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In Support of the TRIPS Agreement

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Citation

JAKOBSSON, Amanda and Segerstrom, Paul S.. In Support of the TRIPS Agreement. (2012). 1-57. Available at: https://ink.library.smu.edu.sg/soe_research/1573

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In Support of the TRIPs Agreement

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Current version: February 6, 2012

Abstract: This paper challenges the conventional wisdom that the TRIPs agreement is bad for developing countries. We present a dynamic general equilibrium model of North-South trade that allows us to study the implications of stronger intellectual property rights (IPR) protection and simultaneous trade liberalization. In our model, stronger IPR protection in the South (TRIPs) leads to more innovation in the North, more technology transfer to the South and higher long-run southern consumer welfare. The South also benefits from trade liberalization but the welfare gains from TRIPs are considerably larger.

JEL classification: F12, F23, F43, O31, O34.

Keywords: Multinational Firms, North-South Trade, Intellectual Property Rights, Foreign Direct Investment, Product Cycles, Economic Growth.

Acknowledgments: We thank seminar participants at Jerusalem Summer School on Economic Growth (Hebrew University), Mainz Workshop on Trade, Growth and Wages (University of Mainz), DEGIT-XVI (St. Petersburg University), Brixen Workshop on International Trade and Finance, Midwest Economic Theory and International Economics Meetings (Vanderbilt University), Brown University, and University of Bologna for helpful comments. Financial support from the Wallander Foundation is gratefully acknowledged.

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Do not steal. Do not covet anything that belongs to your neighbor. Follow the whole instruction the Lord your God has commanded you, so that things will go well with you in the land you are about to possess.

Deuteronomy 5:19,21,33

1 Introduction

Intellectual property rights (IPR) protection in developing countries has been a topic of debate for many years. The Trade-Related Aspects of Intellectual Property Rights (TRIPs) agreement was signed as part of the Uruguay Round in 1994. This agreement formally introduced intellectual property rights into the World Trade Organization (WTO) and the world trading system. The TRIPs agreement covers copyrights and patents but also enforcement procedures and dispute mechanisms. Since most developed countries already had such systems in place, the implied changes in national regulation required by the TRIPs agreement mostly affects developing countries. They have been forced to increase their IPR protection to remain inside the WTO.

The TRIPs agreement has come in for intense criticism. As Irwin (2009, p.231) explains, "Many developing countries complain that, unlike mutually beneficial tariff reductions, the TRIPs agreement merely transfers income from developing to developed countries by strengthening the ability of multinational corporations to charge higher prices in poorer countries." In his book In Defense of Globalization, Bhagwati (2004, p.183) describes TRIPs as "like the introduction of cancer cells into a healthy body." For this influential economist, the otherwise healthy body is the World Trade Organization and TRIPs is killing it. Birdsall, Rodrik and Subramanian (2005) concur. They write "An international community that presides over TRIPs and similar agreements forfeits any claim to being development-friendly. This must change: the rich countries cannot just amend TRIPs; they must abolish it altogether."

Turning to the economics literature, perhaps the best support for this critique is provided by McCalman (2001), who estimates the value of transfers of income between countries implied by the TRIPs agreement. He finds that only a few countries gained from TRIPs (United States, Germany, France, Italy, Sweden, Switzerland) and that all other countries were made worse off, including all developing countries. But it is just assumed in McCalman's cost-benefit analysis that there are no dynamic benefits from TRIPs. Recently, evidence has emerged indicating that there are dynamic benefits from TRIPs. For example, Branstetter, Fisman, Foley and Saggi (2011) study the response of host country industrial production to stronger IPR protection. They find that following patent reform, US-based multinational firms expand the scale of their activities in reforming countries and exports of new goods increase in these reforming countries.

The purpose of this paper is to challenge the conventional wisdom that TRIPs is bad for developing countries. We present a dynamic general equilibrium model that allows us to study the implications of stronger IPR protection and simultaneous trade liberalization. For developing countries that belong to the WTO, patent reforms done to satisfy the conditions of the TRIPs agreement are implemented at the same time as tariffs and other trade barriers are lowered to comply with other WTO decisions.

In the model, firms in the North (developed countries) engage in innovative R&D to develop new product varieties, and once successful, these northern firms earn global monopoly profits from selling the new products. Northern firms also engage in adaptive R&D to learn how to produce their products in the lower-wage South (developing countries), and once successful, their foreign affiliates located in the South earn even higher global monopoly profits. Southern firms engage in imitative R&D to learn how to produce both the product varieties of northern firms and their foreign affiliates. So over time, the production of every product variety moves from the North to the South and international technology transfer occurs both through foreign direct investment (FDI) and imitation. The innovation, FDI and imitation rates are all endogenously determined based on profit-maximization considerations.

We calibrate the model to fit two benchmark cases: the 1990 benchmark (the world prior to the implementation of the TRIPs agreement) and the 2005 benchmark (the world after the implementation of the TRIPs agreement). Going from the 1990 to the 2005 benchmark, we are able to replicate the large 10-fold observed increase in FDI inflows to developing countries from 1990 to 2005. Our results suggest that for plausible parameter values, TRIPs (stronger southern IPR protection) leads to more FDI, more innovation and considerably higher long-run southern consumer welfare. The South also benefits from the trade liberalization that occurred from 1990 to 2005 but the welfare gains from TRIPs are considerably larger. Furthermore, we find that trade liberalization by itself has a negligible effect in stimulating FDI, so most of the 10-fold observed increase in FDI inflows to developing countries (from 1990 to 2005) can be attributed to stronger southern IPR protection. This big increase in FDI is the main reason why TRIPs is good for developing countries in our analysis.

In the related literature, Helpman (1993), Lai (1998) and Branstetter and Saggi (2011) all

study the effects of stronger IPR protection using dynamic general equilibrium North-South trade models with costless FDI. Recently, Gustafsson and Segerstrom (2011) have developed a dynamic general equilibrium North-South trade model with costly FDI and we build on their analysis by also incorporating costly trade. Thus, in our model, there are costs of moving production across regions and there are also costs of moving goods across regions.

The rest of the paper is organized as follows. In Section 2, we present the model and derive seven steady-state equilibrium conditions. In Section 3, we solve the model numerically for different parameter values and present the results. Then in Section 4 we offer some concluding remarks. We discuss the related literature in more detail in Appendix 1 and in Appendix 2, we present calculations that we did to solve the model in more detail.

2 The Model

2.1 Overview

We consider a global economy consisting of two regions: the North and the South. In both regions, labor is the only factor used to manufacture product varieties and to do R&D. Labor is perfectly mobile across activities within a region but cannot move across regions. Since labor markets are perfectly competitive, there is a single wage rate paid to all northern workers w_N and one single wage rate paid to all southern workers w_S . Labor is employed in four distinct activities: manufacturing of final consumption goods, innovative R&D, adaptive R&D and imitative R&D. Although labor cannot move across regions, goods can. We assume iceberg trade costs between the North and the South: $\tau > 1$ units of a good must be produced and exported for one unit to arrive at its destination.

In this global economy, firms can hire northern workers to engage in innovative R&D with the goal of learning how to produce new product varieties. A successful firm earns global monopoly profits from producing a new product variety and selling it to consumers in both regions. We call such a firm a *northern firm* because all production is located in the North. A northern firm can hire southern workers to engage in adaptive R&D with the aim of transferring its manufacturing operations to the lower-wage South $(w_N > w_S)$. When successful in adaptive R&D, a firm earns higher global monopoly profits because of the lower wage rate in the South. We call such a firm a foreign affiliate because production takes place in the South but a fraction of its profits is repatriated back to its northern stockholders. Adaptive R&D can be interpreted as an index of FDI (foreign direct investment) because it represents the cost that northern firms incur to transfer their technology to foreign affiliates, and even when financed by southern savings, northern firms control the amount of adaptive R&D in order to maximize their global expected discounted profits.

In addition, there are *southern firms*, firms that are owned and operate in the South. These firms can hire southern workers to engage in imitative R&D with the goal of learning how to produce foreign affiliate varieties. Once a product variety has been successfully imitated, it is no longer profitable for the foreign affiliate to produce the variety since the southern firm has lower production costs. Instead, the successful southern firm produces the imitated variety and earns global profits from selling to consumers in both regions. Southern firms can also engage in a second more difficult type of imitative R&D aimed at learning how to produce northern varieties in the South. When successful, they earn even larger global profits from selling to consumers in both regions. Thus technology transfer from the North to the South may occur through two different channels: either through FDI by foreign affiliates or through imitation by southern firms of northern products.

As illustrated in Figure 1, the model generates one-way product cycles. Each product variety is initially developed and produced by a northern firm, its production can later shift to a foreign affiliate as a result of adaptive R&D, and eventually its production shifts to a southern firm that imitates the production technology. The innovative R&D activities of northern firms result in the innovation rate g and the adaptive R&D activities of foreign affiliates result in production shifting to foreign affiliates at the FDI rate ϕ . Due to the imitative R&D activities of southern firms, foreign affiliate varieties are imitated at the rate ι_S and northern varieties are imitated at the rate ι_N . The innovation, FDI and imitation rates are all endogenously determined based on profit-maximization considerations.¹

2.2 Households

In both the North and the South, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate g_L , the population growth rate. Let $L_{Nt} = L_{N0}e^{g_L t}$ denote the supply of labor in the North at time t, let $L_{St} = L_{S0}e^{g_L t}$ denote

¹Gustafsson and Segerstrom (2011) present a North-South model with exogenous imitation and costless trade. The reader may find it useful to study this simpler model first, before studying the more complicated model in this paper.

Figure 1: One-Way Product Cycles.

the corresponding supply of labor in the South, and let $L_t = L_{Nt} + L_{St}$ denote the world supply of labor. In addition to wage income, households also receive asset income from their ownership of firms. We assume that R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings, which is consistent with the Feldstein and Horioka (1980) finding that domestic savings finances domestic investments.

Households in both the North and the South share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$
U = \int_0^\infty e^{-(\rho - g_L)t} \ln(u_t) dt
$$
 (1)

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility of an individual at time t. The static constant elasticity of substitution (CES) utility function is given by

$$
u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega \right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1.
$$
 (2)

In (2), $x_t(\omega)$ is the per capita quantity demanded of the product variety ω at time t and n_t is the total number of invented varieties at time t. There are four types of varieties: n_{N_t} varieties produced by northern firms, n_{Ft} varieties produced by foreign affiliates, n_{It} varieties produced by southern firms that have imitated foreign affiliates ("I" for imitation) and n_{C_t} varieties produced by southern firms that have imitated northern firms ("C" for copying). The number for varieties available on the world market n_t is the sum of these four types of varieties: $n_t = n_{Nt} + n_{Ft} + n_{It} + n_{Ct}$. We assume that varieties are gross substitutes. Then with α measuring the degree of product differentiation, the elasticity of substitution between product varieties is $\sigma \equiv \frac{1}{1-\alpha} > 1$.

Solving the static consumer optimization problem yields the demand function:

$$
x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}}
$$
\n(3)

where c_t is individual consumer expenditure at time t, $p_t(\omega)$ is the price of variety ω at time t and $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices. We will shortly define one such price index for each of the two regions. By substituting the demand function (3) into (2) and using the definition of the price index P_t , it can be shown that $u_t = c_t/P_t$. Then maximizing (1) subject to the relevant intertemporal budget constraint yields the intertemporal optimization condition

$$
\frac{\dot{c}_t}{c_t} = r_t - \rho \tag{4}
$$

implying that individual consumer expenditure grows over time only if the market interest rate r_t exceeds the subjective discount rate ρ .

The representative consumer in each region has different wage income $(w_N > w_S)$ and hence different consumer expenditure. Let c_N and c_S denote the representative consumer's expenditure in the North and South, respectively. We treat the southern wage as the numeraire price $(w_S = 1)$, that is, we measure all prices relative to the price of southern labor. Furthermore, we solve the model for a steady-state equilibrium where w_N , w_S , c_N and c_S are all constant over time. Then $\dot{c}_t/c_t = 0$ in (4) and hence $r_t = \rho$. The steady-state market interest rate is thus constant over time and equal in the two regions.²

Due to the positive trade costs, the prices of goods differ between the two regions. Let p_N denote the price charged to northern consumers by each northern firm, p_N^* denote the price charged to southern consumers by each northern firm, p_F denote the price charged to southern consumers by each foreign affiliate, p_F^* denote the price charged to northern consumers by each foreign affiliate, p_I denote the price charged to southern consumers by each southern firm that has imitated a foreign affiliate variety, p_I^* denote the price charged to northern consumers by each southern firm that has imitated a foreign affiliate variety, p_C denote the price charged to southern consumers by each southern firm that has imitated a northern variety and p_C^* denote the price charged to northern consumers by each southern firm that has imitated a northern variety. In our notation, the asterisk refers to a firm's price in its export market, regardless of whether this market is the North or the South. We solve for a steady-state equilibrium where all of these prices are constant over time.

2.3 Steady-State Dynamics

We will now derive some properties that must hold in any steady-state equilibrium.

Let $g \equiv \dot{n}_t/n_t$ denote the steady-state growth rate of the number of varieties. From the variety condition $n_t = n_{Nt} + n_{Ft} + n_{It} + n_{Ct}$, it follows that the number of varieties produced by each type of

²Our earlier assumption that "R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings" implies that there is no international capital mobility. Thus the two regions typically have different interest rates along the transition path leading to a new steady-state equilibrium. But in a steady-state equilibrium, the two regions must have the same interest rate because consumers in both regions have the same subjective discount rate ρ .

firm must grow at the same rate g, that is, $g \equiv \dot{n}_t/n_t = \dot{n}_{Nt}/n_{Nt} = \dot{n}_{Ft}/n_{Ft} = \dot{n}_{It}/n_{It} = \dot{n}_{Ct}/n_{Ct}$. Therefore, the variety shares $\gamma_N \equiv n_{Nt}/n_t$, $\gamma_F \equiv n_{Ft}/n_t$, $\gamma_I \equiv n_{It}/n_t$, $\gamma_C \equiv n_{Ct}/n_t$ are necessarily constant over time in any steady-state equilibrium and satisfy $\gamma_N + \gamma_F + \gamma_I + \gamma_C = 1$.

Let $\phi \equiv (\dot{n}_{Ft} + \dot{n}_{It})/n_{Nt}$ denote the steady-state FDI rate, which is constant over time in any steady-state equilibrium since $\phi \equiv \frac{\dot{n}_{Ft} + \dot{n}_{It}}{n_{Ft}}$ $\frac{\dot{m}_{t}+\dot{n}_{It}}{n_{Nt}}=\frac{\dot{n}_{Ft}}{n_{Ft}}$ $n_{F t}$ $n_{F t}/n_t$ $\frac{n_{Ft}/n_t}{n_{Nt}/n_t} + \frac{\dot{n}_{It}}{n_{It}}$ $\overline{n_{It}}$ n_{It}/n_t $\frac{n_{It}/n_t}{n_{Nt}/n_t} = g \frac{\gamma_F}{\gamma_N}$ $\frac{\gamma_F}{\gamma_N}+g\frac{\gamma_I}{\gamma_N}$ $\frac{\gamma_I}{\gamma_N}$. The FDI rate is the rate at which production of varieties shift from the North to the South due to foreign affiliates engaging in adaptive R&D. It is taken into account in the definition of the FDI rate that moving production to a foreign affiliate in the South exposes the firm to a positive imitation rate by southern firms.³ Let $\iota_S \equiv \dot{n}_{It}/n_{Ft}$ denote this imitation rate of foreign affiliate-produced varieties. ι_S is constant over time in any steady-state equilibrium since $\iota_S \equiv \frac{\dot{n}_{It}}{n_{It}}$ $\frac{\dot{n}_{It}}{n_{Ft}} = \frac{\dot{n}_{It}}{n_{It}}$ n_{It} n_{It}/n_t $\frac{n_{It}/n_t}{n_{Ft}/n_t} = g \frac{\gamma_I}{\gamma_F}$ $\frac{\gamma_I}{\gamma_F}$. Let $i_N \equiv \dot{n}_{Ct}/n_{Nt}$ be the imitation rate of northern-produced varieties. This imitation rate is also constant over time in any steady-state equilibrium since $\iota_N \equiv \frac{\dot{n}_{Ct}}{n_{Nt}}$ $\frac{\dot{n}_{Ct}}{n_{Nt}} = \frac{\dot{n}_{Ct}}{n_{Ct}}$ $\overline{n_{Ct}}$ n_{Ct}/n_t $\frac{n_{Ct}/n_t}{n_{Nt}/n_t} = g \frac{\gamma_C}{\gamma_N}$ $\frac{\gamma_C}{\gamma_N}.$

Taking the time derivative of the variety condition $n_t = n_{Nt} + n_{Ft} + n_{It} + n_{Ct}$, we can solve for the northern variety share γ_N :

$$
\gamma_N = \frac{g}{g + \phi + \iota_N}.\tag{5}
$$

From the steady-state expressions for the imitation rates ι_S and ι_N , and the steady-state expression for the northern variety share (5), it follows that

$$
\gamma_C = \frac{\iota_N}{g + \phi + \iota_N}.\tag{6}
$$

Substituting (5) and (6) into $\gamma_N + \gamma_F + \gamma_I + \gamma_C = 1$ yields $\gamma_F + \gamma_I = \frac{\phi}{g + \phi}$ $\frac{\varphi}{g + \phi + \iota_N}$. Using this expression together with $\iota_S = g\gamma_I/\gamma_F$, we obtain

$$
\gamma_I = \left(\frac{\phi}{g + \phi + \iota_N}\right) \left(\frac{\iota_S}{g + \iota_S}\right) \text{ and } \gamma_F = \left(\frac{\phi}{g + \phi + \iota_N}\right) \left(\frac{g}{g + \iota_S}\right). \tag{7}
$$

As expected, the variety share of foreign affiliates depends positively on the FDI rate ($\phi \uparrow \Rightarrow \gamma_F \uparrow$). An increase in the imitation rate of foreign affiliate varieties increases the variety share of the imitating I-firms at the expense of the foreign affiliates $(\iota_S \uparrow \Rightarrow \gamma_I \uparrow, \gamma_F \downarrow)$ and similarly, an increase in the imitation rate of northern varieties increases the variety share of the imitating C-firms at

³To see that the term \dot{n}_{It} should be included in the FDI rate definition, it is helpful to think about a bathtub with an open drain that is being filled with water from a faucet. The flow of water coming out of the faucet into the bathtub equals the rate of change in the volume of water in the bathtub plus the flow of water going down the open drain. Likewise, the flow number of varieties that firms transfer to the South through FDI $(i_{Ft} + i_{It})$ equals the rate of change in the number of varieties produced by foreign affiliates $(n_F t)$ plus the flow number of foreign affiliate varieties that are imitated by southern firms (\dot{n}_{It}) .

the expense of northern firms $(\iota_N \uparrow \Rightarrow \gamma_C \uparrow, \gamma_N \downarrow).$

Because of trade costs, the price of each product variety is different in the two regions and and we need to define a price index for each region. Let P_{Nt} denote the price index for the North and P_{St} denote the price index for the South. Given the earlier definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, it follows that the northern price index satisfies $P_{Nt}^{1-\sigma}$ $n_{Nt}p_N^{1-\sigma} + n_{Ft} (p_F^*)^{1-\sigma} + n_{It} (p_I^*)^{1-\sigma} + n_{Ct} (p_C^*)^{1-\sigma}$ and the southern price index satisfies $P_{St}^{1-\sigma} =$ $n_{Nt} (p_N^*)^{1-\sigma} + n_{Ft} p_F^{1-\sigma} + n_{It} p_I^{1-\sigma} + n_{Ct} p_C^{1-\sigma}$ $\int_C^{1-\sigma}$. Using the variety shares defined earlier, we can rewrite these expressions as

$$
P_{Nt}^{1-\sigma} = \left[\gamma_N p_N^{1-\sigma} + \gamma_F (p_F^*)^{1-\sigma} + \gamma_I (p_I^*)^{1-\sigma} + \gamma_C (p_C^*)^{1-\sigma} \right] n_t \tag{8}
$$

$$
P_{St}^{1-\sigma} = \left[\gamma_N \left(p_N^* \right)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma} \right] n_t \tag{9}
$$

where the terms in brackets are constant over time. Thus $P_{Nt}^{1-\sigma}$ and $P_{St}^{1-\sigma}$ both grow over time at the rate g in any steady-state equilibrium.

