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Inventory Management and Financial Hedging of Storable Commodities

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Abstract

This paper studies the integrated physical and financial risk management of storable commodities used as inputs in end-products facing uncertain demand. In our stylized model, we study a problem of dual sourcing with financial hedging for a risk averse buyer (the seller of the end product) who procures a single storable commodity from a supplier via a flexible long-term contract and "tops up" via short-term purchases from a spot market. The spot market has adequate supply (i.e., market liquidity is assumed) but a random price. To hedge the uncertainty of the spot price and the end-product customer demand, the buyer can trade financial contracts written on the spot market prices such as forward contracts, call and put options. We obtain multi-period optimal inventory and financial hedging policies for a risk averse buyer with an inter-period mean-variance objective. For most cases, the optimal policies are myopic and easy to compute and implement. We examine different cases of financial hedging, single hedges and portfolio hedges, and characterize their optimal hedging amounts and portfolio structure. For optimal portfolios (use of forwards and call/put options) the allocation of funds to the various hedges can be obtained via the solution of a system of linear equations. We also offer insights on the role and impact of the operational and financial hedging on the profitability, risk control, and service level to the customer.For many cases better operational or financial hedging improves the end-customer service level via allowing more aggressive inventory policies.

Keywords: stochastic inventory, commodity markets, forwards, options, risk management, hedging, risk aversion.

1 Introduction

Sourcing, inventorying and processing of storable commodities to be eventually sold in the form of differentiated goods to end product markets are cornerstone activities of many business strategies. However, commodity risks can jeopardize even the best thought out strategies (Tevelson, Ross, and Paranikas 2007). These days commodity risks are even more pronounced than before. The increasing appetite of rapidly developing economies like China and India has driven up the demand

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and the prices of everything from soybeans to steel and oil. Oil marched towards $150 a barrel largely due to China's boom. Ethanol demand drove up corn prices, and as many soybean fields were switched to corn, soybean prices rose. In short, prices of many commodities, such as corn, soybeans and wheat, are now fluctuating as much in a single day as they did in a year in the early 1990s (Wiggins and Blas 2008).

For companies that rely on such commodities as production inputs and can not pass cost increases to their customers, such volatility substantially increases their working capital needs and risks of financial distress. For example, food companies tended to allocate procurement activities of commodity inputs to logisticians with limited commodity hedging knowledge and skills. However, as ingredients prices that went into corn flakes, chocolate bars and yoghurts squeezed their margins away, food companies are in search of procurement managers with commodity trading skills (Wiggins 2008). Unilever, the multinational food and household products conglomerate, estimates its commodity costs increase in the first half of 2008 to over $1.5B. Hershey, the US chocolate group, saw commodity input costs, such as sugar, peanuts, and cocoa, rise 45% the same year, and is in search for trading skills to implement a $12M hedging strategy. The same challenges remain true in other industries from mature markets like autos to fast growth high technology products (printers, computers, disk drives, consumer electronics etc.). Ford had posted over a $1B loss on precious metals inventory in the early 2000s due to a misplaced bet on rising prices, and HP had a significant risk exposure to flash memory components in the mid-2000s. Under significant commodity risk exposure firms are in search of better ways to hedge such exposure in order to lock in supplies, maintain lower costs, minimize earnings volatility, and in the long run gain competitive edge.

In our paper, we will study integrated physical and financial risk management for hedging storable commodity risks. As argued effectively in Kleindorfer (2008a), the growth of commodity exchanges, and derivative instruments defined on them, has offered opportunities to integrate traditional forms of bilateral contracting with shorter term market driven physical and financial transactions for effective hedging of commodities. However, ways to optimally decide the sourcing allocation between long-term contracts and spot markets, the needed inventory levels of commodity inputs to deal with demand uncertainty of the final product markets, and the simultaneous optimal choice of the portfolio of forward and other derivative contracts written on commodity exchanges is a difficult problem, with only limited answers mostly for non-storable commodities (e.g., electricity). Our work makes the first steps in offering answers to this integrated risk management problem for storable commodities, such as soybeans, metals, and hi-tech components.

The fundamental setting of our problem is for a firm that is going to integrate a long-term input sourcing contract with "top up" contract purchases from a commodity exchange (spot market) for meeting (after pursuing requisite processing of the commodity inputs into final product goods) uncertain demand at differentiated final product markets. The availability of forward and other derivative contracts traded at the commodity exchange allows for better hedging of the cash flow volatility, and it is of the interest of our firm to integrate it with its sourcing and inventory management decisions. Many companies prefer dual input sourcing, especially the long term-short term sourcing contract integration advocated here, because (1) sourcing competition can keep prices under control; (2) a wider supply base can mitigate risk in the event of an accident or other upheaval at one supplier, and (3) suppliers may have limited capacity. Long-term contracting with suppliers is a commonly used source in practice, with its advantages coming from the consistent over time availability of uniform quality goods, and with the needed pre-processing and logistics services the buyer requires. As clearly described in commodity management books (Geman 2008), there are differentiated quality grades even for commodity inputs, with long-term suppliers offering better selection and consistent supply of higher quality grades, and with last minute spot market purchases increasing quality risks and logistics costs (for more justification for the presence and advantages of long-term contracts for commodity inputs see Kleindorfer 2008a). Suppliers usually offer fixed purchase prices over certain contractual horizons, with the prices often at a premium over the ones from forward curves written on existing spot markets. The premiums are reflecting the long-term supplier services of consistent quality, pre-processing and transportation logistics. Often in these bilateral contracts, buyers obtain supplier concessions on prices that reflect their contract volumes over the horizon and some flexibility in the purchased quantity in every period. For example, HP works with a binding long-term agreement with a major flash memory supplier, with some quantity flexibility in it, and with prices that reflect substantial quantity discounts (15% or more). HP writes such contracts with major component suppliers over horizons equivalent to the product life cycles of the end market products served (e.g., 18 to 36 months durations). (See more details for HP storable commodity sourcing practices in Nagali et al. 2008). Short-term contracting with spot market is a source that provides high inventory flexibility, allowing companies to buy and sell at any quantity with zero lead time, but at a random price. Obviously, risk averse decision makers who use both sources should seek best ways to balance their inventory and price risks while pursuing profitability. And optimal ways to achieve it is the essential research question of our study.

In this paper, we study a problem of dual sourcing with financial hedging for a risk averse buyer who procures a single storable commodity product from a supplier and a spot market. The buyer has a long-term contract with the supplier which specifies a fixed price together with a lower and upper limit of the order quantity for each period. The lower and upper limits are functions of the purchasing price. The spot market has adequate supply (i.e., market liquidity is assumed) but a random price. The buyer can buy from or sell to the spot market at the market price at the start of each period. Note that the long-term contract and the spot market participation are sources to fill the customer demand and to hedge the customer demand uncertainty as well. However, spot market participation also brings price uncertainty. To hedge the uncertainty of the spot price and the customer demand, the buyer can trade financial contracts on spot market such as forward contracts, call and put options. Ideally, the buyer should build an optimal portfolio of financial hedging contracts. For the tractability of the analysis, we only focus on the financial hedging contracts whose strike time is a period later after the transaction. In other words, at the start of each period, all the financial contracts the buyer purchase or sell will be exercised at a same time, the start of the next period.

We formulate this multi-period problem using a modified mean-variance utility function, which is an inter-period utility function (for justification of our modeled objective function and other modeling artifices in our study, refer to Sections 3 and 4). At the start of each period, the buyer needs to make operational decisions, which include how much to order from the supplier, and whether to and how much to buy from or sell to the spot market. At the same time, the buyer also needs to make financial hedging decisions, which include selection of the best hedging contract(s) to trade and how much to trade for each contract. We derive an optimal inventory and financial hedging policy for the buyer for various scenarios of available hedging contract choices. We consider the following financial hedging scenarios: (1) use of a single, and of the same, type of hedging contract across all periods, e.g. forward contract, call option, or put option, (2) use of the optimal single hedging contract among all available ones in each period without restricting for type consistency across periods, and (3) an optimal hedging portfolio open to allocate among all the financial hedging contracts(forwards, call or put options) available on the spot market. We investigate the effects of the long-term and spot market contracting and the incorporation of financial hedging to the buyer's mean profit, variance of the profit, and service level offered to customers.

The literature in integrating physical and financial risk management of commodities has seen many interesting research studies in the last 10 years, with them effectively summarized and discussed within an overarching conceptual framework in Kleindorfer (2008b). Most of the presented work deals with integration of long-term and short-term contracts (spot markets) for risk averse buyers, within single period decision settings, or multi-period environments but for typically non-storable commodities (and thus inventory considerations linking decisions across periods are not present, with problem decomposability across periods as an important feature). Most of this work does not explicitly address optimal hedging portfolio questions and their interaction with operational policies, and the few exceptions are mostly for electricity markets (Kleindorfer and Wu 2005). In this paper, we address challenging issues on the integrated inventory and financial hedging policy for a risk averse buyer of storable commodities within a multi-period setting, and offer insights on the structure of such optimal integrated policies within an inter-period mean variance objective function, a definite contribution to the integrated risk management of storable commodities literature.

We provide a quick preview of our model results:

(a) Optimal inventory (base stock) policies are characterized for the single and multiple period setting. The base stock levels are dependent on the type of financial hedging used. However, when we know that the firm hedges using an optimal portfolio we can proceed to obtain the optimal base stock levels without any further details on the structure of the optimal hedging contract (base stock policies the same as if the firm was hedging via a forward contract).

(b) We derive optimal financial hedging policies for single and multi-period settings, and we show that they are heavily dependent on the inventory levels used to meet the uncertain demand. Thus, in structuring the optimal financial hedging portfolios detailed inventory policy information is needed, thus emphasizing the need for cross-functional integration for effective commodity risk management.

(c) In single period settings, the single forward contract is an optimal hedge. We can further observe that call (put) options with low (high) strike prices perform really well in single period models. However, these observations do not hold for the multi-period problem, but the optimal hedging problem is still computationally tractable. Computation of the optimal multi-period hedging portfolios results in easy to handle system of linear equations.

(d) In single period settings, the better the financial hedge (in terms of our mean-variance objective performance) the higher the corresponding optimal inventory, and thus offered service, level. Furthermore, the lower the buyer's risk aversion the higher the inventory level used. However, these results are not necessarily true within a multi-period setting, with our numerical study offering useful insights on some observed behavior for such settings.

(e) Finally, we clearly describe the role of the long-term contract, spot market, and financial hedges in dealing with demand volatility at end markets and price volatility of commodity inputs. Our computational study shows the advantages of integrating physical and financial risk management, with the integrated long-term and short-term contracting delivering the major impact on expected profits. However, such dual sourcing ends up increasing the cash flow volatility and associated risks, and the employment of relevant financial hedges allows controlling variance of cash flows with moderate benefits on expected profits. The latter effect is due to pursuing more aggressive inventory policies when variance control financial hedging is in place.

