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Asymmetric Response of Volatility: Evidence from Stochastic Volatility Models and Realized Volatility

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Asymmetric Response of Volatility: Evidence from Stochastic Volatility Models and Realized Volatility*

Jun Yu[†]

September 17, 2004

Abstract

This paper examines the asymmetric response of equity volatility to return shocks. We generalize the news impact function (NIF), originally introduced by Engle and Ng (1993) to study asymmetric volatility under the ARCH-type models, to be applicable to both stochastic volatility (SV) and ARCH-type models. Based on the generalized concept, we provide a unified framework to examine asymmetric properties of volatility. A new asymmetric volatility model, which nests both ARCH and SV models and at the same time allows for a more flexible NIF, is proposed. Empirical results based on daily index return data support the classical asymmetric SV model with a monotonically decreasing NIF. This empirical result is further reinforced by the realized volatility obtained from high frequency intraday data. We document the option pricing implications of these findings.

JEL classification: C11, C15, G12

Keywords: Bayes factors; Leverage effect; Markov chain Monte Carlo; EGARCH; Realized volatility; Asymmetric volatility

1 Introduction

The intertemporal relation between the equity volatility and return shock has long been an active research topic in the finance literature. It is generally agreed that there is an asymmetry in the relation, that is, a positive return shock has a smaller impact on future volatility than does a negative shock of the same size. Volatility models which allow for such an asymmetric

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property not only improve the ability to describe return dynamics (Engle and Ng, 1993) but also provide more accurate option prices (Heston and Nandi, 2000 and Christoffersen and Jacobs, 2004).

The economic reasons for the asymmetry are not entirely clear. A popular explanation in the literature is the leverage effect (see, for example, Black, 1976 and Christie, 1982). This market folklore suggests that when a negative return shock (i.e., bad news) arrives, it decreases the value of a firm's equity and hence increases its leverage. Consequently, the equity becomes more risky and its volatility increases. By similar arguments the volatility decreases when good news arrives. According to this effect, there must be a negative relationship between future volatility and returns and the volatility of returns should be a decreasing function of return shocks. Although, the leverage effect is synonymous with asymmetric volatility to many researchers, it should be understood as a special type of asymmetric response.

Another explanation for asymmetry is the volatility feedback effect (French, Schwert and Stambaugh 1987, Campbell and Hentschel 1992, and Wu 2001). Under the assumption of volatility clustering, a large piece of news (good or bad) will lead to a high volatility which tends to be followed by another large volatility. According to the volatility feedback effect, since volatility is priced, an increase in volatility should increase the required rate of return and hence decrease the stock price. As a result, when a large piece of bad news arrives, it tends to increase future expected volatility and hence amplify the negative impact of bad news. When a large piece of good news arrives, the increase of future volatility dampens the positive impact of good news.

It is obvious that both effects can explain volatility asymmetry. However, they differ in how volatility responds to good news. In particular, while the leverage effect predicts a downward movement of future expected volatility, the volatility feedback effect does not predict any relationship between volatility and returns when good news arrives.

The most popular empirical method to examine asymmetry is via some form of ARCH-type models. A rather incomplete list is Nelson (1991), Glosten, Jagannathan and Runkle (1993), Rabemananjara and Zokoia (1993), Engle and Ng (1993), Hentschel (1995), Duan (1996), Bekaert and Wu (2000), and Linton and Mammen (2004). Based on the work of Pagan and Schwert (1990), Engle and Ng (1993) introduced the news impact function (NIF) as an empirical tool to examine the effect of return shocks on future volatility in isolation. Using

NIF, Engle and Ng compared various ARCH models and found evidence of asymmetry. A common feature of almost all empirically estimated asymmetric ARCH-type models is that the NIF is asymmetrically U- or V-shaped, although more flexible NIFs are generally allowed. An implication is that the asymmetry cannot be explained by the leverage effect alone as the NIF is not a globally decreasing function.

Another empirical approach to study volatility asymmetry is via the SV models (see Ghysels, Harvey and Renault, 1996 and Shephard, 1996 for reviews of SV models). Work on option pricing based on asymmetric SV models dates back to 1987 (Hull and White, 1987 and Wiggins, 1987), which is several years earlier than the first asymmetric ARCH model (Nelson 1991). There are certain advantages in the SV models over the ARCH-type models for modeling the dynamics of asset returns (see Ghysels et al, 1996 and Andersen, 1994). Yet asymmetric SV models have received far less attention than asymmetric ARCH-type models in the empirical finance literature. For example, no SV model was considered in Engle and Ng (1993) or in any paper surveyed by Bekaert and Wu (2000). We postulate several possible reasons why asymmetric SV models have fewer empirical applications. First and perhaps most obviously, SV models are difficult to estimate. This is the case for the basic SV model and even more so for the asymmetric SV model. Second, how asymmetry should be specified in a SV model had long been an open question. In particular, it was not clear if one should allow for a contemporaneous correlation between return shocks and volatility shocks as suggested in Harvey and Shephard (1996), or for a correlation between return shocks and lagged volatility shocks as advocated in Jacquier, Polson and Rossi (2004). Finally, the NIF commonly used in the ARCH literature is not directly applicable to SV models since there exist two sources of shocks in the SV models.

Fortunately, recent developments in the volatility literature make possible a thorough analysis of asymmetric response of volatility under a more flexible framework. For example, in terms of estimation, Harvey and Shephard (1996) provided one of the first econometric treatments of an asymmetric SV model using a quasi-maximum likelihood method. In terms of specification, Yu (2004) showed that the specification with the contemporaneous correlation is superior to that with the inter-temporal correlation and further offered a full likelihood-based treatment to the model using a Markov chain Monte Carlo (MCMC) method. Moreover, the availability of ultra high frequency data makes it feasible to accurately approximate unobserved volatility based on realized daily volatility. This has prompted researchers to treat volatility as directly observable

in many recent studies. As a result, a new and powerful channel based on realized volatility can be used to examine the validity of parametric volatility models; see Andersen, Bollerslev, Diebold and Labys (2001) (ABDL hereafter), Barndorff-Nielsen and Shephard (2002), and Andersen, Bollerslev, Diebold and Ebens (2001) (ABDE hereafter) for important contributions in this area, and Andersen, Bollerslev and Diebold (2004) for an excellent survey.

The central focus of the present paper is to provide a unified framework to investigate asymmetric response of volatility, applicable to both ARCH-type and SV models. The importance of an appropriate specification of asymmetry is manifest in much of financial decision making, including asset allocation, volatility forecasting, options pricing, risk management, and hedging.

Our contribution to the literature is three-fold. First, we generalize the NIF of Engle and Ng (1993) so that it is applicable to SV models. Based on the generalized NIF, we develop a general framework to examine volatility asymmetry, which makes it feasible to compare asymmetric ARCH with asymmetric SV models for the purpose of finding a better specification outside the ARCH family to explain the observed asymmetry. To the best of our knowledge, such a comparison has not been done before, reflecting the existence of two virtually separate literatures. Studies using ARCH-type models typically suggest an asymmetrically U- or V-shaped NIF, whereas studies using the SV models always assume a monotonically decreasing NIF. We propose a model which nests both the ARCH and SV models and at the same time allows for a flexible NIF. When fitting parametric models to daily index data, we find evidence of monotonically decreasing NIF.

