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Dynamic poverty decomposition analysis: An application to the Philippines

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Dynamic Poverty Decomposition Analysis: An Application to the Philippines

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In this paper, we propose a new method of poverty decomposition. Our method remedies the shortcomings of existing methods and has some desirable properties such as time-reversion consistency and subperiod additivity. Our decomposition integrates the existing methods of growth-redistribution decomposition and sector-based decomposition, because it allows us to decompose the change in poverty into growth and redistribution components for each group (e.g., regions or sectors) in the economy. Our decomposition works well in cases where only partial data are available for some periods. It is also flexible and can be extended to have the following six components: population shift, within-region redistribution, between-region redistribution, nominal growth, inflation, and methodological change components. The empirical application of the six-way decomposition to the Philippines for the period 1985–2009 shows that important policies for poverty reduction may differ across regions. For example, the Autonomous Region in Muslim Mindanao would need growth-enhancing policies, whereas Eastern Visayas would need policies to improve the income distribution. Our decomposition method has a wide applicability and may complement the poverty profile approach.

1. Introduction

Poverty statistics are the most basic piece of information for assessing the poverty situation of a country and for formulating antipoverty policies. With broader recognition of their importance, the availability of poverty statistics has significantly improved over the last four decades. The World Bank's Living Standards Measurement Study (LSMS) website alone lists 40 countries with household surveys,[1] and many other countries not in the list also routinely conduct surveys and publish national poverty statistics without much external assistance.

The quality of poverty statistics has also improved with the accumulation of knowledge and experience. Better survey designs have helped make the measurement of standards of living more accurate and more readily comparable across regions within a country and over years. As a result, we have a better understanding of the profile of the poor and its transition over time.

However, in the standard poverty profile approach, it is often unclear what has caused the observed change in poverty. Adding to this problem, the methodology used to derive national poverty statistics is not always uniform, making the poverty statistics incomparable across regions or over time. To address these issues, we offer a new methodology of poverty decomposition in this paper.[2]

Our method is highly flexible and allows us to decompose the poverty change into several components (e.g., growth and redistribution components) for each region or each sector in a country in a coherent manner, a feature most existing decomposition methods do not possess. While the Shapley decomposition allows us to do similar decompositions, it is still built on the unrealistic assumption that only one of the factors of interest is allowed to change at a time. As a result, even in situations where everyone is always above the poverty line and thus there is no poverty or poverty change at all, the Shapley decomposition may spuriously ascribe non-zero poverty contribution to some factors of interest. Furthermore, the treatment of multiperiod data and partial data are also unclear under the Shapley decomposition. These points will be elaborated subsequently.

[2] Here, we are concerned with the case where the reference standards of living at the poverty line are not comparable across time. However, incomparability can occur for other reasons, such as the variations in survey design over time. See, for example, Lanjouw and Lanjouw (2001). Deaton and Kozel (2005) provide an overview of the related debate in India.

We allow all the factors of interest to change simultaneously instead of fixing all factors but one. Unlike the existing decomposition methods, we use the time derivative of the poverty measure and apply the chain rule. The chain rule essentially allows us to express the total change in poverty as a sum of contributions from the factors of interest at each point in time. We then integrate back over time to find the contribution from each factor in a given period of time. Because the reference period is internalized in this calculation, our method does not suffer from the problems associated with the choice of the reference period. As discussed further, our integral-based approach also has an advantage that there is an obvious way to handle multiperiod and partial data.

The decomposition we propose is not only theoretically sound but also relevant for choosing appropriate policies to fight poverty. For example, in regions where economic growth is pro-poor but slow, policies to enhance regional economic growth (e.g., investment in infrastructure) may be an appropriate poverty reduction policy. On the other hand, in regions with high but anti-poor economic growth, distribution-improving policies (e.g., cash transfers) may be more appropriate.

Our method is also easy to implement, especially when a set of simplifying (but reasonable) assumptions are made. It produces a neat decomposition result that does not have an interaction term or residual, which is difficult to interpret. Further, as discussed subsequently, it satisfies two desirable properties of time-reversion consistency and subperiod additivity unlike the existing decomposition methods and offers a clear and intuitive recommendation about the way subperiod information should be used.

We apply our method to the Philippines for three reasons. First, the poverty reduction process in the Philippines has been slower than that of most other countries in Southeast Asia. It is therefore useful to identify the sources of slow progress in the Philippines. To this end, we decompose the poverty change in each region in the Philippines into six components: population shift (PS), within-region redistribution (WR), between-region redistribution (BR), nominal growth (NG), inflation (IF), and methodological change (MC). Our decomposition shows that most of the poverty reduction achieved by nominal growth is offset by inflation and worsening distribution within each region when we look at overall poverty change in the Philippines during 1985–2009. Our regional disaggregation results show that the sources of poverty change are heterogeneous across regions and thus the suitable poverty reduction policies also vary across regions. For example, we find that growth-enhancing policies are desirable for poverty reduction in the Autonomous Region in Muslim Mindanao (ARMM), whereas distribution-improving policies are also important in Eastern Visayas (Region VIII).

Second, the official poverty statistics in the Philippines are calculated with poverty lines that are specific to a region or a province. Therefore, the changes in the national statistics reflect not only the real changes in poverty but also the superficial changes due to the way official poverty lines are adjusted over time. By applying our method to the Philippines, we can separate the superficial changes from the observed changes. We find that the slow progress in the reduction of official poverty in the Philippines is partly driven by the superficial changes due to the change in methodology.

Finally, the Philippines has collected household income data once every three years since 1985. This allows us to see the poverty change over a relatively long period of time. Therefore, it is possible to see whether the driving force of poverty change has altered over time. We find that worsening distribution severely crippled the progress in poverty reduction in the two periods 1988–91 and 1994–97. In other years, the slow progress in poverty reduction was mainly explained by the lack of high real economic growth.

This paper is organized as follows. In the next section, we briefly review existing methodologies of poverty decomposition and develop a new method of dynamic poverty decomposition. In Section 3, we describe the data and discuss some measurement issues. In Section 4, we present the decomposition results in the Philippines. Section 5 provides some discussion.

2. Methodology

In this section, we develop a new method of dynamic poverty decomposition. To highlight the novelty of our method, we first introduce the notations and review the existing methods in Section 2(a). We then present our general decomposition method in Section 2(b). This method requires that we know the path of the changes in the factors of interest (e.g., mean and distribution of income). However, this requirement is typically not fulfilled in a practical application. Therefore, we will consider approximations that allow us to implement the method in a straightforward manner.

In Section 2(c), we consider a simple linear approximation, in which the relative poverty line (poverty line relative to the mean income) and the cumulative distribution function of the relative income (individual income relative to the mean income) change linearly. This assumption leads to a very simple expression when the poverty measure of interest is the poverty rate. In the Online Appendix B, we alternatively consider a log-linear approximation, where a linear approximation is used for the *logarithmic* relative poverty line and distribution of the *logarithmic* relative income. This approach also has some attractions as it has some relevance to pro-poor growth literature.

In Section 2(d), we compare our decomposition under the linear approximation with the existing poverty decompositions using a graph. We argue that our method has several theoretical and practical advantages. Because the approximation we use affects the decomposition results, it is important to check the robustness of our results. Therefore, we propose to investigate the sensitivity of our decomposition method to the speed of change in the mean income relative to that of the income distribution in Section 2(e).

In Section 2(f), we consider an extension of the method with six components to highlight the flexibility of our decomposition method. In this decomposition, each of the six components can be further divided by groups such as regions or sectors. This extension helps researchers and policy makers decide what poverty reduction policies are suitable for each group. Finally, we discuss some implementation issues in Section 2(g).

(a). Notations and existing methods

We assume that the individual-level poverty measure is determined by the individual income and poverty line. The nominal income per capita y is non-negative[3] and the income distribution at time t for the population of interest is given by the probability density function $f(y,t)$. The corresponding cumulative distribution function is denoted by $F(y,t)$ and we assume that it satisfies $F(0,t)=0$. The poverty line at time t, or the threshold income level below which the individual is deemed poor, is denoted by $z(t)(>0)$.

With some slight abuse of notation, we consider a class of poverty measures M that has the following form:

$$M(t) \equiv M(F(\cdot,t),z(t)) \equiv \int_0^{z(t)} g(y/z(t))f(y,t)\mathrm{d}y \quad (1)$$

where the function $g(\cdot)$ represents the individual-level poverty measure, which we assume is differentiable at any point on the unit

[3] Our decomposition results can be applied without modification to the cases where y is the nominal consumption per capita.

interval except for zero. The class of poverty measures defined in Eqn. (1) is additively decomposable. That is, the poverty measure for any group can be expressed as the mean of subgroup poverty measures weighted by the subgroups' population shares. This is a useful property for poverty analysis, because it allows us to identify the major contributing groups to poverty. Further, additive decomposability is not a restrictive requirement, because any poverty measure that satisfies the subgroup consistency—a property that requires the group poverty measure to increase whenever the poverty measure for any of its subgroups increases—can be expressed as a monotonic transformation of an additively decomposable measure (Foster & Shorrocks, 1991).

The Foster–Greer–Thorbecke (FGT) measure (Foster, Greer, & Thorbecke, 1984), which is the most popular measure in the recent poverty literature, is a special case of Eqn. (1) with $g(\check{y}) = (1-\check{y})^{\alpha}$, where $\check{y} \equiv y/z$ is the income normalized by the poverty line and $\alpha(\geqslant 0)$ is a parameter. The Watts measure (Watts, 1968) is also a special case of Eqn. (1) with $g(\check{y}) = -\ln \check{y}$. While the Watts measure is not as widely used in applied research as the FGT measure, it follows from a set of reasonable axioms (Tsui, 1996; Zheng, 1993) and is closely related to our decomposition analysis as shown in Online Appendix B. The Chakravarty measure (Chakravarty, 1983) can also be obtained as a special case of Eqn. (1) by letting $g(\check{y}) = 1-\check{y}^{\beta}$, where β is a parameter.

Because g is independent of F in Eqn. (1), a number of other poverty indices are excluded from consideration, including those proposed by Sen (1976), Kakwani (1980), Takayama (1979), and Clark, Hemming, and Ulph (1981). While it is possible to modify our analysis to let g depend on F, we maintain the independence for the sake of simplicity of presentation.

In what follows, we focus on the FGT and Watts measures, which are denoted by P_α and W with the following definitions, respectively:

$$P_\alpha(F,z) \equiv \int_0^z \left(1-\frac{y}{z}\right)^{\alpha} \mathrm{d}F \tag{2}$$

$$W(F,z) \equiv \int_0^z \ln\frac{z}{y}\mathrm{d}F. \tag{3}$$

We shall refer to the FGT measure with parameter 0, 1, and 2 as poverty rate (P_0), poverty gap (P_1), and poverty severity (P_2), respectively.

To conduct poverty decomposition, it is useful to introduce a few additional notations. We denote the mean income at time t by $\mu(t) \equiv \int_0^\infty yf(y,t)\mathrm{d}y$, the relative income by $\tilde{y} \equiv y/\mu(t)$, and the relative poverty line by $\tilde{z} \equiv z/\mu(t)$. Here, the tilde notations ($\sim$) are used to emphasize that the quantity is relative to the population mean. The probability density function of the relative income is $\tilde{f}(\tilde{y},t)$, which satisfies $\tilde{f}(\tilde{y},t) = \mu(t)f(y,t)$ for all t and y (see Online Appendix C), and the corresponding cumulative distribution function is $\tilde{F}$. It is straightforward to show $M(F(\cdot,t),z(t)) = M(\tilde{F}(\cdot,t),\tilde{z}(t))$.

The purpose of poverty decomposition is to attribute the actual poverty change $\Delta M(t_0,t_1) \equiv M(t_1) - M(t_0)$ to the components of interest, such as the growth and redistribution components. Formally, we define poverty decomposition as follows:

Definition 1. Let C be the index set for the components of interest and $\Delta^c M$ be the contribution of component $c(\in C)$ to the poverty change. The pair $(C, \{\Delta^c M(t_0,t_1)\}_{c\in C})$ is called a **poverty decomposition** for the poverty change between $t = t_0$ and $t = t_1$ when $\Delta M(t_0,t_1) = \sum_c \Delta^c M(t_0,t_1)$.

