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Abstract

In this article we propose a new multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) model with time-varying correlations. We adopt the vech representation based on the conditional variances and the conditional correlations. Whereas each conditional-variance term is assumed to follow a univariate GARCH formulation, the conditional-correlation matrix is postulated to follow an autoregressive moving average type of analog. Our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and Baba-Engle-Kraft-Kroner models. We report some Monte Carlo results on the finite-sample distributions of the maximum likelihood estimate of the varying-correlation MGARCH model. The new model is applied to some real data sets.

KEY WORDS: BEKK model, Constant correlation, Maximum likelihood estimate, Monte Carlo method, Multivariate GARCH model, Varying correlation

1. INTRODUCTION

After the success of the autoregressive conditional heteroscedasticity (ARCH) model and the generalized ARCH (GARCH) model in describing the time-varying variances of economic data in the univariate case, many researchers have extended these models to multivariate dimension. Applications of the multivariate GARCH (MGARCH) models to financial data have been numerous. For example, Bollerslev (1990) studied the changing variance structure of the exchange rate regime in the European Monetary System, assuming the correlations to be time invariant. Kroner and Claessens (1991) applied the models to calculate the optimal debt portfolio in multiple currencies. Lien and Luo (1994) evaluated the multiperiod hedge ratios of currency futures in a MGARCH framework. Karolyi (1995) examined the international transmission of stock returns and volatility, using different versions of MGARCH models. Baillie and Myers (1991) estimated the optimal hedge ratios of commodity futures and argued that these ratios are nonstationary. Gourieroux (1997, chap. 6) presented a survey of several versions of MGARCH models. See also Bollerslev et al. (1992) and Bera and Higgins (1993) for surveys on the methodology and applications of GARCH and MGARCH models.

Bollerslev et al. (1988) provided the basic framework for a MGARCH model. They extended the GARCH representation in the univariate case to the vectorized conditional-variance matrix. Their specification follows the traditional autoregressive moving average time series analog. This vech representation is very general, and it involves a large number of parameters. Empirical applications require further restrictions and simplifications. A useful member of the vech-representation family is the diagonal form. Under the diagonal form, each variance-covariance term is postulated to follow a GARCH-- type equation with the lagged variance-covariance term and the product of the corresponding lagged residuals as the rightside variables in the conditional-(co)variance equation.

It is often difficult to verify the condition that the conditional-variance matrix of an estimated MGARCH model is positive definite. Engle et al. (1984) presented the necessary conditions for the conditional-variance matrix to be positive definite for a bivariate ARCH model. Extensions of these results to more general models are, however, intractable. Furthermore, such conditions are often very difficult to impose during the optimization of the log-likelihood function. Bollerslev (1990) suggested a constant-correlation MARCH (CC--MGARCH) model that can overcome these difficulties. He pointed out that under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive definite, the optimization will not fail as long as the conditional variances are positive. In addition, when the correlation matrix is concentrated out of the log-likelihood function further simplification is achieved in the optimization.

Because of its computational simplicity, the CC-MGARCH model is widely used in empirical research. However, although the constant-correlation assumption provides a convenient MARCH model for estimation, many studies find that this assumption is not supported by some financial data. Thus, there is a need to extend the MGARCH models to incorporate time-varying correlations and yet retain the appealing feature of satisfying the positive-definite condition during the optimization.

Engle and Kroner (1995) proposed a class of MGARCH model called the BEKK (named after Baba, Engle, Kraft, and Kroner) model. The motivation is to ensure the condition of a positive-definite conditional-variance matrix in the process of optimization. Engle and Kroner provided some theoretical analysis of the BEKK model and related it to the vech-representation form. Another approach examines the conditional variance as a factor model. The works by Diebold and Nerlove (1989), Engel and Rodrigues (1989), and Engle et al. (1990) are along this line. One disadvantage of the BEKK and factor models is that the parameters cannot be easily interpreted, and their net effects on the future variances and covariances are not readily seen. Bera et al. (1997) reported that the BEKK model does not perform well in the estimation of the optimal hedge ratios. Lien et al. (2001) reported difficulties in getting convergence when they used the BEKK model to estimate the conditional-variance structure of spot and futures prices.

In this article we propose a new MGARCH model with time-varying correlations. Basically we adopt the vech representation. The variables of interest are, however, the conditional variances and conditional correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analog to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional--correlation matrix is guaranteed to be positive definite during the optimization. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-- correlation and BEKK models.

The plan of the rest of the article is as follows. In Section 2 we describe the construction of the varying-correlation MGARCH model. As in other MGARCH models, the new model can be estimated by use of the MLE method. Some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model are reported in Section 3. Section 4 describes some illustrative examples of the new model that use some real data sets. These are the exchange rate data, national stock market price data, and sectoral stock price data. The new model is compared against the CC-MGARCH model and the BEKK model. It is found that the new model compares favorably against the BEKK model. Extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. The estimated conditional-correlation path provides a time history that would be lost in a constant-correlation model. Finally, we give some concluding remarks in Section 5.

2. A VARYING-CORRELATION MGARCH MODEL

Consider a multivariate time series of observations $\{ y_t \}$, $t = 1, \ldots, T$, with K elements each, so that $y_t = (y_{1t}, \ldots, y_{Kt})'$. We assume that the observations are of zero (or known) mean. This assumption simplifies the discussions without straining the notations. Additional parameters would be required to represent the conditional-mean equation in the complete model if the mean were unknown. Under certain conditions, the MLE of the parameters in the conditional-mean equation is asymptotically uncorrelated with the MLE of the parameters of the conditional-variance equation. Under such circumstances, we may treat y_t as pre-filtered observations [see Bera and Higgins (1993) for further discussions]. Otherwise, the parameter vector has to be augmented to take account of the parameters in the unknown conditional mean.

The conditional variance of y_t is assumed to follow the time-varying structure given by

$$\mathrm{Var}\,(y_t \mid \Phi_{t\text{-}1}) = \Omega_t, \qquad (1)$$

where Φ_t is the information set at time t. We denote the variance elements of Ω_t by $\sigma^2{}_{ijt}$, for $i = 1, \ldots, K$, and the covariance elements by σ_{ijt}, where $1 \le i \le j \le K$. Denoting D_t as the K x K diagonal matrix where the ith diagonal element is σ_{it}, we let $\varepsilon_t = D_t^{-1} y_t$. Thus, ε_t is the standardized residual and is assumed to be serially independently distributed with mean zero and variance matrix $\Gamma_t = \{ \rho_{ijt} \}$. Of course, Γ_t is also the correlation matrix of y_t. Furthermore, $\Omega_t = D_t \, \Gamma_t \, D_t$.