The structure of our remaining paper is as follows. In Section 2, we review relevant literature and carefully position our work within it. Section 3 introduces all relevant notation and important assumptions behind our multi-period integrated inventory and financial hedging of storable commodities model, while the formal model (single period and dynamic programming formulation) and its early results appear in Section 4. Specifically, Section 4 provides the optimal inventory and hedging policies when we restrict ourselves to using a single, and of the same, type of hedge across all periods. Section 5 deals with the more general hedging policies that allow use of an optimal portfolio of hedges among all possible forwards, call and put options, with the structure of the optimal portfolio changing from one period to the other. From the insights of Sections 4 and 5, we proceed to offer practical suggestions on the use of single contract hedges in each period, but without restricting ourselves to the use of a single type across all periods. Section 7 offers insights on the role and impact of operational and financial hedges on profitability, cash flow variance, and service levels. We conclude with managerial insights and summary of important results in Section 8.

2 Literature Review

Our work falls under the general themes of “integrated physical and financial risk management in supply chains” and “hedging commodity risks in supply management”, which were both expertly reviewed by Kleindorfer (2008 a, b). For an earlier review on the literature on supply contracting and spot markets see Kleindorfer and Wu (2003). The more general field of supply chain contracts is of passing relevance to our work, and we refer the reader to Cachon (2003). In this section, we review in detail work that is closely related to our paper and especially the research on integrated long-term and short-term (including spot markets) sourcing contracts, even though most of it does not include any financial hedging concerns. According to the Kleindorfer and Wu (2003) classification, we are studying an "open spot market for storable goods" and we will review papers with similar spot market models. For review of work on "closed spot markets" see Milner and Kouvelis (2007) and references therein. The general framework with integrated long term-short term contract decisions for mostly non-storable goods is presented in the work of Wu and Kleindorfer (2005). It develops a single-period model to analyze business-to-business (B2B) transactions in supply chains where a buyer and multiple sellers can either contract for delivery in advance (the buyer purchasing "call options" from the sellers) or trade on spot. The authors characterize the structure of the optimal portfolio of contracting with sellers and spot market transactions for the buyer and the sellers. For a more extensive review of related work in the non-storable commodities area we refer the readers in the references in the Wu and Kleindorfer (2005) and Wu, Kleindorfer and Zhang (2002).

Lee and Whang (2002) are the first to integrate after sales spot market considerations within a newsvendor ordering framework, and thus effectively endogenizing the salvage value used in these models. Peleg, et al. (2002) is among the early works in long term-short term integrated sourcing, and it uses a stylized two period model. It considers a risk-neutral manufacturer who can choose between three alternative procurement strategies: (1) a long-term contract with a single supplier; (2) an on-line search, in which multiple suppliers are contacted for a price quote; and (3) a combined strategy, and develops conditions under which each of the three alternatives is preferred. The nature of the optimal inventory policy when such dual sourcing is used, and in the presence of a fixed cost for spot market participation, is studied in Yi and Scheller-Wolf (2003). Martínez-de-Albéniz and Simchi-Levi (2005) addresses the optimal creation of a portfolio of supply contracts (including long-term fixed commitment, flexible, and capacity reservation contracts) which integrated with potential spot market purchases can effectively deal with demand and spot market price risks, and offers insights on the structure of such a portfolio and the optimal replenishment policy under it. The above work is not concerned with the specifics of storable commodities, and it is intended to show benefits of dual sourcing and integration of spot markets in optimizing profitability for risk neutral players. Furthermore, there are no concerns of dealing with cash flow volatility and hedging for risk management purposes of a risk averse buyer, which are issues of prominence in our work.

Recent work explicitly addressing issues specific to commodity sourcing contexts is the Goel and Gutierrez (2006). It analyzes a multi-period procurement problem for a risk-neural manufacturer who procures commodities from spot and futures market, endogenizes convenience yield values and their implications for inventory holding costs from the observed spot and futures market prices, incorporates transaction costs for spot market procurement, and derives an optimal procurement policy. Risk aversion concerns and financial hedging of cash flow volatility are not modeled in this work. Devalkar, Anupindi and Sinha (2007) motivated by issues in soybean processing analyze the integrated procurement, processing and trading decisions of a commodity processor. They study a multi-period decision problem for a risk-neutral/risk-averse manufacturer who procures input commodities from a spot market, and processes and sells the output commodities on a futures market. In their stylized model there is a single input commodity transformed to an identical single output commodity at constant marginal processing cost and in the presence of procurement but not processing capacity constraints. A numerical study illustrates the benefits of the integrated optimal decisions and the impact of risk aversion (modeled with the use of a Value-at-risk constraint). This work differs from ours in terms of lack of financial hedging concerns, the absence of dual input sourcing, and in the modeling of end product demand uncertainty and inter period buyer's risk aversion. Rich in institutional details of the fed-cattle supply chain, Boyabatli, Kleindorfer and Koontz (2008) offer a lucid picture of the beef processor's (meat packers such as Tyson Foods) problem in these environments via a stylized single period model. The risk neutral processor first contracts for a number of fed-cattle with a feed lot operator, facing demand for beef products and spot price uncertainty. He then procures in the fed-cattle spot market after uncertainty is realized, processes under capacity constraint, and then fills demand of two downwardly substitutable products, ground beef and boxed beef. Optimal long term-short term procurement and processing decisions in this proportional production environment are made in the presence of spot market transaction costs, economies of scale in processing, and correlated end product demand. Our work, even though less rich in industry specific institutional details than Boyabatli et al (2008), is able to handle multi-period settings, risk aversion, and financial hedging of storable commodities.

There is very limited amount of research on commodity procurement decisions with financial hedging and most of them are for non-storable commodities, such as electricity and liquid natural gas (for representative such work and related references see Oum, Oren and Deng (2006) and Bodily and Palacios (2007)). Supply contracts with financial hedging are studied in Caldentey and Haugh (2008) in the presence of spot market uncertainty. They study a single-period Stackelberg game between a buyer and a producer, both of them risk-neutral. The buyer purchases from the producer and sells it all at a stochastic clearance price under a budget constraint. The stochastic clearance price depends on some observable financial stochastic process. Two contracts, a flexible contract and the flexible contract with financial hedging, are compared in terms of their supply chain performance. Under the flexible contract, the producer offers the buyer a menu of wholesale prices which depend on the financial market condition observed up to the inventory delivery time. Under the flexible contract with hedging, a variation of the flexible contract, the buyer can hedge his budget by trading in the financial market dynamically up to the inventory delivery time. It is found that the producer always prefers the flexible contract with hedging. However, the buyer may or may not prefer the flexible contract with hedging. Our work differs from the latter paper in its modeling of the end product demand uncertainty, the incorporation of risk aversion, and the offered insights on the interaction of long-term flexible contracts and financial hedges.

3 The Notation and Assumptions for The Multi-Period Model

For $n = 1, \dots, N$:

1. $\alpha \in (0, 1)$: the period discount factor for the buyer's cash flow.
2. D_n: the buyer's customer demand in period n with an increasing[4] cumulative distribution function (cdf) $F_n(\cdot)$.
3. w: the wholesale price the supplier charges under the long-term contract with a lower and upper bound of purchase quantity depending on w, denoted by $l(w)$ and $u(w)$, respectively.
4. S_n: the random spot market price at the start of the period n with s_n denoting its realization. For the convenience of presentation, we assume S_n's support is $[s_l, s_u]$, for $n = 1, \dots, N + 1$.
5. r: the revenue the buyer gains from each unit sold to his customers. We assume $r > S_n$[5].
6. x_n: the on-hand inventory level at the start of period n.
7. q_n: the order quantity under the long-term contract at price w at the start of period n. q_n^* denotes the optimal order quantity.
8. z_n: the inventory level after the purchase from the supplier and buying or selling in the spot market (i.e., the stock level available to fill demand D_n) at the start of period n.

[4]Increasing cdf is assumed for presentation convenience. Our results apply for non-decreasing cdf as well.

[5]Note that $r > S_n$ implies $r > s_u$. In addition, if $r \leq S_n$, the analysis in this paper is still applicable. The optimal policy is that the buyer will not fill any of his customer demand, but sell all his inventory to the spot market. The buyers in this case are spot market speculators. Therefore, in this paper, we focus on buyers whose first priority is operation and spot market is used to improve operation, rather than buyers who are spot market speculators.

9. z_n^{i*}: the optimal inventory level for the buyer when hedging contract i is used, where $i = 0$ (no hedging), $i = f$ (forward), $i = c$ (European call option), or $i = p$ (European put option).

10. z_n^* represents the optimal inventory level for the buyer when portfolio hedge is used.

11. $q_{i,n}$: the quantity of financial contract i, $i = f$ (forward), c (call), and p (put) traded at the start of period n, $n = 0, \ldots, N+1$, where $q_{i,n} < 0$ if contract i is sold and $q_{i,n} > 0$ if contract i is purchased. Assume the financial contracts, with payoff function $\chi_i(S_{n+1})$[6], the buyer considers have a same strike time (the start of period $n+1$).

12. $q_{i,n}^{1*}$: the optimal quantity of financial contract i, $i = f, c, p$, traded at the start of period n when a single hedge is used. $q_{i,n}^*$ denotes the same quantity when portfolio hedge is adopted.

At the start of period n, $n = 1, \ldots, N$, the buyer determines the optimal q_n for the long-term supply, z_n for the spot market, and $q_{i,n}$ for financial contract i, $i = f, c, p$. At the end of the horizon, we assume that the buyer cannot purchase from the long-term supply (i.e., $q_{N+1} = 0$) but can trade in the spot market. Since the financial contracts we consider are exercised a period after the purchase, it is easy to see that no hedge should be purchased at the end of the horizon (i.e., $q_{i,N+1}^{1*} = 0$). We assume that for each period unmet demand is lost, and physical inventory holding cost without loss of generality is negligible. Financial holding costs of inventories, under their usual interpretation of opportunity cost of money invested, are captured in our discounting of relevant cash flows. We also assume that the buyer's customer demand is independent of the spot price. This assumption is reasonable for a buyer whose trades do not have large influence on the spot market price. Usually spot market price is derived under the assumption that all the participants are equally small and thus no individuals have impact on the market price. We consider an inter-period "mean-variance"[7] type of utility function for the risk averse buyer. Specifically, at each decision-making time epoch, i.e., at the start of each period, the utility function equals the sum of the expected profit from the current period and the future periods less λ times of the sum of the variances of the profit from the current period and the future periods. It is important to note that only the use of inter-period utility functions is appropriate for capturing risk-aversion. Indeed, Sobel (2007) shows that the use of intra-period utility functions, which are the other type of utility

[6] Let $K_{i,n} > 0$ denote the strike price and $\beta_{i,n}$ denote the cost paid upon transaction. The payoff function $\chi_i(S_{n+1}) = S_{n+1} - K_{f,n}$ for forward, $(S_{n+1} - K_{c,n})^+ - \beta_{c,n}/\alpha$ for call and $(K_{p,n} - S_{n+1})^+ - \beta_{p,n}/\alpha$ for put. Note that $E[\chi_i(S_{n+1})]$ represents the buyer's risk premium for hedge i for period n, $i = f, c, p$.

[7] For justification of using "mean-variance" type of utility function, please see Ding, Dong and Kouvelis (2007).

functions and commonly using in modeling multi-period risk-averse decision making, surprisingly implies risk-neutrality.

