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Bayesian Analysis of Country Risk Premia in Developing Small Open Economies

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Abstract

This thesis studies a model presented by Neumeyer & Perri (2005), which aims to explain the strong countercyclicality of interest rates and net exports in emerging market economies. The model accomplishes this by decomposing interest rates into an international rate and a country risk component, and by making labor demand sensitive to movements in these rates via a working capital constraint imposed on the representative firm. Moreover, it proposes two approaches to determining the stochastic processes for these interest rates: the independent country risk case and the induced country risk case. The induced country risk model calibrated to Argentine data reproduces the country's business cycle facts well. However, the results are sensitive to certain parameters and specifications which are not properly set in accordance with developing economy data. Hence, a Bayesian approach is used to verify the model results. The estimation results highlight some areas for improvement in the model. First, certain key parameters of the model are difficult to inform by existing data. Second, the mechanisms through which the model tries to explain key developing economy business cycle facts are important, but need to be augmented in some way. In particular, more structure needs to be added into the model of default risk to help capture the dynamics of interest rates and output more fully.

Keywords: Small open economy, Bayesian DSGE, real business cycles, country risk premium, interest rate shocks, countercyclical interest rates

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Ad Majorem Dei Gloriam.

Chapter 1

Introduction

A significant part of small open economy research focuses on trying to explain the similarities and differences in empirical regularities of macroeconomic aggregates between developed and emerging economies. Some of the more well-documented differences are that emerging economies display more volatility in aggregates, have countercyclical interest rates and much more strongly countercyclical net exports as compared to their developed counterparts. On the other hand, the procyclicality and persistence of consumption, investment, hours and productivity are similar across the two groups of countries.

One of the well-known papers in this area is that of Neumeyer & Perri (2005), which analyzes the interest rate that an emerging country faces in the international financial markets and its effect on the country's business cycles. The focus on interest rates is driven by the observation that many developing economies face frequent and large fluctuations in the rate at which they can borrow funds from the rest of the world, and that these swings have usually been associated with large movements in other economic aggregates.

This thesis attempts to analyze the Neumeyer-Perri (NP) model using the more novel Bayesian DSGE approach, and compares the results to the original calibrated model. The rest of this chapter tries to motivate this approach. In the next sections, the related small open economy literature is surveyed briefly. The intuition behind the NP model is presented, and the merits of a Bayesian estimation of this model are discussed. Finally, the dataset used in this study is described, and the differences with the original NP dataset are highlighted.

1.1 Brief Survey of Literature

The earlier attempts to match business cycle facts in developed small open economies were done by Mendoza (1991) and Correia et al. (1995). These papers employed fairly standard RBC models calibrated to the Canadian and Portuguese economies, respectively, that successfully replicated the mild countercyclicality of the trade balance, the procyclicality of interest rates and the less than unity consumption-output ratio in these economies.

Mendoza (1995) was one of the first papers to pose a possible explanation of the differences between developed and emerging economies that were mentioned above. It focused on terms of trade shocks, driven by the observation that developing economies tend to specialize in exporting primary commodities and tend to be small players in the world market for these commodities. The model featured three production sectors: the exportable, importable and nontradable goods sectors. There were four driving forces, consisting of productivity shocks to each of the three sectors and a terms of trade shock. Among other things, the calibrated model successfully replicated the cyclicality of terms of trade in both developed and emerging economies, the countercyclicality (though overstated) of the trade balance and the direction of correlation between the trade balance and the terms of trade.

Aguiar & Gopinath (2007) explore the possibility that the statistical properties of the exogenous shock processes for productivity differ across developed and developing economies. They motivate their study by the observation that emerging economies usually experience frequent reversals in monetary, fiscal and trade policies that can be considered to cause changes in trend growth. They use a standard small open economy RBC model with one international financial asset, except that there are shocks to the labor-augmenting technological progress. Given calibrated values for all other variables, they use a Generalized Method of Moments procedure to estimate the parameters of the mean-reverting total factor productivity process as well as the nonstationary trend growth process using data from Canada and Mexico. They find that for Mexico, shocks to trend growth affect the variation in business cycles more than productivity shocks do, while the reverse is true for Canada.

Another recent strand of research focuses on the effect of financial conditions on emerging economies. An example of these is a paper on sudden stops of capital flows by Christiano et al. (2004). In this model, when an external credit constraint suddenly binds, interest rates go up, output declines and there is a huge increase in the trade balance (Neumeyer & Perri, 2005).

Uribe & Yue (2006) and Neumeyer & Perri (2005) explore the financial conditions explanation in a different way - through interest rates and country risk spreads. Both papers decompose the interest rate that an emerging market faces into two components - an international rate for risky investments and a country-specific risk premium - and show that developing economy business cycles are highly responsive to movements in these two rates, but that it is also very likely that country risk premia also respond to movements in output. They take this into account in their RBC models, and produce simulated interest rates and net exports that are strongly countercyclical, as in the data. Both models share two features (besides the decomposition of interest rates) that deviate from the standard RBC models, and which are crucial for obtaining the results. The first is the use of the preference specification suggested by Greenwood et al. (1988), and the second is the addition of a working capital constraint into the firm's production problem. To explore this specific line of research further, some more details of the NP model are given in the next section.

