 7\}\}$$

where $y(\tau)$ denotes the monetary transfer from the buyer to the seller at state τ.[11]

I can define each agent's state-dependent ex post utility corresponding to a complete contract $f(\cdot) = (x(\tau_\ell), x(\tau_h), y(\tau_\ell), y(\tau_h))$ such that for each τ,

$$u_B(f(\tau); \tau) = \begin{cases} 15 - y(\tau) & \text{if } \tau = \tau_h \text{ and } x(\tau) = 1 \\ -y(\tau) & \text{if } \tau = \tau_\ell \end{cases}$$
$$u_S(f(\tau); \tau) = \begin{cases} y(\tau) - 10 & \text{if } \tau = \tau_h \text{ and } x(\tau) = 1 \\ y(\tau) - 1 & \text{if } \tau = \tau_\ell \text{ and } x(\tau) = 1 \\ y(\tau) & \text{if } x(\tau) = 0 \end{cases}$$

[11] The restrictive nature of Y^τ (which takes only two values here) is assumed for purely expository purpose.

I assume that only the seller makes an investment e_S which increases the probability that the good entails high value. There are only two levels of investment: either $e_S = 1$ (investment) or $e_S = 0$ (no investment). The cost of investment $c(e_S)$ (in terms of money) is given as

$$c(e_S) = \begin{cases} 1 & \text{if } e_S = 1 \\ 0 & \text{if } e_S = 0 \end{cases}$$

It is common knowledge that the likelihood and the way these states of the world depend upon the seller's investment:

$$\begin{array}{l} p(\tau_h | e_S = 1) = 2/3 \\ p(\tau_\ell | e_S = 1) = 1/3 \end{array} \quad \text{and} \quad \begin{array}{l} p(\tau_h | e_S = 0) = 1/2 \\ p(\tau_\ell | e_S = 0) = 1/2 \end{array}$$

Thus, $X^\tau = X$ (state independent) for any $\tau = \tau_\ell, \tau_h$. Define $Z = \{1, 2, \ldots, 16\}$ as the dummy index. Define a deciphering key $\varphi : Z \to A$ as follows:

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$x(\tau_\ell)$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$x(\tau_h)$	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
$y(\tau_\ell)$	0	0	7	7	0	0	7	7	0	0	7	7	0	0	7	7
$y(\tau_h)$	0	7	0	7	0	7	0	7	0	7	0	7	0	7	0	7
$\varphi(k)$	$\varphi(1)$	$\varphi(2)$	$\varphi(3)$	$\varphi(4)$	$\varphi(5)$	$\varphi(6)$	$\varphi(7)$	$\varphi(8)$	$\varphi(9)$	$\varphi(10)$	$\varphi(11)$	$\varphi(12)$	$\varphi(13)$	$\varphi(14)$	$\varphi(15)$	$\varphi(16)$

Define $V_B : Z \to \mathbb{R}$ and $V_S^\tau : Z \to \mathbb{R}$ for each $\tau = \tau_\ell, \tau_h$ as the interim number-based payoff functions. Define $\mathcal{V}_S = \{V_S^{\tau_\ell}, V_S^{\tau_h}\}$ and $\mathcal{V} = \mathcal{V}_B \times \mathcal{V}_S$. The interim utilities are given as below:

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
e_S^*	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
$V_B(k)$	0	−7/2	−7/2	−7	10	16/3	23/3	3	0	−7/2	−7/2	−7	10	23/3	23/3	3
$V_S^{\tau_\ell}(k)$	0	0	7	7	−1	−1	6	6	−1	6	6	6	−2	−2	5	5
$V_S^{\tau_h}(k)$	0	7	0	7	−11	−4	−11	−4	0	0	0	7	−11	5	−11	−4

It can be checked that $V_S^{\tau_\ell}$ and $V_S^{\tau_h}$ are not positive affine transformations of one another. Therefore, every number-based contract $\hat{f} : \{V_S^{\tau_\ell}, V_S^{\tau_h}\} \to Z$ is welfare-neutral. Theorem 1 of this paper shows that welfare-neutrality is the only constraint due to indescribability, while the other constraints (such as Bayesian implementability) remain the same regardless of whether or not the states are indescribable.