4 Optimal Policy for Single Contract Financial Hedging

In this section, we study the operational and hedging policies the buyer should apply in the case where the buyer chooses to employ a single hedge, either forward, a call or a put. The same type of hedge is used across all periods. Note that forward and options are the most commonly used hedging contracts in practice. Using a constructive approach to the presentation of our research results, our analysis with a single hedging contract highlights important aspects of a risk averse buyer's behavior on balancing quantity risk and price risk through the integrated use of operational and financial hedges.

4.1 The Buyer's Utility Function With Single Contract Financial Hedging

At the start of period n, the buyer needs to make operational decisions, q_n and z_n, as well as the hedging decision, $q_{i,n}$, to maximize his utility function based on up-to-date information of inventory, spot market and financial contracts written on spot price. Let $\pi_n^i(s_n, x_n, q_{i,n-1}, q_n, z_n)$ denote the buyer's profit function in period n (if the buyer restricts himself to the use of a single hedge of type i, $i = f, c, p$, across all periods), for any given real-time spot price s_n, on-hand inventory level x_n, the quantity of the hedging contract traded in the previous period $q_{i,n-1}$, order quantity with the supplier $q_n \in [l(w), u(w)]$, and inventory level used to fill demand z_n. Thus, we have

$$\begin{aligned} \pi_n^i(s_n, x_n, q_{i,n-1}, q_n, z_n) &= q_{i,n-1}\chi_i(s_n) - wq_n - s_n[z_n - (x_n + q_n)] + r(z_n \wedge D_n) \\ &= q_{i,n-1}\chi_i(s_n) + q_n(s_n - w) - s_n(z_n - x_n) + r(z_n \wedge D_n), \end{aligned} \tag{1}$$

where $x \wedge y = \min\{x, y\}$. Normally, as indicated in the first equation in (1), the costs come from exercising the hedging contract purchased in the previous period, purchasing q_n from the supplier at price w, and then adjusting the total on-hand inventory level, $x_n + q_n$, up or down to z_n by trading in the spot market at price s_n. Under the assumption that the buying and selling prices are equal in the spot market, we can assess the costs differently as indicated in the second equation in (1). We observe that the transactions with the long-term contract and the spot market are equivalent to purchasing q_n from the supplier and selling it to the spot market, and then adjusting the total on-hand inventory level, x_n, up or down to z_n by participating in the spot market.

This observation helps in characterizing the optimal policies and investigating the impact of each individual transactions to the total utility.

Note that at the end of horizon, the buyer receives no customer demand (i.e., $D_{N+1} = 0$) and thus the buyer should not purchase from the long-term supply. Thus,

$$\pi^i_{N+1}(s_{N+1}, x_{N+1}, q_{i,N}, q_{N+1} = 0, z_{N+1}) = q_{i,N}\chi_i(s_{N+1}) - s_{N+1}(z_{N+1} - x_{N+1}). \tag{2}$$

We next define the buyer's utility function for period n, $n = 1, \ldots, N$, by

$$\begin{aligned}
&U_n(q_n, z_n, q_{i,n}|s_n, x_n, q_{i,n-1}) \\
= &E[\pi^i_n(s_n, x_n, q_{i,n-1}, q_n, z_n)] - \lambda V[\pi^i_n(s_n, x_n, q_{i,n-1}, q_n, z_n)] \\
&+E[\alpha\pi^i_{n+1}(S_{n+1}, (z_n - D_n)^+, q_{i,n}, Q^*_{n+1}, Z^{i*}_{n+1})] - \lambda V[\alpha\pi^i_{n+1}(S_{n+1}, (z_n - D_n)^+, q_{i,n}, Q^*_{n+1}, Z^{i*}_{n+1})] \\
&+\sum_{k=n+2}^{N+1} \Big\{E[\alpha^{k-n}\pi^i_k(S_k, (Z^{i*}_{k-1} - D_{k-1})^+, Q^{1*}_{i,k-1}, Q^*_k, Z^{i*}_k)] \\
&\qquad -\lambda V[\alpha^{k-n}\pi^i_k(S_k, (Z^{i*}_{k-1} - D_{k-1})^+, Q^{1*}_{i,k-1}, Q^*_k, Z^{i*}_k)]\Big\},
\end{aligned} \tag{3}$$

where Q^*_k[8], Z^{i*}_k, $Q^{1*}_{i,k}$, are random variables representing the optimal decisions for the future period k, $k \geq n+1$ (randomness coming from spot prices $S_{n+1}, \ldots, S_k$ and period demands $D_n, \ldots, D_{k-1}$ which are not observable at the decision time, the start of period n).

At the end of horizon, note that the buyer should not purchase any financial hedge, i.e., $q^{1*}_{i,N+1} = 0$, and thus the buyer's utility function can be simplified using (2) as

$$U_{N+1}(q_{N+1} = 0, z_{N+1}, q^{1*}_{i,N+1} = 0|s_{N+1}, x_{N+1}, q_{i,N}) = q_{i,N}\chi_i(s_{N+1}) - s_{N+1}(z_{N+1} - x_{N+1}). \tag{4}$$

It is easy to see that $z^*_{N+1} = 0$ (i.e., the buyer should sell all excess inventory). Therefore, we have $Q^*_{N+1} = Z^{i*}_{N+1} = 0$, for $i = f, c, p$.

It is important to note that our utility functions are inter-period (without utility discounting over periods) and thus do not exhibit the iterative property (commonly seen in dynamic programming) as the intra-period utility functions (with utility discounting over periods). An interesting and unconventional observation reveals that when variance-like (or non-expectation) terms are included

[8] As will be shown, it does not depend on the type of hedging contract adopted and thus we omit superscript i. In addition, we follow the tradition of using upper case letters for random variables and corresponding lower case letters for their realizations.

in utility functions (inter-period or intra-period), the optimal decisions determined for now (based on all the up-to-date information) may not look optimal when evaluated at any earlier time. In standard multi-period inventory models (an example of inter-period utility functions containing means only), the optimal decisions determined for now always look optimal when evaluated at any earlier time. It is due to the fact that taking expectation of the random demands and prices always preserves the optimality of real-time decisions. Taking variance of these random variables, however, can sometimes change the optimality of on-time decisions.

For example, the optimal decision determined at the start of period $N+1$ (i.e., $z^*_{N+1} = 0$ and $q^{1*}_{i,N+1} = 0$) may not look optimal if evaluated one period earlier. Indeed, we next show that it may be worse than a decision with $z_{N+1} = x_{N+1}$ and $q_{i,N+1} = 0$ (which we refer to as do-nothing decision). Without loss of generality, we assume $q_{i,N} = 0$. Evaluating the optimal decision and the do-nothing decision at the start of period N (when spot price S_{N+1} and on-hand inventory $(z_N - D_N)^+$ are both random), we obtain utility values $E[S_{N+1}]E[(z_N - D_N)^+] - \lambda V[S_{N+1}(z_N - D_N)^+]$ and 0, respectively. It is easy to see that for some distributions of S_{N+1} or a large mean-variance ratio λ, the optimal decision may result in a lower utility when evaluated one period early than the do-nothing decision. This observation has received little awareness and attention in the existing literature and greatly complicates our analysis.

4.2 Optimal Policy For The Last Period

In this section, we derive the optimal policy for the last period. It is important to note that although the inventory and hedging decisions are determined simultaneously, due to the one period time lag between the transaction and the exercise of the hedging contract, inventory decisions affect the profit in the last period, while hedging decision affects the profit at the end of the horizon[9].

Proposition 4.1 *Given x_N and s_N, the optimal policy parameters are:*
(1) $q^{1*}_{f,N} = -\int_0^{z^{f*}_N} F_N(\xi)d\xi + \frac{E[\chi_f(S_{N+1})]}{2\lambda\alpha V[S_{N+1}]}$; $q^{1*}_{i,N} = -\int_0^{z^{i*}_N} F_N(\xi)d\xi \frac{Cov(S_{N+1},\chi_i(S_{N+1}))}{V[\chi_i(S_{N+1})]} + \frac{E[\chi_i(S_{N+1})]}{2\lambda\alpha V[\chi_i(S_{N+1})]}$, where $|q^{1*}_{f,N}| \leq |q^{1*}_{i,N}|$, if $E[\chi_j(S_{N+1})] = 0$ for any $j = f, c, p$ (i.e., zero risk premium), $i = c, p$;
(2) $q^*_N = l(w)$ if $w \geq s_N$; $q^*_N = u(w)$ if $w < s_N$;
(3) If zero risk premium is assumed, z^{f*}_N satisfies $\int_0^{z^{f*}_N} F_N(\xi)d\xi = \frac{r - s_N}{2\lambda\left(r^2 + \alpha^2 E[S^2_{N+1}]\right)}$; for $i = c, p$, z^{i*}_N

[9] Note that if $w = s_N$, the buyer gains zero profit from selling the procurement from the supplier to the spot market (i.e., the second term in (1) is zero). Therefore, the buyer should be indifferent of the order quantity. Without loss of generality, we set $q_N = l(w)$ when $w = s_N$.

satisfies

$$2\lambda\alpha^2 \int_0^{z_N^{i*}} F_N(\xi)d\xi \left(V[S_{N+1}] - \frac{Cov^2(S_{N+1}, \chi_i(S_{N+1}))}{V[\chi_i(S_{N+1})]} \right)$$
$$= r\bar{F}_N(z_N^{i*}) \left(1 - \frac{s_N}{r} - 2\lambda \frac{r^2 + \alpha^2 E^2[S_{N+1}] + \alpha^2 \frac{Cov^2(S_{N+1},\chi_i(S_{N+1}))}{V[\chi_i(S_{N+1})]}}{r} \int_0^{z_N^{i*}} F_N(\xi)d\xi \right).$$

We first discuss the effect of the risk premium, denoted by $E[\chi_i(S_{N+1})]$. Note that the hedge quantity deviation $\frac{E[\chi_i(S_{N+1})]}{2\lambda\alpha V[\chi_i(S_{N+1})]}$ in the expression of $q_{i,N}^{1*}$ given above is independent of the demand distribution (and is relatively small when the demand is in large scale or spot price volatility is large). Thus it exists even if the demand is zero. From the perspective of a manufacturing firm, the main objective of trading in the financial hedging market is to minimize the price risk resulted from the spot market participation (which is utilized to reduce the mismatch risk between the demand and inventory), not gaining profit from speculating in the financial market. Thus, we will present the rest of our results in the paper under an assumption of non-speculative motives of our firm, which is operationalized via the assumed zero risk premium. In addition, assuming zero risk premium[10] can increase the analysis tractability, thus offering sharper results and clear insights for the problem.

Next we note that, if zero risk premium is assumed, the buyer should sell $|q_{f,N}^{1*}|$ forward contracts, or sell $|q_{c,N}^{1*}|$ call options, or buy $|q_{p,N}^{1*}|$ put options to hedge the price risk in selling the excess inventory to the spot market. The optimal hedging quantity for forward is the minimum among all the single hedges. In practice, such property will promote the use of forward than others because the hedging market is often illiquid. More importantly, in Section 6.1, we show that forward is indeed the best single hedge. For the last period, the hedging contracts are similar to a return contract, but with a fixed return quantity. The difference between this fixed return quantity and the excess inventory will be cleared by trading in the spot market. We also note that the optimal order quantity from the supplier q_N^* is independent of the spot market participation and financial hedging. It can be easily determined by comparing the spot price to the wholesale price.

