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Sequential Investment, Hold-up and Strategic Delay

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Abstract

We investigate hold-up with simultaneous and sequential investment. We show that if the encouragement effect of sequential complementary investments dominates the delay effect, sequential investment alleviates the underinvestment caused by the hold-up problem. Further, if it is allowed to choose when to invest, strategic delay occurs when the encouragement effect of sequential complementary investments dominates the delay effect.

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1 Introduction

According to Che and Sákovics (2008), "hold-up arises when part of the return on an agents relationship-specific investments is ex post expropriable by his trading partner." With incomplete contract, which arises due to causes such as unforeseen contingencies and inability of enforcement, relationship-specific investments are distorted by the hold-up problem and are therefore insufficient.

The current literature on hold-up (see the survey of Che and Sákovics 2008) mainly focuses on the inefficiency issue due to the hold-up problem and organizational or contract remedies to achieve the first best through some ex post renegotiation design. In their models, relationship-specific investments are usually simultaneously invested. In contrast, we investigate hold-up with simultaneous and sequential investment and focus on the impact of sequential investment on inefficiency issue of underinvestment. We show that if the encouragement effect of sequential complementary investments dominates the delay effect, sequential investment alleviates the underinvestment caused by the hold-up problem. Further, if it is allowed to choose when to invest, strategic delay occurs when the encouragement effect of sequential complementary investments dominates the delay effect.

More specifically, there is a potentially profitable relationship between two parties. Some relationship-specific pre-investments from both sides are often involved, which is a double moral-hazard problem in terms of Laffont and Martimort (2002). The two parties have to rely on bargaining to divide the surplus of investment through the ex post renegotiation, since ex ante contracts are incomplete. With sequential investment, the leader may have incentive to invest more to elicit more investment from the follower – ***encouragement effect***. Meanwhile, due to the delay of the realization of the surplus of investment under sequential investment – ***delay effect***, sequential investment alleviates the underinvestment caused by the hold-up problem if the encouragement effect dominates the delay effect. Further, if parties have the option to choose when to invest, strategic delay occurs when the encouragement effect dominates the delay effect.

Our model is close to Smironov and Wait (2004a, 2004b)'s sequential investment model. Smirnov and Wait (2004a, 2004b) provide a model to allow the flexibility in the timing of investment and show that the overall welfare may be detrimental due to the cost of delay. In their alternative investment regime (sequential investment), renegotiation occurs after the leader makes the relationship-specific investment and therefore there is no role of encouragement effect of sequential complementary investments. In contrast, in our model, contracting is impossible on both relationship-specific investments. Consequently, renegotiation will only occur after both relationship-specific investments are sunk.

Our model is also related to the literature on property rights theory.[1] Nöldeke and Schmidt (1998) and Zhang and Zhang (2010) show that the underinvestment caused by the hold-up problem still exists under the sequential investment setting. Further, Zhang and Zhang (2010) show the alleviation of underinvestment under sequential investment and the consequent impact of sequential investment on the choice of ownership structure. In their model, there is no discount and hence there is no role of delay effect.

Lastly, there is some literature on the dynamics of hold-up (see, for instance, Che and Sákovics (2004)), which allows the parties to continue to invest until they agree on the terms of trade. In contrast, we assume the relationship-specific investments are a one-time irreversible choice. Even if parties can choose when to invest, they can not alter the investment level once it has been sunk.

The rest of the paper is organized as follows. Section 2 provides the setup of our basic model and shows that sequential investment alleviates the underinvestment caused by the hold-up problem if the encouragement effect of sequential complementary investments dominates the delay effect. In section 3, it is allowed to choose when to invest and we show that strategic delay occurs when the encouragement effect of sequential complementary investments dominates the delay effect. Section 4 concludes.

2 The Model

Follow the setup of Smironov and Wait (2004a, 2004b). There is a potentially profitable relationship between two parties that, for convenience, we label as a buyer $M1$ and a seller $M2$. Specifically, if the buyer and seller invest I_1 and I_2 respectively, the two parties share surplus $R(I_1, I_2)$.

Two alternative timing arrangements are considered. First, both players invest simultaneously at date $t = 1$, as shown in Figure 1. At this stage, contracting on either investment is not possible; consequently, renegotiation (or contracting) will occur at date $t = 2$ after both investments are sunk. If there is an agreement, surplus is realized and the payoffs to each party are made. Otherwise, if the renegotiation breaks down, they will stay with their own non-trade payoffs, which are normalized to zero.

Figure 2 illustrates the timing of the alternative investment regime. In this regime, the buyer $M1$ invests I_1 at date $t = 1$. After observing $M1$'s investment, the seller $M2$

[1] They assume ex ante parties could negotiate on the ownership structure (residual rights of control), which determines the status quo payoffs of the parties in the ex post renegotiation. And thus, hold-up problem reduces through this organization remedy. In contrast, we assume ex ante contracting is impossible on both relationship-specific investments. Hence, there is no role of ownership structure in our model.

