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Abstract

On a "marketplace" platform, where two sides of users trade, the platform owner has an incentive to regulate its marketplace for a higher profit. This study focuses on a monopoly platform's nonpricing, regulatory strategies in governing quality heterogeneity of competing sellers. In contrast to related studies, we endogenize strategic interactions among platform users. Our model extends the circular city model to capture seller heterogeneity in both variety and quality. The closed-form equilibrium solution reveals a ripple effect that exerts competitive pressure from seller to seller at a diminishing magnitude. The equilibrium analysis enables us to connect the economic mechanisms in users' trading strategies with the platform's regulatory problem. We find that the platform does not benefit from an equal support to all sellers that increases the average quality. Instead, the platform owner is better off providing discriminatory support in favor of higher-quality sellers to enhance quality heterogeneity. Moreover, the optimal quality support rate is lower for a higher average of seller quality because quality support is more costly; on the other hand, a higher variance make quality levels more responsive to discriminatory support and leads to a higher support rate. A higher transportation cost for buyers diminishes the optimal support rate because quality support is less effective when sellers are more differentiated. Lastly, we also consider non-competitive equilibria of the circular city model to discuss the feasibility of the competitive case.

Keywords: Platform regulation, variety, quality heterogeneity, circular-city model.

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1 Introduction

Many platforms host an online marketplace, where buyers trade with a diversity of sellers. For example, Airbnb joins hosts, who have unused living space, and renters, who need short-term lodging. Its property listings include apartments, houses, and cottages of any size, location, and style. Amazon is another well-known platform, which predominantly serves buyers and third-party sellers who provide products in over a hundred categories from grocery to video games (Wohlesen 2013). Similarly, mobile application markets list applications provided by third-party developers, encompassing numerous categories and varying levels of ratings. The two largest markets, Apple's App Store and Google Play, both passed the 25-billion download mark in 2012 (Miller 2012, Moscaritolo 2012). Some other platforms that engage transactions between buyers and a great diversity of sellers include eBay, video game markets for different consoles (e.g., Sony PlayStation, XBox, Nintendo, etc.), and eBook markets for Kindle and other e-readers.

We refer to this type of platforms as "marketplace" platforms, on which trading between two sides of users involves strategic pricing and purchasing decisions. Here, a seller's/buyer's payoff is not simply based on the number of buyers/sellers to trade with. Characteristics of competing sellers, buyers' preferences, and the number of sellers and buyers jointly determine each platform user's payoff, which then shapes the platform owner's revenues.

Platform owners commonly employ non-pricing regulatory policies to optimize revenues generated from the vibrant economy of buyer-seller transactions (Boudreau and Hagiu 2009). Airbnb dynamically ranks listings according to the quality of their recent activities (e.g., reviews, nights booked, response rate, cancellations, etc.). Both Apple and Google reward high-quality applications through "Apple Design Awards" (WWDC 2012) and "Android Developer Challenge" (ADC 2012), respectively. More interestingly, Taobao Mall (a dominant B2C online marketplace based in China) imposes higher fees on poor-performing sellers (Fletcher 2011). These regulation mechanisms treat sellers according to their performances to create suitable incentives for boosting the platform owner's profits.

This paper examines a "marketplace" platform owner's non-pricing regulation strategies. For this type of platforms, regulatory policies affect the platform's revenues by altering seller heterogeneity and competition, and both sellers' and buyers' incentives. Recent research is drawing more attention to heterogeneity and interactions among platform users—rather than the sheer number of users—to understand the impact of such heterogeneity on values of users and the platform owner (Hagiu 2009, Boudreau 2011, Lin et al. 2011, Ceccagnoli et al. 2012). For instance, Boudreau (2011) underscores that "it is the heterogeneity and diversity of producers, rather than just the added numbers of producers per se, that play a crucial role in generating [this] variety" (p. 1). And in the context of value cocreation, Ceccagnoli et al. (2012) find that differences in independent software vendors' (ISVs') capabilities create heterogeneity in their performance improvements. Our work echoes this emphasis on heterogeneity. We anchor on the microfoundation to account for sellers diversity, buyers' preferences, and their strategic responses to one another. We then tease out the economic mechanisms through which regulations on seller heterogeneity impact sellers' and buyers' payoffs and the platform owner's profits.

We study seller diversity in two dimensions: variety and quality heterogeneity. Variety refers to the number of sellers that are horizontally differentiated. Sellers provide products that carry different attributes—such as color, style, functionality, etc.—to target buyers of different tastes. For example, on a typical day, the eBay market displays more than 2 million postings for women's shoes alone, which can be sorted into different styles, brands, heel heights, sizes, and so on. Similarly, Airbnb listings show apartments located near downtown, country homes in tranquil neighborhoods, studios next to hiking trails, and other locations that suit renters' different needs. Variety in the platform market has strong economic implications. When such variety expands, the new products may attract a untapped buyer segment, and the matching between products and buyers' tastes is improved. Meanwhile, more variety could lead to a crowded marketplace with increasingly similar products; as a result, the substitution effect intensifies competition between sellers.

A variety of sellers tend to also exhibit different levels of quality, or vertical differentiation. For instance, bed sheets of various patterns (variety) may also be of different thread counts (quality). Although buyers often prefer sheets of 300-thread-count over those of 100-thread-count, they may face a tradeoff between sheets of their favorite patterns and those of higher quality among the available products. Sellers can strategically exploit this tradeoff based on characteristics of their products relative to those offered by their competitors. Moreover, critical to online transactions is not only product quality, but also quality-relevant attributes associated with the sellers. In particular, buyers would prefer transacting with a high-quality seller who offers reliable service, speedy shipping, and attractive product guarantee. This is often reflected in the seller's rating based on its past transactions. Overall, quality heterogeneity among a variety of sellers further complicates buyers' choice considerations, sellers' pricing strategies, and the transaction volume and profits on the platform.

This paper addresses the following questions: How does quality heterogeneity among a variety of sellers affect their competition, pricing strategies, and profits? How do both seller variety and quality heterogeneity affect the platform owner's profits? How should the platform owner regulate quality heterogeneity among a variety of sellers? We extend the circular city model (Salop 1979) to differentiated quality levels to capture seller diversity and obtain the closed-form solution for sellers' equilibrium prices and profits. We then derive the platform owner's profits and analyze its regulatory strategies on seller quality heterogeneity. The paper also discusses issues of lower seller variety where competition does not arise.

We derive the equilibrium of seller competition, which shows a ripple effect among sellers' pricing strategies. In other words, changes to any seller's price propagate to its immediate neighbors, then to their neighbors, and so on at a decreasing magnitude. We also find that the average equilibrium price does not depend on sellers' quality because the ripple effect distributes quality effects across all sellers' prices, keeping the sum of prices constant.

Based on the platform owner's percentage royalty revenue model, the equilibrium analysis delivers findings on the impact of seller diversity on the platform's profits. The microfoundation endogenizes how sellers' equilibrium strategies and profits change according to their quality levels and network size. First, a higher degree of quality heterogeneity among sellers leads to more profits for the platform owner. Wider dispersion of quality reduces expected competition intensity between neighboring sellers, which also benefits the other sellers through the ripple effect; thus, the total profit increases. Second, seller variety has mixed effects on the platform owner's profits. In the absence of quality heterogeneity, an increase in seller variety intensifies competition and reduces the platform's profits; however, with different quality levels, variety amplifies the positive effect of quality heterogeneity, which then lifts up the platform's profits.

