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Vertical Differentiation and a Comparison of Online Advertising Models

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Abstract: Designing business models that take into consideration the role of advertising support is critical to the success of online services. In this paper, we address the challenges of these business model strategies and compare different ad revenue models. We use game theory to model vertical differentiation in both monopoly and duopoly settings, in which online service providers may offer an ad-free service, an ad-supported service, or a combination of these services. Offering both ad-free and ad-supported services is the optimal strategy for a monopolist, because ad revenues compensate for the cannibalistic effect of vertical differentiation. In a duopoly equilibrium, exactly one firm offers both services, when the ad revenue rate is sufficiently high. Furthermore, we find that a higher ad revenue rate may lead to lower service prices. Consistent across both monopoly and duopoly settings, such price reductions are more severe in the cost-per-thousand-impressions (CPM) model than in the cost-per-click (CPC) model. Our findings emphasize the role of advertising revenues in vertical differentiation and offer strategic guidance for monetizing online services.

Key Words: Vertical differentiation, ad-supported business models, online advertising, game theory, economic analysis, e-commerce.

1 Introduction

Content distribution is migrating to online channels, where business model strategies are critical for success. For example, Netflix—known for its DVD-by-mail service—has begun to digitize films for instant streaming. It offers multiple levels of paid memberships, and its revenue is based completely on subscriptions. A newer online service, Hulu, provides television content and films on demand. Both the free and paid services offered by Hulu are ad supported, and the company reached $263 million in earnings in 2010 [15]. Other examples include Amazon Prime and iTunes, both of which have extensive libraries of entertainment content for renting and downloading. Among platforms of online games, Pogo.com varies the advertising intensity and mixes free and paid memberships, whereas FreeArcade.com offers a single, free service financed by ads. Furthermore, for Internet music discovery services, Pandora offers multiple subscription levels, with and without ads, whereas a competing service, Jango, relies on a single ad-supported service that is free of charge.

As online services flourish, widely adopted ad-supported business models are leading to an inevitable surge in online advertising. According to the Interactive Advertising Bureau (IAB), online advertising revenues have been growing steadily for the past five consecutive quarters [13]. In 2010, advertising revenues on the Internet reached a record high of $26 billion, surpassing those for many traditional media, including newspaper, radio, and TV cable networks [13]. In this paper, we use game theory to analyze the design of service offerings through subscription-based and/or ad-supported business models, under both monopolistic and competitive market structures. We also provide a comparison of the effects of different ad revenue models on online service providers' pricing and business model decisions.

Two types of ad revenue model contribute most substantially to online advertising revenues: The performance-based model and the cost-per-thousand-impressions (CPM) (i.e., cost-per-mille) model. From 2004 to 2008, each of these two models consistently generated revenues of more than 40% of the total online advertising revenues [13]. The CPM model has commonly been used for many traditional media, such as television and newspaper, and has also been adopted for online advertising. Even though highly ad-averse users are unlikely to respond to ads, the CPM model charges advertisers according to the level of exposure, regardless of ad effectiveness. Performance-based revenue models take into account user ad aversion and charge advertisers on the basis of user actions. For example, in cost-per-click (CPC)—a highly successful and popular performance-based model—advertisers pay only when their ads generate clicks. Performance-based ad revenues surpassed the CPM revenues in 2006 and remain increasingly dominant [13]. Our paper offers an economic perspective on different ways the two models affect the pricing and demand of online services.

Aside from the challenges related to service combinations and advertising models, online service providers may also face different market forces. Competition is fierce for many types of services. For example, Pandora and Jango, along with a number of other online services (e.g., iLike, Mog.com, etc.), stream music with socializing and discovery capabilities. They rely substantially on advertising revenues and encounter increasing difficulties in monetizing online music in a rivalrous landscape. Whereas Netflix, Hulu, Amazon Prime, and iTunes compete in online movie rental, Hulu holds the exclusive license to stream current episodes of television shows from ABC, Comedy Central, FOX, NBCUniversal, MTV, and more, and has monopoly power over the PC and mobile channels. However, it has also struggled with building the user base for the recently introduced Hulu Plus subscription service [28].

Among the ad-free, ad-supported, and mixed (ad-free and ad-supported) business models, we identify the optimal and equilibrium business models in the monopoly and duopoly settings, respectively. The monopolist's optimal business model includes both the ad-free and ad-supported services. The latter service generates additional ad revenues that compensate for the cannibalistic effect of vertical differentiation. The mixed business model allows the monopolist to price discriminate and to serve consumers who would otherwise be excluded under a single ad-supported service. By fully endogenizing the prices, we find that a paid ad-supported service may be sustainable when an ad-free service is available at a higher price. In the duopoly equilibrium, exactly one firm may adopt the mixed business model, whereas the rival offers only the ad-supported service. Competition then drives down both firms' price for the ad-support service, explaining the zero price of ad-supported services in many practical cases. This equilibrium requires a sufficiently high ad revenue rate; otherwise, the firm with the mixed business model will offer only the ad-free service. Thus, given lower ad revenue rates, competing firms can avoid the price war through differentiation.

By comparing the CPC- and CPM-based ad revenue models, we find that, in both monopoly and duopoly settings, the CPC model leads to greater stability in service pricing for all business model configurations. The optimal pricing strategy fluctuates with the ad revenue rate. A higher ad revenue rate induces online service providers to expand the demand of the ad-supported service by cutting price. In the case of the CPM model, this price reduction is severe because the marginal revenue gain is independent of consumer ad aversion and proportional to the demand. In the CPC model, the service provider gains diminishing marginal revenues from pricecuts because the additional consumers captured are more ad averse and less effective in generating ad revenues. This intuition applies to all cases studied in our analysis.

The remainder of the paper is organized as follows: We review the related literature in the next section. In the third section, we present the model and the analysis of the monopolist's optimization problem. We study the duopoly case and derive the equilibrium in the fourth section. In the fifth section, we discuss the theoretical and managerial contributions. Finally, in the sixth section, we conclude the paper.

2 Literature Review

Our work is closely related to theories of vertical differentiation. In a monopoly context, Mussa and Rosen [21], Moorthy [19], Salant [26], and others examine the optimality of vertical differentiation based on different cost structures and consumer heterogeneity. These studies explore whether offering additional lower-quality options induces unfavorable competition within the monopolist's product line, and they offer insights into the scenarios in which such cannibalization is absent or occurs. In particular, Mussa and Rosen [21] show that offering multiple qualities is optimal for the monopolist under a linear consumer utility function and continuous types; Moorthy [19] derives contrasting results by generalizing the consumer utility function. Furthermore, Bhargava and Choudhary [2] extend and contribute to this line of literature by generalizing the consumer distribution and marginal cost functions. They find that only the ratio of the cost of improving quality to the increase in consumers' utility is relevant in the monopolist's optimal strategy, and that offering the highest quality product alone is optimal if it has the lowest cost-to-quality ratio [2].

Several seminal works study vertical differentiation with competition. Shaked and Sutton [27] endogenize competing firms' entry decisions as well as quality and price decisions. They find that in equilibrium exactly two firms enter the market and produce vertically differentiated goods [27]. Moorthy [20] contrasts the results of vertical differentiation in a duopoly setting with those in a monopoly setting and points out that the competition results in inefficient differentiation.

Motivated by the growth in online advertising and in online services, our work focuses on vertical differentiation in the context of ad-supported business models, in which firms offering lower quality products generate additional revenues through ad sponsorship. We follow the common assumption of zero marginal production costs for information goods and model consumer heterogeneity in consumers' sensitivity toward advertisements. Furthermore, our duopoly analysis differentiates from that in the literature by allowing each competing firm to offer multiple qualities of services simultaneously.

Bhargava and Choudhary [2] and other related studies have suggested that researchers investigate the role of advertising revenue in firms' decision to offer multiple product qualities. A body of research in ad-supported business models has developed recently. Riggins [24] examines a monopolist's strategies in pricing and quality design when offering two types of products: Fee-based and sponsorship-based. He studies the effects of ad revenues on the product quality as determined by the content quality and ad relevancy [24]. His results show that decreasing ad revenues may lead to higher quality differentiation between two products, which raises the price of the fee-based product [24]. However, he does not consider the monopolist's inclusion or exclusion of ad sponsorship in its product offerings. Dewan et al. [7] study an ad-supported website manager's problem in dynamically balancing the content and advertisements on a website. They find that it is optimal to bring more content and fewer ads initially to generate a large viewer base [7]. Fan et al. [10] model how channel difference factors (e.g., the online channel access cost, advertising level and revenues, and program quality) affect a monopolist's optimal price and advertising levels in media distribution over traditional and online channels [10]. Kumar and Sethi [16] study the dynamic pricing problem in a hybrid service model that is based on both subscription revenues and advertising revenues with a CPM model, and they identify the optimal subscription fee and advertising level over time. Prasad et al. [23] examine a monopolistic media provider's problem of setting the subscription fee and advertising level when two types of consumers differ in their income. They find the conditions under which a monopoly's optimal strategy is to offer two different options that will each be adopted by a type of consumers (in a separating equilibrium) [23]. Our work connects ad-supported business strategies with the theories of vertical differentiation by accounting for different service combinations, analyzing pricing decisions, and comparing the CPC and CPM ad revenue models. We also model both monopoly and duopoly settings to understand the effect of market forces on firms' strategies.

Peitz and Valletti [22] analyze the symmetric competition among pay-tv media platforms and among free-to-air platforms. By endogenizing the advertising level and content variety, they compare the competitive outcomes of the pay-tv and free-to-air cases [22]. Casadesus-Masanell and Zhu [5] address an incumbent's business model reconfiguration problem in choosing the optimal business strategy when facing an ad-sponsored entrant. They find that, among four choices of business models, the incumbent tends to strategize by using a pure business model that is based on either the subscription revenue or ad revenue, rather than using a hybrid business model that combines a subscription-based approach with advertising. Their explanation is that pure business models create greater differentiation from the entrant's service than do hybrid business models, thereby generating higher revenues for the incumbent [5]. In this paper, we endogenize the business model decisions of both competing firms and find the equilibrium. Moreover, we focus on the comparison of different ad revenue models rather than on optimization of advertising level.

Another stream of literature to which our work is relevant is online advertising. In particular, studies on targeted advertising [8] [14] [11] investigate the marketing strategies that advertisers use to selectively influence consumer demand and the market impact of these strategies. Similarly, our study examines the market impact of different advertising strategies in both monopoly and competitive settings; however, we take the perspective of ad publishers, who design their service offerings to segment the market by second-degree pricing discrimination. In keyword advertising, many studies examine the performance-based auction mechanism for ranking ads and compare the efficiency of this auction mechanism to that of the traditional second-price auction [6] [18] [29]. We look at the interaction between intermediaries/publishers and consumers and study service pricing under different ad revenue models.

An ad-supported service provider acts like a two-sided platform that joins advertisers and users [9]. Studies in two-sided markets literature, such as Rochet and Tirole [25], Caillaud and Jullien [4], and Armstrong [1], investigate issues of pricing and governance structures. Recent works, including Boudreau [3] and Lin et al. [17], explore the the role of innovation in two-sided markets. In this paper, the emphasis is on endogenous business model decisions; thus, our model focuses only on consumer-side pricing. Our contribution to the literature on two-sided markets lies in identifying cases in which exclusion of the advertising side yields higher revenues, given market competition, the ad revenue model, and consumers' ad aversion.

3 Ad-Supported Monopoly

An online service provider may have a monopoly position because of a unique channel presence or an exclusive content license. We consider a monopolistic online service provider whose strategy choices include a single ad-free service, a single ad-supported service, and the mixed business model that includes both services. Moreover, the monopolist also compares the CPC and CPM ad revenue models in its service offering(s). The monopolist first chooses the business model and then sets the optimal price for each type of service offered. Note that the prices of ad-supported services are also endogenous – we do not assume they are zero.

The monopolist's revenue source includes (1) the subscription revenues collected from consumers and, (2) if any ad-supported service is offered, the advertising revenues collected from advertisers according to the demand of the ad-supported service. We assume zero marginal cost, consistent with the literature on information goods, and homogeneous sunk costs for all types of services. Regardless of whether advertising support is adopted, the service provider incurs upfront costs in content acquisition and the technology infrastructure that supports the online service. For example, online content providers must acquire licenses and legal rights from record labels and content owners. Although advertising support may introduce additional costs, we are interested in the case in which the production cost of the service is dominant.[1]

Suppose consumers have homogeneous valuation for the online service at the reservation value $r > 0$. In ad-supported services, ads may cause nuisance, depending on each consumer's sensivity to ads. Denote consumer ad sensitivity by θ that is uniformly distributed in the interval $[0,1]$. A consumer's utility for purchasing an ad-supported service i at the price p_i is $r - \theta k - p_i$, which is a variant of the utility function in [5] and isolates the heterogeneity in consumers' perception of ad nuisance from the value of the service. Furthermore, $i \in \{M, C\}$, where M and C refer to CPM- and CPC-based ad revenue models, respectively, and k is the disutility of the most ad-averse consumer ($\theta = 1$). Note that consumers do not distinguish between CPM- and CPC-based ads because both revenue models display ads in the common

[1]In general, higher advertising costs will introduce an additional condition and simply shift the firm's choice toward the ad-free service. However, in a separate study, exploring a richer cost structure of the advertising support might prove interesting.

forms (e.g., text, image, and video).[2]

Assumption 1 (Reservation Value and Disutility). *Consumers' reservation value (r) for the online service is greater than the disutility of the most ad-averse consumer (k).*

Assumption 1 implies that the service itself offers substantial value, such that all consumers are willing to adopt it with ads if it is free. One might note that for some services, such as the yellow pages, many consumers would choose not to have them even when they are free. We impose Assumption 1 to focus on sufficiently valuable services, for which strategic analysis is meaningful.