One of the most popular decomposition methods was proposed by Datt and Ravallion (1992), which has been used in a number of studies, including Ravallion and Huppi (1991), Grootaert (1995), and Sahn and Stifel (2000). The Datt–Ravallion (DR) decomposition uses the initial time point t_0 as the reference time point. In their study, the poverty line, z, is fixed. Therefore, the change in the relative poverty line, $\tilde{z}$, is driven only by the change in mean income (i.e., growth). By fixing either the relative poverty line or relative income distribution and letting the other change, we can decompose the poverty change into the growth component Δ_{DR}^{GR} and redistribution component Δ_{DR}^{RD} in the following manner with the notations introduced above:[4]

$$\Delta_{DR}^{GR}M(t_0,t_1) = M(\tilde{F}_0,\tilde{z}_1) - M(\tilde{F}_0,\tilde{z}_0)$$

$$\Delta_{DR}^{RD}M(t_0,t_1) = M(\tilde{F}_1,\tilde{z}_0) - M(\tilde{F}_0,\tilde{z}_0)$$

$$\Delta_{DR}^{RS}M(t_0,t_1) = \Delta M(t_0,t_1) - \Delta_{DR}^{GR}M(t_0,t_1) - \Delta_{DR}^{RD}M(t_0,t_1),$$

where we hereafter denote the (relative) income distribution and poverty line at time t_a for $a \in \{0,1\}$ by $\tilde{F}_a(\cdot) \equiv \tilde{F}(\cdot,t_a)$ and $\tilde{z}(t_a) \equiv \tilde{z}_a$, respectively. For example, $\tilde{z}_0$ and $\tilde{z}_1$ are the relative poverty lines at the initial and terminal time points, respectively.

The residual term Δ_{DR}^{RS} represents the poverty change not explained by the growth and redistribution components. It captures the interaction between growth and redistribution components and can be interpreted as the difference between the growth [redistribution] components evaluated under the terminal and initial relative income distributions [mean incomes] (Datt & Ravallion, 1992).

While setting the reference time point at the initial point is a natural choice, the presence of the residual term undermines the usefulness of the decomposition analysis. This is particularly true when the residual term is large in absolute value. As Baye (2006) argues, knowledge of how much of observed changes in poverty are due to changes in the redistribution as distinguished from growth in average incomes is critical for public policy and debate. Thus, if most of the poverty change is inexplicable, the decomposition results do not give much useful information to policy makers.

We can easily avoid this problem if we are willing to assume that the change in mean income and distribution occurs in a certain sequence. In this case, we attribute the residual term to either the growth or redistribution component in effect. For example, Kakwani and Subbarao (1990) implicitly assume that the growth takes place first and the redistribution second, and thus Δ_{DR}^{RS} is attributed to the redistribution component in their decomposition. On the other hand, Jain and Tendulkar (1990) consider a decomposition in which redistribution takes place first and growth second. Formally, the Kakwani–Subbarao (KS) and Jain–Tendulkar (JT) decompositions are defined as follows:

$$\Delta_{KS}^{GR}M(t_0,t_1) = M(\tilde{F}_0,\tilde{z}_1) - M(\tilde{F}_0,\tilde{z}_0) \quad (= \Delta_{DR}^{GR}M(t_0,t_1))$$

$$\Delta_{KS}^{RD}M(t_0,t_1) = M(\tilde{F}_1,\tilde{z}_1) - M(\tilde{F}_0,\tilde{z}_1) \quad (= \Delta_{DR}^{RD}M(t_0,t_1) + \Delta_{DR}^{RS}M(t_0,t_1))$$

$$\Delta_{JT}^{GR}M(t_0,t_1) = M(\tilde{F}_1,\tilde{z}_1) - M(\tilde{F}_1,\tilde{z}_0) \quad (= \Delta_{DR}^{GR}M(t_0,t_1) + \Delta_{DR}^{RS}M(t_0,t_1))$$

$$\Delta_{JT}^{RD}M(t_0,t_1) = M(\tilde{F}_1,\tilde{z}_0) - M(\tilde{F}_0,\tilde{z}\_0) \quad (= \Delta_{DR}^{RD}M(t_0,t_1))$$

[4] Datt and Ravallion (1992) use a discrete time model. On the other hand, our presentation is based on a continuous time model. However, this distinction makes no essential difference. The same remark applies to other decomposition methods discussed in this subsection.

However, the KS and JT decompositions are also unsatisfactory because the assumptions about the sequence of change are arbitrary. Furthermore, neither the DR, the KS, nor the JT decomposition satisfies the *time-reversion consistency* defined below:

Definition 2. The decomposition $(C, \{\Delta^c M(t_0, t_1)\}_{c\in C})$ is **time-reversion consistent** when $\Delta^c M(t_0, t_1) + \Delta^c M(t_1, t_0) = 0$ for all c, t_0, and t_1.

The time-reversion consistency requires that when the poverty line and income distribution revert from the terminal state $(\tilde{z}_1, \tilde{F}_1)$ to the original state $(\tilde{z}_0, \tilde{F}_0)$, the reverse decomposition yields the same decomposition result except that each component has the opposite sign.

To see why the time-reversion consistency is a reasonable requirement, imagine that you are a time traveler. You start the travel at $t = t_0$ and end at $t = t_1$. You observe all the changes between $t = t_0$ and $t = t_1$, and conduct the poverty decomposition. Now, you return from $t = t_1$ to $t = t_0$ along the same path of change such that you experience all the changes backward. If the time-reversion consistency is not satisfied, some components contribute either positively or negatively to the poverty measure during the entire time travel, even though all the changes that you have experienced during the "outgoing" travel have been canceled during the "return" travel.

One way to obtain a time-reversion consistent decomposition is to take the average of KS and JT decompositions. This is the average of all possible sequences (i.e., growth-redistribution and redistribution-growth in the standard two-way decomposition). Because this decomposition is essentially based on the average of the marginal contributions of each component in all the possible sequences, it is similar to the Shapley solution in cooperative games and thus called the Shapley decomposition (Kolenikov & Shorrocks, 2005; Maasoumi & Mahmoudi, 2013; Shorrocks, 2013). Formally, each component in the Shapley decomposition is defined as follows:

$$\Delta^c_S M(t_0, t_1) \equiv (\Delta^c_{KS} M(t_0, t_1) + \Delta^c_{JT} M(t_0, t_1))/2 \quad \text{where } c \in \{RD, GR\}.$$

It is straightforward to verify that the Shapley decomposition is a time-reversion consistent decomposition (see also Kakwani, 2000). Unlike KS and JT decompositions, the Shapley decomposition can also be extended to the case of multiple components. For example, Son (2003) proposes a four-component Shapley-type decomposition method applied to the *rate* of poverty change for a general poverty measure.

These features of the Shapley decomposition are attractive. However, as with DR, KS, and JT decompositions, it does not satisfy the *subperiod additivity* defined below:

Definition 3. Assume that we have observations of the poverty measure and other relevant parameters at time $t = s_0, s_1, \ldots, s_D$ in the time period between $t = t_0$ and $t = t_1$ with $t_0 = s_0 < s_1 < \ldots < s_D = t_1$. Then, the decomposition $(C, \{\Delta^c M(t_0, t_1)\}_{c\in C})$ is **subperiod additive** when the following equation is satisfied for all $c \in C$:

$$\Delta^c M(t_0, t_1) = \sum_{d=1}^{D} \Delta^c M(s_{d-1}, s_d). \tag{4}$$

The subperiod additivity requires that the poverty change due to a particular component for two contiguous subperiods is equal to the sum of the poverty change due to that component in each subperiod.[5]

Datt and Ravallion (1992) propose to address this problem by fixing the reference time point $r \in [t_0, t_1]$ for the decomposition of *all* subperiods. This approach, however, is not ideal because the reference period lies outside most of the subperiods. Kakwani (2000) proposes another method to address this issue but his method is also unsatisfactory because an additional observation changes the decomposition results for *all* subperiods. As discussed below, this issue leads to a problem when incorporating subperiod information under existing decomposition methods. In the next subsection, therefore, we propose a simple decomposition method that addresses all the issues mentioned above.

2.2. New method of dynamic poverty decomposition

To derive a decomposition method that is residual-free, time-reversion consistent, and subperiod additive, we first consider an infinitesimal change of $M(t)$ with respect to time t and find growth and redistribution components for this change. This allows us to ignore the (second-order) interaction effect so that the results are residual-free. By integrating each component over the time interval of interest, we obtain the growth and redistribution components. Because the reference time point is already built-in in this decomposition method, our method is clearly time-reversion consistent. The subperiod additivity follows from the property of integration. Despite this simplicity of the method, this is the first paper to employ time derivative and integration to poverty decomposition.[6]

Following the procedure described above and using the notations introduced in Section 2(a), we can obtain the following results:[7]

Proposition 1. *Let $c \in \{RD, GR\}$ and define the following:*

$$\Delta^{RD}_* M(t_0, t_1) \equiv \int_{t_0}^{t_1} \left[\int_0^{\tilde{z}} g\left(\frac{\tilde{y}}{\tilde{z}}\right) \frac{\partial \tilde{f}(\tilde{y}, t)}{\partial t} \mathrm{d}\tilde{y} \right] \mathrm{d}t \tag{5}$$

$$\Delta^{GR}_* M(t_0, t_1) \equiv \int_{t_0}^{t_1} [g(1)\tilde{f}(\tilde{z}, t) - \int_0^{\tilde{z}} g'\left(\frac{\tilde{y}}{\tilde{z}}\right) \frac{\tilde{y}}{\tilde{z}^2} \tilde{f}(\tilde{y}, t) \mathrm{d}\tilde{y}] \frac{\mathrm{d}\tilde{z}}{\mathrm{d}t} \mathrm{d}t. \tag{6}$$

Then, the pair $(C, \{\Delta^c M(t_0, t_1)\}_{c\in C})$ is a time-reversion consistent and subperiod additive poverty decomposition.

Four points are worth making here. First, it is straightforward to verify that $\Delta^{RD}_* M(t_0, t_1) = 0$ holds when $\tilde{F}(\tilde{y}, t)$ is constant over $t \in [t_0, t_1]$ for given $\tilde{y}$. In other words, $\Delta^{RD}_* M$ is driven by the changes in the distribution and thus we call it the redistribution component. Similarly, we have $\Delta^{GR}_* M(t_0, t_1) = 0$ if $\tilde{z}$ is constant over t. In line with the previous studies, we call $\Delta^{GR}_* M$ the growth component, even though it is driven by the changes in both z and μ. The reason that we do so is that all the changes are due to growth (change in the mean income) once the poverty line is fixed, which is what is assumed in most previous studies on poverty decomposition.

Second, we chose to use the cumulative distribution function of the relative income $\tilde{F}$ to represent our decomposition for simplicity of presentation. Note, however, that the Lorenz curve has been used in a number of previous studies of poverty decomposition. Because the Lorenz curve and $\tilde{F}$ carry the same information, Eqns. (5) and (6) can be rewritten using the Lorenz curve.

Third, the first [second] term in the integral in $\Delta^{GR}_* M$ in Eqn. (6) represents the change in poverty in the extensive [intensive] mar-

[5] If Eqn. (4) holds for any s, the time-reversion consistency follows from subperiod additivity. To see this, let $s_0 = s_2 = t_0$ and $s_1 = t_1$. Then, $\Delta^c M(t_0, t_1) + \Delta^c M(t_1, t_0) = \Delta^c M(s_0, s_1) + \Delta^c M(s_1, s_2) = \Delta^c M(s_0, s_2) = \Delta^c M(t_0, t_0) = 0$, proving the claim.

[6] In the context of source decomposition of changes in inequality, Okamoto (2011) proposes an integration-based approach to justify Shapley-type decomposition.

[7] All the proofs are provided in Appendix A.

gin of poverty. The second term is zero for the poverty rate measure and the first term is zero for the Watts measure and the FGT measure with $\alpha > 0$. Therefore, in our applications, only one of these two terms matters for a given poverty measure.

Fourth, Eqns (5) and (6) show that $\Delta_*^{RD}M$ and $\Delta_*^{GR}M$ include both $\tilde{f}$ and $\tilde{z}$ in their integrations and thus depend on the way $\tilde{f}$ and $\tilde{z}$ vary between $t = t_0$ and $t = t_1$. This means that the decomposition is path-dependent. Therefore, to implement Eqns. (5) and (6), we need the observation of $\tilde{z}$ and $\tilde{F}$ over $t \in [t_0, t_1]$ in general.

In a typical application, however, we observe them only at the beginning and end of the time interval (i.e., $t = t_0$ and $t = t_1$) and possibly a few other time points in between. Therefore, we need to make some assumptions about the path to make the decomposition operational. Once the assumptions are made, we can calculate the integrals in Eqns. (5) and (6) by numerical integration for a general form of $\tilde{f}$ and $\tilde{g}$.