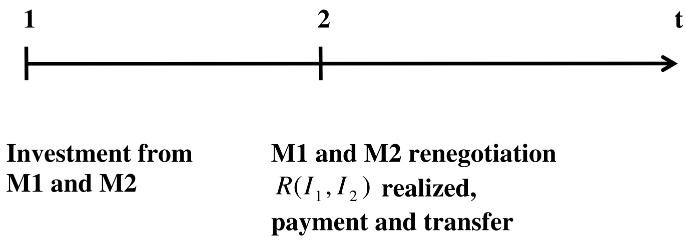

Figure 1: Timing of Simultaneous Investment

invests I_2 at date $t = 2$. At both of these two stages, contracting on either investment is not possible;[2] consequently, renegotiation (or contracting) will occur at date $t = 3$ after both investments are sunk. If there is an agreement, surplus is realized and the payoffs to each party are made. Otherwise, if the renegotiation breaks down, they will stay with their own non-trade payoffs, which are normalized to zero.

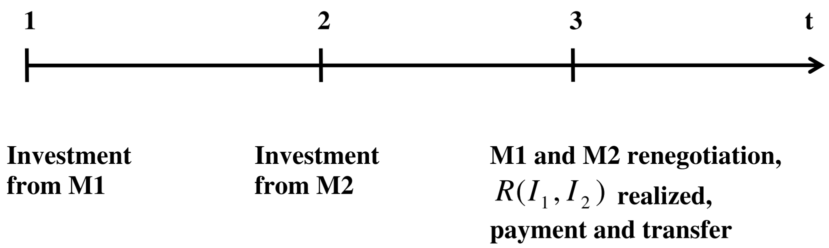

Figure 2: Timing of Sequential Investment

Suppose both parties have the common discount factor $\delta \in (0, 1]$. In addition, we make the following assumptions for $R(I_1, I_2)$.

Assumption 1 *$R(I_1, I_2)$ is twice differentiable, nondecreasing in both variables, and strictly concave.*

Assumption 2

$$\frac{\partial^2 R(I_1, I_2)}{\partial I_1 \partial I_2} \geq 0$$

Assumption 1 is the usual assumption of the surplus function. Assumption 2 says that investments are complementary at the margin. Let α represent the ex post bargaining weight of $M1$, where $\alpha \in (0, 1)$.

[2] In Smironov and Wait (2004a, 2004b), they assume once I_1 has been made, I_2 is contractible. Therefore, in their models, renegotiation is in between I_1 and I_2 for the sequential investment. On the contrary, we assume contracting on either investment is possible only if both investments are sunk.

2.1 The First-Best

In the first-best, $M1$ and $M2$ maximize the date 1 present value of their trading relationship, the ex ante surplus.[3]

$$\max_{I_1, I_2} \delta R(I_1, I_2) - I_1 - I_2$$

The first order conditions are

$$\begin{cases} \delta \frac{\partial R(I_1,I_2)}{\partial I_1} = 1 \\ \delta \frac{\partial R(I_1,I_2)}{\partial I_2} = 1 \end{cases}$$

Let (I_1^*, I_2^*) denote the solution of the maximization problem above.

2.2 Simultaneous Investment

At date 1, $M1$ and $M2$ maximize their own payoffs, net of investment costs.

$$\max_{I_1} \alpha\delta R(I_1, I_2) - I_1$$
$$\max_{I_2} (1-\alpha)\delta R(I_1, I_2) - I_2$$

The first order conditions are

$$\begin{cases} \alpha\delta \frac{\partial R(I_1,I_2)}{\partial I_1} = 1 \\ (1-\alpha)\delta \frac{\partial R(I_1,I_2)}{\partial I_2} = 1 \end{cases}$$

Suppose $(\underline{I_1}, \underline{I_2})$ satisfies the first order conditions above.

The following proposition shows that under simultaneous investment, there is underinvestment in relationship-specific investments due to the hold-up problem.

Proposition 1 *Under simultaneous investment,* $(\underline{I_1}, \underline{I_2}) \leq (I_1^*, I_2^*)$.

Proof. See the Appendix. ■

The response functions and the equilibrium investment pairs under simultaneous investment and at the first-best are illustrated in Figure 3. Here, $I_1^*(I_2)$ is the response function of I_1 with respect to I_2 under the first best; $I_2^*(I_1)$ is the response function of I_2 with respect to I_1 under the first best; $\underline{I_1}(I_2)$ is the response function of I_1 with respect to I_2 under the simultaneous investment; $\underline{I_2}(I_1)$ is the response function of I_2 with respect to I_1 under the simultaneous investment.

[3] It takes one period for $R(I_1, I_2)$ to be realized after both I_1 and I_2 are invested.

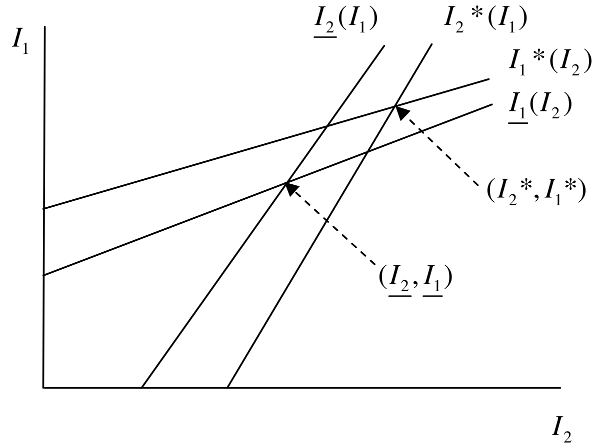

Figure 3: Equilibrium Investment Pairs under Simultaneous Investment

2.3 Sequential Investment

With sequential regime, $M2$ can observe the investment I_1 from $M1$ before his investment. $M1$ chooses I_1 at date 1. After observing $M1$'s investment, $M2$ chooses I_2 at date 2. They maximize their own payoffs, net of investment costs.