1.2 The Role of Interest Rates in Emerging Economy Cycles - Intuition Behind the NP Model

While standard RBC models of small open economies have been successful in replicating volatilities of macroeconomic aggregates and countercyclical net exports, interest rates tend to be quantitatively unimportant in the simulations. To replicate the countercyclicality of interest rates observed in the data, Neumeyer and Perri augment a standard small open economy RBC model with one financial asset in four ways:

1. They employ the Greenwood et al. (1988) specification for preferences, given by

$$u(c_t, l_t) = \frac{[c_t - \psi(1+\gamma)^t l_t^\nu]^{1-\sigma}}{1-\sigma}, \quad \nu > 1,\ \psi > 0$$

   This generates a labor supply that is independent of consumption, and hence is unresponsive to interest rates. From here on, this specification will be referred to as GHH preferences.

2. They introduce a working capital constraint, requiring firms to set aside a fraction of wages before production takes place, to make labor demand sensitive to the interest rate.

3. Each time period is broken up into two parts: the beginning and the end. Interest rates on bonds are revealed at the beginning of the period, and bonds issued at this time mature *at the end of the next period.* Firms hire labor, rent capital, and borrow funds at the beginning, and finish production at the end of the period. Hence, all trades and payments are made at the end, including payment for the firm's loans.

4. In linearized form, the interest rate faced by the economy is decomposed into two components: an international rate for risky assets, $\hat{R}^*$ plus a country-specific risk spread, $\hat{D}$:

$$\hat{R}_t = \hat{R}^*{}_t + \hat{D}_t$$

Exogenous processes for the deviations from steady state of TFP, $\hat{A}_t$, and the international rate are modeled in standard AR(1) form with no intercept.

The country risk spread, interpreted lightly as risk of default is modeled in reduced form in two ways:

1. Independent Country Risk Case: Country risk itself is modeled as an AR(1) process independent of international interest rates.
2. Induced Country Risk Case: Country risk responds to expected future productivity shocks:

$$\hat{D}_t = -\eta E_t(\hat{A}_{t+1}) + \varepsilon_{I\ t}$$

   where η is a constant capturing how much risk responds to expected TFP.

The rest of the economy is formulated as in the standard small open economy RBC model with labor-augmenting technological progress.

To gain an intuition of how this model works, consider how changes in interest rates affect the model economy. Note how with the working capital constraint, an increase in interest rates will make the firm's marginal cost of labor increase, causing it to reduce its demand for labor. Given the GHH specification, labor supply will not be affected by this increase in rates. Hence, equilibrium labor hours will decrease, and the magnitude of this decrease will be determined by the slope of the labor supply curve, i.e. the labor supply elasticity. Because capital does not fluctuate as much at business cycle frequencies, and because the firm is not required to set aside part of the costs of renting capital, the interest rate's effects on capital movements are negligible. Therefore, the decrease in labor will tend to cause output to go down.

The change in consumption will depend on the interest rate in two ways. First, an increase in the interest rate will cause the household to substitute consumption intertemporally from the present to the future. Second, the decrease in equilibrium labor hours means the household has less to spend. Hence, consumption also declines. With the NP baseline parameter values, this response can be stronger than the response of output, producing another facet of emerging market data, which is that consumption is more volatile than output.

Lastly, the intertemporal substitution of consumption also means that savings using financial assets increase, leaving less funds for investment. If the response of consumption is almost as strong as, or even stronger than that of output, the decline of consumption and investment together will cause net exports to increase as domestic absorption shrinks. Because output decreases in response to interest rates, the model is able to produce countercyclical net exports.

Note, too, that firms borrow and repay funds within the same period. Therefore, given the timeline provided in item 3 above, firms face the interest rate revealed in the previous period, so financial shocks affect real variables at a lag (this is also used by Uribe & Yue (2006) as an identification restriction in their study). Consumption, on the other hand, is decided at the same time as the household's bond purchasing decisions are made - at the end of the period, when the previous period's bonds have matured and new bonds are bought at the current interest rate. Hence, consumption goes down on impact after an increase in interest rates.

Consider now the economy under Cobb-Douglas preferences. Consumption and leisure enter in a non-additively-separable manner, so labor and consumption are negatively related. Since the response of consumption to an increase in interest rates is negative, labor supply goes up on impact. In the beginning of the next period, the firm faces this new interest rate, so labor demand decreases. This offsets at least some of the previous increase in equilibrium hours, but the net effect of interest rates on labor (and hence on output) can be positive or negative, depending on the strength of the initial reaction of consumption to the shock, showing that the equilibrium characterization is not quite as clear-cut under alternative preference specifications.

In contrast, the effects of a productivity shock on the output cycle of the economy depends on the way country risk is modeled. In the independent country risk case, productivity does not affect interest rates, so that the economy responds in the way that the standard RBC models do - labor demand increases, inducing hours and output to increase. In the induced country risk case, a positive productivity shock in the current period increases expected productivity in the future, reducing country risk. This serves to increase labor demand even more, showing that the interaction between the working capital requirement and the productivity-induced country risk spreads can amplify the effects of TFP shocks on the economy.