CPC and CPM models generate revenues differently. The CPM model charges advertisers purely on the basis of the number of ad impressions. Thus, the ad revenue is proportional to the number of consumers adopting the service: If the service is priced at p_m and generates demand q_m, then the monopolist's revenue is $\pi_m = q_m(p_m + \beta_m)$, where β_m is the ad revenue rate.[3] The CPC model generates ad revenues only when consumers click on ads.

Assumption 2 (Ad Aversion and Clicking Probability). *Consumers who are less ad-averse (with a lower θ) are more likely to click on ads.*

Assumption 2 suggests that the CPC model relies on the consumers who are less sensitive to ads to generate ad revenues. Their actions (e.g., clicks) signal interest, and advertisers' payments are contingent on such actions. However, because even the least ad-averse consumer would not always click on ads, consumers' probability of clicking on ads should be only a fraction of $1-\theta$. We denote this probability by $(1-\theta)\tau$, where $\tau \in (0,1)$ is the click

[2]In the Appendix "Alternative Model Setup," we examine an extension of the model, where the CPC ad revenue model leads to a higher quality service than the CPM model from consumers' perspective. The key results from the main model continue to hold in that setting.

[3]To focus on the service provider's problem of optimal service offering(s), we use exogenous ad revenue rates and do not explicitly model the interaction between advertisers and the service provider. Modeling the firm as a two-sided platform for advertisers and consumers may reveal other insights into the firm's pricing strategies; however, it is likely to compromise model tractability.

rate parameter that adjusts a consumer's ad aversion to his/her clicking probability. By offering a CPC-based ad-supported service priced at p_c and capturing consumers between θ_0 to θ_1, the monopolist's total revenue is given by $\pi_c = (\theta_1 - \theta_0) p_c + \tau\beta_c \int_{\theta_0}^{\theta_1} (1-\theta) d\theta$, where β_c is the ad revenue rate of the CPC model.

<Insert Table 1 approximately here>

3.1 Single-Service Business Models

In this section, we analyze the monopolist's optimal pricing strategy for any single-service business model.

Since consumers have a homogeneous reservation price for the ad-free service at r, the optimal price of a single ad-free service is simply r. This allows the monopolist to capture all consumers and derive revenues r. If the monopolist offers a single ad-supported service, it may strategically exclude some consumers, depending on the value of r. If the service itself provides adequate value (sufficiently high r), the monopolist's optimal strategy is to serve even the most ad-averse consumer at the price $r-k$, covering the entire market; otherwise, the optimal strategy for the monopolist is to charge a higher price and exclude the consumers who are more ad-averse. The implication is that the value of service is an important factor for assessing price-demand trade-offs. For example, many magazines with little information content but heavy advertising content target only the reader base that tolerates ads. We relegate the model analysis to the Appendix and summarize the results of the monopolist's optimal strategies in Table 2.

<Insert Table 2 approximately here>

3.2 Mixed Business Models

A mixed business model allows the monopolist to segment the market by simultaneously offering the ad-free and ad-supported services at different prices. The monopolist maximizes its collective revenues from both services:

$$\max \pi_{ni}(p_n, p_i) = (1 - \frac{p_n - p_i}{k}) p_n + \pi_i(p_n, p_i) \tag{1}$$

$$\text{s.t.} \quad r - p_n \geq 0, p_n - p_i \geq 0 \,, \tag{2}$$

where $i \in \{m, c\}$. We use lower-case letters for the subscripts of mixed business models.

Mixed business models ensure that all consumers are served because the consumers who are too ad-averse to adopt the ad-supported service can opt for the ad-free service. In the CPM case, the monopolist pursues the mixed business model only when the ad revenue rate is below a certain threshold ($\beta_m \leq 2k$). A higher revenue rate excludes the ad-free service -- the monopolist is better off with only a CPM service and covers the entire market (see Table 2). In the CPC case, the most ad-averse consumers will always purchase the ad-free service; the monopolist has no incentive to include these consumers in the ad-supported service because the revenue of the CPC model is contingent on consumers' interest toward ads. The details of this analysis are provided in the Appendix, and the results are summarized in Table 3.

<Insert Table 3 approximately here>

Proposition 1 ***(Price Discrimination with a Mixed Business Model).*** *The optimal price of an ad-supported service is (weakly) higher under the mixed business model than it is under the single-service business model.*

Regardless of the ad revenue model, a mixed business model provides the leverage for price discrimination, allowing the monopolist to raise the price of the ad-supported service. When offering a single ad-supported service, the monopolist needs to set a sufficiently low price to serve some of the ad-averse consumers who would purchase nothing otherwise. Under a mixed business model, the ad-supported service can be priced higher to target only the consumers who are less ad-averse; the other consumers will purchase the ad-free service at the reservation price. This result suggests that offering only the ad-supported service may lead to underpricing the ad-supported service. For instance, Hulu, by offering only ad-supported services (paid and free), has been unsuccessful in capturing a sufficient user base for the paid ad-supported service, Hulu Plus. This problem may be mitigated by adding an ad-free service.

Proposition 2 ***(Effect of Ad Revenue on Optimal Price).*** *In both single-service and mixed business models, when the ad revenue rate increases, it is optimal for the monoplist to reduce the price of the ad-supported service. Moreover, this price reduction is more severe for the CPM model than for the CPC model.*

In both single-service[4] and mixed business models, the ad revenue rate plays a role in the monopolist's trade-offs between price and demand. A higher ad revenue rate shifts the weight from the monopolist's subscription revenues to its ad revenues. Reducing the ad-supported service price captures more consumers and generates higher ad revenues. In the mixed business model, this shift is achieved by transferring demand from the ad-free service to the ad-supported service. Therefore, the ad revenue rate has an overall negative effect on the optimal price of the ad-supported service.

Furthermore, we find that, as the ad revenue rate increases, the optimal price under the CPC model is more stable than that under the CPM model, in both single-service and mixed cases. Notice that the marginal consumers the monopolist captures by reducing the price are increasingly ad-averse. Under the CPC model, the marginal ad revenue is decreasing, because of the lower probability that these consumers will click on ads. The CPM-based ad revenues are independent of consumers' ad aversion – the monopolist has a constant marginal ad return as its

[4]For the single-service business models, the optimal price of the ad-support service is independent of the ad revenue rate when the market is covered. Thus, the discussion here pertains to the case when the market is not covered.

market segment expands. Therefore, in the CPM case, the monopolist has a stronger price-cutting incentive, whereas in the CPC case, the incentive continually diminishes with the ad revenue rate. This result shows that the ad revenue model has a clear impact on the monopolist's pricing strategies. Given recent steady growth of online advertising, the monopolist service provider is likely to benefit from the pricing stability provided by the CPC ad revenue model.

3.3 Business Model Comparison

To find the monopolist's optimal business model, we first compare the single-service and mixed models and then find the conditions under which the dominant business model emerges.

Proposition 3 ***(Optimal Monopoly Business Model).*** *The monopolist's optimal strategy is to mix the ad-free service with an ad-supported service (nc or nm).*

Given either CPC or CPM revenue models, vertical differentiation achieved by offering services with and without ads generates higher payoffs than any single-service business model. Price discrimination and market segmentation allow the monopolist to set higher optimal prices, eliminating inefficiencies of single-service models due to low subscription revenues in the ad-supported case and zero ad revenue in the ad-free case. Thus, regardless of other factors, the monopolist online service provider is better off diversifying its service offerings in the ad-supported option. By including ads in both the free and paid services, the service provider, such as Hulu, may be playing a suboptimal strategy that not only alienates the ad-averse consumers but also underprices its contents.

Proposition 4 ***(Optimal Ad Revenue Model).*** *If* $\beta_m < \tau\beta_c\sqrt{\frac{2k}{2k+\tau\beta_c}}$ *, then the optimal business model is to mix the ad-free service with the CPC-based ad-supported service; otherwise, adopting the mixed business model with the CPM-based ad-supported service is optimal.*

Proposition 4 establishes the condition under which the monopolist should mix the ad-free service with the CPC ad-supported service rather than with the CPM service. Figure 1 shows the optimal regions, based on the relationship between the CPM ad revenue rate and the CPC ad revenue rate adjusted by the click rate parameter. If the adjusted CPC revenue rate is sufficiently greater than the CPM revenue rate, the mixed business model with the CPC model is optimal.

<Insert Figure 1 approximately here>

Although it is intuitive that the optimal ad revenue model depends on the relative ad revenue rates, we reveal a nonlinear relationship between the revenue rates of the two models that determines the optimal strategy. The concave curve in Figure 1 implies that the CPC model generates revenues at an increasingly higher efficiency than the CPM model, as the revenue rate increases. The driving force is the change in the monopolist's optimal trade-offs between the subscription revenues and ad revenues. A higher CPM ad revenue rate induces the monopolist to lower the price of the CPM service to achieve higher ad returns; the instability in price amplifies as the CPM revenue rate further increases. This tendency to become advertising-driven is much less pronounced in the CPC case because the monopolist, in reducing price, can attract only increasingly ad-averse consumers, who would then generate decreasing marginal ad revenues. Therefore, the stability of the optimal price offered by the CPC model pays off as the revenue rate increases.

4 Ad-Supported Duopoly Competition

In this section, we analyze two competing firms' equilibrium business models. Following the setup in the monopoly setting, each firm may offer a single service that is either ad-free or ad-supported (i.e., *N*, *C*, or *M*), or offer a mix of ad-free and ad-supported services (i.e., *nc* or *nm*). The two firms first choose their business models simultaneously and then set equilibrium prices.

The cases in which both firms offer the ad-free service are trivial. Either the single-service (*N*) or the mixed business model (*nc* or *nm*) leads to the Bertrand-type competition that results in equilibrium prices of zero,[5] assuming non-negative pricing. The two firms then split the market, and both derive zero revenue in equilibrium. The remaining non-trivial cases are shown in Figure 2.

<Insert Figure 2 approximately here>

When firms offer only ad-supported services, consumers are indifferent between purchasing from either firm. Given Assumption 1, all consumers are willing to adopt an ad-supported service for free. Thus, two firms split the market and set the equilibrium price at zero as a result of Bertrand competition. Note that each firm's choice of ad revenue model is independent of the rival's decision, because neither demand nor price is affected by that decision. The CPC ad-supported service yields the revenue $\pi_C^* = \frac{1}{4}\tau\beta_c$, and for the CPM service $\pi_M^* = \frac{1}{2}\beta_m$. In equilibrium, both firms will choose the model that is more profitable on the basis of the relative ad revenue rates. The competition between the basic version of Yahoo! Mail and that of Hotmail is an example of this simple case.

4.1 Single-Service Business Models (B1)

Without loss of generality, suppose that only firm 1 offers the ad-free service. When both firms have a single-service business model, they simply differentiate in quality, with firm 1 providing the premium ad-free service and firm 2 offering a cheaper ad-supported alternative. When firm

[5]When the ad-free service is priced at zero, the ad-supported service also cannot be priced above zero because it is perceived as the lower quality service as compared with the ad-free service.

2's ad revenue rate is lower, both firms set a positive price; otherwise, firm 2's ad-supported service is offered for free.

<Insert Table 4 approximately here>

Proposition 5 ***(Effect of Ad Revenue on Equilibrium Prices).*** *As firm 2 earns a higher ad revenue rate, both firms cut price in equilibrium; moreover, when firm 2 adopts the CPM model, the price reductions are more severe than the case with the CPC model.*

The revenue rate of firm 2's advertising support may intensify price competition. Regardless of firm 2's ad revenue model, both firms' equilibrium prices fall as the ad revenue rate increases. Consistent with the monopoly case, a higher ad revenue rate incentivizes firm 2 to extract more ad revenues by cutting its price and stimulating demand; however, in a duopoly, competition further induces price cuts by the rival, firm 1. As a result, changes in the ad revenue rate for one firm can lead to an industry-wide service price fluctuation, triggered by competition. This, combined with fast-growing online advertising revenues, may help to explain the losses suffered by the traditional print business and the difficulty in monetizing purely subscription-based online services.