2.4 Product Markets

The firms producing different product varieties compete in prices and maximize profits. There is constant returns to scale in production. For each firm operating in the North and for each of the foreign affiliates located in the South, one unit of labor produces one unit of output. Due to their lack of familiarity with the southern economic environment, foreign affiliates have higher production costs than southern firms. For southern firms that have imitated either a foreign affiliate variety or a northern variety, $\zeta \in (0,1)$ units of labor produce one unit of output. Each northern firm has the marginal cost w_N serving the northern market and taking trade costs into account, the marginal cost τw_N serving the southern market. Costs are lower for foreign affiliates: each foreign affiliate has the marginal cost w_S serving the southern market and the marginal cost τw_S serving the northern market. Southern firms have the lowest costs of all: each southern firm has the marginal cost ζw_S serving the southern market and the marginal cost $\tau \zeta w_S$ serving the northern market.

A northern firm earns the flow of global profits $\pi_{Nt} = (p_N - w_N) x_{Nt} L_{Nt} + (p_N^* - \tau w_N) x_{Nt}^* L_{St}$ where x_{Nt} is the quantity demanded by the typical northern consumer of the northern firm's product and x_{Nt}^* is the quantity demanded by the typical southern consumer of the northern firm's product. Given (3), a northern firm faces consumer demands $x_{Nt} = p_N^{-\sigma}$ $\frac{-\sigma}{N}c_N/P_{Nt}^{1-\sigma}$ and

 $x_{Nt}^* = (p_N^*)^{-\sigma} c_S/P_{St}^{1-\sigma}$. A northern firm chooses its price in each market to maximize profits. It is straightforward to verify that the profit-maximizing price in the northern market is the monopoly price $p_N = \frac{w_N}{\alpha}$ and similarily in the export market, the profit-maximizing price is the monopoly price $p_N^* = \frac{\tau w_N}{\alpha}$. Using these prices and some simple algebra, we can reexpress the northern firm's global profit flow as

$$
\pi_{Nt} = \left[\frac{w_N \left(X_N + \tau X_N^*\right)}{(\sigma - 1)\gamma_N}\right] \frac{L_t}{n_t} \tag{10}
$$

where $X_N \equiv \frac{p_N^{-\sigma} c_N L_{Nt} n_{Nt}}{P^{1-\sigma} L}$ $\frac{N_{CN}L_{Nt}n_{Nt}}{P_{Nt}^{1-\sigma}L_t}$ and $X_N^* \equiv \frac{(p_N^*)^{-\sigma}c_{S}L_{St}n_{Nt}}{P_{St}^{1-\sigma}L_t}$ $\frac{p_{SDSt}^{(l)}(x)}{p_{St}^{1-\sigma}L_t}$ are population-adjusted aggregate demand terms for northern product varieties in the North and the South, respectively. X_N and X_N^* are both constant over time in steady-state equilibrium since prices and consumer expenditure are constant over time, L_{Nt} grows at the same rate g_L as the world population L_t , and both $P_{Nt}^{1-\sigma}$ and $P_{St}^{1-\sigma}$ grow at the same rate g as n_{Nt} . Hence the bracketed term in (10) is constant over time in any steady-state equilibrium and the profits earned by a northern firm only changes because L_t/n_t changes over time. L_t/n_t is a measure of the size of the market relevant for each northern firm. Population growth increases the size of the market for firms but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.

A foreign affiliate earns the flow of global profits $\pi_{Ft} = (p_F - w_S) x_{Ft} L_{St} + (p_F^* - \tau w_S) x_{Ft}^* L_{Nt}$ where $x_{Ft} = p_F^{-\sigma}$ $\bar{f}_F^{-\sigma} c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the foreign affiliate's product and $x_{Ft}^* = (p_F^*)^{-\sigma} c_N / P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the foreign affiliate's product. Profit maximization yields the domestic price $p_F = w_S/\alpha$ and the export price $p_F^* = \tau w_S/\alpha$. We assume that $\tau < w_N/w_S$ to ensure that each foreign affiliate exports to the northern market. For large trade costs, a large wage differential is needed to justify exporting. Trade costs cannot be too high. The flow of global profits for a foreign affiliate is

$$
\pi_{Ft} = \left[\frac{w_S \left(X_F + \tau X_F^*\right)}{\left(\sigma - 1\right)\gamma_F}\right] \frac{L_t}{n_t} \tag{11}
$$

where $X_F \equiv \frac{p_F^{-\sigma} c_S L_{St} n_{Ft}}{P_0^{1-\sigma} L}$ $\frac{\sigma_{c_S L_{St} n_{Ft}}}{P_{St}^{1-\sigma} L_t}$ and $X_F^* \equiv \frac{(p_F^*)^{-\sigma} c_N L_{Nt} n_{Ft}}{P_{Nt}^{1-\sigma} L_t}$ $\frac{C_N E_N t^{\mu} t^{\mu}}{P_{Nt}^{1-\sigma} L_t}$ are population-adjusted aggregate demand terms for foreign affiliate products in the South and the North, respectively. Following the same reasoning as for aggregate demand for northern products, it can be seen that X_F and X_F^* are constant over time in any steady-state equilibrium. Therefore the profit flow of a foreign affiliate only changes because the market size term L_t/n_t changes over time.

After a product variety that a foreign affiliate produces is imitated by a southern firm (an I-

firm), the southern firm earns the flow of global profits $\pi_{It} = (p_I - \zeta w_S) x_{It} L_{St} + (p_I^* - \tau \zeta w_S) x_{It}^* L_{Nt}$ where $x_{It} = p_f^{-\sigma}$ $I^{-\sigma} c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the I-firm's product and $x_{It}^* = (p_I^*)^{-\sigma} c_N / P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the I-firm's product. Without competition, an I-firm would set the monopoly price $\zeta w_S/\alpha$ in the southern market. But an I-firm is in competition with the foreign affiliate whose product it has imitated. Given that the foreign affiliate cannot lower its price below its marginal cost w_S , we assume that $\zeta > \alpha$, so $\zeta w_S/\alpha$ exceeds the marginal cost of the foreign affiliate w_S . Then the limit price $p_I = w_S$ is less than the monopoly price and it is profit-maximizing for the I-firm to practice limit-pricing in its domestic market. By setting $p_I = w_S$, the I-firm gets all the consumers and drives the foreign affiliate out of business. In the export market, it is also profit-maximizing for the I-firm to practice limit-pricing by setting $p_I^* = \tau w_S$. With these prices, the global profit flow of a southern firm that has imitated a foreign affiliate's product can be expressed as

$$
\pi_{It} = \left[\frac{w_S\left(1-\zeta\right)\left(X_I + X_I^*\right)}{\gamma_I}\right] \frac{L_t}{n_t} \tag{12}
$$

where $X_I \equiv \frac{p_I^{-\sigma} c_S L_{St} n_{It}}{P_I^{1-\sigma} L}$ $\frac{\sigma_{cS} L_{St} n_{It}}{P_{St}^{1-\sigma} L_t}$ and $X_I^* \equiv \frac{(p_I^*)^{-\sigma} c_N L_{Nt} n_{It}}{P_{Nt}^{1-\sigma} L_t}$ $\frac{P_N E_N t^{\mu} I_t}{P_N^{\perp \sigma} L_t}$ are population-adjusted aggregate demand terms for the I-firm varieties in the South and the North, respectively. Note that the profits of an I-firm only change over time because of the market size term L_t/n_t .

After a variety that a northern firm produces is imitated by a southern firm (a C-firm), the southern firm earns the flow of global profits $\pi_{Ct} = (p_C - \zeta w_S)x_{Ct}L_{St} + (p_C^* - \tau \zeta w_S)x_{Ct}^*L_{Nt}$ where $x_{Ct} = p_C^{-\sigma}$ $C^{\sigma}c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the C-firm's product and $x_{Ct}^* = (p_C^*)^{-\sigma} c_N / P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the C-firm's product. Maximizing these profits with respect to p_C yields the monopoly price $\zeta w_S/\alpha$. But the southern firm faces a competitor in the northern firm whose product it has imitated. We restrict attention to the case where $\tau w_N > \zeta w_S/\alpha$ or equivalently $\tau w_N/w_S > \zeta/\alpha$ to guarantee that the southern firm charges the monopoly price $p_C = \zeta w_S/\alpha$ in its domestic market. When it comes to the C-firm's export market, there are two cases to consider, depending on whether trade costs are small or large. If $w_N \ge \tau \zeta w_S/\alpha$ or $w_N/w_S \ge \tau \zeta/\alpha$ (the small trade cost case), then the northern firm has too high marginal cost w_N to effectively compete in the northern market and the southern firm sets the monopoly export price $p_C^* = \tau \zeta w_S/\alpha$. If $w_N < \tau \zeta w_S/\alpha$ or $w_N/w_S < \tau \zeta/\alpha$ (the large trade cost case), then the northern firm has low enough marginal cost w_N so its presence needs to be taken into account and the southern firm sets the limit price $p_C^* = w_N$ in equilibrium.

With these prices, the global profit flow of a southern firm that has imitated a northern firm's product can be expressed as

$$
\pi_{Ct} = \left[\frac{\zeta w_S \left(X_C + \tau X_C^* \right)}{(\sigma - 1) \gamma_C} \right] \frac{L_t}{n_t} \qquad \text{if } \frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha} \tag{13}
$$

$$
\pi_{Ct} = \left[\frac{\zeta w_S X_C}{(\sigma - 1) \gamma_C} + \frac{(w_N - \tau \zeta w_S) X_C^*}{\gamma_C} \right] \frac{L_t}{n_t} \quad \text{if } \frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha} \tag{14}
$$

where $X_C \equiv \frac{p_C^{-\sigma} c_S L_{St} n_{Ct}}{P^{1-\sigma} L}$ $\frac{\sigma_{c_S L_{St} n_{Ct}}}{P_{St}^{1-\sigma} L_t}$ and $X_C^* \equiv \frac{(p_C^*)^{-\sigma} c_N L_{Nt} n_{Ct}}{P_{Nt}^{1-\sigma} L_t}$ $\frac{C_N D_N \mu_0 C_t}{P_{N_t}^{1-\sigma} L_t}$. Again, profits only change if the market size term L_t/n_t changes over time.

The above analysis implies that as a product shifts from being produced by a northern firm to its foreign affiliate and then by a southern firm, the equilibrium price of the product declines in the North $(p_N = w_N/\alpha > p_F^* = \tau w_S/\alpha > p_I^* = \tau w_S)$ as well as in the South $(p_N^* = \tau w_N/\alpha > p_F =$ $w_S/\alpha > p_I = w_S$). This price pattern is consistent with Vernon's (1966) description of the product life cycle, in which multinational firms play a central role.

2.5 Innovation, FDI and Imitation

There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology. To innovate and develop a new product variety, a representative northern firm i must devote $a_N g^{\beta} / n_t^{\theta}$ units of labor to innovative R&D, where a_N is an innovative R&D productivity parameter and n_t is the disembodied stock of knowledge at time t (the total number of varieties that have been developed in the past). The intertemporal knowledge spillover parameter θ can be positive or negative. For $\theta > 0$, R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knowledge increases. In contrast, innovating becomes more difficult at time passes when $\theta < 0$. Grossman and Helpman (1991) assume that intertemporal knowledge spillovers are quite strong and set $\theta = 1$. We will instead follow Jones (1995) by assuming that intertemporal knowledge spillovers are weaker and satisfy $\theta < 1$. This assumption is the key to ruling out strong scale effects. Finally, the externality parameter $\beta > 0$ captures the duplicative nature of innovative R&D. When all firms do more innovative R&D $(g \equiv \dot{n}_t/n_t$ is higher), $\beta > 0$ means that the individual firm must do more innovative R&D in order to develop a new product variety.

Given this technology, the flow of new products developed by northern firm i is

$$
\dot{n}_{it} = \frac{l_{Rit}}{a_N g^{\beta} / n_t^{\theta}} = \frac{n_t^{\theta} l_{Rit}}{a_N g^{\beta}}
$$

where \dot{n}_{it} is the time derivative of n_{it} and l_{Rit} is the labor used for innovative R&D by firm i ("R" for R&D). Summing over individual northern firms, the aggregate flow of new products developed in the North is

$$
\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N g^{\beta}} = \left[\frac{n_t^{\theta + \beta} L_{Rt}}{a_N} \right]^{1/(1+\beta)}
$$
\n(15)

where $L_{Rt} \equiv \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities. Thus, the parameter β measures the degree of decreasing returns to innovative R&D at the industry level. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the industry level (point estimates of $1/(1 + \beta)$ lie between 0.1 and 0.6 according to Kortum (1993), which corresponds to β values between .66 and 9).

In any steady-state equilibrium, the share of labor employed in innovative R&D must be constant over time. Given that the northern supply of labor grows at the population growth rate g_L , northern R&D employment L_{Rt} must grow at this rate as well. Dividing both sides of $\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{4 N \theta^{\beta}}$ $a_N g^\beta$ by n_t yields

$$
g \equiv \frac{\dot{n}_t}{n_t} = \frac{n_t^{\theta - 1} L_{Rt}}{a_N g^{\beta}}.
$$

Since g is constant over time in any steady-state equilibrium, $n_t^{\theta-1}$ and L_{Rt} must grow at offsetting rates, that is, $(\theta - 1) \frac{\dot{n}_t}{n_t} + \frac{\dot{L}_{Rt}}{L_{Rt}}$ $\frac{L_{Rt}}{L_{Rt}} = (\theta - 1) g + g_L = 0.$ It immediately follows that

$$
g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1 - \theta}.\tag{16}
$$

Thus, the steady-state rate of innovation g is pinned down by parameter values and is proportional to the population growth rate g_L . As in Jones (1995), the parameter restriction $\theta < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite (given that there is positive population growth).

We can now solve for the steady-state rate of economic growth. The representative northern consumer has utility $u_{Nt} = c_N / P_{Nt}$ and the representative southern consumer has utility $u_{St} =$ c_S/P_{St} . In steady-state equilibrium, individual consumer expenditure is constant over time but consumer utility nevertheless grows because the price indexes fall over time. Since both $P_{Nt}^{1-\sigma}$ and

 $P_{St}^{1-\sigma}$ both grow over time at the rate g, it follows that consumer utility growth is

$$
g_u \equiv \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = \frac{g}{\sigma - 1} = \frac{g_L}{(1 - \theta)(\sigma - 1)}.
$$
\n(17)

With consumer utility in both regions being proportional to consumer expenditure holding prices fixed, consumer utility growth equals real wage growth and we use it as our measure of economic growth.

Equation (17) implies that public policy changes like trade liberalization (a decrease in τ) have no effect on the steady-state rate of economic growth. In this model, growth is "semi-endogenous." We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well. Further evidence for equation (17) is provided by Venturini (2010). Looking at US manufacturing industry data for the period 1973-1996, he finds that semi-endogenous growth models (where public policies do not have long-run growth effects) have better empirical support than fully-endogenous growth models (where public policies have long-run growth effects).

We can now define relative R&D difficulty. In the unit labor requirement for innovation $a_N g^{\beta}/n_t^{\theta}$, the term $n_t^{-\theta}$ is a measure of (absolute) R&D difficulty. It increases over time if $\theta < 0$ and decreases over time if $\theta \in (0,1)$. By taking the ratio of R&D difficulty and the market size term L_t/n_t , we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market).

$$
\delta \equiv \frac{n_t^{-\theta}}{L_t/n_t} = \frac{n_t^{1-\theta}}{L_t} \tag{18}
$$

To see that δ is constant over time in steady-state equilibrium, note that $\frac{\dot{\delta}}{\delta} = (1 - \theta) \frac{\dot{n}_t}{n_t}$ $\frac{\dot{n}_t}{n_t} - \frac{\dot{L}_t}{L_t} =$ $(1 - \theta) \frac{g_L}{1 - \theta} - g_L = 0.$

To learn how to produce a northern variety in the South, the foreign affiliate of a northern firm must devote $a_F \phi^{\beta}/n_t^{\theta}$ units of labor to adaptive R&D, where a_F is an adaptive R&D productivity parameter that can be throught of as measuring the friendliness of southern FDI-related policies and $\phi \equiv (\dot{n}_{Ft} + \dot{n}_{It})/n_{Nt}$ is the FDI rate, the rate at which northern varieties shift to the South as a result of adaptive R&D done by foreign affiliates. The externality parameter β now captures the duplicative nature of adaptive R&D. Taking into account that adaptation is followed by imitation, the number of varieties that have been succesfully adapted for southern production by firm i increases over time according to

$$
\dot{n}_{Fit} + \dot{n}_{lit} = \frac{l_{Fit}}{a_F \phi^{\beta} / n_t^{\theta}} = \frac{n_t^{\theta} l_{Fit}}{a_F \phi^{\beta}}
$$

where $\dot{n}_{Fit} + \dot{n}_{lit}$ is the time derivative of the number of varieties that firm i is responsible for moving to the South and l_{Fit} is the labor used for adaptive R&D by firm i ("F" for FDI). Summing over individual foreign affiliates, the aggregate flow of varieties to the South through FDI is given by

$$
\dot{n}_{Ft} + \dot{n}_{It} = \frac{n_t^{\theta} L_{Ft}}{a_F \phi^{\beta}}
$$
\n(19)

where $L_{Ft} = \sum_i l_{Fit}$ is the total amount of southern labor employed in adaptive R&D activities.

To learn how to produce a foreign affiliate variety, a southern firm must devote $a_I \iota_S^{\beta}$ S/n_t^{θ} units of labor to imitative R&D, where a_I is an imitative R&D productivity parameter, $\iota_S \equiv \dot{n}_{It}/n_{Ft}$ is the rate at which a southern firm imitates a foreign affiliate variety and the externality parameter $\beta > 0$ captures the duplicative nature of imitative R&D. When all southern firms do more imitative R&D (the imitation rate ι_S is higher), $\beta > 0$ means that individual southern firms must do more imitative R&D to learn how to produce a foreign affiliate variety. We interpret a_I as measuring the strength of IPR protection in the South and study what happens when a_I changes. An increase in a_I means stronger IPR protection in the South (as in compliance with the regulations of the TRIPs agreement). Given this technology, the flow of foreign affiliate varieties that a southern firm i imitates is

$$
\dot{n}_{Iit} = \frac{l_{Iit}}{a_I \iota_S^{\beta} / n_t^{\theta}} = \frac{n_t^{\theta} l_{Iit}}{a_I \iota_S^{\beta}}
$$

where l_{Iit} is the labor used for imitative R&D by firm *i*. Summing over individual southern firms, the aggregate flow of foreign affiliate varieties that southern firms imitate is

$$
\dot{n}_{It} = \frac{n_t^{\theta} L_{It}}{a_I t_S^{\beta}}
$$
\n(20)

where $L_{It} \equiv \sum_i l_{Iit}$ is the total amount of southern labor employed in imitating foreign affiliate varieties.

To learn how to produce a northern variety in the South, a southern firm must devote $da_I \iota_N^{\beta}$ $\frac{\beta}{N}/n_t^\theta$ units of labor to imitative R&D, where $d > 1$ is a "distance" parameter that captures the extra cost of imitating northern-produced varieties and $\iota_N \equiv \dot{n}_{Ct}/n_{Nt}$ is the rate at which southern firms imitate northern-produced varieties. Given this technology, the flow of northern varieties that southern firm *i* imitates is

$$
\dot{n}_{Cit} = \frac{l_{Cit}}{da_I l_N^{\beta}/n_t^{\theta}} = \frac{n_t^{\theta} l_{Cit}}{da_I l_N^{\beta}}
$$

where l_{Cit} is the labor used by firm i to imitate northern varieties. Summing over individual southern firms, the aggregate flow of northern varieties that southern firms imitate is

$$
\dot{n}_{Ct} = \frac{n_t^{\theta} L_{Ct}}{da_I \iota_N^{\beta}}
$$
\n(21)

where $L_{C_t} \equiv \sum_i l_{Cit}$ is the total amount of southern labor employed in imitating northern varieties.