To quantify the effects mentioned here, the NP model is calibrated to match quarterly Argentine data from 1983 to 2001 using both models of country risk. In the simulations, the induced country risk case is able to explain Argentine business cycle facts well, especially the cyclicality of interest rates. They note that the presence of working capital is important to achieve this, since it makes labor demand respond to borrowing rates and it serves to amplify fundamental total factor productivity (TFP) shocks so as to increase volatility. The results of their quantitative experiments based on this model suggest that shutting down country-specific risk lowers Argentine GDP volatility by 27 percent while stabilizing the international risky rate lowers volatility by less than 3 percent.

In the NP calibration, the portion of labor costs that the firm has to pay in advance is arbitrarily set to a hundred percent. In the sensitivity analysis, it becomes clear that the ability of the model to quantitatively match the countercyclicality of interest rates and volatility of output decreases substantially as we reduce this percentage. Another important parameter is the labor supply elasticity. This should ideally be set according to micro-level estimates. However, there is a lack of studies documenting this parameter for Argentina. The approach taken by Neumeyer & Perri (2005) and Uribe & Yue (2006) is to merely set this parameter to the usual values used in models of the US or other developed countries. These observations serve as the primary motivations for this thesis. These parameters are the main drivers of the favorable results obtained from the model. As such, these should not be kept as free parameters, nor should they be set haphazardly to values consistent with data from other countries. Rather, they should be obtained from data. Admittedly, there have been no well-known studies that provide estimates of these parameters for Argentina, making them hard to calibrate. However, newly developed Bayesian methods are now available for structural macroeconomic models. These methods can help pin down a range of values that are admissible for each parameter given the observable variables at hand, and whatever other information sources are available.

The next section gives a short introduction to Bayesian DSGE methods, its advantages and disadvantages.

1.3 Bayesian Estimation

The calibration tradition does not fully acknowledge parameter uncertainty, as argued for the particular case of the percentage of the wage bill that the firm has to pay in advance in the NP model. In fact there is no statistically sound way of justifying the parameter values used in simulating calibrated models, as these are not treated as data generating processes. The benefit of this is that it fully acknowledges potential misspecification. On the other hand, structural parameters that are not informed by long run data averages or micro-level studies (free parameters), are difficult to defend, especially if they are crucial in quantitatively reproducing certain important aspects of macroeconomic data.

Also, since there is no traditional measure of goodness of fit, comparisons across non-nested models are generally not informative. The following discussion, based mostly on An & Schorfheide (2007), briefly discusses two alternative approaches to analysis of DSGE models.

One recently developed method is Maximum Likelihood Estimation, with the likelihood function derived from the DSGE model solution. Unlike calibration, this provides an econometric framework for inference and specification tests. However, it depends heavily on the joint distribution of parameters implied by the model, and totally ignores information from long run averages and micro-level studies that the model parameters should be consistent with. Hence, likelihood functions for these models often peak at regions which are at odds with this additional information. Also, in the face of identification problems, the likelihood function becomes flat in some directions, leading to difficulties in numerical optimization.

The Bayesian DSGE approach is a popular new method for estimating and evaluating macroeconomic models because of its full acknowledgement of parameter uncertainty and its use of practical computational methods such as Markov Chain Monte Carlo simulations. Further, it enables the researcher to use parameter values from previous studies as a starting point (prior distribution), and then update these values based on the current data at hand (posterior distribution). Like the maximum likelihood method, it depends on the likelihood function generated by the DSGE model and so is senstive to misspecification and identification issues. However, unlike maximum likelihood, the prior helps to introduce some curvature into the likelihood function, so that numerical optimization can be more easily carried out.

An advantage of the Bayesian method over calibration is that it employs a full econometric approach, which enables the researcher to use statistical methods to assess the model's fit with the data. In particular, it enables the researcher to attach probabilistic statements to model predictions, fully acknowledging parameter uncertainty.

In this thesis, a Bayesian approach is applied to the NP model to estimate its parameters and verify the results of the simulations. In terms of key statistics that it should be able to replicate, particular interest will be paid to the countercyclicality of interest rates, countercyclicality of net exports, and volatility of output and consumption. In terms of parameters, special attention will be given to the estimates of the labor supply elasticity and the percentage of labor costs that the firm has to pay in advance. Estimates for these parameters that are too far from the calibrated values will signal a need for a closer examination as to whether the model can indeed explain emerging market business cycles well, or whether other approaches may be more promising for future research on emerging market cycles and country risk premia.

Also, the model will be simulated using the parameters from the simulated posterior draws to assess whether it can still predict key macroeconomic regularities in Argentine data even after using a Bayesian estimation approach.

1.4 Data

Before discussing the NP model in greater detail, the datasets used for both the NP study and this thesis are described. Important features of Argentine data are presented here. These statistics are also presented for, and compared across, a number of developed and emerging small open economies in Neumeyer & Perri (2005). The results for those comparisons agree with the widely accepted empirical regularities mentioned at the beginning of this chapter.