From the regulatory perspective, it is optimal for the platform owner to offer discriminatory quality support rather than uniform quality support. In particular, altering the quality among all sellers by a fixed amount has no effect on the platform's (or sellers') profits. The value of any improvement of each seller's quality is offset by competition between neighboring sellers — the degree of quality heterogeneity remains unchanged. On the other hand, discriminatory regulation that supports higher-quality sellers to a greater extent amplifies the variation of sellers' quality levels, which mitigates competition and raises total profits.

Moreover, a platform's optimal quality support rate depends on the mean and variance of its sellers' quality levels and its buyers' transportation cost. A higher average quality level makes quality support more costly while not affecting the platform owner's profits; thus, a higher average quality reduces the optimal quality support rate. Conversely, a greater quality variance is more responsive to support and leads to a higher quality support rate. Buyers' transportation cost reflects the degree of horizontal differentiation among sellers. A higher transportation cost mitigates neighboring sellers' competition, which in turn reduces the effectiveness of the quality support mechanism and lowers the optimal support rate.

We contribute to the literature on two-sided platforms by developing a theory on platform regulation based on the microfoundation of platform users' transactions, whereas the majority of existing studies focuses on platform pricing and users' entry decisions. Interactions of platform users on the transaction level exert complex economic forces that are not captured by network effects alone. We characterize the equilibrium of sellers' price competition to better understand platform users' behaviors and the platform's decision problem. Our work is also among the first to examine the platform's non-pricing instruments. The connection between platform users' strategic decisions and the platform owner's profit illustrates the economic mechanisms underlying the performance-based regulatory policies.

Furthermore, our equilibrium characterization of platform users offers an important contribution to the literature of spatial competition with heterogeneity. Although a few recent studies incorporate heterogeneity in the circular city model (Alderighi and Piga 2010, Syverson 2004, Vogel 2008), to the best of our knowledge, we are the first to account for quality heterogeneity in this setting and to derive the closed-form equilibrium solution. The availability of explicit solution forms removes the reliance on numerical approximation and permits further analytical studies, such as comparative statics. In our work, the equilibrium solution also indicates the degrees of the ripple effect based on sellers' relative locations, which is fundamental to competition among a diversity of sellers.

Our findings generate several managerial implications. First, specific dimensions of sellers' characteristics—variety and quality—are directly linked to platform profitability. The platform's strategies should not be limited to expanding user base; in fact, accounting for buyers' preferences and seller competition is critical for the platform's profits. Second, altering the average seller quality is ineffective, whereas increasing the degree of quality heterogeneity benefits the platform. An optimal policy is to offer discriminatory quality support that favors high-quality sellers. The platform owner should also adjust its quality support rate according to changes in the average seller quality, and the degrees of quality heterogeneity and horizontal differentiation.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature and highlight the contribution of our work. Section 3 presents the model setup. In Section 4, we characterize and analyze the equilibrium of competing sellers on the platform. Based on these results, we examine the platform's regulatory strategies in Section 5. Then, Section 6 extends the discussion to lower varieties of sellers. Finally, Section 7 concludes the paper.

2 Literature

Our paper is related to two streams of literature: two-sided platforms and spatial competition with heterogeneity. Compared to other studies of two-sided platforms, we focus on the transactions on the platform and examine trading between two sides of users by building a microfoundation rather than explicitly modeling network effects. This allows us to understand the economic forces that connect platform users' pricing and purchasing strategies to the platform's regulatory decisions. We contribute to the Hotelling literature by incorporating quality heterogeneity into a circular city model and solving the closed-form equilibrium solution.

Whereas earlier works in the platform literature focus primarily on two-sided pricing issues (Rochet and Tirole 2003, 2006, Caillaud and Jullien 2003, Hagiu 2006, Armstrong 2006, Armstrong and Wright 2007, Yoo et al. 2002), recent research development has shown a keen interest in platform regulation. Boudreau and Hagiu (2009) emphasize the regulatory role of the platform owner. They provide a conceptual framework and case studies to illustrate that non-pricing regulatory instruments are powerful for the platform to obtain higher profits from its complex ecosystem. Casadesus-Masanell and Halaburda (2011) present a model showing that the platform owner has an incentive to limit the variety of applications. They point out that the platform can limit application choice to help resolve problems that might prevent users from buying the same application, which benefit users through consumption complementarities. Claussen et al. (2013) study a reward mechanism on Facebook based on a natural experiment, which is a rule change that rewards engaging applications. They show several key outcomes of such a reward, including higher user ratings on applications and a slower decline in the number of active users. Their findings underscore the efficacy of platform incentive systems on its users' behaviors. Our work closely connects with these studies by analytically modeling the platform's non-pricing problem in regulating quality heterogeneity among sellers. Our model accounts for both seller variety and quality heterogeneity and leads to sellers' equilibrium prices and profits. Our findings explain the economic mechanisms that incentivize the platform owner to offer quality support that favors higher-quality sellers, a phenomenon empirically examined by Claussen et al. (2013).

Platform-based models are particularly relevant in the context of information technology for better understanding policies and business models on the Internet. Bhargava and Chaudhary (2004) explore versioning strategies of infomediaries. They find that infomediaries who market to two sides of users benefit more from offering quality-differentiated services than sellers of information goods. Their findings offer guidelines for infomediaries in differentiating service levels. To address the well-known debate on net neutrality, Cheng et al. (2011) model Internet service providers (ISPs) as two-sided platforms connecting content providers and consumers. They find that the ISP benefits from the abolishment of net neutrality, whereas content providers' profits suffer; however, net neutrality generally induces higher investment by the ISP in its infrastructure capacity. Although our model emphasizes the non-pricing aspect of the platform's strategies, our approach in analyzing platform users' interactions echoes with that in Cheng et al. (2011) by capturing horizontal differentiation among sellers (e.g., content providers in Cheng et al. (2011)). In addition, we consider sellers' quality heterogeneity and endogenize their pricing decisions.

Several other papers on two-sided platforms also explicitly analyze sellers' price competition in trades with buyers. Lin et al. (2011) study sellers' innovation race which precedes entry into the platform market. They model the interaction between the platform's pricing strategy and sellers' innovation incentives and solve equilibrium pricing decisions of vertically differentiated sellers in a duopoly setting. In the current paper, we consider competition between a variety of sellers that also differ in quality. Economides and Katsamakas (2006) examine two-sided pricing decisions of operating system platforms. They show that such a platform earns more profits when software users have a strong preference for application variety. They use linear functions for the buyer-side demand and build into them the exogenous price effects to capture the price competition among the software applications. Whereas the reduced-form demand function in Economides and Katsamakas (2006) guarantees a positive effect of additional sellers, our model allows the effects of increasing variety to arise endogenously through sellers' and buyers' equilibrium decisions. Although our paper is similar in using the number of sellers to represent variety, we endogenize the price competition through buyers' preferences and sellers' strategic decisions and further address quality heterogeneity among sellers, which is not considered in Economides and Katsamakas (2006). Moreover, Hagiu (2009) analyzes the impact of buyers' preference for variety on the platform's pricing strategies, allowing buyers to purchase multiple products on a platform. His approach differs from our work in that buyers and sellers are heterogeneous in their entry costs but not in their preferences. Thus, he focuses on user heterogeneity in the entry stage, whereas we highlight user heterogeneity in the transaction stage.

