7-2014

Trade and Financial Integration, Extensive Margin, and Income Divergence

Haiping ZHANG
Singapore Management University, hpzhang@smu.edu.sg

SMU ECONOMICS & STATISTICS
WORKING PAPER SERIES

Trade and Financial Integration, Extensive Margin, and Income Divergence

Haiping Zhang

July 2014

Paper No. 05-2014

Trade and Financial Integration, Extensive Margin, and Income Divergence

Haiping Zhang*

July 2014

Abstract

We revisit the classical question on economic integration and income convergence in a two-sector OLG model with financial frictions and sectoral heterogeneity in minimum investment requirements (MIR, hereafter). The *extensive margin* of investment is a critical channel through which aggregate income may become a determinant of comparative advantage. Free trade allows the rich (poor) country to specialize partially or completely in the high-MIR (low-MIR) sector which has a high (low) return endogenously. The specialization effect interacts with the neoclassical effect, which may lead to income divergence among inherently identical countries. Similarly, financial integration may also lead to income divergence through the extensive-margin channel.

We then revisit the Stolper-Samuelson theorem. Antras and Caballero (2009) show that, given cross-country and cross-sector differences in financial frictions, free trade alone cannot deliver factor price equalization, while allowing both trade and capital flows can do so. In our model, if free trade induces the rich countries to specialize completely in the high-return sector, the credit market condition changes fundamentally and so does the interest rate determination. In this case, *moving from autarky to free trade does not reverse the cross-country interest rate differentials and the direction of capital flows, and allowing both trade and capital flows does not lead to factor price equalization and income convergence.* This way, our findings complement Antras-Caballero's results and refine the condition for the Stolper-Samuelson theorem.

Keywords: financial frictions, financial integration, income divergence, minimum investment requirement, symmetry breaking, trade integration
JEL Classification: E44, F41

*School of Economics, Singapore Management University. 90 Stamford Road, Singapore 178903. E-mail: hpzhang@smu.edu.sg
We would like to thank Volker Böhm, Davin Chor, Klaus Desmet, Jiandong Ju, Tomoo Kikuchi, Cheng Wang, and the seminar participants at the National University of Singapore for helpful comments and suggestions. Financial support from Singapore Management University is sincerely acknowledged.

1 Introduction

The recent literature provides the comprehensive empirical evidence that financial development matters for international trade (Ahn, Amiti, and Weinstein, 2011; Beck, 2002, 2003; Manova, 2008, 2013; Svaleryd and Vlachos, 2005). Chor and Manova (2012) analyze the collapse of international trade flows during the global financial crisis and show that credit conditions were an important channel through which the financial crisis affected trade volumes. A small but growing theoretical literature investigates the role of financial sector in determining the patterns of production and trade (Antras and Caballero, 2009, 2010; Ju and Wei, 2005; Kletzer and Bardhan, 1987). Ju and Wei (2011) show that, in the countries with low-quality institutions, the quality of financial system is an independent source of comparative advantage. Wynne (2005) shows that a country's wealth can be an important determinant of comparative advantage when access to credit differs across sectors of the economy. In particular, wealthier nations exhibit a comprehensive advantage towards goods produced in sectors facing more severe financial imperfections.

These theoretical models share three common features. First, the cross-sector and the cross-country differences in financial frictions lead to the cross-country differences in the sectoral output prices, which then drives trade flows. Second, the mass of investors in each sector is **exogenous** so that the sectoral investment adjusts only on the **intensive margin**.[1] Third, there exists a **unique** steady state under autarky as well as under free trade. Thus, the impacts of trade integration are unambiguous.

This paper makes two contributions to the literature. First, we embed the Heckscher-Ohlin model in an OLG framework where the two sectors are subject to the same degree of financial frictions but the different minimum investment requirements (MIR, hereafter). The mass of investors in each sector is **endogenous** so that the sectoral investment adjusts also on the **extensive margin**. The extensive margin is the key channel through which aggregate income may become a determinant of comparative advantage and then, free trade allows the country with a high (low) income to specialize partially or completely in the high-MIR, high-return (low-MIR, low-return) sector. The specialization effect interacts with the neoclassical effect, which may lead to **multiple** steady states for a small open economy and income divergence among inherently identical countries. Similarly, financial integration may also lead to income divergence through the extensive margin.

Second, we revisit the Stolper-Samuelson theorem. Antras and Caballero (2009) show that, given the cross-sector and cross-country differences in financial frictions, allowing both trade and capital flows leads to factor price equalization and income convergence. In our model, free trade may lead to the **complete** specialization of the rich countries in the high-return sector, which fundamentally changes their credit market condition and interest rate determination. In this case, allowing both trade and capital flows *does not* lead to factor price equalization and income convergence. Thus, we complement Antras-Caballero's results and further refine the condition for the Stolper-Samuelson theorem.

The sector-specific MIR and the economy-wide financial frictions are the two key elements of our model. In the literature, the MIR is used to capture the investment indi-

[1]The sectoral investment depends on the investment size of individual agents (the intensive margin) and the mass of investors in a particular sector (the extensive margin).

visibility at the individual level, which is a important feature of business ideas, physical and human capital (Aghion and Bolton, 1997; Banerjee and Moll, 2010; Banerjee and Newman, 1993; Chesnokova, 2007; Galor and Zeira, 1993; Matsuyama, 2000; Piketty, 1997). Recently, Erosa and Hidalgo-Cabrillana (2008), Barseghyan and DiCecio (2011), Buera, Kaboski, and Shin (2011), Manova (2013), and Midrigan and Xu (2014) introduce the fixed cost or the entry cost at the firm level and show that the individual investment is above a minimum scale in equilibrium. In the presence of either the MIR or the fixed cost, the individual production set is non-convex,[2] and, if financial frictions are also present, a change in aggregate income affects the individual's net wealth and the mass of investors so that aggregate investment adjusts on the extensive margin. Assuming the MIR allows us to characterize the dynamic properties in the entire parameter spaces.

1.1 Model Structure and Intuitions

Consider an overlapping-generation model with two-period lived agents who have the labor endowment when young and consume when old. Labor and capital are hired in two sectors, A and B, to produce final good A and B, respectively, which are used for consumption and investment in the CES form. The investment is sector-specific and the resulting capital is available in the next period. The model deviates from the standard OLG framework in three aspects. First, all agent are endowed with the linear investment technology, subject to the MIR, i.e., the individual's investment size must be no less than a specific value. The two sectors differ in the MIR and, for simplicity, the MIR in sector B is normalized at zero. Second, due to limited commitment, agents can borrow only up to a fraction of the investment return and this fraction depends on financial development. Third, agents differ in the labor endowment which is continuously distributed.

Given the wage rate and the level of financial development, the agents with the labor endowment below (equal to or above) a cutoff value cannot (can) meet the MIR in sector A and are called households (entrepreneurs). Households can save their labor income by lending to the credit market and investing in sector B, while entrepreneurs have one more option, i.e., investing in sector A. Thus, if the aggregate investment in sector B turns out to be positive (zero) in equilibrium, the interest rate must be equal to (higher than) the rate of return in sector B. Meanwhile, the rate of return in sector A is no less than the interest rate; otherwise, entrepreneurs would not invest in sector A. Given the level of financial development and the MIR, the higher the aggregate income, the higher the wage rate, the higher the agent's labor income and net wealth, the larger (smaller) the mass of entrepreneurs (households). Thus, the mass of investors in each sector is *endogenous*, depending on aggregate income, financial development, and the MIR.

Let us first consider the model dynamics under autarky. If aggregate income is below a threshold value, the mass of entrepreneurs (households) is so low (high) that the investment in sector A (B) is less (more) than the efficient level and the rate of return in sector A is higher than in sector B. Thus, entrepreneurs invest their entire labor income in sector A and borrow to the limit. The higher aggregate income implies the higher

[2]Despite the nonconvex individual production set, Matsuyama (2007, 2008) argues that assuming a continuum of agents convexifies the aggregate production set.

labor income for individual agents, which affects the sectoral investment through two channels. First, it allows all agents to invest more so that the sectoral investment tends to rise in the equal proportions on the **intensive margin**. Then, the decreasing value of the marginal product of capital (the **neoclassical effect**) is a **convergence** force, making the law of motion for aggregate income concave. Second, given a constant MIR, the higher labor income allows more agents to meet the MIR so that the investment in sector A (B) rises (declines) on the **extensive margin** and so does the aggregate credit demand (supply). The change in the **cross-sector investment composition** improves the aggregate allocation efficiency, which is a **divergence** force and makes the law of motion for aggregate income convex. If the level of financial development is below a threshold value, the interactions between the cross-sector composition effect and the neoclassical effect leads to multiple steady states; otherwise, there exists a unique, stable steady state. In the following, we focus on the parameter configurations that ensures the unique autarkic steady state.

Consider a world economy where all countries are inherently identical except for the initial income level. If the cross-sector investment is inefficient in the autarkic steady state, the price of good A (B) is inefficiently high (low) and so is the rate of return in sector A (B). The higher aggregate income improves the cross-sector investment composition, leading to the lower (higher) price of good A (B). Thus, the initially rich (poor) countries have the comparative advantage in good A (B) and free trade in both final goods allows them to specialize towards sector A (B) which has the high (low) return. Thus, in the next period, the aggregate income of the rich (poor) countries is higher (lower) than otherwise under autarky, which allows even more (less) agents to meet the MIR. Then, the rich (poor) countries specialize on the extensive margin further in the high-return (low-return) sector. Changes in the mass of entrepreneurs and in aggregate income reinforce each other over time. Such a dynamic cycle goes on until the specialization effect is balanced by the neoclassical effect. The lower the level of financial development or the higher the MIR, the larger the cross-sector distortion and the rate-of-return differentials, the stronger the specialization effect, the more likely free trade leads to income divergence.

Matsuyama (2004) embeds financial frictions and fixed investment requirements into an OLG model. He shows that financial integration may also lead to income divergence and he calls it symmetry breaking. There is only one final good in his model, which is freely traded and serves as the vehicle for capital flows. There are two final goods in our model. If only one good is freely traded, our model replicates Matsuyama's result.

Intuitively, financial frictions and the sector-specific MIR distort the intratemporal relative price (the relative final good price) and the intertemporal relative price (the interest rate). If the extensive-margin effect dominates the neoclassical effect, the two relative prices rise in aggregate income at the autarkic steady state[3]. Thus, the rich (poor) countries have the comparative advantage in the constrained but high-return (unconstrained but low-return) sector as well as in borrowing (lending). Free trade allows the rich (poor) countries to specialize in the sector that they have the comparative advantage;

[3] In Antras and Caballero (2009), the sector-specific financial frictions also distort the two relative prices. However, in the absence of the extensive-margin effect, the relative final good price is independent of aggregate income and the interest rate strictly decreases with aggregate income in their model.

financial integration leads to capital flows from the poor to the rich countries. In both cases, economic integration may lead to the income divergence rather than convergence among inherently identical countries. Generally speaking, in the presence with economic distortions, free mobility of either products or factors may amplify rather than reduce the distortions, according to the second-best theory (Lipsey and Lancaster, 1956).

What if both trade and financial flows are allowed simultaneously? Does it lead to income convergence? In our model, *if both goods are freely traded and the rich countries* **do not completely** *specialize in the constrained sector*, adding financial integration on top of free trade leads to income convergence. This way, *moving from the one-sector to the two-sector setting reduces the likelihood of Matsuyama's symmetry breaking.*[4]

If free trade does not lead to the complete specialization in the constrained sector (sector A), the positive investment in the unconstrained sector (sector B) implies the **coupling** of the interest rate with the rate of return in the unconstrained sector. By equalizing the interest rate, financial integration implicitly equalizes the rate of return in the unconstrained sector. In addition, by equalizing the relative price of sectoral output, trade integration implicitly equalizes the the rate-of-return ratio of the two sectors. Thus, allowing both trade and capital flows also equalizes the rate of return in the constrained sector. Then, the complete factor prices equalization lead to income convergence.

The logic mentioned above explains the income convergence result of Antras and Caballero (2009). In their model, the mass of investors in each sector is by assumption **exogenous**. The investors in the constrained sector borrow up to the limit but they cannot fully absorb the entire domestic saving. As both sectors have the positive investment, free trade does not lead to the complete specialization. Since their model always satisfies the condition highlighted above, their result holds unambiguously.

In our model, the mass of investors in each sector is **endogenous**. As mentioned above, the trade-driven specialization creates a dynamic, virtuous cycle between the aggregate income and the mass of entrepreneurs in the rich countries. If the mass of entrepreneurs eventually rises to such a high level that entrepreneurs borrow the entire saving of households, the rich countries specialize completely in sector A and the efficient aggregate credit demand **decouples (couples)** the interest rate from (with) the rate of return in sector B (A). Given the sectoral rate-of-return differential, the interest rate in the rich countries **jumps upwards** upon the complete specialization and it can be higher than in the poor countries. Thus, moving from autarky to free trade does not reverse the cross-country interest rate differentials and the direction of capital flows.[5] Due to the decoupling, the interest rate equalization under financial integration does not imply the equalization of the rate of return in sector B and hence, free trade and capital flows

[4] In Matsuyama (2004), the final good is implicitly freely traded. Thus, the symmetry breaking actually arises under free mobility of both trade and capital flows.

[5] In Antras and Caballero (2009), the two countries differ in the level of financial development. In the autarkic steady state, aggregate income is lower in the less financially developed country and so is the interest rate, due to the larger cross-sector distortion. Financial integration alone leads to "uphill" capital flows from the poor to the rich country. Free trade allows the rich (poor) country to specialize **partially** in the constrained (unconstrained) sector. Thus, the interest rate is always coupled with the rate of return in the unconstrained sector, which is higher in the poor than in the rich country. Then, allowing financial flows on top of free trade leads to "downhill" capital flows from the rich to the poor.

cannot lead to factor price equalization and income convergence.

1.2 Related Literature

Our paper is related to the literature on trade and income convergence. Deardorff (2001), Cunat and Maffezzoli (2004b), and Bajona and Kehoe (2010) assume sector-specific factor intensity and show that trade may prevent inherently identical countries from converging to the same steady-state income through specialization. Matsuyama (1996) shows that commodity trade causes the agglomeration of different economic activities in different regions of the world, leading to income divergence. Matsuyama (2005) introduces sector-specific borrowing constraints in a static model and shows that free trade allows the rich (poor) country to specialize in the sector with tighter (looser) borrowing constraints.

Our paper is also related to a recent literature on the joint analysis of intra- and intertemporal trade. Cunat and Maffezzoli (2004a) embed Heckscher-Ohlin features and the sector-specific capital intensity in a two-country model and analyze the international transmission of productivity shocks through trade in goods. Jin (2012) integrates factor-proportions-based trade and financial capital flows in an OLG model and shows that capital tends to flow to countries that become more specialized in capital-intensive industries. Jiao and Wen (2012) embed the Melitz (2003) model into an incomplete-markets setting and analyze the impacts of financial and non-financial shocks on output and trade flows. Ju, Shi, and Wei (2014) introduce two tradeable sectors with different factor intensity in a small open economy model and show that the current account adjustment with respect to exogenous shocks depends on the factor market flexibility.

Our paper focuses on a real friction, i.e., the sector-specific MIR, rather than the sector-specific factor intensity or the sector-specific financial frictions. In our model, countries differ only in the initial income level. Given financial frictions and the investment indivisibility at the individual level, economic integration may lead to the endogenous income divergence. In the real world, countries differ in many other aspects, e.g., economic, social, and political institutions as well as natural endowments. This way, we propose an amplification mechanism through which even very small exogenous heterogeneities may lead to large heterogeneities in endogenous variables.

The rest of the paper is structured as follows. Section 2 sets up the model and analyzes the distortions of financial frictions and the sector-specific MIR on the cross-sector investment composition and the relative prices. Sections 3-5 shows that economic integration may lead to endogenous inequality of nations. Section 6 checks the robustness of our results under alternative specifications. Section 7 concludes with some final remarks. The appendix collects the supporting materials and technical proofs.

2 The Model under International Autarky

The world economy consists of a continuum of countries, indexed by $i \in [0, 1]$. Countries are inherently identical except for the initial income level. In each country, a continuum of agents indexed by $j \in [0, 1]$ are born every period and live for two periods, young and old; the population size of each generation is constant at one; agents have the labor endowment when young and consume when old; agent j is endowed with $l_j = \frac{\theta+1}{\theta}\frac{1}{\epsilon_j}$ units of labor, where $\epsilon_j \in (1, \infty)$ follows the Pareto distribution with the cumulative distribution function $G(\epsilon_j) = 1 - \epsilon_j^{-\theta}$ and $\theta > 1$. Agents supply the labor endowment inelastically to the market and the aggregate labor supply is constant at $L = \int_1^\infty l_j dG(\epsilon_j) = 1$.

In each country, there are two final good sectors, A and B. In period t, sector $f \in \{A, B\}$ employs $K_t^{i,f}$ units of physical capital and $L_t^{i,f}$ units of labor to produce $Y_t^{i,f}$ units of final good f. Physical capital fully depreciates after the production. Then, $Z_t^{i,A}$ units of final good A and $Z_t^{i,B}$ units of final good B are used as the inputs to produce Y_t^i units of composite goods.[6] The composite good is taken as the numeraire. Old agents consume C_t^i units of composite goods and young agents invest $M_t^{i,f}$ units of composite goods in period t to produce $K_{t+1}^{i,f} = RM_t^{i,f}$ units of physical capital, which is sector-specific and becomes available in period $t+1$. Composite and final goods are tradeable, while physical capital and labor are not. $p_t^{i,f}$ denotes the price of final good f and $q_t^{i,f}$ denotes the value of marginal product of capital (VMPK) in sector f. Labor is mobile across sectors and w_t^i denotes the wage rate. Markets for goods and productive factors are competitive so that the inputs are rewarded at their respective value of marginal product.

$$Y_t^{i,f} = \left(\frac{K_t^{i,f}}{\alpha}\right)^\alpha \left(\frac{L_t^{i,f}}{1-\alpha}\right)^{1-\alpha}, \quad q_t^{i,f} K_t^{i,f} = \alpha p_t^{i,f} Y_t^{i,f}, \quad w_t^i L_t^{i,f} = (1-\alpha) p_t^{i,f} Y_t^{i,f}, \tag{1}$$

$$Y_t^i = \left(\frac{Z_t^{i,A}}{\eta}\right)^\eta \left(\frac{Z_t^{i,B}}{1-\eta}\right)^{1-\eta}, \quad p_t^{i,A} Z_t^{i,A} = \eta Y_t^i, \qquad p_t^{i,B} Z_t^{i,B} = (1-\eta) Y_t^i, \tag{2}$$

where $\alpha, \eta \in (0, 1)$. There is no uncertainty in the model economy. The two sectors are symmetric except for the MIR to be described later.

In this section, we analyze the economic allocation under of international autarky where trade and capital flows are not allowed. Thus, the goods markets clear domestically and domestic investment is financed by domestic savings,

$$Z_t^{i,f} = Y_t^{i,f} \text{ and } M_t^{i,A} + M_t^{i,B} = w_t^i. \tag{3}$$

Let $\chi_t^i \equiv \frac{p_t^{i,B}}{p_t^{i,A}}$ and $\mu_t^i \equiv \frac{q_t^{i,B}}{q_t^{i,A}}$ denote the relative final good price and the sectoral VMPK ratio, respectively. Combine the linear sectoral capital formation function $K_{t+1}^{i,f} = RM_t^{i,f}$ with equations (1)-(3) to get the labor input and the investment in the two sectors

$$L_t^{i,A} = \eta L \qquad \text{and} \qquad L_t^{i,B} = (1-\eta) L, \tag{4}$$

$$M_t^{i,A} = \eta w_t^i \frac{\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i} \qquad \text{and} \qquad M_t^{i,B} = (1-\eta) w_t^i \frac{1}{1-\eta+\eta\mu_{t+1}^i}. \tag{5}$$

[6] Under autarky, the market for good f clears domestically, $Z_t^{i,f} = Y_t^{i,f}$. However, under free trade, the domestic absorption of final good f can be different from its domestic output, $Z_t^{i,f} \neq Y_t^{i,f}$. Antras and Caballero (2009) assume that physical capital and labor are used to produce two final goods which can be consumed or invested into physical capital, according to the Cobb-Douglas aggregator. As a result, agents devote a fraction η of their spending to one good and the rest to the other. Alternatively, one can introduce a composite good as a Cobb-Douglas aggregator of two final goods, which is then used for consumption and investment (Ju and Wei, 2011). The two approaches are technically equivalent and we choose the second approach mainly for the analytical simplicity.

If the sectoral investment were frictionless, the final good price would equalize in the two sectors and so would the VMPK, $\chi_t^i = \mu_t^i = 1$. According to equations (4)-(5), a fraction η of aggregate labor and savings would be allocated in sector A and the rest in sector B efficiently.

However, if the investment at the individual level is subject to financial frictions and the sector-specific MIR, the cross-sector investment may become inefficient. Consider agent j born in country i and period t. As shown in the left and middle panels of figure 1, the agent can invest in period t $m_{j,t}^{i,B}$ units of composite goods in sector B and produce $k_{j,t+1}^{i,B} = Rm_{j,t}^{i,B}$ units of physical capital, while its investment in sector A must be no less than a MIR, $m_{j,t}^{i,A} \geq \mathbb{m}_t^i$, so as to have the linear output as in sector B, $k_{j,t+1}^{i,A} = Rm_{j,t}^{i,A}$. The MIR takes the functional form of $\mathbb{m}_t^i = \mathfrak{m}(Y_t^i)^{1-\sigma}$ with $\mathfrak{m} > 0$. As shown in the right panel of figure 1, the MIR is constant for $\sigma = 1$, while it is proportional to aggregate income for $\sigma = 0$. Such a function form allows for the possibility that the MIR may differ in the rich and in the poor country.[7]

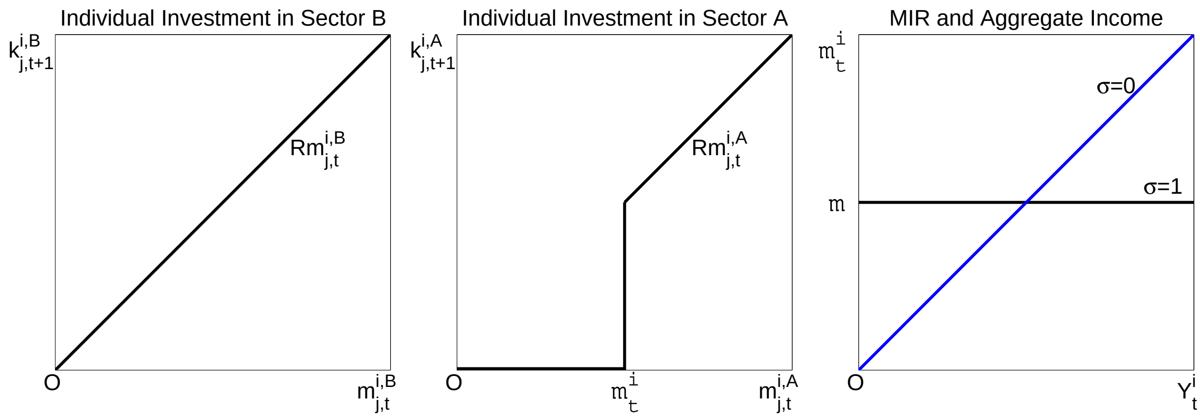

Figure 1: Individual Investment Function, MIR, and Aggregate Income

Agents have three options to save the labor income $n_{j,t}^i = w_t^i l_j$: (1) lending to the credit market for the interest rate r_t^i, (2) investing in sector B for the rate of return $q_{t+1}^{i,B}R$, and (3) investing in sector A for the rate of return $q_{t+1}^{i,A}R$ if they can meet the MIR. Under autarky, both final goods are produced domestically, i.e., $M_t^{i,A} > 0$ and $M_t^{i,B} > 0$. As everyone has the access to option (1) and (2), the interest rate is coupled with the rate of return in sector B, $r_t^i = q_{t+1}^{i,B}R$. Meanwhile, the interest rate cannot exceed the rate of return in sector A, $r_t^i \leq q_{t+1}^{i,A}R$; otherwise, nobody would invest in sector A. To sum up[8]

$$r_t^i = q_{t+1}^{i,B}R \leq q_{t+1}^{i,A}R. \tag{6}$$

[7]It is mainly for the analytical purpose that we allow the MIR to be dependent of aggregate income. As shown in subsection 2.1, for $\sigma = 0$, a change in aggregate income does not affect the mass of investors in each sector and hence, the sectoral investment adjusts only on the intensive margin. For $\sigma \neq 0$, a change in aggregate income affects the mass of investors in each sector and hence, the sectoral investment adjusts on the intensive and extensive margins. This way, we can explicitly highlight the role of the extensive-margin channel by comparing the model results in the two alternative settings. Those who are uncomfortable with this function form may just take the MIR as a constant, i.e, $\sigma = 1$.

[8]As shown in section 3, free trade may induce the country to specialize completely in sector A and the zero investment in sector B $M_t^{i,B} = 0$ implies the decoupling (coupling) of the interest rate from (with) the rate of return in sector B (A), $r_t^i = q_{t+1}^{i,A}R \geq q_{t+1}^{i,B}R$.