Second, we compare the implied NIFs of various parametric volatility models with the NIF obtained from the realized daily volatility based on ultra high frequency index data. Although this nonparametric approach was originally suggested by ABDE (2001), the result obtained in the present paper documents new empirical evidence in the index data – the NIF monotonically decreases and is not V-shaped.

Third, since both daily and intraday data suggest the same kind of volatility asymmetry, we examine the economic significance of the asymmetry. In particular, as in ABDE (2001), we find that the NIF is very flat, featured by the slope estimate being very close to zero. Based on this observation, ABDE argue that the economic importance of the asymmetric effect is marginal. However, we show that it is the correlation between two shocks, not the slope of the NIF, that matters. In the classical asymmetric SV model although the estimated slope of NIF is always

close to zero by construction, the correlation coefficient can be quite different from zero. As a result, we quantify the option pricing implications of asymmetry via simulations.

The remainder of the article is organized as follows. Section 2 reviews certain existing asymmetric ARCH and SV models, and the NIF of Engle and Ng (1993), and then generalizes the NIF to both types of models. In Section 3 a unified framework is formulated by introducing a new asymmetric model with a flexible NIF, together with an explanation of how the proposed model is related to the existing ARCH and SV models. Empirical results based on daily index data are discussed in Section 4, while empirical results based on intraday index data are discussed in Section 5. Section 6 considers the economic implications of the empirical results and Section 7 concludes.

2 Asymmetric ARCH, Asymmetric SV Models and News Impact Function

Researchers beginning with Black (1976) have found empirical evidence that volatility tends to rise in response to bad news and to fall in response to good news. Motivated directly from this leverage effect, numerous forms of asymmetric ARCH models have been introduced. One of earliest and best known models is Nelson's (1991) Exponential GARCH (EGARCH),

$$y_t = \sigma_t \epsilon_t = \exp(h_t/2) \epsilon_t, \quad \epsilon_t \sim \text{iid } N(0, 1) \quad (2.1)$$

$$h_t = \alpha + \phi h_{t-1} + \psi |\epsilon_{t-1}| + \beta \epsilon_{t-1}, \quad (2.2)$$

where y_t is the return, σ_t^2 is the conditional variance, and ϵ_t is the standardized return shock. Another popular asymmetric ARCH model is proposed by Glosten, Jagannathan and Runkle (1993) (GJR-GARCH),

$$\sigma_t^2 = \alpha + \phi \sigma_{t-1}^2 + \beta y_{t-1}^2 + \beta^* y_{t-1}^2 I(y_{t-1} < 0), \quad (2.3)$$

where $I(y_{t-1} < 0) = 1$ if $y_{t-1} < 0$ and 0 otherwise.

To examine how current return shocks affect future expected volatility, Engle and Ng (1993) introduced the news impact function (NIF). The idea is to condition at time $t + 1$ on the information available at t and earlier, and then consider the effect of the return shock, ϵ_t , on σ_{t+1}^2 (or h_{t+1}) in isolation. To be more specific, the NIF is the relation $f(\cdot)$ or $g(\cdot)$ in the

following equations,

$$(\sigma_{t+1}^2 | \epsilon_t, \sigma_t^2 = \bar{\sigma}^2, \sigma_{t-1}^2 = \bar{\sigma}^2, \dots) \equiv f(\epsilon_t), \quad (2.4)$$

or

$$(h_{t+1} | \epsilon_t, h_t = \bar{h}, h_{t-1} = \bar{h}, \dots) \equiv g(\epsilon_t), \quad (2.5)$$

where $\epsilon_t > 0$ corresponds to good news, $\epsilon_t < 0$ corresponds to bad news, and $\bar{\sigma}^2$ (\bar{h}) is the long run mean of σ_t^2 (h_t). Asymmetry occurs if and only if $f(\epsilon_t) \neq f(-\epsilon_t)$ or $g(\epsilon_t) \neq g(-\epsilon_t)$.

It can be seen that the NIF for EGARCH is

$$g(\epsilon_t) = \begin{cases} \alpha + \phi \bar{h} + (\beta + \psi) \epsilon_t, & \text{if } \epsilon_t \geq 0 \\ \alpha + \phi \bar{h} + (\beta - \psi) \epsilon_t, & \text{if } \epsilon_t < 0. \end{cases} \quad (2.6)$$

As long as $\beta \neq 0$, the response of volatility is asymmetric. In particular, if $\beta < 0$ and $\beta + \psi > 0$, the NIF is asymmetrically V-shaped. If $\beta < \psi < -\beta$, the NIF is monotonically decreasing. Although EGARCH can have a flexible NIF, empirical results always suggest that $\beta + \psi > 0$ and $\beta - \psi < 0$, which implies an asymmetrically V-shaped NIF.

The NIF for GJR-GARCH is

$$f(\epsilon_t) = \begin{cases} \alpha + \phi \bar{\sigma}^2 + \beta \bar{\sigma}^2 \epsilon_t^2, & \text{if } \epsilon_t \geq 0 \\ \alpha + \phi \bar{\sigma}^2 + (\beta + \beta^*) \bar{\sigma}^2 \epsilon_t^2, & \text{if } \epsilon_t < 0. \end{cases} \quad (2.7)$$

Empirical estimates always suggest that $\beta > 0$ and $\beta^* > 0$. As a result, a typical curve for NIF is asymmetrically U-shaped. Note that the estimated NIFs from asymmetric ARCH models are inconsistent with the prediction of the leverage effect, although asymmetric ARCH models are motivated by this.

Like the EGARCH model, most asymmetric SV models are also directly motivated by the leverage effect. The well-known asymmetric lognormal (LN) SV model takes the form of, in discrete time,

$$\begin{cases} y_t &= \sigma_t \epsilon_t = \exp(h_t/2) \epsilon_t, \\ h_{t+1} &= \alpha + \phi h_t + \sigma_v v_t, \end{cases} \quad (2.8)$$

ϵ_t and v_t are iid $N(0, 1)$ and $\text{corr}(\epsilon_t, v_t) = \rho$. Different from ARCH-type models, there are two separate shocks in SV models, the return shock ϵ_t and the volatility shock v_t . It is shown in Yu (2004) that when $\rho < 0$, the model implies a negative relationship between $E(h_{t+1} | \epsilon_t)$ and ϵ_t and hence captures the leverage effect.

Another popular asymmetric SV model is the square root SV model proposed by Heston (1993) which takes the form of, in discrete time,¹

$$\sigma_{t+1}^2 = \alpha + \phi\sigma_t^2 + \sigma_v\sigma_tv_t. \quad (2.9)$$

In a more recent study, Jones (2003) extends the square root specification to a constant elastic variance (CEV) specification with the following discrete time representation,

$$\sigma_{t+1}^2 = \alpha + \phi\sigma_t^2 + \sigma_v\sigma_t^\gamma v_t. \quad (2.10)$$

In both Heston's and CEV SV models, ϵ_t and v_t are iid $N(0, 1)$ and $\text{corr}(\epsilon_t, v_t) = \rho$.

In a recent paper, Jacquier et al (2004) consider an asymmetric SV model similar to the one defined in (2.8), but with an important difference. Instead of assuming $\text{corr}(\epsilon_t, v_t) = \rho$, Jacquier et al assume $\text{corr}(\epsilon_t, v_{t-1}) = \rho$. Yu (2004) provides both theoretical and empirical evidence that the correct timing should be $\text{corr}(\epsilon_t, v_t) = \rho$.