With backward induction, at date 2, $M2$ chooses I_2 given $M1$'s choice I_1 at date 1.

$$\begin{aligned} &\max_{I_2} (1-\alpha)\delta R(I_1, I_2) - I_2 \\ &\text{s.t.} \quad I_1 \text{ is some given constant} \end{aligned}$$

The first order condition is

$$(1-\alpha)\delta \frac{\partial R(I_1, I_2)}{\partial I_2} = 1 \tag{1}$$

From the first order condition above, we get the response function of $M2$.

$$I_2 = I_2(I_1)$$

At date 1, $M1$ chooses I_1 given the response function of $M2$ above.

$$\begin{aligned} &\max_{I_1} \ \alpha\delta^2 R(I_1, I_2) - I_1 \\ &\text{s.t.} \quad I_2 = I_2(I_1) \end{aligned}$$

The first order condition is

$$\alpha\delta^2 \frac{\partial R(I_1, I_2)}{\partial I_1} + \alpha\delta^2 \frac{\partial R(I_1, I_2)}{\partial I_2} \frac{dI_2}{dI_1} = 1 \tag{2}$$

Suppose $(\overline{I_1}, \overline{I_2})$ satisfies the first order condition above and the response function $I_2 = I_2(I_1)$ of $M2$.

Since relationship-specific investments are complementary, the first mover has incentive to invest more to encourage the follower to catch up – ***encouragement effect*** à la Zhang and Zhang (2010). Further, with sequential regime, it takes one more period for $R(I_1, I_2)$ to be realized. We call this ***delay effect***. The following proposition shows that if the encouragement effect of sequential complementary investments dominates the delay effect, sequential investment alleviates the underinvestment caused by the hold-up problem. That is, if $M1$ and $M2$ are patient enough, both investment levels will increase with sequential regime.

Proposition 2 *There exists a $\widehat{\delta}$, such that if $\delta \geq \widehat{\delta}$, $(\overline{I_1}, \overline{I_2}) \geq (\underline{I_1}, \underline{I_2})$.*

Proof. See the Appendix. ■

Given some δ, the response functions and the equilibrium investment pairs under sequential investment, under simultaneous investment, and at the first-best are illustrated in Figure 4. Here, $\overline{I_1}(I_2)$ is the response function of I_1 with respect to I_2

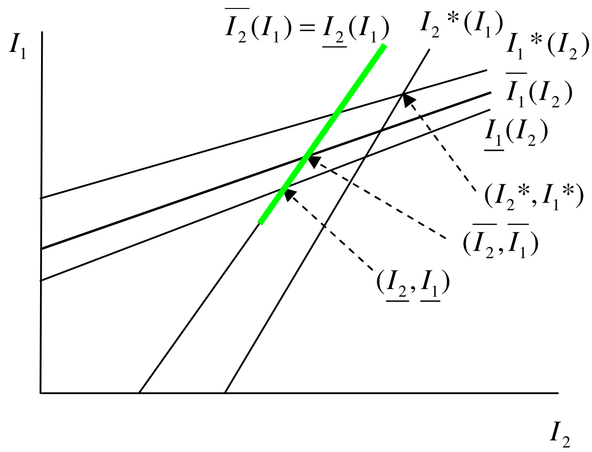

Figure 4: Equilibrium Investment Pairs under Sequential Investment

under the sequential investment; $\overline{I_2}(I_1)$ is the response function of I_2 with respect to I_1 under the sequential investment. With sequential regime, $M2$'s response function remains unchanged, while $M1$'s response function curve could shift up or down depending upon how large δ is. Therefore, the equilibrium investment pairs will reach some point on the $M2$'s response function curve (the bold portion of $I_2(I_1)$ in Figure 4).

Figure 5 illustrates the loci of the equilibrium investment pairs under sequential investment, under simultaneous investment, and at the first-best as δ evolves from 0 to 1. As δ close to zero, both I_1 and I_2 are close to zero for both sequential