In the next subsection, we make a linearity assumption about the path of change for $\tilde{z}$ and $\tilde{F}$. This assumption implies that the changes in average income and income distribution occur smoothly and simultaneously. This assumption is more realistic than the sequential changes (implicitly) assumed in the DR, KS, JT, and Shapley decompositions, because it is highly unlikely that all economic growth occurs before or after all changes in income distribution. While the Shapley decomposition takes into account all the possible sequences of change, this does not make the assumed underlying changes any more realistic. Furthermore, our linearity assumption leads to a convenient expression that does not require numerical integration when the poverty measure of interest is the poverty rate.

(c). Poverty rate decomposition under linear approximation

In this subsection, we assume that both $\tilde{z}$ and $\tilde{F}$ vary linearly between $t = t_0$ and $t = t_1$. This assumption is not very restrictive, because it can be interpreted as a first-order approximation to unknown functions $\tilde{z}$ and $\tilde{F}$ with respect to t. In comparison, sequential changes implicitly assumed in DR, KS, JT, and Shapley decompositions do not permit such an interpretation and are likely to be poor approximation to the true paths of changes.

Using $\tilde{z}_a$ and $\tilde{F}_a$ defined earlier, our linearity assumption is given as follows:

Assumption 1. For $t \in [t_0, t_1]$, $\tilde{z}$ and $\tilde{F}$ respectively satisfy the following equations:

$$\tilde{F}(\tilde{y}, t) = (1 - \tau)\tilde{F}_0(\tilde{y}) + \tau\tilde{F}_1(\tilde{y}) \tag{7}$$

$$\tilde{z}(t) = (1 - \tau)\tilde{z}_0 + \tau\tilde{z}_1, \tag{8}$$

where $\tau \equiv \frac{t - t_0}{t_1 - t_0}$.

We now focus on the poverty rate measure because it is the most frequently used measure of poverty in the literature and leads to a final expression that is simple and easy to implement, as shown in the following proposition:

Proposition 2. *Suppose that Assumption 1, $\tilde{z}_0 \neq \tilde{z}_1$, and $M = P_0$ hold. Then, the poverty decomposition given in Proposition 1 can be written as follows*:

$$\Delta_{l*}^{RD}M(t_0, t_1) = \frac{\tilde{z}_1 P_1(\tilde{F}_1, \tilde{z}_1) - \tilde{z}_0 P_1(\tilde{F}_1, \tilde{z}_0) - \tilde{z}_1 P_1(\tilde{F}_0, \tilde{z}_1) + \tilde{z}_0 P_1(\tilde{F}_0, \tilde{z}_0)}{\tilde{z}_1 - \tilde{z}_0} \tag{9}$$

$$\Delta_{l*}^{GR}M(t_0, t_1) = P_0(\tilde{F}_1, \tilde{z}_1) - P_0(\tilde{F}_0, \tilde{z}_0) - \Delta_*^{RD}M(t_0, t_1). \tag{10}$$

We added the subscript l to the left-hand-side variables to emphasize that Eqns. (9) and (10) are based on the linear approximation. Note also that we excluded the possibility of $\tilde{z}_0 = \tilde{z}_1$ in Proposition 2, because it is not an interesting case. If Eqn. (8) and $\tilde{z}_0 = \tilde{z}_1$ indeed hold, we have $d\tilde{z}(t)/dt = 0$ for $t \in [t_0, t_1]$ under Assumption 1 and thus $\Delta_{l*}^{GR}M = 0$ and $\Delta_{l*}^{RD}M = \Delta M$. However, $\tilde{z}_0 = \tilde{z}_1$ does not imply $\Delta_*^{GR}M = 0$ and $\Delta_*^{RD}M = \Delta M$ without the linearity assumption in general.[8]

One important thing to note in Proposition 2 is that Eqns. (9) and (10) can be implemented without any special software package and without numerical integration. To see this, notice that $P_1(\tilde{F}_a, \tilde{z}_b)$ for $a \in \{0, 1\}$ and $b \in \{0, 1\}$ is simply the poverty gap calculated with the relative income distribution for $t = t_a$ and relative poverty line for $t = t_b$ in Eqn. (9). In Eqn. (10), $P_0(\tilde{F}_a, \tilde{z}_a)$ for $a \in \{0, 1\}$ is just the poverty rate at $t = t_a$. Because the poverty rate is the most widely used poverty measure, the result in Proposition 2 is not only computationally convenient but also highly relevant for practitioners.

(d). Differences from other decompositions

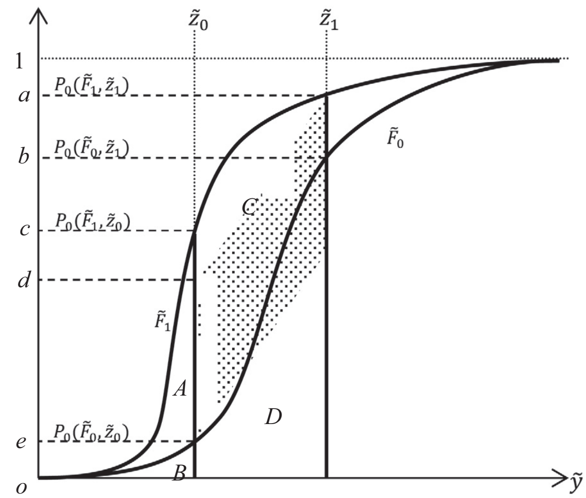
To highlight the difference of our decomposition method from existing methods, we turn to Figure 1. This figure provides a graphical representation of the cumulative distribution function of relative income and the relative poverty line at $t = t_0$ and $t = t_1$. The lengths of line segments *oe* and *oa* respectively represent the poverty rate at $t = t_0$ and $t = t_1$, or $P_0(\tilde{F}_0, \tilde{z}_0)$ and $P_0(\tilde{F}_1, \tilde{z}_1)$. Therefore, poverty has worsened between these two time periods in this figure. The capital letters *A* to *D* are used to represent an area defined by bold lines. Note that areas and *D* include some parts of the shaded areas.

The goal of a conventional two-way decomposition of poverty rate is to split the line segment *ea*, which represents the change in poverty rate, into the growth and redistribution components. If the changes in the relative poverty line and the relative income distribution take place sequentially, it is easy to see each component graphically because we only need to look at one component at a time. In this case, the redistribution [growth] component measures the effect of the change in the relative income distribution [the relative poverty line]. Graphically, the redistribution component is the vertical distance between the two cumulative distributions $\tilde{F}_0$ and $\tilde{F}_1$ at a particular $\tilde{z}$, whereas the growth component is the difference in a particular cumulative distribution between $\tilde{z}_0$ and $\tilde{z}_1$.

If we assume that the change in the relative poverty line precedes [follows] the change in the relative income distribution, we obtain the KS [JT] decomposition. The growth and redistribution components are the lengths of line segments *eb* [*ca*] and *ba* [*ec*], respectively, in Figure 1. In the case of the DR decomposition, the growth and redistribution component are *ec* and *eb*, respectively, and the residual component is what is not explained by these terms, which is *ea* − *ec* − *eb*. The Shapley decomposition is simply the average of the KS and JT decompositions, respectively.

To understand our decomposition, first note that $\tilde{z}_a P_1(\tilde{F}_b, \tilde{z}_a)$ for $a, b \in \{0, 1\}$ in Eqn. (9) represents the average shortfall per person from the poverty line relative to the mean income. Therefore, $\tilde{z}_a P_1(\tilde{F}_b, \tilde{z}_a)$ is the area below the cumulative distribution function $\tilde{F}_b$ and to the left of $\tilde{z}_a$ (see also Eqn. (A2) in Appendix A). For example, $\tilde{z}_0 P_1(\tilde{F}_1, \tilde{z}_0)$ is the area of *A* and *B* combined. It is straightfor-

[8] The linearity assumption can be made for the logarithmic income instead, in which case the assumed income growth rate will be constant between t_0 and t_1 under a fixed nominal poverty line. Further discussion is given in Online Appendix B.



Source: Author's own illustration

Figure 1. Graphical representation of poverty rate decomposition under Assumption 1.

ward to verify that the numerator of the right-hand side of Eqn. (9) is area C. By dividing this area by $\tilde{z}_1 - \tilde{z}_0$, we see that the redistribution component is represented by the vertical distance between $\tilde{F}_0$ and $\tilde{F}_1$ averaged over $\tilde{z} \in [\tilde{z}_0, \tilde{z}_1]$. Suppose now that the redistribution component is ed in Figure 1. Then, the shaded parallelogram has the same area as C. Eqn. (10) shows that the growth component is the part of poverty change not accounted for by the redistribution component, which is da in Figure 1.

Figure 1 also allows us to show the relationship between the Shapley decomposition and our decomposition. While taking the average of possible sequential changes appears arbitrary, our results show that it is not completely unreasonable. To see this, first note that we obtain the Shapley redistribution component if we replace the numerator of Eqn. (9) by the area of the trapezoid (not explicitly drawn) with two bases ec and ba. Therefore, we can consider the Shapley decomposition as a way to approximate the area C by this trapezoid. In particular, if the cumulative distribution functions $\tilde{F}_0$ and $\tilde{F}_1$ are linear between $\tilde{z} = \tilde{z}_0$ and $\tilde{z} = \tilde{z}_1$, the Shapley decomposition is exactly equal to our linear approximation in Proposition 2 but this is unlikely to hold in practice.

It should be emphasized here that our decomposition developed in Proposition 1 satisfies the subperiod additivity only when the underlying path of change is the same. However, as with DR, KS, JT, and Shapley decompositions, Eqns. (9) and (10) generally do not satisfy the subperiod additivity in the sense that $\Delta^c_{l*}M(t_0, t_2) = \Delta^c_{l*}M(t_0, t_1) + \Delta^c_{l*}M(t_1, t_2)$ for $t_0 < t_1 < t_2$ does not hold in general. The reason is that the left-hand side of this equation is based on the assumption that $\tilde{F}$ and $\tilde{z}$ change linearly between t_0 and t_2, whereas the right-hand side is based on the assumption that they change linearly piecewise between t_0 and t_1 and between t_1 and t_2.

This apparent breakdown of the subperiod additivity should *not* be seen as an undesirable property. It merely shows that we should generally prefer $\Delta^c_{l*}M(t_0, t_1) + \Delta^c_{l*}M(t_1, t_2)$ over $\Delta^c_{l*}M(t_0, t_2)$ in the absence of other information, because the piecewise linear approximation is likely to produce more accurate approximation to the underlying path of change than the naïve linear approximation between t_0 and t_2, which ignores the information available at $t = t_1$. In other words, if we have observations for more than two time points, we should conduct poverty decomposition for each of the two closest time points and then aggregate over time to obtain the decomposition result for the entire observation period. This is not only intuitive but also has an advantage that the past decomposition results (between t_0 and t_1) do not depend on the additional data that will become available in the future (at $t = t_2$).

The preceding argument appears obvious but it does not immediately apply to other decompositions even in theory. That is, for, it is not immediately apparent whether we should favor $\Delta^c_d M(t_0, t_1) + \Delta^c_d M(t_1, t_2)$ over $\Delta^c_d M(t_0, t_2)$ because the former entails an implicit change in the reference period or the sequence of change. For example, in the case of *DR*, the former uses two different reference time periods; t_0 for the poverty change between t_0 and t_1 and t_1 for the change between t_1 and t_2. In the latter, the reference time period is always t_0. Therefore, the latter appears to be a more appropriate choice given the spirit of the original formulation but it also means that we completely ignore the subperiod information.

The problem with subperiod additivity also exists for the Shapley decomposition. To see this, consider the standard two-way Shapley decomposition and denote the change in mean income [income distribution] from time t_a and t_b by Γ^{ab} [Ψ^{ab}] (e.g., Ψ^{12} represents the change in income distribution from time t_1 and t_2) under a fixed nominal poverty line. If we compute component $c \in \{RD, GR\}$ for the time period between t_0 and t_2 by $\Delta^c_S M(t_0, t_1) + \Delta^c_S M(t_1, t_2)$, we are essentially averaging the marginal contribution of component c to the poverty change over the following four possible sequences of changes in mean income and income distribution: $\Gamma^{01}\Psi^{01}\Gamma^{12}\Psi^{12}$, $\Psi^{01}\Gamma^{01}\Gamma^{12}\Psi^{12}$, $\Gamma^{01}\Psi^{01}\Psi^{12}\Gamma^{12}$, and $\Psi^{01}\Gamma^{01}\Psi^{12}\Gamma^{12}$. On the other hand, $\Delta^c_S M(t_0, t_2)$ takes the marginal contribution of component c to poverty over the following two sequences: $\Gamma^{01}\Gamma^{12}\Psi^{01}\Psi^{12} (= \Gamma^{02}\Psi^{02})$ and $\Psi^{01}\Psi^{12}\Gamma^{01}\Gamma^{12} (= \Psi^{02}\Gamma^{02})$. One may also argue that we should use the average of all these six sequences above instead in the presence of subperiod information (i.e., the mean income and income distribution at time t_1). If we generalize this argument, we need to compute the marginal contributions of growth and redistribution components along $(2n-1)!/(n-1)!$ sequences when there are n observations. These observations indicate that there is no clear-cut answer as to which implementation of the Shapley decomposition for the time period between t_0 and t_2 is the most desirable.