1.4.1 NP Data

Neumeyer and Perri use data for output, total consumption, gross fixed capital formation and net exports. All series are non-seasonally adjusted time series from 1983:1 to 2002:2, in constant 1993 prices obtained from Ministerio de Economia (MECON). Total consumption includes private consumption, government consumption, inventories and statistical discrepancy.

Employment is measured as total urban employment from Encuesta Permanente de Hogares (Table A3.2, informe economico). Hours are hours worked per employed person per week, from the same source. These are only available at semi-annual frequency from the second half of 1986. Neumeyer and Perri express doubt that these series are reliable, pointing to evidence presented in Kydland & Zarazaga (2002), arguing that public employment figures in Argentina may be a covert form of employment insurance. Moreover, they note that during the 1988-1990 recession, GDP fell by 15 percent while unemployment remained steady, presenting further evidence regarding the low quality of the data.

Nominal interest rates from 1994:2 to 2000:2 are calculated as the sum of the 90-day US Tbill rate and the country spread from EMBI+, and from 2000:3 to 2002:2, nominal rates are the sum of the Tbill rate and the EMBI Global spread. The switch from the EMBI+ to EMBI Global was not explained in the original paper. This interest rate series is extended by using data on bond prices of dollar denominated bonds issued by the Argentine government (BONEX 80, BONEX 81, BONEX 82, BONEX 84, GRA, Brady Discount, Global01, Global03, Global06, Global17, Global27) to estimate spreads. The details of this estimation are given in the appendix of the original paper, and are not delved into here. Real rates are obtained by subtracting expected US inflation, which is the average of US GDP deflator inflation in the current period and the three preceding ones.

These secondary market prices of Argentina's dollar denominated bonds are chosen for two reasons. Since these bonds are traded globally, these rates represent the intertemporal terms of trade that the country faces in the international financial markets. Also, since they are dollar denominated, real interest rates are computed using expected US inflation rather than expected domestic inflation, which in Argentina's case has been known to display erratic behavior in the past. Note that these are rates that the sovereign government faces, instead of the rates that firms actually face. Neumeyer and Perri claim that these rates are close to, and behave like a dataset of dollar-denominated corporate prime rates. The reported correlation between these two is 0.89.

Some key business cycle statistics reported by Neumeyer and Perri from this dataset are presented in Table 3.2, in the rows labeled NP data. Here, all variables are logged and then HP-filtered. From here on, net exports are defined as net exports divided by GDP. Once, again, these confirm the main empirical regularities that this thesis focuses on. Consumption and investment are more volatile than output, interest rates and net exports are countercyclical, and consumption as well as labor hours are procyclical.

1.4.2 New Data

This paper uses a different set of data from the original paper, and this will be referred to in the rest of this thesis as New Data. The Thomson Datastream database was used to gather time series for GDP, total consumption, gross fixed capital formation, net exports, the US Tbill rate and the EMBI+ stripped country spread. All series are non-seasonally adjusted and, except for the Tbill and the country spread, are in constant 1993 prices. All series except for GDP cover the period from 1994:1 to 2009:4, while GDP data extends all the way back to 1980:1. Total consumption is the sum of private consumption, government consumption and the series called "Balance of Supply and Demand".

The calculation of nominal interest rates has been made more straightforward, relative to the original paper. Following Uribe & Yue (2006), interest rates for the current period are the sum of the Tbill rate and the EMBI+ country spread for the whole sample.

As for labor, we managed to obtain quarterly labor data from the CEIC database for the period from 1997:1 to 2008:1. Employment is given as the total volume of employed persons. The database provides an hours worked index with 1997=100. One can back out an estimate of the actual average hours worked per person, per week by averaging over the semi-annual values of hours in 1997 of the NP dataset and multiplying each index value by this ratio divided by 100.

The same summary statistics for the logged, HP-filtered New Data time series are reported in the rows labeled New Data in Table 3.2. Though these statistics differ quantitatively from the published NP statistics, they qualitatively reproduce the aforementioned main empirical regularities. The most obvious improvement is the quality of labor hours data. In particular, the correlation of total hours worked (employment times average hours worked per quarter) with GDP moves from 0.52 in the NP dataset to 0.88 in New Data. Though the trustworthiness of labor data relative to the rest of the national accounts is still in question, we can be confident that this dataset is at least a step in the right direction from the previous paper.

Chapter 2

The Neumeyer-Perri Model

This chapter describes in detail the small open economy real business cycle model that is laid out in Neumeyer & Perri (2005) in order to try to propose an explanation for the business cycle facts mentioned previously. It is a standard neoclassical small open economy with one final good. The only financial asset available is a one-period, non-contingent real bond on the international financial market, which both households and firms can trade. Firms participate in the bond market because of the need to pay for a fraction of the wage bill before production of the final good takes place.

2.1 Timeline

The model is in discrete time. Each period t is split into two parts: the beginning, t^-, and the end of the period, t^+, with t^+ and $(t+1)^-$ arbitrarily close. Let s_t denote the state revealed at period t and s^t denote the history $(s_0, s_1, \ldots, s_t)$. This state determines the two shock processes: total factory productivity (TFP) denoted by $A(s^t)$, and the interest rate $R(s^t)$ on bonds issued either at time t^+ or $(t+1)^-$ which mature at time $(t+1)^+$.