Our study is related to the literature on horizontal differentiation and localized competition. Extending from the classic Hotelling model (1929), Salop (1979) proposes a circular city model to represent multiple competing sellers that are horizontally differentiated. Numerous studies apply Salop's circular city model (1979) to enrich our understanding of various market scenarios. Economides (1993) compares two setups in which firms in a circular city model make entry (location), quality, and pricing decisions; in one setup, firms precommit on quality. He derives symmetric equilibrium among firms in each scenario and finds that precommitment leads to a greater variety but lower quality. Even though quality variation is considered in Economides (1993), in equilibrium quality heterogeneity is not present. Our study tackles the problem of characterizing an equilibrium among horizontally differentiated sellers who also have heterogeneous quality levels.

Syverson (2004), Vogel (2008), and Alderighi and Piga (2012) are directly related to our work. Among these, only Alderighi and Piga (2012) capture the ripple effect highlighted in our model, that is, changes in the price of one seller propagate to the neighboring sellers, then their neighbors, and so on. Alderighi and Piga (2012) generalize the properties of the coefficients in the numerical solution of equilibrium prices and derive implications for geographic concentration of downstream retailers in relation to their upstream wholesalers. Our work makes two major contributions to this stream of studies: First, the heterogeneity that we introduce to the circular city model lies in firms' quality levels, which are factored in to buyers' purchasing decisions, whereas firms' heterogeneity in their efficiency studied by Alderighi and Piga (2012) is not directly relevant to buyers. Second, and more importantly, we obtain the closed-form, analytical solution of the equilibrium prices, market shares and profits, which enables us to further analyze dependencies of the equilibrium on quality heterogeneity.

3 Model Setup

We consider a monopoly platform who regulates variety and quality of its sellers based on a percentage royalty, γ, collected from transactions in its market. The percentage royalty fee is common among "marketplace" platforms, such as Airbnb, Amazon, Apple, and Google. We exclude other types of fee in the main analysis to focus on the platform's non-pricing regulation strategies.[1] The platform first pre-commits to a regulatory policy based on a distribution of sellers' quality levels without the knowledge of individual sellers' quality levels. The uncertainty and delayed realization of sellers' quality are associated with serendipitous nature of R&D and quality-relevant attributes, and information asymmetry in a large marketplace. After quality uncertainties are resolved, the platform applies its regulatory policies on observed quality levels, and then sellers set prices simultaneously while buyers make purchasing decisions; finally, profits are realized.

To model strategic interactions between platform users, we extend the circular-city model (Salop 1979) to allow heterogeneous quality levels, capturing seller diversity in both the variety and quality dimensions. Following Salop (1979), a continuum of buyers is distributed uniformly on a circle of unit circumference. Their locations represent their tastes. Denote by n the number of sellers on the platform, offering products of differentiated variety and quality levels. Because Product variety is generally proportional to the number of sellers on the platform (Boudreau 2011), here n represents the degree of variety on the platform, consistent with the existing literature (Economides and Katsamakas 2006; Hagiu 2009). Suppose sellers' quality levels are independent and identically distributed, denoted by a random variable, v. Since sellers have the same expectation about their quality outcomes, they are located equidistantly on the circle. After positioning their products in the market, sellers' quality levels are realized and observable to all players.

[1] The absence of buyer-side fee aligns with the prevailing practice of online marketplaces (e.g., Amazon, eBay, Airbnb and others), to which buyers have free access.

Without the loss of generality, let seller i be located at $\frac{i}{n}$ and offer products of value $v_i > 0$ at price p_i, where $i \in \{0, 1, ..., n-1\}$. Note that seller $n-1$ is adjacent to seller 0 on the circle. Each buyer purchases one unit of product and incurs a transportation cost due to the difference between the product purchased and that of an ideal match. The transportation cost is linear in the distance between the seller and the buyer at the rate t. Thus, buyer x's utility from purchasing seller i's product is $v_i - p_i - t \cdot |\frac{i}{n} - x|$.

4 Competing Sellers and Equilibrium

By backward induction, we first analyze buyers' purchasing decisions and sellers' equilibrium prices and profits on the platform. Based on these equilibrium results, in Section 5, we examine the platform's regulatory policies. Along the circumference of the circle, each seller's price and quality directly affect its neighboring sellers' decisions; the same effect is then carried to these neighboring sellers' neighboring sellers. Such linkage propagates around the circle, generating *a ripple effect*. To mathematically model this continuous effect around the circle, we extend the range of i, such that $i \in \mathbb{Z}$, where seller i and seller $i \pm n$ are the same entity. For example, seller 0 and seller n refer to the same seller, so do sellers 1, $n+1$, and $2n+1$, and so on.

Following the convention in the literature of spatial competition (Eaton and Lipsey 1978, Syverson 2004, Alderighi and Piga 2012), we examine the equilibrium in which all sellers are locally competitive in the market; in other words, no seller has quality that is too low to attract any buyers. Condition 1 is sufficient for ruling out such cases:

Condition 1. *For any $i \in \mathbb{Z}$, $|v_i - v_{i+1}| < \frac{t}{n}$.*

The buyer located at x between sellers i and $i+1$ is indifferent between the two sellers if

$$v_i - p_i - t\left(x - \frac{i}{n}\right) = v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n} - x\right).$$

From here, we identify the marginal buyer between sellers i and $i+1$:

$$x = \frac{1}{2t}(v_i - v_{i+1} - p_i + p_{i+1}) + \frac{i}{n} + \frac{1}{2n}.$$

Thus, seller i's demand is expressed as,

$$q_i = \frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n}. \tag{1}$$

Its profit is $\pi_i = (1-\gamma)p_i q_i$, which is concave in p_i. The n sellers' optimal prices are given by the following n first order conditions (FOCs):

$$p_i = \frac{t}{2n} + \frac{1}{4}(2v_i - v_{i+1} - v_{i-1} + p_{i+1} + p_{i-1}), \forall i \in \{0, 1, 2, ..., n-1\}. \tag{2}$$

Proposition 1. *(Equilibrium in Platform Market). There exists a unique Nash equilibrium, in which seller i's price is*

$$p_i^* = \frac{t}{n} + v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}, \forall i \in \{0, 1, 2, ..., n-1\}, \tag{3}$$

where $b_d = \frac{\delta^{n-d}+\delta^d}{\sqrt{3}(\delta^n-1)} > 0, \delta = 2+\sqrt{3}$. *Its equilibrium demand is* $q_i^* = \frac{1}{n} + \frac{v_i}{t} - \frac{1}{t}\Sigma_{d=0}^{n-1} b_d v_{i-d}$, *and its equilibrium profit is*

$$\pi_i = (1-\gamma)\frac{p_i^{*2}}{t}. \tag{4}$$

Each seller's equilibrium price, demand, and profit are increasing in its own quality and decreasing in the quality of the other sellers.

Proposition 1 characterizes the closed-form equilibrium solution of the circular city model with seller heterogeneity. To the best of our knowledge, no prior studies provide such an equilibrium characterization. Among closely related works, some models abstract out the role of heterogeneity in sellers' pricing decisions (Syverson 2004; Vogel 2008), while Alderighi and Piga (2012) identifies equilibrium uniqueness and rely on numerical approximations. The significance of the closed-form solution in our work goes beyond mathematical elegance. It empowers us to rigorously illustrate the impact of quality heterogeneity on sellers' equilibrium strategies, profits, and the platform's regulatory decisions.