Similarly, because the Shapley decomposition does not have a strong theoretical foundation, it does not make clear how to make use of partial information available between the initial time period t_0 and terminal time period t_2. For example, suppose that only the mean income (and not the income distribution) is observed at $t_1 \in (t_0, t_2)$. In this case, the marginal contributions can be computed from observations along the following sequences: $\Gamma^{01}\Psi^{02}\Gamma^{12}$, $\Psi^{02}\Gamma^{01}\Gamma^{12} (= \Psi^{02}\Gamma^{02})$, and $\Gamma^{01}\Gamma^{12}\Psi^{02} (= \Gamma^{02}\Psi^{02})$. However, it is not clear how the marginal contributions along these sequences should be weighted to carry out a Shapley decomposition. To understand why an equal weight is not necessarily a reasonable weight to use, suppose that t_1 is very close to t_0. Then, the growth component between t_0 and t_1 is very close to zero. Therefore, the marginal contributions of growth and redistribution in the first sequence above (i.e., $\Gamma^{01}\Psi^{02}\Gamma^{12}$) is similar to those along the sequence of $\Psi^{02}\Gamma^{02}$. In effect, the Shapley decomposition would double the weight attached to the sequence $\Psi^{02}\Gamma^{02}$ relative to the sequence $\Gamma^{02}\Psi^{02}$ just by having one observation of mean income at t_1 arbitrarily close to t_0.

In our decomposition analysis, the way we should handle partial information is clear. In the example discussed above, we simply need to make a piecewise linear assumption on the relative poverty line between t_0 and t_1 and between t_1 and t_2 in Eqn. (8), whereas a linear assumption is made about the relative income distribution between t_0 and t_2 in Eqn. (9). Clearly, the additional information about the mean income at time t_1 will not alter our decomposition results much when t_1 is sufficiently close to t_0 and the mean income changes continuously.

Further, our method does not create spurious poverty unlike the existing methods. To understand this point, suppose that everyone's income was equal and just slightly above the poverty line last year and that the individual incomes stayed the same for half of the population but tripled without ever falling below the poverty line for the other half this year. In this case, the mean income has doubled and the Lorenz curve has shifted from the 45-degree line to a kinked line going through (0, 0), (1/2, 1/4), and (1, 1) over the past one year. When the Shapley decomposition is applied to the poverty change since last year, the growth and redistribution components are negative and positive 25%, respectively. This is because half of the population fall under the poverty line in one of the two possible sequences where redistribution takes place before growth. However, this Shapley decomposition result appears odd given that no one has ever fallen under the poverty line since last year. Existing poverty decomposition methods all suffer from the possibility of spurious poverty like this.

In sum, the preceding discussion shows that the way (potentially partial) subperiod information should be handled in our decomposition method is much clearer and more intuitive than existing decomposition methods. Our method is also free from spurious poverty. Further, as argued subsequently, our decomposition performs better than other decomposition methods empirically. It is also computationally more attractive than the Shapley decomposition particularly when there are many components in the decomposition.

(e). Robustness check with a speed of change parameter

The KS and JT decompositions implicitly assume that the mean and distribution of income change sequentially. The RD and Shapley decompositions do not impose a particular sequence, but their calculations are also based on some sequential changes. Our results presented in Sections 2(c) and 2(d), on the other hand, are based on the assumption that both change simultaneously and smoothly, which is more realistic.

However, one could argue that Assumption 1 is strong because both relative income distribution and relative poverty line are assumed to change at the "same speed." Therefore, we relax Assumption 1 and replace Eqn. (8) with the following equation:

$$\tilde{z}(t) = (1-\tau^{\gamma})\tilde{z}_0 + \tau^{\gamma}\tilde{z}_1, \tag{11}$$

where $\gamma(>0)$ is the parameter that describes the speed of change for $\tilde{F}$ relative to $\tilde{z}$. When γ is large, most of the changes in $\tilde{z}$ occur when $\tilde{F}$ is already close to $\tilde{F}_1$. In fact, when we let $\gamma\to\infty$, the decomposition converges to the JT decomposition. On the other hand, when we let $\gamma\downarrow 0$, the decomposition converges to the KS decomposition. Therefore, by varying γ, we can check the robustness of the results in Proposition 2.

(f). Extension to six-way decomposition

In this subsection, we consider a more detailed decomposition, in which the poverty change in each group in the population is decomposed into six components. While each group represents a region in our application, it may represent other household characteristics such as the household size, the sector in which the household head works, and the ethnicity of the household head. Our decomposition is useful because researchers and policy makers are often interested in finding which group is contributing to national poverty change and why. While we only consider a particular form of six-component decomposition here, our decomposition can be easily modified to have more or fewer components.

It should also be noted that our decomposition presented in Proposition 3 below can be considered as an integration of the sector-based decomposition proposed by Ravallion and Huppi (1991) and the growth-redistribution decomposition discussed earlier. Unlike Ravallion and Huppi (1991), however, our decomposition does not have an interaction term, whose interpretation is not straightforward. Therefore, our results allow researchers and policy makers to identify the source of poverty change more easily and more clearly.

To present our six-way decomposition results, we need to introduce some notations and assumptions. We hereafter assume that there are g groups (e.g., regions, sectors, or ethnic groups) in the country and each group g has a group-specific poverty line $z^g(t)$ at time t. We further assume that the group-specific poverty lines satisfy $z^g=\sum_{j=1}^{J}p_j^g(t)q_j^g(t)$, where $p_j^g(t)$ and $q_j^g(t)$ are the price and quantity of good $j\in\{1,\ldots,J\}$ consumed by a typical household near the poverty line in group g. Therefore, the poverty lines may change not only by the changes in prices but also by the changes in the underlying bundle of goods. This formulation allows us to consider the cases where different poverty lines are set for different groups, which is often the case in practice. Further, we can separate the effect of the changes in underlying prices (i.e., inflation) from the effect of poverty changes in the underlying bundle of goods used for drawing the poverty line.

We denote the population share of group g by w^g. The income distribution of group g has the probability density function f^g and cumulative distribution function F^g. Therefore, we have:

$$f(y,t)=\sum_g w^g(t)f^g(y,t) \text{ and } F(y,t)=\sum_g w^g(t)F^g(y,t) \tag{12}$$

for all t and y. With these notations, we can describe how the income distribution for the whole population can be affected by the changes in the population shares and the changes in the distributions of its subpopulations.

We further denote the mean income for group g at time t by $\mu^g(t)\equiv\int_0^\infty yf^g(y,t)\mathrm{d}y$, its ratio to the population mean by $\hat{\mu}^g(t)\equiv\mu^g(t)/\mu(t)$, the income relative to the group mean by $\hat{y}\equiv y/\mu^g$, and the poverty line relative to the group mean by $\hat{z}^g\equiv z^g/\mu^g=z^g/\mu\hat{\mu}^g$. Notice here that the relative income distribution for group g is characterized by the probability density function $\hat{f}^g$, which satisfies $\hat{f}^g(\hat{y},t)=\mu^g f^g(y,t)$ for all g. We use hat notations ($\hat{\ }$) here to emphasize that the relative income is relative to the group mean. With these notations, we can distinguish between the poverty changes due to the changes in $\hat{\mu}^g$ and $\hat{f}^g$, which respectively represent the between-group [within-group] redistribution component. As with the two-way decomposition, the following six-way decomposition can be obtained by first deriving the time derivative of $M(t_0,t_1)$ and then integrating over $t\in[t_0,t_1]$:

Proposition 3. *Let $C\equiv\{PS, WR, BR, NG, IF, MC\}$ and define the following terms:*

$$\Delta_{**}^{PS}M^g(t_0,t_1)\equiv\int_{t_0}^{t_1}\left[\frac{\mathrm{d}w^g}{\mathrm{d}t}\int_0^{\hat{z}^g}g\left(\frac{\hat{y}}{\hat{z}^g}\right)\hat{f}^g\mathrm{d}\hat{y}\right]\mathrm{d}t$$

$$\Delta_{**}^{WR}M^g(t_0,t_1)\equiv\int_{t_0}^{t_1}\left[w^g\int_0^{\hat{z}^g}g\left(\frac{\hat{y}}{\hat{z}^g}\right)\frac{\partial\hat{f}^g}{\partial t}\mathrm{d}\hat{y}\right]\mathrm{d}t$$

$$\Delta_{**}^{BR}M^g(t_0,t_1)\equiv-\int_{t_0}^{t_1}\left[w^g\left[g(1)\hat{f}^g(\hat{z}^g,t)-\int_0^{\hat{z}^g}g'\left(\frac{\hat{y}}{\hat{z}^g}\right)\frac{\hat{y}}{(\hat{z}^g)^2}\hat{f}^g\mathrm{d}\hat{y}\right]\frac{z^g}{\mu(\hat{\mu}^g)^2}\frac{\mathrm{d}\hat{\mu}^g}{\mathrm{d}t}\right]\mathrm{d}t$$

$$\Delta_{**}^{NG}M^g(t_0,t_1) \equiv -\int_{t_0}^{t_1}\left[w^g[g(1)\hat{f}^g(\hat{z}^g,t)-\int_0^{\hat{z}^g}g'\left(\frac{\hat{y}}{\hat{z}^g}\right)\frac{\hat{y}}{(\hat{z}^g)^2}\hat{f}^g\mathrm{d}\hat{y}]\frac{z^g}{\mu^2\hat{\mu}^g}\frac{\mathrm{d}\mu}{\mathrm{d}t}\right]\mathrm{d}t$$

$$\Delta_{**}^{IF}M^g(t_0,t_1) \equiv \int_{t_0}^{t_1}\left[w^g\left[g(1)\hat{f}^g(\hat{z}^g,t)-\int_0^{\hat{z}^g}g'\left(\frac{\hat{y}}{\hat{z}^g}\right)\frac{\hat{y}}{(\hat{z}^g)^2}\hat{f}^g\mathrm{d}\hat{y}\right]\frac{1}{\mu\hat{\mu}^g}\sum_j q_j^g\frac{\mathrm{d}p_j^g}{\mathrm{d}t}\right]\mathrm{d}t$$

$$\Delta_{**}^{MC}M^g(t_0,t_1) \equiv \int_{t_0}^{t_1}\left[w^g\left[g(1)\hat{f}^g(\hat{z}^g,t)-\int_0^{\hat{z}^g}g'\left(\frac{\hat{y}}{\hat{z}^g}\right)\frac{\hat{y}}{(\hat{z}^g)^2}\hat{f}^g\mathrm{d}\hat{y}\right]\frac{1}{\mu\hat{\mu}^g}\sum_j p_j^g\frac{\mathrm{d}q_j^g}{\mathrm{d}t}\right]\mathrm{d}t,$$

where $\Delta_{**}^{c}M^g$ for $c \in C$ is the contribution of group g to component c. Defining $\Delta_{**}^{c}M \equiv \sum_g \Delta_{**}^{c}M^g$, the pair t_0 is a time-reversion consistent and subperiod additive poverty decomposition.

The first component, $\Delta_{**}^{PS}M$, is the population shift component because it accounts for the poverty changes due to the changes in the relative size of each group. The population shift component represents both inter-group migration and differences in the mortality and fertility across groups. When $M = P_0$, the population shift component positively contributes to poverty reduction when the size of wealthier groups grows faster than the size of poorer groups. For example, if we define groups to be urban and rural areas, we can measure the demographic effect of urbanization on poverty by the population shift component.

The second component, $\Delta_{**}^{WR}M$, is the within-group redistribution component, which accounts for the poverty change due to the change in the relative income distribution in each group. The third component, $\Delta_{**}^{BR}M$, is the between-group redistribution component, because it is driven by the change in the ratio of the group-level mean income to the population mean. These two terms allow us to attribute the observed poverty change to the changes in income distribution across regions and within each region, a feature that does not exist in previous poverty decomposition studies.

The fourth component, $\Delta_{**}^{NG}M$, can be called the nominal growth component because it represents the change in poverty due to the change in the nominal mean income. The fifth component, $\Delta_{**}^{IF}M$, can be considered the inflation component, because it represents the poverty change due to the changes in the price of the bundle of goods for the poverty line. The sum of the fourth and fifth components can be interpreted as the poverty change due to the real growth. However, it should be noted that the bundle of goods used here for the price index is the one for drawing the poverty line and not the average consumption bundle typically used for the consumer price index (CPI).