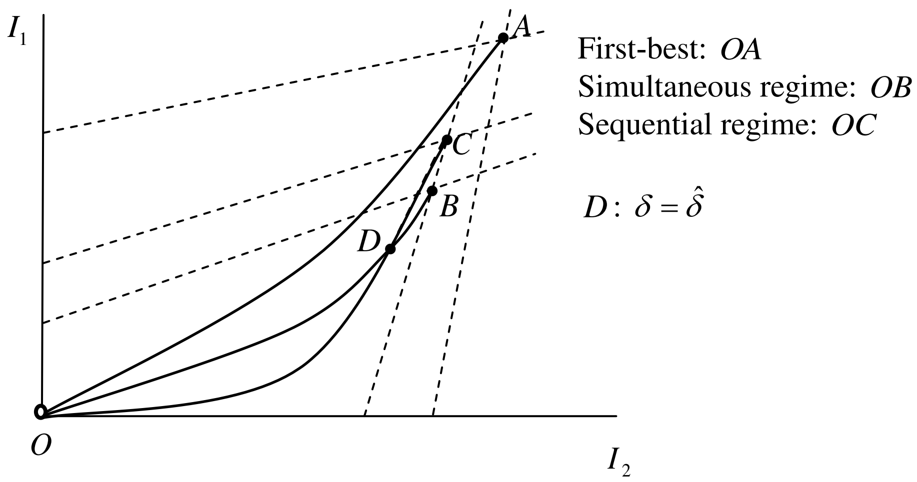

Figure 5: Equilibrium Investment Pairs under Sequential Investment

and simultaneous regimes, as well as at the first-best. As δ approaches to 1, the encouragement effect of sequential investment dominates the delay effect.[4] Therefore, if δ is sufficiently large, the equilibrium investment pairs I_1 and I_2 with sequential regime are larger than those with simultaneous regime.[5]

2.4 Welfare Analysis

In proposition 2, we show that due to both the encouragement effect and the delay effect, there will be more investments under sequential investment if $M1$ and $M2$ are patient enough. The further question is whether more investments are better, or if the ex ante surplus is increasing as I_1 and I_2 increase under sequential investment if $M1$ and $M2$ are patient enough.

Let the ex ante surplus under the simultaneous regime $\underline{S} = \delta R(\underline{I_1}, \underline{I_2}) - \underline{I_1} - \underline{I_2}$; the ex ante surplus under the simultaneous regime $\overline{S} = \delta^2 R(\overline{I_1}, \overline{I_2}) - \overline{I_1} - \delta\overline{I_2}$, the ex ante surplus under the first-best $S^* = \delta R(I_1^*, I_2^*) - I_1^* - I_2^*$. The following lemma shows that $\underline{S}$, $\overline{S}$, and S^* are monotonically increasing as δ evolves from 0 to 1.

Lemma 1 *$\underline{S}$, $\overline{S}$, and S^* are increasing in δ.*

Proof. See the Appendix. ■

[4] Zhang and Zhang (2010) show this for the case $\delta = 1$.

[5] We may not have the the single crossing of the loci of the equilibrium investment pairs under sequential investment and under simultaneous investment as δ evolves from 0 to 1. However, if the encouragement effect is non-decreasing in δ, there exists the single crossing as illustrated in figure 5.

The following proposition shows that if the encouragement effect dominates the delay effect, then the sequential regime will be better than the simultaneous regime in terms of larger ex ante surplus.

Proposition 3

i) *If* $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$, *then* $\overline{S} \leq \underline{S}$.

ii) *There exists a* $\widetilde{\delta} \geq \widehat{\delta}$, *such that if* $\delta \geq \widetilde{\delta}$, $\overline{S} \geq \underline{S}$.

Proof. See the Appendix. ■

Intuitively, with the same or lower level of investments, the sequential regime is worse than the simultaneous regime, due to the delay of the realization of $R(I_1, I_2)$ under sequential regime. Moreover, from proposition 2, if $\delta \geq \widehat{\delta}$, there will be more investment under sequential regime. But this can not guarantee that the ex ante surplus is larger, due to the delay under sequential regime. Similar to Zhang and Zhang (2010) proposition 3, we have $\overline{S} \geq \underline{S}$ if $\delta = 1$. Therefore, we can always find a $\widetilde{\delta} \geq \widehat{\delta}$, such that if $\delta \geq \widetilde{\delta}$, $\overline{S} \geq \underline{S}$.

3 Strategic Delay

3.1 Strategic Delay – One-sided

Suppose now $M2$ has the option when to invest. In this case, both simultaneous and sequential regime are possible. $M1$ invests I_1 at date $t = 1$; $M2$ can choose either to invest I_2 at date $t = 1$ or to wait till date $t = 2$ when $M1$'s investment has been sunk.

The following proposition shows that $M2$ has incentive to delay if the encouragement effect of sequential complementary investments dominates the delay effect.

Proposition 4

i) *If* $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$, *then* $M2$ *does not have incentive to delay.*

ii) *There exists a* $\widehat{\widehat{\delta}} \geq \widehat{\delta}$, *such that if* $\delta \geq \widehat{\widehat{\delta}}$, $M2$ *will wait till date* $t = 2$ *to invest.*

Proof. See the Appendix. ■

Intuitively, if the encouragement effect of sequential complementary investments is dominated by the delay effect such that $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$, $M2$ does not have incentive to delay. Further, if δ is close to one, the encouragement effect of sequential complementary investments dominates the delay effect and $M2$ has incentive to delay.

3.2 Strategic Delay – Two-sided

Suppose now both $M1$ and $M2$ have the option when to invest. In this case, if one party invests at date t, then the other party will invest at date $t+1$, as there is no gain to delay further once the leader's investment has been sunk. The question now is who will initial the investment or both invest at date $t=1$.

The following proposition shows that if the encouragement effect of sequential complementary investments is dominated by the delay effect, both $M1$ and $M2$ do not have incentive to delay. Further, if $M1$ and $M2$ are patient enough, the game becomes an anti-coordination game.