Let us start with the case of $r_t^i < q_{t+1}^{i,A}R$. If agent j can meet the MIR, it prefers to finance its investment in sector A, $m_{j,t}^{i,A}$, with loans. However, due to limited commitment, it can only borrow up to a fraction λ of the present value of its investment return,

$$b_{j,t}^i \leq \lambda \frac{q_{t+1}^{i,A} R m_{j,t}^{i,A}}{r_t^i}, \tag{7}$$

and has to use its own funds as equity capital to cover the gap $m_{j,t}^{i,A} - b_{j,t}^i$, where $\lambda \in (0,1)$ reflects the level of financial development.[9] Let $\psi_{j,t}^i \equiv \frac{m_{j,t}^{i,A} - b_{j,t}^i}{m_{j,t}^{i,A}}$ denote the agent's equity-investment ratio in sector A. In period $t+1$, it gets the investment return, $q_{t+1}^{i,A} R m_{j,t}^{i,A}$, repays the debt, $r_t^i b_{j,t}^i$, and consumes the rest. Its equity rate is defined as the rate of return to equity capital, $\Omega_{j,t}^i \equiv \frac{q_{t+1}^{i,A} R m_{j,t}^{i,A} - r_t^i b_{j,t}^i}{m_{j,t}^{i,A} - b_{j,t}^i}$. Use the borrowing constraint to get,

$$\psi_{j,t}^i \geq 1 - \lambda \frac{q_{t+1}^{i,A} R}{r_t^i}, \tag{8}$$

$$\Omega_{j,t}^i = q_{t+1}^{i,A} R + (q_{t+1}^{i,A} R - r_t^i)(\frac{1}{\psi_{j,t}^i} - 1). \tag{9}$$

The leverage effect $(q_{t+1}^{i,A} R - r_t^i)(\frac{1}{\psi_{j,t}^i} - 1)$ depends positively on the spread $(q_{t+1}^{i,A} R - r_t^i)$ and negatively on $\psi_{j,t}^i$. If $r_t^i < q_{t+1}^{i,A} R$, the positive spread induces the agent to maximize the leverage effect by minimizing the equity-investment ratio, or equivalently, by borrowing to the limit so that the equality sign holds for (8) and $\psi_{j,t}^i$ is independent of agent-j's net wealth; the positive leverage effect, $\Omega_t^i > q_{t+1}^{i,A} R > r_t^i = q_{t+1}^{i,B} R$, induces the agent to invest its entire labor income as equity capital in sector A. If $r_t^i = q_{t+1}^{i,A} R$, the agent does not borrow to the limit so that its investment size is indeterminate; the inequality sign holds for (8) and $\psi_{j,t}^i$ is also indeterminate; due to the zero spread, the leverage effect vanishes and the equity rate is equal to the rate of return in sector A. To sum up,

$$\psi_{j,t}^i \begin{cases} = \psi_t^i \equiv 1 - \lambda \frac{q_{t+1}^{i,A} R}{r_t^i}, & \text{wealth-independent} \qquad \text{if } r_t^i < q_{t+1}^{i,A} R; \\ > 1 - \lambda \frac{q_{t+1}^{i,A} R}{r_t^i}, & \text{indeterminate,} \qquad \text{if } r_t^i = q_{t+1}^{i,A} R; \end{cases} \tag{10}$$

$$\Omega_{j,t}^i = \Omega_t^i = \begin{cases} q_{t+1}^{i,A} R + (q_{t+1}^{i,A} R - r_t^i)(\frac{1}{\psi_t^i} - 1) > q_{t+1}^{i,A} R > q_{t+1}^{i,B} R, & \text{if } r_t^i < q_{t+1}^{i,A} R; \\ q_{t+1}^{i,A} R, & \text{if } r_t^i = q_{t+1}^{i,A} R; \end{cases} \tag{11}$$

$$m_{j,t}^{i,A} \begin{cases} = \frac{n_{j,t}^i}{\psi_t^i} = \frac{w_t^i}{\psi_t^i} \frac{\theta+1}{\theta \epsilon_j}, & \text{and } \frac{\partial m_{j,t}^{i,A}}{\partial \epsilon_j} < 0, \qquad \text{if } r_t^i < q_{t+1}^{i,A} R; \\ < \frac{n_{j,t}^i}{\psi_t^i}, & \text{indeterminate,} \qquad \text{if } r_t^i = q_{t+1}^{i,A} R. \end{cases} \tag{12}$$

If $r_t^i < q_{t+1}^{i,A} R$, there exists a cutoff value $\underline{\epsilon}_t^i$. The agents with $\epsilon_j \in (1, \underline{\epsilon}_t^i]$ can meet the MIR, $m_{j,t}^{i,A} = \frac{w_t^i}{\psi_t^i} \frac{\theta+1}{\theta \epsilon_j} \geq \mathbb{m}_t^i$ and are called *entrepreneurs*. Their total mass is $\tau_t^i = 1 - (\underline{\epsilon}_t^i)^{-\theta}$. The cutoff value is determined by the marginal entrepreneur with $\epsilon_j = \underline{\epsilon}_t^i$,

$$m_{j,t}^{i,A}(\underline{\epsilon}_t^i) = \frac{w_t^i}{\psi_t^i} \frac{1+\theta}{\theta \underline{\epsilon}_t^i} = \mathfrak{m}(Y_t^i)^{1-\sigma}, \Rightarrow \underline{\epsilon}_t^i = \frac{(w_t^i)^\sigma}{\psi_t^i \mathbb{F}}, \text{ where } \mathbb{F} \equiv \frac{\theta \mathfrak{m}}{(1-\alpha)^{1-\sigma}(\theta+1)}. \tag{13}$$

[9] Matsuyama (2008) shows that the strategic default a là Hart and Moore (1994) can give rise to this form of the borrowing constraints.

Young entrepreneurs finance their investment in sector A with the labor income, $n^i_{j,t}$, and the loan $b^i_{j,t} = n^i_{j,t}(\frac{1}{\psi^i_t} - 1)$; when old, they consume, $c^{i,e}_{j,t+1}$, and exit from the economy,

$$n^i_{j,t} = w^i_t l_j \quad \text{and} \quad c^{i,e}_{j,t+1} = n^i_{j,t}\Omega^i_t. \tag{14}$$

The agents with $\epsilon_j > \underline{\epsilon}^i_t$ cannot meet the MIR and are called *households*. Their total mass is $1 - \tau^i_t = (\underline{\epsilon}^i_t)^{-\theta}$. Young households invest $m^{i,B}_{j,t}$ in sector B and lend the rest of their labor income $n^i_{j,t} - m^{i,B}_{j,t}$; when old, they consume, $c^{i,h}_{j,t+1}$, and exit from the economy,

$$n^i_{j,t} = w^i_t l_j \quad \text{and} \quad c^{i,h}_{j,t+1} = n^i_{j,t} r^i_t. \tag{15}$$

The markets for credit, sector-specific physical capital, goods, and labor clear,

$$D^i_t \equiv \int_1^{\underline{\epsilon}^i_t} (m^{i,A}_{j,t} - n^i_{j,t})dG(\epsilon_j),\ S^i_t \equiv \int_{\underline{\epsilon}^i_t}^{\infty} (n^i_{j,t} - m^{i,B}_{j,t})dG(\epsilon_j),\ D^i_t = S^i_t, \tag{16}$$

$$K^{i,A}_{t+1} = \int_1^{\underline{\epsilon}^i_t} R m^{i,A}_{j,t} dG(\epsilon_j) = RM^{i,A}_t,\ K^{i,B}_{t+1} = \int_{\underline{\epsilon}^i_t}^{\infty} R m^{i,B}_{j,t} dG(\epsilon_j) = RM^{i,B}_t, \tag{17}$$

$$C^i_t \equiv \int_1^{\underline{\epsilon}^i_t} c^{i,e}_{j,t} dG(\epsilon_j) + \int_{\underline{\epsilon}^i_t}^{\infty} c^{i,h}_{j,t} dG(\epsilon_j), \quad C^i_t + M^{i,B}_t + M^{i,B}_t = Y^i_t, \tag{18}$$

$$Z^{i,A}_t = Y^{i,A}_t,\ \ Z^{i,B}_t = Y^{i,B}_t,\ \ L^{i,A}_t + L^{i,B}_t = L. \tag{19}$$

where D^i_t and S^i_t denote the aggregate credit demand and supply, respectively.

If $r^i_t = q^{i,A}_{t+1}R$, the agents who can meet the MIR may not invest their entire labor income in sector A or may not borrow to the limit. Despite the indeterminacy of the individual investment size, a fraction η of aggregate saving and labor are allocated to sector A and the rest to sector B.

Definition 1. *Under autarky, a market equilibrium in country i is a set of allocations of agents, $\{n^i_{j,t}, m^{i,f}_{j,t}, c^{i,e}_{j,t}, c^{i,h}_{j,t}, \psi^i_{j,t}\}$, and aggregate variables, $\{Y^i_t, Y^{i,f}_t, K^{i,f}_t, M^{i,f}_t, L^{i,f}_t, Z^{i,f}_t, p^{i,f}_t, q^{i,f}_t, w^i_t, r^i_t, \Omega^i_t, \underline{\epsilon}^i_t\}$, satisfying equations* (1)-(2), (6), (10)-(19).

Under autarky, domestic investment is financed by domestic saving in period t, $M^{i,A}_t + M^{i,B}_t = w^i_t$; according to equations (1)-(2), the total investment return in period $t+1$ is $\sum_{f\in\{A,B\}} q^{i,f}_{t+1}K^{i,f}_{t+1} = \rho w^i_{t+1}$, where $\rho \equiv \frac{\alpha}{1-\alpha}$. The social rate of return is defined as

$$\Upsilon^i_t \equiv \frac{\sum_{f\in\{A,B\}} q^{i,f}_{t+1}K^{i,f}_{t+1}}{\sum_{f\in\{A,B\}} M^{i,f}_t} = \frac{\eta\mu^i_{t+1}}{1-\eta+\eta\mu^i_{t+1}} q^{i,A}_{t+1}R + \frac{1-\eta}{1-\eta+\eta\mu^i_{t+1}} q^{i,B}_{t+1}R = \rho\frac{w^i_{t+1}}{w^i_t}. \tag{20}$$

2.1 Extensive-Margin Effect and Cross-Sector Allocation

Financial frictions and the sector-specific MIR may distort the cross-sector investment, i.e., aggregate saving is allocated inefficiently less (more) in sector A (B). Thus, the rate of return in sector A (B) is higher (lower) than the social rate of return and so is the equity rate (the interest rate), i.e., $\Omega^i_t > q^{i,A}_{t+1}R > \Upsilon^i_t > q^{i,B}_{t+1}R = r^i$ and $\mu^i_{t+1} < 1$. In this case, the borrowing constraints are binding and the aggregate dynamics of country i are characterized by $\{w_t^i, \psi_t^i, \underline{\epsilon}_t^i, \mu_{t+1}^i, \Gamma_t^i, \Upsilon_t^i, r_t^i, \chi_{t+1}^i\}$ satisfying equations (13), (20)-(24),[10]

$$\psi_t^i = 1 - \frac{\lambda}{\mu_{t+1}^i}, \tag{21}$$

$$(\underline{\epsilon}_t^i)^{-(1+\theta)} = 1 - \frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i}\psi_t^i, \tag{22}$$

$$w_{t+1}^i = \left(\frac{R}{\rho}\Gamma_t^i w_t^i\right)^\alpha, \quad \text{where} \quad \Gamma_t^i \equiv \frac{(\mu_{t+1}^i)^\eta}{1-\eta(1-\mu_{t+1}^i)} < 1, \text{ and } \frac{\partial\Gamma_t^i}{\partial\mu_{t+1}^i} > 0, \tag{23}$$

$$r_t^i = \Upsilon_t^i(1-\eta+\eta\mu_{t+1}^i) < \Upsilon_t^i, \quad \chi_{t+1}^i = (\mu_{t+1}^i)^\alpha. \tag{24}$$

Given the aggregate saving w_t^i, the larger the cross-sector distortion, the lower the sectoral capital ratio $\kappa_{t+1}^i \equiv \frac{K_{t+1}^{i,A}}{K_{t+1}^{i,B}} = \frac{RM_t^{i,A}}{RM_t^{i,B}} = \frac{\eta}{1-\eta}\mu_{t+1}^i$, the lower the sectoral rate-of-return ratio μ_{t+1}^i and the sectoral output ratio $\frac{Y_{t+1}^{i,A}}{Y_{t+1}^{i,B}} = \frac{\eta}{1-\eta}\chi_{t+1}^i = \frac{\eta}{1-\eta}(\mu_{t+1}^i)^\alpha$, the lower the aggregate output Y_t^i. μ_{t+1}^i reflects the cross-sector investment composition and Γ_t^i measures the aggregate allocation efficiency.

If the allocation is efficient, the model dynamics are characterized by $\{w_t^i, \Upsilon_t^i, r_t^i, \chi_{t+1}^i\}$ satisfying equations (20), (23)-(24) with $\mu_{t+1}^i = 1$, equations (13) and (22) jointly determine $\underline{\epsilon}_t^i$ and ψ_t^i, and the borrowing constraints (21) are slack with $\psi_t^i > 1-\lambda$.[11]

Define $\Lambda \equiv \frac{(1-\eta+\eta\lambda)^{\frac{1}{1+\theta}}}{1-\lambda}(1-\alpha)(1+\frac{1}{\theta})$ as a function of $\lambda \in (0,1)$ and $\frac{\partial\Lambda}{\partial\lambda} > 0$.

Lemma 1. *Iff $\mathfrak{m} \le (Y_t^i)^\sigma\Lambda$, the cross-sector investment is efficient, $\mu_{t+1}^i = 1$, and the borrowing constraints are slack.*

Iff $\mathfrak{m} > (Y_t^i)^\sigma\Lambda$, the cross-sector investment is inefficient, $\mu_{t+1}^i \in (\lambda, 1)$, and the borrowing constraints are binding. In particular, $\frac{\partial\mu_{t+1}^i}{\partial\lambda} > 0$, $\frac{\partial\underline{\epsilon}_t^i}{\partial\lambda} > 0$; $\frac{\partial\mu_{t+1}^i}{\partial\mathfrak{m}} < 0$, $\frac{\partial\underline{\epsilon}_t^i}{\partial\mathfrak{m}} < 0$; $\operatorname{sgn}\left(\frac{\partial\mu_{t+1}^i}{\partial Y_t^i}\right) = \operatorname{sgn}\left(\frac{\partial\underline{\epsilon}_t^i}{\partial Y_t^i}\right) = \operatorname{sgn}(\sigma)$.

The sectoral rate-of-return ratio μ_{t+1}^i is affected by four factors, i.e., the level of financial development λ, the two MIR parameters $\mathfrak{m}$ and σ, and aggregate income Y_t^i. Consider the case of $\mathfrak{m} > (Y_t^i)^\sigma\Lambda$. First, the lower the λ, the less the entrepreneur can borrow against its investment return, the lower its maximum investment, the lower the cutoff value $\underline{\epsilon}_t^i$, the lower (higher) the mass of entrepreneurs (households), the lower (higher) the investment in sector A (B) on the intensive and **extensive margins**, the larger the cross-sector investment distortion, the lower the μ_{t+1}^i. Second, the larger the $\mathfrak{m}$, the higher the MIR, the lower the cutoff value, the lower (higher) the aggregate investment in sector A (B) on the **extensive margin**, the lower the μ_{t+1}^i. Third, the effects of Y_t^i depends on the sign and the size of σ.

For $\sigma = 0$, a rise in Y_t^i raises the MIR, $\mathbb{m}_t^i = \mathfrak{m}Y_t^i$, and the individual's net wealth, $n_{j,t}^i = l_j w_t^i = l_j(1-\alpha)Y_t^i$, in the equal proportions. Thus, the cutoff value $\underline{\epsilon}_t^i = \underline{\epsilon}_A$ is

[10] See the proofs for lemma 1, 2, and proposition 1 in the appendix for the derivation.

[11] As mentioned above, the zero spread $r_t^i = q_{t+1}^{i,A}R$ leads to the indeterminacy of the investment size and the equity-investment ratio at the individual level. For analytical simplicity, we focus on an equilibrium where all entrepreneurs still invest their entire labor income in sector A and choose the same ψ_t^i.

constant, and so are the mass of entrepreneurs, $\tau_t^i = \tau_A = 1 - \underline{\epsilon}_A^{-\theta}$, the sectoral rate-of-return ratio, $\mu_{t+1}^i = \mu_A$, and the sectoral capital ratio, $\kappa_{t+1}^i = \frac{\eta}{1-\eta}\mu_A$, where X_A denotes the steady-state value of variable X_t^i under autarky. In this case, a change in aggregate income only affects the sectoral investment on the **intensive margin**, with no impacts on the extensive margin, $\frac{\partial \epsilon_t^i}{\partial Y_t^i} = \frac{\partial \mu_{t+1}^i}{\partial Y_t^i} = \frac{\partial \Gamma_t^i}{\partial Y_t^i} = 0$.

For $\sigma > 0$, a rise in Y_t^i raises the individual's net wealth proportionally, while it leads to a less-than-proportional rise or even a decline in the MISR, $\frac{\partial \ln \mathbb{m}_t^i}{\partial \ln Y_t^i} = 1-\sigma < \frac{\partial \ln n_{j,t}^i}{\partial \ln Y_t^i} = 1$. Thus, more agents can meet the MIR and invest in sector A. Besides raising the sectoral investment on the intensive margin, a rise in Y_t^i also improves the cross-sector investment composition $\frac{\partial \mu_{t+1}^i}{\partial Y_t^i} > 0$ and the aggregate allocation efficiency $\frac{\partial \Gamma_t^i}{\partial Y_t^i} > 0$ on the **extensive margin**. The larger the σ, the stronger the extensive-margin effect.[12]

To sum up, the extensive margin is the key channel through which the four factors affect the cross-sector investment composition and the aggregate allocation efficiency. In particular, σ determines the sign and the size of the extensive-margin effect.

2.2 Extensive-Margin Effect and Multiple Steady States

The higher aggregate income leads to the higher individual's labor income and saving. For $\sigma = 0$, the sectoral investment responds on the intensive margin and hence, $\mu_{t+1}^i = \mu_A$; the sectoral investment ratio and the aggregate allocation efficiency indicator are constant at $\frac{M_t^{i,A}}{M_t^{i,B}} = \frac{\eta}{1-\eta}\mu_A$ and $\Gamma_t^i = \Gamma_A$. Due to the decreasing VMPK (the neoclassical effect), the law of motion for wage is concave and log-linear with the slope less than unity,[13]

$$w_{t+1}^i = \left(\frac{R}{\rho} w_t^i \Gamma_A\right)^\alpha, \Rightarrow \frac{\partial \ln w_{t+1}^i}{\partial \ln w_t^i} = \underbrace{\alpha}_{\text{neoclassical effect}} < 1. \tag{25}$$

Proposition 1. *Under autarky, there exists a unique, stable steady state for $\sigma = 0$, while there may exist multiple steady states for $\sigma > 0$.*

For $\sigma > 0$, define $\bar{Y}_A \equiv \left(\frac{\mathbb{m}}{\Lambda}\right)^{\frac{1}{\sigma}}$. According to lemma 1, if $Y_t^i \geq \bar{Y}_A$, the cross-sector investment is efficient and a rise in Y_t^i raises the investment in two sectors proportionally. If $Y_t^i < \bar{Y}_A$, the cross-sector investment is inefficient and a rise in Y_t^i raises the sectoral investment on the intensive margin and improves the cross-sector investment composition on the extensive margin. The intensive-margin adjustment triggers the neoclassical effect, which is a convergence force, while the extensive-margin adjustment affects the aggregate allocation efficiency, which is a divergence force. Use equation (23) to get

$$\frac{\partial \ln w_{t+1}^i}{\partial \ln w_t^i} = \underbrace{\alpha}_{\text{neoclassical effect}} \left(1 + \underbrace{\frac{\partial \ln \Gamma_t^i}{\partial \ln \mu_{t+1}^i} \frac{\partial \ln \mu_{t+1}^i}{\partial \ln w_t^i}}_{\text{cross-sector composition effect} \geq 0}\right). \tag{26}$$

[12]For $\sigma < 0$, the opposite applies and a rise in Y_t^i worsens the cross-sector investment composition and the aggregate allocation efficiency. In this paper, we focus on the case of $\sigma \geq 0$.

[13]Proportional to aggregate income, the wage $w_t^i = (1-\alpha)Y_t^i$ is a sufficient statistics for Y_t^i in our model. Thus, we use the law of motion for wage for the dynamic analysis. Alternatively, one can also use the law of motion for capital but the analysis is technically more complicated.

Let $\bar{w}_A \equiv (1-\alpha)\bar{Y}_A$. For $w_t^i \in (0, \bar{w}_A)$, the law of motion for wage in log is non-linear and multiple steady states may arise, due to the positive cross-sector composition effect.

Figure 2 shows the parameter configuration for multiple steady states in an individual country in the $\{\lambda, \psi_A\}$ space.[14] For the parameter configuration below (above) the diagonal line, the cross-sector investment is inefficient (efficient) in the steady state, $\mu_A < 1$ ($\mu_A = 1$) and the borrowing constraints are binding with $\psi_A = 1 - \frac{\lambda}{\mu_A} \in (0, 1-\lambda)$ (slack with $\psi_A \in [1-\lambda, 1]$). For the parameter configuration to the left (right) of the vertical curve, there exist multiple steady states (an unique steady state). Given σ, the diagonal line and the vertical curve split the $\{\lambda, \psi_A\}$ space into four regions.

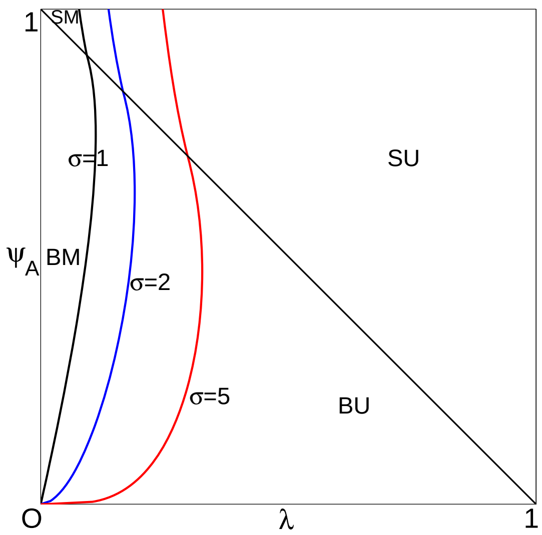

Figure 2: Parameter Configuration for Multiple Steady States under Autarky

Let us start with the region above the diagonal line of figure 2 where the borrowing constraints are **s**lack in the steady state. The dash-dotted curve in the upper-right panel of figure 3 shows the benchmark law of motion for wage $w_{t+1}^i = \left(\frac{R}{\rho} w_t^i\right)^\alpha$ where the cross-sector investment is efficient, while the blue solid curve shows the law of motion for wage with $\{\lambda, \psi_A\}$ in region SU of figure 2. According to lemma 1, for $w_t^i > \bar{w}_A$, the cross-sector investment is efficient so that only the neoclassical effect is active and the law of motion for wage is concave, $w_{t+1}^i = \left(\frac{R}{\rho} w_t^i\right)^\alpha$, crossing the 45° line once and only once at point S with $w_S = \left(\frac{R}{\rho}\right)^\rho$; for $w_t^i \in (0, \bar{w}_A)$, the cross-sector investment is inefficient so that, besides the neoclassical effect, the cross-sector composition effect is also active. The gap

[14]At first sight, it seems wrong to say that figure 2 shows the parameter configuration, because ψ_A on the vertical axis is not a parameter. Instead, one could show the results, for example, in the $\{\lambda, R\}$ space or in the $\{\lambda, \mathfrak{m}\}$ space (Matsuyama, 2004; Zhang, 2013). However, if the results are shown, for example, in the $\{\lambda, R\}$ space, the parameters other than λ and R must be implicitly fixed and it is unclear how changes in the other parameters may affect the shape of the diagram.
ψ_t^i is an endogenous variable. In the autarkic steady state, its value ψ_A depends on all parameters. given λ on the horizontal axis, as long as the parameter combinations give the same value of ψ_A, the shape of the diagram stays unchanged. One can also map the diagram one-to-one from the $\{\lambda, \psi_A\}$ space into the $\{\lambda, R\}$ space or the $\{\lambda, \mathfrak{m}\}$ space. In addition, as both λ and ψ_A can be measured empirically, our results in the $\{\lambda, \psi_A\}$ space can be interpreted meaningfully.