Proposition 1 offers several insights. First, intuitively, each seller's equilibrium price, demand, and profit are increasing in the quality of its own product. When a seller offers a product of a higher value, ceteris paribus, it attracts additional buyers located further away in terms of preference. The quality superiority compensates for a certain degree of mismatch, enabling the seller to raise its price and gain a higher profit.

Second, any seller's equilibrium price, demand, and profit are not only negatively affected by the qualities offered by its adjacent competitors, but also by those of remote sellers. This implies that changes in the quality of one seller affect all other sellers, regardless of the degree of similarity and direct competition. The ripple effect is the force that connects all sellers: When a remote seller offers a product at a higher quality, the direct competition between that seller and its neighbors induces price-cuts by the neighboring sellers, whose neighboring sellers are also pressured to cut prices. This price cutting strategy propagates around the circle riding on the ripple effect and hits every seller. Furthermore, a seller suffers more from an increase in the quality of a more similar product. From (3), $b_d < b'_d$ if $\min\{d, n-d\} < \min\{d', n-d'\}$, implying that the impact of quality diminishes from seller to seller. Sellers located closer (i.e., more similar) to seller i experience a more pronounced negative impact than the sellers who are further away (i.e., more horizontally differentiated) from seller i.

The ripple effect illustrates competitive forces that are absent from both a traditional circular city model and a duopoly. Our equilibrium characterization explicitly identifies the strengths of across-seller quality effect with the coefficient b_d for different sellers. In contrast, a traditional circular city model does not allow any seller to enjoy an advantage over competitors. Such symmetry in the equilibrium imposes a severe limitation for studying the platform's regulatory decisions. Alternatively, whereas duopoly models offer richer analysis of differentiation, the static seller base of size two also substantially undermines discussions of regulatory policies in the platform context where a large number of diverse sellers is present.

Proposition 2. *(Average Price and Quality). The average equilibrium price of sellers,* $\overline{p} = \frac{t}{n}$*, is independent of quality; however, with more sellers on the platform, the average equilibrium price is lower.*

Although quality levels affect individual sellers' pricing strategies, quality exerts no impact on the average price among sellers. Sellers' price adjustments that respond to changes in quality follow a zero-sum game. From Proposition 1, an increase in one seller's quality raises its own price and reduces the price of all other sellers. The ripple effect distributes the shift in one seller's price to remaining sellers and keeps the average price constant. Expectedly, the average price decreases in the number of sellers on the platform. With more sellers on the platform, their products are more similar; the intensified competition then leads to an overall price cut.

5 Platform as a Regulator

In this section, we examine the platform owner's regulatory problem while maximizing the percentage royalties collected from users' transactions. The analysis builds on the equilibrium profits derived in Section 4; thus, sellers' quality plays a vital role in the platform owner's strategies. We identify the regulation policy through which the platform can alter the value sellers provide to buyers.

5.1 Platform Owner's Profit Function

Substituting in sellers' equilibrium profits from Proposition 1 yields the platform's expected profit in the following Lemma:

Lemma 1. *Before quality levels are realized, the platform's expected profit is:*

$$E(\Pi^*) = \frac{\gamma t}{n} + \frac{\gamma n}{t} Var(v) \left(1 - \frac{4}{3\sqrt{3}} \frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3} \frac{\delta^n}{(\delta^n - 1)^2}\right). \tag{5}$$

The second term of Equation (5) shows that the platform owner benefits from heterogeneous quality levels among sellers. In particular, the platform's expected profit increases proportionally with the degree of quality heterogeneity, because a higher degree of quality heterogeneity widens the expected quality gap between neighboring sellers and allows the higher-quality sellers to lift price and attain greater profits. Although lower-quality sellers in turn suffer losses, the overall profit increase among sellers outweighs the loss, which drives up the platform's profit. Furthermore, and interestingly, the platform's expected profit is independent of the expected quality among sellers. Any shifts in the expected quality dissipate in seller competition — only relative quality levels matter in determining the equilibrium profits.

The impact of seller variety on the platform's profit is mixed. The first term of Equation (5) alone shows a negative effect of seller variety on the platform profit. This implies that, in absence of quality heterogeneity, an increase in seller variety leads to greater similarity among sellers, which intensifies competition and reduces each seller's profit. The second term of Equation (5) shows a positive effect of variety that is amplified by the degree of seller heterogeneity. The intuition here is that, with more sellers, more pair-wise competitions are mitigated by the effect of quality heterogeneity, resulting in higher profit gains by the platform.

5.2 Regulating Participating Sellers

The platform can alter existing sellers' quality levels, which in turn reshapes the quality distribution. One example is additional services provided by the platform that affect either all or some of the sellers. For example, many online marketplaces are equipped with a payment service that alleviates risks of fraud and benefits all sellers and buyers. Several platforms also reward their top-performing sellers, such as the developer awards and top charts in mobile application markets for recognizing top-performing applications.

Let us first consider a regulatory policy that uniformly supports all sellers' quality. For any distribution, shifting all quality levels by a fixed amount only affects the expected value, not the variance. Thus, the platform's expected profit remains the same, suggesting that a common service that benefits all sellers by shifting their qualities up a fixed level generates no additional value, positive or negative, to sellers or to the platform. As discussed in Lemma 1, the intuition is that, as a result of competition, only relative—not absolute—quality levels of neighboring sellers determine their profits.[2] The platform's profit, which is a fraction of sellers' total profits, is then independent of this uniform change to all quality levels.

We then discuss a method of asymmetric quality improvement, in which the platform provides quality support at a proportional rate, α, to each seller's original quality level. Reward top-quality sellers in forms of promotions, prizes, or additional auxiliary services that benefit sellers based on their transactional profits are examples of such proportional quality support. Mathematically, we model the quality support as follows. Observing seller i's product quality v_i, the platform may provide an auxiliary service to generate an additional value of αv_i to buyers who purchase from seller i, so that buyers receive a bundle valued at $(1+\alpha)v_i$. Meanwhile, the platform incurs a variable cost for the extent of quality support it offers: $c(\alpha v_i)^2$. By common assumption, it is increasingly more expensive to achieve a higher quality level. The platform's total cost for supporting all sellers

[2] Buyers internalize the value of such uniform quality increase and realize a higher surplus. Thus, in a model of platform competition, which is not the focus of the current paper, uniform quality support to sellers may be effective because platforms also compete for buyers' participation.

proportionally to their quality levels is:

$$E\left(\sum_{i=0}^{n-1} c\left(\alpha v_i\right)^2\right) = nc\alpha^2\left(E\left(v^2\right)\right) = nc\alpha^2\left(Var(v) + E^2\left(v\right)\right) \tag{6}$$

Therefore, the difference in the platform's expected profit (i.e. $E(\Pi^{*Support}) - E(\Pi^*)$) is:

$$\frac{\gamma n}{t} Var(v)\left(\alpha^2 + 2\alpha\right)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n + 1}{\delta^n - 1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n - 1)^2}\right). \tag{7}$$

Proposition 3. *Offering a common support to all sellers' quality levels does not impact the platform's expected profit. Instead, the platform is better off providing discriminatory support to its sellers. The optimal quality support rate is:*

$$\alpha^* = \frac{1}{\frac{ct}{\gamma\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n+1}{\delta^n-1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n-1)^2}\right)}\left(1 + \frac{E(v)^2}{Var(v)}\right) - 1}. \tag{8}$$

Discriminatory quality support increases the platform owner's revenue by increasing quality heterogeneity among sellers. The intuition follows that underlying the platform's expected profit function (Lemma 1). By rewarding higher-quality sellers with an advantage, the expected competition between any pair of neighboring sellers is lessened. Lower-quality sellers' loss due to such asymmetric quality support is outweighed by profit gains of higher-quality sellers. Therefore, the total profits among sellers, hence percentage royalties charged by the platform, are higher. This type of practices is evident on many platform. Through rewarding and publicizing top-quality applications, platform owners such as Apple and Google effectively raise buyers' valuations for these applications. The promoted applications receive monetary support and are perceived by buyers at an even higher quality. Similarly, Airbnb's search algorithm that ranks listings is based on hosts' guest reviews, response rate, cancellations, and other quality-relevant attributes (Airbnb 2013).