Furthermore, in a duopoly, the CPC model offers more price stability than the CPM model, as shown in the monopoly setting. The price competition that arises with an increase in the ad revenue rate intensifies to a higher degree when firm 2 has the CPM model than when it has the CPC model. This shows that the strength of ad revenue effect persists, as competition carries it over to the rival's price.

4.2 Mixing Ad-Free and Ad-Supported Services (B2)

In this case, firm 1 has a mixed business model, while firm 2 offers a single ad-supported service. Bertrand competition occurs between two firms' ad-supported services, resulting in an equilibrium price of zero for both firms' ad-supported service.

Table 5 summarizes the equilibrium in this case: The ad-supported services are free, and firm 1 sets the ad-free service price according to what provides an optimal balance between its own subscription and ad revenues. A striking difference of the duopoly equilibrium as compared with the optimal results in the monopoly is that the ad-free service is not priced at the reservation value r. In most cases, firm 1 sets a price lower than r to compete with its own and the rival's ad-supported services. Only when firm 1 adopts a CPM ad revenue model that has an extremely high ad revenue rate, does firm 1 raise the equilibrium price of the ad-free service to effectively offer only the ad-supported service.

<Insert Table 5 approximately here>

In practice, we rarely observe any online service that is provided in only the ad-supported form. Web mail services (e.g., Yahoo!) have the subscription-based, ad-free version; online games (e.g., Pogo.com) sometimes offer the paid ad-free option; and Internet radio can be streamed without commercial interruptions (e.g., Pandora's subscription service).[6]

Proposition 6 ***(Effect of Ad Revenue on Equilibrium Ad-Free Price).*** *When firm 1 mixes ad-free and ad-supported services, it raises the price of the ad-free service as it earns a higher ad revenue rate from the ad-supported service. Moreover, this price increase is more pronounced when firm 1 adopts the CPM model than the CPC model.*

The effect of the ad revenue rate on price in the present case reverses those in the other cases, which consistently indicate that an increase in the ad revenue rate leads to lower optimal/equilibrium price(s) (Figure 3). A reason for this difference is that, in the present case, firm 1 offers two services that are competing, and it leverages the price of the ad-free service to

[6]The search engine service appears to be an exception, but, in this special case, search users are likely to find ads useful.

shift demand to/from the ad-support service. Thus, as the ad revenue rate of its ad-supported service increases, firm 1 is inclined to transfer some of the ad-free demand to the ad-supported service by inflating the ad-free service price. Also in the present case, the ad-supported service is free; the competition triggered is then more intense than that within a monopolist's mixed business model. These contrasts emphasize the importance of market structure for assessing the impact of ad revenues on firms' strategies.

<Insert Figure 3 approximately here>

Figure 3 also compares CPC and CPM models in different settings. Consistent across all scenarios is that the CPC ad revenue model offers greater price stability. When the ad revenue rate becomes higher, firm 1 raises the price of the ad-free service more aggressively if it offers the CPM service than if it offers the CPC service. The reason, again, is that the CPM model leads to higher marginal ad revenue gains.

4.3 Equilibrium Business Models

We now solve the equilibrium business model by comparing all of the scenarios in the duopoly. The cases in which an ad-free service is present in both firms' business models cannot occur in equilibrium, because either firm would prefer switching to the ad-supported service to achieve a positive revenue. We have also eliminated Case A, in which both firms offer a single ad-supported service. Within the many scenarios of Cases B1 and B2 that are remaining (Figure 4), the equilibrium business models depend on the parameter values and the relationships between β_m, τ, β_c, and k. We examine two specific conditions, $\beta_m > \tau\beta_c$ and $\tau\beta_c > 2\beta_m$, under which key equilibrium results emerge. We refer to the Appendix for further discussions on the equilibrium results and these conditions.

<Insert Figure 4 approximately here>

In equilibrium, the ad-free service is offered by exactly one firm, either as the only option in a single-service business model or as an alternative to the ad-supported service in a mixed business model; the rival's equilibrium strategy is to offer only an ad-supported service. Recall that a monopolist's optimal strategy is vertical differentiation, which effectively segments the market to yield higher subscription and advertising revenues. In a duopoly equilibrium, two firms vertically differentiate and segment the market in a similar manner, provided that the ad revenue rate is below a certain threshold, so that the ad-free firm offers a single service. An excessively high ad revenue rate, however, induces the ad-free firm to also offer an ad-supported service; in this case, the ad-free firm alone creates vertical differentiation, and the subscription revenue of the ad-supported services disappears as a result of Bertrand competition.

The ad revenue rate is the factor determining the equilibrium outcome where vertical differentiation is created by two firms or by a single firm. Higher ad revenue rates lead to the case of vertical differentiation by a single firm, resulting in intense competition and destroying the subscription revenue of ad-supported services. In practice, many online service providers rely primarily on advertising revenues as a result of competition triggered by growing online advertising spending. Although the few online game sites (e.g., Pogo.com) offer a paid option with limited ads, most other competing game sites are financed by advertisers. Similarly, Pandora provides subscription-based, commercial-free online music service, whereas the competing service, Jango, is free and ad-supported.

5 Discussion

5.1 Theoretical Contribution

Our main contributions to the existing literature include four aspects. First, we examine the theories of vertical differentiation with consideration for advertising revenue, which is a key element of online business models with multiple qualities of services; our findings highlight the importance of understanding ad revenue models in monetizing online services. Second, our study takes into account service providers' decisions in both service pricing and business model design, as business strategies emcompass both dimensions; we identify the effect of ad revenue on service pricing that contrasts with past findings. Third, we analyze and compare the revenue models of the CPC- and CPM-based ad-supported services; our findings show sharp differences in their strategic implications for service pricing and business model choice with and without competition. Lastly, our model allows both competing firms to strategize in their business model configurations and proves the optimality of simultaneously offering both ad-free and ad-supported services.

We introduce advertising revenues into the problem of vertical differentiation, which is studied mainly under conditions of cost structure and consumer utility function in the literature [21] [19] [2]. Even though ad revenues mitigate the cannibalistic effect of introducing a lower-quality, ad-supported service, ad revenues may also lower the optimal price of the ad-supported service in the monopoly case. Thus, a monopolistic service provider faces a complex problem of balancing the advertising and subscription revenues. When considering competition, past findings indicate that firms differentiate to avoid competition [27] [20]. However, considering advertising support shows that higher ad revenues may intensify competition by incentivizing strategies that reduce differentiation of business models and by inducing price cuts. In this case, exactly one firm benefits from differentiation by offering both ad-free and ad-supported services, whereas the rival resorts to relying solely on ad revenues.

By endogenizing firms' pricing decisions and analyzing both monopoly and duopoly cases, we gain a more in-depth understanding of the strategic role of advertising support. Riggins

[24] models a monopolistic setting that offers fee-based and sponsorship-based services and finds that decreasing (increasing) ad revenues raises (reduces) the price of the fee-based product. We find a similar result in the duopoly case: When the ad revenue rate of the ad-supported firm increases, the rival that offers the ad-free service reduces its price under competitive pressure. However, the monopoly case of our model reveals that an increase in the ad revenue rate reduces the price of only the *ad-supported service*. One key difference in our work is that the firm also strategize in pricing the ad-supported service, whereas in [24] the sponsorship-based product is assumed to be free.[7] The price endogeneity in our analysis shows that it may be optimal for a monopolist to charge a fee for the ad-supported service according to the ad revenue rate.

By comparing CPC and CPM ad revenue models, we tackle a new area in the theoretical research of online advertising. To the best of our knowledge, existing studies have not examined the effect of different ad revenue models on firms' business strategies. Because the efficacy of these ad revenue models is closely linked with users' ad aversion, our model draws from this connection and suggests that the CPM revenue model leads to greater instability in service pricing and provokes more intense competition. On the other hand, the CPC model relies more on consumers' willingness to receive ads and thus mitigates the trade-off between subscription and advertising revenues.

Our findings mirror some of those discussed by Casadesus-Masanell and Zhu [5], who examine an incumbent's optimal strategy given that a market entrant offers a free, ad-supported service. By allowing both firms to configure their business models and by analyzing multiple service offerings, our study presents additional strategic insights. The consistent result in both our work and [5] is that when the ad revenue rate is sufficiently low, one firm offers the fee-based, ad-free service, while the other offers the ad-supported service. For higher values of

[7] In our model, the ad-supported service price may also be zero, provided that the service value is sufficiently low.

the ad revenue rate, [5] suggests that the incumbent's optimal strategy is to adopt the same business model as the rival, regardless of the trade-offs. In contrast, we take into account multiple-service offerings and find that one firm offers *both* the ad-supported and ad-free services in equilibrium. This result highlights that, even under competitive pressure, a firm can obtain the strategic leverage from offering multiple types of services to segment the market. Although both firms adopt the ad support, the ad-averse consumers nevertheless have access to the ad-free service from the firm offering multiple services.

5.2 Managerial Implications

Our study sheds light on online service providers' business model strategies under market forces, on pricing of ad-supported and ad-free services, and on strategic implications of CPC versus CPM ad revenue models.

By contrasting results in the monopoly and duopoly settings, we emphasize the consideration of firms' market position in their business model choice. Even in the fiercely competitive online service industry, channel diversity, licensing strategies, and the dynamic landscape open monopolistic opportunities for some online service providers. For example, Hulu is privileged to be the exclusive partner of ABC, Comedy Central, FOX, NBCUniversal, and other content owners for current TV episodes. Furthermore, Hulu faces no competition from cable television providers over the mobile media, such as the iPad and Android tablets. Despite the initial buzz generated by its free ad-supported online service, Hulu's paid ad-supported service, Hulu Plus, led to a series of disappointing market performances. It received loud complaints for including ads and resorted to a downward subscription fee adjustment—from $9.99 to $7.99 per month [28]. According to our findings, when a service provider has exclusive access to a market segment, its optimal profile of service offerings should always include both ad-free and ad-supported services, in which the *ad-free* service plays a critical role in price discrimination. Offering only ad-supported services that are vertically differentiated does not effectively segment the market because of consumers' ad aversion. As a result, failure to provide the ad-free service tends to lead to underpricing of the ad-supported service.

Furthermore, contrary to the common assumption that an ad-supported service should always be free, our findings underscore that Hulu's strategic shortcoming was in its failure to introduce an ad-free service rather than in the pricing of its ad-supported services. In fact, a properly priced *ad-free* service might actually help leverage a *paid ad-supported* service, given Hulu's monopoly power. The cable providers are a closely related example in that they often have the monopoly position and have been successful in packaging both paid ad-supported (e.g., ESPN) and ad-free (e.g., HBO) channels. Hulu provides the content of similar quality and variety over a newly emerged medium; given that it secures partnerships with content owners and engages viewers in a quality experience, Hulu's optimal business model should include both paid ad-free and ad-supported services.

Online service providers commonly face intense competition, in which case their business model strategies may not always include both ad-free and ad-supported options. Among the many Internet radio services with the capability of online music discovery, Pandora is one of the few to offer a paid ad-free service, whereas its key competitor, Jango, has only one option: the free ad-supported service. Similarly, most online game providers offer only ad-supported services; a few exceptions provide ad-free or limited-advertising services that are priced higher. Our findings suggest that these providers should indeed rely largely on ad revenues and be cautious about introducing an ad-free service, because direct competition could destroy the subscription revenue. We note that a number of Internet music providers, such as Spotify, Rhapsody, and Slacker, offer the *on-demand* paid service, which is a significantly different premium service than that offered by online music discovery services such as Pandora. In this scenario, the online services explore differentiation outside of advertising to monetize their subscription services.

Analyzing different ad revenue models to reveal and compare their effect can be meaningful to online service providers. Currently, many online service providers receive ads and payment statistics from a third-party platform -- assessing different ad revenue models poses a challenge. For instance, mochimedia hosts a platform that brings together online game developers, advertisers, and publishers. Publishers, who are online game providers, receive both CPM and performance-based ads and generate ad revenues based on eCPM (effective cost per impression), computed by mochimedia [30]. Similarly, Google's AdSense program allocates CPM and performance-based ads on its publishers' sites and provides only aggregate statistics to its publishers. Our analysis presents to online service providers an evaluation of the contingency featured in performance-based models and contrasts it with the traditional CPM model that guarantees “unconditional” ad returns.

In comparison with the CPM ad revenue model, performance-based revenue models (e.g., CPC) offer greater service price stability and more efficiency in generating revenues. These qualities of performance-based models suggest a strong potential for wide adoption. In fact, most online services, including those that are traditionally based on video ads or graphic-intensive ads, have also adopted performance-based ads (e.g., YouTube). Moreover, the Interactive Advertising Bureau reports that the proportion of performance-based ad revenues among total online ad revenues has surpassed that of impression-based ad revenues, at 62% versus 33% in 2010 [13].