The sixth component, $\Delta_{**}^{MC}M$, can be called the methodological change component, because this is the poverty change due to the quantity changes in the underlying bundle of goods for the poverty line, or the change in "real" poverty line. The fifth and sixth components combined represent the changes in poverty due to the shift in the nominal poverty lines. Thus, our method allows us to express the results both in real and nominal terms, depending on how we interpret the fifth (inflation) component.

As with the previous cases, we need to make some assumptions about the path of change to implement the decomposition in Proposition 3. Therefore, we simply assume that w^g, $\hat{f}^g$, μ, $\hat{\mu}^g$, p_j^g, and q_j^g change linearly for each g. That is, we first estimate $\hat{f}^g$ by kernel density estimation and calculate w^g, μ, $\hat{\mu}^g$, p_j^g, and q_j^g for all j at $t = t_0$ and $t = t_1$. Then, we take the linear interpolation. In case of w^g, for example, we assume $w^g(t) = (1-\tau)w^g(t_0) + \tau w^g(t_1)$ for $\tau \equiv (t-t_0)/(t_1-t_0)$. We make a similar assumption for $\hat{f}^g$, μ, $\hat{\mu}^g$, p_j^g, and q_j^g. Note that we are unable to obtain simple closed-form results, because $\hat{f}^g$ is multiplied with another time-varying variable in the integration.

(g). Some implementation issues

To implement the decomposition in Proposition 1 in its general form, we need to estimate $\tilde{f}$ in a typical empirical setup. Therefore, the choice of kernel density function and bandwidth used in the estimation of $\tilde{f}$ affects the results. Following the standard choice in the literature, we use the Epanechnikov kernel density function. Typically, the choice of the kernel density function is not particularly important.[9]

However, one has to be careful about the choice of the bandwidth. If we use a small bandwidth, the resulting poverty estimates are closer to those directly calculated from the observed data. However, the graph of the estimated density function is likely to be spikier. On the other hand, if we use a large bandwidth, the estimate of $\tilde{f}$ is likely to become inaccurate. Therefore, we need to strike a balance.

Given the considerations mentioned above, we set the half-width of kernel at $b = 0.01$ (i.e., one percentage point in the relative income). This bandwidth is small enough to reproduce the poverty statistics that are very close to poverty statistics derived directly from the original sample but large enough to eliminate the spikes in the density estimate from our data. All the empirical results presented in Section 4 that rely on kernel density estimation are based on this choice of bandwidth. However, the results are generally similar even when we use 50% larger or smaller bandwidth (Fujii, 2014).

To implement the numerical integration, we adopted the following procedure. First, we estimate $\tilde{f}(\cdot,t_0)$ and $\tilde{f}(\cdot,t_1)$ on a set of fixed evaluation points $\{a_1,a_2,\ldots,a_V\}$, where V is the number of evaluation points. For all $t \in [t_0,t_1]$, a_1 and a_V satisfy $\tilde{F}(a_1,t)=0$ and $a_V \geqslant \tilde{z}(t)$, respectively. Second, following the interpolation rule specified in the assumption (e.g., Eqn. (7)), we derive $\tilde{f}(\cdot,t)$ on each evaluation point at time $\{b_1,b_2,\ldots,b_N\}$, where $b_1 = t_0$, $b_N = t_1$, and N is the number of evaluation points for the outer integral in Proposition 1. Third, using the estimate of $\tilde{f}(\cdot,t)$, we evaluate the inner integral using a numerical integration method. Once we obtain the inner integral, we evaluate the outer integral in a similar manner.[10] To obtain sufficiently accurate results, we set $N = V = 20{,}000$ in our empirical application.[11]

[9] Note here that we only need the kernel density estimates for the lower tail for our analysis. By focusing on the lower tail, we can reduce the memory usage.

[10] We implemented this with a quadratic interpolation, which is essentially Simpson's rule. We make some adjustments, because the upper end of the integral, $\tilde{z}$, varies over time and does not coincide with an evaluation point in general. Also, because $\mathrm{d}\tilde{z}/\mathrm{d}t$ diverges to infinity at $t = t_0$ when $\gamma < 1$ under the assumption of Eqn. (11), we use a linear approximation of the expression inside the square bracket in Eqn. (6) in this case to calculate the integral over the first interval (i.e., $[b_1, b_2]$). The details of this treatment are given in Online Appendix E.

[11] Comparison of the numerical integration results under linear assumption with the analytical results presented in Section 2(c) indicates that the computational error is at most 0.004 percentage points for two-way decomposition analysis. For other results, we cannot directly evaluate the accuracy of our numerical results. However, the comparison between the sum of each component in the decomposition analysis and the observed change provides some guidance. According to this criterion, the six-way decomposition in Section 4 is slightly less accurate. However, our estimates (in percentage points) are accurate at least up to the first decimal point and up to the second decimal point in most cases. Detailed results are provided in Fujii (2014). While we chose to use a relatively large value for N and V to be conservative about numerical accuracy, decomposition works with practically acceptable accuracy using much smaller values (e.g., $N = V = 2000$) in our experience.

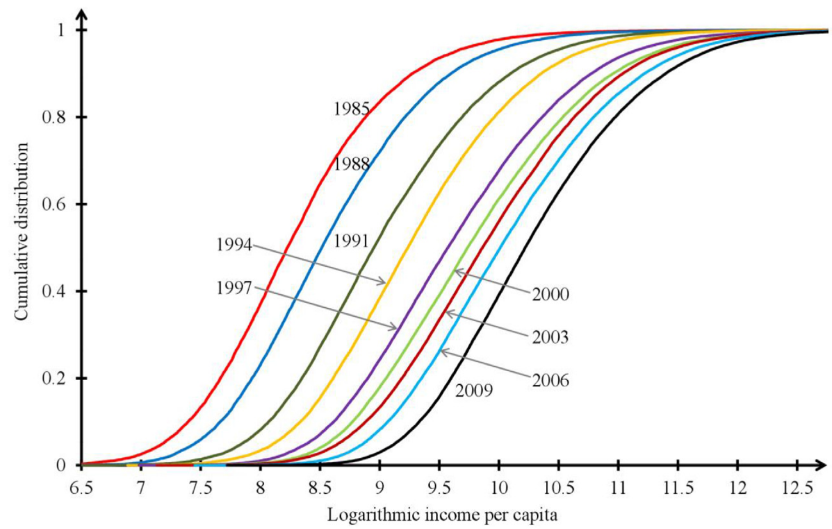


Figure 2. Cumulative distribution function of the logarithmic income per capita per year. *Note:* The income is expressed in Philippine pesos.

3. Data and poverty measurement

We use the public user files for the following nine rounds of the Family Income and Expenditure Survey (FIES): 1985, 1988, 1991, 1994, 1997, 2000, 2003, 2006, and 2009. The FIES was collected by the National Statistics Office (NSO) of the Philippines. The FIES data include income, expenditure, and various other household information. They are used for calculating official poverty statistics published by the National Statistical Coordination Board (NSCB).[12] For the six-way decomposition in Proposition 3, we also use the price data taken from the CPI, also collected by the NSO. As noted earlier, we take each region as a group in the six-way decomposition, but the definition of regions in the Philippines has changed over time. Thus, we choose to adopt the definition in the 2009 FIES data, which has 17 regions, and constructed the region variable under this definition for earlier rounds of FIES from the province variable in the data.

The distribution of the logarithmic nominal annual income per capita in the Philippines is presented in Figure 2. The figure shows that the distribution in each FIES round after 1985 first-order stochastically dominates the previous round, implying that the nominal income has increased for both the rich and the poor in the Philippines. However, this figure ignores inflation and heterogeneity across regions, and thus does not provide a clear picture about the sources of poverty change in the Philippines. Therefore, the poverty decomposition methods developed in the previous section are useful.

To implement the decomposition, we first need to set the poverty lines. A natural choice would be the NSCB's official poverty lines, because the official poverty statistics are widely used by the government and are one of the most important statistics for the formulation of poverty reduction policies in the Philippines.

However, the official methodology for setting poverty lines has been revised three times and the poverty statistics based on different methodologies are not directly comparable. The green line in Figure 3 plots the official poverty statistics during 1985–2000 based on the 1992 revision of official poverty lines. The blue line represents the official poverty statistics during 2000–06 based on the 2003 revision. The red line shows the official poverty statistics for the years 1991, 2003, 2006, and 2009 based on the 2011 revision. As Figure 3 shows, poverty estimates for the same year based on different revisions are different and thus cannot be directly used for our purpose. Figure 3 also shows that no revision of the official methodology covers the entire nine rounds of FIES, making it difficult to understand the nature of the long-term poverty changes in the Philippines. Furthermore, even when the same revision of methodology is used, the comparability of official poverty statistics over time and across regions has been disputed (Balisacan, 2003; Bernales, 2009) and multiple versions of "official" estimates appear to exist for some years.

Hence, we chose to adopt a modified version of the 2011 revision of the official methodology and produced our own back estimates. For the years in which official poverty estimates based on the 2011 revision of the methodology are available, our poverty statistics are very similar to the official poverty statistics as the gray line in Figure 3 shows. They also have a trend very similar to official poverty statistics for other years. Therefore, our poverty statistics capture well the changes in the official poverty statistics over time. Further details on the data and poverty measurement for this study are provided in Fujii (2014).

In Table 1, we provide some summary statistics for the years 1985, 1997, and 2009. For each year and each region, we report the population share W and poverty rate P_0 in percentage point and the poverty line z and average income μ in thousand Pesos per capita per year. The table shows that both the nominal growth and poverty reduction are heterogeneous across regions in the Philippines. The six-way poverty decomposition results presented in the next section sheds light on the sources of this heterogeneity.

4. Results

(a). Two-way decomposition

In this section, we present various decomposition results. We start with the two-way decomposition under the linear approximation discussed in Section 2(c), the results of which are given in Table 2. While convenient analytical results under linear

[12] Following the Philippine Statistical Act of 2013, the NSO and NSCB have been reorganized into the Philippine Statistics Authority.

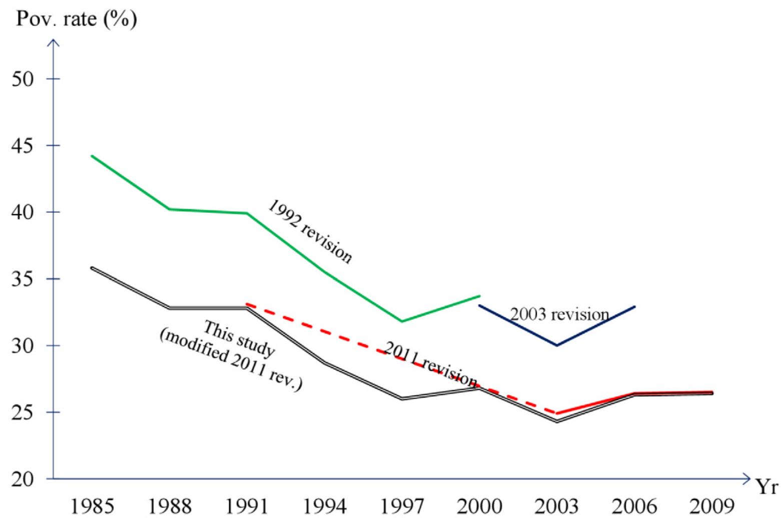


Figure 3. Comparison between official poverty statistics and our poverty statistics. *Note:* The horizontal and vertical axes indicate the calendar year and the poverty rate in percentage, respectively. *Source:* Official figures are compiled from Asian Development Bank (2005) and http://www.nscb.gov.ph/poverty/2009/table_2.asp (Accessed on August 30, 2012).