To specify the conditional variance of y_t, we adopt the vech-diagonal formulation initiated by Bolerslev et al. (1988). Thus, each conditional-variance term follows a univariate GARCH (p, q) model given by the equation

$$\sigma_{it}^2 = \omega_i + \sum_{h=1}^{p} \alpha_{ih} \sigma_{i,\,t-h}^2 + \sum_{h=1}^{q} \beta_{ih} y_{i,\,t-h}^2, \qquad i = 1, \ldots, K, \qquad (2)$$

where ω_i, α_{ih}, and β_{ih} are nonnegative, and $\sum_{h=1}^{p} \alpha_{ih} + \sum_{h=1}^{q} \beta_{ih} < 1$, for $i = 1, \ldots, K$.

Note that we may allow (p, q) to vary with i so that (p, q) should be regarded as the generic order of the univariate GARCH process. Researchers adopting the vech-diagonal form typically assume that the above equation also applies to the conditional-covariance terms in which $\sigma^2{}_{it}$ is replaced by σ_{ijt} and $y^2{}_{it}$ replace by $y_{it}\, y_{jt}$ for $1 \le i \le j \le K$. We shall deviate from this approach, however. Specifically, we shall focus on the conditional-correlation matrix and adopt an autoregressive moving average analog on this matrix. Thus, we assume that the time varying conditional-correlation matrix Γ_t is generated from the recursion

$$\Gamma_t = (1 - \theta_1 - \theta_2)\Gamma + \theta_1 \Gamma_{t-1} + \theta_2 \Psi_{t-1} \qquad (3)$$

$\Gamma = \{ \rho_{ij} \}$ is a (time-invariant) K x K positive definite parameter matrix with unit diagonal elements and $\Psi_{t\text{-}1}$ is a K x K matrix whose elements are functions of the lagged observations of y_t. The functional form of $\Psi_{t\text{-}1}$ will be specified below. The parameters θ_1 and θ_2 are assumed to be non-negative with the additional constraint that $\theta_1 + \theta_2 \le 1$. Thus, Γ_t is a weighted average of Γ, Γ_{t-1}, and $\Psi_{t\text{-}1}$. Hence, if $\Psi_{t\text{-}1}$ and Γ_0 are well-defined correlation matrices (i.e., positive definite with unit diagonal elements), Γ_t will also be a well-defined correlation matrix.

It can be observed that $\Psi_{t\text{-}1}$ is analogous to $y^2_{i,t\text{-}1}$ in the univariate GARCH(1, 1) model. However, as Γ_t is a standardized measure, we also require $\Psi_{t\text{-}1}$ to depend on the (lagged) standardized residuals ϵ_t. Denoting $\Psi_t = \{\psi_{ijt}\}$, we propose to consider the following specification for $\Psi_{t\text{-}1}$:

$$\psi_{ij,t-1} = \frac{\sum_{h=1}^{M} \epsilon_{i,t-h}\epsilon_{j,t-h}}{\sqrt{\left(\sum_{h=1}^{M} \epsilon_{i,t-h}^2\right)\left(\sum_{h=1}^{M} \epsilon_{j,t-h}^2\right)}}, \qquad 1 \le i \le j \le K \qquad (4)$$

Thus, $\psi_{t\text{-}1}$ is the sample correlation matrix of $\{\epsilon_{t-1}, \ldots, \epsilon_{t-M}\}$. We define E_{t-1} as the K x M matrix given by $E_{t-1} = (\epsilon_{t-1}, \ldots, \epsilon_{t-M})$. If B_{t-1} is the K x K diagonal matrix where the ith diagonal element is

$(\sum_{h=1}^{M} \epsilon_{i,t-h}^2)^{1/2}$ or i = 1, … , K, we have

$$\Psi_{t-1} = B_{t-1}^{-1} E_{t-1} E'_{t-1} B_{t-1}^{-1} \qquad (5)$$

Note that when M = 1, $\Psi_{t\text{-}1}$ is identically equal to the matrix of unity. Updating the conditional-correlation matrix with respect to the matrix of unity is of course not meaningful. Thus, taking first-order lag for the formulation of $\Psi_{t\text{-}1}$ is not sufficient. Indeed, M ≥ K is a necessary condition for $\Psi_{t\text{-}1}$ to be positive definite. When positive-definiteness is satisfied, $\Psi_{t\text{-}1}$ is a well-defined correlation matrix. Thus, the condition M ≥ K will be imposed subsequently. In particular, in all of the computations reported in this article we assume M = K.

Equation (3) is analogous to the univariate GARCH equation, with the additional restriction that the sum of the coefficients is equal to 1. Indeed, Γ_t involves updating the conditional-correlation matrix with respect to the latest conditional-correlation matrix Γ_{t-1} and a sample estimate of the conditional-correlation matrix based on the recent M standardized residuals. We shall call the model specified by (2), (3), and (5) the varying-correlation MGARCH (VCMGARCH) model.

Assuming normality, $y_t \mid \Phi_{t\text{-}1} \sim N(0, D_t \Gamma_t D_t)$ so that (ignoring the constant term) the conditional log-likelihood ℓ_t of the observation y_t is given by

$$\ell_t = -\frac{1}{2}\ln|D_t\Gamma_t D_t| - \frac{1}{2} y_t' D_t^{-1}\Gamma_t^{-1}D_t^{-1}y_t \qquad (6)$$

$$= -\frac{1}{2}\ln|\Gamma_t| - \frac{1}{2}\sum_{i=1}^{K}\ln\sigma_{it}^2 - \frac{1}{2} y_t' D_t^{-1}\Gamma_t^{-1}D_t^{-1}y_t, \qquad (7)$$

from which we can obtain the log-likelihood function of the sample as $\ell = \sum_{t=1}^{T} \ell_t$. Here the log-likelihood function is conditional on Γ_0, Ψ_0, and y_0 being fixed. These assumptions have no effects on the asymptotic distribution of the MLE. Denoting $\theta = (\omega_1, \alpha_{11}, \ldots, \alpha_{1p}, \beta_{11}, \ldots, \beta_{1q}, \omega_2, \ldots, \beta_{Kq}, \rho_{12}, \ldots, \rho_{K-1,K}, \theta_1, \theta_2)$ as the parameter vector of the model, the MLE of θ is obtained by maximizing ℓ with respect to θ. We shall denote this value by $\hat{\theta}$.