Proposition 4. *For a higher average seller quality level, the platform's optimal quality support rate is lower; however, for a higher variance of sellers' quality levels, the platform's optimal quality support rate is higher.*

Both the average and the variance of quality levels impact the platform's decision in providing discriminatory quality support. A higher average quality level reduces the optimal support rate because it makes the quality support more costly at the same support rate without affecting the platform's profits. The optimal discriminatory quality support rate is, therefore, adjusted downward to offset the cost increase. On the other hand, quality heterogeneity incentivizes a higher support rate. The reason is that more heterogeneous quality levels are more sensitive to discriminatory quality support. The platform can then increase quality heterogeneity more cost-effectively.

An important caveat here is that, because quality support is not costless, the platform owner should calibrate the extent of discriminatory quality support based on characteristics of its sellers. Expected seller quality levels and quality heterogeneity play starkly different roles on the platform's decisions. A platform with more lower-end sellers or a wider range of seller quality can offer more generous discriminatory quality support. As sellers change over time, it is necessary that platform re-evaluates its regulatory policies.

Proposition 5. *For a higher buyer transportation cost, the platform's optimal quality support rate is lower.*

When sellers are more differentiated in terms of variety, the platform offers a lower rate of discriminatory quality support. Eq. (7) indicates that the transportation cost, t, dampens the platform's gains from quality support. The reason is that, if sellers are more differentiated in terms of variety, any support for their quality will have a lower impact on neighboring sellers. In other words, the platform's effort of quality support becomes less effective. Therefore, the optimal support rate is lower with more differentiation in seller variety. This seems to suggest that differentiations in the dimensions of variety and quality are complementary in their effects on the platform's profits.

In addition to a policy with discriminatory quality improvements discussed above, it is interesting to note that the platform can achieve profit gains by undermining lower-quality sellers. Given that the platform owner yields higher revenues with greater seller quality heterogeneity, extending such heterogeneity toward the lower end has the same effect. As lower-quality sellers experience deteriorated quality, expected quality gaps between neighboring sellers are widened, allowing higher-quality sellers to reap more profits, which again results in higher profits for the platform owner. In fact, imposing a heavier burden on low-quality sellers is observed in practice. For example, Taobao Mall recently revised its policies to raise retailers' security deposits substantially, which, in effect, puts smaller retailers at a competitive disadvantage (Fletcher 2011). This strategy is intriguing because the low-quality sellers' service is impaired due to the tightened financial constraint, and the impairment due the policy revision makes them even less capable than the high-quality sellers in obtaining the deposit refund. Thus, the low-quality sellers bear a higher burden that undermines their quality compared to the high-quality competitors.

6 Discussion of Low Varieties of Sellers

This section presents an extended discussion on seller variety by exploring implications of lower varieties of sellers on the platform's profit. Thus far, our analysis in the main model assumes a sufficient number of sellers (i.e., rich variety) that leads to a competitive marketplace, where quality heterogeneity is a factor in the platform owner's regulatory decisions. On the other hand, low-variety, non-competitive platform marketplaces may pertain to platforms in the transitory, beginning stages, which are less commonly observed. By considering these scenarios, we provide a more in-depth understanding of the optimal scale of platforms. The equilibrium findings show that lower seller varieties that lead to non-competitive marketplaces are suboptimal for the platform owner, which underscores the importance of studying seller competition on a platform.

Modeling non-competitive sellers poses additional analytical challenges. Adding heterogeneity on top of sellers' variety may result in a mixture of competitive and non-competitive pairs of neighboring sellers, which is analytically intractable given a variable number of sellers. With a goal of understanding seller variety in this section, we consider sellers with homogeneous quality level, v. The rest of the model follows that from Section 3.

We first solve for sellers' equilibrium price and profit. The results identify three equilibrium regions based on different levels of variety, n. These equilibrium regions correspond to those identified in Salop (1979): Region I of "monopoly equilibrium," Region II of "kinked equilibrium," and Region III of "competitive equilibrium" (equilibrium terms are adopted from Salop (1979)). Table 1 summarizes the sellers' prices and profits, the platform's profit, consumer surplus and social welfare in the three regions. We relegate the equilibrium analysis to Appendix B.

In Region I, low variety allows sellers to achieve greater horizontal differentiation and to act as local monopolists. Here, differentiation leads to poor matching of sellers' products to buyers' tastes (i.e., buyers incur higher transportation costs). As a result, sellers find it more profitable to exclude some buyers, and a fragmented market emerges.

When variety expands, the equilibrium falls in Region II, and then in Region III, where sellers

Table 1: Equilibrium Characterization of a Variety of Sellers with Homogeneous Quality

Equilibrium Region	**Seller Price p_i^***	**Seller Profit π_i^***	**Platform Profit, Π^***	**Consumer Surplus**	**Social Welfare**
I: $n < \frac{t}{v}$	$\frac{v}{2}$	$\frac{(1-\gamma)v^2}{2t}$	$\frac{\gamma n v^2}{2t}$	$\frac{nv^2}{4t}$	$\frac{3nv^2}{4t}$
II: $\frac{t}{v} \leq n \leq \frac{3}{2}\frac{t}{v}$	$v - \frac{t}{2n}$	$(1-\gamma)\left(\frac{v}{n} - \frac{t}{2n^2}\right)$	$\gamma\left(v - \frac{t}{2n}\right)$	$\frac{t}{4n}$	$v - \frac{t}{4n}$
III: $n > \frac{3}{2}\frac{t}{v}$	$\frac{t}{n}$	$\frac{(1-\gamma)t}{n^2}$	$\frac{\gamma t}{n}$	$v - \frac{5t}{4n}$	$v - \frac{t}{4n}$

compete for buyers. In both regions, all buyers find products sufficiently suitable to their tastes; thus, market is covered. Region II is only semi-competitive in the sense that a seller's marginal buyers—who are located midway between each pair of neighboring sellers—would otherwise be indifferent between not buying at all and buying from the competing neighboring seller. In Region III, however, marginal buyers of a seller would otherwise strictly prefer buying from the competing neighboring seller over not buying. Also, in Region II, neighboring sellers' markets barely touch,