According to Proposition 4.1 (3), the optimal inventory level, z_N^{i*} can be determined by simply increasing the value of z_N until the first order condition is met. With explicit knowledge of type of hedging contract to be used (i.e., forward, call or put), the operational decisions q_N^* and z_N^{i*} can be determined without the details (i.e., the hedging quantity) of the optimal hedge. However, the

[10]For more justification of assuming zero risk premium, please see Ding, Dong and Kouvelis (2007).

inventory decision z_N^{i*} differs depending on the type of hedging contract used. On the other hand, the optimal hedging quantity $|q_{i,N}^{1*}|$ is a convex increasing function of the optimal inventory level z_N^{i*}. Indeed, the higher the inventory level, the higher the hedging quantity the buyer should trade.

To supplement the analytical results provided in this paper, we performed a two-period numerical study with Geometric Brownian motion spot price (i.e., $S_{n+1} = \frac{s_n}{\alpha} e^{\sigma B - \sigma^2/2}$, where $B \sim N(0,1)$) and Poisson demand (i.e., $D_N \sim$ Poisson(μ)). Note that the inventory levels we consider for the numerical study are integers due to the discrete demand distribution used. In order to choose reasonable values for parameters for the spot price, α and σ, we fit the distribution of spot price to the daily return on aluminum futures from year 2000 to 2004 (see Exhibit 3 in Singh (2004)[11]). Note that a higher discount factor α is associated with a longer duration between two adjacent periods and thus should correspond to a higher drift parameter σ. The fitting of the real data implies that for example $\alpha = 0.98$ is associated with 119 days or 4 months and $\sigma = 0.105$. Since 4 months is an appropriate duration between two orders for storable commodities like aluminum or copper, whose spot price has low volatility, we use this set of parameters for a portion of our numerical study. We use another two sets of parameters, $\alpha = 0.9$ and $\sigma = 0.6, 1.0$, for the rest of the numerical study for storable commodities with more volatile prices, such as computer memories and precious metals. In particular, we consider a factorial experiment design[12].

For all the cases we studied, we find that z_N^{f*} is no less than z_N^{c*} and z_N^{p*}. Figure 1 shows a typical relationship among these three optimal inventory levels, as a function of the spot price, for single financial hedging. Figure 2 verifies that the hedging quantity $|q_{f,N}^{1*}|$ is always lower than $|q_{c,N}^{1*}|$ and $|q_{p,N}^{1*}|$ and indicates a large[13] difference among these quantities. We also observe from the numerical study that when the call and put options are more similar to forward, i.e., when the corresponding strike prices $K_{c,N}$ and $K_{p,N}$ are closer to s_l and s_u, respectively, the difference among the optimal hedging quantities are smaller. This observation is proved and presented later in the paper.

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[11]The percentage of the change of the daily return for aluminum was approximately Normally distributed with mean 0.017% and standard deviation 0.963%. This implies that the spot price of aluminum is approximately Geometric Brownian motion with daily discount factor 0.99983 and daily drift parameter 0.009628.

[12]For each set of α and σ, we study a two-period example with $s_{N-1} = 2, 2.5, 3, 3.5, 4.2, 4.4, \ldots, 6.0, 6.5, 7, 7.5, 8$, $w = 4.0$, $r = 6, 9, \ldots, 30$, $\mu = 20$, $l(w) = 1, 5, 10$, $u(w) = 10, 20, 30, 40$, $K_{i,n} =$ 1st,…, 99th percentile of S_n, $i = c, p$, $n = N-1, N$, and $\lambda = 0.003, 0.004, 0.006, 0.01, 0.02$.

[13]$|q_{f,N}^{1*}|$ is around 10, while $|q_{c,N}^{1*}|$ and $|q_{p,N}^{1*}|$ are both in hundreds.

4.3 Optimal Policy For Any Period

Differentiating the utility function with respect to each decision, we can determine the optimal policy for any other period than the last. For notation simplicity, we let

$$H(S_{n+1}, Z_{n+1}^{i*}) = u(w)(S_{n+1} - w)^+ - l(w)(S_{n+1} - w)^- - S_{n+1}Z_{n+1}^{i*} + r(Z_{n+1}^{i*} \wedge D_{n+1}). \quad (5)$$

Proposition 4.2 *Given x_n and s_n, $n = 1, \ldots, N-1$, the optimal policy parameters are:*
(1) $q_{f,n}^{1*} = -\int_0^{z_n^{f*}} F_n(\xi)d\xi - \frac{Cov(S_{n+1}, H(S_{n+1}, Z_{n+1}^{f*}))}{V[S_{n+1}]}$; for $i = c, p$, $q_{i,n}^{1*} = -\int_0^{z_n^{i*}} F_n(\xi)d\xi \frac{Cov\left(S_{n+1}, \chi_i(S_{n+1})\right)}{V[\chi_i(S_{n+1})]} - \frac{Cov(\chi_i(S_{n+1}), H(S_{n+1}, Z_{n+1}^{i*}))}{V[\chi_i(S_{n+1})]}$.
(2) $q_n^* = l(w)$ if $w \geq s_n$; $q_n^* = u(w)$ if $w < s_n$;
(3) z_n^{f*} satisfies $\int_0^{z_n^{f*}} F_n(\xi)d\xi = \frac{r - s_n}{2\lambda\left(r^2 + \alpha^2 E[S_{n+1}^2]\right)}$; for $i = c, p$, z_n^{i*} satisfies the following first order condition, which may have multiple solutions,

$$\begin{aligned}
&2\lambda\alpha^2 \int_0^{z_n^{i*}} F_n(\xi)d\xi \left(V[S_{n+1}] - \frac{Cov^2(S_{n+1}, \chi_i(S_{n+1}))}{V[\chi_i(S_{n+1})]} \right) \\
&= r\bar{F}_n(z_n^{i*}) \left(1 - \frac{s_n}{r} - 2\lambda \frac{r^2 + \alpha^2 E^2[S_{n+1}] + \alpha^2 \frac{Cov^2(S_{n+1}, \chi_i(S_{n+1}))}{V[\chi_i(S_{n+1})]}}{r} \int_0^{z_n^{i*}} F_n(\xi)d\xi \right) \\
&+ 2\lambda\alpha^2 F_n(z_n^{i*}) \left(\frac{Cov(\chi_i(S_{n+1}), H(S_{n+1}, Z_{n+1}^{i*}))Cov(\chi_i(S_{n+1}), \bar{\chi}_i(S_{n+1}))}{V[\chi_i(S_{n+1})]} - Cov(\bar{\chi}_i(S_{n+1}), H(S_{n+1}, Z_{n+1}^{i*})) \right),
\end{aligned}$$

where $\bar{\chi}_c(S_{n+1}) \triangleq -(S_{n+1} - K_{c,n})^-$ and $\bar{\chi}_p(S_{n+1}) \triangleq (S_{n+1} - K_{p,n})^+$.

It is important to note that the optimal inventory policy when forward is the single hedge is myopic[14]. The corresponding hedging policy is two-period myopic, which is due to the one-period lead time for the exercise of the hedging contracts. Indeed, we note that the only parameter that is not from the current period is S_{n+1}. The optimal policy when call or put is the single hedge, however, is not myopic and require calculations of the optimal inventory levels for all the future periods. Forward contracts (or futures) are the most commonly used hedging for storable commodities in practice. Characterization of such an myopic optimal policy, easy to compute and implement, should further boost the popularity of forward contracts and ease off the operation complexities.

[14]Technically, the optimal policy is myopic when the forward contract is used as the single hedge, or included in the used hedge portfolio or can be replicated by the used hedge portfolio.

Similarly as in the optimal policy for the last period, the optimal order quantity from the supplier q_n^* is independent of the spot market participation and financial hedging. The optimal inventory level z_n^{f*} can be determined by simply increasing the value of z_n until the first order condition is met. We note that the search of z_n^{i*}, $i = c, p$, is, however, much more complex[15]. Clearly, the operational decisions, q_n^* and z_n^{i*}, though relying on the type of the hedging contract, can be determined without the details of the optimal hedging decision. On the other hand, the hedging decision $|q_{i,n}^{1*}|$ depends on all the optimal inventory decisions.

Interestingly, we find that, as will be shown in the next section, z_n^{f*} is the same as the optimal inventory level when the buyer adopts multiple hedges rather than a single hedge. Therefore, we present detailed sensitivity results for the optimal inventory level z_n^{f*} as its significance goes beyond the use of forward as a single hedge[16].

Corollary 4.3 *For* $n = 1, \ldots, N$,
(1) z_n^{f*} *is a non-increasing function of* λ, s_n *and* $\frac{E[S_{n+1}^2]}{s_n^2}$.
(2) As r *increases,* z_n^{f*} *is non-decreasing if* $r < s_n + \sqrt{\alpha^2 E[S_{n+1}^2] + s_n^2}$ *and non-increasing otherwise.*
(3) As z_n *increases, the variance term in the utility function* $U_n(\cdot)$, $\sum_{k=n} V[\alpha^{k-n}\pi_k^f(\cdot)]$, *increases.*

Intuitively, the buyer should keep a higher inventory level if it is cheaper to procure from the spot market. A more risk averse buyer should keep a lower inventory level. In other words, a risk neutral buyer will hold the highest inventory level. When facing a higher spot price volatility, a risk-verse buyer should keep a lower inventory level to reduce the variance and thus maximize his utility. For low to moderate selling prices of the buyer's goods, the mean profit is the dominant term in the utility, and thus increases in selling price will lead to keeping more inventory to increase the profit and thus maximize his utility. However, for high selling prices, the variance of the profit is the dominant term in the utility. Note that the lower the inventory level, the lower the variance of the profit. Thus, when selling price increases the buyer should keep less inventory to lower the variance of the profit and thus maximize his utility. Figure 3 indicates a typical behavior of z_n^{f*} as a function of r for different levels of spot price volatility.

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[15]If $\frac{Cov(\chi_i(S_{n+1}),H(S_{n+1},Z_{n+1}^{i*}))Cov(\chi_i(S_{n+1}),\bar{\chi}_i(S_{n+1}))}{V[\chi_i(S_{n+1})]} - Cov(\bar{\chi}_i(S_{n+1}), H(S_{n+1}, Z_{n+1}^{i*})) \leq \frac{r-s_n}{2\lambda\alpha^2}$, the search of z_n^{i*} is also simple: increasing the value of z_n starting from 0 until the first order condition is satisfied. The reason for the simplicity is that this condition implies monotonicity of the right hand side of equation above for z_n^{i*} when it is positive. However, if this condition does not hold, we might need to compute all the solutions that satisfy the first order condition and choose the one that maximizes the utility function $U_n(\cdot)$. Our numerical study indicates that this condition always holds with a comfortable margin in terms of the difference between the two sides of the condition. A typical example of the condition is $12.7169 \leq 1646.09$.