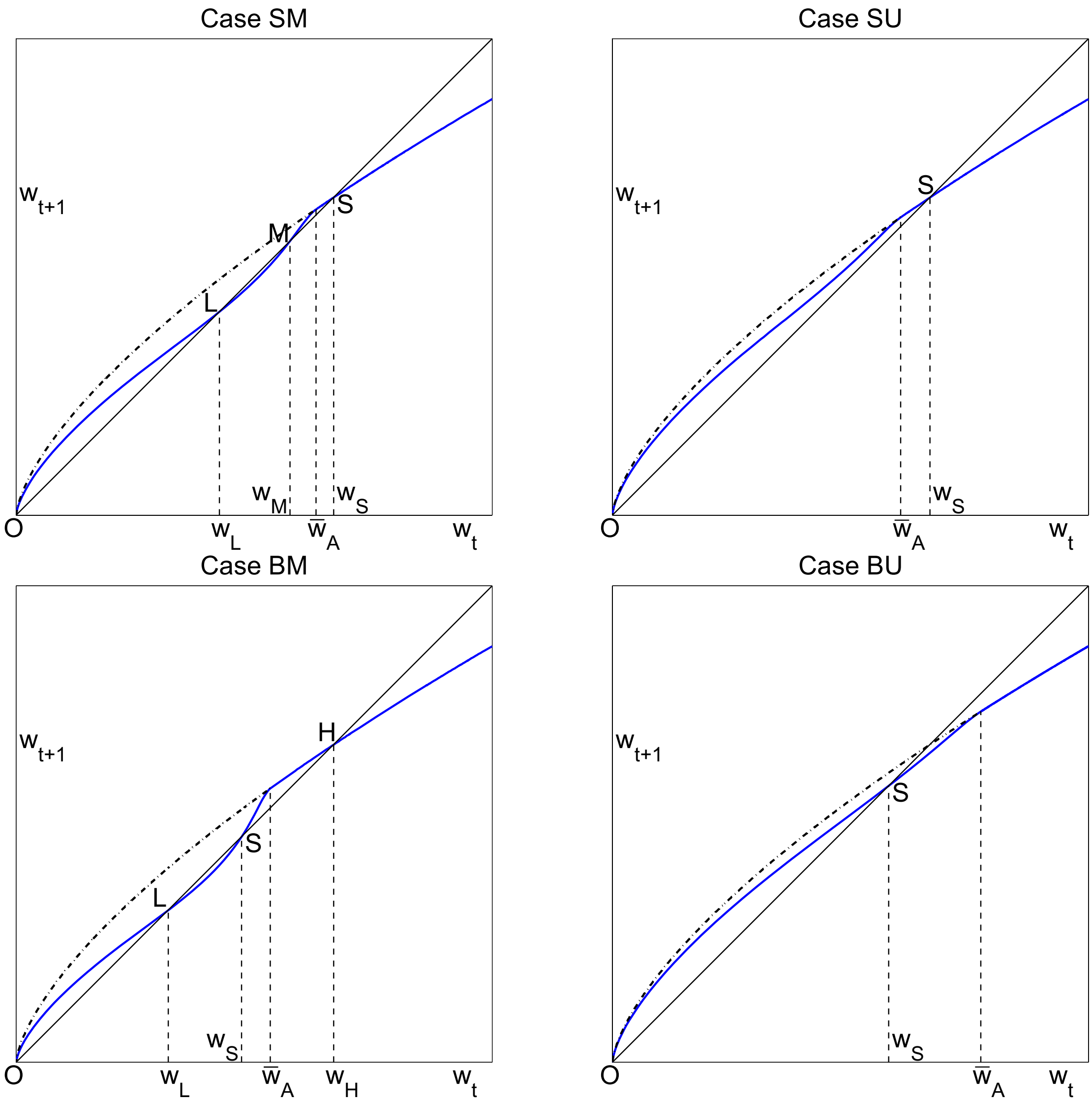

Figure 3: Phase Diagrams of Wage under Autarky: $\sigma > 0$

between the solid and the dash-dotted curves shows the aggregate allocation efficiency loss due to the cross-sector investment distortion, i.e., $\left(\frac{R}{\rho}w_t^i\right)^{\alpha}[1-(\Gamma_t^i)^{\alpha}] > 0$, for $\Gamma_t^i < 1$.

According to lemma 1, the lower the λ, the larger the cross-sector distortion and the efficiency loss. In region SU, the high λ leads to the small cross-sector investment distortion and the small efficiency loss. As shown in the upper-right panel of figure 3, for $w_t^i \in (0, \bar{w}_A)$, the law of motion for wage deviates slightly from its benchmark and does not intersect with the 45° line. Compared to the benchmark case, point S is still the ***u***nique, stable steady state, but the convergence speed to the steady state is slower.

In region SM, the low λ leads to the large cross-sector investment distortion and the large efficiency loss. As shown in the upper-left panel of figure 3, for $w_t^i \in (0, \bar{w}_A)$, the law of motion for wage deviates significantly from its benchmark so that, besides the stable steady state S with $w_S = \left(\frac{R}{\rho}\right)^{\rho}$, there exist another stable steady state L and an unstable steady state M. Starting with a low initial income $w_t^i < w_M$, the country converges to the poverty trap L with a permanently lower income $w_L < w_S$.

Now, consider the region below the diagonal line of figure 2 where the borrowing constraints are ***b***inding in the steady state. According to lemma 1, a higher $\mathfrak{m}$ leads to the less efficient cross-sector investment and hence, μ_{t+1}^i is lower and so is ψ_t^i, according to equation (21). Let us keep λ constant and move from region SU to BU by raising $\mathfrak{m}$. In region BU, the high λ leads to the small cross-sector investment distortion and the small efficiency loss. As shown in the lower-right panel of figure 3, the law of motion for wage deviates slightly from its benchmark so that there exists a ***u***nique steady state S with $w_S = \left(\frac{R}{\rho}\Gamma_A\right)^\rho$ and $\Gamma_A < 1$.

In region BM, the low λ leads to the large efficiency loss. As shown in the lower-left panel of figure 3, for $w_t^i \in (0, \bar{w}_A)$, the law of motion for wage deviates significantly from its benchmark so that there exist multiple steady states, L, S, and H. Starting from a low (high) income with $w_t^i < w_S$ ($w_t^i > w_S$), the country converges to a stable steady state L (H) with $w_L < w_S$ ($w_H > w_S$). Thus, for the parameter configuration in region SM and BM, the initial income matters for the convergence path and the long-run allocation.

As shown in subsection 2.1, the higher the σ, the stronger the extensive-margin effect and the cross-sector composition effect, the more likely the multiple steady states may arise.[15] Thus, the larger the σ, the larger the region SM and BM in figure 2.

In the following sections, we focus on the parameter configurations in regions BU and SU of figure 2 which ensures the existence of a unique steady state under autarky.

3 Trade Integration and Income Divergence

We first specify the condition under which aggregate income may become a determinant of comparative advantage in intratemporal trade. Then, we show that free trade induces countries with different initial incomes to specialize in the sector that they have the comparative advantage, which may lead to income divergence.

3.1 Extensive-Margin Effect and Comparative Advantage

The larger the cross-sector distortion, the lower the sectoral output ratio, $\frac{Y_t^{i,A}}{Y_t^{i,B}}$, the higher (lower) the price of final good A (B), the lower the relative final good price $\chi_t^i = (\mu_t^i)^\alpha \leq 1$. Combine the definition of the relative final good price with equation (2) to get

$$p_t^{i,A} = (\chi_t^i)^{\eta-1} \geq 1, \text{ and } p_t^{i,B} = (\chi_t^i)^{\eta} \leq 1. \tag{27}$$

For $\sigma = 0$, the extensive margin is mute so that $\mu_{t+1}^i = \mu_A$ and $\chi_{t+1}^i = \chi_A = \mu_A^\alpha$ are constant, independent of aggregate income; for $\sigma > 0$ and $Y_t^i > \bar{Y}_A$, the cross-sector

[15] If $\eta = 1$, composite goods are produced one-to-one from final good A and hence, sector B vanishes. The two-sector model degenerates into a one-sector model and there is no cross-sector investment distortion. Aggregate saving w_t^i is entirely invested in sector A, $K_{t+1}^{i,A} = Rw_t^i$, and the law of motion for wage is concave $w_{t+1}^i = \left(\frac{R}{\rho}w_t^i\right)^\alpha$. There exists a unique, stable steady state with $w_A = \left(\frac{R}{\rho}\right)^\rho$ and the initial income level does not matter for the convergence. Thus, the multiple steady states in the two-sector model result essentially from the cross-sector investment distortion on the extensive margin.

investment is efficient so that $\chi^i_{t+1} = \mu^i_{t+1} = 1$ are constant, independent of Y^i_t. In these two cases, χ^i_{t+1} is identical among all countries, independent of Y^i_t.

For $\sigma > 0$ and $Y^i_t \in (0, \bar{Y}_A)$, the cross-sector investment is inefficient and, according to lemma 1, aggregate income affects μ^i_{t+1} and χ^i_{t+1} positively on the extensive margin. Thus, the rich (poor) country has the comparative advantage in sector A (B).

3.2 Trade-Driven Specialization and Multiple Steady States

In period 0, country i announces that two final goods will be freely traded from period 1 onwards.[16] As a small open economy, country i takes the world relative final good price as given, $\chi^i_t = \chi^*$ where $t = 1, 2, 3, ...$ Without loss of generality[17], we assume $\chi^* = \chi_A$.

$Y^{i,f}_t$ and $Z^{i,f}_t$ measure the domestic output and absorbtion of good f, respectively. The export-to-domestic-absorbtion ratio in sector f is $\varsigma^{i,f}_t \equiv \frac{Y^{i,f}_t - Z^{i,f}_t}{Z^{i,f}_t}$, with the negative value for the case of imports. With no international capital flows, trade is balanced, $p^{i,A}_t \varsigma^{i,A}_t Z^{i,A}_t + p^{i,B}_t \varsigma^{i,B}_t Z^{i,B}_t = 0$. Combine it with equation (2) to get

$$\chi^i_t \frac{\eta}{1-\eta} = \frac{Z^{i,A}_t}{Z^{i,B}_t} = -\chi^i_t \frac{\varsigma^{i,B}_t}{\varsigma^{i,A}_t}, \quad \Rightarrow \quad \varsigma^{i,B}_t = -\frac{\eta}{1-\eta} \varsigma^{i,A}_t. \tag{28}$$

If the country specializes completely in sector A (B), it does not produce but imports good B (A) for the domestic production of composition good, $\varsigma^{i,B}_t = -1$ ($\varsigma^{i,A}_t = -1$). Combine them with equation (28) to get the range for $\varsigma^{i,A}_t \in (-1, \frac{1-\eta}{\eta})$ and $\varsigma^{i,B}_t \in (-1, \frac{\eta}{1-\eta})$.

By equalizing the relative final good price, free trade implicitly equalizes the sectoral rate-of-return ratio; if the borrowing constraints are binding, the equity-investment ratio is also equalized.

$$\mu^i_t = (\chi^i_t)^{\frac{1}{\alpha}} = (\chi^*)^{\frac{1}{\alpha}} = \mu^*, \tag{29}$$

$$\psi^i_t = 1 - \frac{\lambda q^{i,A}_{t+1} R}{r^i_t} = 1 - \frac{\lambda q^{i,A}_{t+1} R}{q^{i,B}_{t+1} R} = 1 - \frac{\lambda}{\mu^i_{t+1}} = 1 - \frac{\lambda}{\mu^*} = \psi^*. \tag{30}$$

As shown in subsection 3.1, if $\sigma = 0$, the relative final good price is identical among all countries under autarky, $\chi^i_t = \chi_A$. Thus, given $\chi^* = \chi_A$, free trade does not affect the dynamics and the steady state of the individual country.

If $\sigma > 0$, define $\underline{w}_T \equiv (\psi^* \mathbb{F})^{\frac{1}{\sigma}}$ and $\bar{w}_T \equiv \left(\frac{\mu^*}{\lambda}\right)^{\frac{1}{\sigma(1+\theta)}} \underline{w}_T > \underline{w}_T$. For $w^i_t \in (0, \underline{w}_T]$, nobody can meet the MIR so that the country specializes completely in sector B, i.e., $\varsigma^{i,A}_t = -1$ and $\varsigma^{i,B}_t = \frac{\eta}{1-\eta}$. For $w^i_t \geq \bar{w}_T$, the mass of entrepreneurs is so high that they borrow the entire saving of households and hence, the country specializes completely in sector A, i.e., $\varsigma^{i,A}_t = \frac{1-\eta}{\eta}$ and $\varsigma^{i,B}_t = -1$. For $w^i_t \in (\underline{w}_T, \bar{w}_T)$, some agents can meet the MIR and invest in sector A, but their mass is so low that they cannot borrow the entire saving of households. Thus, both sectors receive the positive investment.

[16]If free trade is announced and implemented in the same period, the relative final good price is determined in the world market immediately, which affects the investment return of the currently old agents and the aggregate income unexpectedly. In the two-period OLG model, announcing free trade one-period in advance avoids creating the uncertainty.

[17]Subsection 3.3 endogenizes the relative final good price in a world economy setting.

Given $\chi_t^i = \chi^*$, $\psi_t^i = \psi^*$, and $\mu_t^i = \mu^*$, the aggregate dynamics of country i are characterized by $\{w_t^i, \underline{\epsilon}_t^i, \varsigma_t^{i,A}, \Gamma_t^i\}$ satisfying equations (31)-(33),[18]

$$\underline{\epsilon}_t^i = \begin{cases} 1, & \text{if } w_t^i \in (0, \underline{w}_T]; \\ \frac{(w_t^i)^\sigma}{\psi^* \mathbb{F}}, & \text{if } w_t^i \in (\underline{w}_T, \bar{w}_T); , \\ \left(\frac{\mu^*}{\lambda}\right)^{\frac{1}{1+\theta}}, & \text{if } w_t^i \geq \bar{w}_T; \end{cases} \tag{31}$$

$$\varsigma_{t+1}^{i,A} = \begin{cases} -1, & \text{if } w_t^i \in (0, \underline{w}_T]; \\ [\eta(1 - \mu^* + \frac{\mu^* - \lambda}{1 - (\underline{\epsilon}_t^i)^{-(1+\theta)}})]^{-1} - 1, & \text{if } w_t^i \in (\underline{w}_T, \bar{w}_T); , \\ \frac{1-\eta}{\eta}, & \text{if } w_t^i \geq \bar{w}_T; \end{cases} \tag{32}$$

$$w_{t+1}^i = \left(\frac{R}{\rho}\Gamma_t^i w_t^i\right)^\alpha, \text{ where } \Gamma_t^i \equiv \frac{(\mu^*)^\eta}{1 - \eta(1 - \mu^*)(1 + \varsigma_t^{i,A})}, \text{ and } \frac{\partial \Gamma_t^i}{\partial \varsigma_t^{i,A}} > 0. \tag{33}$$

Figure 4 shows the parameter configuration for multiple steady states under trade integration in the (λ, ψ_A) space, given $\sigma = 1$ and $\sigma = 0.1$, respectively. The solid and the dash-dotted curves in figure 5 show the law of motion for wage under trade integration versus under autarky, with the parameter configuration in the five regions of figure 4, respectively.

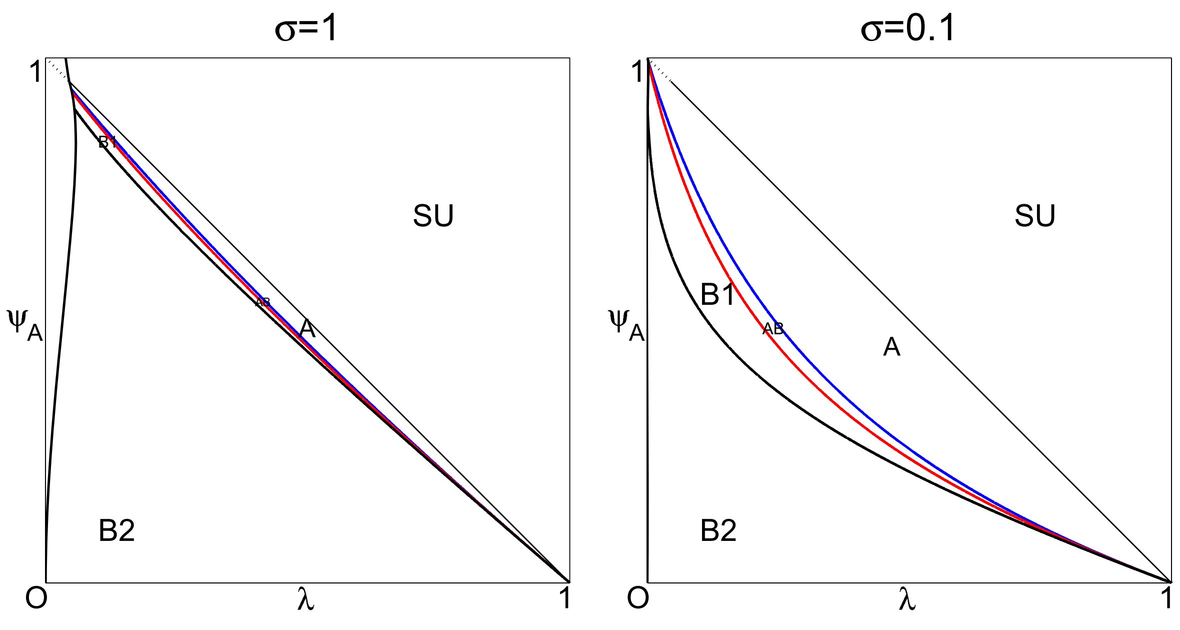

Figure 4: Parameter configuration for Multiple Steady States under Trade Integration

For the parameter configuration in region SU of figure 4, $\chi_A = 1$. Under trade integration, $\chi^* = \chi_A = 1$ implies that the rate of return equalizes in the two sectors, $\mu_t^i = \mu^* = (\chi^*)^{\frac{1}{\alpha}} = 1$. Thus, the cross-sector allocation of domestic savings are irrelevant for the aggregate income in the next period. Due to the neoclassical effect, the law of motion for wage is concave, $w_{t+1}^i = \left(\frac{R}{\rho} w_t^i\right)^\alpha$. See the lower-right panel of figure 5.

[18]See the proof of Proposition 2 in appendix B for the derivation.

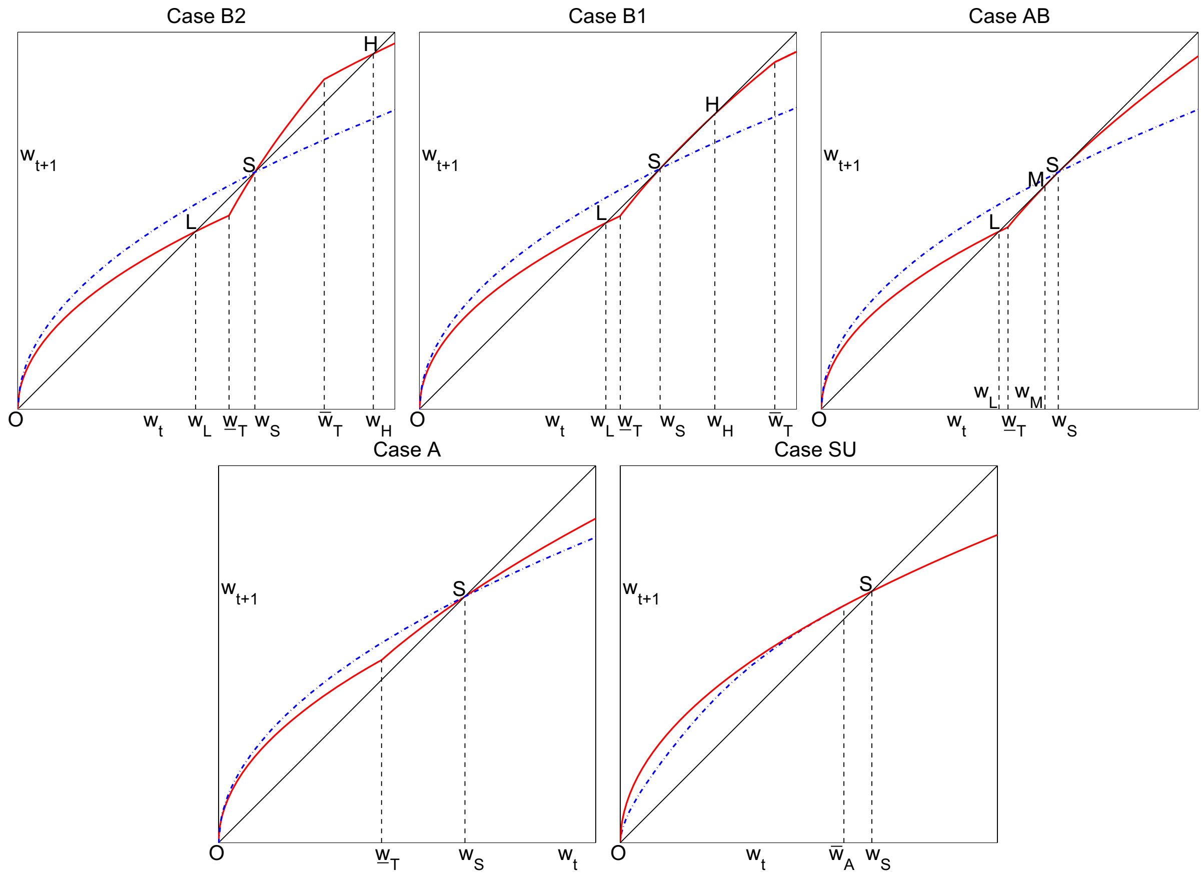

Figure 5: Phase Diagrams of Wage under Trade Integrations

Proposition 2. *Under trade integration, if $\sigma = 0$ or if $\sigma > 0$ and $\chi^* = 1$, the autarkic steady state is still the unique, stable steady state; if $\sigma > 0$ and $\chi^* < 1$, the autarkic steady state may become unstable and there may exist multiple steady states where the country specializes partially or completely in one sector.*

In the following, we focus on region BU of figure 2 where the borrowing constraints are binding, $\mu_A < 1$, and the cross-sector allocation is distorted in the autarkic steady state, $\chi_A = \mu_A^\alpha < 1$.

Consider first the case of $Y_0^i > Y_A$. Had the country stayed under autarky, its relative final good price in period $t = 1$ would be higher than the steady state level, $\chi_1^i > \chi_A$. Given $\chi_t^* = \chi_A < 1$ from period $t = 1$ on, the country has the comparative advantage in good A, i.e., its autarkic price of final good A (B) in period $t = 1$ is lower (higher) than the world level. When the free trade policy is announced in period $t = 0$, the price of final good A (B) in period $t = 1$ is expected to rise (decline) to the world level and so does the rate of return in sector A (B) in period $t = 0$, which affects the sectoral investment in two ways. First, the decline in the rate of return in sector B induces households to invest less in sector B and to lend more to the credit market, leading to a decline in the interest rate. The rise in the unit pledgable value $\lambda q_{t+1}^{i,A} R$ and the decline in the cost of external funds r_t^i allow entrepreneurs to borrow more per unit of the investment, $\frac{\lambda q_{t+1}^{i,A} R}{r_t^i}$ and to invest more in sector A. Thus, the investment in sector A (B) rises (declines) on the **intensive margin**. Second, the decline in the equity-investment ratio $\psi_t^i = 1 - \frac{\lambda q_{t+1}^{i,A} R}{r_t^i}$ allows more agents to meet the MIR and invest in sector A. Thus, the investment in sector A (B) rises (declines) on the **extensive margin**.

The cross-sector investment adjustment enables the country to specialize towards sector A in period $t = 0$ and to export (import) good A (B) in period $t = 1$. Given $\mu^* = \mu_A < 1$, the rate of return is higher in sector A than in sector B. The country benefits from specializing in the high-return sector and its period-1 aggregate income is higher than otherwise under autarky. Then, the higher wage rate in period $t = 1$ allows even more agents to meet the MIR and invest in sector A so that the country specializes even further towards the high-return sector. This way, free trade triggers the dynamic, *virtuous cycles* in the rich countries, through which the rising mass of entrepreneurs and the rising aggregate income reinforcing each other over time through specialization. The dynamic reinforcing process goes on until the the mass of entrepreneurs eventually rises to such a high level that entrepreneurs borrow the entire saving of households. In that case, the country **specializes completely** in sector A and any further rise in the mass of entrepreneurs will not improve the cross-sector investment and the allocation efficiency.

By the same logic, if $Y_0^i < Y_A$, the country has a comparative advantage in sector B and, due to the trade-driven specialization towards the low-return sector (sector B) in period $t = 0$, its aggregate income in period $t = 1$ is lower than otherwise under autarky, which leads to a even lower mass of entrepreneurs and the specialization further towards the low-return sector in period $t = 1$. This way, free trade triggers the dynamic, *vicious cycles* in the poor countries, through which the declining mass of entrepreneurs and the declining aggregate income reinforcing each other over time. The dynamic reinforcing process goes on until the the mass of entrepreneurs eventually declines to zero. In that case, the country **specializes completely** in sector B and any further decline in aggregate income does not worsen the cross-sector investment and the allocation efficiency.

Overall, the **trade-driven specialization** is a **divergence** force, making the law of motion for aggregate income steeper around the autarkic steady state. It interacts with the neoclassical effect, which determines the dynamic stability property.

The lower the level of financial development λ or the larger the $\mathfrak{m}$ or the σ, the larger the cross-sector investment distortion, the larger the cross-sector difference in the final good prices, the stronger the specialization effect, the more likely trade integration may destabilize the autarkic steady state and lead to multiple steady states.

Given the level of financial development, there exist three threshold values,

- $\tilde{\psi}_T \equiv 1 - \frac{\lambda}{1-\eta}\left[\left(\frac{1-\eta}{\lambda} + \eta\right)^{\frac{1}{\sigma\rho(1+\theta)+1}} - \eta\right]$,
- $\hat{\psi}_T = (1-\lambda)\left[1 - \frac{1}{\sigma\rho(1+\theta)(\frac{1-\eta}{\lambda}+\eta)+1}\right]$, and
- $\bar{\psi}_T = 1 - \frac{\eta\lambda}{[1-\eta+\eta\lambda]^{\frac{1}{\sigma\rho(1+\theta)+1}} - (1-\eta)}$,

which split region BU of figure 2 into four subregions of figure 4.