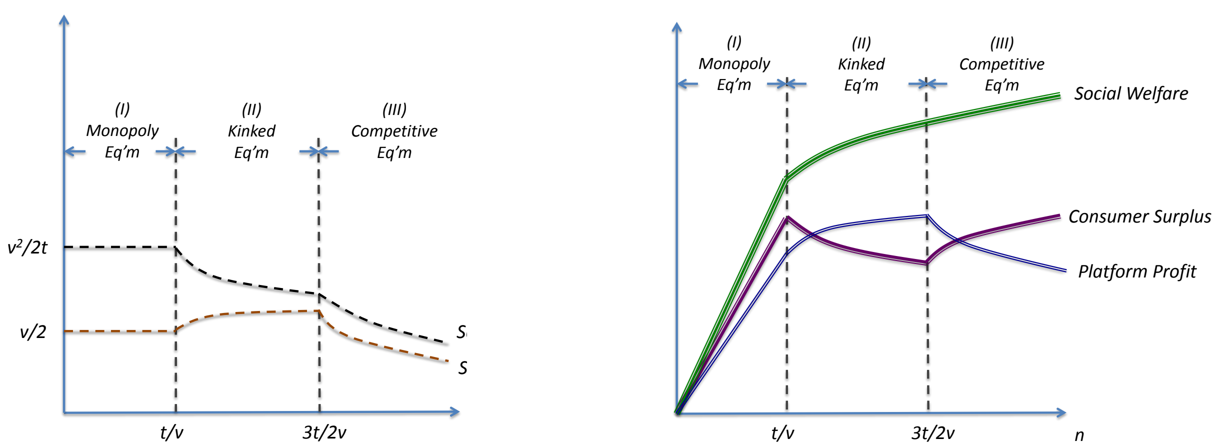

Figure 1: Impact of Variety on the Platform

By comparing the platform's profit in Table 1, we find that the optimal variety is $n^* = \frac{3t}{2v}$, the boundary between Regions II and III (Figure 1). The differences in sellers' pricing strategies in the two regions shape the platform's profit function in attaining this optimal variety. Sellers face two conflicting incentives in the two regions: An increase in variety shrinks each seller's market size, but it also allows better matching with buyers in a tighter neighborhood, creating an incentive for sellers to raise price; Meanwhile, as variety increases, buyers between two neighboring sellers perceive less differentiation between the two sellers, which intensifies seller competition and creates a price-cutting incentive. In the semi-competitive Region II, the incentive to raise price is dominant because each seller still holds a sufficiently large market share and is better off raising price to increase margins. The price-cutting incentive overtakes sellers in the competitive Region III because each seller does not obtain enough market share to gain from price lifts in such a crowded marketplace. Therefore, sellers resort to a price war. The platform benefits from increasing variety in Region II because sellers raise price as variety increases. In Region III, if variety further increases, competition between sellers transfers surplus only to buyers – both sellers and the platform suffer profit losses.

Our main analysis focuses on the competitive equilibrium (Region III), as do the related works (Economides 1993, Syverson 2004, Alderighi and Piga 2012). Thus far, we show that the optimal variety, $n^* = \frac{3t}{2v}$, is at the boundary point of Regions II & III. This indicates that, without quality heterogeneity, the platform owner is better off inducing variety up to a competitive market. In Figure 1, the platform's profit declines as variety further increases in Region III. Notice that, by setting $Var(v) = 0$, the platform's profit (Eq. (5) in Lemma 1) from Section 5 is indeed equivalent to that of Region III from Table 1: $\frac{\gamma t}{n}$. However, the main model examines this region with added quality heterogeneity. In fact, when $Var(v) > 0$, variety (n) has a positive effect that increases with the degree of quality heterogeneity (see the discussion of Lemma 1). Therefore, the competitive equilibrium region is feasible.

For an established marketplace/platform that has achieved a critical mass, the rich variety in the competitive equilibrium is generally more applicable than the other two equilibria. Both monopoly and kinked equilibria (Regions I&II) are transient stages of platform growth that ultimately lead to a platform with a competitive equilibrium (Region III). Region I resembles a platform with a seller-side network small enough to allow all sellers to act as monopolists. Few platforms exhibit this characteristic or maintain this characteristic in the long run. As we have discussed, Region II offers intriguing properties, such as those of marginal buyers' surplus and sellers' pricing strategies. But, they are not sufficiently general or representative for most platforms of interest.[3] The traditional properties of competitive equilibrium are not only more fitting, they are also analytically amenable

[3]Salop (1979) also notes that "the comparative statics at kinked equilibria are all perverse" (p. 149).

to examining additional dimensions of complexity, such as quality heterogeneity in our main model.

7 Conclusion

This paper examines the microfoundation of trades between a diversity of sellers and buyers on a "marketplace platform" and identifies an optimal platform policy for regulating quality heterogeneity of sellers. To understand competition on the platform, we analyze pricing strategies of sellers with differentiated variety and quality and find a ripple effect that drives interdependencies of sellers' pricing strategies. The equilibrium characterization connects platform users' trading strategies with the platform owner's profits and show that seller quality heterogeneity plays an important role in shaping the platform's regulation policies. A higher degree of quality heterogeneity increases the expected quality gap between neighboring sellers and leads to more profit gains than losses. Therefore, the regulation policy that maximizes the platform owner's percentage royalties is a discriminatory quality support that favors higher-quality sellers to enhance quality heterogeneity. The optimal quality support rate also depends on the average and variance of quality levels, which alter the cost-effectiveness of the support mechanism, as well as on buyers' transportation cost.

Our work contributes to the literature on platforms by illustrating the impact of platform user heterogeneity on platform decisions from a theoretical perspective. Several empirical investigations provide strong evidence on the importance of user heterogeneity beyond the sheer size of user networks (Boudreau 2011, Ceccagnoli et al. 2012). Our study is among the few to analytically examine seller diversity on a platform. Our microfoundation differs from that in Lin et al. (2012) in that we allow more than two competing sellers. Also, whereas Lin et al. (2012) solely consider vertical differentiation between two sellers and Hagiu (2009) only models horizontal differentiated sellers, our model captures seller heterogeneity in both variety and quality dimensions. In this setting, we find a ripple effect that sends competitive pressure to all sellers on the platform and explain how variety plays into the effect of quality heterogeneity on sellers' and the platform's profits.

We also provide further understanding on the economic mechanisms of platform regulation through seller competition. In the context of consumption complementarities, Casadesus-Masanell and Halaburda (2011) suggest that the platform should limit the variety of applications. Our findings complement theirs by addressing platform regulation on quality heterogeneity, when a variety of sellers compete in price. Furthermore, the mechanism of supporting sellers proportionally to their quality identified in our result is analogous to the Facebook phenomenon studied by Claussen et al. (2013). They show a number of positive outcomes of this reward mechanism and point out that the efficacy of such incentive systems may eliminate the need for “hard” exclusion policies. Our work serves as the theoretical basis for such implications of their empirical findings.

Several future research directions can be further explored. Whereas the current paper focuses on the transaction-level and the literature focuses more on the entry-level, the problem that combines the analysis of user entry decisions and a microfoundation of trading remains a challenge. Our equilibrium characterization prepares the foundation for such an analysis. A follow-up work could introduce the platform owner's pricing problem for the entry stage to account for indirect network effects while incorporating the transaction stage with a microfoundation. Furthermore, information asymmetry on the platform is another issue not yet well understood. For instance, buyers may not have full information about the platform market until they have gained access. Thus, it may be interesting to introduce uncertainties about product quality and sellers' variety into the problem. The results could shed light on the platform owner's strategies in inducing seller-side innovation efforts.