While southern firms can do imitative R&D, we assume that southern firms are not capable of doing innovative R&D. The conclusions reached in this paper in support of the TRIPs agreement become even stronger if southern firms can do innovative R&D, because then there is an additional benefit of TRIPs: more innovation by southern firms.

2.6 R&D Incentives

Let v_{Nt} denote the expected discounted profits associated with innovating in the North at time t. The R&D labor used to develop one new variety is $a_N g^{\beta}/n_t^{\theta}$ and the cost of developing this variety is $w_N a_N g^{\beta} / n_t^{\theta}$. Since there is free entry into innovative R&D activities in the North, the cost of innovating must be exactly balanced by the benefit of innovating in equilibrium:

$$
v_{Nt} = \frac{w_N a_N g^{\beta}}{n_t^{\theta}}.\tag{22}
$$

Let v_{Ft} denote the expected discounted profits that a foreign affiliate earns from producing a variety in the South at time t. The foreign affiliate uses $a_F \phi^{\beta}/n_t^{\theta}$ units of adaptive R&D labor to transfer production of one variety to the South and the cost of this transfer is $w_S a_F \phi^{\beta}/n_t^{\theta}$. The benefit of the transfer is not the expected discounted profits that a firm could earn from moving its production to the South v_{Ft} but the gain in expected discounted profits $v_{Ft} - v_{Nt}$, since the firm is already earning profits from producing in the North. Since the cost of technology transfer must be exactly balanced by the benefit in steady-state equilibrium, we obtain

$$
v_{Ft} - v_{Nt} = \frac{w_S a_F \phi^\beta}{n_t^\theta}.
$$
\n
$$
(23)
$$

When techology transfer occurs, each foreign affiliate pays its parent firm the royalty payment v_{Nt} for the use of its technology in the South, since the adaptive R&D accounts for the increment in the firm's value $v_{Ft} - v_{Nt}$ which is less than the foreign affiliate's market value v_{Ft} .

Let v_{It} denote the expected discounted profits from producing a foreign affiliate-imitated variety. To learn how to produce a foreign affiliate variety, a southern firm devotes $a_I \iota_S^{\beta}$ $\frac{\beta}{S}/n_t^{\theta}$ units of labor to imitative R&D and incurs the cost $w_S a_I \iota_S^{\beta}$ S/n_t^{θ} . Since the benefit of imitating a foreign affiliate variety equals the cost in steady-state equilibrium, we obtain

$$
v_{It} = \frac{w_S a_I t_S^{\beta}}{n_t^{\theta}}.\tag{24}
$$

Finally, let v_{C_t} denote the expected discounted profits from producing a northern-imitated variety in the South. A southern firm devotes $da_I l_N^{\beta}$ N/n_t^{θ} units of labor to imitative R&D to learn how to produce a northern variety in the South and incurs the cost $w_S da_I \iota_N^{\beta}$ $\frac{\beta}{N}$ / n_t^{θ} . Since the benefit of imitating a northern variety must equal the cost in steady-state equilibrium, we obtain

$$
v_{Ct} = \frac{w_S da_I \iota_N^{\beta}}{n_t^{\theta}}.\tag{25}
$$

We assume that there is a stock market that channels household savings to firms that engage in R&D in each region and helps households to diversify the risk of holding stocks issued by these firms. Since there is no aggregate risk in each region, it is possible for households to earn a safe return by holding the market portfolio in each region. Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate ρ .

For a northern firm i , the relevant no-arbitrage condition is

$$
(\pi_{Nt} - w_{S}l_{Fit}) dt + \dot{v}_{Nt}dt + (\dot{n}_{Fit} + \dot{n}_{Fit}) dt (v_{Ft} - v_{Nt}) - (\iota_N dt)v_{Nt} = \rho v_{Nt}dt.
$$

The northern firm earns the profit flow π_{N_t} during the time interval dt but also incurs the adaptive R&D expenditure flow $w_S l_{Fit} dt$ during this time interval. In addition, the firm experiences the gradual capital gain $\dot{v}_{Nt}dt$ during the time interval dt and its market value jumps up by $(v_{Ft} - v_{Nt})$ for each product that it succeeds in moving to the South. The firm succeeds in moving $(\dot{n}_{Fit} + \dot{n}_{Iit}) dt$ products to the South during the time interval dt. With the probability $\iota_N dt$ the firm's product is successfully imitated by a southern firm during the time interval dt , in which case the northern firm is driven out of business and experiences a total capital loss. To rule out arbitrage opportunities for investors, the rate of return for the northern firm must be the same as the return on an equal sized investment in a risk free bond $\rho v_{N t} dt$. Now

$$
(\dot{n}_{Fit} + \dot{n}_{Fit}) (v_{Fit} - v_{Nt}) = \frac{n_t^{\theta} l_{Fit}}{a_F \phi^{\beta}} \frac{w_S a_F \phi^{\beta}}{n_t^{\theta}} = w_S l_{Fit}
$$

and $v_{Nt} = w_N a_N g^{\beta} / n_t^{\theta}$ implies that $\frac{\dot{v}_{Nt}}{v_{Nt}} = -\theta \frac{\dot{n}_t}{n_t}$ $\frac{n_t}{n_t} = -\theta g$. Thus, after dividing by $v_{Nt}dt$, the noarbitrage condition simplifies to $\frac{\pi_{Nt}}{v_{Nt}} - \theta g - \iota_N = \rho$ or $v_{Nt} = \frac{\pi_{Nt}}{\rho + \theta g + \rho}$ $\frac{\pi_{Nt}}{\rho + \theta g + \iota_N}$. Combining this equation with (22), the northern no-arbitrage condition can be written as

$$
\frac{\pi_{Nt}}{\rho + \theta g + \iota_N} = \frac{w_N a_N g^{\beta}}{n_t^{\theta}}.
$$

In this equation, the left-hand-side is the expected discounted profits from innovating and the righthand-side is the cost of innovating. The northern firm's expected discounted profits or market value is equal to its current profit flow π_{N_t} appropriately discounted by the market interest rate ρ , the capital loss term θg (capital gain if θ is negative) and the imitation rate ι_N . Substituting for $\pi_{N t}$ using (10), dividing both sides by w_N and then by the market size term L_t/n_t yields the steady state northern no-arbitrage condition

$$
\frac{\frac{X_N + \tau X_N^*}{(\sigma - 1)\gamma_N}}{\rho + \theta g + \iota_N} = a_N \delta g^{\beta}.
$$
\n(26)

The left-hand-side of (26) is the market size-adjusted benefit from innovating and the right-handside is the market size-adjusted cost of innovating. In steady-state calculations, we need to adjust for market size L_t/n_t because market size changes over time if $g_L \neq g$ or $\theta \neq 0$. The market sizeadjusted benefit from innovating is higher when the average consumer buys more of each northern variety $(X_N + \tau X_N^*)$, future profits are less heavily discounted $(\rho \downarrow)$, northern firms experience larger capital gains over time $(\theta g \downarrow)$ and northern firms are exposed to a lower imitation rate $(\iota_N \downarrow)$. The market size-adjusted cost of innovating is higher when northern researchers are less productive $(a_N \uparrow)$ and innovating is relatively more difficult $(\delta \uparrow)$.

For a foreign affiliate, the relevant no-arbitrage condition is $\pi_{F t} dt + \dot{v}_{F t} dt - (\iota_S dt) v_{F t} = \rho v_{F t} dt$. The foreign affiliate earns the profit flow π_{F_t} and experiences the gradual capital gain $\dot{v}_{F_t}dt$ during the time interval dt. However, it is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if imitated, which occurs with probability $\iota_S dt$ during the time

interval dt. The earlier equation $v_{Ft} - v_{Nt} = \frac{w_S a_F \phi^{\beta}}{n^{\theta}}$ $\frac{a_F \phi^{\beta}}{n_t^{\theta}}$ implies that $\frac{\dot{v}_{Nt}}{v_{Nt}} = \frac{\dot{v}_{Ft}}{v_{Ft}}$ $\frac{v_{Ft}}{v_{Ft}} = -\theta g$, so dividing the no-arbitrage condition by $v_{Ft}dt$ yields $\frac{\pi_{Ft}}{v_{Ft}} - \theta g - \iota_S = \rho$ or $v_{Ft} = \frac{\pi_{Ft}}{\rho + \theta g + \rho}$ $\frac{\pi_{Ft}}{\rho + \theta g + \iota_S}$. Combining this equation with (22) and (23) , we obtain

$$
v_{Ft} - v_{Nt} = \frac{\pi_{Ft}}{\rho + \theta g + \iota_S} - \frac{w_N a_N g^{\beta}}{n_t^{\theta}} = \frac{w_S a_F \phi^{\beta}}{n_t^{\theta}}
$$

where the left-hand-side is the increase in expected discounted profits from moving production to the South and the right-hand-side is the adaptive R&D cost. The expected discounted profits or market value of the foreign affiliate is equal to its current profit flow π_{F_t} appropriately discounted by the market interest rate ρ , the capital loss term θg and the imitation rate ι_S . Substituting for π_{Ft} using (11), dividing both sides by w_S and then by the market size term L_t/n_t yields the steady state foreign affiliate no-arbitrage condition

$$
\frac{X_F + \tau X_F^*}{(\sigma - 1)\gamma_F} - w a_N \delta g^\beta = a_F \delta \phi^\beta
$$
\n
$$
\rho + \theta g + \iota_S \tag{27}
$$

where $w \equiv w_N/w_S$ is the northern relative wage or the North-South wage ratio. The left-hand-side of (27) is the market size-adjusted benefit from southern adaptation and the right-hand-side is the market size-adjusted cost of southern adaptation. The market size-adjusted benefit is higher when the average consumer buys more of each foreign affiliate variety $(X_F + \tau X_F^* \uparrow)$, future profits are less heavily discounted $(\rho \downarrow)$, foreign affiliates experience larger capital gains over time $(\theta g \downarrow)$ and foreign affiliates are exposed to a lower imitation rate $(\iota_S \downarrow)$. The market size-adjusted cost is higher when foreign affiliate researchers are less productive $(a_F \uparrow)$ and adaptation is relatively more difficult $(\delta \uparrow)$.

For a southern firm imitating a foreign affiliate variety, the relevant no-arbitrage condition is $\pi_{It}dt + \dot{v}_{It}dt = \rho v_{It}dt$. The southern firm earns the profit flow π_{It} and also experiences the gradual capital gain $\dot{v}_{It}dt$ during the time interval dt. The rate of return for an I-firm must equal the rate of return on an equal sized investment in a riskfree bond $\rho v_{It}dt$ in order to rule out arbitrage opportunities for investors. Recall that $v_{It} = w_S a_I l_S^{\beta}$ ${}_{S}^{\beta}/n_t^{\theta}$, which implies that $\frac{\dot{v}_{It}}{v_{It}} = -\theta g$. Dividing the no-arbitrage condition by $v_{It}dt$ yields $\frac{\pi_{It}}{v_{It}} - \theta g = \rho$ and then solving for v_{It} yields $v_{It} = \frac{\pi_{It}}{\rho + \theta g}$. Combining this equation with (24), we obtain

$$
\frac{\pi_{It}}{\rho + \theta g} = \frac{w_S a_I \iota_S^{\beta}}{n_t^{\theta}}.
$$

Substituting for π_{It} using (12), dividing both sides by w_S and then by the market size term L_t/n_t yields the steady state no-arbitrage condition for I-firms

$$
\frac{\frac{(1-\zeta)\left(X_I + \tau X_I^*\right)}{\gamma_I}}{\rho + \theta g} = a_I \delta \iota_S^{\beta}.
$$
\n(28)

For a southern firm that imitates a northern variety, the relevant no-arbitrage condition is $\pi_{C_t}dt + \dot{v}_{C_t}dt = \rho v_{C_t}dt$. During the time interval dt, the C-firm earns the profit flow π_{C_t} and experiences the gradual capital gain $\dot{v}_{Ct}dt$. As for the other types of firms, the rate of return for the C-firm must be the same as the return on an equal-sized investment in a riskfree bond. Recall that $v_{Ct} = w_S da_I \iota_N^{\beta}$ N/N_t^{θ} , which implies that $\frac{\dot{v}_{Ct}}{v_{Ct}} = -\theta g$. Dividing the no-arbitrage condition by $v_{Ct}dt$ yields $\frac{\pi_{Ct}}{v_{Ct}} - \theta g = \rho$ and then solving for v_{Ct} yields

$$
\frac{\pi_{Ct}}{\rho + \theta g} = \frac{w_S da_I \iota_N^{\beta}}{n_t^{\theta}}
$$

.

The southern firm imitating a northern variety will base its pricing decision in the export market on the size of trade costs and hence there will be one steady-state no-arbitrage condition for each of the two trade cost cases described earlier. In the small trade cost case, the southern firm can set the monopoly price in the northern market without having to fear competition from the northern firm whose variety it has imitated. In the large trade cost case, the southern firm needs to take competition from the northern firm into account and practice limit pricing in its export market. Substituting for π_{C_t} using (13) and (14), dividing both sides by w_S and then by the market size term L_t/n_t yields the steady state C-firm no-arbitrage condition:

$$
\frac{\frac{\zeta(X_C + \tau X_C^*)}{(\sigma - 1)\gamma_C}}{\rho + \theta g} = d a_I \delta \iota_N^{\beta} \quad \text{if } \frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}
$$
\n
$$
\frac{\zeta X_C}{(\sigma - 1)\gamma_C} + \frac{(w - \tau \zeta)X_C^*}{\gamma_C} = d a_I \delta \iota_N^{\beta} \quad \text{if } \frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}.\tag{29}
$$

2.7 Labor Markets

Labor markets is perfecty competitive and wages adjust instantaneously to equate labor demand and labor supply. In the North, labor is employed in either innovative R&D or in production. Each innovation requires $a_N g^{\beta}/n_t^{\theta}$ units of labor, so total employment of labor in innovative R&D is $\left(a_{N}g^{\beta}/n_{t}^{\theta}\right)\dot{n}_{t} = a_{N}\frac{n_{t}^{1-\theta}}{L_{t}}g^{\beta}\frac{\dot{n}_{t}}{n_{t}}$ $\frac{\dot{m}_t}{m_t}L_t = a_N \delta g^{1+\beta} L_t$. Northern firms use $\frac{p_N^{-\sigma} c_N L_{Nt}}{p_N^{1-\sigma}}$ $\frac{\sigma_{c_N} L_{Nt}}{P_{Nt}^{1-\sigma}} + \tau \frac{\left(p_N^*\right)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}}$ $\frac{p_{sLst}}{P_{St}^{1-\sigma}}$ units of labor for each variety produced and there are n_{N_t} varieties produced, so total employment in northern production is $\frac{p_N^{-\sigma} c_N L_{Nt} n_{Nt}}{p!-\sigma}$ $\frac{P_N^2 L_{Nt}n_{Nt}}{P_{Nt}^{1-\sigma}} + \tau \frac{\left(p_N^*\right)^{-\sigma} c_S L_{St} n_{Nt}}{P_{St}^{1-\sigma}}$ $\frac{c_{S}L_{St}n_{Nt}}{P_{St}^{1-\sigma}} = (X_N + \tau X_N^*)L_t$. As L_{Nt} denotes the supply of labor in the North, full employment requires that $L_{Nt} = a_N \delta g^{1+\beta} L_t + (X_N + \tau X_N^*) L_t$. Evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the North:

$$
L_{N0} = \left[a_N \delta g^{1+\beta} + X_N + \tau X_N^*\right] L_0.
$$
\n(30)

In the South, labor is employed in adaptive R&D by foreign affiliates, imitative R&D directed towards foreign affiliate varieties, imitative R&D directed towards northern varieties, foreign affiliate production, and southern production by the two types of imitating firms (I-firms and C-firms). Each variety transferred to the South by a foreign affiliate requires $a_F \phi^{\beta}/n_t^{\theta}$ units of labor, so total employment in adaptive R&D is $(a_F \phi^{\beta}/n_t^{\theta}) (\dot{n}_{Ft} + \dot{n}_{It}).$ Each foreign affiliate variety imitated by a southern firm requires $a_I \iota_S^{\beta}$ $\frac{\beta}{S}/n_t^{\theta}$ units of imitative R&D labor, so total employment in imitative R&D directed towards foreign affiliate varieties is $(a_I \iota_S^{\beta}$ $\frac{\beta}{S}/n_t^{\theta}$) \dot{n}_{It} . Similarly, each northern variety imitated by a southern firm requires $da_I l_N^{\beta}$ N/n_t^{θ} units of labor, so total employment in imitative R&D directed towards northern varieties is $(da_I l_N^{\beta}$ $\frac{\beta}{N}$ /n θ /n $_{Ct}$. Turning to production in the South, a foreign affiliate uses $\frac{p_F^{-\sigma} c_S L_{St}}{p_F^{1-\sigma}}$ $\frac{r^{\sigma}c_{S}L_{St}}{P_{St}^{1-\sigma}}+\tau \frac{\left(p_{F}^{*}\right)^{-\sigma}c_{N}L_{Nt}}{P_{Nt}^{1-\sigma}}$ $\frac{c_N L_{Nt}}{P_{Nt}^{1-\sigma}} = \frac{X_F L_t}{n_{Ft}}$ $\frac{K_F L_t}{n_{F t}} + \tau \frac{X_F^* L_t}{n_{F t}}$ $\frac{\Gamma_F L_t}{n_{Ft}}$ units of labor for each variety produced and there are n_{Ft} varieties produced, so total employment in foreign affiliate production is $\left(\frac{X_F L_t}{n_E}\right)$ $\frac{K_F L_t}{n_{Ft}} + \tau \frac{X_F^* L_t}{n_{Ft}} \bigg) n_{Ft} = (X_F + \tau X_F^*) L_t$. A southern firm imitating a foreign affiliate variety uses $\zeta\left(\frac{p_I^{-\sigma}c_S L_{St}}{D^{1-\sigma}}\right)$ $\frac{\sigma_{c_S L_{St}}}{P_{St}^{1-\sigma}} + \tau \frac{\left(p_I^*\right)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}$ $\left(\frac{C_N L_{Nt}}{P_{Nt}^{1-\sigma}} \right) = \zeta \left(X_I \frac{L_t}{n_{It}} \right)$ $\frac{L_t}{n_{It}} + \tau X_I^* \frac{L_t}{n_{It}}$ units of labor for each variety produced, and there are n_{It} such varieties produced, so total employment by I-firms is $\zeta\left(X_I\frac{L_t}{n_I}\right)$ $\frac{L_t}{n_{It}} + \tau X_I^* \frac{L_t}{n_{It}} \Big) n_{It} =$ $\zeta(X_I + \tau X_I^*) L_t$. A southern firm imitating a northern variety uses $\zeta\left(\frac{p_C^{-\sigma} c_S L_{St}}{p_L^{1-\sigma}}\right)$ $\frac{P_{CS}^{\sigma} L_{St}}{P_{St}^{1-\sigma}} + \tau \frac{\left(p_C^*\right)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}$ $\frac{e_N L_{Nt}}{P_{Nt}^{1-\sigma}}\bigg) =$ $\zeta\left(X_C \frac{L_t}{n_C}\right)$ $\frac{L_t}{n_{Ct}} + \tau X_C^* \frac{L_t}{n_{Ct}}$ units of labor for each variety produced, and there are n_{Ct} such varieties produced, so total employment by C-firms is $\zeta\left(X_{C}\frac{L_{t}}{n_{C}}\right)$ $\frac{L_t}{n_{Ct}} + \tau X_C^* \frac{L_t}{n_{Ct}} \bigg) n_{Ct} = \zeta (X_C + \tau X_C^*) L_t.$ As L_{St} denotes the supply of labor in the South, full employment requires that

$$
L_{St} = \frac{a_F \phi^{\beta}}{n_t^{\theta}} (\dot{n}_{Ft} + \dot{n}_{It}) + \frac{a_I \iota_S^{\beta}}{n_t^{\theta}} \dot{n}_{It} + \frac{d a_I \iota_N^{\beta}}{n_t^{\theta}} \dot{n}_{Ct} + (X_F + \tau X_F^*) L_t + \zeta (X_I + \tau X_I^*) L_t + \zeta (X_C + \tau X_C^*) L_t.
$$