In spite of being a useful tool for understanding the impact of return news on volatility, the NIF of Engle and Ng (1993) is not directly applicable to SV models. This is because in SV models there are two sources of shocks and hence both $f(\epsilon_t)$ and $g(\epsilon_t)$ defined in Equations (2.4) and (2.5) are random functions of ϵ_t . As a result, we need to generalize the NIF before applying it to SV models. To achieve this, we propose to fix information dated at t or earlier at a constant, evaluate lagged σ_{t+1}^2 or lagged h_{t+1} at the long run mean of σ_t^2 (say $\bar{\sigma}^2$) or h_t (say \bar{h}), and then define the NIF to be the relation between $E(\sigma_{t+1}^2)$ (or $E(h_{t+1})$) and ϵ_t , Mathematically, the NIF is $F(\cdot)$ or $G(\cdot)$ defined by

$$E(\sigma_{t+1}^2 | \epsilon_t, \sigma_t^2 = \bar{\sigma}^2, \sigma_{t-1}^2 = \bar{\sigma}^2, \dots) \equiv F(\epsilon_t), \quad (2.11)$$

or

$$E(h_{t+1} | \epsilon_t, h_t = \bar{h}, h_{t-1} = \bar{h}, \dots) = G(\epsilon_t). \quad (2.12)$$

That is, instead of examining the relationship between future volatility and return shocks, we consider the relationship between the expected future volatility and return shocks. The NIF defined above is indeed a generalization of the one introduced in Engle and Ng. This is

¹Strictly speaking, the model presented here is the Euler approximation to Heston's model. It is well known that the Lipschitz condition is violated for Heston's model and hence the convergence of the discrete time model to the continuous time model may not be warranted.

obvious because in ARCH-type models σ_{t+1}^2 is a deterministic function and hence conditional expectation of σ_{t+1}^2 is the same as σ_{t+1}^2 .

To derive the NIF in the classical asymmetric LN-SV model, we follow Yu (2004) by rewriting the model in a Gaussian nonlinear state space form with uncorrelated measurement and transition equation errors

$$\begin{cases} y_t &= \sigma_t \epsilon_t = \exp(h_t/2) \epsilon_t, \\ h_{t+1} &= \alpha + \phi h_t + \rho \sigma_v \epsilon_t + \sigma_v \sqrt{1 - \rho^2} w_t, \end{cases} \quad (2.13)$$

where $w_t \equiv (v_t - \rho \epsilon_t) / \sqrt{1 - \rho^2}$ and hence is iid $N(0, 1)$, and $\text{corr}(\epsilon_t, w_t) = 0$. Then it can be shown that

$$G(\epsilon_t) = \alpha + \phi \bar{h} + \rho \sigma_v \epsilon_t. \quad (2.14)$$

This is a linear function in ϵ_t with a slope of $\rho \sigma_v$ (denoted here by λ). As σ_v is always positive, the monotonicity of the NIF is entirely determined by the sign of ρ . When $\rho < 0$, which is typically the case in practice, a piece of bad news to return increases the expected value of future volatility and the reverse is true when news is good, consistent with the prediction of leverage effect.

The same conclusion applies to the other two asymmetric SV models reviewed above. For example, the NIFs for Heston's model and CEV-SV model are, respectively,

$$F(\epsilon_t) = \alpha + \phi \bar{\sigma}^2 + \rho \sigma_v \bar{\sigma} \epsilon_t, \quad (2.15)$$

and

$$F(\epsilon_t) = \alpha + \phi \bar{\sigma}^2 + \rho \sigma_v \bar{\sigma}^\gamma \epsilon_t. \quad (2.16)$$

In these two models the NIF has a slope of $\rho \sigma_v \bar{\sigma}$ and $\rho \sigma_v \bar{\sigma}^{\gamma-1}$, and hence slopes down when $\rho < 0$.

Table 1 summarizes the properties of NIF for various existing asymmetric ARCH and SV models. By comparing the V-shaped or U-shaped asymmetry in ARCH-type models with the downward-sloping asymmetry in SV models, several results emerge. First, both classes make similar predictions when a negative return shock arrives, that is, the future expected volatility tends to increase in response to bad news. Second, the two classes of models make different predictions when good news arrives. In particular, asymmetric ARCH models predict that future expected volatility should increase whereas the existing asymmetric SV models predict

movement in the opposite direction. Third, the risk-return relation holds for every single change with ARCH models, but holds only on average with SV models. This is an important difference and has an implication of a possible misspecification in ARCH models, which we examine below.

It is well known that the ARCH effect becomes insignificant in the context of the basic SV models; see for example, Fridman and Harris (1998) and Danielsson (1994). In light of this evidence, it would be interesting to examine possible misspecification in asymmetric ARCH models. To do so, we simulate 1,000 samples, each with 2,000 observations, from the classical asymmetric LN-SV model with $\alpha = 0$, $\phi = 0.95$, $\sigma_v = 0.1$ and $\rho = -0.3, -0.4$ so that the true data generating process implies a monotonically decreasing NIF. All the parameter values are chosen to be empirically reasonable. For each simulated sequence we fit EGARCH using maximum likelihood. Table 2 shows the proportions that the estimated EGARCH model has a V-shaped NIF, and that V-shape is statistically significant. These two Monte Carlo experiments clearly show that it is very likely that EGARCH would lead to a spurious V-shaped NIF. Similar results occur when GJR-GARCH is estimated. The spuriousness in this experiment is due to the restriction in ARCH-type models where volatility is specified to be a deterministic function of the return shock.

3 A Unified Approach

The distinction in the NIF between the existing asymmetric ARCH and SV models and the concern of possible misspecification in asymmetric ARCH-type models point to the need to develop a unified framework to study the asymmetric response of volatility. On the one hand, as the classical asymmetric SV models have less flexible NIFs than asymmetric ARCH models, we retain the ARCH term in the general model. On the other hand, since ARCH-type models may be misspecified, the general model should allow for the possibility of two error processes. These motivations lead us to introduce the following general model,

$$\begin{cases} y_t &= \sigma_t \epsilon_t = \exp(h_t/2) \epsilon_t, \\ h_{t+1} &= \alpha + \phi h_t + \psi |y_t| + \sigma_v v_t, \end{cases} \quad (3.17)$$

where v_t is iid $N(0, 1)$ and $\text{corr}(\epsilon_t, v_t) = \rho$.²

²Alternatively, we can replace $|y_t|$ with $|\epsilon_t|$ in the volatility equation. Unfortunately, we could not get convergence for the MCMC output in the alternative model.