Finally, I shall claim that the only no-investment-no-trade contract f (i.e., $f(\cdot) = (x(\tau_\ell), x(\tau_h), y(\tau_\ell), y(\tau_h)) = (0, 0, 0, 0)$ and $e_S^*(f) = 0$) can be implementable even when the states are describable. This is indeed an incomplete contract but is merely derived from the standard adverse selection problem. More importantly, albeit its simple structure of the example, indescribability is shown to be irrelevant for this contractual incompleteness.

Assume that S makes an investment ($e_S = 1$) and there is trade at τ_h ($x(\tau_h) = 1$). When S's type is τ_h, individual rationality for τ_h requires that $y(\tau_h) \geq 10 + 1 = 11$.

Incentive compatibility for S requires that $y(\tau_h) = y(\tau_\ell) \geq 11$. In this case, B's interim payoff is $2/3 \times 15 - y(\tau_h)$. Thus, individual rationality for B requires that $y(\tau_\ell) = y(\tau_h) \leq 10$. This is inconsistent with $y(\tau_h) \geq 11$ and therefore I must have $y(\tau_h) = y(\tau_\ell) = 0$. Then, individual rationality for S with each type requires that $x(\tau_h) = x(\tau_\ell) = 0$. Given $(x(\tau_h), x(\tau_\ell), y(\tau_h), y(\tau_\ell)) = (0, 0, 0, 0)$, it is optimal for S to not make investment. On the other hand, assume that S makes no investment ($e_S = 0$) but there is trade at τ_h ($x(\tau_h) = 0$). When S's type is τ_h, individual rationality for τ_h requires that $y(\tau_h) \geq 10$. Incentive compatibility for S requires that $y(\tau_h) = y(\tau_\ell) \geq 10$. In this case, B's interim payoff is $1/2 \times 15 - y(\tau_h)$. Individual rationality for B requires that $y(\tau_h) \leq 7.5$, which contradicts $y(\tau_h) \geq 10$. If $y(\tau_h) \leq 7.5$, individual rationality for τ_h requires that $x(\tau_h) = 0$. Given $x(\tau_h) = 0$, individual rationality for B requires that $y(\tau_h) = y(\tau_\ell) = 0$. Finally, individual rationality for τ_ℓ requires that $x(\tau_\ell) = 0$. Given this, it is indeed optimal for S to not make investment ($e_S = 0$).

6 Concluding remarks

This paper shows that indescribability *by itself* imposes no additional restriction on the set of implementable contracts, even in the presence of asymmetric information at the trading stage. Relying on their irrelevance theorems, MT argued that bounded rationality is an essential ingredient for contractual incompleteness. On the other hand, Segal (1999) (in Theorem 2) proposed (partial) indescribability even at the *trading stage* as a reason for contractual incompleteness in an otherwise standard rational agents model. This paper, in combination with Kunimoto (2008), offers a different proposal: if we show that agents' ability to foresee future payoffs together with inability to describe physical contingencies can generate additional contractual incompleteness, then asymmetric information at the contracting stage should be considered essential for contractual incompleteness. For example, this asymmetric information might be viewed as the agents' previous experiences to the current situation.

This paper takes indescribability as an exogenous constraint. However, this constraint can be endogenously derived from limited communication between the parties. In general, the acts of formulating and absorbing the content of communication are privately costly: the sender's communication effort and the receiver's attention effort are needed for effective communication.[12] For example, I assume that before the contracting stage, the parties' efforts jointly determine the probability q that the sender's information is properly assimilated by the receiver; with probability $1-q$, the information cannot be substantiated. Here, the content of the communication is the "physical" details of the states. Thus, the current paper takes "describability" as a special case in which $q = 1$ no matter what efforts the agents exert and "indescribability" as another special case in which it is infinitely costly for each agent to make q positive.