Using the definitions of ϕ , ι_S , ι_N , δ and evaluating at time $t = 0$, we obtain the steady-state full

employment of labor condition for the South:

$$
L_{S0} = \left(a_F \phi^{1+\beta} \gamma_N + a_I \iota_S^{1+\beta} \gamma_F + d a_I \iota_N^{1+\beta} \gamma_N \right) \delta L_0 + \left[X_F + \tau X_F^* + \zeta \left(X_I + \tau X_I^* + X_C + \tau X_C^* \right) \right] L_0.
$$
 (31)

2.8 Aggregate Demand

To solve the model, we need steady-state values for the aggregate demand terms $X_N, X_N^*, X_I, X_I^*,$ X_C and X_C^* . Solving for the ratio X_N/X_F^* yields

$$
\frac{X_N}{X_F^*} = \frac{\frac{p_N^{-\sigma}c_N L_{Nt}n_{Nt}}{p_{Nt}^{1-\sigma}L_t}}{\frac{(p_F^*)^{-\sigma}c_N L_{Nt}n_{Ft}}{p_{Nt}^{1-\sigma}L_t}} = \left(\frac{p_N}{p_F^*}\right)^{-\sigma} \frac{n_{Nt}/n_t}{n_{Ft}/n_t} = \left(\frac{\frac{w_N}{\alpha}}{\frac{\tau w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_N}{\gamma_F} = \left(\frac{\tau}{w}\right)^{\sigma} \frac{\frac{g}{g + \phi + \iota_N}}{\frac{\phi}{g + \phi + \iota_N} \frac{g}{g + \iota_S}} = \left(\frac{\tau}{w}\right)^{\sigma} \frac{g + \iota_S}{\phi}
$$

and by doing similar calculations looking at other ratios, we obtain that

$$
X_N = X_F^* \left(\frac{\tau}{w}\right)^\sigma \frac{g + \iota_S}{\phi}
$$

$$
X_N^* = X_F \left(\frac{1}{\tau w}\right)^\sigma \frac{g + \iota_S}{\phi}
$$

$$
X_I = X_F \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}
$$

$$
X_I^* = X_F^* \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}
$$

$$
X_C = X_F \left(\frac{1}{\zeta}\right)^\sigma \frac{\left(g + \iota_S\right) \iota_N}{\phi g}
$$

and

$$
X_C^* = \begin{cases} X_F^* \left(\frac{1}{\zeta}\right)^{\sigma} \frac{(g+\iota_S)\iota_N}{\phi g} & \text{if } \frac{w_N}{w_S} \ge \frac{\tau\zeta}{\alpha} \\ X_F^* \left(\frac{\tau}{\alpha w}\right)^{\sigma} \frac{(g+\iota_S)\iota_N}{\phi g} & \text{if } \frac{w_N}{w_S} < \frac{\tau\zeta}{\alpha}. \end{cases}
$$

2.9 Consumer Expenditure and Asset Ownership

To determine consumer expenditures c_N and c_S , we need to specify who owns the firms and how wealth is distributed between the North and the South. We assume that R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings. Then in equilibrium, northern firms end up being owned by northern consumers, southern firms end up being owned by southern consumers and foreign affiliates end up being jointly owned by consumers in both regions.

Let A_{Nt} denote the aggregate value of northern financial assets and A_{St} denote the aggregate value of southern financial assets. The aggregate value of all financial assets is $A_t \equiv A_{Nt} + A_{St} =$ $n_{N t}v_{N t} + n_{F t}v_{F t} + n_{I t}v_{I t} + n_{C t}v_{C t}$. Since consumer savings within the South finance the R&D investments in the South, $A_{St} = n_{Ft} (v_{Ft} - v_{Nt}) + n_{It} v_{It} + n_{Ct} v_{Ct}$. Substituting into this expression using the firm values (23) , (24) and (25) , we obtain

$$
A_{St} = w_S \delta L_t \left[\gamma_F a_F \phi^{\beta} + \gamma_I a_I \iota_S^{\beta} + \gamma_C da_I \iota_N^{\beta} \right].
$$

Since $A_{Nt} = A_t - A_{St} = n_{Nt}v_{Nt} + n_{Ft}v_{Nt}$, substituting into this expression using the firm value (22) yields

$$
A_{Nt} = w_N L_t \left(\gamma_N + \gamma_F \right) a_N \delta g^{\beta}.
$$

Let \tilde{a}_{it} denote the financial asset holdings of the typical consumer in region i $(i = N, S)$. The intertemporal budget constraint of a typical consumer in region i is $\dot{\tilde{a}}_{it} = w_i + \rho \tilde{a}_{it} - c_i - g_L \tilde{a}_{it}$. In any steady-state equilibrium where the wage rates w_i are constant over time, we must have that $\dot{\tilde{a}}_{it} = 0$ and it follows that $c_i = w_i + (\rho - g_L) \tilde{a}_{it}$. For the typical northern consumer, $\tilde{a}_{Nt} = A_{Nt}/L_{Nt}$ and for the typical southern consumer, $\tilde{a}_{St} = A_{St}/L_{St}$. Setting $w_S = 1$ and $w_N = \frac{w_N}{w_S}$ $\frac{w_N}{w_S} \equiv w$, it follows that typical northern and southern consumer expenditure levels are given by

$$
c_N = w \left[1 + (\rho - g_L) \left(\gamma_N + \gamma_F \right) a_N \delta g^\beta \frac{L_0}{L_{N0}} \right]
$$
 (32)

and

$$
c_S = 1 + (\rho - g_L) \delta \left[\gamma_F a_F \phi^\beta + \gamma_I a_I \iota_S^\beta + \gamma_C d a_I \iota_N^\beta \right] \frac{L_0}{L_{S0}}.
$$
 (33)

Having solved for consumer expenditures c_N and c_S , we can determine the ratio X_F^*/X_F and simplifying yields the steady-state asset condition

$$
\frac{X_F^*}{X_F} = \left(\frac{p_F}{p_F^*}\right)^{\sigma} \frac{c_N}{c_S} \frac{L_{N0}}{L_{S0}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} \tag{34}
$$

where

$$
\frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} = \frac{\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}}{\gamma_N p_N^{1-\sigma} + \gamma_F (p_F^*)^{1-\sigma} + \gamma_I (p_I^*)^{1-\sigma} + \gamma_C (p_C^*)^{1-\sigma}}
$$

is constant over time.

Thus, solving this model for a steady-state equilibrium reduces to solving a system of 7 equations [(26), (27), (28), (29), (30), (31), (34)] in 7 unknowns $[w, \delta, \phi, \iota_N, \iota_S, X_F, X_F^*]$, where the 7 equations are: four R&D conditions (innovative, adaptive, two imitative), two labor market conditions (North and South) and one asset condition.

3 Numerical Results

3.1 Parameter Values

The system of 7 equations in 7 unknowns is solved numerically. We calibrate the model to fit the world in 1990, prior to the signing of the TRIPs agreement, and in 2005, after its implementation. In our computer simulations, we used the following benchmark parameter values: $\rho = 0.07$, $\alpha = 0.714$, $g_L = 0.014, \ \theta = 0.72, \ L_{N0} = 1, \ L_{S0} = 2, \ \zeta = 0.9, \ \beta = 1, \ \tau = 1.54 \text{ for } 1990 \text{ and } \tau = 1.33 \text{ for } 2005,$ $a_N = 1, a_F = 100.5, d = 261, a_I = 2.96$ for 1990 and $a_I = 11.8$ for 2005.⁴

The subjective discount rate ρ is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the 20th century (Mehra and Prescott, 1985). The measure of product differentiation α determines the markup of price over marginal cost $1/\alpha$. It is set at 0.714 to generate a northern markup of 40 percent, which is within the range of estimates from Basu (1996) and Norrbin (1993). The parameter g_L is set at 0.014 to reflect a 1.4 percent population growth rate. This was the average annual world population growth rate during the 1990s according to the World Development Indicators (World Bank, 2011). The steady-state economic growth rate is calculated from $g_u = g_L/((\sigma - 1)(1 - \theta))$. In order to generate a steadystate economic growth rate of 2 percent, consistent with the average US GDP per capita growth rate from 1950 to 1994 (Jones, 1995), the R&D spillover parameter θ is set at 0.72. Since only the ratio L_{N0}/L_{S0} matters, we set $L_{N0} = 1$ and $L_{S0} = 2$ so L_{N0}/L_{S0} equals the ratio of working-age population in high-income countries to that in middle-income countries (World Bank, 2003). The cost parameter ζ is set at 0.9 so southern firms have 10 percent lower production costs than foreign affiliates. Empirical studies on patents and R&D suggests that there are significant decreasing returns to R&D at the industry-level. Given that point estimates of $1/(1+\beta)$ lie between 0.1 and 0.6 (Kortum, 1993), we set $\beta = 1$ which yields the intermediate value $1/(1 + \beta) = 0.5$.

During the time period 1990-2005 when the TRIPs agreement was being implemented, North-South trade costs were falling. We use the micro-founded measure of bilateral trade costs developed

⁴The MATLAB files used to solve the model can be obtained from the authors upon request.

by Novy (2011) that indirectly infers trade frictions from observable trade data. By linear extrapolation of the bilateral trade cost estimates between the US and Mexico in 1970 and 2000, we obtain a tariff-equivalent of 54 percent for 1990 ($\tau = 1.54$) and 33 percent in 2005 ($\tau = 1.33$).

The remaining parameters are the R&D productivity parameters a_N (innovation), a_F (adaption), a_I (southern imitation) and the "distance" parameter d that captures the extra cost of imitating northern-produced varieties. Since only the relative difference between the R&D productivity parameters matters, we can normalize a_N to 1. The parameters $a_F = 100.5$ and $d = 261$ are set so FDI and imitation start off being equally important modes of international technology transfer (the FDI rate ϕ equals the northern imitation rate ι_N in the 1990 benchmark), and northern consumer expenditure is 2.5 times as large as southern consumer expenditure $(c_N/c_S = 2.5)$. The average US-Mexico consumption-share adjusted GDP per worker ratio during the time period 1990-2005 was around 2.5 (Heston, Summers and Aten, 2011). The parameter a_I is our measure of IPR protection in the South. Stronger IPR protection makes imitation more difficult; hence stronger IPR protection is captured by a higher a_I . We set a low value for a_I in 1990 to capture weak IPR protection in the South prior to the TRIPs agreement. By setting $a_I = 2.96$, we obtain a high southern imitation rate $\iota_S = .25$ in the 1990 benchmark (one out of four products produced by foreign affiliates is copied each year). Finally, we set a higher value for a_I in 2005 to capture stronger IPR protection after the implementation of the TRIPs agreement. In particular, we set $a_I = 11.8$ so the model is consistent with the evidence of a ten-fold increase in the FDI inflow to developing countries between 1990 and 2005 (UNCTAD, 2011).

In the model, the FDI inflow to developing countries is captured by L_{F_t} (the total amount of southern labor devoted to adaptive R&D activities by foreign affiliates multiplied by the southern wage rate $w_S = 1$). Rewriting (19) using the definitions for the FDI rate ϕ , the relative R&D difficulty δ and the variety share of northern firms γ_N , the FDI inflow measure can be written as $L_{Ft} = \phi^{1+\beta} \gamma_N \delta a_F L_t$. The ratio L_{Ft}/L_t is constant over time in any steady-state equilibrium so we obtain

$$
L_{F0} = \phi^{1+\beta} \gamma_N \delta a_F L_0.
$$

In 1990 the FDI inflow to developing countries (including transition economies) was 34.9 billion US dollars and in 2005 the FDI inflow was 363.4 billion US dollars (UNCTAD, 2011). This represents a roughly ten-fold increase in the FDI inflow to developing countries measured in current prices. Adjusting the FDI inflow in 1990 for population growth and inflation from 1990 to 2005 generates an expected FDI inflow of 59.7 billion US dollars for 2005.⁵ The ratio of the observed FDI inflow to this expected FDI inflow yields a six-fold increase in the FDI inflow to developing countries during the time period 1990-2005 that can be attributed to policy changes (the decrease in τ and increase in a_I). So we set $a_I = 2.96$ in 1990 and $a_I = 11.8$ in 2005 to assure a small FDI inflow L_{F0} in 1990 and a six-fold increase in L_{F0} by 2005.

3.2 The Main Results

The model is solved numerically using the benchmark parameters discussed in Section 3.1. Columns 1 and 2 of Table 1 show the results for the 1990 and 2005 benchmarks. Going from the 1990 to the 2005 benchmark, the model is able to replicate the large 10-fold observed increase in FDI inflows to developing countries from 1990 to 2005 (L_{F0} increases from .015 to .089, a 6-fold increase). The model can also account for large North-South wage differences. In the 1990 benchmark, the northern wage w_N is 2.34 times as high as the southern wage $w_S (w_N/w_S = 2.34)$.

During the time period from 1990 to 2005, WTO-member developing countries implemented stronger IPR protection but also reduced their trade barriers. In order to disentangle the welfare effects of stronger IPR protection and trade liberalization, we solve the model for two counterfactual scenarios. In the first counterfactual, presented in column 3 of Table 1, it is assumed that trade costs are at their 2005 level ($\tau = 1.33$), but that southern IPR protection remains unchanged from its 1990 pre-TRIPs level ($a_I = 2.96$). This would be the case if trade liberalization had followed the same path as in our benchmark scenario, but the TRIPs agreement had not been implemented. In the second counterfactual, presented in column 4, trade costs are kept at the same level as 1990 $(\tau = 1.54)$, but southern IPR protection is at the 2005 post-TRIPs level $(a_I = 11.8)$. This would be the case if the TRIPs agreement had been implemented but not accompanied by lower trade costs.

We find that both the North and the South are better off in the post-TRIPs scenario (comparing columns 1 and 2). Our measure of long-run consumer welfare in the North u_{N0} increases from 364.7 to 394.3 and our measure of long-run welfare for the South u_{S0} increases from 142.0 to 163.1.⁶ The

⁵From 1990 to 2005, the US GDP implicit price deflator increased by 38.4 percent (Federal Reserve Bank of St Louis, 2011). During the same time period, the world population grew by 23.4 percent using the 1.4 percent annual population growth rate. Multiplying the observed FDI inflow in 1990 by the population growth and inflation over the period generates the expected FDI inflow in 2005 in the absence of any policy changes.

⁶Welfare is measured as the consumer utility for the representative consumer at time 0 ($u_{N0} = c_N / P_{N0}$ and $u_{S0} = c_S/P_{S0}$). Steady-state consumer expenditures for the typical northern and southern consumers were derived in (32) and (33). The steady-state price index in each region depends on (i) a constant component of variety shares and product prices, and (ii) the number of varieties n_t . For welfare comparisons across steady-states for different

gain is proportionally larger for the South, where consumer welfare is 15 percent higher in the post-TRIPs case and the northern gain is 8 percent. Taking only lower trade costs into account and setting southern IPR protection at its 1990 level (column 3), the resulting lower price indexes leads to some welfare gains for both northern and southern consumers $(u_{N0}$ increases from 364.7 to 378.4 and u_{S0} increases from 142.0 to 149.2). If instead the model is solved holding North-South

	(1)	(2)	(3)	(4)	(5)
	1990	2005	(A)	$\rm (B)$	(C)
	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.33$	$\tau = 1.54$	$\tau=1.33$
	$a_I = 2.96$	$a_I = 11.8$	$a_I = 2.96$	$a_I = 11.8$	$a_I = 500$
w_N/w_S	2.34	2.17	2.32	2.18	2.22
δ	22.01	22.57	22.01	22.57	23.46
ϕ	.002	.004	.002	.004	.006
ι_S	.250	.049	.250	.049	.001
ι_N	.002	.000	.002	.000	.000
γ_N	.942	.925	.942	.925	.885
γ_F	.005	.035	.005	.035	.112
γ_I	.024	.034	.024	.034	.003
γ_C	.029	.006	.029	.006	.000
L_{F0}	.015	.089	.015	.089	.263
c_N	2.75	2.56	2.73	2.58	2.65
c_S	1.10	1.07	1.10	1.07	1.15
c_N/c_S	2.50	2.39	2.49	2.40	2.31
P_{N0}	.0075	.0065	.0072	.0068	.0063
P_{S0}	.0077	.0066	.0074	.0069	.0064
u_{N0}	364.7	394.3	378.4	379.4	418.2
u_{S0}	142.0	163.1	149.2	155.6	180.4

Table 1: Pre- and post-TRIPs benchmarks and three counterfactual scenarios

trade costs at the 1990 level but assuming that southern IPR protection is at its post-TRIPs level (column 4), consumer utilities in both regions are substantially higher than in the 1990 benchmark scenario, and closer to the 2005 post-TRIPs benchmark scenario $(u_{N0}$ increases from 364.7 to 379.4 and u_{S0} increases from 142.0 to 155.6). Stronger southern IPR protection not only enhances the welfare gains from lower North-South trade costs - most of the welfare gains over the time period can be attributed to stronger IPR protection.

To understand what is driving these welfare gains, we first look at the equilibrium effects of TRIPs. In response to stronger southern IPR protection but not to trade liberalization (comparing columns 1 and 4), the southern imitation rate ι_S falls from 0.250 to 0.049 and the northern imitation

scenarios, we evaluate the price indexes at time 0 using $n_0 = (\delta L_0)^{1/(1-\theta)}$, which follows from the definition of δ .

rate ι_N falls from .002 to almost zero. Faced with this lower imitation risk, northern firms choose to increase their adaptive R&D spending and transfer production to foreign affiliates at a faster rate (the FDI rate ϕ increases from .002 to .004). Consequently, world production is redistributed from the North to the South as the variety share of northern firms γ_N falls from .942 to .925 and the share of world production in the South $(\gamma_F + \gamma_I + \gamma_C)$ increases from .058 to .075. As production is transferred to the South, northern resources are freed up from production activities, causing downward pressure on the northern wage rate (w_N/w_S) decreases from 2.34 to 2.18). Northern firms respond by employing more workers in innovative R&D, leading to a temporary increase in the innovation rate. In the long run, the innovation rate reverts back to its steady-state level $g = g_L/(1-\frac{1}{2})$ θ), which is pinned down by parameter values and is not affected by policy changes. However, the temporary increase in innovation means that relative R&D difficulty becomes permanently higher $(\delta$ increases from 22.01 to 22.57). Comparing relative R&D difficulty in the two counterfactual scenarios (columns 3 and 4) confirms that it is the TRIPs-induced redistribution of production and not trade liberalization that leads to more innovation.

We are now in a position to understand the welfare effects of TRIPs, in particular, why the South benefits from stronger IPR protection. When a_I increases due to TRIPs (from 2.96 to 11.8), southern consumer utility u_{S0} increases by 10 percent (from 142.0 to 155.6). Using the equation $u_{S0} = c_S/P_{S0}$, we can decompose this 10 percent increase into a 2 percent decrease in southern consumer expenditure c_s (from 1.10 to 1.07) and a 12 percent increase in the inverse southern price index P_{S0}^{-1} S_0^{-1} (from 129.87 to 144.93). We discuss first why $c_S = 1 + (\rho - g_L) \delta[\gamma_F a_F \phi^{\beta} + \gamma_I a_I \iota_S^{\beta} + \iota_S^{\beta}]$ $\gamma_C da_I \iota^{\beta}_N$ $\frac{\beta}{N}]\frac{L_0}{L_S}$ $\frac{L_0}{L_{S0}}$ changes and then why $P_{S0}^{-1} = [\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}]$ $\binom{1-\sigma}{C}$ ¹/(σ−1). $n_0^{1/(\sigma-1)}$ 0 changes.