The new model can be equivalently rewritten as

$$h_{t+1} = \alpha + \phi h_t + \psi |y_t| + \rho \sigma_v \epsilon_t + \sigma_v \sqrt{1 - \rho^2} w_t. \quad (3.18)$$

where w_t is iid $N(0, 1)$ and $\text{corr}(\epsilon_t, w_t) = 0$. When $\psi = 0$, it becomes the classical asymmetric LN-SV model and hence has a monotonic NIF. When $\rho = \pm 1$, it becomes an asymmetric ARCH model. This is not surprising because as $\rho = \pm 1$, the two error processes are perfectly related and hence there is essentially one source of shocks. When $\sigma_v = 0$, it becomes a symmetric, although non-standard, ARCH model. When $\rho = 0$, it is the basic SV model augmented by an ARCH term and has a symmetric NIF. Therefore, the proposed general model nests the asymmetric LN-SV model, an asymmetric ARCH model, a symmetric SV model and a symmetric ARCH model. Furthermore, it can be shown that the NIF for the model is

$$G(\epsilon_t) = \begin{cases} \alpha + \phi \bar{h} + (\rho \sigma_v + \psi \bar{\sigma}) \epsilon_t, & \text{if } \epsilon_t \geq 0 \\ \alpha + \phi \bar{h} + (\rho \sigma_v - \psi \bar{\sigma}) \epsilon_t, & \text{if } \epsilon_t < 0. \end{cases} \quad (3.19)$$

Unless $\rho \sigma_v = 0$, i.e., $\lambda = 0$, the NIF is asymmetric. Provided $\rho < 0$, if $\psi \bar{\sigma} > -\rho \sigma_v$, the NIF is asymmetrically V-shaped; if $\rho \sigma_v < \psi \bar{\sigma} < -\rho \sigma_v$, the NIF is monotonically decreasing;³ if $\psi < 0$, a positive return shock will have a larger impact on future volatility than a negative return shock of the same size.

The proposed model is closely related to the asymmetric SV model recently proposed by Asai and McAleer (2004), where the specification of the volatility equation is given by

$$h_{t+1} = \alpha + \phi h_t + \gamma I(y_t < 0) + \rho \sigma_v \epsilon_t + \sigma_v \sqrt{1 - \rho^2} w_t. \quad (3.20)$$

The NIF for this model is

$$G(\epsilon_t) = \begin{cases} \alpha + \phi \bar{h} + \rho \sigma_v \epsilon_t, & \text{if } \epsilon_t \geq 0 \\ \alpha + \gamma + \phi \bar{h} + \rho \sigma_v \epsilon_t, & \text{if } \epsilon_t < 0 \end{cases}$$

Provided $\rho < 0$, if $\gamma \neq 0$, the NIF is a combination of two linear functions with the same slope but different intercepts. As a result, the asymmetric response is also allowed. However, the discontinuous property in the NIF is not desirable. Another difference between the present

³It should be stressed that, compared with the existing asymmetric ARCH models, the proposed model has additional flexibility in terms of the NIF. This is because the shape of the NIF depends on the value of $\bar{\sigma}$ in the proposed model but not in the existing ARCH models.

paper and Asai and McAleer (2004) is that we use MCMC to estimate the model which allows us to go beyond the estimation problem to compare non-nested models while Asai and McAleer (2004) only estimate the model using the Monte Carlo likelihood method. Furthermore, we motivate our specification in terms of NIF while no motivation is provided in Asai and McAleer (2004).

Equation (3.17) also looks similar to Equation (2) in Wu and Xiao (2002), where the implied volatility is assumed to follow a partial linear model. However, there are two important differences between these two models. First, while volatility is treated as an unobserved variable and hence will be learned from returns in our model, it is approximated by the implied volatility in Wu and Xiao (2002). Second, our model is in essence a SV model whereas the model of Wu and Xiao is an ARCH-type model with the error term being interpreted as the estimation error from implied volatility.

4 Empirical Results from Daily Index Returns

To compare and demonstrate the empirical asymmetric properties of ARCH-type and classical SV models with the newly proposed model, we apply three models, namely EGARCH, asymmetric LN-SV, and the newly proposed model, to a daily return series of the S&P500 index.

4.1 Methods for Estimation

It is well-known that SV models are more difficult to estimate than ARCH models since evaluation of the likelihood function is non-trivial for SV models. Parameter estimation for SV models have been done, in recent years, by the simulated or numerical maximum likelihood (ML) methods or by Bayesian MCMC methods, which are supposed to provide full likelihood-based inference. Examples of ML methods include Lisenfeld and Richard (2003), Sandmann and Koopman (1999), and Fridman and Harris (1998). Examples of MCMC methods include Jacquier, Polson and Rossi (1994) and Kim, Shephard and Chib (1998). In this paper we use a Bayesian MCMC method to estimate the three models. There are at least two reasons why we use MCMC. First, we have found that for the basic SV model MCMC provides almost identical estimates to the simulated ML method of Lisenfeld and Richard (2003). This is not surprising as both methods are full-likelihood based. With flat priors, the Bayesian estimates should be

the same as the ML estimates. Second, as a Bayesian method, MCMC provides direct and easy model comparison of non-nested competing models via Bayes factors and devian information criterion (DIC).

The idea behind MCMC is based on the construction of an irreducible and aperiodic Markov chain with realizations $\theta^1, \theta^2, \dots$ in the parameter space, equilibrium distribution $\pi(\theta|y)$, and transition probability $\pi(\theta^{t+1}|\theta^t)$. Under regularity conditions, θ^t tends in distribution to a random quantity with density $\pi(\theta|y)$ as $t \rightarrow \infty$. As a result, when the chain converges, the chain of simulated values is regarded as a correlated sample obtained from the joint posterior and hence can be used for statistical inferences.

To estimate EGARCH via MCMC, we employ the Metropolis-Hasting algorithm used in Vrontos, Dellaportas and Politis (2000) with the proposal distribution being multivariate normal. The prior distributions are: $\alpha \sim N(0, 25)$; $\phi \sim U(-1, 1)$; $\psi \sim N(0, 25)$; $\beta \sim N(0, 25)$. None of these priors is informative.

To estimate the asymmetric LN-SV and newly proposed models, we need to augment the parameter vector from θ to $(\theta, h_1, \dots, h_T)$ for otherwise the computation of likelihood requires the latent variables to be integrated out, which is numerically time consuming. Meyer and Yu (2000) and Lancaster (2004) have shown that MCMC estimation of SV models can be easily done using the all purpose Bayesian software package BUGS. In this paper we use BUGS to estimate the two models based on the representations in (2.13) and (3.18). Following Kim, Shephard and Chib (1998) and Meyer and Yu (2000), we choose the following prior distributions: $\sigma_v^2 \sim \text{Inverse-Gamma}(2.5, 0.025)$ which has a mean of 0.167 and a standard deviation of 0.024; $\phi^* \sim \text{Beta}(20, 1.5)$ which has a mean of 0.93 and a standard deviation of 0.055, where $\phi^* = (\phi + 1)/2$; $\mu \sim N(0, 25)$, where $\mu = \alpha/(1 - \phi)$; $\rho \sim U(-1, 1)$; $\psi \sim N(0, 25)$.

In all cases we choose a burn-in period of 10,000 iterations and a follow-up period of 100,000. For EGARCH, the MCMC sampler is initialized at the ML estimates. For the LN-SV and newly proposed models, the MCMC sampler is initialized by setting $\alpha = 0$, $\phi = 0.95$, $\sigma_v^2 = 0.025$, $\rho = -0.4$, and $\psi = 0$. For all the samples we check convergence using the Heidelberger and Welch convergence test.

4.2 Methods for Model Comparison

The first and perhaps the easiest method of comparing the proposed model with the encompassed SV and ARCH models is to examine the significance of relevant estimated coefficients in the proposed model.