[16]The proof either involves simple algebra or follows directly from Proposition 4.2 and thus is omitted.

5 An Optimal Hedging Portfolio and The Optimal Operational Policy

In practice, many commodities buyers would prefer to choose an optimal portfolio of all the available hedging contracts. We note that it is sufficient to study multiple hedges that are restricted to the use among the forward, call and put options only (see Carr and Madan 2001).

We first define some additional parameters and decision variables. Let $K_{c,n,j}$, $i = 1, \ldots, n_c$, and $K_{p,n,j}$, $j = 1, \ldots, n_p$, denote the strike prices for available call and put options at the start of period n, respectively. Let $\mathbf{q}_{c,n} = [q_{c,n,1} \cdots q_{c,n,n_c}]$ and $\mathbf{q}_{p,n} = [q_{p,n,1} \cdots q_{p,n,n_p}]$ denote their corresponding hedging quantity arrays. Let $U_n(q_n, z_n, q_{f,n}, \mathbf{q}_{c,n}, \mathbf{q}_{p,n} | s_n, x_n, q_{f,n-1}, \mathbf{q}_{c,n-1}, \mathbf{q}_{p,n-1})$[17] denote the buyer's utility function given that the hedging portfolio is $(q_{f,n}, \mathbf{q}_{c,n}, \mathbf{q}_{p,n})$.

Let H denote the Hessian matrix of the utility function. Since $H = -2\lambda\alpha^2\mathbf{\Sigma}$, where $\mathbf{\Sigma}$ is the covariance matrix for random variables $S_{n+1}, (S_{n+1} - K_{c,n,1})^+, \ldots, (S_{n+1} - K_{c,n,n_c})^+, (S_{n+1} - K_{p,n,1})^-, \ldots, (S_{n+1} - K_{p,n,n_p})^-$, we know that H is negative semi-definite and thus the utility function is concave. The concavity leads to the following optimal policy.

Proposition 5.1 *The optimal policy parameters for period n are:*
(1) For $n = N$, $q^*_{f,N} = -\int_0^{z^*_N} F_N(\xi)d\xi \equiv q^{1*}_{f,N}$, $\boldsymbol{q}^*_{c,N} = \boldsymbol{q}^*_{p,N} = \boldsymbol{0}$. Let $S \triangleq S_{n+1}$, $S^+_{c,i} \triangleq (S_{n+1} - K_{c,n,i})^+$, and $S^-_{p,j} \triangleq (S_{n+1} - K_{p,n,j})^-$. For $n < N$ and all $i = 1, \ldots, n_c$ and $j = 1, \ldots, n_p$

$$\begin{cases} q^*_{f,n} V[S] + \sum_{k=1}^{n_c} q^*_{c,n,k} Cov(S^+_{c,k}, S) + \sum_{k=1}^{n_p} q^*_{p,n,k} Cov(S^-_{p,k}, S) = \psi_n(S) \\ q^*_{f,n} Cov(S^+_{c,i}, S) + \sum_{k=1}^{n_c} q^*_{c,n,k} Cov(S^+_{c,i}, S^+_{c,k}) + \sum_{k=1}^{n_p} q^*_{p,n,k} Cov(S^+_{c,i}, S^-_{p,k}) = \psi_n(S^+_{c,i}) \\ q^*_{f,n} Cov(S^-_{p,j}, S) + \sum_{k=1}^{n_c} q^*_{c,n,k} Cov(S^-_{p,j}, S^+_{c,k}) + \sum_{k=1}^{n_p} q^*_{p,n,k} Cov(S^-_{p,j}, S^-_{p,k}) = \psi_n(S^-_{p,j}) \end{cases}, \quad (6)$$

where $\psi_n(\chi(S)) = -E(z^*_n - D_n)^+ Cov(\chi(S), S) - Cov(\chi(S), H(S_{n+1}, Z^*_{n+1}))$;
(2) $q^*_n = l(w)$ if $w \geq s_n$; $q^*_n = u(w)$ if $w < s_n$;
(3) $z^*_n \equiv z^{f*}_n$ satisfies $\int_0^{z^*_n} F_n(\xi)d\xi = \frac{r - s_n}{2\lambda(r^2 + \alpha^2 E[S^2_{n+1}])}$, which is equivalent to

$$r\bar{F}_n(z^*_n)\left(1 - \frac{s_n}{r} - 2\lambda\frac{r^2 + \alpha^2 E^2[S_{n+1}]}{r}\int_0^{z^*_n} F_n(\xi)d\xi\right) = 2\lambda\alpha^2 \left\{ V[S_{n+1}] \int_0^{z^*_n} F_n(\xi)d\xi \right.$$

[17] It is similarly defined as $U_n(q_n, z_n, q_{i,n} | s_n, x_n, q_{i,n-1})$ except that the profit from the hedging here is the sum of the profits from each hedging contract.

$$+F_n(z_n^*)\left(q_{f,n}^*V[S]+\sum_{i=1}^{n_c}q_{c,n,i}^*Cov(S_{c,i}^+,S)+\sum_{j=1}^{n_p}q_{p,n,j}^*Cov(S_{p,j}^-,S)+Cov(S,H(S_{n+1},Z_{n+1}^*))\right)\Bigg\}. \quad (7)$$

Note that the optimal hedging portfolio has a unique solution when Hessian matrix H has full rank (i.e., when any hedging contract cannot be replicated by other hedging contracts in the portfolio). In this case, the hedging portfolio can be easily determined by multiplying the inverse of H to the right-hand-side vector in (6). This proposition implies that for a single period case, the buyer's optimal hedging portfolio contains forward only. In other words, the buyer can completely ignore all the other financial contracts available in the market. For multi-period case, however, the buyer's optimal hedging portfolio comprises forward, all the call options with strike prices lower than the forward price, and all the put options with strike prices higher than the forward price.

An important result to understand is that the optimal inventory level z_n^* when using multiple hedge containing forward is identical to the optimal inventory level z_n^{f*} when using forward as the single hedge. Let us compare the formula of z_n^*, given in (7), to the formula of z_n^{f*}, given in (14), using the first equation in (6). It is easy to note that the part of the variances of the payoffs that is affected by the inventory level for these two cases are equal[18]. Therefore, the optimal inventory levels for these two cases are equal. However, the other part of the variances of the payoffs also controls the total utility function and therefore using forward alone is not optimal for any period besides the last period.

Lastly, we consider a special case in which $n=-\infty,\ldots,-1,0,1,\ldots,\infty$ (infinite horizon) and forward is included in the hedging portfolio. Note that the optimal policy we derive in the previous sections for finite horizon when forward is used has two-period myopic property. In other words, the optimal decisions for each period are independent of the future optimal decisions. Therefore, the optimal policy for any period is indeed determined by maximizing the buyer's utility for the current period and the next period only. This observation implies that a same optimal policy for the finite horizon is also optimal for the infinite horizon case.

6 How To Select A Single Hedge And A Multiple Hedge

Our analysis so far has assumed that the choice of the single hedging contract and the choice of the multiple hedge were exogenous. We now endogenize this decision and first answer the question

[18] It is because $q_{f,n}^*V[S_{n+1}]+\sum_{i=1}^{n_c}q_{c,n,i}^*Cov(S_{c,i}^+,S)+\sum_{j=1}^{n_p}q_{p,n,j}^*Cov(S_{p,j}^-,S)=q_{f,n}^{1*}V[S_{n+1}]$.

of how to order the hedging contracts and what is the optimal for buyers who decide to restrict themselves to the use of a single hedging contract at any one period, but without constraining their choice to the same type hedge across all periods. A single hedge always of the same type across all periods was studied in Section 4. We next numerically study the efficiency of a number of multiple hedges and the factors that affect the choice of the call in the portfolio. We also recommend a simple yet effective heuristic multiple hedge which consists of forward and a call.

6.1 Optimal Single and Multiple Hedging Contract

In this section, we compare and order the financial contracts written on the spot market prices to select the best hedge for the buyer. For the last period, we have shown, in Section 5, that the optimal hedge is forward. Here we prove a monotonic relationship between the ordering of call or put options and their strike prices. For any other period, however, such result may or may not hold and thus an exhaustive computation of the buyer's utility may be required for comparison.

Let χ denote any financial hedge, a single hedge or a multiple hedge[19]. Let $\chi_f, \chi_{c(K_c)}, \chi_{p(K_p)}$ denote forward, the call option with strike price K_c, and the put option with strike price K_p, respectively. Let $U(q_n, z_n, q_{\chi1,n}|s_n, x_n, q_{\chi,n-1})$[20] denote the buyer's utility function given that hedges χ_1 and χ are purchased at the start of period n and $n-1$, respectively.

We next formally define the ordering of the financial hedges based on their contribution to the buyer's utility. For any two hedges χ_1, χ_2 (single or multiple), we say $\chi_1 \succeq \chi_2$ (i.e., χ_1 is not worse than χ_2) if $U(q_n, z_n^{\chi1*}, q^*_{\chi1,n}|s_n, x_n, q_{\chi,n-1}) \geq U(q_n, z_n^{\chi2*}, q^*_{\chi2,n}|s_n, x_n, q_{\chi,n-1})$. We say $\chi_1 \succ \chi_2$ (i.e., χ_1 is better than χ_2) if $>$ holds and $\chi_1 \simeq \chi_2$ (i.e., χ_1 and χ_2 are equivalent) if $=$ holds.

Unlike forward, there are many call and put options with a same strike time, but different strike prices. Applying the definition of ordering, we can characterize an optimal call (with the best strike price $K^*_{c,n}$ or an optimal put (with the best strike price $K^*_{p,n}$) for the buyer who restrict himself to the use of call or put only. It is important to note that for the last period, the reduction of the profit variance from using single hedge χ is characterized by $E^2[(z_n - D_n)^+]\frac{Cov^2(\chi(S_{N+1}),S_{N+1})}{V[\chi(S_{N+1})]}$.

Proposition 6.1 *For the last period, we have the following results for single hedges:*
(1) $\chi_1 \succeq \chi_2$ *if and only if* $\frac{Cov^2(\chi_1(S_{N+1}),S_{N+1})}{V[\chi_1(S_{N+1})]} \geq \frac{Cov^2(\chi_2(S_{N+1}),S_{N+1})}{V[\chi_2(S_{N+1})]}$.

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[19]When χ_i, $i = 1, 2, \ldots$, represents a multiple hedge, $\chi_i = [\chi_{i,1}, \ldots, \chi_{i,m}]$ and the hedging quantity $q_{\chi_i,n} = [q_{\chi_{i,1},n}, \ldots, q_{\chi_{i,m},n}]$, where $\chi_{i,j}$ represents a single hedge, $j = 1, \ldots, m$.

[20]It is similarly defined as $U(q_n, z_n, q_{f,n}|s_n, x_n, q_{f,n-1})$ except that the payoff functions of the hedging contracts are replaced by the corresponding payoff functions.