At t^- the firm hires labor and rents capital from the household for production. A working capital constraint, discussed below, creates the need for the firm to borrow funds to set aside part of the labor costs before the final good is completed.

At t^+, the firm completes production and is able to pay for its inputs, along with the loans it took earlier in the period. The household receives its income and makes purchasing decisions. It also issues or purchases bonds at this time.

2.2 The Firm

The representative firm operates in perfectly competitive markets for inputs and final goods. There is assumed to be a friction in the transfer of resources from the firm to the worker, which requires the firm to set aside a fraction θ of the wage bill, $w(s^t)l(s^t)$, at t^- before production takes place, and pay the rest at t^+. Here, $w(s^t)$ denotes the wage rate given the history until time t and $l(s^t)$ denotes the equilibrium amount of labor hired. Because of this working capital requirement, the firm needs to borrow $\theta w(s^t)l(s^t)$ from the bond market between t^- and t^+ at the rate $R(s^{t-1})$. The market for capital services is assumed to be frictionless, so that owners of capital only need to be paid at the end of the period after the production process.

Therefore, when output becomes available at the end of the period, the firm uses the proceeds from sales to pay for the rest of its wage bill obligations, $(1-\theta)w(s^t)l(s^t)$, to pay for rental services to the capital owners, $r(s^t)k(s^{t-1})$, and to repay working capital loans plus the interest due, $\theta w(s^t)l(s^t)R(s^{t-1})$.

Technology is subject to a constant returns to scale production function

$$y(s^t) = A(s^t)[k(s^{t-1})]^\alpha[(1+\gamma)^t l(s^t)]^{1-\alpha}, \tag{2.1}$$

where γ is the deterministic growth rate of labor-augmenting technological change. Given market prices $w(s^t)$, $r(s^t)$ and $R(s^{t-1})$, the firm's problem is then to maximize profit (measured at time t^+) as given by

$$A(s^t)k(s^{t-1})^\alpha[(1+\gamma)^t l(s^t)]^{1-\alpha} - w(s^t)l(s^t) - r(s^t)k(s^{t-1}) - [R(s^{t-1}) - 1]\theta w(s^t)l(s^t), \qquad (2.2)$$

which has the first order conditions:

$$(1-\alpha)\frac{y(s^t)}{l(s^t)} = w(s^t)\{1 + \theta[R(s^{t-1}) - 1]\} \qquad (2.3)$$

and

$$\alpha\frac{y(s^t)}{k(s^{t-1})} = r(s^t) \qquad (2.4)$$

Here, $[R(s^{t-1}) - 1]\theta w(s^t)l(s^t)$ represents net interest on the portion of wages paid for using borrowed funds. The first equation states that the marginal product of labor (left hand side) must be equal to its marginal cost, given by the wage rate plus the net interest paid on the borrowed part of the last unit of labor. Similarly, the second equation says that the marginal product of capital should be equal to the rental rate.

2.3 Household

Let $\pi(s^t)$ be the probability that the history s^t occurs conditional on $t = 0$ information, $\beta \in (0, 1)$ be the discount factor, and $c(s^t)$, $x(s^t)$ and $b(s^t)$ be consumption, investment and bond holdings conditional on the realization of the history, respectively. The household's expected lifetime utility is given by

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u(c(s^t), l(s^t)). \qquad (2.5)$$

At the beginning of each period t, the representative household supplies labor and capital rental services to the firm and receives the proceeds from these at the end of the period, at which time it also obtains income from bond holdings $b(s^{t-1})R(s^{t-1})$. It spends these on consumption, investment, bond purchases and the cost of holding bonds, $\kappa(b(s^{t-1}))$. The household's per-

period budget constraint is then

$$c(s^t) + x(s^t) + b(s^t) + \kappa\big(b(s^{t-1})\big) \leq w(s^t)l(s^t) + r(s^t)k(s^{t-1}) + b(s^{t-1})R(s^{t-1}). \tag{2.6}$$

Bond holding costs are introduced to ensure that the model is stationary, as small open economy models with incomplete asset markets imply steady states that depend on initial conditions and dynamics that display random walk features (Schmitt-Grohe & Uribe, 2003).

The law of motion for capital is given by

$$x(s^t) = k(s^t) - (1-\delta)k(s^{t-1}) + \Phi\big(k(s^{t-1}), k(s^t)\big) \tag{2.7}$$

where $\Phi\big(k(s^{t-1}), k(s^t)\big)$ is a capital adjustment cost, commonly used to avoid excessive investment volatility.

Then, given a sequence for prices, $\{w(s^t), r(s^t), R(s^t)\}_{t=0}^{\infty}$ and initial conditions $k(s_0)$ and $b(s_0)$, the household maximizes its expected utility subject to the sequence of budget constraints, the law of motion and a no-Ponzi condition by choosing infinite sequences of consumption, investment and bond holdings.