The second method for model comparison is to calculate the posterior model probability which allows for evaluation of posterior odd. Specifically, we calculate the Bayes factors using the marginal likelihood approach of Chib (1995) and Chib and Jeliazhov (2001). Let θ be the parameters in a competing model (excluding the latent variables). Define $m(y)$, $f(y|\theta)$, $\pi(\theta|y)$, $\pi(z)$ as the marginal likelihood, likelihood, posterior distribution, and prior distribution, respectively. Bayes' theorem implies that

$$\ln L = \ln m(y) = \ln f(y|\theta) + \ln \pi(\theta) - \ln \pi(\theta|y). \quad (4.21)$$

This expression can be evaluated at the posterior means, say $\bar{\theta}$. While the calculation of $\ln \pi(\bar{\theta})$ is trivial and an approximation to $\ln \pi(\bar{\theta}|y)$ can be obtained by using a multivariate kernel density estimate, the difficult part in the calculation of $\ln L$ lies in the evaluation of $\ln f(y|\theta)$ for the LN-SV and newly proposed models. This is because $\ln f(y|\theta)$ has no analytical form for these two models as it is marginalized over the latent states $\{h_t\}$. In this paper we employ the particle filter algorithm of Kitigawa (1996) to calculate $\ln f(y|\theta)$. This method is applicable to a broad class of nonlinear non-Gaussian state space models with uncorrelated measurement and transition errors and has been successfully applied to a variety of SV models by Berg, Meyer and Yu (2004).

The third method for model comparison is DIC developed by Spiegelhalter, Best, Carlin and van der Linde (2002). While the Bayes factor addresses how well the prior predicts the observed data, DIC addresses how well the posterior might predict future data generated by the same mechanism that gives rise to the observed data. Assume, in general, that the distribution of the data, $y = (y_1, \dots, y_T)$, depends on a parameter vector θ . (In the context of SV models, θ includes the vector of log-volatilities (h_1, \dots, h_T)). Similar to AIC and BIC, DIC consists of two compromising components,

$$\text{DIC} = \bar{D} + p_D. \quad (4.22)$$

The first component is a Bayesian measure of model fit

$$\bar{D} = -2E_{\theta|y}[\ln f(y|\theta)]. \quad (4.23)$$

The second component is a penalty which measures the complexity of the model and defined by

$$p_D = -2E_{\theta|y}[\ln f(y | \theta)] + 2 \ln f(y|\bar{\theta}). \quad (4.24)$$

From the definition it can be seen that DIC is almost trivial to compute and particularly suited for comparing Bayesian models when posterior distributions have been obtained using MCMC simulation. This is in sharp contrast to Chib's marginal likelihood method. Moreover, for complex hierarchical models, such as the SV models, the number of unknown parameter exceeds the number of observations and hence the number of free parameters is not well defined. As a result, AIC and BIC are not applicable. However, DIC is always well defined and applicable to both hierarchical and non-hierarchical models. Berg et al. (2004) compared the performances of Chib's method and DIC and found that both methods are effective for comparing SV models.

Given the marginal likelihood estimate and DIC value for each model, we can rank all the competing models, as explained in Berg et al. (2004). Moreover, we can obtain Bayes factor for pairs of competing models using Jeffrey's rule.

4.3 Data

The daily data used are 1,892 daily returns of S&P500 from January 4, 1993 to June 30, 2000 and plotted in the first panel of Fig 1.⁴ This sample period is chosen because we have high frequency intraday data over the same sample period. The index data are used because it is known that asymmetry is more pronounced in indices (ABDE, 2001).⁵

Estimation results, including the posterior means, standard deviations and 95% Bayes credible intervals are reported in Table 3. Several conclusions can be drawn when the significance of certain coefficients is examined. First, EGARCH implies a strong asymmetrically V-shaped NIF. On the other hand, the LN-SV model implies a significantly downward sloping NIF, with the 95% credible interval of ρ being $(-0.6622, -0.362)$. This inconsistency is re-examined in the proposed general model, which suggests that there is little evidence against the hypothesis of $\psi = 0$, featured by a nearly zero posterior mean and a 95% credible interval which contains

⁴Returns on June 27, 1995 and December 18, 1998 have been deleted because the high frequency cash index data on these two days are missing. A close look at the two daily returns suggests that they are too small to make important contributions to empirical results.

⁵It worths pointing out that we have experimented with other index data, such as daily S&P500 returns over a different sample period and daily CRSP returns. The empirical results are all very similar to what is reported here.

zero. The result supports the classical asymmetric LN-SV specification. Indeed, the two sets of estimates of common parameters from the LN-SV and the newly proposed models are very close to each other. Furthermore, we cannot find evidence to support either $\rho = \pm 1$ or $\sigma_v = 0$ in the proposed model, which indicates the importance of stochastic volatility over the ARCH term, in line with Fridman and Harris (1998) and Danielsson (1994). Moreover, the 95% posterior credibility interval for ρ in the proposed model is $(-0.6617, -0.3805)$ which indicates the presence of a significant negative correlation between the return shocks and volatility shocks. It is useful to point out that MCMC allows for a straightforward estimation and inference of λ ($= \rho\sigma_v$) without resorting to the delta method. Although the posterior means of λ are close to zero in both models, the 95% credible intervals do not include zero and hence indicate strong evidence of asymmetry.

Also reported in Table 3 are the log-marginal likelihood and DIC values. Both criteria rank the classical LN-SV model ahead of the newly proposed model, followed by EGARCH. Moreover, using the log marginal likelihood values we obtain the Bayes factor of the classical LN-SV over the EGARCH model which is 6.1×10^{10} . This indicates decisive evidence in favor of the LN-SV model against EGARCH. When we compare the classical LN-SV with the proposed model, we find no support for the latter. On the contrary, a value of 12.06 for the Bayes factor indicates that there is strong evidence in favor of the classical LN-SV model. All these results are consistent with the finding of an insignificant ARCH term in the new model and our conjecture that EGARCH may be misspecified.

Fig. 2 plots the three estimated NIFs. It is seen that NIFs of the LN-SV and newly proposed models almost coincide and slope downward, albeit quite flatly. This compares with the NIF of EGARCH which slopes downward sharply to the left of the origin and slopes upward less sharply to the right of the origin. Relative to the other two models, EGARCH tends to overstate h_t for extreme ϵ_{t-1} , but understate h_t for less extreme ϵ_{t-1} .

5 Empirical Results from Intraday Index Returns

Over the last several years realized daily volatility has become a popular empirical measure of daily volatility. Under mild conditions ABDL (2001) show that the realized daily volatility yields a perfect estimate of daily volatility when prices are observed in continuous time and there is no measurement error. However, it is also well known that noise exists due to the imper-

fections of the trading processes. These two results motivate ABDE (2001) and ABDL (2001) to calculate the realized daily volatility using returns over artificially constructed five-minute sampling intervals, a frequency that was argued to be high enough to have effective continuous time record while at the same time being low enough to mitigate the market microstructure frictions. The use of realized volatility enables ABDE and ABDL to treat daily volatility as directly observable without the need of fitting parametric ARCH or SV models.