(2) $\frac{Cov^2(\chi_c(S_{N+1}),S_{N+1})}{V[\chi_c(S_{N+1})]}$ *decreases as* $K_{c,N}$ *increases and stays unchanged if and only if* $(S_{N+1} - K_{c,N})^+$ *is a constant;* $\frac{Cov^2(\chi_p(S_{N+1}),S_{N+1})}{V[\chi_p(S_{N+1})]}$ *increases as* $K_{p,N}$ *increases and stays unchanged if and only if* $(S_{N+1} - K_{p,N})^-$ *is a constant. Thus,* $K^*_{c,N} = s_l$*;* $K^*_{p,N} = s_u$.

(3) $\chi_1 \succeq \chi_2$ *if and only if* $z^*_N(\chi_1) \geq z^*_N(\chi_2)$.

(4) $\chi_f \succeq \chi$*, where* χ *is any contingent claim available on the market, and* $\chi_f \simeq \chi_{c(s_l)} \simeq \chi_{p(s_u)}$.

This proposition implies that the optimal hedge for the last period is forward or equivalent contracts, such as the call with strike price s_l and the put with strike price s_u. We characterize a monotonic relationship: the lower (higher) the strike price, the better the call (put) option for hedging. Furthermore, we learn that the use of a better hedge (single or multiple) leads to a higher inventory level (thus higher service level) and a higher profit variance reduction.

We now understand intuitively why forward is the best hedge for the last period. Note that financial hedges are adopted to hedge the price uncertainty in the profit earned at the end of horizon, $S_{N+1}(z_N - D_N)^+$. Note that we assume demand is independent of spot price and thus financial hedging cannot hedge the demand uncertainty directly. Since the price uncertainty comes from term S_{n+1} only and thus forward, whose payoff function carries the same uncertainty, should be the best hedge. In addition, we note that the pay off function for the optimal call (put) option is $(S_{N+1} - s_l) - \frac{\beta_{c,N}}{\alpha}$ $((s_u - S_{N+1}) - \frac{\beta_{p,N}}{\alpha})$ also involves the same price uncertainty. Therefore, it is not surprising to see that they are indeed equivalent to forward.

Since in general the lower (the higher) the strike price the higher the utility, the buyer should sell the call (buy the put) with the lowest (highest) strike price available on the market in practise if a single option is pursued. Intuitively, if the buyer does so, the risk premium he receives (pays) should be the highest among all the available options. For the same reason, the call (put) option is most likely to be exercised. If his excess inventory at the end of the period is not enough for the transaction, the buyer has to buy from the spot market at the spot price, which is most likely higher (lower) than the strike price, and thus incurring a negative (positive) unit profit. Therefore, a risk averse buyer would sacrifice the benefits of the price protection or high price margin in exchange for the certainty that the option is exercised (most variance reduction of the payoff for the option).

These comparison and optimal selection results, however, may not apply to any other period. In general, calculated utility for the whole planning horizon is needed for comparison between any two financial hedges and thus characterizing an optimal single hedge is not feasible. Therefore, in the rest of this section, we derive conditions which determine the best single hedging contract for any other period, given that the buyer implements an optimal hedging contract for each of the following periods, which may or may not be the same type. Assume that χ is the best financial hedge for period $n+1$.

Proposition 6.2 *For period n, $n = 1, \ldots, N-1$, we have the following results for single hedges:*
(1) $\chi_1 \succeq \chi_2$ if $\frac{Cov^2(\chi_1(S_{n+1}), S_{n+1}(z_n^{\chi_2*} - D_n)^+ + H(S_{n+1}, Z_{n+1}^{\chi*}))}{V[\chi_1(S_{n+1})]} \geq \frac{Cov^2(\chi_2(S_{n+1}), S_{n+1}(z_n^{\chi_2*} - D_n)^+ + H(S_{n+1}, Z_{n+1}^{\chi*}))}{V[\chi_2(S_{n+1})]}$.
(2) $\chi_f \simeq \chi_{c(s_l)} \simeq \chi_{p(s_u)}$ and $z_n^*(\chi_f) = z_n^*(\chi_{c(s_l)}) = z_n^*(\chi_{p(s_u)})$.
(3) If $l(w) = u(w)$ (i.e., fixed quantity commitment contract) and D_{n+1} is assumed constant, $\chi_f \succeq \chi$, where χ is any contingent claim available on the market.

Note that the condition we provide for comparison is sufficient, not necessary[21]. Unlike for the last period, the profit variance reduction quantity for any other period may not be monotonic to the inventory level z_n (as observed in our numerical study) or the strike price if call or put is used as the single hedge (as shown in Figure 4). Therefore, the observed monotonic relationship among the ordering of the hedging contracts, the strike price of call and put options, and the optimal inventory levels for the last period may or may not hold for any other period. Thus, a better hedging contract may or may not lead to a higher service level, although the buyer's utility is improved. However, the equivalence between forward and the call with strike price s_l and the put with strike price s_u preserves for any period. This is because the payoff functions of the equivalent options have exactly the same price uncertainty as the payoff function of the forward.

Note that financial hedges are adopted to hedge against the uncertainty of the next period's profit, where $\pi_{n+1} = S_{n+1}(z_n - D_n)^+ + [u(w)(S_{n+1} - w)^+ - l(w)(S_{n+1} - w)^-] - S_{n+1}Z_{n+1}^* + r(Z_{n+1}^* \wedge D_{n+1})$. If $l(w) = u(w)$ (i.e., fixed quantity for long-term contract) and D_{n+1} is assumed constant, denoted by d_{n+1}, the next period's profit becomes $S_{n+1}(z_n - D_n)^+ + [l(w)(S_{n+1} - w)] - S_{n+1}d_{n+1} + rd_{n+1}$. In this new profit function, the uncertainty comes from term S_{n+1} only and thus forward should be the best hedge for this case. It is important to note that this case may resemble the actual interface of financial hedging and operations management in practice. In reality, many storable commodities suppliers only accept fixed quantity orders to stabilize their production. Financial hedging decisions are made separately in absence of the optimal operation decisions. They are often determined using assumed constant demand (e.g., moving average demand forecast) and thus constant inventory level. Our result clearly explains that why forward is the only hedge commonly

[21]There are cases where we must compare by computing the contribution each hedge χ_i, $i = 1, 2$, makes to the utility, which is characterized by $\frac{Cov^2(\chi_i(S_{n+1}), S_{n+1}(z_n^{\chi_i*} - D_n)^+ + H(S_{n+1}, Z_{n+1}^{\chi*}))}{V[\chi_i(S_{n+1})]} - V[S_{n+1}(z_n^{\chi_i*} - D_n)^+ + H(S_{n+1}, Z_{n+1}^{\chi*})]$.

and popularly used in practice where inventory and financial hedging decisions are not integrated. Our numerical results for the integrated model, however, show that forward may not be optimal for periods other than the last. Thus we next study what hedge we should recommend to the risk-averse buyer to use.

6.2 Heuristic Hedge

We start by continuing the discussion on the price uncertainty in next period's profit function, where $\pi_{n+1} = S_{n+1}(z_n - D_n)^+ + [(u(w) - l(w))(S_{n+1} - w)^+ + l(w)(S_{n+1} - w)] + (r - S_{n+1})Z^*_{n+1} - r(Z^*_{n+1} - D_{n+1})^+$. Note that the price uncertainty carrying terms in the profit include S_{n+1}, $(S_{n+1} - w)^+$, Z^*_{n+1} and $(Z^*_{n+1} - D_{n+1})^+$ (where Z^*_{n+1} is a decreasing function of S_{n+1} when forward is included in the hedging portfolio), for $n < N$. Obviously, the use of forward or the equivalent call or put may not be optimal for any period other than the last and we need to at least combine the use of forward with a call. To neutralize the price uncertainty, based on the analytical discussion above and more below and the observation in the numerical study (the use of forward and the best call achieves at least 99.90% of the maximum utility), we would suggest the use of forward and a call[22] with carefully selected strike price (which normally depends on model parameters $l(w)$, $u(w)$, $E[(z_n - D_n)^+]$, and D_{n+1}). Specifically, we suggest that the strike price should be higher than w and should decreases as the upper limit of the order quantity $u(w)$ increases.

Note that the quantity of inventory involved with $(S_{n+1} - w)^+$ is $u(w) - l(w)$, which we refer to as "quantity of $(S_{n+1} - w)^+$". Similarly, "quantity of S_{n+1}" is $(z_n - D_n)^+ + l(w)$. Note that there exist other sources of uncertainties in the profit function which result from Z^*_{n+1} and $(Z^*_{n+1} - D_{n+1})^+$. Though similar to the uncertainties of S_{n+1} and $(S_{n+1} - K)^+$ for any $K \geq 0$, respectively, they are difficult to quantify. As a result, offering a general guideline on choosing a good strike price for the call is impossible. In special cases such as D_{n+1} is constant (discussed in Section 6.1), however, uncertainties of Z^*_{n+1} and $(Z^*_{n+1} - D_{n+1})^+$ disappear and thus the use of forward and a call with strike price w is optimal. If the quantity of $(S_{n+1} - w)^+$ is zero, using forward only is optimal.

We next utilize the numerical study to enhance our understanding of how to choose the best call for different scenarios. Figure 5 show that the best strike price for call decreases as $u(w)$ increases[23]. We verify that the more the financial contracts we use, the higher the buyer's utility.

[22] Given the use of forward, adding a call is equivalent to adding a put with the same strike price.

[23] This observation coincides with the numerical result for the cases in which call options are used as the single hedge.

We also investigate the marginal benefits of adding more financial contract to the hedging portfolio. As Figure 6 indicates, the marginal benefit generally decreases most of the time as more and more call options are added. In addition, we observe that as $u(w)$ increases, the marginal benefit of adding forward goes up and so does the marginal benefit of adding the first call in addition to forward, but with much slower speed. This implies that when $u(w)$ increases, the use of forward and any call becomes more powerful in hedging.

7 Role and Impact of Operational and Financial Hedges

In this section we discuss the impact of the use of operational and financial hedges on the mean and variance of profits and on the buyer's service level. First of all, we note that the long-term contract is exercised (with payoff function $u(w)(S_{n+1}-w)^+ - l(w)(S_{n+1}-w)^-$) as a combination of a call and a put with the same strike price w. Therefore, it is similar to financial hedging contracts purchased at the beginning of the horizon. However, the optimal decisions for the long-term contract q_n^* are robust to the rest of the optimal inventory and hedging decisions.

7.1 Impact of Operational and Financial Hedges

We now discuss the impacts of the parameters of the operational hedge on the optimal service levels and financial hedging decisions. For the long-term contract, its flexibility (i.e., $u(w)$ and $l(w)$) affects the service levels only when forward is not used. It, however, always affects the optimal financial hedging decisions. The real-time spot price directly affects the service levels and indirectly affects the financial hedging quantity via service levels. When forward is used, we have closed form formula for the service levels. Indeed, as Corollary 4.3 indicates the spot price has linear impact on the service levels. In contrast, the spot price volatility directly affects both the service levels and the financial hedging decisions.

Next, we investigate the property of the optimal service levels for different financial hedging strategies, which include DS (Dual Sourcing without hedging), DS+i (Dual Sourcing with a single hedge i, i = F (forward), C (call), P (put)) and DS+HP (Dual Soucing with a Hedging Portfolio). Note that z_n^{0*}[24] denote the buyer's optimal inventory level period n, $n = 1, \ldots, N$, for DS.