As a final remark, I do not claim that the mechanism constructed in this paper will be employed in real contracting situations. Rather, this mechanism should be used to determine which allocations are consistent with contracting by "rational" agents and

[12] See Dewatripont and Tirole (2005) for the detail of this type of argument.

which are not. The contribution of the current paper lies in clarifying where bounded rationality and/or other considerations will be important for explaining contractual incompleteness.

Appendix

In what follows, I will develop the notions needed for Bayesian implementation based on Mookherjee and Reichelstein (1990). I will start by introducing a special class of mechanisms. By a *revelation* mechanism I set $M_i = \mathcal{T}_i$ for all $i \in N$. For any complete contract f, the induced revelation mechanism $(\mathcal{T}, f)$ will be denoted by Γ_f. The interim payoff of agent i generated by a Bayesian game $\Gamma_f(p)$ if agent i sends τ_i' to the mechanism while his true type is τ_i is given:

$$U_i\left(f|\tau_i, \tau_i'\right) \equiv \sum_{\tilde{\tau}_j \in \mathcal{T}_j} p\left(\tilde{\tau}_j, \tau_i|e^*(f)\right)u_i\left(f\left(\tilde{\tau}_j, \tau_i'\right); \tilde{\tau}_j, \tau_i\right) - c_i(e_i^*(f)).$$

The interim payoff of agent i generated by a Bayesian game $\Gamma_f(p)$ when agent i tells the truth as his message is denoted as follows:

$$U_i(f|\tau_i) \equiv U_i(f|\tau_i, \tau_i).$$

I define f to be incentive compatible at the trading stage.

Definition 2 A complete contract f is *incentive compatible* if

$$U_i(f|\tau_i) \geq U_i\left(f|\tau_i, \tau_i'\right) \quad \text{for all } \tau_i, \tau_i' \in \mathcal{T}_i, \text{ and } i \in N.$$

f is incentive compatible if and only if telling the truth constitutes a Bayesian equilibrium of the revelation mechanism Γ_f. Due to the standard revelation principle, incentive compatibility is a necessary conditions for Bayesian implementation. I will introduce a much richer class of mechanisms below:

Definition 3 An augmented revelation mechanism is a mechanism $\Gamma = (M, g)$ such that for all $i \in N$, $M_i = \mathcal{T}_i \cup B_i$, where B_i is an arbitrary set.

In the above definition, B_i represents the set of additional message options made available to agent i, in addition to messages about his type. See the result below.

Theorem 2 (Mookherjee and Reichelstein 1990) *If $f : \mathcal{T} \rightarrow A$ is Bayesian implementable, then f is implementable in Bayesian equilibrium by an augmented revelation mechanism, in which truthful reporting is an equilibrium.*

This is what Mookherjee and Reichelstein call the *Augmented Revelation Principle*: If this revelation mechanism admits an equilibrium which results in undesired outcomes, it must be true that some agent has a non-type message (in the augmented mechanism) which eliminates this suboptimal equilibrium without upsetting the truth-telling equilibrium. I need two definitions to state another necessary condition for Bayesian implementation.

Definition 4 A Bayesian equilibrium σ in a revelation mechanism Γ_f can be *selectively eliminated* if there exist $i \in N$ and $h : \mathcal{T}_j \to A$ $(j \neq i)$ with the following two properties:

1. $U_i(h \circ \sigma_j|\bar{\tau}_i) > U_i(f \circ \sigma|\bar{\tau}_i)$ for some $\bar{\tau}_i \in \mathcal{T}_i$,
2. $U_i(f|\tau_i) \geq U_i(h|\tau_i)$ for all $\tau_i \in \mathcal{T}_i$.