Southern consumer expenditure c_s falls because there is a drop in the total market value of the firms that southern consumers own. This change is mainly driven by what happens to southern firms that have imitated northern products (the C-firms). Due to stronger IPR protection, the number of such firms shrinks drastically (γ_C decreases from .029 to .006) and their total market value falls as a result $(\delta \gamma_C da_I \iota^{\beta}_N)$ N_N decreases from .76 to .12). When there is a drop in the market value of the assets that southern consumers own, these consumers have to cut back on their expenditures.

However, the 2 percent decrease in southern consumer expenditure is dominated by the 12 percent increase in the inverse southern price index, implying that TRIPs makes southern consumers better off. There are two reasons for the favorable change in the southern price index. First, because stronger IPR protection leads to more FDI (ϕ increases from .002 to .004), more production moves

from the higher-wage North to the lower-wage South (γ_N decreases from .942 to .925) and southern consumers benefit from being able to buy more products at lower prices $([\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} +$ $\gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}$ $\binom{1-\sigma}{C}$ ^{1/(σ -1)</sub> increases from .322 to .348). Second, because stronger IPR protection leads} to more innovation (δ increases from 22.01 to 22.57), southern consumers benefit from being able to buy a greater variety of products $(n_0^{1/(\sigma-1)})$ $\frac{1}{0}^{1/(0-1)}$ increases from 401.13 to 415.70). And the increase in innovation is ultimately driven by the increase in FDI, which leads to more production moving from the North to the South, depressing the wages of northern workers and stimulating northern firms to do more R&D.

To summarize, we find that TRIPs (stronger southern IPR protection) leads to more FDI (ϕ increases from .002 to .004), more innovation (δ increases from 22.01 to 22.57), and considerably higher long-run southern consumer welfare $(u_{S0}$ increases from 142.0 to 155.6). The South also benefits from the trade liberalization that occurred from 1990 to 2005 but the welfare gains from TRIPs are considerably larger $(u_{S0}$ only increases from 142.0 to 149.2 due to trade liberalization). Furthermore, we find that trade liberalization by itself has a negligible effect in stimulating FDI $(L_{F0}$ increases from .0149 to .0150), so most of the 10-fold observed increase in FDI inflows to developing countries from 1990 to 2005 can be attributed to stronger southern IPR protection (L_{F0}) increases from .0149 to .0892 when only a_I is raised). This big increase in FDI is the main reason why TRIPs is good for developing countries in our analysis.

The final column of Table 1 shows what happens when southern IPR protection is very strong $(a_I = 500,$ which has approximately the same effect as $a_I = +\infty$). Both imitation rates fall to almost zero ($\iota_N = .000008$, $\iota_S = .001$) and we get the highest FDI rate ($\phi = .006$). Consequently, a large part of production is transferred from the North to foreign affiliates in the South (γ_F increases to .112) and the share of southern firms almost disappears ($\gamma_I + \gamma_C = .003$). The interesting thing about this case is that we obtain the best long-run outcome for southern consumers (u_{S0}) increases to 180.4). Developing countries benefit from TRIPs and they would benefit even more with very strong southern IPR protection, essentially eliminating imitation.

3.3 The Implications of Alternative Parameter Values

In other dynamic general equilibrium models of North-South trade by Helpman (1993), Lai (1998), and Branstetter and Saggi (2011), it is assumed that FDI is costless. In these models, a northern firm can transfer production to a foreign affiliate in the South without incurring any costs. We assume that FDI, just like innovation and imitation, is a costly activity. A foreign affiliate must

devote labor to adaptive R&D to learn how to produce the parent firm's product variety in the South.

For comparison with these earlier models, we solve our model for low-cost and costless FDI. Costless FDI corresponds to setting $a_F = 0$ in our model. In this exercise, we set $\tau = 1$ to facilitate comparison with the above-mentioned literature (all of which assume costless trade) and to guarantee that the assumption $\tau < w_N/w_S$ continues to be satisfied (this assumption guarantees that foreign affiliates export their products back to the North). As shown in columns 1 and 2 of Table 2, the assumption of costless trade hardly changes the variety shares but generates lower price indexes as expected. Columns 2 to 5 shows the results of assuming less costly FDI, captured by lower values of a_F . We find that lowering a_F causes the North-South wage ratio to fall drastically (w_N/w_S) decreases from 2.134 to 1.788 to 1.199). In the special case of costless FDI (column 5), the northern wage rate is only 1.3 percent higher than the southern wage rate $(w_N/w_S = 1.013)$. Thus, the standard assumption of costless FDI appears to be very strong —an issue that to our knowledge has not been pointed out in the related literature. When FDI is costless, so much production is transferred from the North to the South that the North-South wage difference becomes very small, negligible compared to the large wage differences that we observe between developed and developing countries.

Technology in our model is transferred from the North to the South through both FDI and imitation. We began this section by choosing 1990 benchmark parameter values to guarantee that FDI and imitation are equally important modes of international technology transfer $(\iota_N/\phi = 1)$. We now explore how things change when this property is relaxed. In particular, we are interested in what happens when imitation is more important than FDI as a mode of international technology transfer. Do we still find that stronger southern IPR protection (TRIPs) increases consumer welfare in both regions? In Table 3, we present results from studying three cases $(\iota_N/\phi = 5, \iota_N/\phi = 2$ and $\iota_N/\phi = 1$) to show that the model can generate three different outcomes.

To obtain the 1990 benchmark parameter values (underlying the results in column 1 of Table 1), we used three properties $(\iota_N/\phi=1, c_N/c_S=2.5, \iota_S=.25)$ to determine three parameter values $(a_F = 100.5, d = 261, a_I = 2.96)$. To obtain an alternative 1990 benchmark where $\iota_N/\phi = 5$ (the imitation rate ι_N is five times as large as the FDI rate ϕ), we proceed correspondingly and solve for the values of a_F , d, and a_I that satisfy $\iota_N/\phi = 5$, $c_N/c_S = 2.5$ and $\iota_S = .25$. These calculations yield $a_F = 373$, $d = 149$ and $a_I = 3.52$, given that the other parameter values are the same as in the original 1990 benchmark. The results with this alternative benchmark are presented in column 1

	(1)	(2)	(3)	(4)	(5)
	2005				Costless FDI
	$\tau = 1.33$	$\tau = 1$	$\tau = 1$	$\tau=1$	$\tau=1$
	$a_F = 100.5$	$a_F = 100.5$	$a_F=25$	$a_F=0.5$	$a_F=0$
w_N/w_S	2.166	2.134	1.788	1.199	1.013
δ	22.57	22.57	24.22	38.55	56.68
ϕ	.004	.004	.009	.058	.146
ι_S	.049	.049	.026	.006	.004
ι_N	.000	.000	.000	.000	.000
γ_N	.925	.925	.850	.464	.255
γ_F	.035	.035	.097	.474	.694
γ_I	$.034\,$.035	.051	.061	.050
γ_C	.006	.006	.003	.000	.000
L_{F0}	.089	.090	.116	.089	.000
c_N	2.56	2.52	2.13	1.56	1.47
$c_{\mathcal{S}}$	1.07	1.07	1.08	1.06	1.01
c_N/c_S	2.39	2.35	1.98	1.47	1.46
P_{N0}	.0065	.0058	.0043	.0016	.0009
P_{S0}	.0066	.0058	.0043	.0016	.0009
u_{N0}	394.3	437.9	493.2	961.6	1668.1
u_{S0}	163.1	186.2	249.2	652.1	1144.9

Table 2: The steady-state equilibrium implications of less costly and costless FDI

of Table 3 and the effects of a marginal increase in a_I from 3.52 to 4.0 (holding all other parameter values fixed) are presented in column 2.

Interestingly, columns 1 and 2 show that it is no longer the case that stronger southern IPR protection increases consumer welfare in both regions. In fact, we find that stronger IPR protection decreases consumer welfare in both the South and the North (when a_I increases from 3.52 to 4.0, u_{S0} decreases from 139.6 to 137.1 and u_{N0} decreases from 359.83 to 359.76). There is a problem with these numerical results, however. When a_I increases from 3.52 to 4.0, γ_N increases from .939 to .942 and $\gamma_F + \gamma_I + \gamma_C$ decreases from .061 to .058. Thus stronger IPR protection leads to the North exporting more products and the South exporting fewer products. This is the exact opposite of what Branstetter, Fisman, Foley and Saggi (2011) find when they look at the effects of stronger IPR protection in developing countries. As we mentioned in the introduction, they find that developing countries export more products after IPR reforms.

Next we consider the case where $\iota_N/\phi = 2$ (the imitation rate ι_N is two times as large as the FDI rate ϕ). To obtain benchmark parameter values that generate this outcome, we follow the same procedure as described above. We solve for the values of a_F , d, and a_I that satisfy $\iota_N / \phi = 2$, $c_N/c_S = 2.5$ and $\iota_S = .25$. These calculations yield $a_F = 166$, $d = 190$ and $a_I = 3.2$, given that

	(1)	(2)	(3)	(4)	(5)	(6)
	$\iota_N/\phi=5$		$\iota_N/\phi=2$		$\iota_N/\phi=1$	
	$a_F = 373$		$a_F = 166$		$a_F = 100.5$	
	$d = 149$		$d = 190$		$d = 261$	
	$a_I = 3.52$	$a_I = 4.0$	$a_I = 3.2$	$a_I = 4.0$	$a_I = 2.96$	$a_I = 4.0$
w_N/w_S	2.45	2.47	2.39	2.39	2.34	2.29
δ	21.87	21.85	21.94	21.96	22.01	22.11
ϕ	.0005	.0006	.0011	.0013	.0015	.0021
ι_S	.250	.226	.250	.199	.250	.172
ι_N	.0027	.0024	.0021	.0017	.0015	.0011
γ_N	.939	.942	.940	.943	.942	.941
γ_F	.002	.002	.003	.005	.005	.009
γ_I	.008	.010	.016	.020	.024	.030
γ_C	.051	.046	.040	.032	.029	.020
L_{F0}	.007	.009	.011	.018	.015	.027
$c_{\cal N}$	2.87	2.90	2.80	2.80	2.75	2.69
c_S	1.15	1.14	1.12	1.11	1.10	1.08
c_N/c_S	2.50	2.55	2.50	2.54	2.50	2.49
P_{N0}	.0080	.0081	.0077	.0077	.0075	.0073
P_{S0}	.0082	.0083	.0079	.0079	.0077	.0075
u_{N0}	359.83	359.76	362.3	363.4	364.7	367.7
u_{S0}	139.6	137.1	140.9	139.3	142.0	144.2

Table 3: Alternative 1990 benchmarks

the other parameter values are the same as in the original 1990 benchmark. The results with this alternative 1990 benchmark are presented in column 3 of Table 3 and the effects of a marginal increase in a_I from 3.2 to 4.0 (holding all other parameter values fixed) are presented in column 4.

In the $\iota_N/\phi = 2$ case, we find that stronger IPR protection decreases consumer welfare in the South but increases consumer welfare in the North (when a_I increases from 3.2 to 4.0, u_{S0} decreases from 140.9 to 139.3 and u_{N0} increases from 362.3 to 363.4). Thus, without changing any assumptions, just by appropriately choosing benchmark parameter values, our model can support the conventional wisdom that TRIPs is bad for developing countries and only benefits developed countries. However, we encounter the same problem with the $\iota_N/\phi = 2$ case as with the $\iota_N/\phi = 5$ case. Stronger IPR protection leads the North to export more products and the South to export fewer products (when a_I increases from 3.2 to 4.0, γ_N increases from .940 to .943 and $\gamma_F + \gamma_I + \gamma_C$ decreases from .060 to .057).

Finally, we present the $\iota_N/\phi = 1$ case in columns 5 and 6. Column 5 just reproduces the earlier results from column 1 of Table 1 and column 6 shows the effects of a marginal increase in a_I . In this case, we find that stronger IPR protection increases consumer welfare in both regions (when

 a_I increases from 2.96 to 4.0, u_{S0} increases from 142.0 to 144.2 and u_{N0} increases from 364.7 to 367.7). Furthermore, stronger IPR protection leads to the North exporting fewer products and the South exporting more products (when a_I increases from 2.96 to 4.0, γ_N decreases from .942 to .941 and $\gamma_F + \gamma_I + \gamma_C$ increases from .058 to .059). Thus, of the three cases that we study in Table 5, only the $\iota_N/\phi = 1$ case is consistent with the evidence that developing countries export more products after IPR reforms.

4 Concluding Comments

This paper challenges the conventional wisdom that the TRIPs agreement is bad for developing countries. We present a dynamic general equilibrium model of North-South trade that allows us to study the implications of stronger IPR protection and simultaneous trade liberalization. In the model, firms engage in innovative R&D to develop new product varieties in the North and foreign affiliates of these northern firms engage in adaptive R&D to learn how to produce these product varieties in the South. There are also southern firms that can engage in imitative R&D to learn how to produce both the product varieties of northern firms and their foreign affiliates. So over time, the production of every product variety moves from the North to the South and international technology transfer occurs both through FDI and imitation.

We find that stronger IPR protection in the South (TRIPs) induces foreign affiliates of northern firms to increase their R&D expenditures and results in a faster rate of technology transfer within these multinational firms, consistent with the empirical evidence in Branstetter, Fisman and Foley (2006). As a result of TRIPs, more product varieties end up being produced in the South and exports of new products increase, consistent with the empirical evidence in Branstetter, Fisman, Foley and Saggi (2011). TRIPs also stimulates innovative R&D spending by northern firms and results in faster economic growth in the South, consistent with the empirical evidence in Gould and Gruben (1996). When we solve the model numerically for plausible parameter values, we find that TRIPs leads to significantly higher long-run southern consumer welfare. The welfare gains from the trade liberalization that has occurred are not nearly as large as the welfare gains from TRIPs.

Many years ago, Deardorff (1992) developed a partial equilibrium model where imitation is immediate and costless in the absence of IPR protection. He showed that as more of the world is covered by IPR protection, the additional innovation that can be stimulated by the resulting monopoly profits of patent-holding innovators becomes smaller and smaller. Eventually the benefits

from more innovation are outweighed by the loss of consumer surplus from monopoly pricing. Thus, when the innovating country is sufficiently large, the imitating country is better off not protecting intellectual property. Based on this analysis, Deardorff (1990) concluded that "patent protection is almost certain to redistribute welfare away from developing countries" and he added, "If I could be convinced that a patent system would be the magical key to unlocking the secret of development for those who need it most, then I would gladly change the conclusion of this paper. But at the moment I do not see how that case can credibly be made." It has taken a long time but this paper is our response.

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Appendix 1: Related Literature

In this appendix, we review the related literature about TRIPs in more detail. We discuss models without FDI, models with costless FDI, models with costly FDI and evidence about TRIPs.

Models without FDI

The first models of North-South trade and IPR protection provided support for the conventional wisdom that TRIPs is harmful for developing countries. The first such model was Chin and Grossman (1990), who used a global duopoly structure where a northern firm had the capacity to innovate and a southern firm could costlessly imitate the production process invented by the northern firm in absence of IPR protection. There is innovation and imitation in this model, but no FDI. The gain from IPR protection in the Chin and Grossman model comes from additional expected monopoly profits for the northern firm, which stimulates further innovation. While the North always benefits from IPR protection in the South, the South itself is found to benefit only when the prospects of cost-savings through R&D are quite substantial or when the South comprises a sufficiently large share of the overall market for the good. The interests of the North and the South generally conflict, with the South benefiting from the ability to pirate technology and the North harmed by such actions.

Deardorff (1992) presents another partial equilibrium model where imitation is immediate and costless in absence of IPR protection. In his model, imitation results in perfect competition and there is a continuum of new products that can be invented. All consumers have identical preferences and all firms have the same marginal cost of production. Deardorff finds that as more of the world is covered by IPR protection, the additional innovation that can be stimulated by the resulting monopoly profits of the patent-holding innovators becomes smaller and smaller. Eventually the benefits from more innovation are outweighed by the loss in consumer surplus from monopoly pricing. Thus, when the innovating country is sufficiently large, the imitating country is better off not protecting intellectual property. Based on this analysis, Deardorff (1990, p.507) concludes that "patent protection is almost certain to redistribute welfare away from developing countries."⁷

Diwan and Rodrik (1992) present a partial equilibrium model of IPR protection with the new feature that consumers in the North and South have different preferences. The two regions therefore

⁷At the end of his paper, Deardorff (1992, p.50) does point out that "information about many new inventions is not costlessly transmitted abroad, and that extending patent protection may therefore be beneficial to the extent that it stimulates the transfer of technology." However, he does not develop this insight further.

compete with each other to encourage the development of technologies most suited to their needs. Innovation only takes place in the North while imitation takes place in both regions. The reason for the South to implement IPR protection in their model is to encourage innovation in the South's preferred range of products. Diwan and Rodrik find that a reduction in preference differences between the two regions reduces the optimal level of patent protection in the South and that "free riding on the North makes eminently good sense" in the absence of preference differences.⁸

Partial equilibrium models do not allow for any feedback effects on labor demand, wage rates and consumer expenditures, or any general equilibrium price effects which in turn have real effects on consumer welfare. Helpman (1993) presented the first welfare analysis of IPR protection using a dynamic general equilibrium model of North-South trade. In this model with innovation, imitation but no FDI, innovation is a costly activity that takes place in the North and imitation in the South is costless. The typical northern consumer buys goods for consumption but also saves and invests in firms that engage in innovative R&D (the development of new product varieties). The typical southern consumer spends all of her income on consumption. Starting from a steadystate equilibrium, Helpman finds that the South loses from a marginal decrease in the exogenous imitation rate and the North loses as well if the imitation rate is sufficiently small. Thus stronger IPR protection (a lower exogenous imitation rate) is bad for developing countries in this model.

Grossman and Lai (2004) study a dynamic general equilibrium model with two countries (North and South) that are both capable of innovating. The two countries differ in market size and in their capacity for conducting innovative R&D. The North has a larger market for newly innovated products and a larger endowment of human capital which is necessary for innovation. Imitation is costless when it occurs and due to special assumptions about consumer preferences, no economic growth occurs in equilibrium (regardless of what patent policies countries adopt). Solving for a Nash equilibrium in patent policies, Grossman and Lai show that patent protection will be stronger in the North than in the South. Among patent policies that generate the same overall incentives for global innovation, the North fares better and the South worse, the stronger is patent protection in the South. Grossman and Lai also conclude that a treaty like TRIPs that essentially harmonizes patent policy may well benefit the North at the expense of the South.

From our perspective, the main drawback of the above-mentioned models is that there is no FDI. In these models, no international technology transfer takes place within multinational firms.

⁸We consider it to be a strength of our model that we assume identical consumer preferences in the two regions and can still show that the South benefits from stronger IPR protection (TRIPs). Given the analysis in Diwan and Rodrik (1992), the case for TRIPs becomes stronger if there are North-South differences in consumer preferences.

FDI is central to our explanation for why TRIPs is good for developing countries.

Models with costless FDI

Helpman (1993) also presents a second model with costless innovation, costless imitation and costless FDI. Innovation takes place at a constant exogenous rate in the North and once a northern firm has innovated, it can costlessly form a manufacturing subsidiary in the South to take advantage of lower labor costs there. For a firm that innovates, the risk of imitation is assumed to be independent of whether the firm produces in the North or in the South. Starting from a steady-state equilibrium, Helpman finds that the South loses from a marginal decrease in the exogenous imitation rate (stronger IPR protection) and the North loses as well if the imitation rate is sufficiently small. In this model, costless FDI leads to factor price equalization: there are no wage differences between the North and the South in equilibrium.