For parameter parsimony, (p, q) is usually taken to be of low order. For $p = q = 1$, the total number of parameters in the VC-MGARCH model is 3K + K(K + 1) / 2 + 2. In comparison, an unrestricted BEKK(1 , 1) model has K(K +1) / 2 + $2K^2$ parameters. For example, for K = 2, 3, and 4, the number of parameters in the VC-MGARCH model is 9, 14, and 20, respectively, whereas that for the BEKK model is 11, 24, and 42, respectively. The number of parameters in the VC-MGARCH model always exceeds that of the constant-correlation model by 2, because of the parameters θ_1 and θ_2. Indeed the CC-MGARCH model is nested within the VCMGARCH model under the restrictions $\theta_1 = \theta_2 = 0$.

The conditions $0 \le \theta_1, \theta_2 \le 1$, and $\theta_1 + \theta_2 \le 1$ pose some problems in the optimization. One way to get around this difficulty is through transformation. For example, we may define $\theta_i = (\lambda_i^2/(1+\lambda_1^2+\lambda_2^2)$ for $i = 2$, where λ_1 and λ_2 are unrestricted parameters. The log-likelihood function may be initially optimized with respect to λ_1 , λ_2, and other parameters of interest. The optimization is then shifted to the original vector θ when convergence with respect to λ_1 , λ_2, and other parameters has been achieved. This technique is used in the computations reported in this article.

3. SOME MONTE CARLO RESULTS

Research on the asymptotic theory of conditional heteroscedasticity models has been lagging behind their empirical applications. Weiss (1986), Pantula (1989), Bollerslev and Wooldridge (1992), Lee and Hansen (1994), Lumsdaine (1996), and Ling and Li (1997b) investigated the asymptotic distribution of the quasi-MLE (QMLE) of the univariate ARCH/GARCH models. Sufficient conditions for consistency and asymptotic normality have been established. Recently, Ling and McAleer (2000) examined the asymptotic distribution of a class of vector ARMA-GARCH models. They established conditions for strict stationarity and ergodicity and proved the consistency and asymptotic normality of the QMLE under some mild moment conditions. Although the models considered by Ling and McAleer are quite general, the CCGARCH framework is adopted, and time-varying conditional correlation is not allowed. An extension of the results by Ling and McAleer to the VC-MGARCH model will be interesting. This, however, is beyond the scope of this article.

An interesting issue for empirical applications concerns the properties of the MLE of the conditional heteroscedasticity models in small and moderate samples. In the univariate case, Engle et al. (1985) and Lumsdaine (1995) examined the small sample properties of the MLE of the ARCH and GARCH models. In this section we report some results on the small sample properties of the MLE of the VC-MGARCH model based on a small-scale Monte Carlo experiment. It is not our intention to provide a comprehensive Monte Carlo study of the MLE. We shall focus our interest on the small-sample bias and mean squared error only. The reliability of the inference concerning the model parameters will not be examined. Our results, however, will provide some preliminary evidence with respect to the small-sample properties of the MLE of the VCMGARCH model.

We consider bivariate VC-MGARCH models in which the conditional-variance equations are given by

$$\sigma_{it}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{i,t-1}^2, \qquad i = 1, 2, \tag{8}$$

with

$$\rho_t = (1 - \theta_1 - \theta_2)\rho + \theta_1 \rho_{t-1} + \theta_2 \psi_{t-1}, \tag{9}$$

where ψ_{t-1} is specified as

$$\psi_{t-1} = \frac{\sum_{h=1}^{2} \epsilon_{1,t-h} \epsilon_{2,t-h}}{\sqrt{\left(\sum_{h=1}^{2} \epsilon_{1,t-h}^2\right)\left(\sum_{h=1}^{2} \epsilon_{2,t-h}^2\right)}}, \tag{10}$$

with $\epsilon_{it} = y_{it}/\sigma_{it}$ for $i = 1, 2$.

We consider four experimental setups. The true parameter values of the data-generating processes of these experiments, labelled E1 through E4, are given in Tables 1 and 2. Observations $\{y_t\}$, are generated from these models assuming the errors are normally distributed. We consider T = 500, 1,000, and 1,500. The MLEs are calculated for each generated sample. Using Monte Carlo samples of 1,000 runs, we estimate the bias and mean squared error (MSE) of the MLE. All calculations reported in this section and the next are coded in GAUSS.

Table 1. Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1,1) Models

	Experiment: E1				Experiment: E2			
Parameters	True value	Sample size	Bias	MSE	True value	Sample size	Bias	MSE
ω_1	.4	500	.0907	.0687	.4	500	.1166	.0993
		1,000	.0363	.0194		1,000	.0487	.0273
		1,500	.0266	.0116		1,500	.0328	.0157
α_1	.8	500	−.0135	.0033	.8	500	−.0183	.0043
		1,000	−.0056	.0012		1,000	−.0070	.0016
		1,500	−.0046	.0008		1,500	−.0050	.0010
β_1	.15	500	−.0007	.0013	.15	500	.0005	.0017
		1,000	−.0005	.0006		1,000	−.0010	.0008
		1,500	.0005	.0004		1,500	−.0004	.0005
ω_2	.2	500	.0313	.0095	.2	500	.0364	.0118
		1,000	.0132	.0031		1,000	.0123	.0040
		1,500	.0076	.0017		1,500	.0089	.0024
α_2	.7	500	−.0170	.0062	.7	500	−.0230	.0079
		1,000	−.0094	.0023		1,000	−.0075	.0031
		1,500	−.0043	.0015		1,500	−.0047	.0018
β_2	.2	500	−.0018	.0023	.2	500	.0011	.0030
		1,000	.0013	.0010		1,000	−.0003	.0013
		1,500	−.0005	.0008		1,500	−.0005	.0009
ρ	.7	500	−.0011	.0028	.2	500	−.0008	.0077
		1,000	−.0027	.0084		1,000	−.0012	.0034
		1,500	.0010	.0009		1,500	.0001	.0022
θ_1	.8	500	−.0018	.0014	.8	500	−.0358	.0181
		1,000	−.0090	.0023		1,000	−.0194	.0065
		1,500	.0011	.0004		1,500	−.0111	.0029
θ_2	.1	500	−.0006	.0008	.1	500	.0043	.0016
		1,000	−.0064	.0014		1,000	.0023	.0008
		1,500	.0005	.0003		1,500	.0011	.0004

NOTE: See Equations (8), (9), and (10) for the data-generating processes.