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Appendix

A Proofs

Proof of Proposition 1

Proof. From Eq. (2), $p_{i+1} = 4p_i - p_{i-1} - \frac{2t}{n} - (2v_i - v_{i+1} - v_{i-1})$, or

$$p_i = 2p_{i-1} - p_{i-2} - \frac{2t}{n} - (2v_{i-1} - v_i - v_{i-2}), \tag{9}$$

which can be rewritten as

$$\begin{aligned} p_i - (2-\sqrt{3})p_{i-1} = (2+\sqrt{3})(p_{i-1} - (2-\sqrt{3})p_{i-2}) - \frac{2t}{n} \\ - (2v_{i-1} - v_i - v_{i-2}), \end{aligned} \tag{10}$$

$$\begin{aligned} (2+\sqrt{3})(p_{i-1} - (2-\sqrt{3})p_{i-2}) = (2+\sqrt{3})\Big[(2+\sqrt{3})(p_{i-2} - (2-\sqrt{3})p_{i-3}) - \frac{2t}{n} \\ - (2v_{i-2} - v_{i-1} - v_{i-3})\Big], \end{aligned} \tag{11}$$

$$\dots$$

$$\begin{aligned} (2+\sqrt{3})^{n-1}(p_{i+1} - (2-\sqrt{3})p_i) = (2+\sqrt{3})^{n-1}\Big[(2+\sqrt{3})(p_i - (2-\sqrt{3})p_{i-1}) - \frac{2t}{n} \\ - (2v_i - v_{i+1} - v_{i-1})\Big]. \end{aligned} \tag{12}$$

Summing up Eq. (10) through Eq. (12), we have,

$$\left[1-(2+\sqrt{3})^n\right](p_i-(2-\sqrt{3})p_{i-1}) = \frac{2t}{n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k - \Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-1}-v_{i-k}-v_{i-k-2}), \tag{13}$$

or,

$$p_i-(2-\sqrt{3})p_{i-1} = \frac{1}{1+\sqrt{3}}\frac{2t}{n} - \frac{1}{1-(2+\sqrt{3})^n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-1}-v_{i-k}-v_{i-k-2}). \tag{14}$$

From here, we derive the following:

$$p_i = (2-\sqrt{3})p_{i-1} + \frac{1}{1+\sqrt{3}}\frac{2t}{n} - \frac{1}{1-(2+\sqrt{3})^n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-1}-v_{i-k}-v_{i-k-2}) \tag{15}$$

$$(2-\sqrt{3})p_{i-1} = (2-\sqrt{3})\left[(2-\sqrt{3})p_{i-2} + \frac{1}{1+\sqrt{3}}\frac{2t}{n} - \frac{1}{1-(2+\sqrt{3})^n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-2}-v_{i-k-1}-v_{i-k-3})\right] \tag{16}$$

...

$$(2-\sqrt{3})^{n-1}p_{i+1} = (2-\sqrt{3})^{n-1}\left[(2-\sqrt{3})p_i + \frac{1}{1+\sqrt{3}}\frac{2t}{n} - \frac{1}{1-(2+\sqrt{3})^n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k}-v_{i-k+1}-v_{i-k-1})\right] \tag{17}$$

Summing up Eq. (15) to Eq. (17), we have:

$$[1-(2-\sqrt{3})^n]p_i = \frac{1}{1+\sqrt{3}}\frac{2t}{n}\Sigma_{l=0}^{n-1}(2-\sqrt{3})^l - \Sigma_{l=0}^{n-1}\frac{(2-\sqrt{3})^l}{1-(2+\sqrt{3})^n}\Sigma_{k=0}^{n-1}(2+\sqrt{3})^k(2v_{i-k-l-1}-v_{i-k-l}-v_{i-k-l-2}). \tag{18}$$

Therefore, seller i's equilibrium price is,

$$p_i = \frac{t}{n} - \frac{1}{[1-(2+\sqrt{3})^n][1-(2-\sqrt{3})^n]}\Sigma_{l=0}^{n-1}\Sigma_{k=0}^{n-1}(2-\sqrt{3})^l(2+\sqrt{3})^k(2v_{i-k-l-1}-v_{i-k-l}-v_{i-k-l-2}). \tag{19}$$

By reindexing the v's, we can write Eq. (19) as:

$$p_i^* = \frac{t}{n} - \frac{1}{[1-(2+\sqrt{3})^n][1-(2-\sqrt{3})^n]} \Sigma_{m=0}^{2n} b_m v_{i-m}, \tag{20}$$

where

$$\begin{aligned}
b_0 &= -1; \\
b_m &= -\frac{1}{\sqrt{3}}\left((2+\sqrt{3})^m - (2-\sqrt{3})^m\right), \text{ for } m = 1, 2, ..., n-1; \\
b_n &= \frac{\sqrt{3}-1}{\sqrt{3}}(2+\sqrt{3})^n + \frac{\sqrt{3}+1}{\sqrt{3}}(2-\sqrt{3})^n; \\
b_m &= -\frac{1}{\sqrt{3}}\left((2+\sqrt{3})^{2n-m} - (2-\sqrt{3})^{2n-m}\right), \text{ for } m = n+1, n+2, ..., 2n-1; \\
b_{2n} &= -1.
\end{aligned}$$

Note that v_i, v_{i-n}, v_{i-2n} refer to the same seller, and that for $m = 1, 2, ..., n-1$, v_{i-m} and v_{i-m-n} refer to the same seller. Therefore, combining the coefficients of the same v_i, for $i = 0, 1, ..., n-1$, we have:

$$p_i^* = \frac{t}{n} + v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}, \tag{21}$$

where $b_d = \frac{(2+\sqrt{3})^{n-d} + (2+\sqrt{3})^d}{\sqrt{3}((2+\sqrt{3})^n - 1)} > 0$. From Eq. (2), we derive the equilibrium demand, $q_i^* = \frac{1}{n} + \frac{v_i}{t} - \frac{1}{t}\Sigma_{d=0}^{n-1} b_d v_{i-d}$. Thus, seller i's equilibrium profit is $\pi_i^* = (1-\gamma)p_i^* q_i^* = (1-\gamma)\frac{p_i^{*2}}{t}$. □

Proof of Proposition 2

Proof. The result follows immediately from $\overline{p} = \frac{t}{n}$. □

Proof of Lemma 1

Proof. The platform owner's expected profit is the expectation of the aggregate of sellers' profits multiplied by the fraction γ:

$$
\begin{aligned}
E(\Pi^*) &= \frac{\gamma}{t} E\left[\Sigma_{i=0}^{n-1} p_i^{*2}\right] \\
&= \frac{n\gamma}{t} E\left[\left(\frac{t}{n} + v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right)^2\right] \\
&= \frac{n\gamma}{t} E\left[\frac{t^2}{n^2} + \frac{2t}{n}\left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right)^2\right] \\
&= \frac{\gamma t}{n} + \frac{n\gamma}{t} E\left[\frac{2t}{n}\left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right)^2\right]
\end{aligned}
$$