Of the papers that we have discussed so far, all support the perspective that TRIPs is probably bad for developing countries. With Lai (1998), we start to encounter a more favorable view of TRIPs. He presents a dynamic general equilibrium model of North-South trade with costly innovation, costless imitation and costless FDI. Northern firms do innovative R&D to develop new product varieties and then choose whether to produce in the higher-wage North or in the lowerwage South. Unlike in Helpman (1993), Lai assumes that for a firm that innovates, there is only a risk of imitation if the firm produces in the South. If the firm produces in the North, it earns monopoly profits forever. With these new assumptions, Lai finds that stronger IPR protection (a lower southern imitation rate) leads to a higher rate of innovation and a higher rate of technology transfer to the South. These conclusions continue to hold in a more general model with imitation of northern products if the FDI rate is sufficiently high. But the opposite conclusions hold if imitation is the only mode of international technology transfer. Lai does not comment on which case is more relevant (FDI or imitation as the dominant mode of international technology transfer) and does not study the welfare effects of stronger IPR protection.⁹

Building on Lai (1998), Branstetter and Saggi (2011) have recently developed a dynamic general equilibrium North-South trade model where imitation is treated as a costly activity and the

⁹We showed in Table 3 that when imitation and FDI start off being equally important modes of international technology transfer, stronger IPR protection leads to a higher rate of innovation and a higher rate of technology transfer to the South, making southern consumers better off. However, when imitation starts off being five times more important than FDI as a mode of international technology transfer, stronger IPR protection leads to a lower rate of innovation and a lower rate of technology transfer to the South, making southern consumers worse off. We argued that the first case is more relevant.

southern rate of imitation is endogenously determined. They assume that imitation only targets multinationals in the South and in effect shut down the imitation channel for international technology transfer. Focusing on the case where costless FDI is the only mode of international technology transfer, Branstetter and Saggi find that stronger IPR protection results in a lower southern imitation rate, a higher northern innovation rate and a higher rate of technology transfer to the South. These effects are interesting and suggest that stronger IPR protection (TRIPs) may benefit southern consumers, but Branstetter and Saggi do not provide any welfare analysis.

One paper that does provide welfare analysis is Iwaisako, Tanaka and Futagami (2011). They present a North-South quality-ladders model without transitional dynamics where innovating northern firms can choose to engage in costless FDI and become multinationals. All technology transfer occurs through FDI and they study a special case of IPR protection which is patent breadth. Iwaisako, Tanaka and Futagami show that, as long as the northern labor force is large enough relative to the southern labor force and innovative R&D is productive enough, consumers in both the North and the South experience welfare gains from stronger patent protection. Broadening patent breadth allows multinational firms to charge higher quality-adjusted prices and obtain higher profits, stimulating northern innovation. In this model, imitating firms do not themselves produce in equilibrium but their presence forces quality leaders to practice limit pricing.

From our perspective, the main drawback of the models by Lai (1998), Branstetter and Saggi (2011) and Iwaisako, Tanaka and Futagami (2011) is that FDI is costless. In the real world, FDI is measured in terms of its costs. From 1990 to 2005, the FDI inflow to developing countries increased from 34.9 to 363.4 billion US dollars (UNCTAD, 2011). We calibrate our model to match this evidence and it would not be possible to do this using a model with costless FDI.¹⁰

Models with costly FDI

Recently, Gustavsson and Segerstrom (2011) have developed two North-South trade models with costly FDI. In the first model, there is costly innovation, costly FDI and costless imitation. This model is relatively simple and can be solved analytically. Gustavsson and Segerstrom find that stronger IPR protection leads to more innovation and more FDI. In the second model, there is costly innovation, costly FDI and costly imitation. This model is more complicated and is solved

¹⁰Also, when we solved our model for the costless FDI special case, we found that this assumption leads to so much production being transferred from the North to the South that the northern wage rate was only 1.3 percent higher than the southern wage rate. Explaining large North-South wage differences becomes a problem when FDI is assumed to be costless.

numerically but the results are the same for plausible parameter values. Again, they find that stronger IPR protection leads to more innovation and more FDI.

In an earlier paper, Glass and Saggi (2002) obtained the exact opposite result. They developed a North-South trade model with costly FDI and found that stronger IPR protection leads to less innovation and less FDI. However, as Gustavsson and Segerstrom (2011) point out, the Glass-Saggi result is not robust to allowing for decreasing returns to R&D. The Glass-Saggi result holds when constant returns to R&D is assumed but ceases to hold when a realistic degree of decreasing returns to R&D is assumed (see their Table 5). According to the empirical literature, R&D is subject to significant decreasing returns at the industry level (Kortum, 1993).

In this paper, we build on the analysis in Gustavsson and Segerstrom (2011). As is usual in the North-South trade literature, they assume that there are no costs to international trade. We extend their second model by incorporating trade costs. We present a dynamic general equilibrium model of North-South trade where there are costs of moving production across regions (costly FDI, costly imitation) and there are also costs of moving goods across regions (costly trade). With this model, we can study the implications of stronger IPR protection and simultaneous trade liberalization.

Evidence about TRIPs

Empirical support for the view that TRIPs generates dynamic benefits by promoting innovation and economic growth can be found in Gould and Gruben (1996). They use cross-country data on patent protection, trade regime and country-specific characteristics and find evidence that IPR protection is a significant determinant of economic growth. Countries with stronger IPR protection tend to have higher average yearly per capita GDP growth from 1960 to 1988. Furthermore, the effects are slightly stronger in relatively open economies. They attribute this to the linkage between innovation and IPR protection playing a stronger role in more competitive (open) markets. These results are consistent with our model's implications $(a_I \uparrow \Rightarrow \delta \uparrow, u_{S0} \uparrow)$.

Recently evidence has emerged suggesting that TRIPs also generates other dynamic benefits. Branstetter, Fisman and Foley (2006) study how international technology transfer within US-based multinational firms changes in response to IPR reforms in developing countries. They find that due to IPR reform, royalty payments for technology that has been transferred to foreign affiliates increase and the R&D expenditures of these foreign affiliates increase. In another paper, Branstetter, Fisman, Foley and Saggi (2011) study the response of host country industrial production to stronger IPR protection. They find that following patent reform aimed at strengthening IPR protection, US-based multinational firms expand the scale of their activities in reforming countries and exports of new goods increase in reforming countries. This evidence suggests that IPR reform enhances, rather than retards, southern industrial development.

These results provide further empirical support for our model's implications. In our model, stronger southern IPR protection leads to a permanent increase in the rate of technology transfer to the South within multinational firms $(a_I \rightrightarrows \phi \rightrightarrows a_I)$ and a permanent increase in adaptive R&D spending in the South by foreign affiliates $(a_I \uparrow \Rightarrow L_{F0} \uparrow)$. Whenever technology transfer within a multinational firm occurs, the foreign affiliate pays its parent firm the royalty payment v_{Nt} for the use of its technology in the South, since the adaptive R&D accounts for the increment in the firm's value $v_{Ft} - v_{Nt}$ which is less than the foreign affiliate's market value v_{Ft} . It follows that stronger IPR protection leads to more royalty payments as Branstetter, Fisman and Foley (2006) find. Stronger southern IPR protection also leads to more goods being produced in the South $(a_I \rightrightarrows \gamma_F + \gamma_I + \gamma_C \rightrightarrows)$. Thus, our model is consistent with the evidence in Branstetter, Fisman, Foley and Saggi (2011) that patent reform is associated with an overall expansion of industrial activity and higher exports of new goods in developing countries.

Empirical support for the view that TRIPs is bad for developing countries can be found in McCalman (2001). He estimates the value of income transfers between countries implied by the TRIPs agreement. For each country, the net transfer is defined as the increase in the present value of patent rights held by residents of the country less the increase in the present value of patent rights granted by that country. Using data for 29 countries, McCalman finds that only a few countries gained from TRIPs (United States, Germany, France, Italy, Sweden and Switzerland) and all the other countries were made worse off, including all developing countries. However, it is important to note that this analysis is conducted for a given set of innovations, so the benefits of any increase in innovation in response to the TRIPs agreement are not taken into account. Also the probability of technology diffusion does not depend on the strength of patent protection, so the benefits of any increase in FDI in response to the TRIPs agreement are not taken into account. It is just assumed in McCalman's empirical estimation that TRIPs does not have any effect on innovation or FDI.

Appendix 2: Solving The Model

In this appendix, calculations done to solve the model are spelled out in more detail.

Households

The static consumer optimization problem is

$$
\max_{x_t(\cdot)} \int_0^{n_t} x_t(\omega)^\alpha d\omega \qquad \text{s.t.} \qquad \dot{y}(\omega) = p_t(\omega) x_t(\omega), \ y(0) = 0, \ y(n_t) = c_t.
$$

where $y(\omega)$ is a new state variable and $\dot{y}(\omega)$ is the derivative of y with respect to ω . The Hamiltonian function for this optimal control problem is

$$
H = x_t(\omega)^{\alpha} + \gamma(\omega)p_t(\omega)x_t(\omega)
$$

where $\gamma(\omega)$ is the costate variable. The costate equation $\frac{\partial H}{\partial y} = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω . $\frac{\partial H}{\partial x} = \alpha x_t(\omega)^{\alpha - 1} + \gamma \cdot p_t(\omega) = 0$ implies that

$$
x_t(\omega) = \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)}.
$$

Substituting this back into the budget constraint yields

$$
c_t = \int_0^{n_t} p_t(\omega) x_t(\omega) d\omega = \int_0^{n_t} p_t(\omega) \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)} \right)^{1/(1-\alpha)} d\omega = \left(\frac{\alpha}{-\gamma} \right)^{1/(1-\alpha)} \int_0^{n_t} p_t(\omega)^{\frac{1-\alpha-1}{1-\alpha}} d\omega.
$$

Now $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1-\sigma = \frac{1-\alpha-1}{1-\alpha} = \frac{-\alpha}{1-\alpha}$ $\frac{-\alpha}{1-\alpha}$, so

$$
\frac{c_t}{\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega} = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)}
$$

It immediately follows that the consumer demand function is

$$
x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}}
$$
\n(3)

.

where $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices.

Substituting this consumer demand function back into the consumer utility function yields

$$
u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega\right]^{\frac{1}{\alpha}} = \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha} c_t^\alpha}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}} = c_t \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}}.
$$

Taking into account that $-\sigma\alpha = \frac{-\alpha}{1-\alpha} = 1 - \sigma$, consumer utility can be simplified further to

$$
u_t = \frac{c_t}{P_t^{1-\sigma}} \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{\alpha}} = \frac{c_t}{P_t^{1-\sigma}} \left[P_t^{1-\sigma} \right]^{\frac{1}{\alpha}} = \frac{c_t}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{c_t}{P_t}
$$

$$
\ln u_t = \ln c_t - \ln P_t.
$$

The individual household takes the prices of all products as given, as well as how prices change over time, so the $\ln P_t$ term can be ignored in solving the household's dynamic optimization problem. This problem simplifies to:

$$
\max_{c_t} \int_0^\infty e^{-(\rho - g_L)t} \ln c_t \, dt \qquad \text{s.t.} \qquad \dot{\tilde{a}}_t = w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - c_t,
$$

where \tilde{a}_t represents the asset holding of the representative consumer, w_t is the wage rate and r_t is the interest rate.

The Hamiltonian function for this optimal control problem is

$$
H = e^{-(\rho - g_L)t} \ln c_t + \lambda_t \left[w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - c_t \right]
$$

where λ_t is the relevant costate variable. The costate equation $-\dot{\lambda}_t = \frac{\partial H}{\partial \tilde{a}} = \lambda_t [r_t - g_L]$ implies that

$$
\frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.
$$

 $\partial H/\partial c_t = e^{-(\rho-g_L)t}\frac{1}{c_t} - \lambda_t = 0$ implies that $e^{-(\rho-g_L)t}\frac{1}{c_t} = \lambda_t$. Taking logs of both sides yields $-(\rho - g_L)t - \ln c_t = \ln \lambda_t$ and then differentiating with respect to time yields

$$
-(\rho - g_L) - \frac{\dot{c}_t}{c_t} = \frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.
$$

$$
\frac{\dot{c}_t}{c_t} = r_t - \rho.
$$
 (4)

It immediately follows that

Steady-State Dynamics

We will now derive some steady-state equilibrium implications of the model.

First, we solve for γ_N . By differentiating the variety condition $n_t = n_{Nt} + n_{Ft} + n_{It} + n_{Ct}$, we obtain that

$$
\dot{n}_t = \dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{It} + \dot{n}_{Ct}
$$
\n
$$
\frac{\dot{n}_t}{n_t} = \frac{\dot{n}_{Nt}}{n_{Nt}} \frac{n_{Nt}}{n_t} + \frac{\dot{n}_{Ft} + \dot{n}_{It}}{n_{Nt}} \frac{n_{Nt}}{n_t} + \frac{\dot{n}_{Ct}}{n_{Nt}} \frac{n_{Nt}}{n_t}
$$
\n
$$
g = g\gamma_N + \phi\gamma_N + \iota_N\gamma_N
$$
\n
$$
g = (g + \phi + \iota_N)\gamma_N
$$

and solving for γ_N yields

$$
\gamma_N = \frac{g}{g + \phi + \iota_N}.\tag{5}
$$

To solve for γ_C , note that

$$
\iota_N = g \frac{\gamma_C}{\gamma_N} = g \gamma_C \frac{g + \phi + \iota_N}{g} = \gamma_C (g + \phi + \iota_N),
$$

45

or

from which it follows that

$$
\gamma_C = \frac{\iota_N}{g + \phi + \iota_N}.\tag{6}
$$

We can also solve for γ_F and γ_I . Substituting into $\gamma_N + \gamma_F + \gamma_I + \gamma_C = 1$ yields

$$
\frac{g}{g+\phi+\iota_N}+\gamma_F+\gamma_I+\frac{\iota_N}{g+\phi+\iota_N}=1
$$

or

$$
\gamma_F + \gamma_I = \frac{\phi}{g + \phi + \iota_N}
$$

Since $\iota_S = g\gamma_I/\gamma_F$ implies that $\gamma_F = g\gamma_I/\iota_S$, substituting yields

$$
\frac{g}{\iota_S} \gamma_I + \gamma_I = \left(\frac{g + \iota_S}{\iota_S}\right) \gamma_I = \frac{\phi}{g + \phi + \iota_N}.
$$

It immediately follows that

$$
\gamma_I = \left(\frac{\phi}{g + \phi + \iota_N}\right) \left(\frac{\iota_S}{g + \iota_S}\right) \text{ and } \gamma_F = \left(\frac{\phi}{g + \phi + \iota_N}\right) \left(\frac{g}{g + \iota_S}\right). \tag{7}
$$

.

Because prices differ between the North and the South, we need to define a different price index for each region. Let P_{Nt} be the price index for the North and let P_{St} be the price index for the South. Given the earlier definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, it follows that the northern price index satisfies

$$
P_{Nt}^{1-\sigma} = \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega
$$

= $n_{Nt} p_N^{1-\sigma} + n_{Ft} (p_F^*)^{1-\sigma} + n_{Ht} (p_I^*)^{1-\sigma} + n_{Ct} (p_C^*)^{1-\sigma}$
= $\gamma_N n_t p_N^{1-\sigma} + \gamma_F n_t (p_F^*)^{1-\sigma} + \gamma_I n_t (p_I^*)^{1-\sigma} + \gamma_C n_t (p_C^*)^{1-\sigma}$
= $[\gamma_N p_N^{1-\sigma} + \gamma_F (p_F^*)^{1-\sigma} + \gamma_I (p_I^*)^{1-\sigma} + \gamma_C (p_C^*)^{1-\sigma}] n_t$

where the term in brackets is constant over time. Likewise, the southern price index satisfies

$$
P_{St}^{1-\sigma} = \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega
$$

= $n_{Nt}(p_N^*)^{1-\sigma} + n_{Ft} p_F^{1-\sigma} + n_{Ht} p_I^{1-\sigma} + n_{Ct} p_C^{1-\sigma}$
= $\gamma_N n_t(p_N^*)^{1-\sigma} + \gamma_F n_t p_F^{1-\sigma} + \gamma_I n_t p_I^{1-\sigma} + \gamma_C n_t p_C^{1-\sigma}$
= $[\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}] n_t$

where the term in brackets is constant over time. It follows that both $P_{Nt}^{1-\sigma}$ and $P_{St}^{1-\sigma}$ grow over time at the rate g in any steady-state equilibrium.

Product markets

A northern firm earns the flow of global profits

$$
\pi_{Nt} = (p_N - w_N) x_{Nt} L_{Nt} + (p_N^* - \tau w_N) x_{Nt}^* L_{St}
$$

where x_{N_t} is the quantity demanded by the typical northern consumer of the northern firm's product and x_{Nt}^* is the quantity demanded by the typical southern consumer of the northern firm's product. From the earlier demand function, it follows that $x_{Nt} = p_N^{-\sigma}$ $\frac{-\sigma}{N} c_N / P_{Nt}^{1-\sigma}$ and $x_{Nt}^* = (p_N^*)^{-\sigma} c_S / P_{St}^{1-\sigma}$. Hence, we can write a northern firm's global profit flow as:

$$
\pi_{Nt} = (p_N - w_N) \frac{p_N^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} + (p_N^* - \tau w_N) \frac{(p_N^*)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}}.
$$

Maximizing π_{N_t} with respect to p_N yields the first-order condition

$$
\frac{\partial \pi_{Nt}}{\partial p_N} = \left[(1 - \sigma) p_N^{-\sigma} + \sigma w_N p_N^{-\sigma - 1} \right] \frac{c_N L_{Nt}}{P_{Nt}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) p_N^{-\sigma} + \sigma w_N p_N^{-\sigma-1} = 0$ since we know that $\frac{c_N L_{Nt}}{p_{Nt}^{1-\sigma}} \neq 0$. Dividing by $p_N^{-\sigma}$ N yields $\frac{\sigma w_N}{p_N} = \sigma - 1$ or

$$
p_N = \frac{\sigma w_N}{\sigma - 1} = \frac{w_N}{\alpha}.
$$

To demonstrate the second equality, first note that $\sigma \equiv \frac{1}{1-\alpha}$ implies that $\sigma - 1 = \frac{1-(1-\alpha)}{1-\alpha} = \frac{\alpha}{1-\alpha}$. It follows that $\frac{\sigma}{\sigma-1} = \left(\frac{1}{1-\alpha}\right)/\left(\frac{\alpha}{1-\alpha}\right) = \frac{1}{\alpha}$. Similarly, maximizing π_{Nt} with respect to p_N^* yields the first-order condition

$$
\frac{\partial \pi_{Nt}}{\partial p_N^*} = \left[(1 - \sigma) (p_N^*)^{-\sigma} + \sigma \tau w_N (p_N^*)^{-\sigma - 1} \right] \frac{c_S L_{St}}{P_{St}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) (p_N^*)^{-\sigma} + \sigma \tau w_N (p_N^*)^{-\sigma-1} = 0$. Dividing by $(p_N^*)^{-\sigma}$ yields $\frac{\sigma \tau w_N}{p_N^*} = \sigma - 1$ or $p_N^* = \frac{\sigma \tau w_N}{\sigma_1}$ $\frac{\sigma \tau w_N}{\sigma - 1} = \frac{\tau w_N}{\alpha}$ $\frac{\omega_{I\text{V}}}{\alpha}$.