- For $\psi_A \in (0, \tilde{\psi}_T)$, the parameter configurations are in region B2. Given the level of financial development, the high MIR leads to the severe cross-sector distortion under autarky so that the cross-sector rate-of-return differential is large. Thus, the trade-driven specialization effect is strong enough to dominate the neoclassical effect in the autarkic steady state. As shown in the upper-left panel of figure 5, the autarkic steady state becomes unstable and, for $w_0^i > w_S$ ($w_0^i < w_S$), the country converges to a new steady state H (L) where it specializes completely in sector A (B) with the income higher (lower) than in the autarkic steady state.

- For $\psi_A \in (\tilde{\psi}_T, \hat{\psi}_T)$, the parameter configurations are in region B1. Compared with case B2, the lower $\mathbb{m}$ leads to a smaller cross-sector distortion and hence, the specialization effect is weaker. Although trade integration still destabilizes the autarkic steady state, the aggregate dynamics differ slightly from case B2. As shown in the upper-middle panel of figure 5, for $w_0^i > w_S$, the country converges to a new steady state H with $w_H > w_S$ where it partially specializes in sector A, $w_H < \bar{w}_T$.

- For $\psi_A \in (\hat{\psi}_T, \bar{\psi}_T)$, the parameter configurations are in region AB. With an even lower MIR, the specialization effect is weaker than in case B1. Free trade does not destabilize the autarkic steady state but it generates the other two steady states, M and L. As shown in the upper-right panel of figure 5, for $w_0^i < w_M$, the country converges to a new steady state L where it specializes completely in sector B; otherwise, it converges to the autarkic steady state.

- For $\psi_A \in (\bar{\psi}_T, 1 - \lambda)$, the parameter configurations are in region A. The specialization effect is so weak that free trade does not lead to multiple steady states. However, As shown in the lower-left panel of figure 5, the convergence is slower.

To sum up, the extensive margin is the key channel through which aggregate income may become a determinant of comparative advantage. Free trade affects the mass of investors in each sector and triggers the sectoral investment adjustment on the extensive margin, which may lead to specialization and multiple steady states.[19]

As shown in subsections 2.1 and 2.2, σ affects the size of the extensive-margin effect. Compare the two panels of figure 4. The larger the σ, the larger the cross-sector distortion under autarky, the larger the cross-sector difference in the final good prices, the stronger the specialization effect, the more likely free trade may lead to multiple steady states, and hence, the larger the region B2-B1-AB.

So far, we have taken the world relative final good price as given at $\chi^* = \chi_A$ and analyzed the impacts of trade integration for a small open economy. The model helps explain why countries which are inherently identical except for the initial income may possibly converge to different steady states, but it does not tell whether this is inevitable. In subsection 3.3, we endogenize χ^* in a world economy model and show the condition under which trade integration inevitably leads to income divergence.

[19]In Antras and Caballero (2009), the mass of investors in each sector is exogenous and hence, the extensive margin is inactive. In our model, for $\sigma = 0$, the mass of investors in each sector is endogenous but constant under autarky so that the extensive margin is also inactive. If the cross-country difference in financial development is then introduced into our setting with $\sigma = 0$, our model becomes analytically equivalent to theirs. In both models, free trade affects the sectoral investment only on the intensive margin so that it cannot lead to the complete specialization and multiple steady states.

3.3 Income Divergence in A World Economy

As shown in subsection 2.2, an individual country converges to a unique, stable steady state under autarky with the aggregate income at Y_A, if the parameter configurations are in region SU and BU of figure 2. As a collection of autarkic countries, the world economy has a unique, stable steady state which is symmetric, i.e., all countries end up with the same income level Y_A in the long run.

In the case of trade integration, the two final goods are traded globally at the relative price χ_t^* and the markets clear at the world level. Although the symmetric steady state mentioned above is still a steady state for the world economy, it may not be stable and there may exist stable, asymmetric steady states where the world economy is polarized into two groups of countries with the different incomes in the long run.

The Symmetric Steady State

For the parameter configuration in region SU-A-AB of figure 4, trade integration does not destabilize the autarkic steady state for the small open economy so that the world economy has a stable, symmetric steady state where all countries end up with the same steady-state income as under autarky; for the parameter configuration in region B1-B2, trade integration destabilizes the autarkic steady state for a small open economy so that the world economy does not have the stable, symmetric steady state.

The Asymmetric Steady States

According to figure 5, in the cases of multiple steady states, if a country ends up in the steady state L, it specializes completely in sector B with $w_L = \left[\frac{R}{\rho}(\mu^*)^\eta\right]^\rho$ and import $\frac{\eta Y_L}{p^{A,*}}$ units of good A; if it ends up in the steady state H, it may specialize completely or partially in sector A with $w_H = \left[\frac{R}{\rho}\frac{(\mu^*)^\eta}{1-\eta(1-\mu^*)(1+\varsigma_H^A)}\right]^\rho$ and export $\varsigma_H^A \frac{\eta Y_H}{p^{A,*}}$ units of good A. Suppose that the world economy is in a stable, asymmetric steady state where the fraction δ of countries have the steady-state income Y_L and the rest have Y_H. χ^* is determined by the market clearing condition for final good A at the world level,[20]

$$\delta\frac{\eta Y_L}{p^{A,*}} = (1-\delta)\varsigma_H^A\frac{\eta Y_H}{p^{A,*}}, \Rightarrow \delta = \frac{\varsigma_H^A}{\varsigma_H^A + [1-\eta(1-\mu^*)(1+\varsigma_H^A)]^\rho} \tag{34}$$

Thus, there exists a δ that supports the world relative final good price $\chi^* = (\mu^*)^\alpha$.

According figure 4, $\bar{\psi}_T$ is the threshold value for the border between the region with multiple steady states (B2-B1-AB) and the region with unique steady states (A-SU), given $\chi^* = \chi_A$. As there always exists a δ that supports $\chi^* = \chi_A$, an asymmetric steady state exists for the parameter configurations in region B2-B1-AB, i.e., $\psi_A \in (0, \bar{\psi}_T)$. Besides, given $\psi_A \in (0, \bar{\psi}_T)$, there exists a continuum of χ^* in the neighborhood of χ_A such that, for each χ^*, the world economy has a stable asymmetric steady state. Furthermore,

[20]Given the balanced trade at the country level, if the market for one final good clears at the world level, the market for the other one must also clear, according to the Walras' law. Thus, we only need to analyze the market clearing condition for one final good.

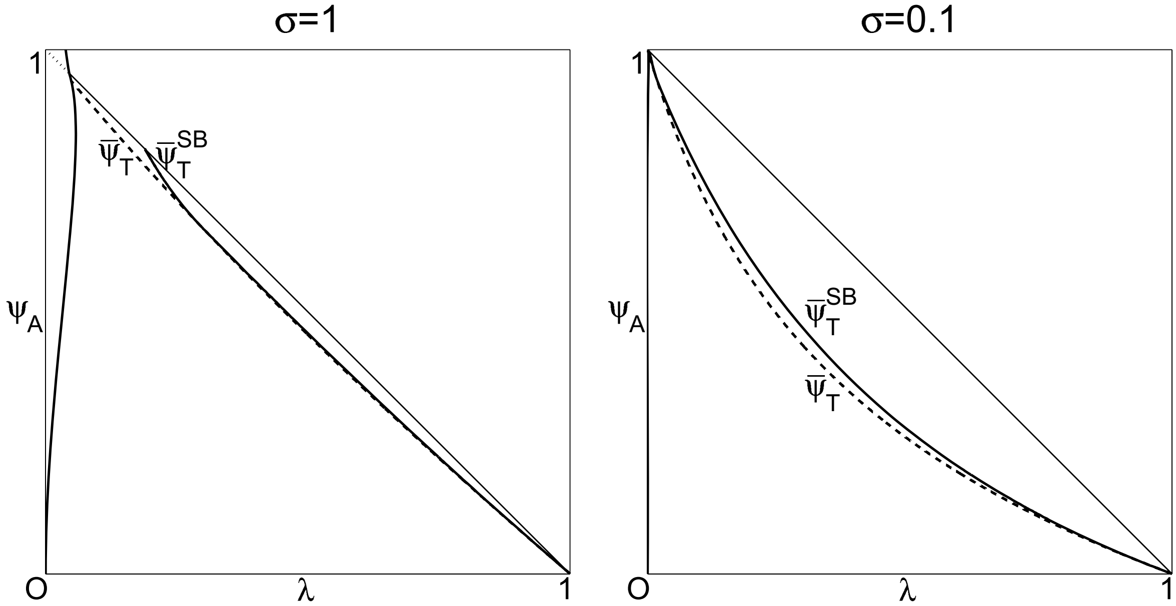

Figure 6: Parameter Configuration for Symmetry Breaking under Trade Integration

without the restriction of $\chi^* = \chi_A$, there may exist asymmetric steady states even for parameter configurations in region A, i.e., $\psi_A > \bar{\psi}_T$.

Proposition 3. *Given the level of financial development, if the MIR is sufficiently high so that $\psi_A < \bar{\psi}_T^{SB}$, the world economy has a continuum of stable, asymmetric steady states under trade integration where a fraction $\delta \in (\delta^-, \delta^+) \subset (0,1)$ of the countries have the income $Y_L < Y_A$ and the rest have the income $Y_H > Y_A$.*

The solid (dashed) curves in figure 6 shows $\bar{\psi}_T^{SB}$ ($\bar{\psi}_T$) in the $\{\lambda, \psi_A\}$ space for $\sigma \in \{1, 0.1\}$. Intuitively, if we do not impose the restriction of $\chi^* = \chi_A$, it is more likely that trade integration may lead to multiple steady states for the individual country and the asymmetric steady states for the world economy, i.e., $\bar{\psi}_T^{SB} > \bar{\psi}_T$.

If the asymmetric steady state is stable, free trade leads to income divergence rather than convergence among inherently identical countries. Thus, the world economy is inevitably polarized into the rich and the poor. This way, we offer a theoretical support for the view that international trade is a mechanism through which rich countries become richer at the expense of poor countries.

4 Financial Integration and Income Divergence

Under autarky, due to financial frictions and the sector-specific MIR, the mass of entrepreneurs (households) is inefficiently low (high) and so is the aggregate credit demand (supply). Thus, the interest rate is below the social rate of return, as shown in equation (24). The higher aggregate income raises the sectoral investment on the intensive margin. The **neoclassical effect** tends to reduce the social rate of return and the interest rate.

If $\sigma > 0$, the higher aggregate income also allows more agents to become entrepreneurs and the aggregate credit demand (supply) rises (declines) on the **extensive margin**, which tends to raise the interest rate. If the extensive-margin effect dominates the neoclassical effect, the interest rate rises in aggregate income under autarky. Thus, the interest rate is higher in the rich than in the poor countries. Free capital mobility leads to capital flows from the poor to the rich countries, which directly raises (reduces) the size and indirectly improves (worsens) the composition of the aggregate investment in the rich (poor) on the **extensive margin**. Thus, financial integration may lead to income divergence. If $\sigma = 0$, the extensive margin is inactive. Due to the neoclassical effect, the interest rate is lower in the rich than in the poor and free capital mobility leads to capital flows from the rich to the poor countries, which narrows the cross-country income gaps and leads to income convergence.

Similar as in the case of trade integration, the extensive margin is the key channel through which aggregate income may become a determinant of "comparative advantage" in the intratemporal trade. Furthermore, it is also through the extensive margin that free capital mobility may affect the allocation efficiency and lead to income divergence.

As the analysis is similar as that of trade integration, we leave it in appendix A.

5 Trade and Financial Integration

Sections 3 and 4 have shown that, in the case of $\sigma > 0$, either trade or financial integration may lead to income divergence. Can trade and financial integration jointly lead to income convergence, as argued in Antras and Caballero (2009)?

5.1 Interest Rate Patterns under Trade Integration

Under trade integration, domestic investment in period t is funded by domestic saving, $M_t^{i,A} + M_t^{i,B} = w_t^i$, and the investment revenue in period $t+1$ is $q_{t+1}^{i,A} R M_t^{i,A} + q_{t+1}^{i,B} R M_t^{i,B} = \rho w_{t+1}^i$. Thus, the social rate of return is $\Upsilon_t^i = \frac{\rho w_{t+1}^i}{w_t^i}$. Combine it with equations (31)-(33) to get the interest rate as a piecewise function of aggregate income over three intervals.

1.) For $w_t^i \in (0, \underline{w}_T]$, no one meets the MIR in sector A and the country specializes completely in sector B. As all agents invest in sector B with the same linear technology, the (underlying) interest rate is equal to the social rate of return,

$$\ln r_t^i = \ln R q_{t+1}^{i,B} = \ln \Upsilon_t^i = -(1-\alpha)\ln w_t^i + \ln \rho^{1-\alpha} R^{\alpha} + \alpha\eta \ln \mu_{t+1}^*. \quad (35)$$

2.) For $w_t^i \in (\underline{w}_T, \bar{w}_T)$, some agents meet the MIR and invest in sector A as entrepreneurs, $\varsigma_t^{i,A} \in (-1, \frac{1-\eta}{\eta})$. If $\mu_{t+1}^* < 1$, entrepreneurs borrow to the limit but their mass is inefficiently low and so is the aggregate credit demand. Thus, the interest rate is below the social rate of return.

$$\ln r_t^i = \ln R q_{t+1}^{i,B} = \ln \Upsilon_t^i [1 - \eta(1-\mu_{t+1}^*)(1+\varsigma_t^{i,A})] < \ln \Upsilon_t^i,$$
$$\ln r_t^i = -(1-\alpha)\ln w_t^i \left[\frac{\frac{1}{\mu_{t+1}^*} - 1}{\psi_t^*} \left(1 - \frac{(\psi_t^* \mathbb{F})^{1+\theta}}{(w_t^i)^{\sigma(1+\theta)}} \right) + 1 \right] + \ln \rho^{1-\alpha} R^{\alpha} + \alpha\eta \ln \mu_{t+1}^*. \quad (36)$$

3.) For $w_t^i > \bar{w}_T$, the mass of entrepreneurs is so high that they borrow the entire saving of households. Thus, the country specializes completely in sector A and the aggregate credit demand is so high that the interest rate is equal to the social rate of return,

$$\ln r_t^i = \ln Rq_{t+1}^{i,A} = \ln \Upsilon_t^i = -(1-\alpha)\ln w_t^i + \ln \rho^{1-\alpha}R^{\alpha} + \alpha\eta \ln \mu_{t+1}^* - \alpha \ln \mu_{t+1}^*. \quad (37)$$

According to equations (35)-(37), $\frac{\partial r_t^i}{\partial w_t^i} < 0$ within each interval, mainly due to the neoclassical effect. For $w_t^i \in (0, \bar{w}_T)$, the positive investment in sector B implies the **coupling** of the interest rate with the rate of return in sector B, $r_t^i = Rq_{t+1}^{i,B}$, according to equation (6); for $w_t^i > \bar{w}_T$, the complete specialization in sector A fundamentally changes the credit market condition, which **decouples** (**couples**) the interest rate from (with) the rate of return in sector B (A), $r_t^i = Rq_{t+1}^{i,A}$. If $\mu_{t+1}^* < 1$, the sectoral rate-of-return differential $Rq_{t+1}^{i,A} > Rq_{t+1}^{i,B}$ implies an upward jump in the interest rate upon the complete specialization at $w_t^i = \bar{w}_T$; if $\mu_{t+1}^* = 1$, $Rq_{t+1}^{i,A} = Rq_{t+1}^{i,B}$ so that the interest rate pattern is **continuous** at $w_t^i = \bar{w}_T$.

Figure 7 shows the interest rate patterns under free trade in the five cases of figure 5, with the wage in log on the horizontal axis. The red solid (blue dash-dotted) curve shows the interest rate (the social rate of return) in log. In case SU, $\mu^* = \mu_A = 1$ and hence, the interest rate is continuous and equal to the social rate of return. In other cases, $\mu^* = \mu_A < 1$ and hence, the interest rate jumps upwards at $w_t^i = \bar{w}_T$. For $w_t^i \in (0, \underline{w}_T) \cup (\bar{w}_T, \infty)$, the complete specialization implies $r_t^i = \Upsilon_t^i$; for $w_t^i \in (\underline{w}_T, \bar{w}_T)$, both sectors are active and $r_t^i < \Upsilon_t^i$. In the steady state, $w_{t+1}^i = w_t^i$ so that $\Upsilon_t^i = \rho$.

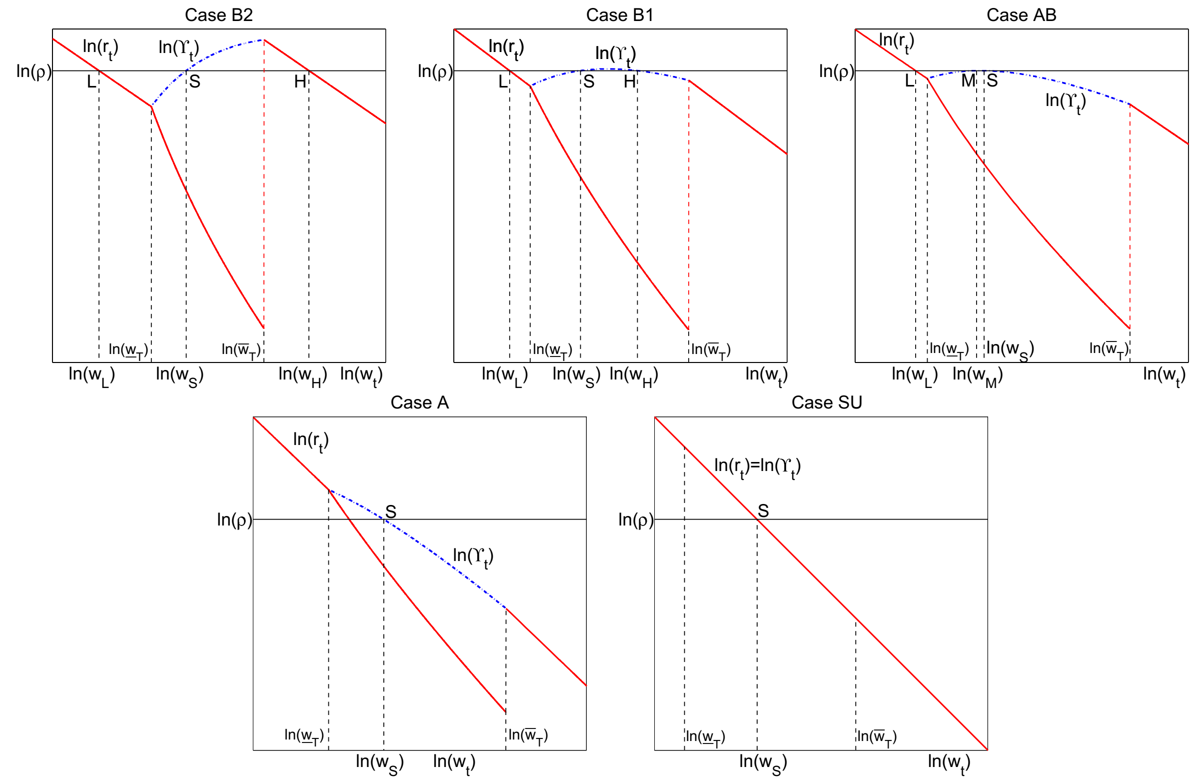

Figure 7: Interest Rate Patterns under Trade Integrations

5.2 Factor Price Equalization and Income Convergence

Consider the parameter configurations in region B1 of figure 4. Under trade integration, the world economy may end up in the asymmetric steady states where the poor (rich) countries specialize completely (**partially**) in sector B (A). According to equations (35)-(36) and the upper-middle panels of figure 5 and 7, the poor end up at point L with $r_t^i = \Upsilon_t^i = \rho$ and the rich at point H with $r_t^i < \Upsilon_t^i = \rho$.

In the asymmetric steady state, since sector B has a positive investment in all countries, the interest rate is coupled with the rate of return in sector B in all countries, $r_t^i = q_{t+1}^{i,B} R$, which is lower in the rich than in the poor. If financial integration is allowed, financial capital flows from the rich to the poor, which equalizes directly the interest rate $r_t^i = r_t^*$ and indirectly the VMPK in sector B, $q_{t+1}^{i,B} = \frac{r_t^i}{R} = \frac{r_t^*}{R} = q_{t+1}^{*,B}$. Given that trade integration has already equalized the sectoral rate-of-return ratio $\mu_{t+1}^i = \mu_{t+1}^*$, allowing financial integration also equalizes the VMPK in sector A, $q_{t+1}^{i,A} = \frac{q_{t+1}^{i,B}}{\mu_{t+1}^i} = \frac{q_{t+1}^{*,B}}{\mu_{t+1}^*} = q_{t+1}^{*,A}$. Thus, although labor is internationally immobile, free mobility of goods and financial capital equalizes the wage rate and aggregate income,

$$w_{t+1}^i = [(q_{t+1}^{i,A})^\eta (q_{t+1}^{i,B})^{1-\eta}]^{-\rho} = [(q_{t+1}^{*,A})^\eta (q_{t+1}^{*,B})^{1-\eta}]^{-\rho} = w_{t+1}^*, \; Y_{t+1}^i = \frac{w_{t+1}^i}{1-\alpha} = \frac{w_{t+1}^*}{1-\alpha} = Y_{t+1}^*.$$

In this case, the world economy behaves like a large autarkic economy and there exists a unique, symmetric steady state where all countries have the same income as under autarky. This result also holds for the parameter configuration in region AB of figure 4.

In Matsuyama (2004), there is only one final good, which serves as the vehicle for capital flows and is freely traded. Thus, symmetry breaking arises in a one-sector model with free mobility of trade and capital flows. Here, we show that *moving from the one-sector to the two-sector setting may reduce the likelihood of symmetry breaking.*[21]

However, allowing free trade and capital flows does not necessarily eliminate symmetry breaking. For the parameter configurations in region B2 of figure 4, trade integration induces the world economy to end up in the asymmetric steady states where the rich (poor) specialize **completely** in sector A (B). According to equations (35) and (37) as well as the upper-left panels of figure 5 and 7, the rich (poor) end up at point H (L) with $r_t^i = \Upsilon_t^i = \rho$. Since the rich and the poor countries have the same interest rate in the asymmetric steady state under trade integration, adding financial integration does not create any capital flows and the income gap between the rich and the poor still exists.

The intuition has been explained in subsection 1.1. In Antras and Caballero (2009), the exogenous mass of investors in each sector allows the sectoral investment to adjust only on the **intensive margin** so that trade integration does not lead to the complete specialization. Thus, free mobility of trade and capital flows unambiguously leads to income convergence. In our model, the endogenous mass of investors in each sector allows the sectoral investment to adjust also on the **extensive margin** so that free trade

[21] Our result holds for a sufficiently large sectoral heterogeneity in the MIR. As shown in subsection 6.1, if the sectoral heterogeneity in the MIR is small enough, the two-sector model behaves analytically identical as the one-sector model around the autarkic steady state so that moving from the one-sector to the two-sector setting does not affect Matsuyama's symmetry breaking.

may lead to the complete specialization. Thus, free mobility of trade and capital flows may lead to income convergence only conditionally.

6 Robustness Check and Extensions

Many assumptions are made in our model for tractability. In this section, we check the robustness of our model results by reconsidering some alternative assumptions.

6.1 Sector-Specific MIR

For simplicity, we normalize the MIR in sector B at zero. In the presence of financial frictions, the positive MIR in sector A becomes an entry barrier and, given the zero MIR in sector B, those who cannot meet the MIR in sector A still can freely invest in sector B and lend to the credit market. Thus, the MIR in sector A distorts the allocation in two dimensions. First, it distorts the intratemporal relative price (the relative final good price) through affecting the cross-sector investment composition, as shown in subsection 3.1; second, it distorts the intertemporal relative price (the interest rate) through affecting the credit market equilibrium, as shown in appendix A.1.

We can decompose the distortions in these two dimensions by allowing for a positive MIR in sector B. Consider the case of the constant MIR, i.e., $\sigma = 1$. Let $\mathbb{m}$ and $\gamma\mathbb{m}$ denote the MIR in sector A and in sector B, respectively, where $\gamma \in [0, 1]$ measures the sectoral MIR ratio. For $\gamma = 0$, the model is the one we have analyzed so far.

For $\gamma = 1$, the two sectors are subject to the same real friction $\mathbb{m}$ and the same financial frictions λ so that the cross-sector investment is efficient in equilibrium and the intratemporal relative price is constant at unity, $\chi^i_{t+1} = 1$.[22] The agents who cannot meet the MIR can only lend their savings to the credit market. Given $\lambda < 1$, the higher the $\mathbb{m}$, the lower (higher) the mass of agents who can (cannot) invest in the two sectors, the lower (higher) the aggregate credit demand (supply), the larger the deviation of the interest rate from the social rate of return.

We can extend the analysis to the intermediate case of $\gamma \in (0, 1)$. Given a sufficiently high $\mathbb{m}$, the lower the γ, the larger the sectoral heterogeneity and the cross-sector investment distortion, the lower the relative final good price, the more likely free trade may lead to income divergence. Given the sectoral MIR ratio γ, the higher the $\mathbb{m}$, the smaller the mass of agents who can meet the MIR, the less (more) the borrowers (lenders) on the credit market, the larger the interest rate distortion. Thus, the size of the MIR $\mathbb{m}$ is a key determinant for the intertemporal distortion, while the sectoral MIR ratio γ is a key determinant for the intratemporal distortion.