This model-free approach has recently prompted more and more research on using realized volatility to study the properties of daily volatility in different markets over different time periods. For example, using realized volatility ABDE (2001) obtained the NIF non-parametrically by plotting $\ln \sigma_{rv,t}$ against $y_{t-1}/\sigma_{rv,t-1}$ based on high frequency intraday returns on individual stocks, where $\sigma_{rv,t}$ is the realized volatility on day t and hence $y_{t-1}/\sigma_{rv,t-1}$ is the lagged standardized return shock. They found that the NIF for typical stocks in the Dow Jones Industrial Average is asymmetrically V-shaped, but very flat to both sides of the origin. This observation leads them to conclude that the economic importance of asymmetric effect is marginal. Ebens (1999) found the similar V-shape asymmetry in intraday Dow Jones Industrial Average index.

In this section we obtain the NIF non-parametrically using the high frequency intraday returns on S&P500 cash index over the period from January 4, 1993 to June 30, 2000. The data are provided by Chicago Merchantile Exchange (CME) and pre-processed by Qianqiu Liu.⁶ Although our daily record covers 9:30 EST to 16:00 EST, the data used are from 10:00 EST to 16:00 EST in order to avoid extra high volatility in the beginning of the day, leaving a total of 72 five-minute returns in each trading day. The five-minute returns are constructed as the logarithmic difference between the cash index levels at each five-minute interval and the overnight returns are done at each overnight interval.⁷ The realized daily volatility is constructed from the filtered 72 five-minute returns within a day. The second and third panels of Fig. 1 plot daily returns obtained from the 72 five-minute returns within a day and realized daily volatility. The main results found in this section hold when the overnight returns and the returns in the first thirty minutes of the day are also used to calculate the realized daily volatility.

⁶We are grateful to Qianqiu Liu for providing the filtered data and CME for allowing us to use the data. We refer to Liu (2004) and ABDE (2001) for details regarding how to filter the intraday returns.

⁷If anything, one would expect that aggregation should decrease market microstructural effects. As a result, using five-minute returns should be more conservative for the index than for individual stocks.

Fig. 3 and 4 display the scatter plot for $\ln \sigma_{rv,t}$ against $y_{t-1}/\sigma_{rv,t-1}$. In Fig. 3 a nonparametric regression curve for the NIF is fitted while in Fig. 4 a regression line for the NIF is fitted. Comparison of these two graphs with Fig. 10 in ABDE (2001) reveals an important difference. In ABDE, the NIF is asymmetrically V-shaped when two regression lines are fitted, consistent with the implication of asymmetric ARCH models. In Fig. 3 and 4, we find new empirical evidence in the high frequency index data – NIFs monotonically decrease. The findings are consistent with the classical asymmetric SV model and the proposed model. Furthermore, there is an important similarity between Fig. 3 and 4 and Fig. 10 in ABDE, that is, the NIFs are all very flat. Because of this feature, ABDE argue that the economic importance of the asymmetric effect is marginal. However, our interpretation of economic significance is different. We know that in the context of LN-SV models, it is the correlation between two shocks (ρ) but not the slope of NIF (λ) that determines the economic importance of asymmetry. As σ_v always takes a small, positive value and ρ is bounded between -1 and 1, being a product of σ_v and ρ , λ inevitably takes a value close to zero. To examine economic significance, therefore, we have to investigate the sensitivity of financial variables with respect to ρ . In the next section, we will provide the economic implications of asymmetry.

6 The Economic Significance of Asymmetric Volatility

One important application of volatility models lies in their use for pricing options. In this section we investigate whether volatility asymmetry in the preferred model is economically important. For the asymmetric LN-SV model, however, option prices have no closed form formula and hence need to be approximated. A flexible way for approximating option prices is via Monte Carlo simulations (Hull and White 1987).

Let C be the value of a European call option on a stock with maturity τ (or T if measured in number of days), strike price X , current volatility σ_0^2 , current stock price S_0 , and interest rate r . The Monte Carlo algorithm for calculating the value of a European call option may be summarized as follows:

1. Obtain the initial value of h_0 based on the initial value of σ_0^2 ;
2. Draw a bivariate normal variable, denoted by ϵ_t and v_t , with 0 means, unit variances and correlation coefficient ρ ;

3. Generate h_t and hence $\sigma_t = \exp(h_t/2)$ according to

$$h_t = \alpha + \phi h_{t-1} + \sigma_v v_t, \text{ for } t = 1, \dots, T;$$

4. Generate S_t according to

$$S_t = S_{t-1} \exp\{r - \sigma_{t-1}^2/2 + \sigma_{t-1}\epsilon_t\}, \text{ for } t = 1, \dots, T;$$

5. Calculate $e^{-rT} \max\{S_T - X, 0\}$ and call it p_1 ;

6. Repeat Steps 3-5 using $\{-\epsilon_t, -v_t\}$ and define $e^{-rT} \max\{S_T - X, 0\}$ by p_2 ;

7. Calculate the average value of p_1 and p_2 and call it c ;

8. Repeat Steps 2-7 for K times and hence obtain a sequence of c 's;

9. Calculate the mean of c 's and this is the estimate of the option price C .

The algorithm is then applied to price a half-year call option based on the classical asymmetric LN-SV model with $K = 10,000$. The parameters used are $S_0 = 10$, $r = 0.1$, $\sigma_0 = 0.3$, $\alpha = -0.033$, $\phi = 0.986$, $\sigma_v = 0.15$, and $S_0/(Xe^{-rT})$ takes each of the following values, 0.9, 0.95, 1, 1.05, 1.1. Moreover, $\rho = 0, -0.1, -0.3, -0.5, -0.7$. These parameters are chosen to be empirically relevant.

Table 4 compares the differences, in percentage term, of the prices between the symmetric and asymmetric LN-SV models. Several results emerge from this table. First, in all cases the symmetric model tends to overprice options. The more the asymmetry, the larger the bias. The intuition is that the negative correlation between the return shocks and volatility shocks makes the terminal stock price distribution more peaked. Second, the bias is more serious in-the-money than out-of-the-money, and can be as large as -20% in-the-money when ρ takes an empirically reasonable value. Since in-the-money options where the strike price is about 10% of the spot price are traded frequently on exchanges, our results suggest that the bias cannot be ignored and hence asymmetry in SV has important practical implications.

7 Conclusion

In this paper we examine the NIF in the context of SV models using daily index data on S&P500 and non-parametrically using realized daily volatility based on the high frequency data on the same index. As the NIF of Engle and Ng (1993) is not well defined for SV models, we generalize it so that it is applicable to both ARCH and SV models. Comparison of the NIFs of existing asymmetric ARCH-type and SV models shows that two classes of models make different prediction about how volatility responds to good news.

Model comparisons of EGARCH, asymmetric LN-SV, and a new model which nests both the ARCH effect and stochastic volatility, reveal that the classical asymmetric LN-SV model fits the daily data the best. Both the asymmetric LN-SV and the newly proposed models suggest a monotonically decreasing NIF, while EGARCH suggests a spurious V-shaped NIF. This result is confirmed by the realized volatility calculated from the high frequency index data and suggests that it is better to study volatility asymmetry by examining the average value of volatility and return shocks. The economic significance of asymmetry on option pricing is examined and results suggest that the degree of asymmetry obtained in real data can lead to substantially different prices, which is important in practical applications.