In the above definition, agent i is offered a new message option, which Mookherjee and Reichelstein refer to as a "flag". When agent i chooses the flag rather than a type message, and other agent j report τ_j, the outcome chosen by the mechanism is given by $h(\tau_j)$. The first condition in the above definition says that agent i prefers to deviate from σ_i to the flag in some state, thereby destroying σ as an equilibrium. However, the second condition ensures that agent i does not deviate from truth-telling to the flag, provided that the other agent j is also truthful. The truthful equilibrium is thus preserved.

Definition 5 The revelation mechanism Γ_f satisfies the *selective elimination* condition if every Bayesian equilibrium σ in Γ_f satisfying $f \circ \sigma \neq f$ can be selectively eliminated.

Here $f \circ \sigma = f$ if $f(\sigma(\tau)) = f(\tau)$ for each $\tau \in \mathcal{T}$ and $f \circ \sigma \neq f$ otherwise. Mookherjee and Reichelstein (1990) show that incentive compatibility and the selective elimination conditions are necessary for Bayesian implementation.

Finally, I am ready to state the sufficient conditions for Bayesian implementation for the number-based contracts.

Theorem 3 *Suppose Assumption A holds and the number-based contract $\hat{f}$ corresponds to a welfare-neutral complete contract f. Then, $\hat{f}$ is Bayesian implementable if there exists an incentive compatible revelation mechanism Γ_f which satisfies the selective elimination condition relative to f.*

Proof of Theorem 3 We deal with a specified set X and so standard implementation theory applies. Let us assume that for any $i \in N$, any $V_i \in \mathcal{V}_i$ with $V_i : Z \times Y_i \to \mathbb{R}$, for any X for which $|Z| = |X|$, and for any deciphering key $\varphi : Z \to X$, there exists $\tau_i \in \mathcal{T}_i$ such that

$$U_i\left(\cdot, \cdot|\tau_i\right) = V_i\left(\varphi^{-1}(\cdot), \cdot\right).$$

This condition was not hypothesized. However, if it is violated, then there are fewer payoff functions to deal with, and so implementation is all the easier. That is, the claim holds a fortiori. For any X, let $\mathcal{T}_1^X \times \mathcal{T}_2^X \equiv \mathcal{T}^X = \{\tau \in \mathcal{T}|X^\tau = X\}$.

The proof proceeds inductively. We start with the revelation mechanism $\Gamma_f^X = ((\mathcal{T}_1^X, \mathcal{T}_2^X), f)$. Since f is incentive compatible, the truth-telling is a Bayesian equilibrium of the revelation mechanism Γ_f^X. In the rest of the argument, we selectively eliminate its suboptimal equilibria by augmenting Γ_f^X. The resulting augmented mechanism will have no new equilibria. If there are any suboptimal equilibria remaining, we construct another augmentation. At any given stage, we begin with an augmented mechanism Γ^X in which the truth-telling is an equilibrium, and all equilibria involve "type" messages only. At the next stage, we obtain $\bar{\Gamma}^X$, an augmentation of Γ^X, which possesses the above properties and has at least one less suboptimal equilibrium. Since Γ_f^X can have at most a finite number of equilibria, a finite number of augmentations will achieve the desired mechanism.

We describe a representative stage of this iterative procedure. Suppose we start with $\Gamma^X = (M, g)$, an augmentation of Γ_f^X with the following properties:

- $M_i = \mathcal{T}_i^X \cup B_i$ for each $i \in N$;
- $g(m) \in X \times Y$ for each $m \in M$;
- For every equilibrium σ of Γ^X, $\sigma_i(\tau_i) \in \mathcal{T}_i^X$ for each $\tau_i \in \mathcal{T}_i^X$ and each $i \in N$.