6.2 Sector-Specific Financial Frictions

In our model, an entrepreneurs can borrow against a fraction $\lambda \in [0, 1]$ of its future investment revenue. Generally speaking, this fraction depends on the institutional factors

[22] In this case, the model is equivalent to a one-sector model, which is analyzed in Zhang (2013).

(i.e, the legal enforcement, the sophistication of financial markets, the liquidity of asset markets, etc.), the sector-specific factors (e.g., the project tangibility and liquidity), and the individual-specific factors (e.g., the borrower's credit record).

In our current setting, both sectors are subject to the same λ[23], which reflects the institution-related factors. In so doing, we can focus on the sectoral heterogeneity in a real friction, i.e., the MIR. Alternatively, one can assume that both sectors are subject to the same MIR and introduce the sectoral heterogeneity in the financial frictions by assigning $\lambda^f \in [0,1]$ to sector $f \in \{A, B\}$,[24] which does not affect our results. However, with λ^f reflecting both the institution- and the sector-related factors, one cannot decompose the implications of the financial frictions from these two sources.

Zhang (2013a) models explicitly the sector-specific project tangibility, rather than conveniently capturing it with the sector-specific λ^f. Suppose that the individual's project investment in sector $f \in \{A, B\}$, m_t^f, consists of the tangibles, $m_t^{f,T}$, which determines the project scale, and the intangibles, $m_t^{f,I}$, which determines the project productivity, i.e., $m_t^f = m_t^{f,T} + m_t^{f,I}$ and $k_{t+1}^f = m_t^{f,T} R(\varpi_t^f)$, where $\varpi_t^f \equiv \frac{m_t^{f,I}}{m_t^{f,T}}$ denotes the intangibles-tangibles ratio with $R(0) = 1$, $R' > 0$ and $R'' < 0$. Upon default, the intangibles are completely lost and the tangibles have the liquidation value $\lambda q_{t+1}^f m_t^{f,T}$, where λ measures the institutional factors and applies equally to both sectors. Thus, the agents can borrow less per unit of total investment in the sector with a higher intangible-tangible ratio. This way, one can analyze the implications of the sector-specific factors that affects the firm's external financing in a more micro-founded way.

6.3 Sector-Specific Capital Intensity

A recent literature analyzes the implications of the sector-specific capital intensity on trade flows (Bajona and Kehoe, 2010; Cunat and Maffezzoli, 2004b; Deardorff, 2001; Jin, 2012; Ju, Shi, and Wei, 2014; Ju and Wei, 2009, 2011). In our current setting, the two sectors have the same capital share, α. Under autarky, the capital-labor ratio is endogenous and lower in the more financially constrained sector; under trade integration, the rich (poor) country exports the labor-intensive (capital-intensive) goods. Antras and Caballero (2009) get the similar result and argue that credit constraints may provide an explanation for the so-called Leontief paradox. see Wynne (2005) for more on this. Ju and Wei (2011) introduce the exogenous, sector-specific capital-labor ratio and fixed costs in a static Heckscher-Ohlin model with the financial frictions. They show that the capital intensive sector can become more financially dependent. Following Acemoglu and Guerrieri (2008) and Jin (2012), we can introduce the sector-specific capital share in our current setting[25] and show that the capital intensity can be higher or lower in the more

[23] Given the zero MIR in sector B, the agents who cannot invest in sector A, i.e., households, can freely invest in sector B and lend to the credit market. In equilibrium, $r_t \geq q_{t+1}^{i,B} R$; otherwise, no agents would lend. Thus, households do not strictly prefer borrowing and hence, financial frictions in sector B are irrelevant. As shown in subsection 6.1, if the MIR in sector B is also positive, the financial frictions in sector B matters for the equilibrium allocation.

[24] Following Antras and Caballero (2009), one may assume that the financial contracting in sector B is perfect, i.e., $\lambda^B = 1$, while there is a financial friction in sector A, $\lambda^A < 1$.

[25] Acemoglu and Guerrieri (2008) find a huge dispersion of the average capital share among 22 sectors.

financially constrained sector.

Consider the case of the constant MIR, $\sigma = 1$. Let α^f denote the capital share in sector $f \in \{A, B\}$. For $\alpha^A = \alpha^B$, the model is the one we have analyzed so far. For $\alpha^A \neq \alpha^B$, define some auxiliary parameters $\tilde{\eta} \equiv \frac{\alpha^A \eta}{\alpha^A \eta + \alpha^B (1-\eta)}$, $\tilde{\alpha} \equiv \alpha^A \eta + \alpha^B (1-\eta)$, and $\tilde{\rho} \equiv \frac{\tilde{\alpha}}{1-\tilde{\alpha}}$. Under autarky, the law of motion for wage is $w_{t+1} = \left(\frac{R}{\tilde{\rho}} w_t \Gamma_t\right)^{\tilde{\alpha}}$ with $\Gamma_t \equiv \frac{\tilde{\eta}\mu_{t+1}}{1-\tilde{\eta}(1-\mu_{t+1})}$, which are analytically identical as equation (23).

Let $\Bbbk_t^f \equiv \frac{K_t^f}{L_t^f}$ denote the capital-labor ratio in sector f. Let $\rho^f \equiv \frac{\alpha^f}{1-\alpha^f}$. The sectoral capital-intensity ratio $\frac{\Bbbk_{t+1}^A}{\Bbbk_{t+1}^B} = \frac{\rho^A}{\rho^B}\mu_{t+1}$ depends on two factors, i.e., the cross-sector difference in the capital share, $\frac{\rho^A}{\rho^B}$, and the cross-sector investment distortion, μ_{t+1}. In our current setting, $\alpha^A = \alpha^B = \alpha$ and hence, the sectoral capital intensity ratio depends only on the cross-sector investment distortion, $\frac{\Bbbk_{t+1}^A}{\Bbbk_{t+1}^B} = \mu_{t+1}$. In the frictionless case, the cross-sector investment is efficient, $\mu_{t+1} = 1$, and the sectoral capital intensity equalizes, $\Bbbk_{t+1}^A = \Bbbk_{t+1}^B$; in the frictional case, the cross-sector investment is inefficient, $\mu_{t+1} < 1$, and the sectoral capital intensity lower in the more financially constrained sector, $\Bbbk_{t+1}^A < \Bbbk_{t+1}^B$, due to the under- (over-) investment in sector A (B).

Suppose that sector A not only has a higher MIR but also a higher capital share than sector B, $\alpha^A > \alpha^B$. In the frictionless case, $\mu_{t+1} = 1$, so that the capital intensity is strictly higher in the sector with a higher capital share, $\Bbbk_{t+1}^A = \frac{\rho^A}{\rho^B}\Bbbk_{t+1}^B > \Bbbk_{t+1}^B$. In the frictional case, $\mu_{t+1} < 1$. If the cross-sector distortion dominates (is dominated by) the cross-sector difference in the capital share, the capital intensity is lower (higher) in the more financially constrained sector under autarky.

7 Final Remarks

The main message of this paper is to highlight the *extensive margin* as a critical channel through which aggregate income may become a determinant of comparative advantage, given financial frictions and the sectoral heterogeneity in the MIR. It is also through the extensive margin channel that free trade may allow the initially rich (poor) countries to specialize completely in the high-MIR (low-MIR) sector. Given the sectoral rate-of-return differentials, the trade-driven specialization effect interacts with the neoclassical effect, which may lead to income divergence among inherently identical countries. Financial integration may also lead to income divergence through the extensive-margin channel.

If free trade allows the rich countries to specialize completely in the high-MIR sector, the credit market condition changes fundamentally and the interest rate in the rich countries jumps upward, which can be higher than in the poor countries. Thus, moving from autarky to free trade *does not necessarily* reverse the direct of capital flows across countries and free mobility of trade and capital flows *does not necessarily* lead to income convergence. Here, the extensive-margin channel plays a critical role in explaining these results. This way, we complement the results of Antras and Caballero (2009) and refine the condition for the Stolper-Samuelson theorem.

Our model has two policy implications. First, the countries with a high level of financial development and/or aggregate income (e.g., developed countries) benefit from economic integration in terms of the long-run income level and/or the convergence speed. Second, the countries with the moderately low level of financial development and/or aggregate income (e.g., middle-income countries) should be cautious of the timing for trade or capital account liberalization as well as the partners with whom they are integrated. Policies aiming at improving domestic financial institutions are more relevant than those simply aiming at reducing the barriers to trade or financial transactions.

References

Acemoglu, D., and V. Guerrieri (2008): "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116(3), 467–498.

Aghion, P., and P. Bolton (1997): "A Theory of Trickle-Down Growth and Development," *Review of Economic Studies*, 64(2), 151–172.

Ahn, J., M. Amiti, and D. Weinstein (2011): "Trade Finance and the Great Trade Collapse," *American Economic Review Papers and Proceedings*, 101(3), 298–302.

Antras, P., and R. Caballero (2009): "Trade and Capital Flows: A Financial Frictions Perspective," *Journal of Political Economy*, 117(4), 701–744.

——— (2010): "On the Role of Financial Frictions and the Saving Rate during Trade Liberalizations," *Journal of the European Economic Association*, 8(2-3), 442–455.

Bajona, C., and T. J. Kehoe (2010): "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," *Review of Economic Dynamics*, 13(3), 487.

Banerjee, A. V., and B. Moll (2010): "Why Does Misallocation Persist?," *American Economic Journal: Macroeconomics*, 20(1), 189–206.

Banerjee, A. V., and A. F. Newman (1993): "Occupational Choice and the Process of Development," *Journal of Political Economy*, 101(2), 274–298.

Barseghyan, L., and R. DiCecio (2011): "Entry costs, industry structure, and cross-country income and TFP differences," *Journal of Economic Theory*, 146(5), 1828–1851.

Beck, T. (2002): "Financial development and international trade: Is there a link?," *Journal of International Economics*, 57(1), 107–131.

——— (2003): "Financial Dependence and International Trade," *Review of International Economics*, 11(2), 296–316.

Buera, F. J., J. Kaboski, and Y. Shin (2011): "Finance and development: a tale of two sectors," *American Economic Review*, 101(5), 1964–2002.

Chesnokova, T. (2007): "Immiserizing deindustrialization: A dynamic trade model with credit constraints," *Journal of International Economics*, 73(2), 407–420.

Chor, D., and K. Manova (2012): "Off the cliff and back? Credit conditions and international trade during the global financial crisis," *Journal of International Economics*, 87(1), 117–133.

Cunat, A., and M. Maffezzoli (2004a): "Hecksher-Ohlin Business Cycles," *Review of Economic Dynamics*, 7(3), 555–585.

——— (2004b): "Neoclassical Growth and Commodity Trade," *Review of Economic Dynamics*, 7(3), 707–736.

Deardorff, A. V. (2001): "Rich and Poor Countries in Neoclassical Trade and Growth," *Economic Journal*, 111(470), 277–94.

Erosa, A., and A. Hidalgo-Cabrillana (2008): "On finance as a theory of TFP, cross-industry productivity differences, and economic rents," *International Economic Review*, 491(2), 437–473.

Galor, O., and J. Zeira (1993): "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60(1), 35–52.

Hart, O., and J. Moore (1994): "A Theory of Debt Based on the Inalienability of Human Capital," *The Quarterly Journal of Economics*, 109(4), 841–79.

Jiao, Y., and Y. Wen (2012): "Capital, Finance, and Trade Collapse," St; Louis Fed Working Paper 2012-003A.

Jin, K. (2012): "Industrial Structure and Financial Capital Flows," *American Economic Review*, 102(5), 2111–46.

Ju, J., K. Shi, and S.-J. Wei (2014): "On the Connection between Intra-temporal and Intertemporal Trades," *Journal of International Economics.*, 92(S1), S36.

Ju, J., and S.-J. Wei (2005): "Endowment versus Finance: A Wooden Barrel Theory of International Trade," IMF Working Papers No 05/123.

——— (2009): "Current Account Adjustment in a Model with Multiple Tradable Sectors and Labor Market Rigidities," unpublished.

——— (2011): "When is quality of financial system a source of comparative advantage?," *Journal of International Economics*, 84(2), 178–187.

Kletzer, K., and P. K. Bardhan (1987): "Credit markets and patterns of international trade," *Journal of Development Economics*, 27(1-2), 57–70.

Lipsey, R. G., and K. Lancaster (1956): "The General Theory of Second Best," *Review of Economic Studies*, 24, 11–32.

Manova, K. (2008): "Credit constraints, equity market liberalizations and international trade," *Journal of International Economics*, 76(1), 33–47.

——— (2013): "Credit Constraints, Heterogeneous Firms, and International Trade," *Review of Economic Studies*, 80(2), 711–744.

Matsuyama, K. (1996): "Why Are There Rich and Poor Countries? Symmetry-Breaking in the World Economy," *Journal of the Japanese and International Economies*, 10(4), 419–439.

——— (2000): "Endogenous Inequality," *Review of Economic Studies*, 67(4), 743–59.

——— (2004): "Financial Market Globalization, Symmetry-Breaking, and Endogenous Inequality of Nations," *Econometrica*, 72(3), 853–884.

——— (2005): "Credit Market Imperfections and Patterns of International Trade and Capital Flows," *Journal of the European Economic Association*, 3(2-3), 714–723.

——— (2007): "Credit Traps and Credit Cycles," *American Economic Review*, 97(1), 503–516.

——— (2008): “Aggregate Implications of Credit Market Imperfections,” in *NBER Macroeconomics Annual 2007*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, vol. 22. MIT Press.

Melitz, M. J. (2003): “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, 71(6), 1695–1725.

Midrigan, V., and D. Y. Xu (2014): “Finance and Misallocation: Evidence from Plant-Level Data,” *American Economic Review*, 104(2), 422–58.

Piketty, T. (1997): “The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing,” *Review of Economic Studies*, 64(2), 173–89.

Svaleryd, H., and J. Vlachos (2005): “Financial markets, the pattern of industrial specialization and comparative advantage: Evidence from OECD countries,” *European Economic Review*, 49(1), 113–144.

Wynne, J. (2005): “Wealth as a Determinant of Comparative Advantage,” *American Economic Review*, 95(1), 226–254.

Zhang, H. (2013): “Minimum Investment Requirement, Financial Integration and Economic (In)stability: A Refinement to Matsuyama (2004),” SMU Economics & Statistics Working Paper No. 09-2013.

——— (2013a): “Financial Development and Intangible Investment,” work in progress.

Appendix

A Financial Integration and Income Divergence

Similar as in section 3, we show that financial integration may lead to income divergence among inherently identical countries. We first derive the condition under which aggregate income may become a determinant of “comparative advantage” for intertemporal trade, i.e., borrowing or lending. Then, we show that, under financial integration, capital may flow from the poor to the rich, widening the initial income gap.

A.1 Extensive-Margin Effect and Comparative Advantage

Financial frictions and the sector-specific MIR may distort the interest rate. For notational simplicity, we suppress the country index.

In the case of the efficient cross-sector investment $\mu_{t+1} = 1$, the interest rate coincides with the social rate of return. According to equations (20) and (23)-(24), the higher the aggregate income, the higher the aggregate saving and investment, the lower the social rate of return and the interest rate, due to the neoclassical effect.

In the case of the inefficient cross-sector investment $\mu_{t+1} < 1$, the borrowing constraints are binding so that, due to the inefficiently low aggregate credit demand, the interest rate is below the social rate of return. Combining the binding borrowing constraints with equations (5), (16), and (22), the aggregate credit demand and supply are,

$$D_t = \lambda \frac{q_{t+1}^A R}{r_t} M_t^A = \lambda \frac{q_{t+1}^A R}{r_t} w_t \left[1 - (1-\tau_t)^{\left(1+\frac{1}{\theta}\right)} \frac{1-\eta}{1-\eta+\eta\lambda} \right], \quad \frac{\partial D_t}{\partial r_t} < 0, \tag{38}$$

$$\begin{aligned} \ln D_t = & \underbrace{\ln w_t}_{\text{net-wealth effect}} + \underbrace{\ln \left[1 - (1-\tau_t)^{\left(1+\frac{1}{\theta}\right)} \frac{1-\eta}{1-\eta+\eta\lambda} \right]}_{\text{demand-side extensive-margin effect}} + \underbrace{\ln q_{t+1}^A R}_{\text{neoclassical effect}} \\ & + \underbrace{\ln \lambda}_{\text{financial-development effect}} - \underbrace{\ln r_t}_{\text{interest-rate effect}} \end{aligned} \tag{39}$$

$$S_t = w_t \underline{\epsilon}_t^{-(1+\theta)} - M_t^{i,B} = w_t \left[(1-\tau_t)^{\left(1+\frac{1}{\theta}\right)} - \frac{1-\eta}{1-\eta+\eta\frac{r_t}{q_{t+1}^A R}} \right], \quad \frac{\partial S_t}{\partial r_t} > 0, \tag{40}$$

$$\ln S_t = \underbrace{\ln w_t}_{\text{net-wealth effect}} + \ln \left[\underbrace{(1-\tau_t)^{\left(1+\frac{1}{\theta}\right)}}_{\text{supply-side extensive-margin effect}} - \underbrace{\frac{1-\eta}{1-\eta+\eta\mu_{t+1}}}_{\text{alternative investment effect}} \right]. \tag{41}$$

According to equation (38), a rise in the interest rate reduces the present value of the entrepreneurs' pledgeable investment return so that the credit demand curve is downward sloping; according to equation (40), a rise in the interest rate induces households to cut their investment in sector B and lend more so that the credit supply curve is upward sloping. As shown in equations (39) and (41), the credit demand and the credit supply are also affected by the following factors.

- The net-wealth effect: the higher the aggregate income, the higher the agents' labor income and net wealth, the higher the credit demand and the credit supply.
- The extensive-margin effect: the larger the mass of entrepreneurs τ_t, the smaller the mass of households $1-\tau_t$, the higher (lower) the credit demand (supply).
- The neoclassical effect: the higher the aggregate investment in sector A in period t, the lower the VMPK in sector A in period $t+1$, the lower the pledgeable value of the individual entrepreneur's investment return, the lower the credit demand.
- The financial-development effect: the higher the level of financial development, the more the individual entrepreneur can borrow, the higher the credit demand.
- The alternative-investment effect: the more the households invest in sector B, the lower the credit supply.

Figure 8 shows the credit market equilibrium under autarky. Consider the case of the inefficient cross-sector investment. The downward-sloping credit demand curve D_t and the upward-sloping credit supply curve S_t cross at point E with the equilibrium interest rate at r_t. If aggregate income rises marginally from Y_t to $\tilde{Y}_t$, the aggregate saving rises proportionally from $w_t = (1-\alpha)Y_t$ to $\tilde{w}_t = (1-\alpha)\tilde{Y}_t$. Define $\Delta \ln X_t \equiv \ln \tilde{X}_t - \ln X_t$ as the percentage change in variable X_t.

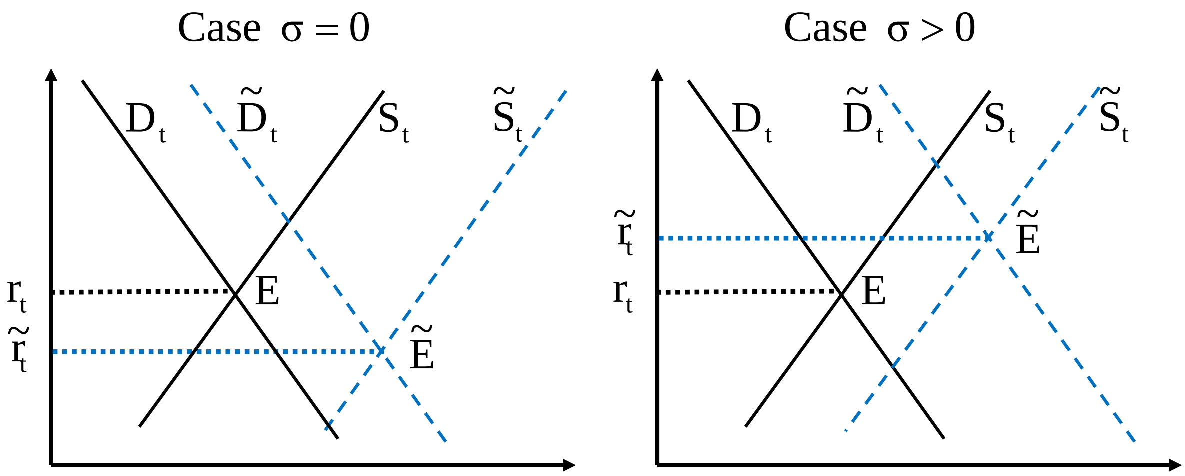

Figure 8: Interest Rate Response to An Increase in Aggregate Income

If $\sigma = 0$, the extensive margin is mute $\mu_{t+1} = \mu_A$ so that higher Y_t raises the sectoral investment only on the intensive margin, without affecting the mass of entrepreneurs $\tau_t = \tau_A$. According to equations (39) and (41), the positive net-wealth effect raises the credit supply and demand in the equal proportions, while the neoclassical effect (the decreasing VMPK) reduces the credit demand. With the net wealth effect exactly canceling out on both sides, the interest rate is purely driven by the neoclassical effect,

$$\begin{aligned} &\Delta \ln D_t = \Delta \ln w_t + \Delta \ln q^A_{t+1} R - \Delta \ln r_t, \quad \Delta \ln S_t = \Delta \ln w_t, \\ &\Delta \ln D_t = \Delta \ln S_t, \quad \Rightarrow, \quad \Delta \ln r_t = \underbrace{\Delta \ln q^A_{t+1} R}_{\text{the neoclassical effect (-)}} \quad . \end{aligned} \tag{42}$$

As shown in the left panel of figure 8, the rightward shift of the credit demand curve is dominated by that of the credit supply curve and hence, the credit market equilibrium moves from point E to Ẽ with a lower interest rate $\tilde{r}_t < r_t$.

If $\sigma > 0$, higher Y_t affects the sectoral investment on the intensive and the extensive margins. In particular, the extensive-margin effect raises (reduces) the credit demand (supply). As shown in the right panel of figure 8, the rightward shift of the credit demand (supply) curve is larger (smaller) than in the case of $\sigma = 0$. Combining equations (39) and (41), the interest rate is affected by four factors in the case of $\sigma > 0$,

$$\begin{aligned} \Delta \ln r_t = \; &\underbrace{\Delta \ln q^A_{t+1} R}_{\text{neoclassical effect (-)}} + \underbrace{\Delta \ln \left[1 - (1-\tau_t)^{\left(1+\frac{1}{\theta}\right)} \frac{1-\eta}{1-\eta+\eta\lambda}\right]}_{\text{demand-side extensive-margin effect (+)}} \\ &- \Delta \ln \left[\underbrace{(1-\tau_t)^{\left(1+\frac{1}{\theta}\right)}}_{\text{supply-side extensive-margin effect (-)}} - \underbrace{\frac{\frac{1-\eta}{\eta}}{\frac{1-\eta}{\eta} + \mu_{t+1}}}_{\text{alternative investment effect (?)}} \right]. \end{aligned} \tag{43}$$

If the demand- and the supply-side extensive-margin effects dominate the neoclassical effect, the rightward shift of the credit demand curve dominates that of the credit supply curve. If so, the right panel of figure 8 shows that the credit market equilibrium moves from point E to Ẽ with a higher interest rate, $\tilde{r}_t > r_t$.

Define $\mathbb{B} \equiv \sigma\rho\eta + 2 + \frac{\eta\lambda}{1-\eta}[\sigma(\rho\eta + 1) + \frac{\theta}{1+\theta}]$ and $\hat{\psi}_A \equiv \frac{\mathbb{B}-\sqrt{\mathbb{B}^2-4(\sigma\rho\eta+1)(\frac{\eta\lambda}{1-\eta}+1)}}{2(\sigma\rho\eta+1)}$ as a function of λ. Define $\hat{\lambda}$ as the solution to the function of $1 - \lambda = \hat{\psi}_A$.

According to lemma 1, the equity-investment ratio increases in aggregate income. Thus, ψ_t can be used as a proxy for Y_t.

Lemma 2. *If $\sigma > 0$ and $\lambda \in (\hat{\lambda}, 1)$ or if $\sigma = 0$, the interest rate is lower in the country with the higher income.*

If $\sigma > 0$ and $\lambda \in (0, \hat{\lambda})$, the interest rate is higher in the country with the marginally higher income for $\psi_t \in (\hat{\psi}_A, 1 - \lambda)$, while the interest rate is lower in the country with the marginally higher income for $\psi_t \in (0, \hat{\psi}_A) \bigcup (1 - \lambda, 1)$.