Proposition 5

i) *If* $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$ *and* $(\overline{\overline{I_1}}, \overline{\overline{I_2}}) \leq (\underline{I_1}, \underline{I_2})$*, then both* $M1$ *and* $M2$ *will invest at date* $t = 1$*.*[6]

ii) *There exists a* $\widetilde{\widetilde{\delta}} \geq \widehat{\delta}$*, such that if* $\delta \geq \widetilde{\widetilde{\delta}}$*, the game becomes an anti-coordination game. There are three possible equilibria:*

- **(1)** $M1$ *invests at date* $t = 1$*, followed by* $M2$ *investing at date* $t = 2$*;*
- **(2)** $M2$ *invests at date* $t = 1$*, followed by* $M1$ *investing at date* $t = 2$*;*
- **(3)** $M1$ *and* $M2$ *invest at date* $t = 1$ *with probability* (p^*, q^*)*, where* $p^*, q^* \in (0, 1)$*; for any date* $t > 1$*, if no one has invested before,* $M1$ *and* $M2$ *invest at date* t *with probability* (p^*, q^*)*.*

Proof. See the Appendix. ■

Intuitively, if the encouragement effect of sequential complementary investments is dominated by the delay effect such that $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$ and $(\overline{\overline{I_1}}, \overline{\overline{I_2}}) \leq (\underline{I_1}, \underline{I_2})$, both $M1$ and $M2$ do not have incentive to delay. Further, if δ is close to one, the encouragement effect of sequential complementary investments dominates the delay effect. The benefit from sequential regime is so large that $M1$ and $M2$ end up with an anti-coordination game: if one waits, it is better for the other to invest immediately.

4 Conclusion

We investigate hold-up with simultaneous and sequential investment and focus on the impact of sequential investment on inefficiency issue of underinvestment. We show

[6]Here, by a slight abuse of notation, for sequential regime, denote the equilibrium investment pairs when $M2$ is the leader as $(\overline{\overline{I_1}}, \overline{\overline{I_2}})$, which is different from the equilibrium investment pair $(\overline{I_1}, \overline{I_2})$ when $M1$ is the leader.

that if the encouragement effect of sequential complementary investments dominates the delay effect, sequential investment alleviates the underinvestment caused by the hold-up problem. Further, if it is allowed to choose when to invest, strategic delay occurs when the encouragement effect of sequential complementary investments dominates the delay effect.

Appendix

Proof of Proposition 1 Let $x = (I_1, I_2)$. Similar to the proof of proposition 1 in Hart and Moore (1990) and proposition 1 in Zhang and Zhang (2010), define $g(x) = \delta R(I_1, I_2) - I_1 - I_2$ and $h(x)$ such that

$$\nabla g(x) = \begin{pmatrix} \delta \frac{\partial R(I_1,I_2)}{\partial I_1} - 1 \\ \delta \frac{\partial R(I_1,I_2)}{\partial I_2} - 1 \end{pmatrix}$$

$$\nabla h(x) = \begin{pmatrix} \alpha\delta \frac{\partial R(I_1,I_2)}{\partial I_1} - 1 \\ (1-\alpha)\delta \frac{\partial R(I_1,I_2)}{\partial I_2} - 1 \end{pmatrix}$$

From the first order conditions in section 2.1 and 2.2, we have

$$\nabla g(x)\big|_{x=(I_1^*,I_2^*)} = 0$$
$$\nabla h(x)\big|_{x=(\underline{I_1},\underline{I_2})} = 0$$

From assumption 1, we have $\nabla g(x) \geq \nabla h(x)$ for any investments I_1, I_2. Define $f(x,\lambda) = \lambda g(x) + (1-\lambda)h(x)$. Also define $x(\lambda) = (i(\lambda), e(\lambda))$ to solve $\nabla f(x,\lambda) = 0$. Total differentiating, we obtain

$$H(x,\lambda)dx(\lambda) = -[\nabla g(x) - \nabla h(x)]d\lambda$$

where $H(x,\lambda)$ is the Hessian of $f(x,\lambda)$ with respect to x. From assumption 1 and 2, $H(x,\lambda)$ is negative definite. Also, from assumption 2, the off-diagonal elements of $H(x,\lambda)$ are non-negative. From Takayama (1985), p.393, theorem 4.D.3 [III"] and [IV"], $H(x,\lambda)^{-1}$ is nonpositive. Therefore, $dx(\lambda)/d\lambda \geq 0$, and $x(1) \geq x(0)$, which implies $\underline{I_1} \leq I_1^*$ and $\underline{I_2} \leq I_2^*$. ■

Proof of Proposition 2 With backward induction, at date 2, $M2$ maximizes his own payoffs, net of investment costs, by choosing I_2 given $M1$'s choice I_1 at date 1.1. Total differentiating the first order condition (equation 4), we obtain

$$(1-\alpha)\frac{\partial^2 R(I_1,I_2)}{\partial I_2^2}dI_2 + (1-\alpha)\frac{\partial^2 R(I_1,I_2)}{\partial I_2 \partial I_1}dI_1 = 0$$

Rearranging and from assumption 1 and 2, we have

$$\frac{dI_2}{dI_1} = -\frac{\frac{\partial^2 R(I_1,I_2)}{\partial I_2 \partial I_1}}{\frac{\partial^2 R(I_1,I_2)}{\partial I_2^2}} \geq 0$$