2.3.1 Functional forms

The functional forms for the adjustment costs are given by

$$\kappa\big(b(s^{t-1})\big) = \frac{\kappa}{2}y(s^t)\left[\frac{b(s^{t-1})}{y(s^t)} - \frac{b}{y}\right]^2 \tag{2.8}$$

for the bond holdings and

$$\Phi\big(k(s^{t-1}), k(s^t)\big) = \frac{\phi}{2}k(s^{t-1})\left[\frac{k(s^t) - k(s^{t-1})(1+\gamma)}{k(s^{t-1})}\right]^2 \tag{2.9}$$

for capital. These forms ensure that the costs are zero along a balanced growth (steady state) path, and that they remain constant as the economy grows. Note that variables without s^t denote steady state values.

Two alternative specifications are used for preferences. The first is known as GHH preferences (Greenwood et al., 1988), commonly used in small open economy models such as Mendoza (1991) and Correia et al. (1995). It takes the form

$$u(c_t, l_t) = \frac{1}{1-\sigma}[c_t - \psi(1+\gamma)^t l_t^\nu], \quad \nu > 1, \psi > 0. \tag{2.10}$$

These preferences generate a labor supply curve that is independent of consumption, and is crucial to obtaining the NP results. Here, technological progress is assumed to increase the utility of leisure. This is necessary for the model to have a balanced growth path.

The second specification utilizes the more standard Cobb-Douglas (CD) preferences, and is used to determine the sensitivity of the results to the functional form of preferences. CD preferences are given by

$$u(c_t, l_t) = \frac{1}{1-\sigma}[c_t^\mu (1-l_t)^{1-\mu}], \quad \mu \in (0,1) \tag{2.11}$$

Since the recursive methods used in macroeconomics require variables to be stationary, all growing variables are detrended. For a trend-stationary variable z_t, define $\tilde{z}_t = \frac{z_t}{(1+\gamma)^t}$. Then we can rewrite the preferences as

$$u(\tilde{c}, l) = \frac{(1+\gamma)^{t(1-\sigma)}}{1-\sigma}\Big(\tilde{c}(s^t) - \psi l(s^t)^\nu\Big)^{1-\sigma} \tag{2.12}$$

for GHH and

$$u(\tilde{c}, l) = \frac{(1+\gamma)^{t\mu(1-\sigma)}}{1-\sigma}\Big(\tilde{c}(s^t)^\mu [1 - l(s^t)]^{1-\mu}\Big)^{1-\sigma} \tag{2.13}$$

for CD.

The budget constraint and law of motion for capital can also be rewritten in terms of detrended variables as follows:

$$\tilde{c}(s^t) + \tilde{x}(s^t) + \frac{\kappa}{2}\tilde{y}(s^t)\Big[\frac{\tilde{b}(s^{t-1})}{\tilde{y}(s^t)} - \frac{b}{y}\Big]^2 \leq \tilde{w}(s^t)l(s^t) + r(s^t)\tilde{k}(s^{t-1}) + \tilde{b}(s^{t-1})R(s^{t-1}) - (1+\gamma)\tilde{b}(s^t) \tag{2.14}$$

$$\tilde{x}(s^t) = (1+\gamma)\tilde{k}(s^t) - (1-\delta)\tilde{k}(s^{t-1}) + \frac{\phi}{2}\tilde{k}(s^{t-1})(1+\gamma)^2\Big[\frac{\tilde{k}(s^t) - \tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\Big]^2. \tag{2.15}$$

2.3.2 Household's Problem

After detrending, the household's problem can now be expressed by the following Lagrangian:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\sum_{s^t}\Big\{&\beta^{*t}\pi(s^t)u[\tilde{c}(s^t), l(s^t)] + \lambda(s^t)\Big(\tilde{w}(s^t)l(s^t) + r(s^t)\tilde{k}(s^{t-1}) \\ &+ \tilde{b}(s^{t-1})R(s^{t-1}) - (1+\gamma)\tilde{b}(s^t) - \tilde{c}(s^t) - (1+\gamma)\tilde{k}(s^t) + (1-\delta)\tilde{k}(s^{t-1}) \\ &- \frac{\phi}{2}\tilde{k}(s^{t-1})(1+\gamma)^2\Big[\frac{\tilde{k}(s^t) - \tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\Big]^2 - \frac{\kappa}{2}\tilde{y}(s^t)\Big[\frac{\tilde{b}(s^{t-1})}{\tilde{y}(s^t)} - \frac{b}{y}\Big]^2\Big)\Big\},\end{aligned} \tag{2.16}$$

where $\beta^* = \beta(1+\gamma)^{(1-\sigma)}$ for GHH preferences and $\beta(1+\gamma)^{\mu(1-\sigma)}$ for CD.