6 Conclusion

In this paper, we examine vertical differentiation in online service provision with advertising support, while comparing the CPC and CPM ad revenue models. In both monopoly and duopoly settings, we model service providers' business model and pricing decisions. We show that a monopolist's optimal business model always includes both ad-free and ad-supported services, whereas in a channel with competing online services, the ad-free service is offered by exactly one provider. Furthermore, the level of ad revenue rate is an important factor determining whether this provider also offers the ad-supported service that would lead to an intense price competition with the rival. Moreover, while a higher ad revenue rate leads to lower service prices, the CPC ad revenue model offers greater price stability than the CPM revenue model in both monopoly and competitive cases.

We suggest several future extensions to address the limitations of this paper. First, considering *multiple* ad-supported options that are vertically differentiated may be a promising direction. In the Appendix, section "Alternative Model Setup" moves in this direction by tying service quality to the ad revenue model in a monopoly setting, assuming that the CPC ad-supported service is the higher quality service, relative to the CPM service. To capture the full picture, however, it may be necessary to endogenize the quality of ad-supported services by letting advertisers set ad effectiveness and choose the ad revenue model. In this extension, the advertiser-side pricing will be a critical instrument for creating suitable incentives among advertisers. Offering both the CPM-based and CPC-based services could be an optimal strategy, which runs counter to Proposition 9 in the Appendix. It is possible that an endogenous advertiser-side price would induce the ad revenue rate and quality gap close to those specified in the conditions of Proposition 10, where the optimal business model includes all types of services. Furthermore, this two-sided model would also have the power to illustrate the relationship between the service price and advertising fees.

Another future direction is in the dynamics of product introduction. In this paper, we use static settings to capture the market equilibrium; however, to understand strategies to adapt to the increasing demand for online services, it may be useful to consider a sequential decision problem. A dynamic pricing model would capture the transition from a traditional, subscription-based business model to an ad-supported model with demand uncertainty. Furthermore, firms may also need to decide whether to offer the new service as a separate option or as an add-on to the existing service.

Lastly, it may be interesting to explore different cost structures of adopting an advertising support, since we consider only the case in which the cost of the service is dominant. This would help separate service providers who manage their own advertiser relations from those who receive ads from an intermeidary service, such as Google AdSense.

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Appendix

Monopoly: Single-Service Business Models

Ad-free, N

Without advertisements, consumers have the same surplus, $u(p_N) = r - p_N$. It is intuitive that the monopolist then fully extracts consumers' surplus and sets the optimal price of the ad-free service at $p_N^* = r$, and its revenue is $\pi_N^* = r$.

CPC-Based Ad Support

Consumers prefer purchasing the ad-supported service over nothing when $r - \theta k - p_C > 0$, which implies that the indifference consumer is $\theta^* = \frac{r - p_C}{k}$. The monopolist's profit maximization problem is the following:

$$\max \pi_C(p_C) = \frac{r - p_C}{k} p_C + \tau\beta_c \int_0^{\frac{r - p_C}{k}} (1-\theta) d\theta \tag{3}$$

$$\text{s.t.} \quad 0 \le \frac{r - p_C}{k} \le 1. \tag{4}$$

The Lagrangian equation can be written as:

$$L(p_C, \lambda_1, \lambda_2) = \frac{r - p_C}{k} p_C + \tau\beta_c \int_0^{\frac{r - p_C}{k}} (1-\theta) d\theta + \lambda_1 (r - p_C) + \lambda_2 (1 - \frac{r - p_C}{k})$$

The first-order condition (FOC) with respect to p_C is:

$$k(r - 2p_C) + \tau\beta_c(-k + r - p_C) - \lambda_1 k^2 + \lambda_2 k = 0.$$

If $\lambda_1 > 0$, then $p_C = r$ and $\lambda_2 = 0$. From the FOC we have $\lambda_1 = -\frac{1}{k}(r + \tau\beta_c) < 0$, which contradicts $\lambda_1 > 0$; thus, $\lambda_1 = 0$.

If $\lambda_2 = 0$, then $\frac{r - p_C}{k} < 1$, and the market is not covered. We get $p_C^* = \frac{k(r - \tau\beta_c) + \tau\beta_c r}{2k + \tau\beta_c}$ and $\pi_C^* = \frac{(r + \tau\beta_c)^2}{2(2k + \tau\beta_c)}$. The indifference consumer is given by $\theta^* = \frac{r - p_C^*}{k} = \frac{kr + k\tau\beta_c}{k(2k + \tau\beta_c)}$. Since the market is not covered, we need the condition $\frac{kr + k\tau\beta_c}{k(2k + \tau\beta_c)} < 1$, which holds if and only if $r < 2k$. If $\lambda_2 > 0$, then $\frac{r - p_C}{k} = 1$, and the market is covered. The optimal price is $p_C^* = r - k$ and $\pi_C^* = r - k + \frac{1}{2}\tau\beta_c$.

By Assumption 1, $p_C^* > 0$ in both cases; therefore, the monopolist always sets a positive optimal price when offering a single CPC-based ad-supported service.

CPM-Based Ad Support

The analysis here is similar to the CPC case. The indifference consumer is $\theta^* = \frac{r - p_M}{k}$, and the monopolist's Lagrangian equation is:

$$L(p_M, \lambda_1, \lambda_2) = \frac{r - p_M}{k}(p_M + \beta_m) + \lambda_1 (r - p_M) + \lambda_2 (1 - \frac{r - p_M}{k}), \tag{5}$$

where we also must have $\lambda_1 = 0$. When $\lambda_2 = 0$, market is not covered, we derive the optimal results: $p_M^* = \frac{r - \beta_m}{2}$, $\pi_M^* = \frac{(r + \beta_m)^2}{4k}$, and the demand is $\frac{r + \beta_m}{2k}$. $\frac{r + \beta_m}{2k} < 1$ implies $r < 2k - \beta_m$. When $\lambda_2 > 0$, the market is covered, and the monopolist then sets the optimal price $p_M^* = r - k$ and earns the revenue $\pi_M^* = r - k + \beta_m$.

Therefore, when offering a single CPM-based ad-supported service, if $r < 2k - \beta_m$, the market is not covered, $p_M^* = \frac{r - \beta_m}{2}$, and $\pi_M^* = \frac{(r + \beta_m)^2}{4k}$; if $r \geq 2k - \beta_m$, the market is covered, $p_M^* = r - k$, and $\pi_M^* = r - k + \beta_m$.

Monopoly: Mixed Business Models

Ad-Free and CPC-Based Ad-Supported Services

When the monopolist offers both ad-free and CPC-based ad-supported services, the maximization problem is the following:

$$\max \pi_{nc}(p_n, p_c) = (1 - \frac{p_n - p_c}{k}) p_n + (\frac{p_n - p_c}{k}) p_c + \int_0^{\frac{p_n - p_c}{k}} (1 - \theta) \tau \beta_c d\theta \tag{6}$$

$$\text{s.t. } r - p_n \geq 0, p_n - p_c \geq 0. \tag{7}$$

From Eq. (6) and Eq. (7), the Lagrangian equation can be written as:

$$L(p_n, p_c, \lambda_1, \lambda_2) = (1 - \frac{p_n - p_c}{k}) p_n + (\frac{p_n - p_c}{k}) p_c + \int_0^{\frac{p_n - p_c}{k}} (1 - \theta) \tau \beta_c d\theta + \lambda_1 (r - p_n) + \lambda_2 (p_n - p_c).$$

The FOCs with respect to p_n and p_c are:

$$\frac{\partial \pi_{nc}}{\partial p_n} = 1 - \frac{2p_n}{k} + \frac{2p_c}{k} + \tau\beta_c \frac{1}{k}(1 - \frac{p_n - p_c}{k}) - \lambda_1 + \lambda_2 = 0,$$

$$\frac{\partial \pi_{nc}}{\partial p_c} = \frac{2p_n - 2p_c}{k} + \tau\beta_c(-\frac{1}{k} + \frac{p_n - p_c}{k^2}) - \lambda_2 = 0.$$

Adding the two FOCs shows that $\lambda_1^* = 1 > 0$, so the constraint is active, and $p_n^* = r$. By solving the FOCs, we then get that $p_c^* = r - \frac{k\tau\beta_c}{2k + \tau\beta_c}$ and $\pi_{nc}^* = r + \frac{\tau^2 \beta_c^2}{2(2k + \tau\beta_c)}$. Given p_n^* and p_c^*, the indifference consumer is $\theta^* = \frac{\tau\beta_c}{2k + \tau\beta_c}$, which is between 0 and 1. Therefore, when mixing the ad-free and the CPC-based ad-supported services, the monopolist serves the entire market, $p_n^* = r$, $p_c^* = r - \frac{k\tau\beta_c}{2k + \tau\beta_c}$, and $\pi_{nc}^* = r + \frac{\tau^2 \beta_c^2}{2(2k + \tau\beta_c)}$.

Ad-Free and CPM-Based Ad-Supported Services

Here, the monopolist's revenue function takes the following form:

$$\max \pi_{nm}(p_n, p_m) = (1 - \frac{p_n - p_m}{k}) p_n + \frac{p_n - p_m}{k}(p_m + \beta_m) \quad (8)$$

$$\text{s.t.} \quad r - p_n \geq 0, p_n - p_m \geq 0. \quad (9)$$

Similar to the analysis in the CPC case, we derive that the monopolist also covers the entire market here and that the optimal solution is $p_n^* = r, p_m^* = r - \frac{\beta_m}{2}$, and $\pi_{nm}^* = r + \frac{\beta_m^2}{4k}$. The indifference consumer here is $\theta^* = \frac{\beta_m}{2k}$, which is between 0 and 1 when $\beta_m \leq 2k$. When $\beta_m > 2k$, the monopolist offers only the CPM ad-supported service at the optimal price p_M^* (see Table 2).

Duopoly: Single-Service Business Models (B1)

By offering the ad-free service only, firm 1's revenue function is:

$$\pi_1(p_1, p_2) = (1 - \frac{p_1 - p_2}{k}) p_1. \quad (10)$$

Firm 2's revenue function depends on the revenue model of the ad-supported service. For the CPC model:

$$\pi_2(p_1, p_2) = \frac{p_1 - p_2}{k} p_2 + \int_0^{\frac{p_1 - p_2}{k}} (1-\theta)\tau\beta_c d\theta; \quad (11)$$

and for the CPM revenue model:

$$\pi_2(p_1, p_2) = \frac{p_1 - p_2}{k}(p_2 + \beta_m). \quad (12)$$

Consumers choose firm 1's ad-free service if $r - p_1 > r - \theta k - p_2$, or $\theta > \frac{p_1 - p_2}{k}$. The FOC of firm 1's revenue function (Eq. (10)) is $1 - \frac{2p_1 - p_2}{k} = 0$. When firm 2 uses the CPC-based ad-supported service, the FOC is $\frac{p_1 - 2p_2}{k} + \tau\beta_c\left(\frac{p_1 - p_2}{k^2} - \frac{1}{k}\right) = 0$. Solving the two FOCs for p_1 and p_2 gives $p_1^* = \frac{2k^2}{3k + \tau\beta_c}$ and $p_2^* = \frac{k^2 - k\tau\beta_c}{3k + \tau\beta_c}$, which give the equilibrium revenues: $\pi_1^* = \frac{4k^3}{(3k + \tau\beta_c)^2}$ and $\pi_2^* = \frac{(2k + \tau\beta_c)(k + \tau\beta_c)^2}{2(3k + \tau\beta_c)^2}$. $p_2^* > 0$ requires the condition $\tau\beta_c < k$. Therefore, when $\tau\beta_c \geq k$, $p_2^* = 0$ and $p_1^* = k/2$, the equilibrium revenues are $\pi_2^* = \frac{3}{8}\tau\beta_c$ and $\pi_1^* = \frac{k}{4}$.

When firm 2 adopts the CPM ad revenue model, firm 1's FOC remains unchanged, while the FOC firm 2's revenue function (Eq. (12)) becomes $\frac{p_1 - 2p_2 - \beta_m}{k} = 0$. Solving the two FOCs yields the equilibrium prices $p_1^* = \frac{2k - \beta_m}{3}$ and $p_2^* = \frac{k - 2\beta_m}{3}$, and then the equilibrium revenues can be derived: $\pi_1^* = \frac{(2k - \beta_m)^2}{9k}$ and $\pi_2^* = \frac{(k + \beta_m)^2}{9k}$. Positive p_2^* implies that $\beta_m < k/2$. Thus, when $\beta_m \geq k/2$, then $p_2^* = 0$, $p_1^* = k/2$, $\pi_2^* = \beta_m/2$, and $\pi_1^* = k/4$.