[24] Following a similar reasoning we stated for the uniqueness of z_n^{i*}, $i = c, p$, we note that a unique z_n^{0*} may or may not exist depending on the quantity of term $Cov(S_{n+1}, H(S_{n+1}, Z_{n+1}^{0*}))$. z_n^{i*} is the solution of the following equation. $r\bar{F}_n(z_n^{0*})\left(1 - \frac{s_n}{r} - 2\lambda\frac{r^2+s_n^2}{r}\int_0^{z_n^{0*}} F_n(\xi)d\xi\right) = 2\lambda\alpha^2 V[S_{n+1}]\int_0^{z_n^{0*}} F_n(\xi)d\xi + 2\lambda\alpha^2 F_n(z_n^{0*})Cov(S_{n+1}, H(S_{n+1}, Z_{n+1}^{0*}))$.

Proposition 7.1 $(z^*_N =)z^{f*}_N \geq z^{c*}_N, z^{p*}_N > z^{0*}_N$, *and* z^{f*}_n, $n = 1, \ldots, N$, z^{0*}_N, z^{i*}_N, $i = c, p$, *are decreasing functions of* λ.

For the last period, a risk neutral buyer (i.e., $\lambda = 0$) holds the highest inventory level, i.e., his inventory level serves as an upper bound for any risk averse buyer. Note that financial hedging has no contribution to the risk neutral buyer's utility when zero risk premium is assumed. As the buyer's risk aversion increases and in absence of financial hedging, his inventory level drops. By employing financial hedging, the buyer can better control the variance of the profit, and thus it allows him to raise his inventory level. The better the hedging contract employed, the higher the inventory level held by the buyer.

For any other period, a risk neutral buyer still holds the highest inventory level. First, the optimal inventory level for any risk averse buyer may behave differently. For example, our numerical study for moderate to high spot price volatility indicates that the optimal inventory when a single call (put) is used is sometimes (always) higher (lower) than the optimal inventory for DS+HP. The use of a better call does not necessarily lead to a higher service level[25]. Furthermore, we observe that impact of hedging contracts on service level increases as the volatility of the spot price increases. Second, a more risk averse buyer may or may not hold less inventory unless forward is the only contract used. A closer examination of the formula for z^{i*}_n reveals that its sensitivity on the risk aversion, λ, depends on the quantity of $Cov(S_{n+1}, H(S_{n+1}, Z^{0*}_{n+1}))$. If this quantity is negative, z^{i*}_n is a non-increasing function of λ. Otherwise, however, z^{i*}_n is not necessarily non-increasing as λ increases. Unfortunately, our numerical study with many cases of positive $Cov(S_{n+1}, H(S_{n+1}, Z^{0*}_{n+1}))$ did not capture a case in which z^{i*}_n is a non-decreasing function of λ.

When call or put options are used as the single hedge, our numerical study shows that the optimal inventory levels z^{c*}_{N-1} and z^{p*}_{N-1} have the same behavior as z^{f*}_n in response to the change of risk aversion, spot price, and volatility of the spot price. Figure 7 illustrates a typical comparison among the inventory levels z^{i*}_{N-1}, $i = f, c, p$, as a function of the spot price. Note that z^{p*}_{N-1} is always the lowest, while the highest switches between z^{f*}_{N-1} and z^{c*}_{N-1}. Since $z^{f*}_n = z^*_n$, we learn that the use of a suboptimal financial hedge may lead to a higher customer service level.

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[25]For example, for the cases with $u(w) = 10$, we find that the put option with 30th percentile strike price is better than the put option with 20th percentile strike price, but has a lower optimal inventory level.

7.2 Role of Operational Hedges and Financial Hedges

Here we compare the impacts of different hedging strategies for buyers with different risk aversion. Figure 8 reveals the composition of the mean and variance of the profit as a function of the risk aversion for the buyer, λ, for a variety of hedging strategies, which include the use of a Single Source (the wholesale price contract)(SS), DS, DS+i (i = F, C, P), and DS+HP. As his risk aversion increases, the buyer moves to the optimal position with lower mean and lower variance for any case. We observe that the curves for SS and DS+HP are both at the bottom of the graph area, but are apart from each other. The curve for SS lies in the zone with low mean and low variance, while the curve for DS+HP lies in the zone with high mean and low variance. This clearly indicates the benefits of using the financial hedge - increase the service level and thus the mean profit and maintain a low variance. Moreover, we find that in right of the graph area, the ordering of the curves from the top to the bottom is the curves for DS, DS+P, DS+F, DS+C, and DS+HP. This implies that it is always beneficial to adopt financial hedges and it is essential to choose a good hedge. Also, multiple hedges are generally more effective than single hedges. Moreover, we note that the optimal policy when multiple hedge (which includes forward) is used is myopic and thus is computationally easier to derive than that for the other cases. The optimal hedging quantities can also be easily determined by solving a system of linear equations. The above observations backup our recommendation for the use of multiple hedges, specifically the use of forward and a call with carefully selected strike price.

8 Managerial Insights and Conclusions

Effective ways to manage highly volatile commodity prices when sourcing, inventorying and processing commoditized inputs to be finally sold in differentiated form in end product markets is a concern of companies in industries as diverse as food processing, autos, household products, and hi-tech electronics. While most of the previous supply chain management literature focused on the structuring of supply chain contracts (minimum quantity commitment, flexible, capacity reservation, option contracts) and the use of portfolio of such contracts in dealing with associated demand and price risks (see representative work in Li and Kouvelis (1999), Martínez-de-Albéniz and Simchi-Levi (2005)), recent efforts reported in Kleindorfer (2008 a, b) has emphasized the need for integrating long-term bilateral contracts(fixed commitment or flexible) with access to reasonably liquid spot markets for short-term responses to realized uncertainties. The effectiveness of such integration of long term-short term contracting leads to increases in expected profits, but also increases the cash flow volatility. This can be of a concern to risk averse buyers (loss averse owners of private firms or managers/agents of public firms evaluated with risk measures relying on cash flow volatility), thus raising the need to complement them with financial hedges that can effectively deal with both the profit maximizing concerns and the risk control of operational decision for commodity inputs. However, these days financial intermediaries offer a variety of financial contracts (futures, call or put options) written on commodity spot market prices, thus offering the opportunity to better hedge risk exposure in commodity procurement decisions. Our paper is the first to offer an integrated risk management framework for storable commodities deploying dual sourcing via flexible long-term supply contracts and short-term commodity exchange purchases combined with a portfolio of financial hedges for cash flow volatility control. Our results offer useful insights on how much to source from the spot market, optimal inventory policies in the presence of both end product demand and commodity price uncertainties, and structure of the optimal financial hedging portfolio.

Effective implementation of such policies requires cross-functional decision coordination and information sharing among operations and financial managers. Our results indicate that the information burden is lower for operations managers in effectively executing sourcing and inventory decisions. The setting of optimal base stock inventory policy levels requires awareness of the firm's commitment to financially hedge cash flow volatility and the type of hedging contract to be used, without however requiring the details of the hedged amounts of the financial hedge contracts. Especially, if the firm will pursue an optimal portfolio of hedges, the base stock calculations are greatly simplified from a formulaic perspective and the set base stock levels are set in a similar way to when single forward contracts are used for hedging (see Proposition 5.1, and especially part (3) of it). For such cases, the base stock levels are decreasing in risk aversion, observed spot market prices, and the variance of spot market prices.

The information burden on the financial manager in implementing the integrated commodity risk management results is higher as explicit knowledge of the base stock levels is required in choosing the structure and the optimal hedge amounts of the optimal hedging portfolio. The good news is that the optimal financial hedging portfolio composition can be easily obtained via the solution of a straightforward system of linear equations (see Proposition 5.1 (1)). For short-term horizons, more or less resembling our single (last) period results, forward (or futures) contracts are all that are needed to hedge the variance of cash flows. However, this is not the case for longer horizons (unless we are working with fixed quantity commitment contracts and using demand forecast, rather than random demand, for future periods), and our heuristic solution (if managers decide not to solve for the optimal portfolio) recommends use of forward and a call option with strike price higher than the long-term contract per unit price for moderate to high flexible contract upper limits. The strike price should be even higher the lower the contract upper limits are. The institutional reality of many commodity markets (lack of financial intermediaries that offer option contracts or the limited liquidity of the market for such instruments) might be forcing the consideration of only forward contracts as the available hedges, and in that case the needed hedge amount calculations are described in closed form in Propositions 4.1 and 4.2.

Our theoretical results and our computational study (refer in particular to Figure 8) clearly outline the benefits of integrated commodity risk management. The long-term contract-spot market dual sourcing deliver strong expected profit benefits. As usual in risk management optimization settings (see for example the conceptual framework and problem formulation in Kleindorfer 2008a) policies that optimize profits are often limited in their feasible execution by cash flow risk constraints, or in the case of our model by the penalizing features of the variance term included in the objective function. However, when financial hedges enter the picture through their cash flow volatility controls they allow more aggressive execution of profit maximizing policies without feasibility (or variance penalty effects) concerns.

Our work clearly shows the role played by the operational and financial hedges. Our two operational hedges, the long-term contract and access to the spot market, are effectively used to deal with the demand uncertainty. By its nature, the long-term contract protects the buyer against spot market price volatility, while the financial derivative contracts hedge directly the spot price uncertainty and only indirectly the demand uncertainty (via correlation to the demand). For single period settings, use of financial hedges drives up inventory levels. Better financial hedges lead to higher inventory levels and increased service offered to the end-product customer. Furthermore, the inventory levels decrease as the buyer becomes more risk averse. However, these are not easily generalizable, or even true, for multi-period settings as our numerical study illustrated.