Let σ be a suboptimal equilibrium of Γ^X and suppose that agent 1 is the agent designated by the selective elimination condition relative to f. Consider the mechanism $\bar{\Gamma}^X = (\bar{M}, \bar{g})$ with:

- $\bar{M}_1 = M_1 \cup FL$;
- $\bar{M}_2 = M_2 \cup \{CFL^1, \ldots, CFL^{k_2}\}$, where $k_2 = |\mathcal{T}_2^X|$.

Thus, agent 1 is given a new message called "flag," denoted FL, and agent 2 is given a set of corresponding messages called "counterflags". Naturally, outcomes associated with messages in the previous stage are left unchanged. Outcomes associated with the new messages satisfy the following four rules:

1. If agent 1 chooses FL and agent 2 chooses type message, then the outcome is given by $h(\tau_2) = (x^h(\tau_2), y_1^h(\tau_2), y_2^h(\tau_2))$ where h is provided by the selective elimination condition corresponding to the equilibrium σ. Here $x^h : \mathcal{T}_2^X \to X$ and $y_i^h : \mathcal{T}_2^X \to Y_i$ for each $i \in N$. This gives rise to:

$$\begin{aligned} \bar{g}(m) &= g(m) \quad \text{for all } m \in M \\ \bar{g}(FL, \tau_2) &= h(\tau_2) \; \forall \, \tau_2 \in \mathcal{T}_2^X, \\ &\qquad \text{where } h(\tau_2) = \left(x^h(\tau_2), y_1^h(\tau_2), y_2^h(\tau_2)\right) \end{aligned}$$

2. If agent 1 chooses the current flag and agent 2 is choosing a previous flag or counterflag, then every agent is treated as if agent 1 had reported τ_1^1 instead of FL except that agent 1 is charged $\varepsilon > 0$ for choosing FL.

$$\begin{aligned} \bar{g}_1(FL, b_2) &= \left(g_x\left(\tau_1^1, b_2\right), g_{y_1}\left(\tau_1^1, b_2\right) - \varepsilon\right) \quad \forall \, b_2 \in B_2 \\ \bar{g}_2(FL, b_2) &= g_2\left(\tau_1^1, b_2\right) \quad \forall \, b_2 \in B_2 \end{aligned}$$

   Here we shall use the notation $g(m) = (g_1(m), g_2(m))$; $g_i(m) = (g_x(m), g_{y_i}(m))$ for each $i \in N$.

3. If agent 2 raises a new CFL^j, but agent 1 does not choose FL, we have two cases: (a) If agent 1 chooses a message $b_1 \in B_1$, then the outcome is as if agent 2 had announced τ_2^j instead of CFL^j; (b) If agent 1 chooses $\tau_1 \in \mathcal{T}_1^X$, agent 2 is treated as if agent 2 had announced τ_2^j, except that agent 2 is charged $\varepsilon > 0$. The same

principle applies to agent 1, except that he gets the private transfer $y_1^*(\tau_2^j) > 0$ (to be determined below) if he announces τ_1^1.

$$\begin{aligned}\bar{g}\left(b_1, CFL^j\right) &= g\left(b_1, \tau_2^j\right)\\ \bar{g}_1\left(\tau_1^1, CFL^j\right) &= \left(g_x\left(\tau_1^1, \tau_2^j\right), y_1^*\left(\tau_2^j\right)\right)\\ \bar{g}_1\left(\tau_1^k, CFL^j\right) &= g_1\left(\tau_1^k, \tau_2^j\right) \quad \text{for} \quad k \neq 1\\ \bar{g}_2\left(\tau_1, CFL^j\right) &= \left(g_x\left(\tau_1, \tau_2^j\right), g_{y_2}\left(\tau_1, \tau_2^j\right) - \varepsilon\right)\end{aligned}$$