Duopoly: Firm 1 Adopts a Mixed Business Model (B2)

Since both firms' ad-supported services have a price of zero, consumers are indifferent between the ad-free and ad-supported services when $r - p_n = r - \theta k$; thus, the demand for the ad-free service is $1 - \frac{p_n}{k}$. Regardless of firm 2's model choice, if firm 1 adopts the CPC model, its revenue function has the form, $\pi_{nc}(p_n) = (1 - \frac{p_n}{k}) p_n + \frac{1}{2}\int_0^{\frac{p_n}{k}} (1-\theta)\tau\beta_c d\theta$; with the CPM model, it has the revenue function, $\pi_{nm}(p_n) = (1 - \frac{p_n}{k}) p_n + \frac{1}{2}\frac{p_n}{k}\beta_m$. Firm 2's revenue is also a function of firm 1's pricing strategy: $\pi_C(p_n) = \frac{1}{2}\int_0^{\frac{p_n}{k}} (1-\theta)\tau\beta_c d\theta$ and $\pi_M(p_n) = \frac{1}{2}\frac{p_n}{k}\beta_m$ for CPC- and CPM-based ad revenue models, respectively.

If firm 1 has the business model nc, the FOC is $1 - \frac{2p_n}{k} + \frac{\tau\beta_c}{2}\left(\frac{1}{k} - \frac{p_n}{k^2}\right) = 0$, which yields $p_n^* = \frac{k(2k + \tau\beta_c)}{4k + \tau\beta_c}$. Notice that the indifference consumer (between purchasing the ad-free and ad-supported services) is $\theta^* = \frac{2k + \tau\beta_c}{4k + \tau\beta_c} < 1$, implying that the ad-free demand is positive; also, $p_n^* < r$, since $k < r$. From p_n^*, we derive firm 1's revenue $\pi_{nc}^* = \frac{(2k + \tau\beta_c)^2}{4(4k + \tau\beta_c)}$ and firm 2's revenue for the CPC and CPM models: $\pi_C^* = \frac{(2k + \tau\beta_c)(6k + \tau\beta_c)\tau\beta_c}{4(4k + \tau\beta_c)^2}$ and $\pi_M^* = \frac{(2k + \tau\beta_c)\beta_m}{2(4k + \tau\beta_c)}$.

Similarly, if firm 1 chooses the nm business model, the FOC is $1 - \frac{2p_n}{k} + \frac{\beta_m}{2k} = 0$, which yields $p_n^* = \frac{2k + \beta_m}{4}$, which gives firm 1's revenue $\pi_{nm}^* = \frac{(2k + \beta_m)^2}{16k}$ and, for firm 2, $\pi_C^* = \frac{\tau\beta_c}{64k^2}(2k + \beta_m)(6k - \beta_m)$ and $\pi_M^* = \frac{\beta_m}{8k}(2k + \beta_m)$. This equilibrium requires the condition $\beta_m \leq 2k$, which implies $\theta^* \leq 1$.

When $\beta_m > 2k$, no consumer purchases the ad-free service; thus, firm 1 would set p_n^* such that $r - p_n^* > r - \theta k$ for all θ. Therefore, when $\beta_m > 2k$, the entire market is split between the two firms' ad-supported services: For firm 1, $p_n^* > k$ and $\pi_{nm}^* = \frac{\beta_m}{2}$, and, for firm

2, $\pi_C^* = \frac{\tau\beta_c}{4}$ and $\pi_M^* = \frac{\beta_m}{2}$.

Notice that Case B2 is a general case that endogenizes firms' decisions in Case A, where both firms only offer ad-supported services. In Case B2, firm 1 can adjust the price of the ad-free service to include or eliminate the demand for the ad-free service. Therefore, we can rule out Case A in the analysis of the equilibrium business model, which reduces our analysis to the matrix shown in Figure 4. It can be verified that, given the rival's strategy, firm 1's optimal revenue under the mixed business model (Table 5) is indeed higher than its revenue without the ad-free service: $\pi_{nc}^* - \pi_C^* > 0$ and $\pi_{nm}^* - \pi_M^* \geq 0$.

Duopoly: Discussion of Equilibrium Business Models

Proposition 7 ***(CPM Equilibrium).*** *When $\beta_m > \tau\beta_c$, in equilibrium firm 2 chooses the CPM-based ad-supported service, and firm 1 never chooses the mixed business model with a CPC ad revenue model (nc is a dominated strategy). In particular, if $\beta_m > \beta_m^*$, in equilibrium firm 1 adopts the mixed business model with the CPM model, otherwise, firm 1 offers a single ad-free service.*

Proposition 8 ***(CPC Equilibrium)***. *When $\tau\beta_c > 2\beta_m$, in equilibrium firm 2 chooses the CPC-based ad-supported service (C is a dominant strategy), and firm 1 never chooses the mixed business model with a CPM ad revenue model (nm is a dominated strategy). In particular, if $\tau\beta_c > \beta_c^*$, in equilibrium firm 1 adopts the mixed business model with the CPC model, otherwise, firm 1 offers a single ad-free service.*

The conditions in Propositions 7 and 8 highlight the cases of a unique equilibrium but also do not preclude any strategy sets that would otherwise emerge in equilibrium. The remaining two strategy sets, (nc, M) and (nm, C), cannot occur in equilibrium, because two firms will always choose the same, more profitable ad revenue model. Without the conditions in Propositions 7 and 8, multiple equilibria may be established; the equilibria will consist of a combination of the equilibrium strategy sets indicated in these propositions.

Alternative Model Setup: Vertically Differentiated Ad-Supported Services

Here we consider an extension of the main model, where two ad revenue models result in different qualities of service experience for consumers. For model tractability, we focus on the case in which CPC ads create a higher quality service than CPM ads (or, equivalently, CPM ads induce higher disutility than CPC ads). This assumption is driven by the emphasis on contextual relevance and targeting precision in the CPC model to convert views to immediate user actions and recognizes that the CPM model traditionally focuses on increasing brand awareness and influencing consumers' future purchase behavior.[8] Denote by k_L and k_H the disutility induced by CPM and CPC ads, respectively; thus, $k_L > k_H$. To simplify notations, we normalize k_L to k and let $k_H = \alpha k_L = \alpha k$ where $\alpha \in (0,1)$. Following Assumption 1, $r > k_i$ for $i \in \{L, H\}$.

For single-service business models, only the optimal strategies for the ad-supported

[8]It is also possible that a service with CPM ads has a higher quality than a service with CPC ads. For example, Super Bowl commercials are some of the most popular ads, adding to the excitement of the NFL game. Thus, ultimately, advertisers choose ad effectiveness and the service quality. We further discuss this point as a future direction in the conclusion.

services may change from those in the main model (Table 6).

<Insert Table 6 approximately here>

For mixed business models, when the monopolist offers both the ad-free and CPC ad-supported services, $p_n^* = r$ and $p_h^* = r - \frac{\alpha k \tau \beta_c}{2\alpha k + \tau \beta_c}$, and $\pi_{nh}^* = r + \frac{\tau^2 \beta_c^2}{2(2\alpha k + \tau\beta_c)}$. Its optimal strategies when offering ad-free and CPM ad-supported services are identical to those in the main model.

We can easily verify that Proposition 2 from the main model holds here: In both single-service and mixed business models, the optimal price of the ad-supported service decreases with the ad revenue rate; also, the CPC ad revenue model leads to greater price stability than the CPM model.

Multiple Ad-Supported Services

In this section, we consider the monopolist's strategies when combining two qualities of ad-supported services.

hl Business Model

The monopolist offers both high- and low-quality ad-supported services but does not include the ad-free service. Its maximization problem is:

$$\max_{p_h, p_l} \pi_{hl}(p_h, p_l) = \left(\frac{r-p_h}{\alpha k} - \frac{p_h - p_l}{(1-\alpha)k}\right)p_h + \int_{\frac{p_h-p_l}{(1-\alpha)k}}^{\frac{r-p_h}{\alpha k}} (1-\theta)\tau\beta_c d\theta + \frac{p_h-p_l}{(1-\alpha)k}(p_l + \beta_m) \qquad (13)$$

$$\text{s.t. } 1 \geq \frac{r-p_h}{\alpha k} \geq \frac{p_h-p_l}{(1-\alpha)k} \geq 0.$$

By summing the FOCs of Eq. (13), we get $\frac{r-2p_h}{\alpha k} + \tau\beta_c\left(-\frac{1}{\alpha k} + \frac{r-p_h}{\alpha^2 k^2}\right) = 0$.

In the interior solution, $\frac{r-p_h}{\alpha k} > \frac{p_h-p_l}{(1-\alpha)k} > 0$, the monopolist has positive demand for both ad-supported services. We get the following results:

- $r \leq 2\alpha k$,

$$p_h^* = \frac{\alpha k(r-\tau\beta_c) + r\tau\beta_c}{2\alpha k + \tau\beta_c},\ p_l^* = p_h^* - \frac{(1-\alpha)k(\beta_m - \tau\beta_c)}{2(1-\alpha)k - \tau\beta_c},\ \pi_{hl}^* = \frac{(r+\tau\beta_c)^2}{2(2\alpha k + \tau\beta_c)} + \frac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]}$$

- $r > 2\alpha k$, $p_h^* = r - \alpha k,\ p_l^* = p_h^* - \frac{(1-\alpha)k(\beta_m - \tau\beta_c)}{2(1-\alpha)k - \tau\beta_c},\ \pi_{hl}^* = r - \alpha k + \frac{1}{2}\tau\beta_c + \frac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]}$.

To ensure that the above interior solution exists, we need the following condition:

$$\frac{\beta_m - \tau\beta_c}{2(1-\alpha)k - \tau\beta_c} > 0. \qquad (14)$$

There are also two corner solutions that are equivalent to the *H* case and the *L* case of single-service business models: When $p_h = p_l$ (i.e., $\frac{p_h - p_l}{(1-\alpha)k} = 0$), all consumers will buy the high-quality service, which is equivalent to not offering the low-quality ad-supported service at all; and when p_h is sufficiently high (i.e., $\frac{r - p_h}{\alpha k} = \frac{p_h - p_l}{(1-\alpha)k}$), all consumers resort to the low-quality service.

"all" Business Model

When offering all three types of service, the monopolist's maximization problem is:

$$\max \pi_{all}(p_n, p_h, p_l) = (1 - \frac{p_n - p_h}{\alpha k}) p_n + (\frac{p_n - p_h}{\alpha k} - \frac{p_h - p_l}{(1-\alpha)k}) p_h + \int_{\frac{p_h - p_l}{(1-\alpha)k}}^{\frac{p_n - p_h}{\alpha k}} (1-\theta)\tau\beta_c d\theta + \frac{p_h - p_l}{(1-\alpha)k}(p_l + \beta_m)$$

$$\text{s.t. } r - p_n \geq 0 \text{ and } \frac{p_n - p_h}{\alpha k} \geq \frac{p_h - p_l}{(1-\alpha)k} \geq 0. \qquad (15)$$

The Lagrangian function can be written as

$$L(p_n, p_h, p_l, \lambda) = (1 - \frac{p_n - p_h}{\alpha k}) p_n + (\frac{p_n - p_h}{\alpha k} - \frac{p_h - p_l}{(1-\alpha)k}) p_h + \frac{p_h - p_l}{(1-\alpha)k}(p_l + \beta_m)$$
$$+ \tau\beta_c [\frac{p_n - p_h}{\alpha k} - \frac{p_h - p_l}{(1-\alpha)k} - \frac{1}{2}(\frac{p_n - p_h}{\alpha k})^2 + \frac{1}{2}(\frac{p_h - p_l}{(1-\alpha)k})^2]$$
$$+ \lambda_1 (r - p_n) + \lambda_2 (\frac{p_n - p_h}{\alpha k} - \frac{p_h - p_l}{(1-\alpha)k}) + \lambda_3 (p_h - p_l)$$

Adding the FOCs shows that $\lambda_1 > 0$, so $p_n^* = r$.

If $\frac{p_n - p_h}{\alpha k} > \frac{p_h - p_l}{(1-\alpha)k} > 0$, we have

$$p_h^* = r - \frac{\alpha k \tau\beta_c}{2\alpha k + \tau\beta_c}, \quad p_l^* = p_h^* - \frac{(1-\alpha)k(\beta_m - \tau\beta_c)}{2(1-\alpha)k - \tau\beta_c}, \quad \pi_{all}^* = r + \frac{\tau^2\beta_c^2}{2(2\alpha k + \tau\beta_c)} + \frac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]}.$$

This interior solution also requires condition (14). The two corner solutions correspond exactly to the two cases of mixing each ad-supported service with the ad-free service. The first is $p_h^* = r - \frac{\alpha k \tau\beta_c}{2\alpha k + \tau\beta_c}$, $p_l^* = p_h^*$, and $\pi_{all}^* = r + \frac{\tau^2\beta_c^2}{2(2\alpha k + \tau\beta_c)}$, which is equivalent to the case of offering only the CPC-based service with the ad-free service because no consumer would adopt the lower-quality CPM service; the other one is $p_h^* = (1-\alpha)r + \alpha p_l^*$, $p_l^* = r - \frac{\beta_m}{2}$, and $\pi_{all}^* = r + \frac{\beta_m^2}{4k}$, equivalent to the case where only the ad-free and CPM-based services are offered.