It can be shown that $\Sigma_{d=0}^{n-1} b_d = 1$, based on which we further simplify:

$$
\begin{aligned}
& E\left[\frac{2t}{n}\left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right) + \left(v_i - \Sigma_{d=0}^{n-1} b_d v_{i-d}\right)^2\right] \\
= \;& \frac{2t}{n} E(v)\left(1 - \Sigma_{d=0}^{n-1} b_d\right) + E\left(v_i^2 (1-b_0)^2 - 2(1-b_0) v_i \Sigma_{d=1}^{n-1} b_d v_{i-d} + \left(\Sigma_{d=1}^{n-1} b_d v_{i-d}\right)^2\right) \\
= \;& (1-b_0)^2 E\left(v^2\right) - 2(1-b_0)\Sigma_{d=1}^{n-1} b_d E^2(v) + E\left(\Sigma_{d=1}^{n-1} b_d v_{i-d}\right)^2 \\
= \;& (1-b_0)^2 E\left(v^2\right) - 2(1-b_0)^2 E^2(v) + \Sigma_{d=1}^{n-1} b_d^2 E\left(v^2\right) + \Sigma_{j,k=1;j\neq k}^{n-1} b_j b_k E^2(v) \\
= \;& \left[(1-b_0)^2 + \Sigma_{d=1}^{n-1} b_d^2\right]\left(Var(v) + E^2(v)\right) - 2(1-b_0)^2 E^2(v) + \Sigma_{j,k=1;j\neq k}^{n-1} b_j b_k E^2(v) \\
= \;& Var(v)\left((1-b_0)^2 + \Sigma_{d=1}^{n-1} b_d^2\right) + E^2(v)\left((1-b_0)^2 - 2(1-b_0)^2 + \left(\Sigma_{d=1}^{n-1} b_d\right)^2\right) \\
= \;& Var(v)\left((1-b_0)^2 + \Sigma_{d=1}^{n-1} b_d^2\right) \\
= \;& Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n+1}{\delta^n-1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n-1)^2}\right)
\end{aligned}
$$

Therefore, $E(\Pi^*) = \frac{\gamma t}{n} + \frac{\gamma n}{t} Var(v)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n+1}{\delta^n-1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n-1)^2}\right)$.

□

Proof of Proposition 3

Proof. The platform is interested in solving α to maximize the gains from offering quality support minus the costs incurred:

$$
\frac{\gamma n}{t} Var(v)\left(\alpha^2 + 2\alpha\right)\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^n+1}{\delta^n-1} + \frac{2n}{3}\frac{\delta^n}{(\delta^n-1)^2}\right) - nc\alpha^2\left(Var(v) + E^2(v)\right). \qquad (22)
$$

The first-order condition w.r.t. α is:

$$2n\alpha\left[\frac{\gamma}{t}Var\left(v\right)\left(1-\frac{4}{3\sqrt{3}}\frac{\delta^{n}+1}{\delta^{n}-1}+\frac{2n}{3}\frac{\delta^{n}}{(\delta^{n}-1)^{2}}\right)-c\left(Var\left(v\right)+E^{2}\left(v\right)\right)\right]$$
$$+\frac{2n\gamma}{t}Var\left(v\right)\left(1-\frac{4}{3\sqrt{3}}\frac{\delta^{n}+1}{\delta^{n}-1}+\frac{2n}{3}\frac{\delta^{n}}{(\delta^{n}-1)^{2}}\right)=0. \tag{23}$$

Suppose the second-order condition is satisfied, such that $c > \frac{\frac{r}{t}Var(v)\left(1-\frac{4}{3\sqrt{3}}\frac{\delta^{n}+1}{\delta^{n}-1}+\frac{2n}{3}\frac{\delta^{n}}{(\delta^{n}-1)^{2}}\right)}{Var(v)+E^{2}(v)}$, we have an internal solution $\alpha^{*} > 0$. □

Proof of Proposition 4

Proof. It is straightforward to see that Eq. (8) decreases in $E(v)$ and increases in $Var(v)$. □

Proof of Proposition 5

Proof. It is straightforward to see that Eq. (8) decreases in t. □

B Equilibrium Analysis for Lower Varieties of Sellers

Consider the case of local monopolies, with at least some consumers not purchasing between any pair of neighboring sellers. Seller i's profit function is $\pi_i(p_i) = (1-\gamma)p_i \cdot q_i$, where q_i is the demand. Since sellers do not compete, consumer x would purchase from seller i if $v - p_i - t \cdot |\frac{i}{n} - x| > 0$. Thus, $q_i = \frac{2(v-p_i)}{t}$. By taking the first-order condition (FOC) of $\pi_i(p_i)$, we find that $p_i^* = \frac{v}{2}$, $q^* = \frac{v}{t}$, and $\pi_i^* = (1-\gamma)\frac{v^2}{2t}$.

To ensure that in this equilibrium sellers are indeed local monopolies, we need a condition under which neighboring sellers' demands do not overlap: $\frac{q_i^*}{2} + \frac{q_{i+1}^*}{2} < \frac{1}{n}$, which implies $n < \frac{t}{v}$. Therefore, when $n < \frac{t}{v}$, the above equilibrium holds and sellers are local monopolies.

When the sellers compete for the consumers, between seller i and seller $i+1$, consumer x purchases from seller i when $v - p_i - t(x - \frac{i}{n}) > v - p_{i+1} - t(\frac{i+1}{n} - x)$, implying $x < \frac{p_{i+1}-p_i}{2t} + \frac{2i+1}{2n}$; similarly, between seller $i-1$ and seller i, consumer y purchases from seller i when $y > \frac{p_i - p_{i-1}}{2t} + \frac{2i-1}{2n}$. The demand to seller i is $q_i = \frac{p_{i+1}-2p_i+p_{i-1}}{2t} + \frac{1}{n}$. Thus, seller i maximizes $(1-\gamma)p_i q_i$, subject to $v - p_i - \frac{t}{2n} \geq 0$; so that the consumer located midway between seller i and its neighbors has a positive surplus. We then take the FOC of the following Lagrangean equation:

$$L = (1-\gamma)\left[\frac{p_{i+1}-2p_i+p_{i-1}}{2t}+\frac{1}{n}\right]p_i+\lambda\left(v-p_i-\frac{t}{2n}\right) \tag{24}$$

$$\frac{\partial L}{\partial p_i} = (1-\gamma)\left[\frac{1}{n}+\frac{p_{i+1}+p_{i-1}}{2t}-\frac{2p_i}{t}\right]-\lambda=0 \tag{25}$$

$$\frac{\partial L}{\partial \lambda} = v-p_i-\frac{t}{2n}=0 \tag{26}$$

If $\lambda > 0$, then $v - p_i - \frac{t}{2n} = 0$; thus, $p_i^* = v - \frac{t}{2n}$ and $\pi_i^* = (1-\gamma)\frac{p_i^*}{n} = (1-\gamma)(\frac{v}{n} - \frac{t}{2n^2})$. If $\lambda = 0$, then $\frac{1}{n} + \frac{p_{i+1}+p_{i-1}}{2t} - \frac{2p_i}{t} = 0$. Since sellers are homogeneous except their locations, we look for the symmetric equilibrium, thus $p_i^* = \frac{t}{n}$. We can then derive that $\pi_i^* = \frac{(1-\gamma)t}{n^2}$. Since in this case $v-p_i^*-\frac{t}{2n} > 0$, we must satisfy $n > \frac{3}{2}\frac{t}{v}$. Therefore, we have a kinked equilibrium when $\frac{t}{v} \leq n \leq \frac{3}{2}\frac{t}{v}$, and a competitive equilibrium when $n > \frac{3}{2}\frac{t}{v}$. Given that the platform collects γ percentage of sellers' profits. Its profit (Table 1) for each equilibrium follows immediately. Consumer surplus and social welfare can also be easily derived.

Furthermore, the platform's profit is increasing in Regions I and II and decreasing in Region III. Therefore, its profit reaches the maximum at $n^* = \frac{3t}{2v}$.