Table 1
Summary statics for years 1985, 1997, and 2009

Year		1985				1997				2009			
Region	Description of region	W	z	μ	P_0	W	z	μ	P_0	W	z	μ	P_0
NCR	National Capital Region	14.02	3.36	9.86	12.33	14.09	10.61	52.69	2.69	12.95	19.80	77.46	3.96
CAR	Cordillera Administrative Region	1.98	2.67	6.37	16.94	1.86	8.67	21.52	31.34	1.73	16.12	46.69	23.11
Reg. I	Ilocos Region	5.84	3.00	5.47	30.42	5.51	9.51	19.94	31.94	5.33	17.77	40.24	23.12
Reg. II	Cagayan Valley	4.00	2.76	5.02	32.08	3.89	7.92	18.03	25.10	3.32	15.31	40.79	18.94
Reg. III	Central Luzon	10.09	3.13	6.86	21.87	10.57	9.50	25.40	11.69	10.92	18.98	46.94	15.56
Reg. IV-A	CALARARZON	9.48	3.00	6.18	25.50	10.14	9.85	29.37	14.03	12.91	17.78	53.03	13.98
Reg. IV-B	MIMAROPA	2.87	2.43	4.43	42.28	2.98	7.98	16.92	27.72	3.21	15.77	29.73	35.72
Reg. V	Bicol Region	6.76	2.93	3.69	57.18	7.06	8.85	14.59	47.46	6.15	17.15	30.26	44.62
Reg. VI	Western Visayas	8.35	2.88	4.23	52.23	8.60	8.26	17.36	31.56	7.76	16.04	34.15	30.83
Reg. VII	Central Visayas	7.07	3.06	3.98	61.20	7.17	8.07	17.16	36.48	7.63	17.85	37.85	34.24
Reg. VIII	Eastern Visayas	5.40	2.65	3.45	53.13	5.11	7.04	13.76	37.25	4.78	15.91	33.16	41.65
Reg. IX	Zamboanga Peninsula	3.66	2.77	4.30	48.04	3.55	7.67	16.78	36.16	3.48	15.16	30.10	43.50
Reg. X	Northern Mindanao	4.86	2.77	4.84	44.90	4.82	8.16	18.58	36.30	4.59	16.57	34.50	39.23
Reg. XI	Davao Region	4.86	2.96	5.10	37.32	4.62	8.54	18.79	33.15	4.67	17.04	35.84	31.06
Reg. XII	SOCCSKSARGEN	3.53	2.85	4.61	37.37	3.90	8.63	15.89	39.28	4.27	15.76	32.98	35.78
ARMM	Autonomous Region in Muslim Mindanao	4.26	2.57	4.95	23.70	3.30	7.14	13.37	23.86	3.61	16.33	21.71	43.84
Caraga	Caraga Region	2.96	2.94	4.07	46.09	2.84	8.59	14.09	47.32	2.71	16.86	29.53	48.88
Philippines		100.00	2.81	5.64	35.83	100.00	8.49	24.07	25.99	100.00	16.84	43.54	26.36

Note: The population share W and poverty rate are P_0 expressed in percentage points. The poverty line z and average income μ are expressed in thousand Philippine pesos per capita per year. The poverty line for the Philippines is the average across regions weighted by the population share. The PPP conversion factor for private consumption in 2009 is 1 USD = 18.2 PHP.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

approximation are only available for the poverty rate P_0, we have also carried out the decomposition for poverty gap P_1, poverty severity P_2, and the Watts measure W by numerical integration.[13] The first two columns in Table 2 provide the initial year t_0 and the terminal year t_1. For each poverty measure, we report the initial level of poverty $M(t_0)$, the change Δ in the poverty measure between t_0 and t_1, the growth component GR, and the redistribution component RD. For example, the growth component of the change in poverty gap during 1994–97 is −3.40 percentage points. The last row (all periods) is the sum of all the changes in the eight 3-year periods.

Table 2 shows that the poverty changes in the Philippines have been largely driven by the growth component. Notice here that the growth component in this analysis refers to the change in poverty due to the relative poverty line, or the poverty line over the mean income. Therefore, the effect of inflation at the poverty line is already accounted for in the growth component.

Table 2 also shows that the patterns of poverty change are similar across all the poverty measures considered here. Over the periods during 1985–2009, about 30% of the poverty reduction achieved by economic growth has been offset by worsened income inequality, regardless of the poverty measure used. Most of the effects of worsening income inequality took place in the two periods 1988–91 and 1994–97.

One concern about this analysis is that the results may be driven by our linearity assumption. Therefore, we have carried out a robustness check assuming Eqn. (11) as described in Section 2(e).

[13] The decomposition result for P_0 reported in Table 2 is based on Proposition 2 and thus derived without kernel density estimation. For all other decomposition results, kernel density estimation is used. In the case of two-way decomposition, $\tilde{f}$ is estimated for the Philippines. In the case of six-way decomposition, $\tilde{f}^g$ is estimated for each region of the Philippines. This leads to a small discrepancy in Δ across tables.

Table 2
Comparison of two-way decomposition results across various poverty measures

t_0	t_1	Poverty rate (P_0)				Poverty gap (P_1)				Poverty severity (P_2)				Watts measure (W)			
		$M(t_0)$	Δ	GR	RD	$M(t_0)$	Δ	GR	RD	$M(t_0)$	Δ	GR	RD	$M(t_0)$	Δ	GR	RD
1985	1988	35.83	−3.05	−2.82	−0.23	11.13	−1.46	−1.12	−0.34	4.82	−0.85	−0.56	−0.29	15.08	−2.28	−1.61	−0.67
1988	1991	32.78	0.05	−3.16	3.21	9.66	0.24	−1.33	1.57	3.97	0.21	−0.66	0.87	12.80	0.48	−1.90	2.39
1991	1994	32.84	−4.18	−2.42	−1.75	9.90	−1.45	−1.01	−0.44	4.18	−0.66	−0.50	−0.16	13.28	−2.01	−1.44	−0.57
1994	1997	28.66	−2.67	−8.46	5.79	8.45	−0.84	−3.40	2.56	3.52	−0.39	−1.67	1.28	11.27	−1.19	−4.83	3.64
1997	2000	25.99	0.81	0.98	−0.16	7.61	0.22	0.39	−0.17	3.13	0.06	0.19	−0.13	10.08	0.26	0.55	−0.29
2000	2003	26.81	−2.55	−1.02	−1.53	7.83	−0.75	−0.39	−0.36	3.20	−0.28	−0.19	−0.09	10.34	−0.96	−0.55	−0.41
2003	2006	24.25	2.02	1.73	0.29	7.08	0.52	0.68	−0.16	2.91	0.14	0.33	−0.19	9.39	0.58	0.96	−0.38
2006	2009	26.27	0.08	1.04	−0.96	7.60	−0.34	0.42	−0.77	3.05	−0.23	0.20	−0.43	9.97	−0.55	0.59	−1.14
All periods			−9.48	−14.13	4.66		−3.87	−5.75	1.88		−2.00	−2.86	0.87		−5.66	−8.23	2.57

Note: $M(t_0)$ is in percentage. Δ, GR, and RD are all in percentage points.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

Table 3
Decomposition of poverty rate with various methods and values of γ

t_0	t_1	Δ	Δ^{GR}_{KS}	$\Delta^{GR}_{*,\gamma=1/4}$	$\Delta^{GR}_{*,\gamma=1/2}$	Δ^{GR}_{l*}	$\Delta^{GR}_{*,\gamma=2}$	$\Delta^{GR}_{*,\gamma=4}$	Δ^{GR}_{JT}	Δ^{GR}_{S}	Δ^{RS}_{DR}
1985	1988	−3.07	−2.92	−2.91	−2.87	−2.82	−2.77	−2.73	−2.67	−2.80	0.25
1988	1991	0.08	−3.19	−3.19	−3.16	−3.15	−3.14	−3.14	−3.12	−3.16	0.06
1991	1994	−4.21	−2.49	−2.51	−2.49	−2.49	−2.49	−2.50	−2.50	−2.49	−0.01
1994	1997	−2.68	−8.38	−8.48	−8.45	−8.47	−8.48	−8.48	−8.46	−8.42	−0.08
1997	2000	0.85	0.95	0.97	0.97	0.98	0.99	0.99	1.00	0.97	0.05
2000	2003	−2.51	−1.03	−1.03	−1.02	−1.01	−1.00	−0.99	−0.98	−1.00	0.05
2003	2006	1.98	1.76	1.75	1.73	1.71	1.70	1.69	1.68	1.72	−0.08
2006	2009	0.09	1.03	1.05	1.06	1.07	1.09	1.11	1.13	1.08	0.10

Note: All the figures for poverty decomposition are in percentage points.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

Table 3 shows the decomposition results for the poverty rate using various values of γ and a few other methods described in Section 2. The third column, Δ, is the change in poverty rate between t_0 and t_1. The fourth column, Δ^{GR}_{KS}, is the growth component in the KS decomposition, which corresponds to $\gamma \downarrow 0$. The fifth column, $\Delta^{GR}_{*,\gamma=1/4}$, is the growth component for $\gamma = 1/4$. The results in the seventh column, Δ^{GR}_{l*}, are essentially the same as those presented in the fifth column of Table 2, except that they are based on kernel density estimation (see also Endnote 13). The tenth column, Δ^{GR}_{JT}, is the growth component for the JT decompositions, which corresponds to $\gamma \to \infty$. As shown in the fourth to tenth columns of Table 3, the decomposition results are quite stable with respect to the changes in γ.

The eleventh column, Δ^{GR}_{S}, is the growth component in the Shapley decomposition, which gives results similar to Δ^{GR}_{l*}. The last column, Δ^{GR}_{DR}, is the residual component in the DR decomposition, which turns out to be small. Incidentally, the growth component under the log-linear approximation (unreported) is also similar to Δ^{GR}_{l*}. Therefore, our decomposition results and the decomposition results based on the existing methods are generally close in the Philippines for each of the three-year periods we considered.

However, our finding does not imply that the choice of decomposition method does not matter. To see how much the choice of method may matter, we carry out an experiment for poverty rate decomposition. We assume that the linearity assumption is satisfied piecewise for all eight 3-year periods from 1985 to 2009. Because this assumption is based on the best estimate of the true change path, the decomposition under this assumption serves as a reasonable benchmark.

We then drop from the data some years in between and calculate the growth component of poverty change for each period in the data (e.g., if years 1988, 1991, 1994, 2003, and 2006 are dropped from the data, there are three periods: 1985–97, 1997–2000, and 2000–09) and add the growth component for these peri-

Table 4
The mean and maximum absolute deviations from the benchmark decomposition results

# dropped obs.	# combination	Stat	KS	JT	S	Eq. (10)
1	7	Max	1.09	0.49	0.36	0.25
		Mean	0.31	0.27	0.12	0.12
2	21	Max	1.05	0.68	0.41	0.41
		Mean	0.36	0.34	0.16	0.14
3	35	Max	1.05	0.98	0.45	0.45
		Mean	0.4	0.43	0.18	0.15
4	35	Max	1.01	1.1	0.49	0.38
		Mean	0.39	0.5	0.21	0.15
5	21	Max	0.96	0.97	0.47	0.41
		Mean	0.34	0.54	0.23	0.16
6	7	Max	0.5	0.98	0.45	0.36
		Mean	0.22	0.53	0.25	0.19

Note: All the decomposition results are derived directly from the data without density estimation.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

Table 5
Six-way decomposition of P_0 by time period

t_0	t_1	PS	WR	BR	NG	IF	MC	ΔP_0
1985	1988	0.23	0.41	0.83	−17.92	9.65	3.74	−3.06
1988	1991	−0.29	1.40	2.21	−26.25	21.05	1.95	0.07
1991	1994	0.24	−1.97	−0.23	−12.83	12.74	−2.14	−4.19
1994	1997	−0.13	4.64	1.47	−20.29	9.18	2.43	−2.70
1997	2000	−0.38	0.40	0.14	−7.61	7.84	0.46	0.86
2000	2003	−0.21	0.38	−2.26	−3.71	5.42	−2.13	−2.52
2003	2006	0.20	−0.25	−0.13	−7.00	8.21	0.96	1.99
2006	2009	0.06	−0.55	−1.35	−9.48	8.64	2.77	0.10
All periods		−0.28	4.46	0.67	−105.09	82.73	8.05	−9.45

Note: All the figures for poverty decomposition are expressed in percentage points.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

ods to arrive at an estimate of the growth component for the entire period 1985–2009 for KS, JT, and Shapley decompositions as well as our decomposition.

We do this for all the possible combinations for each number of observations dropped, where the number of observations dropped is varied between 1 and 6 so that multiple combinations are available. We then calculate the maximum and mean absolute deviation of the estimated growth component from the benchmark growth component. It should be noted here that whether we use the growth component or redistribution component makes no difference in the two-way decomposition because the change in poverty during 1985–2009 is fixed. Therefore, the absolute deviations are the same between growth and redistribution components.

Table 4 compares the performance of our decomposition with other decompositions based on this experiment. The KS, JT, and S columns respectively show the maximum and mean absolute deviations of the growth component in the KS, JT and Shapley decompositions from the benchmark growth component, whereas the Eqn. (10) column shows the corresponding statistics for our method under the linearity assumption between observed time periods. It should be reminded that that KS, JT, and Shapley decompositions do not satisfy the subperiod additivity as discussed in Section 2. Therefore, it is at best debatable whether these decomposition results are meaningful on their own.

The first row of Table 4 shows the case in which only one observation strictly during 1985–2009 is dropped. Because one of the years 1988, 1991, 1994, 1997, 2000, 2003, and 2006 is dropped, there are a total of seven possible combinations in this case. Table 4 shows that the growth component under the linearity assumption during 1985–2009 with one observation dropped is different from that in the benchmark case of piecewise linearity assumption by 0.12 percentage points on average across the seven combinations and up to 0.25 percentage points.