Proposition 9 *If $\tau\beta_c > \beta_m$, or $\beta_m \geq \tau\beta_c$ and $2(1-\alpha)k - \tau\beta_c < 0$, simultaneously offering the CPC and CPM ad-supported services (either the hl or all business model) is suboptimal.*

Proposition 9 suggests that offering both CPM and CPC ad-supported services may lead to cannibalization. The CPM service, the low-quality service, cannibalizes the CPC service by tapping into the consumer segment that has a lower ad aversion, which the CPC model primarily relies on for generating ad revenues. This result contrasts with the finding in Casadesus-Masanell and Zhu [5], where offering two vertically differentiated, ad-supported services (termed a "dual model") does not lead to cannibalization. The main difference in our model is that we isolate consumer heterogeneity in ad aversion from the valuation for the service, whereas in [5], consumers value service quality and advertising level based on the same taste parameter. We also account for the inclusion of ad-free service and find that such cannibalization still occurs with the ad-free service.

There exists a scenario in which the optimal strategy allows for the coexistence of CPC and CPM ad-supported services. It requires a sufficiently high ad revenue rate in the CPM model ($\beta_m \geq \tau\beta_c$) and considerable quality differentiation between the two ad-supported services ($2(1-\alpha)k - \tau\beta_c \geq 0$). The ad revenues from the CPM model combined with mitigated competition (due to the quality gap) compensate for the loss in the CPC-generated ad revenues; thus, vertically differentiated ad-supported services may be sustainable.

Proposition 10 *If* $\tau\beta_c > \beta_m$, *or* $\beta_m \geq \tau\beta_c$ *and* $2(1-\alpha)k - \tau\beta_c < 0$, *the monopolist's optimal strategy is to offer the ad-free service with the CPC ad-supported service when* $\beta_m < \tau\beta_c\sqrt{\frac{2k}{2\alpha k + \tau\beta_c}}$ *and with the CPM service otherwise. If* $\beta_m \geq \tau\beta_c$ *and* $2(1-\alpha)k - \tau\beta_c \geq 0$, *offering the ad-free service with both ad-supported services (all) is optimal.*

Proposition 10 mirrors Proposition 4 in the main model and uncovers a new finding. The monopolist may choose between the two mixed business models as the optimal strategy, and the condition follows that in the optimal strategy of the main model (Proposition 4). In addition, the business model that combines all three services may be optimal here, given a high CPM ad revenue rate and a substantial quality gap. The intuition builds on that of Proposition 9: Offering all three services is superior than offering ad-free and CPC services, because introducing the CPM ad-supported service will boost the ad revenues ($\beta_m \geq \tau\beta_c$); the *all* business model also dominates only offering ad-free and CPM services, because the CPC ad-supported service can contribute substantially to subscription revenues given the quality difference ($2(1-\alpha)k - \tau\beta_c \geq 0$).

Proofs

Proof of Proposition 1.

Proof. In the CPC case, for $r < 2k$, $p_C^* - p_c^* = \frac{-kr}{2k + \tau\beta_c} < 0$; and for $r \geq 2k$, $p_C^* - p_c^* = \frac{-2k^2}{2k + \tau\beta_c} < 0$. In the CPM case, for $r < 2k - \beta_m$, $p_M^* - p_m^* = \frac{-r}{2} < 0$; for $r \geq 2k - \beta_m$, $p_M^* - p_m^* = \frac{\beta_m}{2} - k \leq 0$, given $\beta_m \leq 2k$.

Proof of Proposition 2.

Proof. *Single-Service Case*: First, for $r < 2k$, $\frac{\partial p_C^*}{\partial \beta_c} = \frac{\tau k(r-2k)}{(2k+\tau\beta_c)^2} \le 0$; for $r \ge 2k$, p_C^* is independent of β_c. And then, for $r < 2k - \beta_m$, $\frac{\partial p_M^*}{\partial \beta_m} = -\frac{1}{2} < 0$; and for $r \ge 2k - \beta_m$, p_M^* is independent of β_m. Therefore, both prices are decreasing in the corresponding revenue rate.

To show that the optimal price under the CPM-based model is more sensitive than that under the CPC-based model, we need $\frac{\tau k(2k-r)}{(2k+\tau\beta_c)^2} < \frac{1}{2}$. Since $k(2k-r) < \frac{1}{2}(2k+\tau\beta_c)^2$ and $0 < \tau < 1$, $\frac{\tau k(2k-r)}{(2k+\tau\beta_c)^2} < \frac{1}{2}$ is then satisfied.

Mixed Case: We derive that $\frac{\partial p_c^*}{\partial \beta_c} = \frac{-2\tau k^2}{(2k+\tau\beta_c)^2} < 0$ and $\frac{\partial p_m^*}{\partial \beta_m} = -\frac{1}{2} < 0$. Since $2\tau k^2 < \frac{1}{2}(2k+\tau\beta_c)^2$, $\left|\frac{\partial p_c^*}{\partial \beta_c}\right| < \left|\frac{\partial p_m^*}{\partial \beta_m}\right|$.

Proof of Proposition 3.

Proof. By comparing the equilibrium revenues, it is straightforward to see that $\pi_{nc}^* > \pi_N^*$, $\pi_{nm}^* > \pi_N^*$. We then compare π_{nc}^* and π_{nm}^* with π_C^* and π_M^*, respectively.

When $r < 2k$, $\pi_{nc}^* - \pi_C^* = r + \frac{\tau^2\beta_c^2}{2(2k+\tau\beta_c)} - \frac{(r+\tau\beta_c)^2}{2(2k+\tau\beta_c)} = \frac{r(4k-r)}{2(2k+\tau\beta_c)} > 0$. When $r \ge 2k$, $\pi_{nc}^* - \pi_C^* = r + \frac{\tau^2\beta_c^2}{2(2k+\tau\beta_c)} - (r - k + \frac{1}{2}\tau\beta_c) = \frac{2k^2}{2k+\tau\beta_c} > 0$. When $r < 2k - \beta_m$, $\pi_{nm}^* - \pi_M^* = r + \frac{\beta_m^2}{4k} - \frac{(r+\beta_m)^2}{4k} = \frac{(4k-r-2\beta_m)r}{4k}$; since $r > 0$ and $4k - r - 2\beta_m > 0$ for $r < 2k - \beta_m$, $\frac{(4k-r-2\beta_m)r}{4k} > 0$. When $r \ge 2k - \beta_m$, $\pi_{nm}^* - \pi_M^* = \frac{\beta_m^2}{4k} + k - \beta_m = \frac{(\beta_m - 2k)^2}{4k} \ge 0$. Therefore, $\pi_{nc}^* > max\{\pi_N^*, \pi_C^*\}$ and $\pi_{nm}^* > max\{\pi_N^*, \pi_M^*\}$.

Proof of Proposition 4.

Proof. Let $\Delta\pi = \pi_{nc}^* - \pi_{nm}^* = \frac{2k\tau^2\beta_c^2 - 2k\beta_m^2 - \beta_m^2\tau\beta_c}{4k(2k+\tau\beta_c)}$. It can be derived that $\Delta\pi = 0$ when $\beta_m = \tau\beta_c\sqrt{\frac{2k}{2k+\tau\beta_c}}$, and that $\frac{\partial \Delta\pi}{\partial(\tau\beta_c)} > 0$ and $\frac{\partial \Delta\pi}{\partial \beta_m} < 0$. Therefore, when $\beta_m < \tau\beta_c\sqrt{\frac{2k}{2k+\tau\beta_c}}$, the nc business model dominates nm, and vice versa.

Proof of Proposition 5.

Proof. By taking the derivatives with respect to β_c for the CPC-based model and with respect to β_m for the CPM-based model, we derive that for firm 2, $\frac{\partial p_C^*}{\partial \beta_c} = \frac{-4\tau k^2}{(3k+\tau\beta_c)^2} < 0$ and $\frac{\partial p_M^*}{\partial \beta_m} = -\frac{2}{3} < 0$. For firm 1, if the rival uses the CPC-based model, $\frac{\partial p_N^*}{\partial \beta_c} = \frac{-2\tau k^2}{(3k+\tau\beta_c)^2} < 0$; if the rival uses the CPM-based model, $\frac{\partial p_N^*}{\partial \beta_m} = -\frac{1}{3} < 0$. Therefore, both firms' equilibrium prices are decreasing in firm 2's ad revenue rate.

Since $4\tau k^2 < \frac{2}{3}(3k+\tau\beta_c)^2$, thus, $\left|\frac{\partial p_C^*}{\partial \beta_c}\right| < \left|\frac{\partial p_M^*}{\partial \beta_m}\right|$, which implies that firm 2's equilibrium price in the CPM model is more sensitive to the ad revenue rate than that in the CPC model. And from $2\tau k^2 < \frac{1}{3}(3k+\tau\beta_c)^2$, we have $\left|\frac{\partial p_N^*}{\partial \beta_c}\right| < \left|\frac{\partial p_N^*}{\partial \beta_m}\right|$, implying that firm 1's equilibrium price is more sensitive given the rival has the CPM-based model than when it has the CPC-based model.

Proof of Proposition 6.

Proof. When firm 1 chooses nc, $\frac{\partial p_n^*}{\partial \beta_c} = 2\tau k^2/(4k+\tau\beta_c)^2$. When firm 1 chooses nm and $\beta_m \le 2k$, $\frac{\partial p_n^*}{\partial \beta_m} = \frac{1}{4}$; when $\beta_m > 2k$, p_n^* is independent of β_m. Both $\frac{\partial p_n^*}{\partial \beta_c}$ and $\frac{\partial p_n^*}{\partial \beta_m}$ are clearly positive, thus, the price of the ad-free service is increasing in the ad revenue rate. Furthermore, $\frac{\partial p_n^*}{\partial \beta_c} < \frac{\partial p_n^*}{\partial \beta_m}$, since $2\tau k^2 < \frac{1}{4}(4k+\tau\beta_c)^2$; therefore, the ad-free service price increases at a higher rate with the CPM model.

Proof of Proposition 7.

Proof. To prove firm 2's dominant strategy is M when $\beta_m > \tau\beta_c$, we need to show that given firm 1 choosing N, nc, and nm, firm 2 is better off choosing M over C:

• When firm 1 chooses N, we need to show for firm 2, $\pi_M^* > \pi_C^*$ in all cases.

$$\pi_M^* = \begin{cases} \frac{(k+\beta_m)^2}{9k} & \beta_m < \frac{k}{2} \\ \frac{\beta_m}{2} & \beta_m \ge \frac{k}{2} \end{cases}, \quad \pi_C^* = \begin{cases} \frac{(2k+\tau\beta_c)(k+\tau\beta_c)^2}{2(3k+\tau\beta_c)^2} & \tau\beta_c < k \\ \frac{3}{8}\tau\beta_c & \tau\beta_c \ge k \end{cases}$$

There are four cases:

1) $\tau\beta_c < \beta_m < \frac{k}{2}$. Since π_M^* is increasing in β_m, if $\pi_M^* > \pi_C^*$ at $\beta_m = \tau\beta_c$, then it holds for $\beta_m > \tau\beta_c$.

$$\pi_M^*(\tau\beta_c) - \pi_C^* = \frac{(k+\tau\beta_c)^2}{9k} - \frac{(2k+\tau\beta_c)(k+\tau\beta_c)^2}{2(3k+\tau\beta_c)^2} = \frac{(k+\tau\beta_c)^2}{18k(3k+\tau\beta_c)^2}[2\tau^2\beta_c^2 + 3k\tau\beta_c] > 0$$

2) $\tau\beta_c < \frac{k}{2} \le \beta_m$. $\pi_M^*(\frac{k}{2}) = \frac{k}{4} > \pi_C^*(\frac{k}{2}) = \frac{45}{196}k$. Since π_M^* is increasing in β_m and π_C^* is increasing in $\tau\beta_c$ ($\frac{\partial \pi_C^*}{\partial(\tau\beta_c)} = \frac{2(3k+\tau\beta_c)(k+\tau\beta_c)(11k^2+8k\tau\beta_c+\tau^2\beta_c^2)}{4(3k+\tau\beta_c)^4} > 0$), $\pi_M^* > \pi_C^*$ at $\beta_m = \tau\beta_c$ holds for $\beta_m > \tau\beta_c$.

3) $\frac{k}{2} \le \tau\beta_c < k$. We need to show that $\pi_M^*(\beta_m = \tau\beta_c) > \pi_C^*$, so for $\beta_m > \tau\beta_c$, $\pi_M^* > \pi_C^*$.

$$\pi_M^*(\tau\beta_c) - \pi_C^* = \frac{\tau\beta_c}{2} - \frac{(2k+\tau\beta_c)(k+\tau\beta_c)^2}{2(3k+\tau\beta_c)^2} = \frac{1}{(3k+\tau\beta_c)^2}[4k^2\tau\beta_c + 2k\tau^2\beta_c^2 - 2k^3],$$

which is positive, given $\tau\beta_c \ge \frac{k}{2}$.