E1 and E2 represent models with higher volatility persistence (as measured by $\alpha_i + \beta_i$), and E3 and E4 represent models with lower volatility persistence. The selected values of ρ in the experiments are .2 and .7. It can be seen from the Monte Carlo results that the biases of the MLE are generally quite small. The bias decreases with the sample size, although in some cases not steadily. Likewise, the same is true for the MSE. Overall, for the sample sizes and models considered, the bias and MSE appear to be small. In the next section, we illustrate the application of the VC-MGARCH model with some real data sets.

4. SOME ILLUSTRATIVE EXAMPLES

We examine three sets of financial data, denoted by DSI, DS2, and DS3. DS1 consists of two exchange rate (versus U.S. dollar) series, namely, the deutsche mark (D) and the Japanese yen (J). These series represent 2,131 daily observations from January 1990 through June 1998. DS2 covers the stock market indices of the Hong Kong and the Singapore markets. We use the Hang Seng Index (H) for the Hong Kong market and the SES Index (S) for the Singapore market. There are 1,942 daily (closing) prices for each series, covering the period from January 1990 through March 1998. DS3 consists of three sectoral price indices of the Hong Kong stock market. These are the Finance (F), Properties (P), and Utilities (U) sectors. Each series includes 2,440 daily observations covering the period from October 1990 through August 2000. DS1 was downloaded from the website of the Federal Reserve Bank of New York. DS2 was compiled from various issues of the Stock Exchange of Singapore Journal. Some adjustments were made to account for the differences in the holidays of the two exchanges. DS3 was downloaded from Datastream.

Figures 1 through 3 present the plots of the seven series in the three data sets. In Figure 1 the Japanese yen (Y) series have been rescaled for easy presentation. This is similarly done for the Hang Seng Index (H) series in Figure 2. We can see that the exchange rates of the Deutsche mark and the Japanese yen generally moved in tandem against the U.S. dollar during the sample period. As expected, the three sectoral indices in the Hong Kong stock market moved quite closely together. This is especially true for the Finance and Properties Indices. In contrast, the Utilities Index was quite sluggish in the mid-1990s, whereas the Finance and Properties Indices underwent a bull run during this period. It is quite clear from Figure 2 that the national stock markets of Hong Kong and Singapore experienced different phases of bulls and bears. The general impression is that Hong Kong has a more volatile market compared with Singapore.

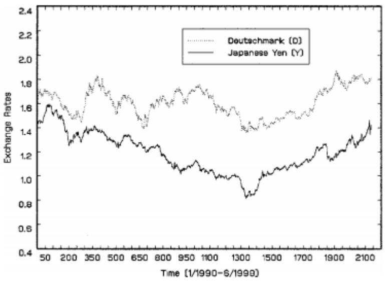

Figure 1. Exchange Rates of Deutsche Mark and Japanese Yen.

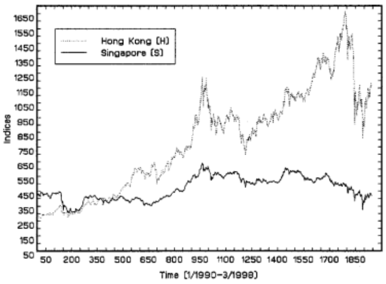

Figure 2. National Stock Market Indices.

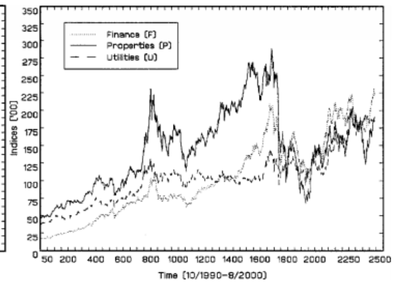

Figure 3. Hang Seng Sectoral Indices.

Table 2. Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1,1) Models

	Experiment: E3				*Experiment: E4*			
Parameters	*True value*	*Sample size*	*Bias*	*MSE*	*True value*	*Sample size*	*Bias*	*MSE*
ω_1	.4	500	.0293	.0148	.4	500	.0315	.0188
		1,000	.0137	.0062		1,000	.0114	.0088
		1,500	.0090	.0037		1,500	.0051	.0052
α_1	.5	500	−.0131	.0092	.5	500	−.0181	.0109
		1,000	−.0077	.0036		1,000	−.0067	.0051
		1,500	−.0053	.0022		1,500	−.0025	.0031
β_1	.3	500	−.0050	.0037	.3	500	−.0032	.0042
		1,000	−.0007	.0017		1,000	−.0019	.0021
		1,500	.0003	.0011		1,500	−.0022	.0015
ω_2	.2	500	.0197	.0067	.2	500	.0219	.0081
		1,000	.0089	.0026		1,000	.0109	.0032
		1,500	.0054	.0016		1,500	.0089	.0024
α_2	.5	500	−.0291	.0216	.5	500	−.0352	.0268
		1,000	−.0125	.0090		1,000	−.0188	.0110
		1,500	−.0083	.0054		1,500	−.0089	.0074
β_2	.2	500	−.0028	.0030	.2	500	−.0008	.0034
		1,000	−.0017	.0014		1,000	.0016	.0017
		1,500	.0001	.0009		1,500	.0021	.0013
ρ	.7	500	.0011	.0064	.2	500	.0002	.0139
		1,000	.0014	.0025		1,000	.0007	.0068
		1,500	−.0003	.0015		1,500	.0001	.0041
θ_1	.6	500	−.0026	.0034	.6	500	−.0137	.0055
		1,000	−.0015	.0014		1,000	−.0035	.0023
		1,500	−.0011	.0010		1,500	−.0058	.0016
θ_2	.3	500	−.0048	.0019	.3	500	.0035	.0023
		1,000	−.0019	.0009		1,000	−.0009	.0010
		1,500	−.0006	.0006		1,500	.0019	.0007

NOTE: See Equations (8), (9), and (10) for the data-generating processes.