The household's first order conditions are as follows. In each of the following two pairs of equations, the first corresponds to the GHH specification and the second to the CD specification. For $\tilde{c}(s^t)$, we have

$$\beta^{*t}\pi(s^t)[\tilde{c}(s^t) - \psi l(s^t)^{\nu}]^{-\sigma} = \lambda(s^t) \tag{2.17}$$

$$\beta^{*t}\pi(s^t)\frac{\mu}{\tilde{c}(s^t)}\Big[\tilde{c}(s^t)^{\mu}(1 - l(s^t))^{1-\mu}\Big]^{1-\sigma} = \lambda(s^t). \tag{2.18}$$

For $l(s^t)$,

$$\beta^{*t}\pi(s^t)\left[\tilde{c}(s^t) - \psi l(s^t)^\nu\right]^{-\sigma}\left[\psi\nu l(s^t)^{\nu-1}\right] = \lambda(s^t)\tilde{w}(s^t) \tag{2.19}$$

$$\beta^{*t}\pi(s^t)\frac{1-\mu}{1-l(s^t)}\left[\tilde{c}(s^t)^\mu\left(1-l(s^t)\right)^{1-\mu}\right]^{1-\sigma} = \lambda(s^t)\tilde{w}(s^t) \tag{2.20}$$

Since the utility function does not involve bonds or capital, the first order conditions for these are the same under both specifications. For $\tilde{b}(s^t)$, this is

$$\lambda(s^t)(1+\gamma) = \sum_{s^{t+1}} \lambda(s^{t+1})\left[R(s^t) - \kappa\left(\frac{\tilde{b}(s^t)}{\tilde{y}(s^{t+1})} - \frac{b}{y}\right)\right], \tag{2.21}$$

and for $\tilde{k}(s^t)$,

$$\begin{aligned} &\lambda(s^t)(1+\gamma)\left\{1 + \phi(1+\gamma)\left[\frac{\tilde{k}(s^t) - \tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\right]\right\} \\ &= \sum_{s^{t+1}} \lambda(s^{t+1})\left\{r(s^{t+1}) + 1 - \delta - \frac{\phi}{2}(1+\gamma)^2\left[1 - \frac{\tilde{k}(s^{t+1})^2}{\tilde{k}(s^t)^2}\right]\right\}. \end{aligned} \tag{2.22}$$

2.4 Equilibrium

Given initial conditions for capital and bond holdings and state-contingent sequences of interest rates and TFP, the state-contingent sequences of allocations $\{c(s^t), x(s^t), b(s^t), k(s^t), l(s^t)\}_{t=0}^{\infty}$ and prices $\{w(s^t), r(s^t)\}_{t=0}^{\infty}$ constitute an equilibrium if

1. the allocations solve the household's problem given equilibrium prices and

2. the markets for factor inputs clear.

A balanced growth path (BGP or steady state) is an equilibrium along which $R(s^t)$ and $A(s^t)$ are constant. Along a BGP, labor hours $l(s^t)$ are constant while all other quantities grow at the same constant rate γ.

From the consumption first order conditions, the lagrange multiplier for the history s^t can be obtained. Substituting this into each of the other first order conditions of the household, and replacing the prices $r(s^{t+1})$ and $\tilde{w}(s^t)$ with their corresponding expressions from the firm's solution, we obtain the equilibrium equations. For GHH preferences, we have

1. Labor

$$\begin{aligned} \psi\nu l(s^t)^{\nu-1} &= \tilde{w}(s^t) = (1-\alpha)\frac{\tilde{y}(s^t)}{l(s^t)\{1+\theta[R(s^{t-1})-1]\}} \\ \Longrightarrow \psi\nu l(s^t)^{\nu} &= (1-\alpha)\frac{\tilde{y}(s^t)}{1+\theta[R(s^{t-1})-1]} \end{aligned} \tag{2.23}$$

2. Bonds

$$1 = \frac{\beta^*}{1+\gamma}\mathbb{E}_t\left\{\left[\frac{\tilde{c}(s^{t+1})-\psi l(s^{t+1})^{\nu}}{\tilde{c}(s^t)-\psi l(s^t)^{\nu}}\right]^{-\sigma}\left[R(s^t)-\kappa\left(\frac{\tilde{b}(s^t)}{\tilde{y}(s^{t+1})}-\frac{b}{y}\right)\right]\right\} \tag{2.24}$$

3. Capital

$$\begin{aligned} &\left[1+\phi(1+\gamma)\left(\frac{\tilde{k}(s^t)-\tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\right)\right] \\ &= \frac{\beta^*}{(1+\gamma)}\mathbb{E}_t\left\{\left[\frac{\tilde{c}(s^{t+1})-\psi l(s^{t+1})^{\nu}}{\tilde{c}(s^t)-\psi l(s^t)^{\nu}}\right]^{-\sigma}\left[\alpha\frac{\tilde{y}(s^{t+1})}{\tilde{k}(s^t)}+1-\delta-\frac{\phi}{2}(1+\gamma)^2\left(1-\frac{\tilde{k}(s^{t+1})^2}{\tilde{k}(s^t)^2}\right)\right]\right\} \end{aligned} \tag{2.25}$$

and for CD,

1. Labor

$$\begin{aligned} \frac{1-\mu}{\mu}\frac{\tilde{c}(s^t)}{1-l(s^t)} &= \tilde{w}(s^t) = (1-\alpha)\frac{\tilde{y}(s^t)}{l(s^t)\{1+\theta[R(s^{t-1})-1]\}} \\ \Longrightarrow \frac{1-\mu}{\mu}\frac{l(s^t)}{1-l(s^t)} &= (1-\alpha)\frac{\tilde{y}(s^t)}{\tilde{c}(s^t)\{1+\theta[R(s^{t-1})-1]\}} \end{aligned} \tag{2.26}$$