4) $\tau\beta_c \ge k$. Since $\pi_M^*(\tau\beta_c) - \pi_C^* = \frac{1}{2}\tau\beta_c - \frac{3}{8}\tau\beta_c > 0$, then for $\beta_m > \tau\beta_c$, $\pi_M^* > \pi_C^*$.

Therefore, we can conclude that given firm 1 chooses N, for firm 2, M dominates C, if $\beta_m > \tau\beta_c$.

• When firm 1 chooses nc, firm 2's equilibrium revenues for M and C are $\pi_C^* = \frac{(2k+\tau\beta_c)(6k+\tau\beta_c)\tau\beta_c}{4(4k+\tau\beta_c)^2}$ and $\pi_M^* = \frac{(2k+\tau\beta_c)\beta_m}{2(4k+\tau\beta_c)}$, respectively. Since $\pi_M^*(\tau\beta_c) - \pi_C^* = \frac{(2k+\tau\beta_c)^2\tau\beta_c}{4(4k+\tau\beta_c)^2} > 0$, then $\pi_M^* > \pi_C^*$ for $\beta_m > \tau\beta_c$. Therefore, given firm 1 chooses nc, for firm 2, M dominates C, if $\beta_m > \tau\beta_c$.

• When firm 1 chooses nm, we need to compare the revenues under two cases, $\beta_m \le 2k$ and $\beta_m > 2k$. When $\beta_m \le 2k$, $\pi_M^* \ge \pi_C^* \Leftarrow \beta_m \ge \frac{(6k-\beta_m)\tau\beta_c}{8k}$, which holds since $\frac{6k-\beta_m}{8k} < 1$ and $\beta_m > \tau\beta_c$. When $\beta_m > 2k$, $\frac{\beta_m}{2} > \frac{\tau\beta_c}{4}$ for $\beta_m > \tau\beta_c$. Therefore, given firm 1 chooses nm, for firm 2, M dominates C, if $\beta_m > \tau\beta_c$.

We have shown that, if $\beta_m > \tau\beta_c$, firm 2's dominant strategy is M. Now given firm 2 choosing M, we compare firm 1's payoffs.

• We first compare firm 1's strategies nc and nm, the revenues under which are

$$\pi_{nc}^* = \frac{(2k+\tau\beta_c)^2}{4(4k+\tau\beta_c)}, \quad \pi_{nm}^* = \begin{cases} \frac{(2k+\beta_m)^2}{16k} & \beta_m \le 2k \\ \frac{1}{2}\beta_m & \beta_m > 2k \end{cases}$$

There are three cases:

1) $\tau\beta_c < \beta_m \le 2k$. $\pi_{nm}^* > \pi_{nc}^* \Leftarrow \frac{(2k+\beta_m)^2}{16k} > \frac{(2k+\tau\beta_c)^2}{16k+4\tau\beta_c}$, which holds given $\beta_m > \tau\beta_c$.

2) $\tau\beta_c \le 2k < \beta_m$. $\pi^*_{nm}(2k) - \pi^*_{nc} = \dfrac{12k^2 - \tau^2\beta_c^2}{4(4k+\tau\beta_c)} > 0$, so $\pi^*_{nm} > \pi^*_{nc}$ for $\tau\beta_c \le 2k < \beta_m$.

3) $2k < \tau\beta_c < \beta_m$. $\pi^*_{nm}(\tau\beta_c) - \pi^*_{nc} = \dfrac{4k\tau\beta_c + \tau^2\beta_c^2 - 4k^2}{4(4k+\tau\beta_c)} > 0$, thus $\pi^*_{nm} > \pi^*_{nc}$ for $\beta_m > \tau\beta_c$.

Therefore, nm dominates nc, given $\beta_m > \tau\beta_c$.

• We now compare firm 1's payoffs under nm and N. When $\beta_m < \dfrac{k}{2}$,

$$\pi^*_{nm} - \pi^*_N = \frac{(2k+\beta_m)^2}{16k} - \frac{(2k-\beta_m)^2}{9k} = \frac{1}{144k}\left[9(2k+\beta_m)^2 - 16(2k-\beta_m)^2\right],$$

which is increasing in β_m; it is also negative when $\beta_m = 0$ and positive when $\beta_m = \dfrac{k}{2}$.

Therefore, there exists $0 < \beta^*_m < \dfrac{k}{2}$, such that when $\beta_m > \beta^*_m$, firm 1 prefers nm, otherwise, it prefers N.

For $\beta_m \ge \dfrac{k}{2}$, $\pi^*_N = \dfrac{k}{4}$; at $\beta_m = \dfrac{k}{2}$, $\pi^*_{nm} = \dfrac{25k}{64} > \pi^*_N$. And it is straightforward to show that $\dfrac{\beta_m}{2} > \dfrac{k}{4}$ for the case $\beta_m > 2k$. Therefore, $\pi^*_{nm} > \pi^*_N$, when $\beta_m \ge \dfrac{k}{2}$.

Thus, we conclude that given $\beta_m > \tau\beta_c$, if $\beta_m > \beta^*_m$, the equilibrium is (nm, M), otherwise, it is (N, M).

Proof of Proposition 8.

Proof. To prove firm 2's dominant strategy is C when $\tau\beta_c > 2\beta_m$, we need to show that given firm 1 choosing N, nc, and nm, firm 2 is better off choosing C over M:

• When firm 1 chooses N, we need to show for firm 2, $\pi^*_C > \pi^*_M$ in all cases. Since π^*_C is increasing in $\tau\beta_c$ (proven in the proof of Proposition 7), if $\pi^*_C > \pi^*_M$ at $\tau\beta_c = 2\beta_m$, then it holds for $\tau\beta_c > 2\beta_m$.

When $\beta_m < \dfrac{k}{2}$, comparing π^*_C at $\tau\beta_c = 2\beta_m$ and π^*_M, we have:

$$\pi^*_C(2\beta_m) - \pi^*_M = \frac{(2k+2\beta_m)(k+2\beta_m)^2}{2(3k+2\beta_m)^2} - \frac{(k+\beta_m)^2}{9k} = \frac{\beta_m(k+\beta_m)}{9k(3k+2\beta_m)^2}[15k^2 + 20k\beta_m - 4\beta_m^2] > 0.$$

Thus, this inequality holds for all $0 \le \beta_m < \dfrac{k}{2}$.

When $\beta_m \ge \dfrac{k}{2}$, at $\tau\beta_c = 2\beta_m$, $\pi^*_C = \dfrac{3\beta_m}{4}$, which is greater than $\pi^*_M = \dfrac{\beta_m}{2}$. Therefore, given firm 1 chooses N, for firm 2, C dominates M, if $\tau\beta_c > 2\beta_m$.

• When firm 1 chooses nc, comparing firm 2's equilibrium revenues for C and M, we have $\pi^*_C \ge \pi^*_M \Leftarrow \dfrac{(6k+\tau\beta_c)\tau\beta_c}{4k+\tau\beta_c} \ge 2\beta_m$, which holds, since $\dfrac{6k+\tau\beta_c}{4k+\tau\beta_c} > 1$ and $\tau\beta_c > 2\beta_m$. Therefore, given firm 1 chooses nc, for firm 2, C dominates M, if $\tau\beta_c > 2\beta_m$.

• When firm 1 chooses nm, we need to compare the revenues under two cases, $\beta_m \le 2k$ and $\beta_m > 2k$. When $\beta_m \le 2k$, $\pi_C^* \ge \pi_M^* \Leftarrow \frac{(6k-\beta_m)\tau\beta_c}{4k} \ge 2\beta_m$, which holds since $\frac{6k-\beta_m}{4k} \ge 1$ (because $\beta_m \le 2k$) and $\tau\beta_c > 2\beta_m$. When $\beta_m > 2k$, $\frac{\tau\beta_c}{4} \ge \frac{\beta_m}{2}$ for $\tau\beta_c > 2\beta_m$. Therefore, given firm 1 chooses nm, for firm 2, C dominates M, if $\tau\beta_c > 2\beta_m$.

We have shown that, if $\tau\beta_c > 2\beta_m$, firm 2's dominant strategy is C. Now given firm 2 choosing C, we compare firm 1's payoffs under different business models.

• We first compare firm 1's strategies nc and nm. Since π_{nc}^* is increasing in $\tau\beta_c$: $\frac{\partial\pi_{nc}^*}{\partial(\tau\beta_c)} = \frac{(2k+\tau\beta_c)(6k+\tau\beta_c)}{(4k+\tau\beta_c)^2} > 0$. Thus, we only need to show $\pi_{nc}^* \ge \pi_{nm}^*$ at $\tau\beta_c = 2\beta_m$.

For $\beta_m \le 2k$, $\pi_{nc}^*(2\beta_m) - \pi_{nm}^* = \frac{\beta_m}{16k(2k+\beta_m)}[2k(2k+\beta_m) - \beta_m^2] > 0$. For $\beta_m > 2k$, $\pi_{nc}^*(2\beta_m) - \pi_{nm}^* = \frac{k^2}{2(2k+\beta_m)} > 0$. Therefore, nc dominates nm, given $\tau\beta_c > 2\beta_m$.

• We now compare firm 1's payoffs under nc and N.

$$\pi_{nc}^* = \frac{(2k+\tau\beta_c)^2}{4(4k+\tau\beta_c)},\ \pi_N^* = \begin{cases} \frac{4k^3}{(3k+\tau\beta_c)^2} & \tau\beta_c < k \\ \frac{k}{4} & \tau\beta_c \ge k \end{cases}$$

Notice that π_N^* is decreasing in $\tau\beta_c$ for $\tau\beta_c < k$, and π_{nc}^* is monotonous and increasing in $\tau\beta_c$. When $\tau\beta_c = 0$, $\pi_N^* = \frac{4k}{9}$ and $\pi_{nc}^* = \frac{k}{4}$, $\pi_N^* > \pi_{nc}^*$. When $\tau\beta_c \ge k$, $\pi_N^* = \frac{k}{4} < \pi_{nc}^*$. Therefore, there exists $0 < \beta_c^* < k$, such that when $\tau\beta_c < \beta_c^*$, $\pi_N^* > \pi_{nc}^*$, otherwise, $\pi_N^* \le \pi_{nc}^*$.

Thus, we conclude that given $\tau\beta_c > 2\beta_m$, if $\tau\beta_c > \beta_c^*$, the equilibrium is (nc, C), otherwise, it is (N, C).

Proof of Proposition 9.

Proof. If $\tau\beta_c > \beta_m$, considering the interior solutions requires condition (14), which implies that $2(1-\alpha)k - \tau\beta_c < 0$ and $\frac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]} < 0$. π_{hl}^* in the interior solution of the hl case is clearly lower than π_H^* in the single-service case. Thus, the interior solution of the hl business model is not optimal. Also, π_{all}^* in the interior solution of the all business model is lower than π_{nh}^* in the case of the mixed business model. Thus, the interior solution of the all business model cannot be optimal.

If $\beta_m \ge \tau\beta_c$ and $2(1-\alpha)k - \tau\beta_c < 0$, condition (14) for the interior solution cannot be satisfied; therefore, only the corner solutions are feasible. In the case of the hl business model, the corner solutions are equivalent to either of the single-service ad-supported business models (H and L). In the case of the all business model, the corner solutions are equivalent to the mixed business models (nh and nl). Therefore, offering both CPC and CPM ad-supported services is not optimal.

Proof of Proposition 10.

Proof.

1. If $\tau\beta_c > \beta_m$, or $\beta_m \geq \tau\beta_c$ and $2(1-\alpha)k - \tau\beta_c < 0$, we have $\pi^*_{hl} = \max\{\pi^*_H, \pi^*_L\}$ and $\pi^*_{All} = \max\{\pi^*_{nh}, \pi^*_{nl}\}$ by Proposition 9.

Similar to the proof of Proposition 3, we can show that $\pi^*_{nh} > \max\{\pi^*_H, \pi^*_N\}$ and $\pi^*_{nl} > \max\{\pi^*_L, \pi^*_N\}$. Thus, the maximum revenue could only be π^*_{nh} or π^*_{nl}. Since $\Delta\pi = \pi^*_{nh} - \pi^*_{nl} = \dfrac{2k\tau^2\beta_c^2 - 2\alpha k\beta_m^2 - \beta_m^2\tau\beta_c}{4k(2\alpha k + \tau\beta_c)}$, $\pi^*_{nh} > \pi^*_{nl}$ if and only if $2k\tau^2\beta_c^2 - 2\alpha k\beta_m^2 - \beta_m^2\tau\beta_c > 0$, or equivalently $\beta_m < \tau\beta_c\sqrt{\dfrac{2k}{2\alpha k + \tau\beta_c}}$.