Plugging the prices back into the profit expression, we obtain

$$
\pi_{Nt} = (p_N - w_N) \frac{p_N^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} + (p_N^* - \tau w_N) \frac{(p_N^*)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}}
$$
\n
$$
= \left(\frac{w_N}{\alpha} - w_N\right) \frac{p_N^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} + \left(\frac{\tau w_N}{\alpha} - \tau w_N\right) \frac{(p_N^*)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}}
$$
\n
$$
= \frac{w_N}{\sigma - 1} \left[\frac{p_N^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} + \tau \frac{(p_N^*)^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} \right]
$$

where we have used that $\frac{1}{\alpha} - 1 = \frac{\sigma}{\sigma - 1} - \frac{\sigma - 1}{\sigma - 1} = \frac{1}{\sigma - 1}$. It turns out to be convenient to reexpress profits by multiplying the RHS by $\frac{L_t}{L_t}$ n_{Nt} $\overline{n_{Nt}}$ $\frac{n_t}{t}$ $\frac{n_t}{n_t}$:

$$
\pi_{Nt} = \frac{w_N}{\sigma - 1} \left[\frac{p_N^{-\sigma} c_N L_{Nt} n_{Nt}}{P_{Nt}^{1-\sigma} L_t} + \tau \frac{(p_N^*)^{-\sigma} c_S L_{St} n_{Nt}}{P_{St}^{1-\sigma} L_t} \right] \frac{L_t}{n_t \frac{n_{Nt}}{n_t}}.
$$

Now $\gamma_N \equiv \frac{n_{Nt}}{n_t}$ $\frac{n_{Nt}}{n_t}$ is constant over time, $X_N \equiv \frac{p_N^{-\sigma} c_N L_{Nt} n_{Nt}}{p_{Nt}^{1-\sigma} L_t}$ $\frac{c_N L_{Nt} n_{Nt}}{P_{Nt}^{1-\sigma} L_t}$ is constant over time since $P_{Nt}^{1-\sigma}$ grows at the same rate g as n_{Nt} , and $X_N^* \equiv \frac{(p_N^*)^{-\sigma} c_S L_{St} n_{Nt}}{P_{\sigma}^{1-\sigma} L_{t}}$ $\frac{c_S L_{St} n_{Nt}}{P_{St}^{1-\sigma} L_t}$ is constant over time since $P_{St}^{1-\sigma}$ grows at the same rate g as n_{N_t} . Thus we can write π_{N_t} more simply as:

$$
\pi_{Nt} = \left[\frac{w_N \left(X_N + \tau X_N^*\right)}{\left(\sigma - 1\right)\gamma_N}\right] \frac{L_t}{n_t}.\tag{10}
$$

A foreign affiliate earns the flow of global profits:

$$
\pi_{Ft} = (p_F - w_S) x_{Ft} L_{St} + (p_F^* - \tau w_S) x_{Ft}^* L_{Nt}
$$

where $x_{Ft} = p_F^{-\sigma}$ $\int_{F}^{-\sigma} c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the foreign affiliate's product and $x_{Ft}^* = (p_F^*)^{-\sigma} c_N / P_{Nt}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the foreign affiliate's product. Hence, we can write a foreign affiliate's profit flow as

$$
\pi_{Ft} = (p_F - w_S) \frac{p_F^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (p_F^* - \tau w_S) \frac{(p_F^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}.
$$

Maximizing π_{Ft} with respect to p_F yields the first-order condition

$$
\frac{\partial \pi_{Ft}}{\partial p_F} = \left[(1 - \sigma) p_F^{-\sigma} + \sigma w_S p_F^{-\sigma - 1} \right] \frac{c_S L_{St}}{P_{St}^{1 - \sigma}} = 0
$$

which implies that $(1 - \sigma) p_F^{-\sigma} + \sigma w_S p_F^{-\sigma - 1} = 0$. Dividing by $p_F^{-\sigma}$ $\frac{-\sigma}{F}$ yields $\frac{\sigma w_S}{p_F} = \sigma - 1$ or

$$
p_F = \frac{\sigma w_S}{\sigma - 1} = \frac{w_S}{\alpha}.
$$

Similarly, maximizing π_{Ft} with respect to p_F^* yields the first-order condition

$$
\frac{\partial \pi_{Ft}}{\partial p_F^*} = \left[(1 - \sigma) (p_F^*)^{-\sigma} + \sigma \tau w_S (p_F^*)^{-\sigma - 1} \right] \frac{c_N L_{Nt}}{P_{Nt}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) (p_F^*)^{-\sigma} + \sigma \tau w_S (p_F^*)^{-\sigma-1} = 0$. Dividing by $(p_F^*)^{-\sigma}$ yields $\frac{\sigma \tau w_S}{p_F^*} = \sigma - 1$ or $p_F^* = \frac{\sigma \tau w_S}{\sigma - 1}$ $\frac{\sigma \tau w_S}{\sigma - 1} = \frac{\tau w_S}{\alpha}$ $\frac{\omega S}{\alpha}$.

We assume that
$$
\tau w_S < w_N
$$
 so each foreign affiliated exports to the northern market. The trade costs parameter τ cannot be too high. Plugging the prices back into the profit expression, we obtain

$$
\pi_{Ft} = \left(\frac{w_S}{\alpha} - w_S\right) \frac{p_F^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + \left(\frac{\tau w_S}{\alpha} - \tau w_S\right) \frac{(p_F^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}
$$

$$
= \frac{w_S}{\sigma - 1} \left[\frac{p_F^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + \tau \frac{(p_F^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \right].
$$

We reexpress profits by multiplying the RHS by $\frac{L_t}{L_t}$ $n_{F t}$ $\overline{n_{F t}}$ $\frac{n_t}{t}$ $\frac{n_t}{n_t}$:

$$
\pi_{Ft} = \frac{w_S}{\sigma - 1} \left[\frac{p_F^{-\sigma} c_S L_{St} n_{Ft}}{P_{St}^{1-\sigma} L_t} + \tau \frac{(p_F^*)^{-\sigma} c_N L_{Nt} n_{Ft}}{P_{Nt}^{1-\sigma} L_t} \right] \frac{L_t}{n_t \frac{n_{Ft}}{n_t}}.
$$

Now $\gamma_F \equiv \frac{n_{Ft}}{n_t}$ $\frac{n_{Ft}}{n_t}$ is constant over time, $X_F \equiv \frac{p_F^{-\sigma} c_S L_{St} n_{Ft}}{P_{cr}^{1-\sigma} L_t}$ $\frac{P_{CS}L_{StnFt}}{P_{st}^{1-\sigma}L_t}$ is constant over time since $P_{St}^{1-\sigma}$ grows at the same rate g as n_{Ft} , and $X_F^* \equiv \frac{(p_F^*)^{-\sigma} c_N L_{Nt} n_{Ft}}{P_{\infty}^{1-\sigma} L_{\infty}}$ $\frac{c_N L_{Nt} n_{Ft}}{P_{Nt}^{1-\sigma} L_t}$ is constant over time since $P_{Nt}^{1-\sigma}$ grows at the same rate g as n_{Ft} . Thus we can write π_{Ft} more simply as:

$$
\pi_{Ft} = \left[\frac{w_S \left(X_F + \tau X_F^*\right)}{(\sigma - 1)\gamma_F}\right] \frac{L_t}{n_t}.\tag{11}
$$

After a product variety that a foreign affiliate produces is imitated by a southern firm (an "I-firm", where "I" stands for Imitating), the southern firm earns the flow of global profits

$$
\pi_{It} = (p_I - \zeta w_S) x_{It} L_{St} + (p_I^* - \tau \zeta w_S) x_{It}^* L_{Nt}
$$

where $x_{It} = p_f^{-\sigma}$ $I^{-\sigma} c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the I-firm's product and $x_{It}^* = (p_I^*)^{-\sigma} c_N / P_{Nt}^{\overline{1-\sigma}}$ is the quantity demanded by the typical northern consumer of the I-firm's product. Thus,

$$
\pi_{It} = (p_I - \zeta w_S) \frac{p_I^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (p_I^* - \tau \zeta w_S) \frac{(p_I^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}.
$$

Maximizing π_{It} with respect to p_I yields the first-order condition

$$
\frac{\partial \pi_{It}}{\partial p_I} = \left[(1 - \sigma) p_I^{-\sigma} + \sigma \zeta w_S p_I^{-\sigma - 1} \right] \frac{c_S L_{St}}{P_{St}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) p_I^{-\sigma} + \sigma \zeta w_S p_I^{-\sigma-1} = 0$. Dividing by $p_I^{-\sigma}$ $I_I^{-\sigma}$ yields $\frac{\sigma \zeta w_S}{pI} = \sigma - 1$ or $p_1 = \frac{\sigma \zeta w_S}{\sigma - 1} = \frac{\zeta w_S}{\alpha}$. This is the monopoly price that the southern firm would charge if it had no competitors. We assume that $\zeta > \alpha$, so $\frac{\zeta w_S}{\alpha}$ exceeds the marginal cost of the foreign affiliate w_S . Then the limit price

$$
p_I = w_S
$$

is less than the monopoly price and it is profit-maximizing for the southern firm to practice limit pricing in its domestic market. (Once the product is imitated, the imitator is in competition with the foreign affiliate. The foreign affiliate cannot lower its price below its marginal cost w_S . At $p_I = w_S$ the imitator still makes positive profits but drives the foreign affiliate out of business.) Maximizing π_{It} with respect to p_I^* yields the other first-order condition:

$$
\frac{\partial \pi_{It}}{\partial p_I^*} = \left[(1 - \sigma) (p_I^*)^{-\sigma} + \sigma \tau \zeta w_S (p_I^*)^{-\sigma - 1} \right] \frac{c_N L_{Nt}}{P_{Nt}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) (p_I^*)^{-\sigma} + \sigma \tau \zeta w_S (p_I^*)^{-\sigma - 1} = 0$. Dividing by $(p_I^*)^{-\sigma}$ yields $\frac{\sigma \tau \zeta w_S}{p_I^*} = \sigma - 1$ or $p_I^* = \frac{\sigma \tau \zeta w_S}{\sigma - 1} = \frac{\tau \zeta w_S}{\alpha}$. This is the monopoly price that the southern firm would charge in the export market if it had no competitors. But it is competing against the foreign affiliate whose product it has imitated. Since $\frac{\tau \zeta w_S}{\alpha}$ exceeds the marginal cost of the foreign affiliate τw_S , it is profit-maximizing for the southern firm to set the limit price

$$
p_I^* = \tau w_S
$$

in its export market. Plugging the limit prices back into the profit expression, we obtain

$$
\pi_{It} = (w_S - \zeta w_S) \frac{p_I^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (\tau w_S - \tau \zeta w_S) \frac{p_I^* c_N L_{Nt}}{P_{Nt}^{1-\sigma}}
$$
\n
$$
= w_S (1 - \zeta) \left[\frac{p_I^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + \tau \frac{(p_I^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \right]
$$
\n
$$
= w_S (1 - \zeta) \left[\frac{p_I^{-\sigma} c_S L_{St} n_{It}}{P_{St}^{1-\sigma} L_t} + \tau \frac{(p_I^*)^{-\sigma} c_N L_{Nt} n_{It}}{P_{Nt}^{1-\sigma} L_t} \right] \frac{L_t}{n_t \frac{n_{It}}{n_t}}.
$$

Now $\gamma_I \equiv \frac{n_{It}}{n_t}$ $\frac{n_{It}}{n_t}$ is constant over time, $X_I \equiv \frac{p_I^{-\sigma} c_S L_{St} n_{It}}{P_{cr}^{-\sigma} L_t}$ $\frac{\sigma_{CS}L_{St}n_{It}}{P_{St}^{1-\sigma}L_t}$ is constant over time since $P_{St}^{1-\sigma}$ grows at the same rate g as n_{It} , and $X_I^* \equiv \frac{(p_I^*)^{-\sigma} c_N L_{Nt} n_{It}}{P_N^{1-\sigma} L_t}$ $\frac{c_N L_{Nt} n_{It}}{P_{Nt}^{1-\sigma} L_t}$ is constant over time since $P_{Nt}^{1-\sigma}$ grows at the same rate g as n_{It} . Thus we can write $\pi_{It}^{(n)}$ more simply as:

$$
\pi_{It} = \left[\frac{w_S\left(1-\zeta\right)\left(X_I + X_I^*\right)}{\gamma_I}\right] \frac{L_t}{n_t}.\tag{12}
$$

After a variety that a northern firm produces is imitated by a southern firm (a "C-firm", where "C" stands for Copying), the southern firm earns the flow of global profits

$$
\pi_{Ct} = (p_C - \zeta w_S)x_{Ct}L_{St} + (p_C^* - \tau \zeta w_S)x_{Ct}^*L_{Nt}
$$

where $x_{Ct} = p_C^{-\sigma}$ $C^{\sigma} c_S/P_{St}^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the Cfirm's product, and $x_{C_t}^{*} = (p_C^*)^{-\sigma} c_N / P_{N_t}^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the C-firm's product. We write the C-firm's profit flow as

$$
\pi_{Ct} = (p_C - \zeta w_S) \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (p_C^* - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}.
$$

Maximizing π_{C_t} with respect to p_C yields the first-order condition

$$
\frac{\partial \pi_{Ct}}{\partial p_C} = \left[(1 - \sigma) p_C^{-\sigma} + \sigma \zeta w_S p_C^{-\sigma - 1} \right] \frac{c_S L_{St}}{P_{St}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma) p_C^{-\sigma} + \sigma \zeta w_S p_C^{-\sigma - 1} = 0$. Dividing by $p_C^{-\sigma}$ $\frac{\sigma\zeta w_S}{\rho_C} = \sigma - 1$, and rearranging gives the monopoly price

$$
p_C = \frac{\sigma \zeta w_S}{\sigma - 1} = \frac{\zeta w_S}{\alpha}.
$$

But the southern firm faces a competitor in the northern firm whose product it has imitated. We restrict attention to the steady-state equilibrium where $\tau w_N > \frac{\zeta w_S}{\alpha}$ or $\tau \frac{w_N}{w_S}$ $\frac{w_N}{w_S} > \frac{\zeta}{\alpha}$ $\frac{\zeta}{\alpha}$ to guarantee that the southern firm charges the monopoly price $p_C = \zeta w_S/\alpha$ in its domestic market. Maximizing π_{Ct} with respect to p_C^* yields the first-order condition

$$
\frac{\partial \pi_{Ct}}{\partial p_C^*} = \left[\left(1 - \sigma \right) (p_C^*)^{-\sigma} + \sigma \tau \zeta w_S (p_C^*)^{-\sigma - 1} \right] \frac{c_N L_{Nt}}{P_{Nt}^{1 - \sigma}} = 0,
$$

which implies that $(1 - \sigma)(p_C^*)^{-\sigma} + \sigma\tau \zeta w_S (p_C^*)^{-\sigma-1} = 0$. Dividing by $(p_C^*)^{-\sigma}$ yields $\sigma\tau \zeta w_S / p_C^* =$

 $\sigma-1$ and rearranging gives the monopoly export price $p_C^* = \frac{\sigma \tau \zeta w_S}{\sigma - 1} = \frac{\tau \zeta w_S}{\alpha}$. But southern firm faces a competitor in the northern firm whose product it has imitated. There are two cases to consider, depending on whether trade costs are small or large. If $w_N \ge \frac{\tau \zeta w_S}{\alpha}$ or $\frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}$ $rac{\tau \zeta}{\alpha}$ (the small trade cost case), then the northern firm has too high marginal cost w_N to effectively compete in the northern market and the southern firm sets the monopoly export price in equilibrium:

$$
p_C^* = \frac{\tau \zeta w_S}{\alpha}
$$
 if $\frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}$.

If $w_N < \frac{\tau \zeta w_S}{\alpha}$ or $\frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}$ $\frac{r_0}{\alpha}$ (the large trade cost case), then the northern firm has low enough marginal cost w_N so its presence needs to be taken into account and the southern firm sets the limit price in equilibrium:

$$
p_C^* = w_N
$$
 if $\frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}$.

In the small trade cost case,

$$
\pi_{Ct} = (p_C - \zeta w_S) \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (p_C^* - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \n= \left(\frac{\zeta w_S}{\alpha} - \zeta w_S\right) \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + \left(\frac{\tau \zeta w_S}{\alpha} - \tau \zeta w_S\right) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \n= \frac{\zeta w_S}{\sigma - 1} \left[\frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + \tau \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}}\right] \n= \frac{\zeta w_S}{\sigma - 1} \left[\frac{p_C^{-\sigma} c_S L_{St} n_{Ct}}{P_{St}^{1-\sigma} L_t} + \tau \frac{(p_C^*)^{-\sigma} c_N L_{Nt} n_{Ct}}{P_{Nt}^{1-\sigma} L_t}\right] \frac{L_t}{n_t \frac{n_{Ct}}{n_t}}.
$$

Since $X_C \equiv \frac{p_C^{-\sigma} c_S L_{St} n_{Ct}}{P_0^{1-\sigma} L}$ $\frac{P_{CS}^{5} L_{St}^{n_{Ct}}}{P_{St}^{1-\sigma} L_t}, X_C^* \equiv \frac{(p_C^*)^{-\sigma} c_N L_{Nt}^{n_{Ct}}}{P_{Nt}^{1-\sigma} L_t}$ $\frac{c_N L_{Nt} n_{Ct}}{P_{Nt}^{1-\sigma} L_t}$ and $\gamma_C \equiv \frac{n_{Ct}}{n_t}$ $\frac{n_{Ct}}{n_t}$ are all constant over time in steadystate equilibrium, the profit flow for a C-firm is:

$$
\pi_{Ct} = \left[\frac{\zeta w_S \left(X_C + \tau X_C^* \right)}{(\sigma - 1) \gamma_C} \right] \frac{L_t}{n_t} \quad \text{if } \frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}.
$$
\n(13)

In the large trade cost case,

$$
\pi_{Ct} = (p_C - \zeta w_S) \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (p_C^* - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \n= \left(\frac{\zeta w_S}{\alpha} - \zeta w_S\right) \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (w_N - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \n= \frac{\zeta w_S}{\sigma - 1} \frac{p_C^{-\sigma} c_S L_{St}}{P_{St}^{1-\sigma}} + (w_N - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt}}{P_{Nt}^{1-\sigma}} \n= \left[\frac{\zeta w_S}{\sigma - 1} \frac{p_C^{-\sigma} c_S L_{St} n_{Ct}}{P_{St}^{1-\sigma} L_t} + (w_N - \tau \zeta w_S) \frac{(p_C^*)^{-\sigma} c_N L_{Nt} n_{Ct}}{P_{Nt}^{1-\sigma} L_t}\right] \frac{L_t}{n_t \frac{n_{Ct}}{n_t}}.
$$

and it follows that the profit flow for a C-firm is:

$$
\pi_{Ct} = \left[\frac{\zeta w_S X_C}{(\sigma - 1)\gamma_C} + \frac{(w_N - \tau \zeta w_S) X_C^*}{\gamma_C} \right] \frac{L_t}{n_t} \quad \text{if } \frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}.
$$
\n
$$
\tag{14}
$$

Innovation, FDI and Imitation

Given the innovative R&D technology, the aggregate flow of new products developed in the North is θ

$$
\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N g^{\beta}}
$$

where $L_{Rt} \equiv \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities. Substituting for $g \equiv \frac{\dot{n}_t}{n_t}$ $\frac{n_t}{n_t}$, we obtain

$$
\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N (\dot{n}_t)^{\beta} / (n_t)^{\beta}}
$$
\n
$$
(\dot{n}_t)^{1+\beta} = \frac{n_t^{\theta+\beta} L_{Rt}}{a_N}
$$
\n
$$
\dot{n}_t = \left[\frac{n_t^{\theta+\beta} L_{Rt}}{a_N}\right]^{1/(1+\beta)}
$$

R&D Incentives

For a northern firm, the relevant no-arbitrage condition is

$$
v_{Nt} = \frac{\pi_{Nt}}{\rho + \theta g + \iota_N} = \frac{w_N a_N g^{\beta}}{n_t^{\theta}}.
$$

Substituting for π_{Nt} yields

$$
\frac{\frac{w_N(X_N + \tau X_N^*)}{(\sigma - 1)\gamma_N} \frac{L_t}{n_t}}{\rho + \theta g + \iota_N} = \frac{w_N a_N g^{\beta}}{n_t^{\theta}}
$$

$$
\frac{\frac{X_N + \tau X_N^*}{(\sigma - 1)\gamma_N}}{\rho + \theta g + \iota_N} = a_N \frac{n_t^{1-\theta}}{L_t} g^{\beta}.
$$

Thus the steady-state northern no-arbitrage condition is

$$
\frac{\frac{X_N + \tau X_N^*}{(\sigma - 1)\gamma_N}}{\rho + \theta g + \iota_N} = a_N \delta g^{\beta}.
$$
\n(26)

.