We use Assumption A2 to choose $y_1^*(\tau_2^j)$ big enough such that for all $\tau_1 \in \mathcal{T}_1^X$:

$$V_1^{\tau_1}\left(\varphi^{-1}\left(g_x\left(\tau_1^1, \tau_2^j\right), y_1^*\left(\tau_2^j\right)\right)\right) > V_1^{\tau_1}\left(\varphi^{-1}\left(g\left(FL, \tau_2^j\right)\right)\right)$$

and

$$y_1^*\left(\tau_2^j\right) > g_{y_1}\left(\tau_1^1, \tau_2^j\right)$$

4. Finally, if FL meets CFL^j, we apply the following rule with the reward $\delta > 0$ to agent 2:

$$\begin{aligned}\bar{g}_1(FL, CFL^j) &= \bar{g}_1\left(FL, \tau_2^j\right)\\ \bar{g}_2(FL, CFL^j) &= \left(\bar{g}_x\left(FL, \tau_2^j\right), \bar{g}_{y_2}\left(FL, \tau_2^j\right) + \delta\right)\end{aligned}$$

This completes the specification of the augmented mechanism $\bar{\Gamma}^X$. The proof is established by the following claims, which state that σ is not an equilibrium in $\bar{\Gamma}^X$, and that there is no equilibrium in $\bar{\Gamma}^X$ where agents 1 and 2 choose any of their new message options.

Claim 1 *σ is not an equilibrium in $\bar{\Gamma}^X$.*

Proof of Claim 1 Note first that $\sigma_i(\tau_i) \in \mathcal{T}_i^X$ for all $\tau_i \in \mathcal{T}_i^X$ and all $i \in N$. Given Rule 2 and the selective elimination condition for f, agent 1 has an incentive to raise a flag. This implies that σ is not an equilibrium. □

To establish that there are no new equilibria we introduce the following notation. Let $p_* \equiv \min\{p(\tau_j|\tau_i, e^*(f)) \mid i \neq j, p(\tau_j|\tau_i, e^*(f)) > 0\}$ denote the minimum positive probability assigned by a type of one agent to a type of the other agent. Since the set of types is finite, it is clear that p_* is well defined and strictly positive. Then, it follows that

1. For any $\tau_1 \in \mathcal{T}_1^X$, there exists $\tau_2 \in \mathcal{T}_2^X$ such that $p(\tau_1|\tau_2, e^*(f)) > 0$.
2. For any $\tau_1 \in \mathcal{T}_1^X$ and $\tau_2 \in \mathcal{T}_2^X$, $p(\tau_2|\tau_1, e^*(f)) > 0$ if and only if $p(\tau_1|\tau_2, e^*(f)) > 0$.[13]

[13] The common prior assumption, more precisely, the common support assumption, is important for this property to hold.

Claim 2 *There does not exist an equilibrium in* $\bar{\Gamma}^X$ *where any type of agent* 1 *chooses FL.*

Proof of Claim 2 Suppose, on the contrary, that there is such an equilibrium. Let $\mathcal{T}_1^* \subset \mathcal{T}_1^X$ denote the set of types τ_1 of agent 1 that choose FL in some equilibrium. Let $\mathcal{T}_2^* \subset \mathcal{T}_2^X$ denote the set of types τ_2 of agent 2 that assign positive probability to the event that agent 1's type is in $\mathcal{T}_1^*$. Since types in $\mathcal{T}_2^*$ assign at least probability p_* that agent 1 is choosing FL, every type $\tau_2 \in \mathcal{T}_2^*$ will prefer CFL^j to τ_2, provided ε (used in Rule 2) is sufficiently small relative to δ (used in Rule 3). Hence in the given equilibrium, every type $\tau_2 \in \mathcal{T}_2^*$ will choose either some CFL^j, or messages in B_2.