We see that generally the last two columns perform better than the KS and JT columns. This is not surprising because both the KS and JT decompositions rely on a particular sequential change. While the Shapley decomposition is simply the average of these two decompositions, it is close to the benchmark case because it can approximate the linear assumption reasonably well, as argued in Section 2(c). Eqn. (10) is close to the benchmark by construction, because the assumed path of change is the same as the benchmark case except for the periods that involve dropped observations. If the piecewise linearity assumption is an accurate approximation to the actual change, Table 4 shows that our method is generally better than other methods, including the Shapley decomposition.

Another important advantage of our method is that it allows for more detailed decompositions. Unlike the KS, JK, and DR decompositions, we can neatly decompose the poverty change into population shift, within-group redistribution, between-group redistribution, nominal growth, inflation, and methodological change components for each group in the population of interest. While the Shapley decomposition also allows us to produce residual-free decomposition results, the appropriate treatment of multiperiod data is not clear as pointed out earlier. Furthermore, the Shapley decomposition can be computationally demanding to carry out complex decomposition with regional disaggregation.[14]

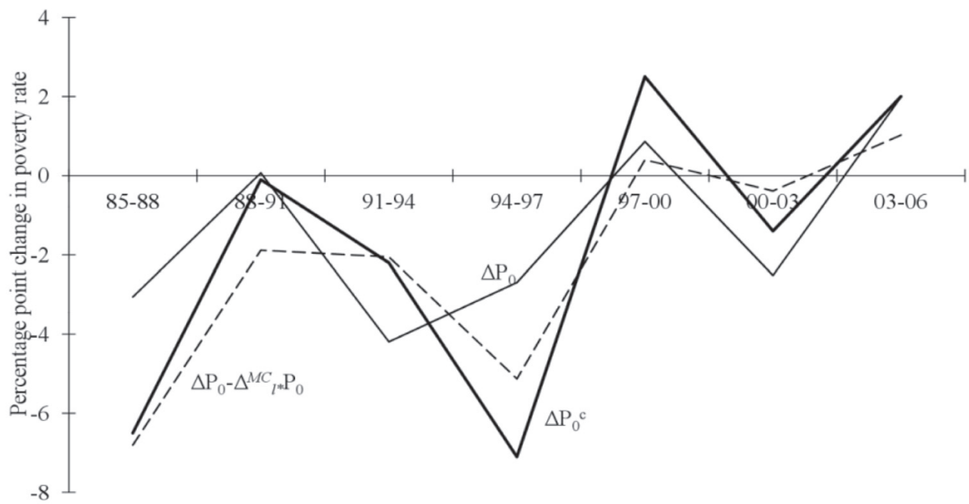
(b). Six-way decomposition

In Table 5, we report the results of six-way decomposition of poverty rate described in Section 2(f). The last row is the sum of the eight three-year periods and the last column, which represents the change in poverty rate, is equal to the sum of all six components. There are three important points to note in this table. First, Table 5 shows that the nominal growth has contributed to a more than 100-percentage-point reduction in poverty rate during 1985–2009. This is possible because nominal growth can eliminate poverty created by other factors such as inflation. In fact, Table 5 shows that much of the poverty reduction by nominal growth has been offset by inflation. If we define the effect of real growth as the combined effects of nominal growth and inflation components, we see that the effect of real growth has contributed to the reduction of poverty by more than 20 percentage points during 1985–2009 in the Philippines. This overwhelming effect of real growth would not be surprising given that countries with higher growth have tended to reduce poverty at a faster rate. It is also worth noting that much of the real-growth effect has taken place before 1997 and real growth has not contributed to poverty reduction since then (see also Online Appendix D).

Second, Table 5 also shows that poverty has increased due to both within-group redistribution and between-group redistribution effects, but the former effect is much larger than the latter. It also shows that their relative importance has changed over time. For example, the main driver of poverty increase due to worsening distribution was between-region inequality for 1988–91, but it was within-region inequality for 1994–97. We also see from Table 5 that the between-region inequality has changed favorably for poverty reduction since 2000. The population shift component did not have much impact on the national poverty rate in the Philippines.

Third, the methodological change component is not negligible. Poverty has increased by as much as eight percentage points due to this component. The interpretation of the methodological component is slightly tricky. The methodological component reflects the changes in the quantity of goods at the level of the poverty line. Therefore, if the standards of living at the poverty line go up over

[14] Consider the six-way decomposition in the Philippines discussed in Section 4(b). In this case, there are 17 regions with a total of 102 = 17 × 6 components. Therefore, the number of states in which poverty has to be evaluated is $2^{102} \approx 5.1 \times 10^{30}$ and the possible sequences of change is $102! \approx 9.6 \times 10^{161}$. While the computation can be much simplified in this particular example by exploiting the independence across regions for some factors, it remains true that the amount of computation explodes with the number of components for the Shapley decomposition.



Source: Author's calculation based on Family Income Expenditure Survey (FIES) data; Balisacan (2003); ADB (2009).

Figure 4. Comparison with Balisacan's Estimate of Consumption Poverty.

time (e.g., because of the inconsistency in the way poverty lines are drawn), poverty goes up even when there is no change in the mean and distribution of income in the Philippines. Hence, it is possible that the methodological change component reflects a spurious change due to methodological inconsistency. It is also possible, however, that the poor have systematically increased consumption of goods that are getting expensive.

The latter possibility, however, is unlikely to be true in the Philippines because the poor typically tend to shift away from goods that are getting rapidly expensive (Fujii 2013). To further address this point, we also compare in Figure 4 the changes in consumption poverty (ΔP_0^C, bold line) calculated by Professor Balisacan (see Balisacan (2003) and Asian Development Bank (2009)) against the "raw" change in poverty rate (ΔP_0, solid line) and the one adjusted for the methodological change ($\Delta P_0 - \Delta^{MC}_{l*} P_0$, dashed line) for the period during 1985–2006. As Figure 4 shows, the changes in our poverty measure with the adjustment for the methodological change are closer to the changes in Balisacan's consumption poverty rate than those without the adjustment. Because Balisacan uses consumption poverty lines that are supposed to be comparable over time, the methodological change component indeed appears to reflect the spurious change in poverty arising from the change in the methodology used for setting poverty lines. Therefore, the slow progress in poverty reduction in our poverty measure is partly because of the increases in the standards of living at the poverty line. This in turn means that actual poverty reduction may have been faster than what Figure 3 suggests, once we fix the standards of living at the poverty line.

Table 6 also shows that there is substantial heterogeneity in the way each region has contributed to poverty change in the Philippines. Although nominal growth and inflation are the largest components in absolute value for all the regions, the impact of real growth (NG + IF) on national poverty varies quite substantially, ranging from −2.58 in Region VI to −0.07 in ARMM. The magnitudes of within-group and between-group redistribution components also vary over regions. We find that the within-group redistribution component has contributed to an increase in the poverty rate in most regions, whereas the impact of the between-group redistribution component is quite diverse. To our knowledge, this is the first paper to find the geographical difference in the contribution of between- and within-group inequalities to the observed poverty change. While we only discussed the six-way decomposition results for the poverty rate, the results for the poverty gap, poverty severity, and Watts measure are qualitatively similar as reported in Fujii (2014).

(c). Comparisons with other studies

To demonstrate the features of our decomposition, it is useful to compare our results with existing studies in the Philippines. The closest study we are aware of is Reyes and Tabuga (2011). They also use the FIES and report the results of DR decomposition for the 2003–06, 2006–09, and 2003–09 periods. Despite the fact that the poverty measures they use are different from ours, the order of magnitude of each of growth and redistribution components is comparable between this study and theirs. However, it is apparent from their results that none of the components in the poverty decomposition for the 2003–09 period can be expressed as the sum of these components for the 2003–06 and 2006–09 periods. Yet, their study does not make it clear which choice is more preferable. Reyes and Tabuga (2011) also conduct the DR decomposition for each region but there is no discussion on how it relates to the overall poverty change in the Philippines. In our decomposition, neither of these issues exists. For the former, we clearly prefer to use the sum of the decompositions for the 2003–06 and 2006–09 periods to obtain the decomposition results for the 2003–09 period.[15] For the latter, the change in poverty in the Philippines as a whole can be decomposed, for example, into the growth, redistribution, and population shift components for each region by using a variant of our decomposition.

Another relevant study is Balisacan and Fuwa (2004). They estimate the growth elasticity of poverty using the FIES data for the 1985–97 period at the provincial level. While the provinces with the highest and lowest estimated growth elasticity of poverty depend on the model assumption, the provinces with the highest elasticities include West Samar, Bicol, and Misamis Oriental and the provinces with the lowest elasticities include East Samar, Pampanga, and Tawi-Tawi.

[15] The growth, redistribution, and residual components of the change in poverty rate are 2.06, −0.72, and 0.20 percentage points, respectively, when they are computed only with 2003 and 2009 data. If these components are computed as a sum of the decompositions for the 2003–06 and 2006–09 periods, they are 2.14, −0.73, and 0.14 percentage points, respectively (Reyes & Tabuga, 2011). In our results in Table 2, the growth and redistribution components for the 2003–09 period are 2.77 and −0.67 points, respectively.

Table 6
Six-way decomposition of P_0 by region, 1985–2009

Region	Description of region	PS	WR	BR	NG	IF	MC	ΔP_0
NCR	National Capital Region	−0.06	−0.02	−0.53	−6.15	5.21	0.37	−1.17
CAR	Cordillera Administrative Region	−0.06	0.21	0.14	−1.75	1.32	0.20	0.05
Reg. I	Ilocos Region	−0.19	0.22	0.29	−6.93	5.23	0.84	−0.55
Reg. II	Cagayan Valley	−0.17	0.36	−0.04	−4.85	3.73	0.32	−0.66
Reg. III	Central Luzon	0.17	−0.27	0.66	−9.98	7.69	1.20	−0.53
Reg. IV-A	CALARARZON	0.50	0.40	−0.42	−8.84	7.06	0.66	−0.64
Reg. IV-B	MIMAROPA	0.12	−0.12	0.23	−3.46	2.59	0.60	−0.04
Reg. V	Bicol Region	−0.27	0.68	−0.18	−9.01	7.22	0.45	−1.12
Reg. VI	Western Visayas	−0.14	0.32	−0.22	−10.82	8.23	0.64	−1.98
Reg. VII	Central Visayas	0.16	0.22	−0.76	−8.31	6.79	0.19	−1.71
Reg. VIII	Eastern Visayas	−0.22	0.89	−0.68	−6.53	5.12	0.53	−0.89
Reg. IX	Zamboanga Peninsula	−0.07	0.24	0.28	−4.03	3.09	0.23	−0.26
Reg. X	Northern Mindanao	−0.10	0.26	0.22	−5.15	3.94	0.45	−0.39
Reg. XI	Davao Region	−0.07	0.29	0.25	−5.62	4.17	0.61	−0.36
Reg. XII	SOCCSKSARGEN	0.30	0.58	0.07	−4.73	3.23	0.79	0.23
ARMM	Autonomous Region in Muslim Mindanao	−0.09	−0.34	1.33	−5.27	5.20	−0.27	0.57
Caraga	Caraga Region	−0.11	0.53	0.05	−3.66	2.92	0.24	−0.03
Philippines	National Capital Region	−0.28	4.46	0.67	−105.09	82.73	8.05	−9.45

Note: All the figures for poverty decomposition are expressed in percentage points.
Source: Author's calculation based on Family Income Expenditure Survey (FIES) data.

While it is not possible to strictly compare our results with Balisacan and Fuwa (2004), our six-way decomposition for the 1985–97 period, which are reported in Table D.1 in the Online Appendix D, corroborates with their findings. Bicol and Northern Mindanao Regions, which Bicol and Misamis Oriental provinces are located, respectively, have relatively high WR components, suggesting that the poverty reduction due to growth is offset by widening within-region inequality. On the other hand, Central Luzon and ARMM, which Pampanga and Tawi-Tawi are located, respectively, have among the lowest WR component. Western Visayas, where both West Samar and East Samar are located, have a close to average WR component.

In Table D.2 in the Online Appendix D, we also report the six-way decomposition for the 1997–2009 period. It is interesting to note that the pattern we noted above for the 1985–97 period does not hold for the 1997–2009 period. This point highlights one advantage of our method over the growth elasticity approach. The latter typically requires some form of uniformity across time periods and/or areas conditional on observable variables but the former does not rely on such assumptions.