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Table 1: Typical Parameter Values and the NIF for Asymmetric ARCH and SV Models

Models	EGARCH	GJR-GARCH	LN SV	Heston's SV	CEV-SV
Typical Parameter Values	$\beta < 0$ $\psi + \beta > 0$	$\beta, \beta^* > 0$	$\rho < 0$	$\rho < 0$	$\rho < 0$
Shape of NIF	Asymmetric-V for $g(\cdot)$	Asymmetric-U for $f(\cdot)$	Slope down for $G(\cdot)$	Slope down for $F(\cdot)$	Slope down for $F(\cdot)$

Table 2: Estimated Shaped for NIF When EGARCH is Fitted to Data Simulated from LN-SV

ρ	-0.3	-0.4
Proportion of V-shaped NIF	98.5%	99.7%
Proportion of significant V-shaped NIF	66.6%	73.8%

Table 3: Estimation using daily data

		EGARCH	LN-SV	New Model
α	Post Mean	-0.1192	-0.01355	-0.02423
	Post SD	(0.0196)	(0.0052)	(0.0144)
	95% Cred Int	[-.158,-.079]	[-.0248,-.0047]	[-.058,-.0016]
ϕ	Post Mean	0.9809	.9780	.9731
	Post SD	(0.004)	(.0069)	(.0088)
	95% Cred Int	[.967,.988]	[.9632,.990]	[.9539,.9882]
ψ	Post Mean	0.1476		0.01322
	Post SD	(0.029)		(0.01684)
	95% Cred Int	[.088,.201]		[-.0152,.0518]
β	Post Mean	-0.094		
	Post SD	(0.018)		
	95% Cred Int	[-.129,-.060]		
σ_v	Post Mean		0.1638	0.1644
	Post SD		(.02408)	(.02174)
	95% Cred Int		[.1185,.2113]	[.1463,.2457]
ρ	Post Mean		-0.5250	-0.5258
	Post SD		(0.07684)	(0.07262)
	95% Cred Int		[-.6622,-.3620]	[-.6617,-.3805]
λ (= $\rho\sigma_v$)	Post Mean		-0.1018	-0.1026
	Post SD		(0.02122)	(0.02104)
	95% Cred Int		[-.143,-.061]	[-.146,-.0637]
Log Marg Likelihood		-2333.19	-2306.05	-2308.54
DIC		4703.85	4501.04	4508.18

Table 4: Difference in Percentage of Call Option Prices Based on symmetric SV and asymmetric SV Models; Option Parameters: $\tau = 0.5$ years, $S_0 = 10$, $r = 0.1$, $\sigma_0 = 0.3$, $\alpha = -0.033$, $\phi = 0.986$, $\sigma_v = 0.15$

$\frac{S_0}{Xe^{-r\tau}}$	$\rho = -0.1$	$\rho = -0.3$	$\rho = -0.5$	$\rho = -0.7$	$\rho = -0.9$
0.9	-3.104	-9.477	-16.141	-23.239	-30.990
0.95	-1.721	-5.326	-9.182	-13.395	-18.189
1	-0.900	-2.953	-5.225	-7.851	-10.884
1.05	-0.463	-1.566	-2.838	-4.303	-6.252
1.1	-0.182	-0.609	-1.265	-2.122	-3.431

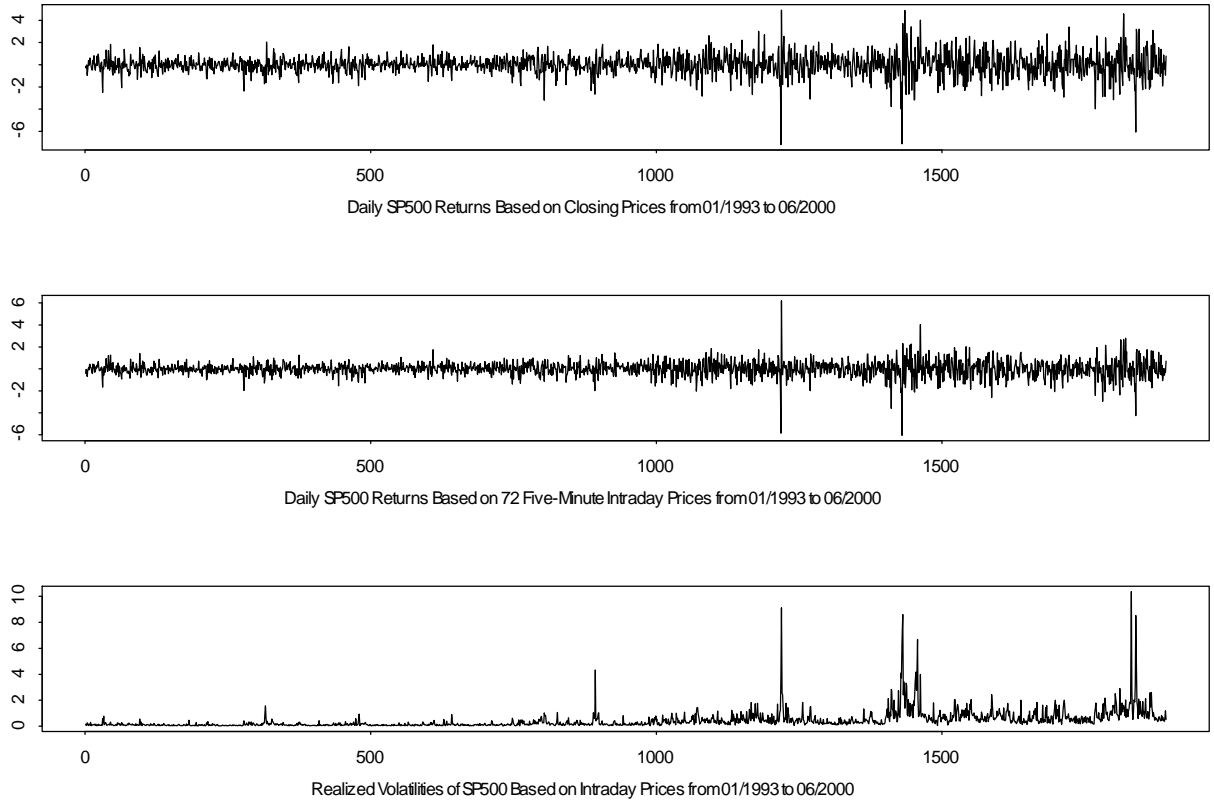


Figure 1: The first panel shows the time series plot of daily SP500 returns calculated from daily closing prices over the period from January 1993 to June 2000. The second panel shows the time series plot of daily SP500 returns calculated from 72 five-minutes intraday prices within each day over the period from January 1993 to June 2000. The third panel shows the time series plot of realized daily volatilities for SP500 cash index over the period from January 1993 to June 2000. The realized volatilities are calculated from 72 five-minute intraday returns from 10:00 EST to 16:00 EST.