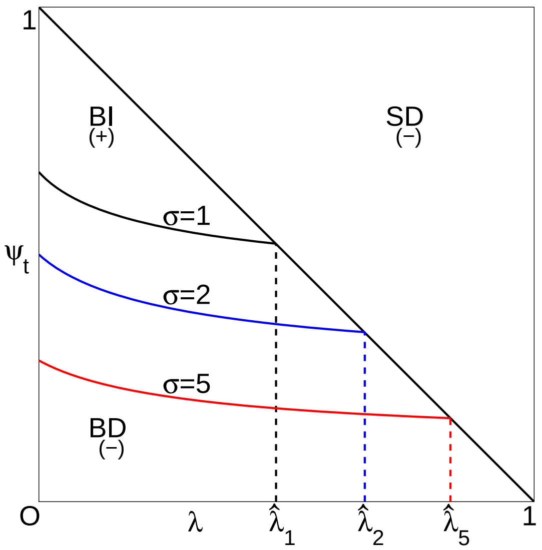

Figure 9: The Direction of Interest Rate Responses to Income Changes: $\sigma > 0$

Figure 9 shows the sign of the interest rate response to the aggregate income change in the (λ, ψ_t) space. The solid curves between region BI and BD show the threshold values $\hat{\psi}_A$ in the cases of $\sigma = 1, 2, 5$, respectively. Consider the case of $\sigma > 0$ and $\lambda \in (0, \hat{\lambda})$. Keeping λ constant, if the country starts with a very low level of income, ψ_t is so low that the allocation is initially in region BD where the borrowing constraints are ***b***inding. Due to the very low income level, the neoclassical effect dominates the extensive-margin effect so that the interest rate ***d***eclines in Y_t. Along the convergence path to the steady state, Y_t rises and so does ψ_t. If $\psi_t > \hat{\psi}_A$, the country enters into region BI where the borrowing constraints are still ***b***inding. Given the intermediate level of income, the neoclassical effect is dominated by the extensive-margin effect so that the interest rate ***i***ncreases in Y_t. If Y_t rises further such that $\psi_t > 1 - \lambda$, the country enters into region SD where the borrowing constraints are ***s***lack. Then, the extensive margin is mute and, due to the neoclassical effect is active, the interest rate ***d***eclines in Y_t. As shown in subsection 2.1, the larger the σ, the stronger the extensive-margin effect, the more likely the interest rate responds positively to income changes, the larger the region BI. See figure 9.

Figure 10 shows that for $\lambda \in (0, \hat{\lambda})$, the interest rate is non-monotonic with aggregate income. Let us focus on the interest rate response to income change around the steady state. Given $\lambda \in (0, \hat{\lambda})$, if the parameter configuration makes ψ_A in region BI, the interest rate rises in aggregate income around the steady state, as shown in the middle panel of figure 10; if $\{\lambda, \psi_A\}$ is in region BD or SD, the interest rate declines in aggregate income around the steady state, as shown in the left and the right panels of figure 10.

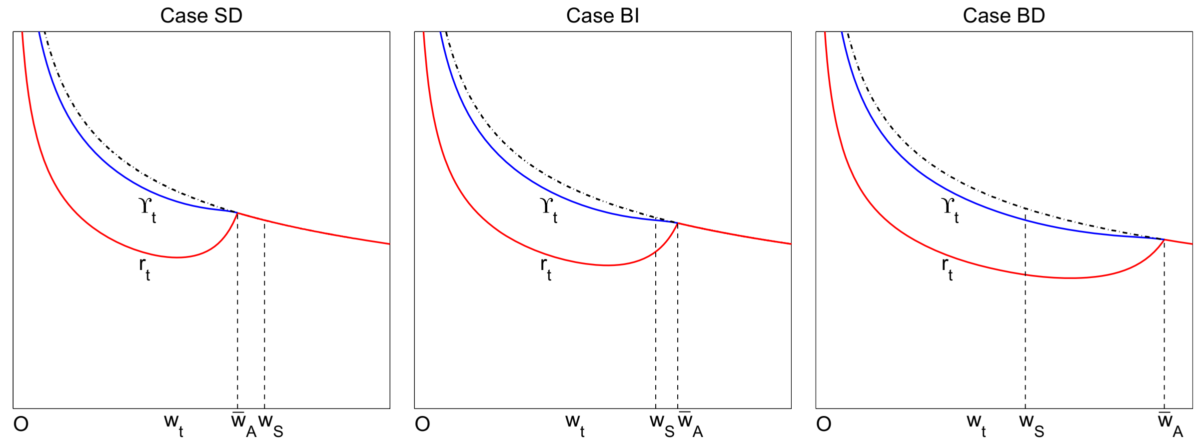

Figure 10: Interest Rate Responses to Income Changes

For $\lambda \in (\hat{\lambda}, 1)$, λ is sufficiently high and hence, the cross-sector investment distortion is mild. A rise in Y_t only leads to a small extensive-margin effect which is always dominated by the neoclassical effect. Thus, the interest rate always declines in aggregate income.

To sum up, if financial frictions and the sector-specific MIR distort the cross-sector investment $\mu_{t+1} < 1$, the relative final good price reflects the distortion on the intratemporal dimension, $\chi_{t+1} = (\mu_{t+1})^{\alpha} < 1$, while the interest rate reflects the distortion on the intertemporal dimension, $r_t = \Upsilon_t(1 - \eta + \eta\mu_{t+1}) < \Upsilon_t$. The two relative prices are linked through the sectoral rate-of-return ratio μ_{t+1}. In the case of $\sigma > 0$, a rise in aggregate income may raise them through the extensive-margin channel.

A.2 Financial Integration and Multiple Steady States

From period $t = 0$ on, agents in country i are allowed to borrow and lend abroad. As a small open economy, country i takes the world interest rate as given, $r_t^i = r^*$. Without loss of generality, we assume $r^* = r_A$.

Let ϕ_t^i denote the ratio of financial outflow over domestic saving, with the negative value for the case of financial inflows. Capital mobility affects the total funds for domestic investment, $M_t^{i,A} + M_t^{i,B} = w_t^i(1 - \phi_t^i)$. The composite good is freely traded and serves as the vehicle for international borrowing/lending, while two final goods are not traded.[26]

Under financial integration, there exists a threshold value $\bar{w}_F$ such that, given $r_t^i = r^*$, for $w_t^i \in (0, \bar{w}_F)$, the borrowing constraints are binding, $\mu_{t+1}^i < 1$, and the aggregate dynamics of the country are characterized by $\{w_t^i, \psi_t^i, \underline{\epsilon}_t^i, \mu_{t+1}^i, \Gamma_t^i, \phi_t^i, \Upsilon_t^i, \chi_{t+1}^i\}$ satisfying

[26] In our model, there are three goods, i.e., a composite good and two final goods. Our results in this subsection hold if and only if one of them is freely traded. It does not have to be the composite good.

equations (13), (21), (44)-(46),[27]

$$\phi_t^i = 1 - \frac{[1-(\underline{\epsilon}_t^i)^{-(1+\theta)}][\frac{1}{\eta}-(1-\mu_{t+1}^i)]}{\mu_{t+1}^i\psi_t^i}, \tag{44}$$

$$w_{t+1}^i = \left[\frac{R}{\rho}\Gamma_t^i w_t^i(1-\phi_t^i)\right]^{\alpha}, \text{ where } \Gamma_t^i \equiv \frac{(\mu_{t+1}^i)^{\eta}}{1-\eta(1-\mu_{t+1}^i)} < 1, \text{ and } \frac{\partial\Gamma_t^i}{\partial\mu_{t+1}^i} > 0, \tag{45}$$

$$\Upsilon_t^i = \rho\frac{w_{t+1}^i}{w_t^i(1-\phi_t^i)}, \quad r_t^i = r^* = \Upsilon_t^i(1-\eta+\eta\mu_{t+1}^i) < \Upsilon_t^i, \quad \chi_{t+1}^i = (\mu_{t+1}^i)^{\alpha}. \tag{46}$$

For $w_t^i > \bar{w}_F$, the cross-sector investment is efficient $\mu_{t+1}^i = 1$ and the borrowing constraints are slack. A rise (fall) in aggregate income affects domestic saving, leading to financial capital outflows (inflows). Thus, the law of motion for wage is flat at $w_{t+1}^i = \left(\frac{R}{r^*}\right)^{\rho}$. One can solve for $\bar{w}_F$ by putting $\mu_{t+1}^i = 1$ in equations (13), (21), (44)-(46).

Consider the case of $\sigma = 0$. According to lemma 2, the extensive margin is mute. If the country has the period-0 income $Y_0^i > Y_A$, the autarkic interest rate would be lower than the world level, $r_0^i < r^* = r_A$, due to the neoclassical effect. Upon financial integration, households lend abroad for a higher interest rate and financial capital outflows reduces the total funds available for domestic investment. Meanwhile, the rise in the interest rate reduces the entrepreneurs' borrowing capacity so that the investment in sector A declines. Due to the decline in the domestic investment and the worsening of the cross-sector composition, aggregate output in period $t = 1$ is lower than under autarky. The law of motion for wage is globally concave and flatter around the autarkic steady state.

Proposition 4. *Under financial integration, if $\sigma = 0$ and $r^* = r_A$, the autarkic steady state is still the unique, stable steady state but the convergence to the steady state is faster than under autarky; if $\sigma > 0$ and $r^* = r_A$, the autarkic steady state may become unstable so that multiple steady states may arise.*

Consider the case of $\sigma > 0$. Figure 11 shows the parameter configuration for multiple steady states under financial integration in the (λ, ψ_A) space, given $\sigma = 1$ and $\sigma = 2$, respectively. The blue dashed curve shows the threshold value $\hat{\psi}_A$ defined for lemma 2 in subsection 3.1. The solid and the dash-dotted curves in figure 12 show the laws of motion for wage under financial integration versus under autarky, with the parameter configuration in the five regions of figure 11, respectively.

Consider the parameter configuration in region B. As shown in the upper-left panel of figure 12, if the country's initial income is higher (lower) than in the autarkic steady state $w_0^i > w_S$ ($w_0^i < w_S$), financial integration makes it converge to a new stable steady state H (L) with $w_H^i > w_S$ ($w_L^i < w_S$). Thus, financial integration destabilizes the autarkic steady state and creates multiple steady states. The intuition is as follows.

According to lemma 2, if $\sigma > 0$ and the cross-sector investment is inefficient under autarky, a rise in aggregate income may raise or reduce the interest rate, depending on the relative magnitude of the extensive-margin effect and the neoclassical effect. For the parameter configuration $\{\lambda, \psi_A\}$ in region BI of figure 9, an increase in Y_t^i raises the

[27] See the proof of Proposition 4 in appendix B for the derivation.

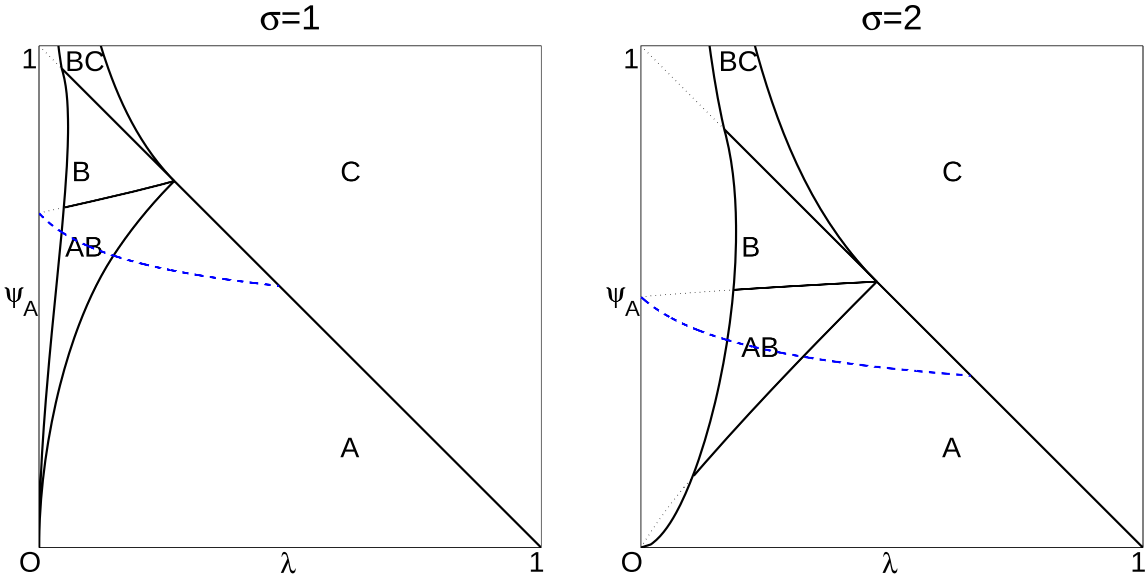

Figure 11: Parameter configuration for Multiple Steady States under Financial Integration

autarkic interest rate and, according to equation (39), the interest rate effect dampens the rises in the aggregate credit demand and the investment in sector A.

Consider first the case of $Y_0^i > Y_A$. Had the country stayed under autarky, the interest rate would be higher than the world level $r_0^i > r^* = r_A$. Upon financial integration, financial capital flows into this country, which affects domestic investment in two ways. First, capital inflows directly raise the size of the total funds available for domestic investment so that the sectoral investment rises on the intensive margin; second, capital inflows push the interest rate down to the world level and the entrepreneurs can borrow and invest more, which improves the cross-sector investment composition on the extensive margin. By the same logic, if $Y_0^i < Y_A$, the country witnesses financial capital outflows, which directly reduces the size of domestic investment and indirectly worsens the cross-sector investment composition and the aggregate allocation efficiency.

For the parameter configuration in region B of figure 11, the low λ implies the severe cross-sector investment distortion and the strong cross-sector composition effect under autarky. Under financial integration, the direct size effect and the indirect composition effect are so large that the slope of the law of motion for wage around the autarkic steady state exceeds unity, as shown in the upper-left panel of figure 12.

To sum up, financial integration affects directly the size and indirectly the composition of domestic investment. In particular, by keeping the intertemporal relative price (the interest rate) constant, financial integration eliminates the dampening effect (i.e., the positive interest rate response to the aggregate income change under autarky) on the sector-A investment, which amplifies the cross-sector composition effect. The size effect and the composition effect jointly destabilize the autarkic steady state. The positive interest rate response to the aggregate income change results from the extensive-margin effect and so does the cross-sector composition effect. Thus, the existence of multiple steady states depends on the magnitude of the extensive-margin effect.

Starting from region B of figure 11, let us reduce $\mathfrak{m}$ so that ψ_A rises and the parameter

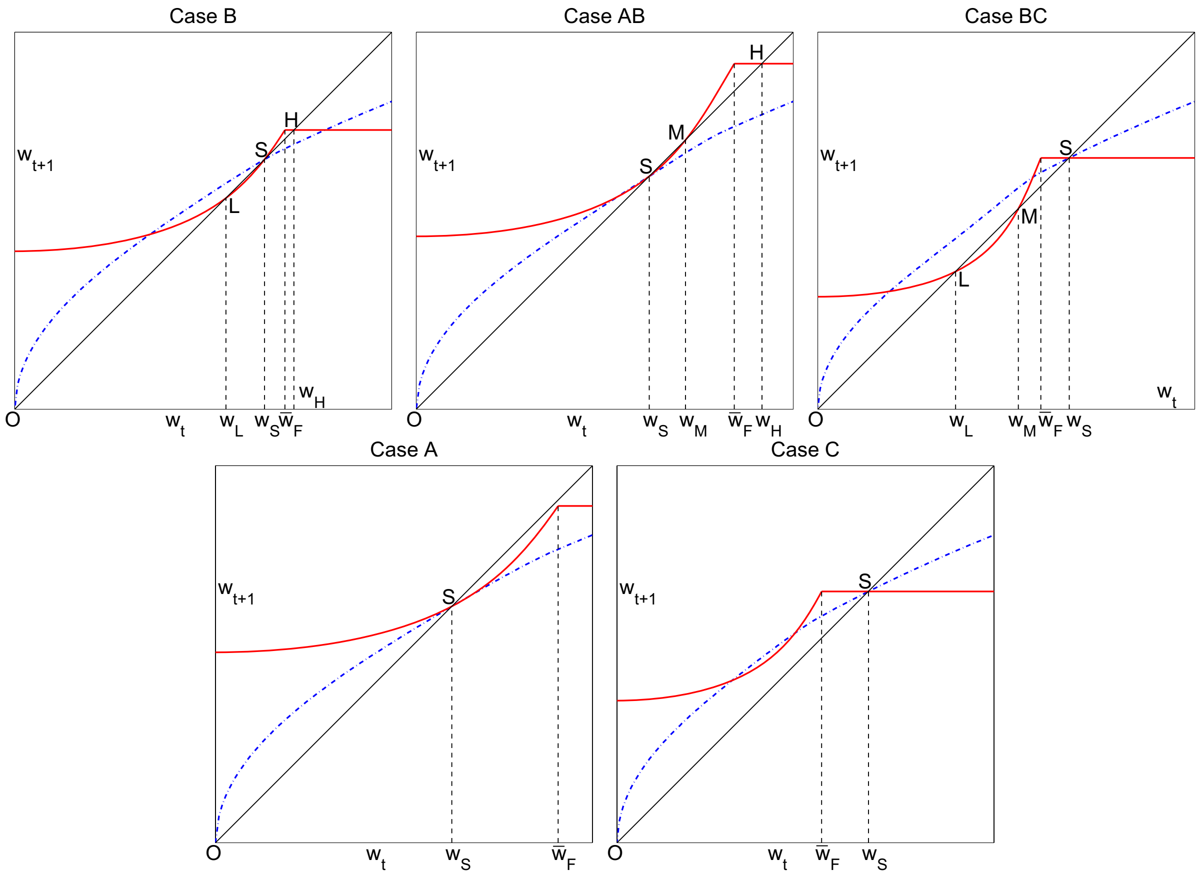

Figure 12: Phase Diagrams of Wage under Financial Integrations

configuration moves upwards into region BC where the borrowing constraints are slack and the cross-sector investment is efficient in the autarkic steady state $\mu_A = 1$. The autarkic interest rate, which coincides with the social rate of return, declines in aggregate income, due to the neoclassical effect. A marginal increase in aggregate income above the autarkic steady state tends to reduce the autarkic interest rate. Given $r^* = r_A$, financial integration leads to financial capital outflow so that domestic investment and output decline in period $t+1$. Thus, the law of motion for wage is flat at the autarkic steady state with $w^i_{t+1} = \left(\frac{R}{r^*}\right)^\rho = \left(\frac{R}{\rho}\right)^\rho = w_S$ and hence, the autarkic steady state is locally stable. However, for $w^i_t \ll w_S$, ψ^i_t enters into region BI of figure 9 where the interest rate responds positively to income change and financial integration affects the size and the composition of domestic investment in the same way as in case B. As shown in the upper-right panel of figure 12, besides the stable autarkic steady state S, there are another stable steady state L and an unstable steady state M.

Starting from region B of figure 11, let us raise $\mathfrak{m}$ so that ψ_A declines and the parameter configuration moves downwards into region AB where the borrowing constraints are binding in the autarkic steady state. In region AB, the interest rate response to income change is either negative or slightly positive around the autarkic steady state so that financial integration does not destabilize the autarkic steady state. However, for $w^i_t \gg w_S$, ψ^i_t enters into region BI in figure 9 where the interest rate response to income change is strongly positive so that financial integration affects the size and the cross-sector composition of domestic investment in the same way as in case B. As shown in the upper-middle panel of figure 12, besides the stable autarkic steady state S, there are another stable steady state H and an unstable steady state M with $w_H > w_M > w_S$.

In region AB-B-BC, financial integration generates multiple steady states and hence, the initial income matters for the convergence path and the long-run allocation.

The higher the λ, the less the sectoral investment distortion, the smaller the efficiency loss, the weaker the extensive-margin effect and the cross-sector composition effect. Thus, for the parameter configurations in region A and C of figure 11, financial integration does not generate multiple steady states but it affects the convergence path. See the lower-left and lower-right panels of figure 12.

Relationship to Matsuyama (2004)

Matsuyama (2004) shows in a one-sector OLG model that financial integration may lead to income divergence. He assumes that all agents have the identical labor endowment and the individual investment project is indivisible with a fixed size at unity. Aggregate investment adjusts only on the extensive margin and agents who can borrow and invest are randomly determined by lottery.

In our model, if $\theta \to \infty$, the distribution of labor endowment degenerates into a unit mass at $l_j = 1$ so that all agents have the identical labor endowment; if $\mathfrak{m} = 1$ and $\sigma = 1$, the MIR is constant at one; if $\eta = 1$, only sector A is active. Putting them together, our model degenerates into the model of Matsuyama (2004). In particular, figure 11 essentially coincides with figure 5 in Matsuyama (2004).[28]

In our model, we set $\theta < \infty$ and assume the MIR so that the adjustment of aggregate investment takes place on the intensive and the extensive margins; we set $\sigma > 0$ so that the MIR becomes aggregate-income dependent and one can control the magnitude of the extensive-margin effect by changing σ; we set $\eta \in (0, 1)$ so that one can analyze the impacts of trade integration.

In Matsuyama (2004), if the interest rate responds positively to income change around the autarkic steady state, financial integration destabilizes the autarkic steady state purely through the aggregate investment size effect. In our model, besides the direct size effect, financial capital flows also indirectly affect the cross-sector composition and the aggregate allocation efficiency, which is another amplification mechanism.

A.3 Income Divergence in A World Economy

As shown in subsection 2.2, given the parameter configurations in region SU and BU of figure 2, an individual country converges monotonically to a unique, stable steady state with aggregate income at Y_A under autarky. As a collection of autarkic countries, the world economy has a unique, stable steady state under autarky which is symmetric, i.e., all countries end up with the same income level Y_A in the long run.

In the case of financial integration, the interest rate is determined globally at r_t^* and the credit market clears at the world level. Although the symmetric steady state

[28]See Zhang (2013) for the detailed analysis of the one-sector version of our model.

mentioned above is still a steady state for the world economy, it may not be stable and there may exist stable, asymmetric steady states, i.e., the world economy is polarized into two groups of countries with the different steady-state income.

By the same logic as for the case of trade integration, given the parameter configuration in region B of figure 11, the world economy has a continuum of stable asymmetric steady states under financial integration where a fraction $\delta \in (\delta^-, \delta^+)$ of countries have the income $Y_L < Y_A$ and the rest have the income $Y_H > Y_A$. The proof follows that of Proposition 4 of Matsuyama (2004).

If the asymmetric steady state is stable under financial integration, the world economy is inevitably polarized into the rich and the poor. This way, financial integration may lead to income divergence rather than convergence among nations. It offers a theoretical support for the view that international capital flow is a mechanism through which rich countries become richer at the expense of poor countries.

B Proofs

Proof of Lemma 1

Proof. The proof consists of two steps. First, we prove that, given the aggregate income Y_t^i, or equivalently the wage w_t^i, if the borrowing constraints are binding, $q_{t+1}^{i,A} R > r_t^i$, or equivalently, $\mu_{t+1}^i < 1$, one can solve $\underline{\epsilon}_t^i$, ψ_t^i, and μ_{t+1}^i by using equations (13), (21)-(22). Second, we derive the condition under which the borrowing constraints are binding.

Combine equations (6) and (10) and use the definition of μ_{t+1}^i to get (21). Combine equations (5), (12), (17) to get (22) as follows,

$$\frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i} = \frac{M_t^{i,A}}{w_t^i} = \int_1^{\underline{\epsilon}_t^i} \frac{n_{j,t}^i}{\psi_t^i} dG(\epsilon_j) = \frac{1-(\underline{\epsilon}_t^i)^{-(1+\theta)}}{\psi_t^i}.$$

With the aggregate labor supply constant at $L_t = 1$, equations (1)-(2) imply that the wage is proportional to aggregate income, $w_t^i = (1-\alpha)Y_t^i$. Combine equations (13), (21)-(22) to solve for μ_{t+1}^i and $\underline{\epsilon}_t^i$ as the functions of Y_t^i,

$$\begin{aligned}\sigma \ln Y_t^i =& \ln(1-\frac{\lambda}{\mu_{t+1}^i}) + \frac{1}{1+\theta}\ln(1-\eta+\eta\mu_{t+1}^i) - \frac{1}{1+\theta}\ln(1-\eta+\eta\lambda) + \ln\mathfrak{m} \\ &+ \ln\frac{\theta}{(\theta+1)} - \ln(1-\alpha),\end{aligned} \tag{47}$$

$$\frac{\partial \ln \mu_{t+1}^i}{\partial \ln Y_t^i} = \frac{\sigma}{\frac{\lambda}{\mu_{t+1}^i-\lambda} + \frac{1}{1+\theta}\frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i}}, \Rightarrow \quad \text{sgn}\left(\frac{\partial \mu_{t+1}^i}{\partial Y_t^i}\right) = \text{sgn}(\sigma), \tag{48}$$

$$\frac{\partial \ln \mu_{t+1}^i}{\partial \ln \lambda} = \frac{\frac{\eta}{1-\eta+\eta\lambda} + \frac{1+\theta}{\mu_{t+1}^i-\lambda}}{\frac{1}{\lambda}\frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i} + \frac{1+\theta}{\mu_{t+1}^i-\lambda}} > 0, \quad \frac{\partial \ln \mu_{t+1}^i}{\partial \ln \mathfrak{m}} = \frac{-1}{\frac{\lambda}{\mu_{t+1}^i-\lambda} + \frac{1}{1+\theta}\frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i}} < 0, \tag{49}$$

$$\underline{\epsilon}_t^i = \left(\frac{1-\eta+\eta\mu_{t+1}^i}{1-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}}, \quad \frac{\partial \ln \underline{\epsilon}_t^i}{\partial \ln \mu_{t+1}^i} = \frac{1}{1+\theta}\frac{\eta\mu_{t+1}^i}{1-\eta+\eta\mu_{t+1}^i} > 0. \tag{50}$$

Consider the boundary case where the borrowing constraints are weakly binding and the cross-sector investment is efficient $\mu^i_{t+1} = 1$. Rewrite equation (47) as

$$\mathfrak{m} = (Y^i_t)^\sigma \Lambda, \quad \text{where } \Lambda \equiv \frac{(1-\eta+\eta\lambda)^{\frac{1}{1+\theta}}}{1-\lambda}(1-\alpha)(1+\frac{1}{\theta}) \text{ and } \frac{\partial \Lambda}{\partial \lambda} > 0. \tag{51}$$

Given Y^i_t, equations (49)-(50) show that μ^i_{t+1} rises (declines) in λ ($\mathfrak{m}$) and so does the cutoff value $\underline{\epsilon}^i_t$. Thus, for $\mathfrak{m} > (Y^i_t)^\sigma \Lambda$, $\mu^i_{t+1} < 1$ so that the borrowing constraints are binding and the cross-sector investment is inefficient; otherwise, for $\mathfrak{m} < (Y^i_t)^\sigma \Lambda$, the borrowing constraints are slack and the cross-sector investment is efficient $\mu^i_{t+1} = 1$. □

Proof of Proposition 1

Proof. Combine equations (1)-(5) to get the law of motion for wage (23) under autarky.