Table 3 provides a summary of the descriptive statistics of the data. The summary statistics refer to those of the differences of the logarithmic series (expressed as a percentage). It can be seen that all differenced logarithmic series exhibit excess kurtosis (compared with the normal distribution) in the unconditional distribution. Whereas the exchange rate data (DS1) demonstrate no evidence of serial correlation, the stock return data (DS2 and DS3) show significant serial correlation, as suggested by the Q, statistics. The Q_2 statistics show that there is serial correlation in the conditional variance for all data sets, and GARCH-type modeling may be required. In the subsequent analysis, we apply MGARCH models to the data sets. Autoregressive filters are used for the conditional mean equations. Thus, the following conditional-mean equations are considered:

Table 3. Summary Statistics of the Differenced Logarithmic Series of Various Data Sets

Variable (Code)	*Mean*	*Std. dev.*	*Minimum*	*Maximum*	*Std. skewness*	*Std. kurtosis*	$Q_1(20)$	$Q_2(20)$	*No. of obs.*
A: Forex market data (DS1), 90/1–98/6									
Deutsche mark (D)	.0025	.6746	−2.8963	3.1030	.3715	16.6655	21.9957	464.2324	2121
Japanese yen (J)	−.0023	.6750	−4.5228	3.2269	−9.5384	33.4012	27.6373	112.5759	2131
B: Stock market data (DS2), 90/1–98/3									
Hong Kong (H)	.0721	1.7093	−14.7347	17.2471	−.0533	120.2852	36.6618	759.7676	1942
Singapore (S)	−.0010	1.0768	−7.7236	8.7867	−1.6643	95.7543	116.8250	846.2872	1942
C: Hang Seng sectoral indices data (DS3), 90/10–00/8									
Finance (F)	.1055	1.8196	−17.6894	18.0011	−2.7550	109.8381	57.6920	986.0076	2440
Properties (P)	.0561	2.1585	−14.2739	20.6846	5.5556	88.3062	93.1470	986.3667	2440
Utilities (U)	.0667	1.6940	−14.4889	16.6176	6.6282	92.1913	42.5415	609.1001	2440

NOTE: $Q_1(20)$ is the Box–Pierce portmanteau statistic of the differenced logarithmic series based on the autocorrelation coef' cients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared differenced logarithmic series.

$$y_{jt} = \mu_j + \sum_{i=1}^{p_j} \phi_{ji} y_{j,t-i} + \epsilon_{jt}, \quad j = 1, \ldots, K, \tag{11}$$

where $\epsilon_t | \Phi_{t-1} \sim N(0, \Omega_t)$. The types of MGARCH model we consider are the CC-MGARCH(1, 1) model, the VC-MGARCH(1, 1) model, and the BEKK(1, 1) model. The parameters of the conditional-mean and conditional-variance equations are estimated simultaneously with the use of MLE assuming normality.

We fit the CC-MGARCH(1, 1) model to the data sets, using Bolerslev's (1990) algorithm. The results are summarized in Table 4. The standard errors reported are calculated using the robust QMLE covariance matrix of the parameters. It can be seen that the estimates of α, β, and ρ are statistically significant at the 5% level for all data sets. In comparison, the exchange rate data have the highest intensity of persistence in volatility as measured by $\hat{\alpha} + \hat{\beta}$. With respect to the correlation coefficients, the returns of the national stock markets of Hong Kong and Singapore have the lowest correlation. In contrast, the correlations between the various sectoral indices of the Hong Kong stock market are the highest.

Table 5. Estimation Results of the Varying-Correlation Models: VC-MGARCH(1,1)

Variable	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ω	α	β	θ_1	θ_2	Correlations
A: Forex market data (DS1)											
D	.0013	.0513				.0055	.9376	.0504	.9705	.0158	ρ_{DJ} =.6292
	(.0136)	(.0198)				(.0030)	(.0141)	(.0098)	(.0081)	(.0046)	(.0457)
J	−.0011	.0494				.0103	.9314	.0473			
	(.0155)	(.0219)				(.0070)	(.0303)	(.0180)			
B: National Stock Market data (DS2)											
H	.1342	.0629				.1715	.8035	.1246	.9570	.0312	ρ_{HS} =.4829
	(.0282)	(.0241)				(.0723)	(.0513)	(.0301)	(.0138)	(.0099)	(.0772)
S	.0312	.1641				.0874	.7273	.1872			
	(.0168)	(.0261)				(.0267)	(.0605)	(.0471)			
C: Hang Seng sectoral indices data (DS3)											
F	.1401	.0744	.0552			.1128	.8617	.0993	.9743	.0132	ρ_{FP} =.8256
	(.0321)	(.0164)	(.0156)			(.0388)	(.0300)	(.0215)	(.0069)	(.0033)	(.0236)
P	.0885	.1338	.0407	−.0275	−.0202	.0982	.8822	.0908			ρ_{FU} =.7453
	(.0336)	(.0165)	(.0147)	(.0137)	(.0122)	(.0306)	(.0209)	(.0155)			(.0283)
U	.0617	.0732		−.0394		.1508	.8209	.1204			ρ_{PU} =.7884
	(.0290)	(.0171)		(.0150)		(.0393)	(.0284)	(.0198)			(.0258)

NOTE: The parameter estimates are the MLE assuming normality. The ' gures in parentheses are the standard errors. They are calculated using the robust QMLE covariance matrix of the parameters.

Table 5 summarizes the estimation results of the VC-MGARCH(1, 1) models for the three data sets. Again, it can be seen that the estimates of α, β, and ρ are statistically significant at the 5% level for all data sets. In addition, al estimates of θ_1 and θ_2 are statistically significant at the 5% level, indicating that the correlations are significantly time varying. We note that the intensity of the volatility persistence remains approximately unchanged compared with the CC-MGARCH models. Indeed, incorporating time-varying correlations does not have much effect on the estimates of α and β. The estimates of ρ in the varying-correlation models are al larger than the corresponding estimates of ρ in the constant-correlation models. This, however, does not imply that the correlations are on average higher in the varying-correlation model. It should be noted that the time-invariant component of the conditional correlation coefficient in the VC-MGARCH(1, 1) model is $(1 - \theta_1 - \theta_1)\rho$. A comparison of the correlations in the two models will be provided below.