Similar to the proof of proposition 1, let $x = (I_1, I_2)$. From equation 4 and 2, define $h(x)$ and $l(x)$ such that

$$\nabla h(x) = \begin{pmatrix} \alpha\delta\frac{\partial R(I_1,I_2)}{\partial I_1} - 1 \\ (1-\alpha)\delta\frac{\partial R(I_1,I_2)}{\partial I_2} - 1 \end{pmatrix}$$

$$\nabla l(x) = \begin{pmatrix} \alpha\delta^2\frac{\partial R(I_1,I_2)}{\partial I_1} + \alpha\delta^2\frac{\partial R(I_1,I_2)}{\partial I_2}\frac{dI_2}{dI_1} - 1 \\ (1-\alpha)\delta\frac{\partial R(I_1,I_2)}{\partial I_2} - 1 \end{pmatrix}$$

From the first order conditions in section 2.2 and 2.3, we have

$$\nabla h(x)\big|_{x=(\underline{I_1},\underline{I_2})} = 0$$
$$\nabla l(x)\big|_{x=(\overline{I_1},\overline{I_2})} = 0$$

From the first order conditions in section 2.1, 2.2, and 2.3, there exist corresponding unique investment pairs (I_1^*, I_2^*), $(\underline{I_1}, \underline{I_2})$, and $(\overline{I_1}, \overline{I_2})$, for any given δ. Same logic as the proof in proposition 1, (I_1^*, I_2^*), $(\underline{I_1}, \underline{I_2})$, and $(\overline{I_1}, \overline{I_2})$ are increasing as δ increases.

If δ is close to zero, all investments will be close to zero because it takes one period for $R(I_1, I_2)$ to be realized after both I_1 and I_2 are invested. Further, if $\delta = 1$, we have $\nabla l(x) \geq \nabla h(x)$ for any investments I_1, I_2 since $\frac{dI_2}{dI_1} \geq 0$. That is, for $\delta = 1$

$$\alpha\delta^2\frac{\partial R(I_1,I_2)}{\partial I_1} + \alpha\delta^2\frac{\partial R(I_1,I_2)}{\partial I_2}\frac{dI_2}{dI_1} \geq \alpha\delta\frac{\partial R(I_1,I_2)}{\partial I_1}$$

Same logic as the proof of proposition 1, we have $\overline{I_1} \geq \underline{I_1}$ and $\overline{I_2} \geq \underline{I_2}$. Since all functions are continuous and differentiable, we can always find a $\widehat{\delta}$, such that if $\delta \geq \widehat{\delta}$, $\overline{I_1} \geq \underline{I_1}$ and $\overline{I_2} \geq \underline{I_2}$. ■

Proof of Lemma 1 Total differentiating the ex ante surplus under the first-best $S^* = \delta R(I_1^*, I_2^*) - I_1^* - I_2^*$,

$$dS^* = R(I_1^*, I_2^*)d\delta + \left[\delta\frac{\partial R(I_1^*, I_2^*)}{\partial I_1^*} - 1\right]dI_1^* + \left[\delta\frac{\partial R(I_1^*, I_2^*)}{\partial I_2^*} - 1\right]dI_2^*$$

From the first order conditions in section 2.1, we have

$$\frac{dS^*}{d\delta} = R(I_1^*, I_2^*) \geq 0$$

Similarly, total differentiating the ex ante surplus under the simultaneous regime $\underline{S} = \delta R(\underline{I_1}, \underline{I_2}) - \underline{I_1} - \underline{I_2}$,

$$d\underline{S} = R(\underline{I_1}, \underline{I_2})d\delta + \left[\delta\frac{\partial R(\underline{I_1}, \underline{I_2})}{\partial \underline{I_1}} - 1\right]d\underline{I_1} + \left[\delta\frac{\partial R(\underline{I_1}, \underline{I_2})}{\partial \underline{I_2}} - 1\right]d\underline{I_2}$$

From the first order conditions in section 2.2, we have

$$\frac{d\underline{S}}{d\delta} = R(\underline{I_1}, \underline{I_2}) + \left[\frac{1}{\alpha} - 1\right]\frac{d\underline{I_1}}{d\delta} + \left[\frac{1}{1-\alpha} - 1\right]\frac{d\underline{I_2}}{d\delta} \geq 0$$

Here, $(\underline{I_1}, \underline{I_2})$ are increasing in δ from From proposition 2.

Same logic, total differentiating the ex ante surplus under the sequential regime $\overline{S} = \delta^2 R(\overline{I_1}, \overline{I_2}) - \overline{I_1} - \delta\overline{I_2}$,

$$d\overline{S} = \left[2\delta R(\overline{I_1}, \overline{I_2}) - \overline{I_2}\right] d\delta + \left[\delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_1}} - 1\right] d\overline{I_1} + \delta\left[\delta \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}} - 1\right] d\overline{I_2}$$

From the first order conditions under sequential investment in section 2.3, we have

$$\frac{d\overline{I_2}}{d\overline{I_1}} = \frac{1 - \alpha\delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_1}}}{\alpha\delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}}} = \frac{1 - \alpha\delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_1}}}{\delta\left[\delta \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}} - 1\right]} \geq \frac{1 - \delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_1}}}{\delta\left[\delta \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}} - 1\right]}$$

which implies

$$\left[\delta^2 \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_1}} - 1\right] d\overline{I_1} + \delta\left[\delta \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}} - 1\right] d\overline{I_2} \geq 0$$