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Appendix: Figures

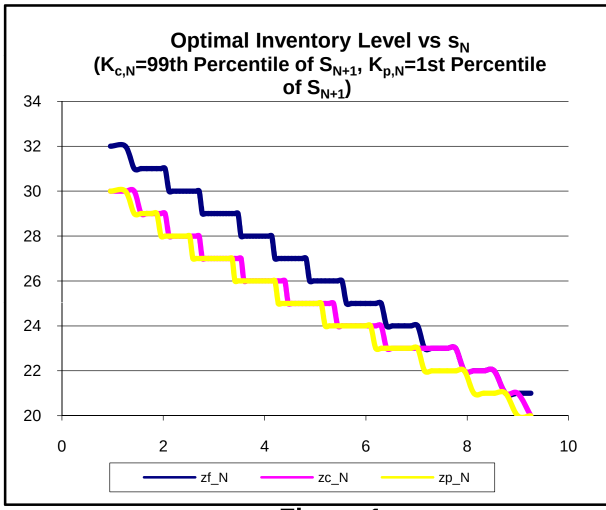

Figure 1

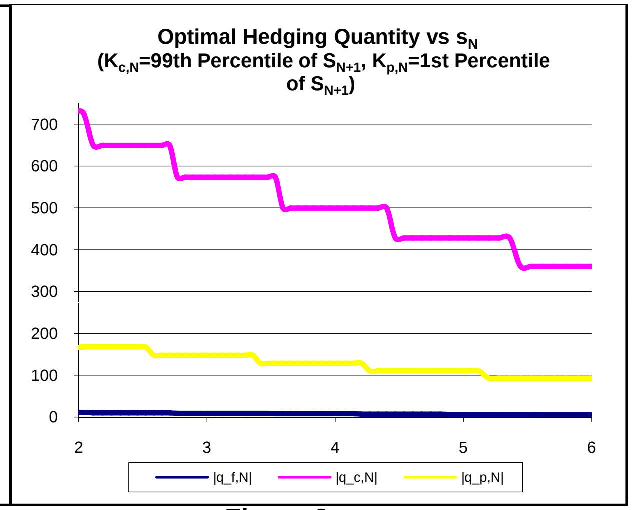

Figure 2

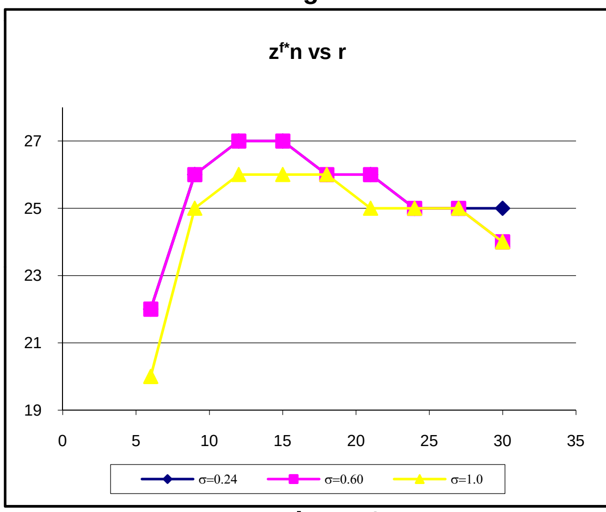

Figure 3

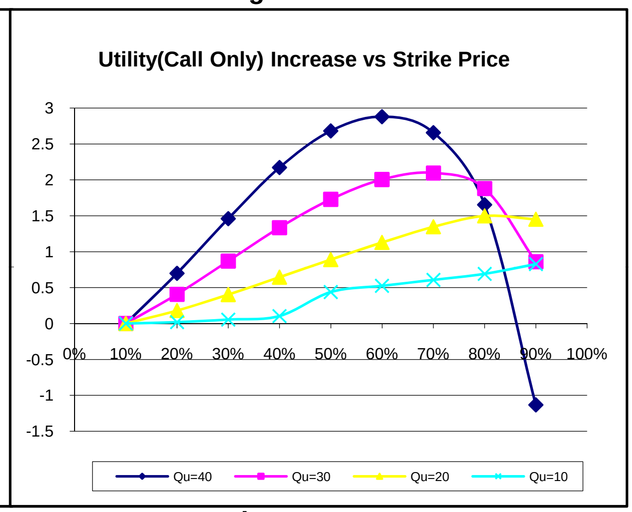

Figure 4

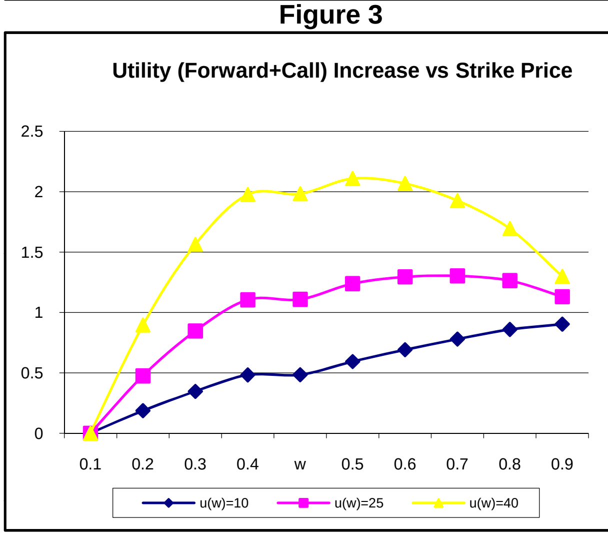

Figure 5

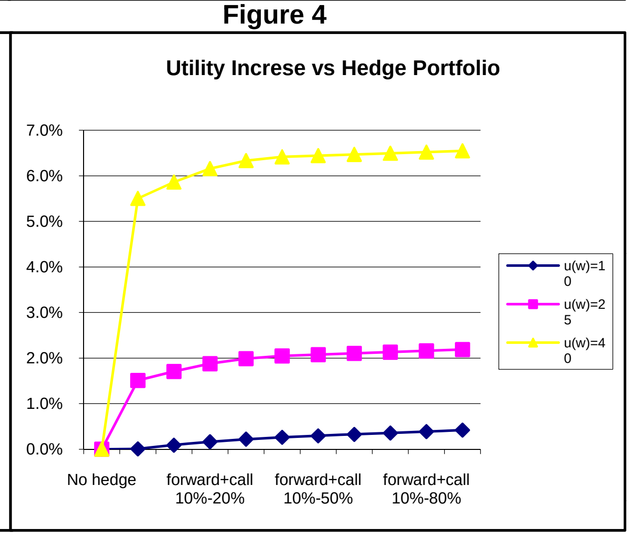

Figure 6

Note: (1) 10%, …, 90% in x-axis in Figure 4 refer to the correponding percentiles of the spot price for the call option strike price. (2) "forward+call 10%-i%", i=10,…,90, in Figure 6 refers to the use of the forward cotract and call options with strike prices 10th, …, ith percentile of the spot price.

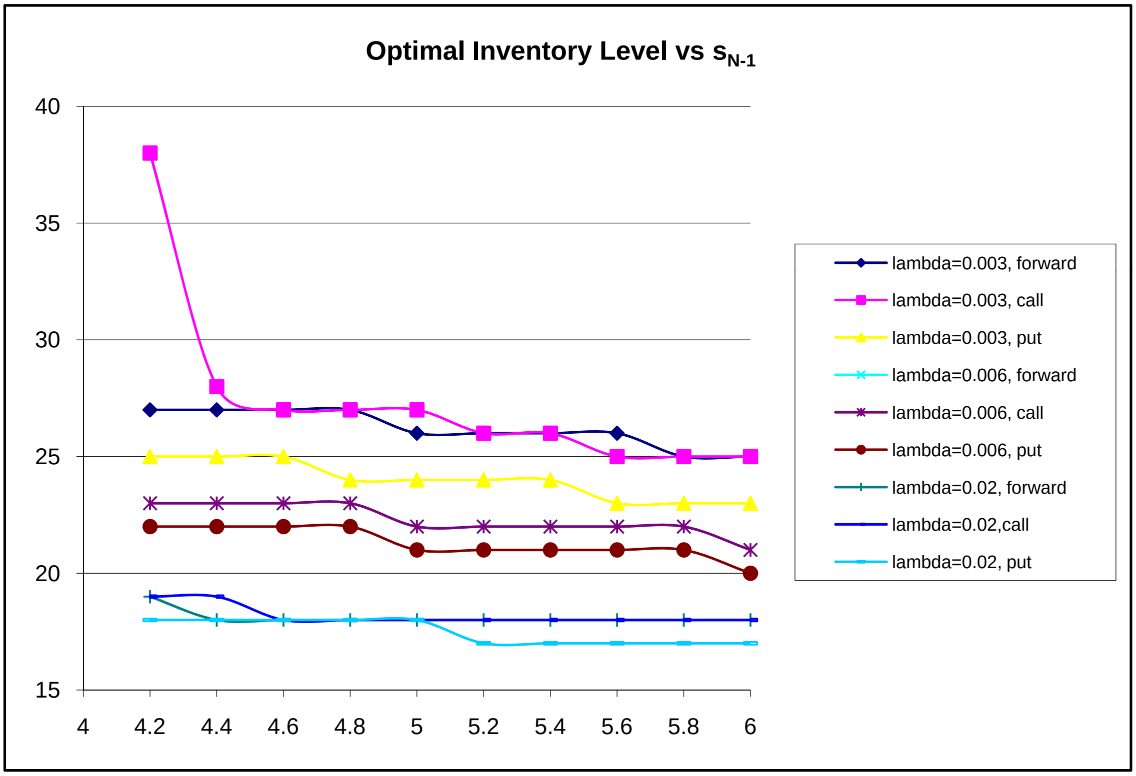

Figure 7

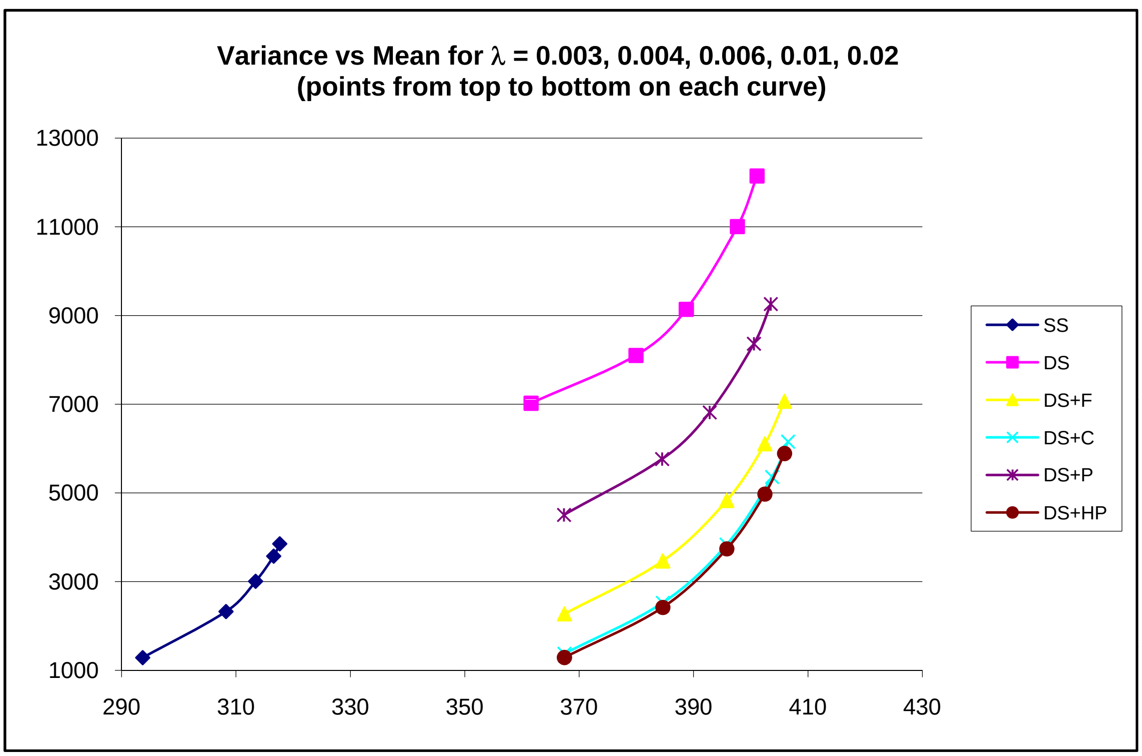

Figure 8