5. Discussion

In this paper, we proposed a method of dynamic poverty decomposition that is subperiod additive and time-reversion consistent. Our decomposition analysis consistently integrates the conventional dynamic poverty decomposition such as Datt and Ravallion (1992) and group-based decomposition such as Ravallion and Huppi (1991). Our method has an additional advantage in that there are no residual or interaction terms and does not suffer from the possibility of spurious poverty. Our method also works well when information pertinent to some components is missing for some periods or collected less frequently than other components. While our method requires the specification of the path of change, we have provided a practical way to implement the decomposition under a set of reasonable assumptions. Under these assumptions, our method performs better than other existing methods.

As with the Shapley decomposition, our decomposition is flexible and time-reversion consistent and has no residual. Our decomposition method is subperiod additive once the underlying path of changes is specified. Therefore, unlike the Shapley and other decomposition methods, it provides users with a clear and intuitive instruction regarding how to use subperiod information. Our decomposition has a computational advantage over the Shapley decomposition when we disaggregate by a large number of groups.

Another potential advantage of our method is that it does not necessarily require all the components of interest to independently vary over time. For example, if inequality determines growth through the process of human and physical capital accumulation (e.g., Galor & Moav, 2004) while z is held constant, we would have the following relationship instead of, say, Eqn. (8) in Assumption 1: $\mathrm{d}\tilde{z}(t)/\mathrm{d}t = H(\tilde{F}(\cdot,t))$ for some function $H(\cdot)$ with its definite integral from $t = t_0$ to $t = t_1$ being equal to $\tilde{z}_1 - \tilde{z}_0$. In a case like this, poverty change is completely determined by the change in income distribution. However, it is still possible to identify the growth and redistribution components in our approach, because Proposition 1 still holds and thus the growth component can be computed by simply replacing $\mathrm{d}\tilde{z}(t)/\mathrm{d}t$ by $H(\tilde{F}(\cdot,t))$ in Eqn. (6). Similarly, it is also possible to consider the effects of demographic dividend by explicitly modeling the relationship between demographic characteristics of the population and economic growth. While it is beyond the scope of this paper to provide details of model-based decomposition, the possibility of model-based decomposition gives our method an additional advantage over existing decomposition methods.

In our empirical application to the Philippines, we considered a six-way decomposition in which the national poverty change is decomposed into population shift, within-group redistribution, between-group redistribution, nominal growth, inflation, and methodological change components for each of the 17 regions in the Philippines. We find that nominal growth and inflation are by far the largest components in absolute value in each region and that the impacts of other components are heterogeneous, which indicates that the appropriate poverty reduction policies may vary from region to region. For example, the results reported in Table 6 suggest that some regions, such as the Autonomous Region in Muslim Mindanao, would require growth-enhancing policies to reduce poverty effectively, whereas other regions, such as Eastern Visayas, may need policies to improve the income distribution within the region. This point is true even if the pre- and

[15] The growth, redistribution, and residual components of the change in poverty rate are 2.06, −0.72, and 0.20 percentage points, respectively, when they are computed only with 2003 and 2009 data. If these components are computed as a sum of the decompositions for the 2003–06 and 2006–09 periods, they are 2.14, −0.73, and 0.14 percentage points, respectively (Reyes & Tabuga, 2011). In our results in Table 2, the growth and redistribution components for the 2003–09 period are 2.77 and −0.67 points, respectively.

post-1997 periods are considered separately (see also Online Appendix D). We also find that poverty reduction in the Philippines has been slowed substantially by worsening inequality for the periods 1988–91 and 1994–97. For other periods, the apparent slow progress in poverty reduction was mostly because of the lack of real economic growth but also partly because of the methodological change.

In this study, we chose regions as a unit of the group for empirical illustration, because the Philippines is spatially heterogeneous in terms of consumption patterns, growth rate, and inflation rate. However, our analysis can also be applied to a number of other issues by using other variables as a unit of group, such as the ethnic groups, the education of household head, the employment status or sector of the household head, and the household size. Using these variables, we can expand the scope of the standard poverty profile approach. That is, instead of simply comparing the poverty rate, poverty gap, and poverty severity across different groups for various years, as is done in the standard poverty profile approach, using our method, we can decompose the change in national poverty into various components for each group in the population. Hence, our decomposition method can complement and enhance the usefulness of the poverty profile approach.

While we chose to apply our method to a developing country, it is also applicable to poverty analysis in developed countries, where poverty lines are typically drawn separately for each household category and defined as a fraction of mean or median income. Therefore, even if everyone's income is rising, poverty may still increase if the income at the center of the distribution rises faster than incomes in the lower tail of the distribution. Using our method (with a slight modification), it is possible to separate the effects of increasing income at the bottom and middle on poverty.

As with other decomposition methods, our decomposition is descriptive and essentially an accounting exercise. Therefore, it does not in general describe the causal relationship between poverty and the factors of interest. However, we can potentially make meaningful inferences if we are willing to make some assumptions about the underlying relationship among the factors and carry out model-based decompositions. Even if not, our decomposition still provides researchers and policy makers with useful information for the understanding of the sources of poverty change and formulating poverty reduction policies that are suitable for different groups in the population.

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Appendix A. Proofs of propositions

Proof of Proposition 1. Note first that $y/z = \tilde{y}/\tilde{z}$ holds. Therefore, using the change of variables, we have $M(\tilde{F}(\tilde{y},t),\tilde{z}(t)) = M(F(y,t),z(t))$ for all t. Using this and because the poverty change between t_0 and t_1 can be written as the integral of the time derivative of $M(t)$, we can make the following transformation:

$$\begin{aligned}
\Delta M(t_0,t_1) &= \int_{t_0}^{t_1} \tfrac{\mathrm{d}M(t)}{\mathrm{d}t}\mathrm{d}t = \int_{t_0}^{t_1} \tfrac{\mathrm{d}}{\mathrm{d}t}\left[\int_0^{\tilde{z}} g(\tilde{y}/\tilde{z})\tilde{f}(\tilde{y},t)\mathrm{d}\tilde{y}\right]\mathrm{d}t \\
&= \int_{t_0}^{t_1}\left[\int_0^{\tilde{z}} g(\tfrac{\tilde{y}}{\tilde{z}})\tfrac{\partial\tilde{f}(\tilde{y},t)}{\partial t}\mathrm{d}\tilde{y}\right]\mathrm{d}t + \int_{t_0}^{t_1}\left[\tfrac{\partial}{\partial\tilde{z}}\left[\int_0^{\tilde{z}} g(\tilde{y}/\tilde{z})\tilde{f}(\tilde{y},t)\mathrm{d}\tilde{y}\right]\tfrac{\mathrm{d}\tilde{z}}{\mathrm{d}t}\right]\mathrm{d}t \\
&= \Delta_*^{RD}M(t_0,t_1) + \Delta_*^{GR}M(t_0,t_1),
\end{aligned} \tag{A1}$$

where the third line follows from the chain rule. It is clear from Eqn. (A1) that $(C,\{\Delta_*^{c}M(t_0,t_1)\}_{c\in C})$ is a poverty decomposition. The time-reversion consistency and subperiod additivity follow immediately from the basic properties of integrals. □

Proof of Proposition 2. By setting $\alpha = 1$ and using integration by parts in Eqn. (2), we have:

$$P_1(\tilde{F}(\cdot),\tilde{z}) = \tilde{z}^{-1}\int_0^{\tilde{z}} \tilde{F}(\tilde{y})\mathrm{d}\tilde{y}. \tag{A2}$$

First, notice that the following equation follows from Eqn. (8):

$$\frac{\mathrm{d}\tilde{z}}{\mathrm{d}t} = \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tau}\cdot\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{\tilde{z}_1-\tilde{z}_0}{t_1-t_0} \tag{A3}$$

Second, defining the probability density function for $\tilde{F}_a$ by $\tilde{f}_a$ and taking the derivative of Eqn. (7) with respect to t and $\tilde{y}$, we obtain:

$$\frac{\mathrm{d}\tilde{f}(\tilde{y},t)}{\mathrm{d}t} = \frac{\tilde{f}_1(\tilde{y})-\tilde{f}_0(\tilde{y})}{t_1-t_0} = \frac{\tilde{f}(\tilde{y},t_1)-\tilde{f}(\tilde{y},t_0)}{t_1-t_0} \tag{A4}$$

Noting that $g(\cdot) = 1$ holds for the poverty rate measure and that t_0 and t_1 are just constants, substituting Eqn. (A4) in Eqn. (5) leads to the following transformation:

$$\begin{aligned}
\Delta_{I*}^{RD}M(t_0,t_1) &= \int_{t_0}^{t_1}\left[\int_0^{\tilde{z}}\frac{\tilde{f}(\tilde{y},t_1)-\tilde{f}(\tilde{y},t_0)}{t_1-t_0}\mathrm{d}\tilde{y}\right]\mathrm{d}t \\
&= \int_{t_0}^{t_1}\left[\frac{\tilde{F}(\tilde{z},t_1)-\tilde{F}(\tilde{z},t_0)}{t_1-t_0}\right]\mathrm{d}t
\end{aligned}$$

$$\begin{aligned}
&= \int_{\tilde{z}_0}^{\tilde{z}_1}\left[\frac{\tilde{F}(\tilde{z},t_1)-\tilde{F}(\tilde{z},t_0)}{\tilde{z}_1-\tilde{z}_0}\right]\mathrm{d}\tilde{z} \\
&= \frac{\tilde{z}_1P_1(\tilde{F}_1,\tilde{z}_1)-\tilde{z}_0P_1(\tilde{F}_1,\tilde{z}_0)-\tilde{z}_1P_1(\tilde{F}_0,\tilde{z}_1)+\tilde{z}_0P_1(\tilde{F}_0,\tilde{z}_0)}{\tilde{z}_1-\tilde{z}_0},
\end{aligned}$$

where the third and fourth equalities follow from Eqns. (A3) and (A2), respectively. The result for $\Delta_{I*}^{GR}M(t_0,t_1)$ follows immediately from this. □

Proof of Proposition 3.. The proof is similar to that of Proposition 1. First, by $\hat{z}^g = \sum_j p_j^g q_j^g/\mu\hat{\mu}^g$, we have the following relationship:

$$\frac{\mathrm{d}\hat{z}^g}{\mathrm{d}t} = -\frac{\hat{z}^g}{\hat{\mu}^g}\frac{\mathrm{d}\hat{\mu}^g}{\mathrm{d}t} - \frac{\hat{z}^g}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}t} + \frac{1}{\mu\hat{\mu}^g}\sum_{j=1}^{J} q_j^g\frac{\mathrm{d}p_j^g}{\mathrm{d}t} + \frac{1}{\mu\hat{\mu}^g}\sum_{j=1}^{J} p_j^g\frac{\mathrm{d}q_j^g}{\mathrm{d}t}. \tag{A5}$$

Now, consider the time-derivative of $M(\mathrm{t})$:

$$\begin{aligned}
\tfrac{\mathrm{d}M(t)}{\mathrm{d}t} &= \tfrac{\mathrm{d}}{\mathrm{d}t}\left[\sum_g w^g(t)\int_0^{z^g} g(\tfrac{y}{z^g})f^g(y,t)\mathrm{d}y\right] \\
&= \tfrac{\mathrm{d}}{\mathrm{d}t}\left[\sum_g w^g(t)\int_0^{\hat{z}^g} g(\tfrac{\hat{y}}{\hat{z}^g})\hat{f}^g(\hat{y},t)\mathrm{d}\hat{y}\right] \\
&= \sum_g\left[\tfrac{\mathrm{d}w^g(t)}{\mathrm{d}t}\int_0^{\hat{z}^g} g(\tfrac{\hat{y}}{\hat{z}^g})\hat{f}^g(\hat{y},t)\mathrm{d}\hat{y} + w^g(t)\int_0^{\hat{z}^g} g(\tfrac{\hat{y}}{\hat{z}^g})\tfrac{\partial\hat{f}^g(\hat{y},t)}{\partial t}\mathrm{d}\hat{y}\right. \\
&\quad\left. + \tfrac{\mathrm{d}}{\mathrm{d}\hat{z}^g}\left[\int_0^{\hat{z}^g} g(\tfrac{\hat{y}}{\hat{z}^g})\hat{f}^g(\hat{y},t)\mathrm{d}\hat{y}\right]\cdot\left[\tfrac{\mathrm{d}\hat{z}^g}{\mathrm{d}t}\right]\right]
\end{aligned}$$

Substituting Eqn. (A5) in the equation above and integrating the equation above over $t \in [t_0, t_1]$, we obtain $\Delta M = \sum_c \Delta_*^c M$, proving that $(C, \{\Delta_*^c M(t_0, t_1)\})$ is a poverty decomposition. The time-reversion consistency and subperiod additivity follow immediately from the properties of integration. $\square$

Appendix B. Online appendix

Online appendix associated with this article can be found at http://dx.doi.org/10.1016/j.worlddev.2017.07.031.

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