Here,

$$\delta\left[\delta \frac{\partial R(\overline{I_1}, \overline{I_2})}{\partial \overline{I_2}} - 1\right] = \delta\left[\frac{1}{1-\alpha} - 1\right] > 0$$

In addition, $2\delta R(\overline{I_1}, \overline{I_2}) - \overline{I_2} \geq \delta R(\overline{I_1}, \overline{I_2}) - \overline{I_2} \geq \delta(\delta R(\overline{I_1}, \overline{I_2}) - \overline{I_2}) \geq \delta^2 R(\overline{I_1}, \overline{I_2}) - \delta\overline{I_2} - \overline{I_1} \geq 0$, as the ex ante surplus is non-negative. Therefore, we have $\frac{d\overline{S}}{d\delta} \geq 0$. ■

Proof of Proposition 3

i) With the same level of investment, $\overline{S} \leq \underline{S}$, as $[\delta R(I_1, I_2) - I_1 - I_2] - [\delta^2 R(I_1, I_2) - I_1 - \delta I_2] = (1-\delta)[\delta R(I_1, I_2) - I_2] \geq 0$. Here, $[\delta R(I_1, I_2) - I_2] \geq 0$ to ensure a non-negative ex ante surplus. In addition, $\underline{S}$ and $\overline{S}$ are increasing in δ from lemma 1. Therefore, if $\overline{I_1} \leq \underline{I_1}$ and $\overline{I_2} \leq \underline{I_2}$, then $\overline{S} = \delta^2 R(\overline{I_1}, \overline{I_2}) - \overline{I_1} - \delta\overline{I_2} \leq \delta R(\overline{I_1}, \overline{I_2}) - \overline{I_1} - \overline{I_2} \leq \delta R(\underline{I_1}, \underline{I_2}) - \underline{I_1} - \underline{I_2} = \underline{S}$.

ii) From lemma 1, $\underline{S}$ and $\overline{S}$ are monotonically increasing as δ evolves from 0 to 1. Moreover, from part i) of this proposition, with the same or lower level of investments, the sequential regime is worse than the simultaneous regime. Further, similar to Zhang and Zhang (2010) proposition 3, we can show that $\overline{S} \geq \underline{S}$ if $\delta = 1$. Finally, all functions are continuous and differentiable. Therefore, we can always find a $\widetilde{\delta} \geq \widehat{\delta}$, such that if $\delta \geq \widetilde{\delta}$, $\overline{S} \geq \underline{S}$. ■

Proof of Proposition 4

i) At date $t = 1$, $M2$ has the option when to invest. The present value of payoff for $M2$ to invest at date $t = 1$, net of investment cost, is

$$\pi_2^S = (1-\alpha)\delta R(\underline{I_1}, \underline{I_2}) - \underline{I_2}$$

The present value of payoff for $M2$ to wait till date $t = 2$, net of investment cost, is

$$\pi_2^F = (1-\alpha)\delta^2 R(\overline{I_1}, \overline{I_2}) - \delta\overline{I_2}$$

Total differentiating $(1-\alpha)\delta R(I_1, I_2) - I_2$,

$$d[(1-\alpha)\delta R(I_1, I_2) - I_2] = (1-\alpha)\delta\frac{\partial R(I_1, I_2)}{\partial I_1}dI_1 + \left[(1-\alpha)\delta\frac{\partial R(I_1, I_2)}{\partial I_2} - 1\right]dI_2$$

From the first order conditions in section 2.2 and 2.3, we have

$$(1-\alpha)\delta\frac{\partial R(I_1, I_2)}{\partial I_2} = 1$$

which implies $(1-\alpha)\delta R(I_1, I_2) - I_2$ is increasing in I_1 and I_2.

Therefore, if $\overline{I_1} \leq \underline{I_1}$ and $\overline{I_2} \leq \underline{I_2}$,

$$\begin{aligned}\pi_2^F = (1-\alpha)\delta^2 R(\overline{I_1}, \overline{I_2}) - \delta\overline{I_2} &= \delta\left[(1-\alpha)\delta R(\overline{I_1}, \overline{I_2}) - \overline{I_2}\right] \\ &\leq \delta\left[(1-\alpha)\delta R(\underline{I_1}, \underline{I_2}) - \underline{I_2}\right] \\ &\leq \left[(1-\alpha)\delta R(\underline{I_1}, \underline{I_2}) - \underline{I_2}\right] = \pi_2^S\end{aligned}$$

That is to say, $M2$ does not have incentive to delay.

ii) If $\delta = 1$, from proposition 2, we have $\overline{I_1} \geq \underline{I_1}$ and $\overline{I_2} \geq \underline{I_2}$. From part i) of this proposition, $(1-\alpha)\delta R(I_1, I_2) - I_2$ is increasing in I_1 and I_2. In this case,

$$\pi_2^F = (1-\alpha)R(\overline{I_1}, \overline{I_2}) - \overline{I_2} \geq (1-\alpha)R(\underline{I_1}, \underline{I_2}) - \underline{I_2} = \pi_2^S$$

Therefore, $M2$ will wait till date $t = 2$ to invest I_2. Finally, all functions are continuous and differentiable. Therefore, we can always find a $\widehat{\widehat{\delta}} \geq \widehat{\delta}$, such that if $\delta \geq \widehat{\widehat{\delta}}$, $M2$ will wait till date $t = 2$ to invest I_2. ■