Consider the case of $\sigma = 0$. According to lemma 1, for $\mathfrak{m} > \Lambda$, the borrowing constraints are binding and equation (47) implies that the sectoral rate-of-return ratio is constant and independent of aggregate income $\mu^i_{t+1} = \mu_A < 1$; for $\mathfrak{m} \leq \Lambda$, the borrowing constraints are slack and $\mu_{t+1} = \mu_A = 1$. Combine $\mu^i_{t+1} = \mu_A$ with equation (23) to get the law of motion for wage (25), which is strictly concave and crosses the 45° line once and only once from the left. Thus, there exists a unique, stable steady state.

Consider the case of $\sigma > 0$. According to lemma 1, for $w^i_t \geq \bar{w}_A \equiv (1-\alpha)\bar{Y}_A$, the cross-sector investment is efficient $\mu^i_{t+1} = 1$ and the law of motion for wage is strictly concave $w^i_{t+1} = \left(\frac{R}{\rho} w^i_t\right)^\alpha$; for $w^i_t \in (0, \bar{w}_A)$, the cross-sector investment is inefficient, $\mu^i_{t+1} \in (\lambda, 1)$, and the law of motion for wage is determined jointly by equations (52)-(53),

$$(w^i_t)^\sigma = \left(\frac{1-\eta+\eta\mu^i_{t+1}}{1-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}} (1 - \frac{\lambda}{\mu^i_{t+1}})\mathbb{F}, \tag{52}$$

$$w^i_{t+1} = \left(\frac{R}{\rho} w^i_t \Gamma^i_t\right)^\alpha, \quad \text{where } \Gamma^i_t \equiv \frac{(\mu^i_{t+1})^\eta}{1-\eta+\eta\mu^i_{t+1}} \tag{53}$$

Evaluate the first derivative of the law of motion for wage at any steady state if exists,

$$\frac{\partial w^i_{t+1}}{\partial w^i_t}\|_{w^i_{t+1}=w^i_t} = \underbrace{\alpha}_{\text{neoclassical effect}} \left[1 + \underbrace{\frac{\sigma(1-\mu^i_{t+1})\frac{\eta(1-\eta)}{1-\eta+\eta\mu^i_{t+1}}}{\frac{\lambda}{\mu^i_{t+1}-\lambda} + \frac{1}{1+\theta}\frac{\eta\mu^i_{t+1}}{1-\eta+\eta\mu^i_{t+1}}}}_{\text{cross-sector composition effect}}\right]. \tag{54}$$

The necessary and sufficient condition for a steady state to be stable is $\frac{\partial w^i_{t+1}}{\partial w^i_t}\|_{w^i_{t+1}=w^i_t} < 1$. In the case of $\sigma > 0$, if the cross-sector composition effect is so strong that $\frac{\partial w^i_{t+1}}{\partial w^i_t}\|_{w^i_{t+1}=w^i_t} > 1$, the steady state is unstable and there may exist multiple steady states, as discussed in subsection 2.2. The border between regions BU and BM as well as between regions SU and SM in figure 2 show the parameter configurations in the $\{\lambda, \psi_A\}$ space with which the law of motion for wage is tangent with the 45° line at $w^i_t \in (0, \bar{w}_A)$. The parameter configurations for the boundary are calculated in three steps:

- set $\frac{\partial w_{t+1}^i}{\partial w_t^i}\|_{w_{t+1}^i = w_t^i} = 1$ to solve μ_{t+1}^i as a function of λ;

- plug μ_{t+1}^i into equations (52)-(53) and compute w_{t+1}^i and w_t^i, respectively;

- equalize w_{t+1}^i with w_t^i to solve ψ_A as a function of λ.

□

Proof of Proposition 2

Proof. The proof consists of three steps.

Step 1: derive the model solutions (31)-(33) **under trade integration**

Given the relative final good price determined globally $\chi_t^i = \chi^*$ from period $t = 1$ on, the market clearing condition for final good f in country i is $Z_t^{i,f}(1 + \varsigma_t^{i,f}) = Y_t^{i,f}$. Combine it with equations (1)-(2) to get

$$\frac{q_{t+1}^{i,A} R M_t^{i,A}}{q_{t+1}^{i,B} R M_t^{i,B}} = \frac{M_t^{i,A}}{\mu_{t+1}^i M_t^{i,B}} = \frac{\eta}{1-\eta} \frac{(1 + \varsigma_{t+1}^{i,A})}{(1 + \varsigma_{t+1}^{i,B})}. \tag{55}$$

With no international borrowing and lending, domestic investment is financed by domestic saving, $M_t^{i,A} + M_t^{i,B} = w_t^i$. Combine it with equation (55) and (28) to get

$$M_t^{i,A} = \frac{\eta \mu_{t+1}^i (1 + \varsigma_{t+1}^{i,A})}{1 - \eta(1 - \mu_{t+1}^i)(1 + \varsigma_{t+1}^{i,A})} w_t^i \text{ and } M_t^{i,B} = \frac{(1-\eta)(1 + \varsigma_{t+1}^{i,B})}{1 - \eta(1 - \mu_{t+1}^i)(1 + \varsigma_{t+1}^{i,A})} w_t^i. \tag{56}$$

Combine equation (56) with (1)-(2), (28) to get (33) as the law of motion for wage.

According to equations (29)-(30), the relative final good price is determined at the world level $\chi_t^i = \chi^*$ and so are the sectoral rate-of-return ratio and the equity-investment ratio, $\mu_t^i = \mu^*$ and $\psi_t^i = \psi^*$. According to equation (13), the cutoff value is loglinear in the wage. For $w_t^i \le \underline{w}_T \equiv (\psi^* \mathbb{F})^{\frac{1}{\sigma}}$, nobody can meet the MIR, $\underline{\epsilon}_t^i = 1$, and the country specializes completely in sector B, $\varsigma_{t+1}^{i,A} = -1$.

For the parameter configuration in region BU of figure 2, the cross-sector investment is inefficient at the autarkic steady state $\mu_A < 1$. Given $\mu_{t+1}^i = \mu^* = \mu_A < 1$, the borrowing constraints are binding and the investment in sector A is

$$\int_1^{\underline{\epsilon}_t^i} \frac{n_{j,t}^i}{\psi_t^i} dF(\epsilon_j) = w_t^i \frac{1 - (\underline{\epsilon}_t^i)^{-(1+\theta)}}{1 - \frac{\lambda}{\mu_{t+1}^i}} = M_t^{i,A} = w_t^i \frac{\mu_{t+1}^i}{\frac{1}{\eta(1+\varsigma_{t+1}^{i,A})} - (1 - \mu_{t+1}^i)} \tag{57}$$

$$\Rightarrow \quad \varsigma_{t+1}^{i,A} = [\eta(1 - \mu^* + \frac{\mu^* - \lambda}{1 - (\underline{\epsilon}_t^i)^{-(1+\theta)}})]^{-1} - 1. \tag{58}$$

Given $\psi_t^i = \psi^*$ under trade integration, the higher aggregate income allows more agents to invest in sector A, $\frac{\partial \underline{\epsilon}_t^i}{\partial w_t^i} > 0$, which reduces the imports or raises the exports of good A. There is a threshold value $\bar{w}_T$ such that for $w_t^i = \bar{w}_T$, the mass of entrepreneurs is so high that they borrow the entire saving of households and invest in sector A $M_t^{i,A} = w_t^i$. Combine it with (57) to get $\varsigma_{t+1}^{i,A} = \frac{1-\eta}{\eta}$, implying that the country specializes completely in sector A. Combine it with equations (58) and (31) to get $\underline{\epsilon}_t^i = \left(\frac{\mu^*}{\lambda}\right)^{\frac{1}{1+\theta}}$ and $\bar{w}_T = \left(\frac{\mu^*}{\lambda}\right)^{\frac{1}{\sigma(1+\theta)}} \underline{w}_T$. Thus, equations (31)-(32) characterize the solutions to $\underline{\epsilon}_t^i$ and $\varsigma_{t+1}^{i,A}$.

Step 2: the shape of the law of motion for wage under trade integration

For simplicity, we suppress the country index i. Under trade integration, the law of motion for wage is a piecewise function. Given the world relative final good price χ^* and the related μ^*, combine equations (31)-(33) to get the law of motion for wage in log,

$$\ln w_{t+1} = \alpha(\ln w_t + \ln \Gamma_t + \ln \frac{R}{\rho}), \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \underbrace{\alpha}_{\text{neoclassical effect}} \left(1 + \underbrace{\frac{\partial \ln \Gamma_t}{\partial \ln \underline{\epsilon}_t} \frac{\partial \ln \underline{\epsilon}_t}{\partial \ln w_t}}_{\text{specialization effect}}\right);$$

$$\ln \Gamma_t = \begin{cases} \eta \ln \mu^*, & \text{if } w_t \in (0, \underline{w}_T]; \\ \eta \ln \mu^* + \ln \left[\frac{1-\lambda}{\mu^*-\lambda} - \frac{1-\mu^*}{\mu^*-\lambda} \underline{\epsilon}_t^{-(1+\theta)}\right], \text{ where } \underline{\epsilon}_t = \frac{w_t^\sigma}{\psi^* \mathbb{F}}, & \text{if } w_t \in (\underline{w}_T, \bar{w}_T); \\ (\eta - 1) \ln \mu^*, & \text{if } w_t \geq \bar{w}_T. \end{cases}$$

$$\frac{\partial \ln \Gamma_t}{\partial \ln \underline{\epsilon}_t} = \begin{cases} 0, & \text{if } w_t \in (0, \underline{w}_T]; \\ \frac{(1+\theta)\frac{1-\mu^*}{\mu^*-\lambda}\underline{\epsilon}_t^{-(1+\theta)}}{\frac{1-\lambda}{\mu^*-\lambda} - \frac{1-\mu^*}{\mu^*-\lambda}\underline{\epsilon}_t^{-(1+\theta)}} > 0 \text{ and } \frac{\partial \ln \underline{\epsilon}_t}{\partial \ln w_t} = \sigma, & \text{if } w_t \in (\underline{w}_T, \bar{w}_T); \\ 0, & \text{if } w_t \geq \bar{w}_T. \end{cases}$$

For $w_t \in (0, \underline{w}_T]$, the country specializes completely in sector B; for $w_t > \bar{w}_T$, it specializes completely in sector A. In either case, the change in aggregate income does not affect aggregate allocation efficiency $\frac{\partial \ln \Gamma_t}{\partial \ln w_t} = 0$ so that, due to the neoclassical effect, the law of motion for wage is increasing and concave, or equivalently, log-linear with the slope $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha < 1$.

For $w_t \in (\underline{w}_T, \bar{w}_T)$, the country produces both final goods, $\underline{\epsilon}_t > 1$.

$$\mathbb{J} \equiv \frac{\partial w_{t+1}}{\partial w_t} = \alpha \left[1 + \sigma \frac{(1+\theta)\mathbb{P}\mathbb{X}}{1 + \mathbb{P}(1-\mathbb{X})}\right] \frac{w_{t+1}}{w_t}, \text{ where } \mathbb{X} \equiv \underline{\epsilon}_t^{-(1+\theta)} \in (0,1), \ \mathbb{P} \equiv \frac{1-\mu^*}{\mu^*-\lambda} \geq 0,$$

$$\frac{\partial w_{t+1}}{\partial (w_t)^2} = -\left\{\frac{\sigma(1+\theta)\mathbb{P}\mathbb{X}[\sigma(1+\theta)-1]}{[1+\mathbb{P}-\mathbb{P}\mathbb{X}+\mathbb{P}\mathbb{X}\sigma(1+\theta)]} + (1-\alpha)\frac{1+\mathbb{P}+\mathbb{P}\mathbb{X}[\sigma(1+\theta)-1]}{1+\mathbb{P}-\mathbb{P}\mathbb{X}}\right\} \frac{\mathbb{J}}{w_t}$$

- In the case of $\sigma = 0$, the change in aggregate income does not affect the cross-sector investment composition $\frac{\partial \ln \underline{\epsilon}_t}{\partial \ln w_t} = 0$; in the case of $\sigma > 0$ and $\mu^* = 1$, the rate of return equalizes in the two sectors so that the change in aggregate income does not affect aggregate allocation efficiency $\Gamma_t = 1$. In either case, the law of motion for wage is increasing and concave, or equivalently, log-linear with the slope $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha < 1$, due to the neoclassical effect.

- In the case of $\sigma > 0$ and $\mu^* < 1$, sector A has a higher return than sector B so that agents who can meet the MIR invest their entire labor income in sector A and borrow to the limit. For $w_t > w_A$, a rise in w_t allows more agents to meet the MIR and invest in sector A, $\frac{\partial \ln \underline{\epsilon}_t}{\partial \ln w_t} > 0$; the country specializes towards the higher return sector (A), which improves the allocation efficiency of domestic saving $\frac{\partial \ln \Gamma_t}{\partial \ln \underline{\epsilon}_t} > 0$ and w_{t+1}. Thus, the trade-driven specialization amplifies the income change through the extensive-margin channel, making the law of motion for wage steeper $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} > \alpha$ around the autarkic steady state. In the case of $\sigma > 0$ and $\mu^* < 1$, the law of motion for wage is concave.

Overall, given $\sigma > 0$ and $\mu^* < 1$, the law of motion for wage is a piecewise function over three intervals and there are two kinks at $w_t = \underline{w}_T$ and $w_t = \bar{w}_T$. Within each interval, it is increasing and concave.

Step 3: the threshold values for multiple steady states under trade integration

For the parameter configuration in region SU, $\psi_A \geq 1 - \lambda$ and $\chi_A = \mu_A = 1$. Given $\chi^* = \chi_A$ and hence $\mu^* = \mu_A = 1$, the law of motion for wage is log-linear with the slope $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha < 1$ under trade integration and hence, there exists a unique steady state.

Given $\chi^* = \chi_A$ and accordingly, $\mu^* = \mu_A$, figure 4 shows that three threshold values split region BU of figure 2 into four regions. Figure 5 shows the law of motion for wage in the five cases, respectively. In the following, we derive the three threshold values.

For the parameter configuration in region BU, $\psi_A < 1-\lambda$ and $\mu_A < 1$ so that $\chi_A < 1$. If the specialization effect is sufficiently strong, the slope of the law of motion for wage at the autarkic steady state is larger than unity so that multiple steady states arise. Use equations (21)-(22) to solve μ_A and $\underline{\epsilon}_A$ as the implicit functions of λ and ψ_A. Combine them with $\mu^* = \mu_A$ and $\frac{\partial w_{t+1}}{\partial w_t} \|_{w_{t+1}=w_t} = 1$ to get,

$$\hat{\psi}_T = (1-\lambda)\left[1 - \frac{1}{\sigma\rho(1+\theta)(\frac{1-\eta}{\lambda} + \eta) + 1}\right],$$

which defines the border between region B1 and AB of figure 4.

Given λ, for $\psi_A < \hat{\psi}_T$, the country with the initial income $Y_0 < Y_A$ specializes completely in sector B under trade integration. If the kink point of the law of motion for wage at $w_t = \bar{w}_T$ is below (above) the 45° line, the country with $Y_0 > Y_A$ specialize **partially** (**completely**) in sector A. Use equations (13), (21)-(23) to solve μ_A, $\underline{\epsilon}_A$, $\mathbb{F}$, and w_A as the functions of λ and ψ_A. Combine them with $\mu^* = \mu_A$ and $\left[\frac{R}{\rho}(\mu^*)^{\eta-1}\right]^{\rho} = \bar{w}_T$ to get,

$$\tilde{\psi}_T = 1 - \frac{\lambda}{1-\eta}\left[\left(\frac{1-\eta}{\lambda} + \eta\right)^{\frac{1}{\sigma\rho(1+\theta)+1}} - \eta\right],$$

which defines the border between region B2 and B1 of figure 4.

Given λ, for $\psi_A \in (\hat{\psi}, 1-\lambda)$, the law of motion for wage under trade integration has a slope less than unity at the autarkic steady state. Thus, the autarkic steady state is still stable, but it may not be unique. There exist other steady states under trade integration if the kink point of the law of motion for wage at $w_t = \underline{w}_T$ is below the 45° line, i.e., $\left[\frac{R}{\rho}(\mu^*)^{\eta}\right]^{\rho} \leq \underline{w}_T$. Use equations (13), (21)-(23) to solve μ_A, $\underline{\epsilon}_A$, $\mathbb{F}$, and w_A as the functions of λ and ψ_A. Combine them with $\mu^* = \mu_A$ and $\left[\frac{R}{\rho}(\mu^*)^{\eta}\right]^{\rho} = \bar{w}_T$ to get,

$$\bar{\psi}_T = 1 - \frac{\eta\lambda}{[1-\eta+\eta\lambda]^{\frac{1}{\sigma\rho(1+\theta)+1}} - (1-\eta)},$$

which defines the border between region AB and A of figure 4. □

Proof of Proposition 3

Proof. We focus on the case of $\sigma > 0$ and $\mu^* < 1$.[29] For simplicity, we suppress the country index i. According to figure 5, multiple steady states arise iff

1. the kink point at $w_t = \underline{w}_T$ is below the 45° line and
2. the law of motion for wage intersects at least once with the 45° line for $w_t \in (\underline{w}_T, \bar{w}_T)$.

Let $w_M = \left[\frac{R}{\rho}(\mu^*)^\eta \frac{1-\lambda}{\mu^*-\lambda}\left(1 - \frac{1-\mu^*}{1-\lambda}\underline{\epsilon}_M^{-(1+\theta)}\right)\right]^\rho$ denote the unstable steady state in the interval of $w_t \in (\underline{w}_T, \bar{w}_T)$ and $w_L \equiv \left[\frac{R}{\rho}(\mu^*)^\eta\right]^\rho$ denote the stable steady state in $w_t \in (0, \underline{w}_T)$, where $\underline{\epsilon}_M$ is the cutoff value related to w_M. The two conditions are formulated technically as

$$w_L < \underline{w}_T, \ \text{ and } \ \frac{\partial w_{t+1}}{\partial w_t}\|_{w_M} = \alpha + \alpha\sigma\frac{(1+\theta)\frac{1-\mu^*}{1-\lambda}\underline{\epsilon}_M^{-(1+\theta)}}{1 - \frac{1-\mu^*}{1-\lambda}\underline{\epsilon}_M^{-(1+\theta)}} \geq 1 \tag{59}$$

Let $\mathbb{x} \equiv \frac{\lambda}{\mu^*} \in (\lambda, 1)$, $\mathbb{A} \equiv \rho\sigma(1+\theta)$, and $\mathbb{B} \equiv \rho\sigma\eta$. Conditions (59) are simplified as

$$\mathbb{x} \geq \mathbb{x}^c \equiv \frac{(\mathbb{A}+1)\lambda}{\mathbb{A}+\lambda} > \lambda, \ \text{ and } \ \mathbb{L} \equiv \frac{1}{\mathbb{F}}\left(\frac{R}{\rho}\lambda^\eta\right)^{\rho\sigma} \leq \mathbb{R} \equiv \mathbb{x}^{\mathbb{B}}(1-\mathbb{x}) \tag{60}$$

$\mathbb{R}$ is a hump-shaped function of $\mathbb{x}$ with the unconstrained maximum value $\mathbb{R}^o \equiv \frac{\mathbb{B}^{\mathbb{B}}}{(\mathbb{B}+1)^{\mathbb{B}+1}}$ at $\mathbb{x}^o = \frac{\mathbb{B}}{\mathbb{B}+1} \in (0,1)$ and with the minimum value $\underline{\mathbb{R}} = 0$ at $\mathbb{x} = 0$ and $\mathbb{x} = 1$. Let $\mathbb{R}^c \equiv (\mathbb{x}^c)^{\mathbb{B}}(1-\mathbb{x}^c)$. Given $\mathbb{x} \in (\mathbb{x}^c, 1)$, figure 13 shows the results in fives cases. The horizontal axis shows $\mathbb{x}$ and the vertical axis shows $\mathbb{R}$ and $\mathbb{L}$.

- If $\lambda \in (0, \frac{\mathbb{A}\mathbb{B}}{\mathbb{A}\mathbb{B}+\mathbb{A}+1})$, $\mathbb{x}^c \in (0, \mathbb{x}^o)$ and $\mathbb{R}$ has the unconstrained maximum $\mathbb{R}^o$ at $\mathbb{x}^o$.
  - If $\mathbb{L} > \mathbb{R}^o$, condition (60) does not hold so that there does not exist the stable asymmetric steady state. See the upper-left panel of figure 13.
  - If $\mathbb{L} \in (\mathbb{R}^c, \mathbb{R}^o)$, there are two threshold values, $\mathbb{x}^-$ and $\mathbb{x}^+$. For $\mu^* \in (\frac{\lambda}{\mathbb{x}^+}, \frac{\lambda}{\mathbb{x}^-})$, there exists a stable asymmetric steady state with $\chi^* = (\mu^*)^\alpha$ and μ^* is supported by a unique value of δ. See the upper-middle panel of figure 13.
  - If $\mathbb{L} \leq \mathbb{R}^c$ there is a threshold value $\mathbb{x}^+$ such that for $\mu^* \in (\frac{\lambda}{\mathbb{x}^+}, \frac{\lambda}{\mathbb{x}^c})$, there exists a stable asymmetric steady state with $\chi^* = (\mu^*)^\alpha$ and μ^* is supported by a unique value of δ. See the upper-right panel of figure 13.
- If $\lambda \in (\frac{\mathbb{A}\mathbb{B}}{\mathbb{A}\mathbb{B}+\mathbb{A}+1}, 1)$, $\mathbb{x}^c \in (\mathbb{x}^o, 1)$ so that $\mathbb{R}$ has the constrained maximum $\mathbb{R}^c$ at $\mathbb{x}^c$.
  - If $\mathbb{L} > \mathbb{R}^c$, condition (60) does not hold so that there does not exist the stable asymmetric steady state. See the lower-left panel of figure 13.

[29] As shown in step 2 of the proof for proposition 2, given the world relative final good price χ^* and the corresponding μ^*, the law of motion for wage in a small open economy under trade integration is a piecewise function over three intervals. If either $\sigma = 0$ or $\mu^* = 1$, the law of motion for wage is globally concave and differentiable, $w_{t+1} = \left(\frac{R}{\rho}w_t\Gamma^*\right)^\alpha$, so that there exists a unique steady state under trade integration. If $\sigma > 0$ and $\mu^* < 1$, the law of motion for wage has two kinks and is concave within each interval, which may give rise to multiple steady states under trade integration.

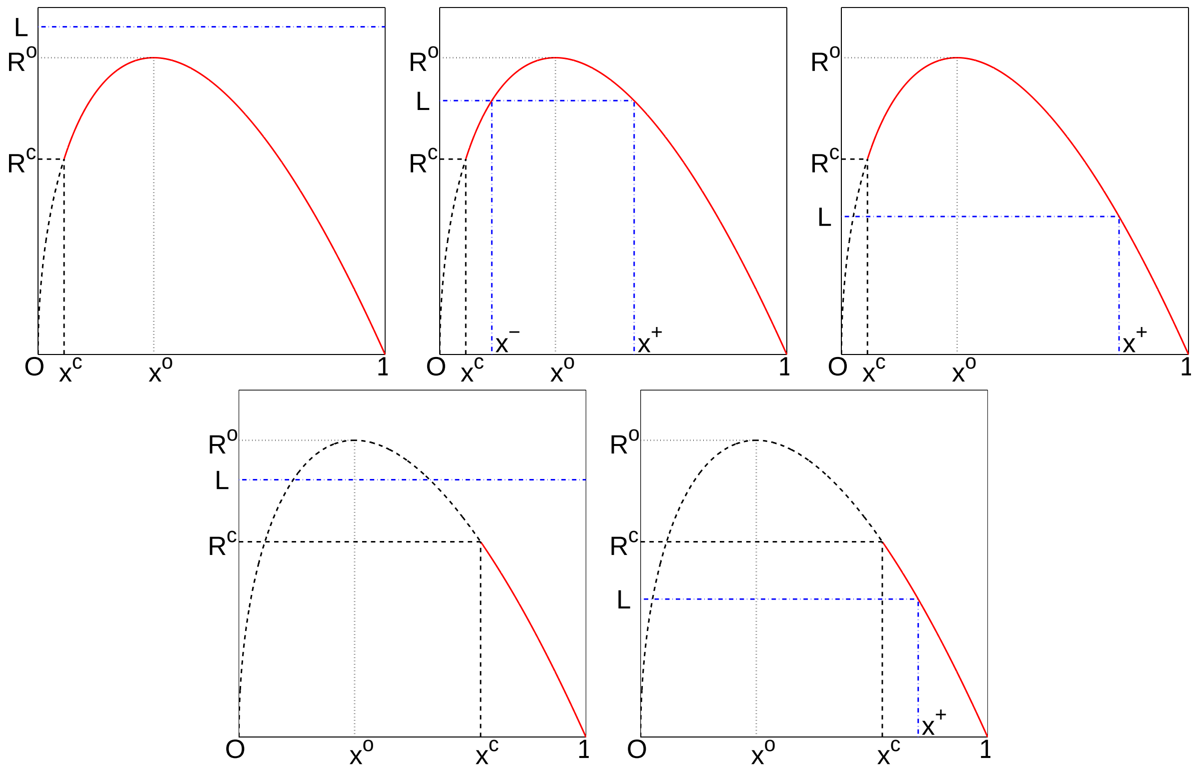

Figure 13: The Existence of Asymmetric Steady States

- If $\mathbb{L} \in (0, \mathbb{R}^c)$ there is a threshold value $\mathbb{x}^+$. For $\mu^* \in (\frac{\lambda}{\mathbb{x}^+}, \frac{\lambda}{\mathbb{x}^c})$, there exists a stable asymmetric steady state with $\chi^* = (\mu^*)^\alpha$ and μ^* is supported by a unique value of δ. See the lower-right panel of figure 13.