We also estimate the BEKK(1, 1) model defined by the conditional-variance equation

$$\Omega_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}' \Omega_{t-1} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}' \epsilon_{t-1}\epsilon_{t-1}' \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \tag{12}$$

This equation is for the bivariate case. The trivariate model is similarly defined. Tables 6 and 7 summarize the estimation results. It is found that all off-diagonal elements of the GARCH (i.e., g_{ij}) and ARCH (i.e., a_{ij}) terms of the conditional-variance equations are insignificant. Thus, Table 7 represents the diagonal BEKK(1, 1) models.

As the CC-MGARCH(1, 1) model is nested within the VCMGARCH (1, 1) model, ignoring the extension would induce model misspecification. We now proceed to examine the model diagnostics of the estimated models. Table 8 summarizes the maximized log-likelihood value 4LF5 and a battery of diagnostic tests for the fitted models. The constant-correlation assumption is tested using a Lagrange multiplier test (LMC) based on the estimates of the CC-MGARCH(1, 1) model and the likelihood ratio (*LR*) test based on the estimates of the VC-MGARCH(1, 1) model. LMC is the Lagrange multiplier test suggested by Tse (2000) for the assumption of (joint) constant correlation in a MGARCH model. It is asymptotically distributed as χ^2_R, where R = K (K - 1)/2, under the null. From part A of Table 8 we can see that the constant-correlation assumption is rejected for all data sets at the 5% level of significance. In part B of Table 8 we present the likelihood ratio statistic LR, which tests for the restriction $H_0 : \theta_1 = \theta_2 = 0$. It can be seen that the constant-correlation assumption is rejected for all data sets at any conventional level of significance.

To further test for misspecification in the MGARCH models we adopt the regression-based diagnostics suggested by Wooldridge (1990, 1991). The methodology developed by Wooldridge applies to a wide class of possible misspecification. Here we focus on the problem of misspecification in the conditional heteroscedasticity. As shown by Wooldridge, the suggested tests are robust to departure from distributional assumptions that are not being tested. Since our main concern is misspecification in the conditional variance, we use the squared standardized residuals and the cross-products of the squared standardized residuals as the indicators.

We first consider tests based on the squared standardized residuals. We denote $\hat{\epsilon}_{it}$ as the estimate of the standardized residual ϵ_{it} and σ^2_{it} as the estimated conditional variance of y_{it}. We define $\lambda_{it} = (\hat{\epsilon}^2_{i,t-1}, \hat{\epsilon}^2_{i,t-2}, \ldots, \hat{\epsilon}^2_{i,t-Q})'$; the vector of indicator variables and $\nabla_\theta \sigma^2_{it}$ as the gradient vector of σ^2_{it} with respect to θ evaluated at $\hat{\theta}$. Denoting $(\nabla_\theta \sigma^2_{it})/\hat{\sigma}^2_{it}$ as $\nabla_\theta \hat{\sigma}^{-2}_{it}$, we regress each element of $\hat{\lambda}_{it}$ on $\nabla_\theta \hat{\sigma}^{-2}_{it}$ to obtain the Q-element residuals $\hat{r}_{it}$. Finally, we regress unity on the vector of Q regressors $\hat{\phi}_{it}\hat{r}_{it}$, where $\hat{\phi}_{it} = \hat{\epsilon}^2_{it} - 1$. We calculate $W_{ii}(Q)$ = T - SSR, where SSR is the sum of squares of the residuals of the last regression. If there is no model misspecification, $W_{ii}(Q)$ is asymptotically distributed as χ^2_Q.

The preceding diagnostic statistic can be calculated for the cross-products of the standardized residuals from different equations as tests for pairwise correlations. Specifically, we define $\hat{\lambda}_{it} = (\hat{\epsilon}_{i,t-1}\hat{\epsilon}_{j,t-1}, \hat{\epsilon}_{i,t-2}\hat{\epsilon}_{j,t-2}, \ldots, \hat{\epsilon}_{i,t-Q}\hat{\epsilon}_{j,t-Q})'$; $\nabla_\theta \hat{\phi}_{it}$ as the gradient vector of $\phi_{it} = \epsilon_{it}\epsilon_{jt} - \hat{\rho}_{ijt}$ with respect to θ evaluated at $\hat{\theta}$. We regress each element of $\hat{\lambda}_{it}$ on $\nabla_\theta \hat{\sigma}_{ijt}$ to obtain the Q-element residuals $\hat{r}_{ijt}$ and then regress unity on the Q regressors $\hat{\phi}_{ijt}\hat{r}_{ijt}$, where $\phi_{ijt} = \hat{\epsilon}_{jt}\hat{\epsilon}_{ijt} - \hat{\rho}_{ijt}$. We define the test statistic as $W_{ii}(Q)$ = T - SSR for $1 \le i \le j \le K$, which is asymptotically distributed as χ^2_Q when there is no misspecification.

Table 6. Estimation Results of the Conditional-Mean Equations of the BEKK(1,1) Models

Variable	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
A: Forex market data (DS1)					
D	−.0017	.0472			
	(.0128)	(.0207)			
J	−.0052	.0524			
	(.0134)	(.0215)			
B: National Stock Market data (DS2)					
H	.1188	.0574			
	(.0299)	(.0269)			
S	.0270	.1574			
	(.0186)	(.0262)			
C: Hang Seng sectoral indices data (DS3)					
F	.1354	.0720	.0507		
	(.0305)	(.0184)	(.0159)		
P	.0804	.1305	.0402	−.0279	−.0214
	(.0325)	(.0169)	(.0151)	(.0137)	(.0133)
U	.0701	.0723		−.0483	
	(.0322)	(.0263)		(.0172)	

NOTE: The parameter estimates are the MLE assuming normality. The ' gures in parentheses are the standard errors. They are calculated using the robust QMLE covariance matrix of the parameters.

We apply the W statistics to the MGARCH models with Q = 4. From the results in Table 8 we can see that both the CC-MGARCH and the VC-MGARCH models pass the diagnostic checks of the W statistics. Indeed, the W statistics of the two models are quite similar. As the constant-correlation assumption is not supported by the LMC and the LR statistics, one might expect the W statistics of the CC-MGARCH model to be significant. The fact that this is not the case may be an indication of loss in power when the test has no specific alternative. As for the BEKK model, most diagnostics are insignificant, with the exception of W11 in DS3. It is noted that the CC-MGARCH model has the lowest log-likelihood for all models. The VC-MGARCH model has the highest log-likelihood for DS2 and DS3, and the BEKK model has the highest log-likelihood for DS1. Based on penalized likelihood criteria such as the AIC, VC-MGARCH is the preferred model for DS2 and DS3, and BEKK is the preferred model for DS1.