2. If $\beta_m \geq \tau\beta_c$ and $2(1-\alpha)k - \tau\beta_c \geq 0$, condition (14) is satisfied. So we have:

$$\pi^*_{all} = r + \frac{\tau^2\beta_c^2}{2(2\alpha k + \tau\beta_c)} + \frac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]}$$

$$\pi^*_{hl} = \begin{cases} \dfrac{(r+\tau\beta_c)^2}{2(2\alpha k + \tau\beta_c)} + \dfrac{(\beta_m - \tau\beta_c)^2}{2[2(1-\alpha)k - \tau\beta_c]}, & if\ r \leq 2\alpha\, k \\ r - \alpha k + \dfrac{1}{2}\tau\beta_c + \dfrac{(\beta_m - \tau\beta_c)^2(1-\alpha)k}{2[2(1-\alpha)k - \tau\beta_c]}, & if\ r > 2\alpha\, k \end{cases}$$

π^*_{hl} is (weakly) higher than π^*_H in the single-service case. Similarly, π^*_{all} is (weakly) higher than π^*_{nh} in the case of the mixed business model. Furthermore, when $r \leq 2\alpha k$, $\pi^*_{all} - \pi^*_{hl} = \dfrac{r(4\alpha k - r)}{2(2\alpha k + \tau\beta_c)} > 0$; when $r > 2\alpha k$, $\pi^*_{all} - \pi^*_{hl} = \dfrac{4\alpha^2 k^2}{2(2\alpha k + \tau\beta_c)} > 0$. Thus, offering all three types of services is optimal.

Tables and Figures

Table 1: Parameters and Decision Variables

Notation	Definition	Comment
r	Consumers' reservation value of the ad-free service	Consumers have homogeneous reservation value
θ	Consumer ad sensitivity	Consumers differ in their sensitivity toward ads: $\theta \sim U[0,1]$
k	Consumers' disutility from ads	Consumers who are more sensitive to ads experience a higher reduction in utility, θk;

		also, $k < r$.
N, *M*, *C*	Subscripts to prices and revenues; *M* and *C* are also subscripts to ad revenue rates	*N* denotes the ad-free service, *M* the CPM ad-supported service, and *C* the CPC ad-supported service; *N*, *M*, and *C* are capitalized to refer to the single-service business model, where only one of three services is offered.
n, *m*, *c*	Subscripts to prices and revenues; *m* and *c* are also subscripts to ad revenue rates	Similarly, they denote the ad-free service, the CPM-based, and the CPC-based services, respectively; however, these are lower-case letters to represent services offered in a mixed business model with two or more services.
β_m, β_c	Ad revenue rate for the CPM or CPC model	In the CPM case, ad revenues are proportional to service demand, whereas in the CPC case, they depend on consumers' ad sensitivity.
τ	Ad click rate parameter	Adjustment from consumers' ad sensitivity to the clicking probability, and $\tau \in (0,1)$.
p_N, p_n, p_C, p_c, p_M, p_m	Price of the ad-free service, the CPC-based ad-supported service, or the CPM-based ad-supported service	In a mixed business model, services can be offered at different prices, which are represented separately with the corresponding subscripts.
π_N, π_C, π_M, π_{nc}, π_{nm}	Revenues of the service provider with any of the single-service or mixed business models	In a mixed business model, the revenue accounts for the total revenues from all types of services offered.

Table 2: Monopolist's Optimal Price and Revenue Under Single-Service Business Models

Service Type	**Optimal Price**	**Optimal Revenue**
Ad-free	$p_N^* = r$	$\pi_N^* = r$
CPC-based ads	$p_C^* = \begin{cases} \dfrac{kr + \tau\beta_c(r-k)}{2k + \tau\beta_c} & if\ r < 2k; \\ r - k & if\ r \geq 2k. \end{cases}$	$\pi_C^* = \begin{cases} \dfrac{(r + \tau\beta_c)^2}{2(2k + \tau\beta_c)} & if\ r < 2k; \\ r - k + \dfrac{1}{2}\tau\beta_c & if\ r \geq 2k. \end{cases}$
CPM-based ads	$p_M^* = \begin{cases} \dfrac{r - \beta_m}{2} & if\ r < 2k - \beta_m; \\ r - k & if\ r \geq 2k - \beta_m. \end{cases}$	$\pi_M^* = \begin{cases} \dfrac{(r + \beta_m)^2}{4k} & if\ r < 2k - \beta_m; \\ r - k + \beta_m & if\ r \geq 2k - \beta_m. \end{cases}$

Table 3: Monopolist's Optimal Price and Revenue Under Mixed Business Models

Service Types	**Optimal Prices**	**Optimal Revenue**
Ad-free and CPC	$p_n^* = r\,, \quad p_c^* = r - \frac{k\tau\beta_c}{2k + \tau\beta_c}$	$\pi_{nc}^* = r + \frac{\tau^2\beta_c^2}{2(2k + \tau\beta_c)}$
Ad-free and CPM	$p_n^* = r\,, \quad p_m^* = r - \frac{\beta_m}{2}$	$\pi_{nm}^* = r + \frac{\beta_m^2}{4k}$

Table 4: Duopoly Equilibrium Results Under Single-Service Business Models (B1)

Revenue Model of Firm 2's Ad-Supported Service	
CPC-Based Model, C	**CPM-Based Model,** M
$\begin{cases} \tau\beta_c < k \\ p_N^* = \frac{2k^2}{3k + \tau\beta_c}, \pi_N^* = \frac{4k^3}{(3k + \tau\beta_c)^2}, \\ p_C^* = \frac{k^2 - k\tau\beta_c}{3k + \tau\beta_c}, \pi_C^* = \frac{(2k + \tau\beta_c)(k + \tau\beta_c)^2}{2(3k + \tau\beta_c)^2}; \\ \\ \tau\beta_c \geq k \\ p_N^* = \frac{k}{2}, \pi_N^* = \frac{k}{4}, \\ p_C^* = 0, \pi_C^* = \frac{3}{8}\tau\beta_c. \end{cases}$	$\begin{cases} \beta_m < \frac{k}{2} \\ p_N^* = \frac{2k - \beta_m}{3}, \pi_N^* = \frac{(2k - \beta_m)^2}{9k}, \\ p_M^* = \frac{k - 2\beta_m}{3}, \pi_M^* = \frac{(k + \beta_m)^2}{9k}; \\ \\ \beta_m \geq \frac{k}{2} \\ p_N^* = \frac{k}{2}, \pi_N^* = \frac{k}{4}, \\ p_M^* = 0, \pi_M^* = \frac{1}{2}\beta_m. \end{cases}$

Table 5: Duopoly Equilibrium Results Under Mixed Business Models (B2)

Revenue Model of Firm 2's Ad-Supported Service		
	CPC-Based Model, C	**CPM-Based Model,** M
Firm 1 nc	$p_n^* = \frac{k(2k + \tau\beta_c)}{4k + \tau\beta_c}, p_c^* = 0, \pi_{nc}^* = \frac{(2k + \tau\beta_c)^2}{4(4k + \tau\beta_c)}$	
	$p_C^* = 0, \pi_C^* = \frac{(2k + \tau\beta_c)(6k + \tau\beta_c)\tau\beta_c}{4(4k + \tau\beta_c)^2}$	$p_M^* = 0, \pi_M^* = \frac{(2k + \tau\beta_c)\beta_m}{2(4k + \tau\beta_c)}$
Firm 1 nm **when** $\beta_m \leq 2k$	$p_n^* = \frac{2k + \beta_m}{4}, p_m^* = 0, \pi_{nm}^* = \frac{(2k + \beta_m)^2}{16k}$	
	$p_C^* = 0, \pi_C^* = \frac{\tau\beta_c}{64k^2}(2k + \beta_m)(6k - \beta_m)$	$p_M^* = 0, \pi_M^* = \frac{\beta_m}{8k}(2k + \beta_m)$

when $\beta_m > 2k$	$p_n^* \geq k,\ p_m^* = 0, \pi_{nm}^* = \frac{1}{2}\beta_m$	
	$p_C^* = 0, \pi_C^* = \frac{1}{4}\tau\beta_c$	$p_M^* = 0, \pi_M^* = \frac{1}{2}\beta_m$

Table 6: Optimal Price and Revenue under Single-Service Business Models for the Alternative Model Setup (Appendix)

Service Type	**Optimal Price**	**Optimal Revenue**
High-quality only	$p_H^* = \begin{cases} \dfrac{\alpha kr + \tau\beta_c(r - \alpha k)}{2\alpha k + \tau\beta_c} & if\ \ r < 2\alpha k; \\ r - \alpha k & if\ \ r \geq 2\alpha k. \end{cases}$	$\pi_H^* = \begin{cases} \dfrac{(r + \tau\beta_c)^2}{2(2\alpha k + \tau\beta_c)} & if\ \ r < 2\alpha k; \\ r - \alpha k + \dfrac{1}{2}\tau\beta_c & if\ \ r \geq 2\alpha k. \end{cases}$
Low-quality only	$p_L^* = \begin{cases} \dfrac{r - \beta_m}{2} & if\ \ r > \beta_m; \\ 0 & if\ \ r \leq \beta_m. \end{cases}$	$\pi_L^* = \begin{cases} \dfrac{(r + \beta_m)^2}{4k} & if\ \ r > \beta_m; \\ \dfrac{r\beta_m}{k} & if\ \ r \leq \beta_m. \end{cases}$

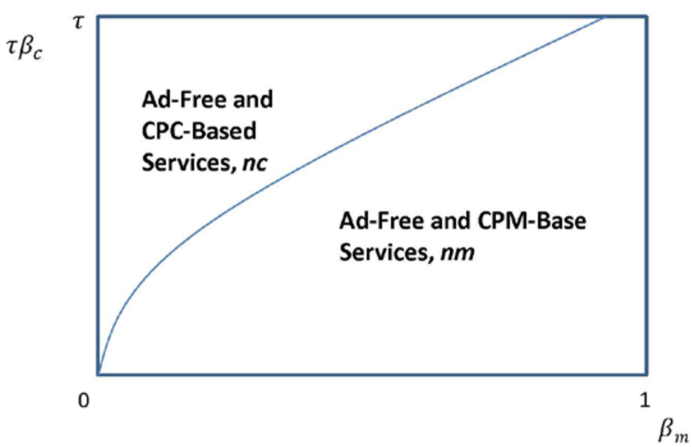

Figure 1: Monopolist's Optimal Business Model

Firm 2

Firm 1 — Business Models	Ad-Free (N)	CPC (C)	CPM (M)	Ad-Free + CPC (nc)	Ad-Free + CPM (nm)
Ad-Free (N)	(0,0)	***Single-Service (B1)***		(0,0)	(0,0)
CPC (C)	***Single-Service (B1)***	***Ad-Supported Only (A)***		***Firm 2 Mixed (B2)***	
CPM (M)					
Ad-Free + CPC (nc)	(0,0)	***Firm 1 Mixed (B2)***		(0,0)	(0,0)
Ad-Free + CPM (nm)	(0,0)			(0,0)	(0,0)

Figure 2: Duopoly Payoff Matrix for Cases A, B1, and B2

Business Model / *Model Setting*	***Single-Service***	***Mixed (Ad-Free + Ad-Supported)***
Monopoly	CPC service price shows more stability $\frac{\partial p_M^*}{\partial \beta_m} < \frac{\partial p_C^*}{\partial \beta_c} < 0$	CPC service price shows more stability $\frac{\partial p_m^*}{\partial \beta_m} < \frac{\partial p_c^*}{\partial \beta_c} < 0$
Duopoly	CPC leads to more price stability for *self and rival (ad-free)* $\frac{\partial p_M^*}{\partial \beta_m} < \frac{\partial p_C^*}{\partial \beta_c} < 0;\ \frac{\partial p_N^*}{\partial \beta_m} < \frac{\partial p_N^*}{\partial \beta_c} < 0$	CPC leads to more price stability for self *(ad-free)* (note: $p_m^* = p_c^* = p_M^* = p_C^* = 0$) $\frac{\partial p_n^*}{\partial \beta_m} > \frac{\partial p_n^*}{\partial \beta_c} > 0$

Figure 3: Comparison of Price Stability Results

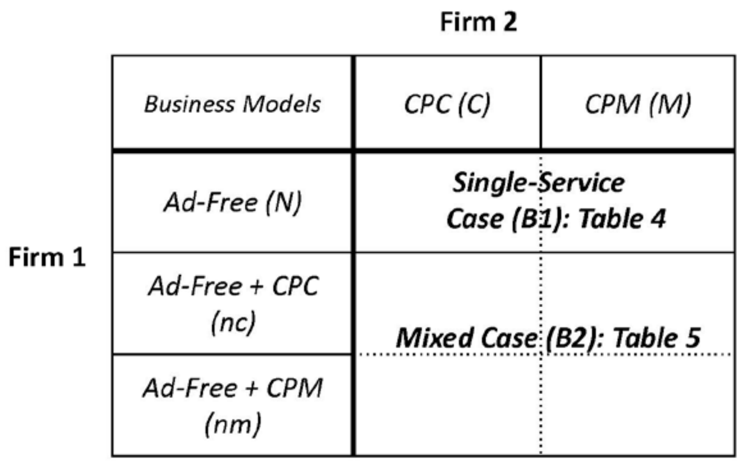

Figure 4: Reduced Duopoly Payoff Matrix