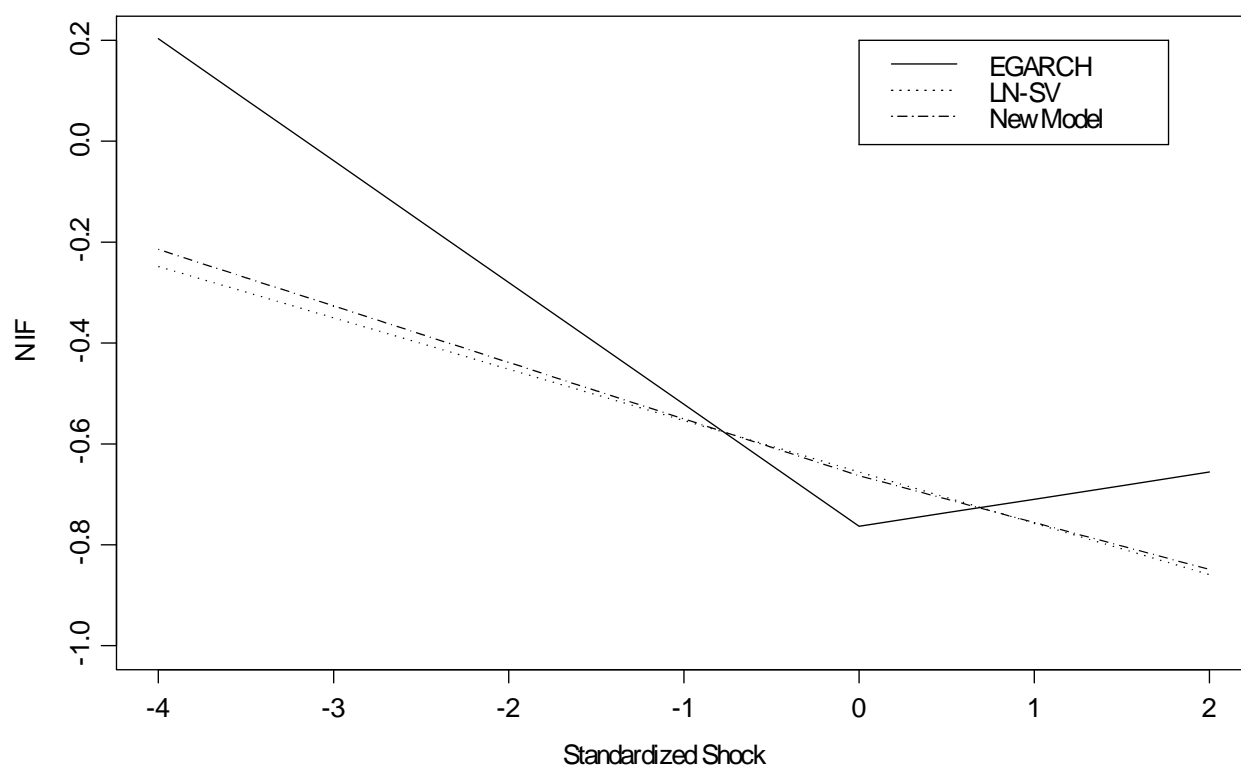


Figure 2: Estimated NIFs for EGARCH, the classical asymmetric LN-SV model and the newly proposed model.

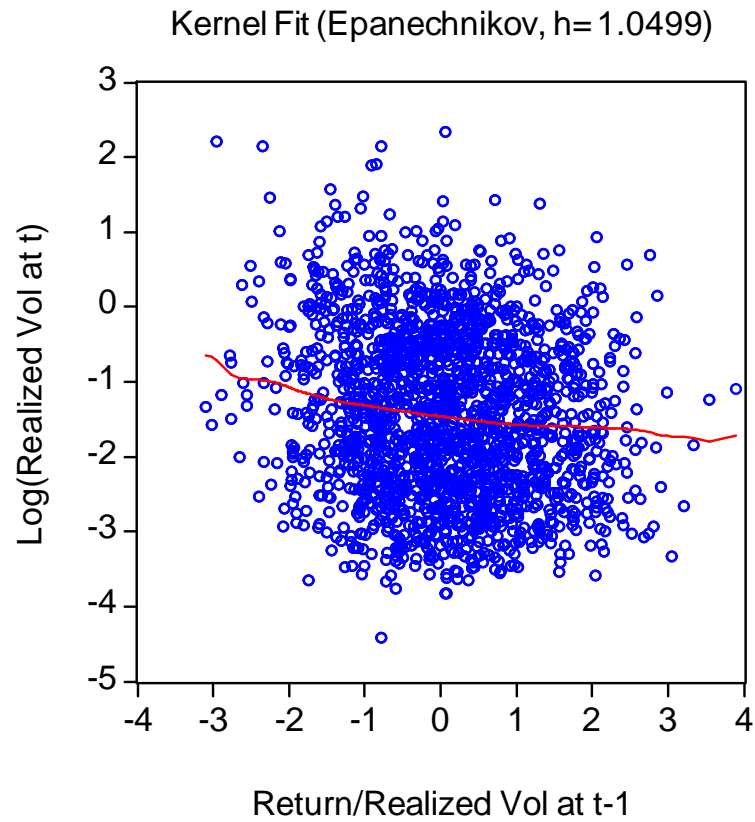


Figure 3: News impact function. The figure shows the scatter plot of the realized daily volatility against the lagged standardized return shock for SP500. The solid curve is a kernel regression smoother. The realized volatilities are calculated from 72 five-minute intraday returns from 10:00 EST to 16:00 EST.

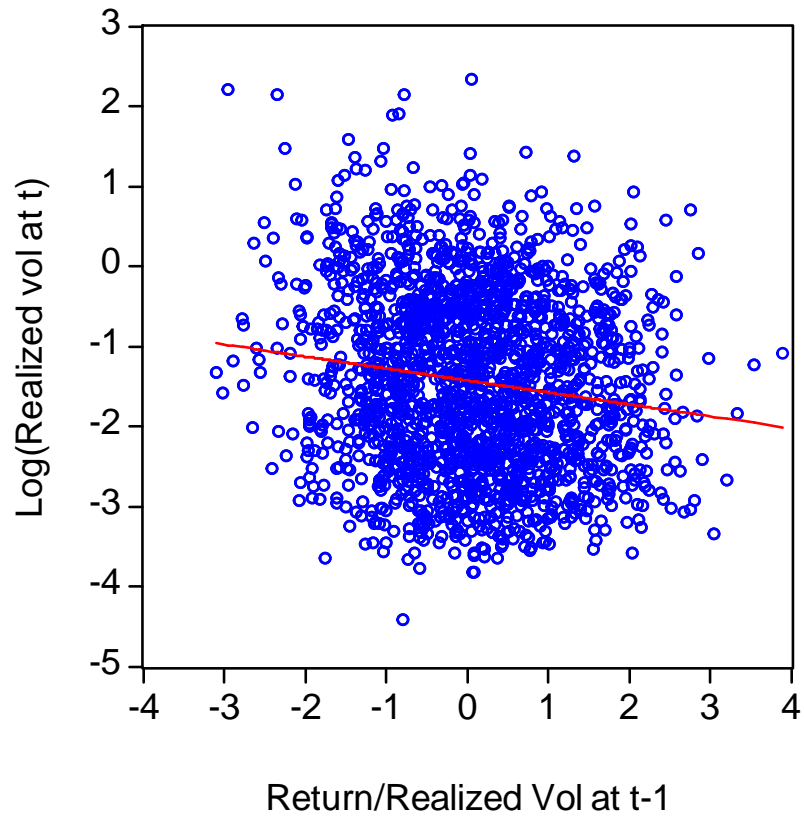


Figure 4: News impact function. The figure shows the scatter plot of the realized daily volatility against the lagged standardized return shock for SP500. The solid curve refers to the estimated regression line. The realized volatilities are calculated from 72 five-minute intraday returns from 10:00 EST to 16:00 EST.