For a foreign affiliate, the relevant no-arbitrage condition is

$$
v_{Ft} - v_{Nt} = \frac{\pi_{Ft}}{\rho + \theta g + \iota_S} - \frac{w_N a_N g^{\beta}}{n_t^{\theta}} = \frac{w_S a_F \phi^{\beta}}{n_t^{\theta}}.
$$

Using the foreign affiliate profits from earlier, we can write this as:

$$
\frac{\frac{w_S(X_F + \tau X_F^*)}{(\sigma - 1)\gamma_F} \frac{L_t}{n_t}}{\rho + \theta g + \iota_S} - \frac{w_N a_N g^{\beta}}{n_t^{\theta}} = \frac{w_S a_F \phi^{\beta}}{n_t^{\theta}}
$$

$$
\frac{\frac{X_F + \tau X_F^*}{(\sigma - 1)\gamma_F}}{\rho + \theta g + \iota_S} - \frac{w_N}{w_S} a_N \frac{n_t^{1-\theta}}{L_t} g^{\beta} = a_F \frac{n_t^{1-\theta}}{L_t} \phi^{\beta}.
$$

It follows that the steady-state foreign affiliate no-arbitrage condition is

$$
\frac{X_F + \tau X_F^*}{(\sigma - 1)\gamma_F} - w a_N \delta g^\beta = a_F \delta \phi^\beta
$$
\n
$$
(27)
$$

where $w \equiv \frac{w_N}{w_S}$ $\frac{w_N}{w_S}$ is the northern relative wage.

For a southern firm imitating a foreign affiliate variety, the relevant no-arbitrage condition is

$$
v_{It} = \frac{\pi_{It}}{\rho + \theta g} = \frac{w_S a_I \iota_S^{\beta}}{n_t^{\theta}}.
$$

Using the southern firm profits from earlier, we can write this as:

$$
\frac{\frac{w_S(1-\zeta)\left(X_I + \tau X_I^*\right)}{\gamma_I} \frac{L_t}{n_t}}{\rho + \theta g} = \frac{w_S a_I \iota_S^{\beta}}{n_t^{\theta}}
$$

$$
\frac{\frac{(1-\zeta)\left(X_I + \tau X_I^*\right)}{\gamma_I}}{\rho + \theta g} = a_I \frac{n_t^{1-\theta}}{L_t} \iota_S^{\beta}
$$

It follows that the steady-state I-firm no-arbitrage condition is

$$
\frac{\frac{(1-\zeta)\left(X_I + \tau X_I^*\right)}{\gamma_I}}{\rho + \theta g} = a_I \delta \iota_S^{\beta}.
$$
\n(28)

.

.

For a southern firm imitating a northern variety, the relevant no-arbitrage condition is

$$
v_{Ct} = \frac{\pi_{Ct}}{\rho + \theta g} = \frac{w_S da_I \iota_N^{\beta}}{n_t^{\theta}}
$$

In the small trade cost case,

$$
\frac{\pi_{Ct}}{\rho + \theta g} = \frac{w_S da_I \iota_N^{\beta}}{n_t^{\theta}}
$$

$$
\frac{\zeta w_S (X_C + \tau X_C^*)}{(\sigma - 1)\gamma_C} \frac{L_t}{n_t}}{p + \theta g} = \frac{w_S da_I \iota_N^{\beta}}{n_t^{\theta}}
$$

$$
\frac{\zeta (X_C + \tau X_C^*)}{(\sigma - 1)\gamma_C}}{p + \theta g} = da_I \frac{n_t^{1 - \theta}}{L_t} \iota_N^{\beta}.
$$

It follows that the steady-state C-firm no-arbitrage condition is

$$
\frac{\zeta(X_C + \tau X_C^*)}{\rho + \theta g} = da_I \delta \iota_N^{\beta} \qquad \text{if } \frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}.
$$
\n(29a)

In the large trade cost case,

$$
\frac{\pi_{Ct}}{\rho + \theta g} = \frac{w_S da_I t_N^{\beta}}{n_t^{\theta}}
$$
\n
$$
\frac{\left[\frac{\zeta w_S X_C}{(\sigma - 1)\gamma_C} + \frac{(w_N - \tau \zeta w_S) X_C^*}{\gamma_C}\right] \frac{L_t}{n_t}}{\rho + \theta g} = \frac{w_S da_I t_N^{\beta}}{n_t^{\theta}}
$$
\n
$$
\frac{\frac{\zeta X_C}{(\sigma - 1)\gamma_C} + \frac{(w - \tau \zeta) X_C^*}{\gamma_C}}{\rho + \theta g} = da_I \frac{n_t^{1 - \theta}}{L_t} t_N^{\beta}
$$

It follows that the steady-state C-firm no-arbitrage condition is

$$
\frac{\frac{\zeta X_C}{(\sigma - 1)\gamma_C} + \frac{(w - \tau \zeta)X_C^*}{\gamma_C}}{\rho + \theta g} = da_I \delta \iota_N^{\beta} \qquad \text{if } \frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}.\tag{29b}
$$

.

Labor Markets

Full employment of labor in the South implies that

$$
L_{St} = \frac{a_F \phi^{\beta}}{n_t^{\theta}} (\dot{n}_{Ft} + \dot{n}_{It}) + \frac{a_I \iota_S^{\beta}}{n_t^{\theta}} \dot{n}_{It} + \frac{d a_I \iota_N^{\beta}}{n_t^{\theta}} \dot{n}_{Ct} + (X_F + \tau X_F^*) L_t + \zeta (X_I + \tau X_I^*) L_t + \zeta (X_C + \tau X_C^*) L_t.
$$

Now, using $\delta \equiv \frac{n_t^{1-\theta}}{L_t}$, $\iota_N \equiv \frac{\dot{n}_{Ct}}{n_{Nt}}$ $\frac{\dot{n}_{Ct}}{n_{Nt}}, \phi \equiv \frac{\dot{n}_{Ft} + \dot{n}_{It}}{n_{Nt}}$ $\frac{n_t + n_{It}}{n_{Nt}}$ and $\iota_S \equiv \frac{\dot{n}_{It}}{n_{Ft}}$ $\frac{n_{It}}{n_{Ft}}$, southern R&D employment can be written as

$$
= \frac{a_F \phi^{\beta}}{n_t^{\theta}} (\dot{n}_{Ft} + \dot{n}_{It}) + \frac{a_I \iota_S^{\beta}}{n_t^{\theta}} \dot{n}_{It} + \frac{d a_I \iota_N^{\beta}}{n_t^{\theta}} \dot{n}_{Ct}
$$

\n
$$
= \frac{a_F \phi^{\beta} (\dot{n}_{Ft} + \dot{n}_{It})}{n_{Nt}} \frac{n_{Nt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t + a_I \iota_S^{\beta} \frac{\dot{n}_{It}}{n_{Ft}} \frac{n_{Ft}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t + d a_I \iota_N^{\beta} \frac{\dot{n}_{Ct}}{n_{Nt}} \frac{n_{Nt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t
$$

\n
$$
= a_F \phi^{1+\beta} \gamma_N \delta L_t + a_I \iota_S^{1+\beta} \gamma_F \delta L_t + d a_I \iota_N^{1+\beta} \gamma_N \delta L_t.
$$

It follows that

$$
L_{St} = \left[\left(a_F \phi^{1+\beta} \gamma_N + a_I t_S^{1+\beta} \gamma_F + d a_I t_N^{1+\beta} \gamma_N \right) \delta + X_F + \tau X_F^* + \zeta \left(X_I + \tau X_I^* + X_C + \tau X_C^* \right) \right] L_t
$$

and evaluating at $t = 0$ yields the steady-state full employment of southern labor condition:

$$
L_{S0} = \left(a_F \phi^{1+\beta} \gamma_N + a_I l_S^{1+\beta} \gamma_F + da_I l_N^{1+\beta} \gamma_N \right) \delta L_0 + \left[X_F + \tau X_F^* + \zeta \left(X_I + \tau X_I^* + X_C + \tau X_C^* \right) \right] L_0.
$$
 (31)

Aggregate demand

We need to solve for steady-state values of the aggregate demand expressions X_N , X_N^* , X_I , X_I^* , X_C and X_C^* . The calculations

$$
\frac{X_N}{X_F^*} = \frac{\frac{p_N^{-\sigma}c_N L_N t n_N}{p_N^{1-\sigma}L_t}}{\frac{(p_F^*)^{-\sigma}c_N L_N t n_{Ft}}{p_N^{1-\sigma}L_t}} = \left(\frac{p_N}{p_F^*}\right)^{-\sigma} \frac{n_{Nt}/n_t}{n_{Ft}/n_t} = \left(\frac{\frac{w_N}{\alpha}}{\frac{\alpha}{\alpha}}\right)^{-\sigma} \frac{\gamma_N}{\gamma_F} = \left(\frac{\tau}{w}\right)^{\sigma} \frac{\frac{g}{g+\phi+\iota_N}}{\frac{\phi}{g+\phi+\iota_N}\frac{g}{g+\iota_S}} = \left(\frac{\tau}{w}\right)^{\sigma} \frac{g+\iota_S}{\phi}
$$
\n
$$
\frac{X_N^*}{X_F} = \frac{\frac{(p_N^*)^{-\sigma}c_S L_{St} n_{Nt}}{p_{St}^{1-\sigma}L_t}}{\frac{p_{St}^{-\sigma}c_S L_{St} n_{Ft}}{p_{St}^{1-\sigma}L_t}} = \left(\frac{p_N^*}{p_F}\right)^{-\sigma} \frac{n_{Nt}/n_t}{n_{Ft}/n_t} = \left(\frac{\frac{\tau w_N}{\alpha}}{\frac{\alpha}{\alpha}}\right)^{-\sigma} \frac{\gamma_N}{\gamma_F} = \left(\frac{1}{\tau w}\right)^{\sigma} \frac{\frac{g}{g+\phi+\iota_N}}{\frac{\phi}{g+\phi+\iota_N}\frac{g}{g+\iota_S}} = \left(\frac{1}{\tau w}\right)^{\sigma} \frac{g+\iota_S}{\phi}
$$
\n
$$
\frac{X_I}{X_F} = \frac{\frac{p_I^{-\sigma}c_S L_{St} n_{It}}{p_{St}^{1-\sigma}L_t}}{\frac{p_{St}^{1-\sigma}L_t}{p_{St}^{1-\sigma}L_t}} = \left(\frac{p_I}{p_F}\right)^{-\sigma} \frac{n_{It}/n_t}{n_{Ft}/n_t} = \left(\frac{w_S}{\frac{w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_I}{\gamma_F} = \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\frac{\phi}{g+\phi+\iota_N}\frac{\iota_S}{g+\iota_S}}{\frac{\phi}{g+\phi+\iota_N}\frac{g}{g+\iota_S}} = \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}
$$
\n
$$
\frac{X_I^*}{X_F^*} = \frac{\
$$

imply that

$$
X_N = X_F^* \left(\frac{\tau}{w}\right)^\sigma \frac{g + \iota_S}{\phi}
$$

$$
X_N^* = X_F \left(\frac{1}{\tau w}\right)^\sigma \frac{g + \iota_S}{\phi}
$$

$$
X_I = X_F \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}
$$

$$
X_I^* = X_F^* \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}
$$

$$
X_C = X_F \left(\frac{1}{\zeta}\right)^\sigma \frac{(g + \iota_S)\iota_N}{\phi g}.
$$

In the small trade cost case ($\frac{w_N}{w_S} \geq \frac{\tau \zeta}{\alpha}$ $\frac{r\varsigma}{\alpha}$),

$$
\frac{X_C^*}{X_F^*} = \frac{\frac{\left(p_C^*\right)^{-\sigma} c_N L_N t n_{Ct}}{P_{Nt}^{1-\sigma} L_t}}{\frac{\left(p_F^*\right)^{-\sigma} c_N L_N t n_{Ft}}{P_{Nt}^{1-\sigma} L_t}} = \left(\frac{p_C^*}{p_F^*}\right)^{-\sigma} \frac{\frac{n_{Ct}}{n_t}}{\frac{n_{Ft}}{n_t}} = \left(\frac{\frac{\tau \zeta w_S}{\alpha}}{\frac{\tau w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_C}{\gamma_F} = \left(\frac{1}{\zeta}\right)^{\sigma} \frac{\frac{\iota_N}{g + \phi + \iota_N}}{\frac{\phi}{g + \phi + \iota_N} \frac{g}{g + \iota_S}} = \left(\frac{1}{\zeta}\right)^{\sigma} \frac{\left(g + \iota_S\right) \iota_N}{\phi g}.
$$

In the large trade cost case ($\frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}$ $\frac{r\varsigma}{\alpha}$),

$$
\frac{X_C^*}{X_F^*} = \left(\frac{p_C^*}{p_F^*}\right)^{-\sigma} \frac{n_{Ct}/n_t}{n_{Ft}/n_t} = \left(\frac{w_N}{\frac{\tau w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_C}{\gamma_F} = \left(\frac{\tau}{\alpha w}\right)^{\sigma} \frac{\left(g + \iota_S\right)\iota_N}{\phi g}.
$$

Thus

$$
X_C^* = X_F^* \left(\frac{1}{\zeta}\right)^\sigma \frac{(g + \iota_S) \iota_N}{\phi g} \qquad \text{if } \frac{w_N}{w_S} \ge \frac{\tau \zeta}{\alpha}
$$

and

$$
X_C^* = X_F^* \left(\frac{\tau}{\alpha w}\right)^{\sigma} \frac{\left(g + \iota_S\right) \iota_N}{\phi g} \qquad \text{if } \frac{w_N}{w_S} < \frac{\tau \zeta}{\alpha}
$$

.

Consumer Expenditure and Asset Ownership

The aggregate value of all financial assets is the total value of firms:

$$
A_t = A_{Nt} + A_{St} = n_{Nt}v_{Nt} + n_{Ft}v_{Ft} + n_{It}v_{It} + n_{Ct}v_{Ct}.
$$

The aggregate value of southern financial assets \mathcal{A}_{St} is given by

$$
A_{St} = n_{Ft} (v_{Ft} - v_{Nt}) + n_{It}v_{It} + n_{Ct}v_{Ct}
$$

\n
$$
= n_{Ft} \frac{w_{SAF}\phi^{\beta}}{n_t^{\theta}} + n_{It} \frac{w_{SAI}t_S^{\beta}}{n_t^{\theta}} + n_{Ct} \frac{w_{S}da_{I}t_N^{\beta}}{n_t^{\theta}}
$$

\n
$$
= w_{S} \frac{n_t}{n_t^{\theta}L_t} L_t \frac{n_{Ft}}{n_t} a_F \phi^{\beta} + w_{S} \frac{n_t}{n_t^{\theta}L_t} L_t \frac{n_{It}}{n_t} a_I t_S^{\beta} + w_{S} \frac{n_t}{n_t^{\theta}L_t} L_t \frac{n_{Ct}}{N_t} da_I t_N^{\beta}
$$

\n
$$
= w_{S} \delta L_t \gamma_{F} a_F \phi^{\beta} + w_{S} \delta L_t \gamma_{I} a_I t_S^{\beta} + w_{S} \delta L_t \gamma_{C} da_I t_N^{\beta}
$$

\n
$$
= w_{S} \delta L_t \left[\gamma_{F} a_F \phi^{\beta} + \gamma_{I} a_I t_S^{\beta} + \gamma_{C} da_I t_N^{\beta} \right].
$$

The aggregate value of northern financial assets \mathcal{A}_{Nt} is given by

$$
A_{Nt} = A_t - A_{St}
$$

\n
$$
= [n_{Nt}v_{Nt} + n_{Ft}v_{Ft} + n_{It}v_{It} + n_{Ct}v_{Ct}] - [n_{Ft}(v_{Ft} - v_{Nt}) + n_{It}v_{It} + n_{Ct}v_{Ct}]
$$

\n
$$
= n_{Nt}v_{Nt} + n_{Ft}v_{Nt}
$$

\n
$$
= (n_{Nt} + n_{Ft})v_{Nt}
$$

\n
$$
= (\gamma_N n_t + \gamma_F n_t) \frac{w_N a_N g^{\beta} L_t}{n_t^{\theta} L_t}
$$

\n
$$
= w_N L_t (\gamma_N + \gamma_F) a_N \frac{n_t}{n_t^{\theta} L_t} g^{\beta}
$$

\n
$$
= w_N L_t (\gamma_N + \gamma_F) a_N \delta g^{\beta}.
$$

Northern consumer expenditure c_N is given by

$$
c_N = w_N + (\rho - g_L) \frac{\tilde{a}_{Nt}}{L_{Nt}}
$$

= $w + (\rho - g_L) \frac{A_{Nt}}{L_{Nt}}$
= $w + (\rho - g_L) \frac{w_N L_t (\gamma_N + \gamma_F) a_N \delta g^{\beta}}{L_{Nt}}$

$$
c_N = w \left[1 + (\rho - g_L) \left(\gamma_N + \gamma_F \right) a_N \delta g^\beta \frac{L_0}{L_{N0}} \right]. \tag{32}
$$

Likewise, southern consumer expenditure $c_{\cal S}$ is given by

$$
c_S = w_S + (\rho - g_L) \tilde{a}_{St}
$$

\n
$$
= 1 + (\rho - g_L) \frac{A_{St}}{L_{St}}
$$

\n
$$
= 1 + (\rho - g_L) \frac{w_S \delta L_t \left[\gamma_F a_F \phi^{\beta} + \gamma_I a_I \iota_S^{\beta} + \gamma_C d a_I \iota_N^{\beta} \right]}{L_{St}}
$$

\n
$$
c_S = 1 + (\rho - g_L) \delta \left[\gamma_F a_F \phi^{\beta} + \gamma_I a_I \iota_S^{\beta} + \gamma_C d a_I \iota_N^{\beta} \right] \frac{L_0}{L_{S0}}.
$$
\n(33)

Having solved for consumer expenditures c_N and c_S , we can determine the relationship between X_F and X_F^* :

$$
\frac{X_{F}^{*}}{X_{F}} = \frac{\frac{\left(p_{F}^{*}\right)^{-\sigma}c_{N}L_{Nt}n_{Ft}}{p_{Nt}^{1-\sigma}L_{t}}}{\frac{p_{F}^{-\sigma}c_{S}L_{St}n_{Ft}}{p_{St}^{1-\sigma}L_{t}}} = \left(\frac{p_{F}}{p_{F}^{*}}\right)^{\sigma}\frac{c_{N}}{c_{S}}\frac{L_{N0}}{L_{S0}}\frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} = \left(\frac{\frac{1}{\alpha}}{\frac{\tau}{\alpha}}\right)^{\sigma}\frac{c_{N}}{c_{S}}\frac{L_{N0}}{L_{S0}}\frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}}
$$

and simplifying yields the steady-state asset condition

$$
\frac{X_F^*}{X_F} = \left(\frac{1}{\tau}\right)^{\sigma} \frac{c_N}{c_S} \frac{L_{N0}}{L_{S0}} \frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}},\tag{34}
$$

where

$$
\frac{P_{St}^{1-\sigma}}{P_{Nt}^{1-\sigma}} = \frac{\left(\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}\right) n_t}{\left(\gamma_N p_N^{1-\sigma} + \gamma_F (p_F^*)^{1-\sigma} + \gamma_I (p_I^*)^{1-\sigma} + \gamma_C (p_C^*)^{1-\sigma}\right) n_t}
$$
\n
$$
= \frac{\gamma_N (p_N^*)^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_I p_I^{1-\sigma} + \gamma_C p_C^{1-\sigma}}{\gamma_N p_N^{1-\sigma} + \gamma_F (p_F^*)^{1-\sigma} + \gamma_I (p_I^*)^{1-\sigma} + \gamma_C (p_C^*)^{1-\sigma}}.
$$