Since $p(\tau_2|\tau_1, e^*(f)) > 0$ if and only if $p(\tau_1|\tau_2, e^*(f)) > 0$, any type $\tau_1 \in \mathcal{T}_1^*$ assigns probability one to the event that agent 2's type is in $\mathcal{T}_2^*$, and therefore that agent 2 is choosing either some CFL^j or messages in B_2. If agent 2 chooses messages in B_2, Rule 2 dictates that agent 1 can avoid penalty ε by choosing τ_1^1. If agent 2 chooses CFL^{τ_2}, Rule 3 dictates that agent 1 is better off by choosing τ_1^1 instead of FL. It then follows that every type $\tau_1 \in \mathcal{T}_1^*$ will choose τ_1^1 instead of FL. This is a contradiction. □

Claim 3 *There is no equilibrium in* $\bar{\Gamma}^X$ *where some type of agent* 2 *chooses some* CFL^j.

Proof of Claim 3 We argue by contradiction. Using Claim 2, it suffices to consider any equilibrium $\bar{\beta}$ in $\bar{\Gamma}^X$ where no type of agent 1 ever uses *FL*, but there is a non-empty set $\mathcal{T}_2^* \subset \mathcal{T}_2^X$ of types of agent 2 choosing some CFL^j. It then follows that given any $\tau_2 \in \mathcal{T}_2^*$, every type τ_1 of agent 1 who is assigned positive probability by τ_2 (i.e., $p(\tau_1|\tau_2, e^*(f)) > 0$) must be choosing a message $b_1 \in B_1$. Otherwise, we must consider the case in which agent 1 chooses a type message $\tau_1 \in \mathcal{T}_1^X$. Then, Rule 3 dictates that type τ_2 of agent 2 would be better off reporting τ_2 instead of CFL^j, and thereby avoid the ε-charge imposed whenever agent 1 chooses type messages in $\mathcal{T}_1^X$.

We now argue that the strategy profile β obtained from $\bar{\beta}$ by replacing the message CFL^j with τ_2^j is an equilibrium in the previous stage mechanism Γ^X. Since any type of agent 2 choosing CFL^j in $\bar{\beta}$ assigns probability one to the event that agent 1 is choosing some "non-type" messages in B_1, Rule 1 and Rule 3 imply that all types of agent 2 are playing best responses at β in Γ^X.

It remains to show that there is no type of agent 1 that can profitably deviate from β in Γ^X. Rule 3 requires that announcing any message other than τ_1^1 induces the same payoffs for agent 1 in β (in Γ^X) as in $\bar{\beta}$ (in $\bar{\Gamma}^X$). Suppose that type τ_1 of agent 1 chooses τ_1^1 in both β and $\bar{\beta}$. Because of Rule 1 and Rule 3 and the construction of the private transfer $y_1^*(\tau_2^j)$, the payoff of agent 1 of type τ_1 is uniformly lower in β compared to $\bar{\beta}$. For any type of agent 1 that does not choose τ_1^1 in $\bar{\beta}$ (in $\bar{\Gamma}^X$), it therefore does not pay to deviate to τ_1^1 in β (in Γ^X). Finally, consider a type τ_1 of agent 1 that does not choose the payoff message τ_1^1 in $\bar{\beta}$ (in $\bar{\Gamma}^X$). By the above reasoning, that type τ_1 will assign zero probability to the event that agent 2 chooses some CFL^j. Hence, the payoffs in β associated with all possible messages in M_1, including τ_1^1, are exactly what they were in $\bar{\beta}$.

In summary, we have established that an equilibrium $\bar{\beta}$ involving some CFL^j's but not FL would give rise to a corresponding equilibrium β in the previous stage mechanism Γ^X in which τ_2^j is substituted for CFL^j. However, in this equilibrium, some types of agent 1 must be choosing "non-type" messages in B_1, a contradiction to our initial hypothesis that every equilibrium in Γ^X involves type messages only. □

Since the set $\mathcal{T}$ is finite, there can only be a finite number of equilibria in Γ_f^X, requiring only a finite number of augmentation stages to eliminate all suboptimal equilibria. Throughout the whole process of augmentations, on the other hand, we continue to keep the truth-telling as a Bayesian equilibrium. This completes the proof of Theorem 3. □

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