Proof of Proposition 5

i) Similar to the proof of part i) of proposition 4, let us see the best response of $M2$ if $M1$ invests at date $t = 1$. The present value of payoff for $M2$ to invest at date $t = 1$, net of investment cost, is

$$\pi_2^S = (1-\alpha)\delta R(\underline{I_1}, \underline{I_2}) - \underline{I_2}$$

The present value of payoff for $M2$ to wait till date $t = 2$, net of investment cost, is

$$\pi_2^F = (1-\alpha)\delta^2 R(\overline{I_1}, \overline{I_2}) - \delta\overline{I_2}$$

Similar to the proof in part i) of proposition 4, if $(\overline{I_1}, \overline{I_2}) \leq (\underline{I_1}, \underline{I_2})$, $\pi_2^F \leq \pi_2^S$. That is to say, if $M1$ invests at date $t = 1$, $M2$'s best response is to invest at date $t = 1$.

Further, let us see the best response of $M2$ if $M1$ waits at date $t = 1$. The present value of payoff for $M2$ to invest at date $t = 1$, net of investment cost, is

$$\pi_2^L = (1 - \alpha)\delta^2 R(\overline{\overline{I_1}}, \overline{\overline{I_2}}) - \overline{\overline{I_2}}$$

Similar to the proof in part i) of proposition 4, if $(\overline{\overline{I_1}}, \overline{\overline{I_2}}) \leq (\underline{I_1}, \underline{I_2})$, $\pi_2^L \leq \pi_2^S$.

The present value of payoff for $M2$ to wait till date $t = 2$, net of investment cost, is the continuation payoff when both $M1$ and $M2$ wait at date $t = 1$, denoted as X_2. The following table illustrates the payoff matrix at date $t = 1$ for $M1$ and $M2$.[7]

		$M2$	
		Invest	Wait
$M1$	Invest	π_1^S, π_2^S	π_1^L, π_2^F
	Wait	π_1^F, π_2^L	X_1, X_2

Clearly, both $M1$ and $M2$ wait at date $t = 1$ is not an equilibrium, as at date $t = 2$ they are facing the same game as date $t = 1$ game. If it is optimal for both $M1$ and $M2$ waiting at date $t = 1$, then it is also optimal for both $M1$ and $M2$ waiting at date $t = 2$. Same logic applies to any future period, and the continuation payoff $X_1 = X_2 = 0$. Therefore, if $M1$ waits at date $t = 1$, the best response for $M2$ is to invest at date $t = 1$ with some probability $q \in (0, 1]$, in which $X_2 \leq \pi_2^L$.

Same reasoning applies to $M1$ and we have $\pi_1^F \leq \pi_1^S$, $\pi_1^L \leq \pi_1^S$, and $X_1 \leq \pi_1^L$. If $M2$ invests at date $t = 1$ with some probability $q \in (0, 1)$, then X_1 is some convex combination of π_1^F, π_1^S, π_1^L, and X_1 itself, multiplying the discount factor. If $\delta < 1$, $X_1 < \pi_1^L$, and also $X_2 < \pi_2^L$. In this case, there exists an unique equilibrium such that both $M1$ and $M2$ invest at date $t = 1$.

If $\delta = 1$, we could have the equilibrium such that $M1$ and/or $M2$ invest at date $t = 1$ with some probability in between $(0, 1)$. Still, investing at date $t = 1$ is a weakly dominant strategy for both $M1$ and $M2$.

ii) Similar to the proof in part ii) of proposition 4, if $\delta = 1$, from proposition 2, we have $(\overline{I_1}, \overline{I_2}) \geq (\underline{I_1}, \underline{I_2})$. Analogously, $(\overline{\overline{I_1}}, \overline{\overline{I_2}}) \geq (\underline{I_1}, \underline{I_2})$. Similar to the proof of part i) of this proposition, we have $\pi_1^L \geq \pi_1^S$, $\pi_1^F \geq \pi_1^S$, $\pi_2^L \geq \pi_2^S$, and $\pi_2^F \geq \pi_2^S$. For the continuation payoff, $X_1 \leq \pi_1^L$ and $X_2 \leq \pi_2^L$.

Therefore, the game becomes an anti-coordination game. There are three possible equilibria:

(1) $M1$ invests at date $t = 1$, followed by $M2$ investing at date $t = 2$;

[7] π_1^S, π_1^L, π_1^F, and X_1, the net payoffs for $M1$, are the counterparts of π_2^S, π_2^L, π_2^F, and X_2.

(2) $M2$ invests at date $t = 1$, followed by $M1$ investing at date $t = 2$;

(3) $M1$ and $M2$ invest at date $t = 1$ with probability (p^*, q^*), where $p^*, q^* \in (0, 1)$; for any date $t > 1$, if no one has invested before, $M1$ and $M2$ invest at date t with probability (p^*, q^*).

Finally, all functions are continuous and differentiable. Therefore, we can always find a $\widetilde{\widetilde{\delta}} \geq \widehat{\delta}$, such that if $\delta \geq \widetilde{\widetilde{\delta}}$, the game becomes an anti-coordination game. ■

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