Table 7. Estimation Results of the Conditional-Variance Equations of the BEKK(1, 1) Models

c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	g_{11}	g_{22}	g_{33}	a_{11}	a_{22}	a_{33}
A: Forex market data (DS1)											
.0618	.0403		.0863			.9715	.9728		.2103	.1973	
(.0138)	(.0094)		(.0228)			(.0053)	(.0093)		(.0178)	(.0307)	
B: National Stock market data (DS2)											
.3488	.0988		.2589			.9195	.8789		.3215	.4093	
(.0699)	(.0286)		(.0428)			(.0187)	(.0303)		(.0338)	(.0531)	
C: Hang seng sectoral indices data (DS3)											
.1316	−.0938	−.1467	−.1741	−.1812	−.2528	.9654	.9552	.9548	.2309	.2709	.2561
(.0250)	(.0313)	(.0458)	(.0358)	(.0486)	(.1525)	(.0060)	(.0109)	(.0382)	(.0183)	(.0297)	(.0876)

NOTE: The parameter estimates are the MLE assuming normality. The ' gures in parentheses are the standard errors. They are calculated using the robust QMLE covariance matrix of the parameters.

Table 8. Diagnostic Checks for Various Models

Tests	*Forex market DS1: D-J*	*National stock markets DS2: H-S*	*Hang Seng sectoral indices DS3: F-P-U*
A: CC-MGARCH(1, 1) model			
LF	−1878.23	−4128.73	−7374.26
LMC	4.9138*	10.3442*	8.3589*
$W_{11}(4)$	4.7921	3.9518	6.4373
$W_{22}(4)$	3.8319	1.0694	4.9579
$W_{33}(4)$			5.8914
$W_{12}(4)$	7.6383	5.2031	5.0129
$W_{13}(4)$			7.5183
$W_{23}(4)$			7.6974
B: VC-MGARCH(1, 1) model			
LF	−1855.80	−4088.90	−7293.40
LR	44.8504*	79.6608*	161.7632*
$W_{11}(4)$	4.7263	3.8607	7.0386
$W_{22}(4)$	3.8157	1.0672	5.0164
$W_{33}(4)$			5.9008
$W_{12}(4)$	6.9215	2.0592	5.2156
$W_{13}(4)$			7.7455
$W_{23}(4)$			6.9466
C: BEKK(1, 1) model			
LF	−1851.15	−4104.45	−7324.24
$W_{11}(4)$	4.4679	4.0134	12.2758*
$W_{22}(4)$	3.8566	.7477	6.6085
$W_{33}(4)$			7.8584
$W_{12}(4)$	7.1350	5.5407	.5775
$W_{13}(4)$			1.2633
$W_{23}(4)$			1.2316

NOTE: $W_{ij}(4)$ is Wooldridge's (1991) regression-based diagnostic statistic computed from the standardized residuals of variables i and j based on indicator variables up to lag 4. The indices are according to the order of the coded variables. Thus, $W_{13}(4)$ in the system (F-P-U) is $W_{FU}(4)$. LF is the maximized log-likelihood value. LMC is the Lagrange multiplier test statistic for constant correlation due to Tse (2000). It is approximately distributed as χ_1^2 for a bivariate system and χ_3^2 for the trivariate system when the correlations are time-invariant. LR is the likelihood ratio statistic for H_0: $\theta_1 = \theta_2 = 0$ in the VC-MGARCH(1, 1) model.
*Signi' cance at the 5% level.

Table 9. Summary Statistics of the Standardized Residuals of Various Models

Variable (Code)	Mean	Std. dev.	Minimum	Maximum	Std. skewness	Std. kurtosis	$Q_1(20)$	$Q_2(20)$	No. of obs.
A: CC-MGARCH(1, 1) model									
Forex market data (DS1)									
Deutsche mark (D)	−.0001	.9995	−4.9758	4.1334	−1.3760	10.3514	16.8359	14.7136	2131
Japanese yen (J)	.0004	.9984	−5.8404	4.1482	−11.0287	27.8389	19.1773	11.6364	2131
National Stock Market data (DS2)									
Hong Kong (H)	−.0397	.9991	−8.2506	4.8277	−9.1688	44.4917	22.9629	7.0862	1942
Singapore (S)	−.0343	1.0000	−6.3858	5.6050	−.4392	33.4205	31.5455	11.1104	1942
Hang Seng sectoral indices data (DS3)									
Finance (F)	−.0047	.9993	−6.2435	4.3125	−2.2710	20.4027	18.5158	21.3844	2440
Properties (P)	−.0026	.9992	−6.8961	4.2048	−3.4370	22.2583	36.4885	18.2433	2440
Utilities (U)	.0054	.9995	−7.2480	4.4854	−3.9386	27.8046	25.4859	10.8179	2440
B: VC-MGARCH(1, 1) model									
Forex market data (DS1)									
Deutsche mark (D)	−.0019	.9976	−5.1997	4.0989	−1.5961	10.6562	16.8138	13.6033	2131
Japanese yen (J)	−.0063	1.0015	−5.8539	4.1495	−11.0059	27.8081	19.1706	11.6673	2131
National Stock Market data (DS2)									
Hong Kong (H)	−.0482	1.0019	−8.4556	4.5876	−9.7967	46.5185	25.6588	7.3562	1942
Singapore (S)	−.0410	1.0086	−6.4275	5.6201	−.3672	33.1772	32.0569	11.1440	1942
Hang Seng sectoral indices data (DS3)									
Finance (F)	−.0271	1.0012	−6.4599	4.3445	−2.1722	20.8953	17.6054	25.0739	2440
Properties (P)	−.0207	1.0034	−7.2261	4.0554	−3.6343	23.2231	36.6489	13.4232	2440
Utilities (U)	.0155	.9975	−7.5899	4.3811	−4.2861	29.6450	25.2002	9.0371	2440
C: BEKK(1, 1) model									
Forex market data (DS1)									
Deutsche mark (D)	.0030	.9932	−5.0577	4.0577	−1.5112	10.5148	16.6838	15.1533	2131
Japanese yen (J)	−.0007	.9954	−5.9277	4.1335	−11.5370	28.4453	19.5022	12.5101	2131
National Stock Market data (DS2)									
Hong Kong (H)	−.0359	.9874	−8.4438	4.3589	−10.3784	48.5443	26.2642	9.8984	1942
Singapore (S)	−.0357	.9996	−6.5838	5.5635	−.8210	33.5270	33.7262	10.5492	1942
Hang Seng sectoral indices data (DS3)									
Finance (F)	−.0224	.9822	−6.6095	4.0304	−3.2424	25.1857	20.2667	64.4039	2440
Properties (P)	−.0143	.9849	−7.2597	4.0862	−3.9564	24.4061	36.8795	16.9989	2440
Utilities (U)	−.0159	.9792	−7.6392	4.5386	−3.5271	33.4623	25.2280	25.3953	2440