According to the five cases mentioned above, for $\lambda \in (0, \frac{\mathbb{A}\mathbb{B}}{\mathbb{A}\mathbb{B}+\mathbb{A}+1})$, the threshold value $\bar{\psi}_T^{SB}$ is the solution to $\mathbb{L} \equiv \frac{1}{\mathbb{F}}\left(\frac{R}{\rho}\lambda^\eta\right)^{\rho\sigma} = \mathbb{R}^o = (\mathbb{x}^o)^{\mathbb{B}}(1-\mathbb{x}^o)$; for $\lambda \in (\frac{\mathbb{A}\mathbb{B}}{\mathbb{A}\mathbb{B}+\mathbb{A}+1}, 1)$, the threshold value $\bar{\psi}_T^{SB}$ is the solution to $\mathbb{L} \equiv \frac{1}{\mathbb{F}}\left(\frac{R}{\rho}\lambda^\eta\right)^{\rho\sigma} = \mathbb{R}^c = (\mathbb{x}^c)^{\mathbb{B}}(1-\mathbb{x}^c)$. Figure 6 shows $\bar{\psi}_T^{SB}$ as the function of λ in the $\{\lambda, \psi_A\}$ space. $\square$

Proof of Lemma 2

Proof. As the wage rate is linear in aggregate income $w_t = (1-\alpha)Y_t$, we use the wage rate and aggregate income interchangeably as follows.

Combine equations (1)-(2) to get $q_{t+1}^B = w_{t+1}^{-\frac{1}{\rho}}\mu_{t+1}^\eta$. Combine it with equations (6), (20), and (23) to get (24) as the solution to the interest rate under autarky.

In the case of $\sigma = 0$, the extensive margin is mute so that the sectoral rate-of-return ratio is constant $\mu_{t+1} = \mu_A$. Thus, the interest rate is proportional to the social rate of return, which, due to the neoclassical effect, is a decreasing, log-linear function of the wage. Combine equations (20), (23), and (24) to get,

$$\ln \Upsilon_t = \ln w_{t+1} - \ln w_t + \ln \rho = (\alpha - 1)\ln w_t + \alpha \ln \frac{R}{\rho}\Gamma_A + \ln \rho, \tag{61}$$

$$\ln r_t = \ln \Upsilon_t + \ln(1-\eta+\eta\mu_A), \quad \frac{\partial \ln r_t}{\partial \ln w_t} = \frac{\partial \ln \Upsilon_t}{\partial \ln w_t} = \underbrace{\alpha}_{\text{neoclassical effect}} -1 < 0. \tag{62}$$

In the case of $\sigma > 0$ and $w_t \geq \bar{w}_A$, the cross-sector investment is efficient $\mu_{t+1} = 1$ and $r_t = \Upsilon_t$. Due to the neoclassical effect, the social rate of return declines in aggregate income and so does the interest rate. In both cases, the autarkic interest rate is lower in the rich than in the poor country.

If $\sigma > 0$ and $w_t < \bar{w}_A$, the cross-sector investment is inefficient $\mu_{t+1} \in (\lambda, 1)$ and the borrowing constraints are binding, $\psi_t = 1 - \frac{\lambda}{\mu_{t+1}} \in (0, 1-\lambda)$. In the following, we derive the condition under which the interest rate is a non-monotonic function of the wage for $w_t \in (0, \bar{w}_A)$. Since ψ_t increases in w_t under autarky, it is equivalent to derive the condition under which r_t is a non-monotonic function of ψ_t.

Combine equations (13), (20)-(24) to get,

$$\ln r_t = (\alpha - 1)\ln w_t + \alpha \ln \Gamma_t + \ln(1 - \eta + \eta\mu_{t+1}) + \alpha \ln \frac{R}{\rho} + \ln \rho \tag{63}$$

$$\frac{\partial \ln w_t}{\partial \ln \psi_t} = \frac{1}{\sigma}\frac{\partial \ln \epsilon_t}{\partial \ln \psi_t} + \frac{1}{\sigma}, \quad \frac{\partial \ln \epsilon_t}{\partial \ln \psi_t} = \frac{\frac{\lambda}{1+\theta}\frac{\psi_t}{1-\psi_t}}{\lambda + \frac{1-\eta}{\eta}(1-\psi_t)}, \quad \frac{\partial \ln \mu_{t+1}}{\partial \ln \psi_t} = \frac{\psi_t}{1-\psi_t} \tag{64}$$

$$\frac{\partial \ln r_t}{\partial \ln \psi_t} = \alpha\eta\frac{\psi_t}{1-\psi_t} + (1-\alpha)[(\theta + 1 - \frac{1}{\sigma})\frac{\frac{\lambda}{1+\theta}\frac{\psi_t}{1-\psi_t}}{\lambda + \frac{1-\eta}{\eta} - \frac{1-\eta}{\eta}\psi_t} - \frac{1}{\sigma}] = 0 \tag{65}$$

$$\Rightarrow \mathbb{A}\psi_t^2 - \mathbb{B}\psi_t + \mathbb{C} = 0, \tag{66}$$

$$\mathbb{A} \equiv \sigma\rho\eta + 1, \quad \mathbb{B} \equiv \sigma\rho\eta + 2 + \frac{\eta\lambda}{1-\eta}[\sigma(\rho\eta + 1) + \frac{\theta}{1+\theta}], \quad \mathbb{C} \equiv \frac{\lambda\eta}{1-\eta} + 1. \tag{67}$$

Given the model parameters, equation (66) is a quadratic function of $\psi_t \in (0,1)$. For $\psi_t = 0$, the left-hand-side of equation (66) is positive; for $\psi_t = 1$, the left-hand-side of equation (66) is negative. Thus, for $\psi_t \in (0,1)$, there exists a unique solution $\hat{\psi}_A = \frac{\mathbb{B} - \sqrt{\mathbb{B}^2 - 4\mathbb{A}\mathbb{C}}}{2\mathbb{A}}$ making $\frac{\partial \ln r_t}{\partial \ln \psi_t} = 0$. For $\psi_t \in (0, \hat{\psi}_A)$, $\frac{\partial \ln r_t}{\partial \ln \psi_t} < 0$; for $\psi_t \in (\hat{\psi}_A, 1-\lambda)$, $\frac{\partial \ln r_t}{\partial \ln \psi_t} > 0$. □

Proof of Proposition 4

Proof. The proof consists of three steps. For simplicity, we suppress the country index i.
Step 1: derive the model solutions (44)-(46) **under financial integration**

Given the interest rate determined globally $r_t = r^*$ from period $t = 0$ on, financial flows affects the total funds available for domestic investment, $M_t^A + M_t^B = (1-\phi_t)w_t$. Combine it with equation (1)-(5) to get the sectoral investment

$$M_t^A = \frac{\eta\mu_{t+1}}{1-\eta+\eta\mu_{t+1}}(1-\phi_t)w_t \text{ and } M_t^B = \frac{1-\eta}{1-\eta+\eta\mu_{t+1}}(1-\phi_t)w_t. \tag{68}$$

Thus, the law of motion for wage is characterized by equation (45). For $w_t \in (0, \bar{w}_F)$, the borrowing constraints are binding and the investment in sector A is

$$\frac{\eta\mu_{t+1}}{1-\eta+\eta\mu_{t+1}}(1-\phi_t)w_t = M_t^A = \int_1^{\epsilon_t} \frac{n_{j,t}}{\psi_t} dF(\epsilon_j) = w_t \frac{1 - \epsilon_t^{-(1+\theta)}}{\psi_t}, \tag{69}$$

which gives equation (44) as the solution to ϕ_t. Following the proof of lemma 2, one can get equations (46) as the solutions to the social rate of return and the interest rate.
Step 2: the shape of the law of motion for wage under financial integration

Under financial integration, the law of motion for wage is piecewise. Given the world interest rate r^*, for $w_t > \bar{w}_F$, the borrowing constraints are slack, $\mu_{t+1} = 1$, and the law of motion for wage is flat at $w_{t+1} = \bar{w}_{t+1} \equiv \left(\frac{R}{r^*}\right)^\rho$; for $w_t \in (0, \bar{w}_F)$, the borrowing constraints are binding, $\mu_{t+1} \in (\lambda, 1)$, and the law of motion for wage is implicitly defined by four equations for $\{w_t, \psi_t, \mu_{t+1}, \underline{\epsilon}_t\}$

$$\mu_{t+1} = \frac{\lambda}{1-\psi_t}, \quad \frac{R\mu_{t+1}^\eta}{w_{t+1}^{\frac{1}{\rho}}} = Rq_{t+1}^B = r_t = r^*, \quad w_t^\sigma = \psi_t \underline{\epsilon}_t \mathbb{F}, \quad \frac{w_{t+1}}{w_t} = \frac{1-\underline{\epsilon}_t^{-(1+\theta)}}{\psi_t \mu_{t+1}} \frac{r^*}{\eta\rho}, \tag{70}$$

$$\frac{\partial \mu_{t+1}}{\partial \psi_t} = \frac{\lambda}{(1-\psi_t)^2} > 0, \quad \frac{\partial \psi_t}{\partial w_t} = \frac{\mathbb{S} + \sigma(1-\mathbb{S})}{\mathbb{G}+1} \frac{\psi_t}{w_t} > 0, \tag{71}$$

where $\mathbb{S} \equiv \frac{1-\underline{\epsilon}_t^{-(1+\theta)}}{1+\theta\underline{\epsilon}_t^{-(1+\theta)}}$ and $\mathbb{G} \equiv (1+\eta\rho)\frac{\psi_t}{1-\psi_t}\mathbb{S}$. As both final goods are essential for the composition good production, both sectors are active, $\underline{\epsilon}_t > 1$ so that $\mathbb{S} \in (0,1)$. Given $\frac{\partial \psi_t}{\partial w_t} > 0$, for $w_t \to 0$, $\psi_t \to 0$ so that $\mu_{t+1} \to \lambda$ and $w_{t+1} \to \underline{w}_{t+1} \equiv \left(\frac{R\lambda^\eta}{r^*}\right)^\rho$. Thus, the law of motion for wage has a positive intercept on the vertical axis at $\underline{w}_{t+1}$. Let $\mathbb{Z} \equiv 1 - \psi_t - \frac{\mathbb{S}}{1-\mathbb{S}} - (1+\eta\rho)\theta\psi_t\mathbb{S}^2$.

$$\mathbb{J} \equiv \frac{\partial w_{t+1}}{\partial w_t} = \frac{\eta\rho[\mathbb{S} + \sigma(1-\mathbb{S})]}{\mathbb{G}+1} \frac{\psi_t}{1-\psi_t} \frac{w_{t+1}}{w_t} > 0, \text{ if } \sigma \geq 0; \tag{72}$$

$$\text{for } \sigma = 0, \; \mathbb{H} \equiv \frac{\partial^2 w_{t+1}}{\partial w_t^2} = -\left[\frac{1-\mathbb{S}}{\mathbb{G}\mathbb{S}}(2+\theta\mathbb{S}) + \frac{\eta\rho + \mathbb{G}}{\mathbb{G}} \frac{\psi_t}{1-\psi_t}\right] \frac{\mathbb{S}}{\mathbb{G}+1} \frac{\mathbb{J}}{w_t} < 0; \tag{73}$$

$$\text{for } \sigma = 1, \; \mathbb{H} \equiv \frac{\partial^2 w_{t+1}}{\partial w_t^2} = \mathbb{Z} \frac{1-\mathbb{S}}{\mathbb{G}+1} \frac{1+\eta\rho}{\eta\rho} \frac{1}{1-\psi_t} \frac{\mathbb{J}^2}{w_{t+1}} \quad \Rightarrow \quad sgn\,(\mathbb{H}) = sgn(\mathbb{Z}). \tag{74}$$

In the case of $\sigma = 0$, the law of motion for wage is piecewise with a positive intercept on the vertical axis at $\underline{w}_{t+1}$, concave for $w_t \in (0, \bar{w}_T]$, and flat at $\bar{w}_{t+1}$ for $w_t > \bar{w}_F$.

In the case of $\sigma = 1$,

$$\frac{\partial \mathbb{Z}}{\partial w_t} = -\left\{\frac{[1+(1+\eta\rho)\theta\mathbb{S}_t^2]\psi_t}{(\mathbb{G}+1)w_t} + \frac{(1-\mathbb{S})(1+\theta\mathbb{S})\mathbb{G}}{(\mathbb{G}+1)w_t}\left[\frac{1}{(1-\mathbb{S})^2} + 2\theta(1-\psi_t^i)\mathbb{G}\right]\right\} < 0.$$

Given $\frac{\partial \psi_t}{\partial w_t} > 0$, for $w_t \to 0$, $\psi_t \to 0$, so that $\mathbb{Z} > 0$ and the law of motion for wage is convex. Since $\frac{\partial \mathbb{Z}}{\partial w_t} < 0$, it is possible that, for $w_t \to \bar{w}_F$, $\psi_t \to 1-\lambda$ so that $\mathbb{Z} < 0$ and the law of motion for wage becomes concave. Let $\check{w}_t$ define the threshold value such that $\mathbb{Z} = 0$, i.e., the inflection point of the law of motion for wage. There are two cases.

- Case 1: if $\check{w}_t > \bar{w}_F$, the law of motion for wage is piecewise with a positive intercept on the vertical axis at $\underline{w}_{t+1}$, convex for $w_t \in (0, \bar{w})$, and flat at $\bar{w}_{t+1}$ for $w_t > \bar{w}_F$.

- Case 2: if $\check{w}_t < \bar{w}_F$, the law of motion for wage is piecewise with a positive intercept on the vertical axis at $\underline{w}_{t+1}$, convex for $w_t \in (0, \check{w})$, concave for $w_t \in (\check{w}, \bar{w}_F)$, and flat at $\bar{w}_{t+1}$ for $w_t > \bar{w}_F$.

Step 3: the threshold values for multiple steady states under financial integration

Under financial integration, in the case of $\sigma = 0$, the law of motion for wage has a concave-flat shape so that there exists a unique, stable steady state; in the case of $\sigma > 0$, the law of motion for wage has a convex-flat or convex-concave-flat shape so that multiple steady states may arise in three cases, as shown in figure 12. Given $\sigma > 0$ and $r^* = r_A$, we derive as follows the threshold values that split region BU and SU of figure 2 into five regions of figure 11.

Case 1: consider region SU of figure 2 where $\mu_A = 1$ and $r_A = \rho$. Given $r^* = r_A = \rho$, the law of motion for wage at the autarkic steady state (S) is flat so that the autarkic steady state is still stable under financial integration. Compare the upper-right and the lower-right panels of figure 12. Multiple steady states arise if the law of motion for wage intersects with the 45° line at $w_t \in (0, \bar{w}_F)$. The boundary between region BC and C is defined as the case where the law of motion is tangent with the 45° line at point M, i.e., $w^i_{t+1} = w^i_t = w_M < w_A$, $r_M = r^* = \rho$, and $\mathbb{J}_M \equiv \frac{\partial w_{t+1}}{\partial w_t}\|_{w_M} = 1$. Let $\mathbb{D}_M \equiv 1 - \underline{\epsilon}_M^{-(1+\theta)}$ and $\mathbb{N} \equiv \eta\lambda$. Combine the three conditions with equations (44)-(46) to get

$$w_M < w_A, \Rightarrow \left(\frac{\psi_M \underline{\epsilon}_M}{\psi_A \underline{\epsilon}_A}\right)^{\frac{1}{\sigma}} = \frac{w_M}{w_A} = \left(\frac{\lambda}{1-\psi_M}\right)^{\rho\eta}, \tag{75}$$

$$r_M = \frac{\rho[1-\eta(1-\mu_M)]}{1-\phi_M} = r^* = \rho, \Rightarrow \mathbb{D}_M = \frac{\mathbb{N}\psi_M}{1-\psi_M} < \eta\psi_M, \tag{76}$$

$$\mathbb{J}_M = \frac{\eta\rho[\$_M + \sigma(1-\$_M)]}{(1+\eta\rho)\$_M + \frac{1-\psi_M}{\psi_M}} = 1, \Rightarrow 1 - \frac{1}{\psi_M(\eta\rho\sigma+1)} = \$_M = \frac{\mathbb{D}_M}{1+\theta(1-\mathbb{D}_M)}. \tag{77}$$

Combine equations (76) and (77) to get

$$\left[\sigma + \frac{1}{\eta\rho(\theta+1)}\right]\mathbb{D}_M^2 - \left[\frac{\mathbb{N}}{\eta\rho(1+\frac{1}{\theta})} + \sigma\right]\mathbb{D}_M + \frac{\mathbb{N}}{\eta\rho} = 0. \tag{78}$$

$\mathbb{D}_M$ is a root of equation (78).[30] Combine the solution to $\mathbb{D}_M$ with equation (76) to solve for ψ_M and $\underline{\epsilon}_M = (1-\mathbb{D}_M)^{-\frac{1}{1+\theta}}$. Plug them and $\underline{\epsilon}_A = (1-\eta\psi_A)^{-\frac{1}{1+\theta}}$ in equation (75) to solve ψ_A as a function of λ, which defines the boundary between region BC and C.

Case 2: consider region BU of figure 2 where $\mu_A \in (\lambda, 1)$ and $r_A = \frac{\rho}{1-\eta(1-\mu_A)} < \rho$. See the upper-left panel of figure 12. Under financial integration, given $r^* = r_A$, case B arises if $\mathbb{J}_A \equiv \frac{\partial w_{t+1}}{\partial w_t}\|_{w_A} > 1$. Solve the boundary condition $\mathbb{J}_A = 1$ to get a threshold value as the function of λ,

$$\hat{\psi}_F = \frac{\mathbb{B} - \sqrt{\mathbb{B}^2 - 4\mathbb{C}}}{2}, \text{ where } \mathbb{C} = \frac{1}{(1+\sigma\eta\rho)}, \ \mathbb{B} = 1 + \mathbb{C}\left[1 - \frac{1}{(\theta+1)(1+\frac{1-\eta}{\lambda\eta})}\right],$$

which defines the border between region AB and B of figure 11.

Case 3: consider the region with $\psi_A < \hat{\psi}_F$ in figure 11. Since $\mathbb{J}_A < 1$, the autarkic steady state is still stable under financial integration. Compare the upper-middle and the lower-left panel of figure 12. As proved above, the law of motion for wage $w_t \in (0, \bar{w}_F)$ can be either convex or convex-concave. Taking that into account, financial integration may lead to multiple steady states in two subcases.

- Case 3.1: multiple steady states arise if the kink point of the law of motion for wage is on or above the 45° line. Given $r^* = r_A = \rho(1 - \eta + \eta\mu_A)$, the kink point is

[30] According to equation (78), there are two roots for $\mathbb{D}_t$. However, only one root satisfies the condition of $\mathbb{D}_M < \eta\psi_M < \eta\psi_A$.

characterized by $w_t = \bar{w}_F$, $w_{t+1} = \bar{w}_{t+1} = \left(\frac{R}{r^*}\right)^\rho$, $\psi_t = \psi_K \equiv 1-\lambda$, $\mu_{t+1} = \mu_K = 1$. As the boundary case, the kink point is on the 45° line, i.e., $\bar{w}_{t+1} = \bar{w}_F$. Combine them with equations (70) to get,

$$\bar{w}_{t+1}^{\frac{1}{\rho}} = \frac{R}{r^*} = \frac{R}{r_A} = \frac{R}{\rho(1-\eta+\eta\mu_A)}, \qquad \bar{w}_F^\sigma = \mathbb{F}\psi_K \underline{\epsilon}_K = \mathbb{F}(1-\lambda)\underline{\epsilon}_K \tag{79}$$

$$\frac{r^*}{\eta\rho}\frac{1-\underline{\epsilon}_K^{-(1+\theta)}}{\mu_K\psi_K} = \frac{\bar{w}_{t+1}}{\bar{w}_F} = 1 \;\Rightarrow\; \underline{\epsilon}_K = \left(\frac{1-\eta+\eta\mu_A}{1-\eta+\eta\mu_A-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}} \tag{80}$$

$$\left[\frac{R}{\rho(1-\eta+\eta\mu_A)}\right]^\rho = \bar{w}_{t+1} = \bar{w}_F = [\mathbb{F}(1-\lambda)\underline{\epsilon}_K]^{\frac{1}{\sigma}} \tag{81}$$

$$\left(\frac{R\mu_A^\eta}{\rho(1-\eta+\eta\mu_A)}\right)^\rho = w_A = [\mathbb{F}\psi_A\underline{\epsilon}_A]^{\frac{1}{\sigma}}, \;\; \underline{\epsilon}_A = \left(\frac{1-\eta+\eta\mu_A}{1-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}} \tag{82}$$

$$\mu_A^{\sigma\rho\eta}(1-\lambda) = \psi_A\left(\frac{1-\eta+\eta\lambda+\eta\mu_A-\eta}{1-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}}, \;\; \mu_A = \frac{\lambda}{1-\psi_A} \tag{83}$$

$$\Rightarrow (1-\lambda)\lambda^{\sigma\eta\rho} = \left(1-\frac{\eta(1-\frac{\lambda}{1-\psi_A})}{1-\eta+\eta\lambda}\right)^{\frac{1}{1+\theta}}\psi_A(1-\psi_A)^{\sigma\eta\rho}. \tag{84}$$

Let $\tilde{\psi}_{F,1}$ denote the solution to equation (84), which is a function of λ.

- Case 3.2: Multiple steady states arise if the concave part of the law of motion is at least tangent with the 45° line at point M, i.e., $w_{t+1} = w_t = w_M \in (w_A, \bar{w}_F)$, $\mathbb{J}_M \equiv \frac{\partial w_{t+1}}{\partial w_t}\|_{w_M} = 1$, and $r^* = r_A = \rho(1-\eta+\eta\mu_A)$.[31] Let $\mathbb{D}_M \equiv 1-\underline{\epsilon}_M^{-(1+\theta)}$ and $\mathbb{N} \equiv \frac{\eta\lambda}{1-\eta+\frac{\eta\lambda}{1-\psi_A}}$. Combine the three conditions with equations (44)- (46) to get

$$w_M \in (w_A, \bar{w}_F), \;\Rightarrow\; \left(\frac{\psi_M\underline{\epsilon}_M}{\psi_A\underline{\epsilon}_A}\right)^{\frac{1}{\sigma}} = \frac{w_M}{w_A} = \left(\frac{\mu_M}{\mu_A}\right)^{\rho\eta} = \left(\frac{1-\psi_A}{1-\psi_M}\right)^{\rho\eta}, \tag{85}$$

$$r_M = r^* = r_A = \rho(1-\eta+\eta\mu_A), \;\Rightarrow\; \mathbb{D}_M = \frac{\mathbb{N}\psi_M}{1-\psi_M} > \eta\psi_M, \tag{86}$$

$$\mathbb{J}_M = 1, \;\Rightarrow\; 1-\frac{1}{\psi_M(\eta\rho\sigma+1)} = \$_M = \frac{\mathbb{D}_M}{1+\theta(1-\mathbb{D}_M)}. \tag{87}$$

Combine equations (86) and (87) to get

$$\left[\sigma+\frac{1}{\eta\rho(\theta+1)}\right]\mathbb{D}_M^2 - \left[\frac{\mathbb{N}}{\eta\rho(1+\frac{1}{\theta})}+\sigma\right]\mathbb{D}_M + \frac{\mathbb{N}}{\eta\rho} = 0. \tag{88}$$

$\mathbb{D}_M$ is a root of equation (88).[32] Combine it with equation (86) to solve for ψ_M and $\underline{\epsilon}_M = (1-\mathbb{D}_M)^{-\frac{1}{1+\theta}}$. Plug them and $\underline{\epsilon}_A = \left(\frac{1-\eta+\eta\lambda}{1-\eta+\eta\frac{\lambda}{1-\psi_A}}\right)^{-\frac{1}{1+\theta}}$ in equation (85) to solve $\tilde{\psi}_{F,2}$ as a function of λ.

The boundary between region AB and A is characterized by $\tilde{\psi}_F = \min\{\tilde{\psi}_{F,1}, \tilde{\psi}_{F,2}\}$.

□

[31] The analysis is almost identical as deriving the boundary between region BC and C, except for $r^* = r_A = \rho(1-\eta+\eta\mu_A)$.

[32] According to equation (88), there are two roots for $\mathbb{D}_t$. However, only one root satisfies the condition of $\mathbb{D}_M > \eta\psi_M$.