In Table 9 we present the summary statistics of the standardized residuals of the fitted models. It can be seen that the standardized kurtosis and the Q_2 statistics have dropped significantly compared with those of the raw data in Table 3. We note that the Q_1 and Q_2 statistics are presented here for completeness. As pointed out by Li and Mak (1994) and Ling and Li (1997a), these statistics are not distributed as χ^2 under the null of no misspecification. Although some of the Q_1 statistics appear to be large, we report that none of the lagged autocorrelation coefficients are larger than .08 in absolute value. Although most Q statistics seem to be low, there is an exception for Q_2 in the F series of DS3. This result agrees with the fact that W_{11} in part C of Table 8 is also found to be significant.

To obtain a clearer picture of the time history of the conditional correlations, we plot the time paths of the conditional correlations based on the VC-MGARCH and BEKK models. The plots are presented in Figures 4-8. It can be seen from the graphs that the conditional correlations estimated from the VC-MGARCH and BEKK models follow each other quite closely. However, the paths based on the BEKK model have much larger variability than those estimated by the VC-MGARCH model. In what follows we describe the conditional-correlation paths as provided by the VC-MGARCH model in some detail.

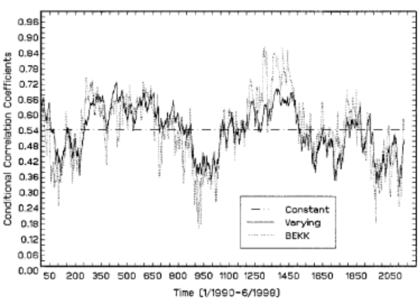

Figure 4. Conditional-Correlation Coefficients of (D, J).

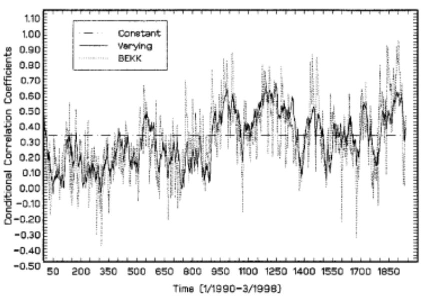

Figure 5. Conditional-Correlation Coefficients of (H, S).

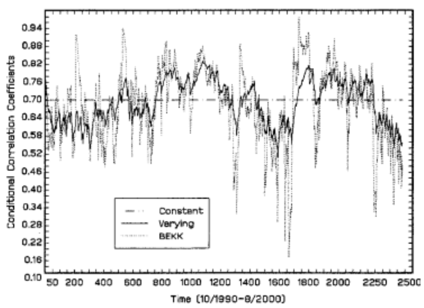

Figure 7. Conditional-Correlation Coefficients of (F, U).

Figure 4 presents the correlations between the deutsche mark and the Japanese yen. Largely, there are two sub-periods when the conditional correlations of these two currencies were mostly above the average (constant) level, namely, October 1991 to June 1993 and March 1994 to October 1996. From October 1996 to June 1998, the conditional correlations were mostly below the average level.

Figure 5 presents an interesting case in which we can see that the conditional correlations between the Hong Kong and the Singapore stock markets experienced an upward shift. From 1994 onward, the conditional correlations were mostly above the average level, whereas the reverse was true before 1994. This finding has important implications for the international diversification of equity portfolios. The increasing conditional correlation means that the two national markets were becoming more closely integrated and implies that there are diminishing benefits from international diversification. Using moving windows of unconditional correlations, Longin and Solnik (1995) showed that there was evidence of increasing correlation between international stock markets in 1960-1990. Further results were updated by Longin and Solnik (2001). Our similar finding for the Hong Kong and the Singapore markets is commensurate with the increasing importance of intra-Asian business in the 1990s. Indeed, in the second half of the 1990s, many companies with business activities in Hong Kong were listed on the Singapore exchange.

Figures 6-8 show that the pairwise correlations between the three sectors in the Hong Kong stock market are quite similar. Broadly speaking, the conditional correlations were above average in the sub-periods of 1993-1994 and mid-1997 to mid-1999. These two sub-periods coincide with the time when the Hong Kong stock market was experiencing a downturn. In contrast, during the sub-periods of the bull runs from 1995 to mid-1997 and post-mid-1999, the conditional correlations were below average. At the risk of oversimplification, this casual observation agrees with the hypothesis that contagion is stronger for negative returns than for positive returns. In a recent study, Bae et al. (2000) examined the financial contagion among Asian and Latin American economies with the use of a multinomial logit model. They reported that the evidence of contagion being stronger for negative returns than for positive returns is mixed. Finally, we note that for the BEKK model the conditional correlations are quite unstable in some periods.

We shall end this section by stating that it is not our intention to claim that the VC-MGARCH models as presented here represent the best MARCH models for the data. Other MARCH models could also provide the conditional-- correlation structure. The VC-MGARCH model, however, does provide a viable alternative that is relatively easy to estimate. As the examples have illustrated, modeling correlations as a time-varying structure provides some interesting results that are not obtainable from constant-correlation models.

5. CONCLUSIONS

In this article we propose a new MGARCH model with time-varying correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analog to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimization.

We report some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model. It is found that the bias and MSE of the MLE are small for sample sizes of 500 or above. The new model is applied to three data sets, namely, the exchange rate data, the national stock market data, and the sectoral price data. The new model is found to pass the model diagnostics satisfactorily and compare favorably against the BEKK model, whereas the constant-correlation MGARCH model is found to be inadequate. Extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. In particular, the estimated conditional-correlation path provides an interesting time history that would not be available in a constant-correlation model.

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