2. Bonds

$$1 = \frac{\beta^*}{1+\gamma}\mathbb{E}_t\left\{\frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})}\left[\frac{\tilde{c}(s^{t+1})^{\mu}(1-l(s^{t+1}))^{1-\mu}}{\tilde{c}(s^t)^{\mu}(1-l(s^t))^{1-\mu}}\right]^{1-\sigma}\left[R(s^t)-\kappa\left(\frac{\tilde{b}(s^t)}{\tilde{y}(s^{t+1})}-\frac{b}{y}\right)\right]\right\} \tag{2.27}$$

3. Capital

$$
\begin{aligned}
&\left[1+\phi(1+\gamma)\left(\frac{\tilde{k}(s^t)-\tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\right)\right] \qquad (2.28)\\
&=\frac{\beta^*}{(1+\gamma)}\mathbb{E}_t\left\{\frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})}\left[\frac{\tilde{c}(s^{t+1})^{\mu}\left(1-l(s^{t+1})\right)^{1-\mu}}{\tilde{c}(s^t)^{\mu}\left(1-l(s^t)\right)^{1-\mu}}\right]^{1-\sigma}\left[\alpha\frac{\tilde{y}(s^{t+1})}{\tilde{k}(s^t)}+1-\delta-\frac{\phi}{2}(1+\gamma)^2\left(1-\frac{\tilde{k}(s^{t+1})^2}{\tilde{k}(s^t)^2}\right)\right]\right\}
\end{aligned}
$$

The equilibrium equation for labor can be interpreted as stating that the marginal disutility of an additional hour of work should be equal to the marginal gain given by the wage rate. The equation for bonds can be understood better by rewriting it as

$$
u_1'(\tilde{c}_t,l_t)=\frac{\beta^*}{1+\gamma}\mathbb{E}_t\left\{u_1'(\tilde{c}_{t+1},l_{t+1})\left[R(s^t)-\kappa\left(\frac{\tilde{b}(s^t)}{\tilde{y}(s^{t+1})}-\frac{b}{y}\right)\right]\right\}, \qquad (2.29)
$$

which requires the marginal utility lost by postponing consumption to buy bonds to be equal to the discounted expected gain, given by the gross interest rate net of the marginal cost of holding on to the bonds. Similarly, the equation for capital can be written as

$$
\begin{aligned}
&u_1'(\tilde{c}_t,l_t)\left[1+\phi(1+\gamma)\left(\frac{\tilde{k}(s^t)-\tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\right)\right] \qquad (2.30)\\
&=\frac{\beta^*}{1+\gamma}\mathbb{E}_t\left\{u_1'(\tilde{c}_{t+1},l_{t+1})\left[\alpha\frac{\tilde{y}(s^{t+1})}{\tilde{k}(s^t)}+1-\delta-\frac{\phi}{2}(1+\gamma)^2\left(1-\frac{\tilde{k}(s^{t+1})^2}{\tilde{k}(s^t)^2}\right)\right]\right\},
\end{aligned}
$$

requiring the marginal utility lost by giving up current consumption to accumulate capital (and pay the marginal cost of adjusting it) to be equal to the discounted expected additional consumption possibility in the future due to the additional output that can be produced from it ($\alpha\tilde{y}_{t+1}/\tilde{k}_t$ is the marginal product of capital), plus the capital good's continuation value net of depreciation and adjustment costs.

In addition to these equations, the production function and budget constraint must hold at equality.

$$
\tilde{y}(s^t)=A(s^t)\tilde{k}(s^{t-1})^{\alpha}l(s^t)^{1-\alpha} \qquad (2.31)
$$

$$\tilde{c}(s^t) = \tilde{w}(s^t)l(s^t) + r(s^t)\tilde{k}(s^{t-1}) + \tilde{b}(s^{t-1})R(s^{t-1}) - (1+\gamma)\tilde{b}(s^t) \tag{2.32}$$
$$- (1+\gamma)\tilde{k}(s^t) + (1-\delta)\tilde{k}(s^{t-1}) - \frac{\phi}{2}\tilde{k}(s^{t-1})(1+\gamma)^2\Big[\frac{\tilde{k}(s^t) - \tilde{k}(s^{t-1})}{\tilde{k}(s^{t-1})}\Big]^2$$
$$- \frac{\kappa}{2}\tilde{y}(s^t)\Big[\frac{\tilde{b}(s^{t-1})}{\tilde{y}(s^t)} - \frac{b}{y}\Big]^2$$

2.5 Shock Processes

To close the model, the shock processes for TFP and interest rates need to be defined.

2.5.1 Productivity Shocks

The process for productivity is assumed to follow an AR(1) process given by

$$\hat{A}(s^t) = \rho_A \hat{A}(s^{t-1}) + \varepsilon_{A\ t}, \quad \varepsilon_{A\ t} \sim \mathcal{N}(0, \sigma_A) \tag{2.33}$$

where a hat denotes deviation from the steady state.

2.5.2 Interest Rates and Country Risk

Assume a large mass of international investors willing to lend any amount to the developing economy at the given rate $R(s^t)$. Although the domestic agents in this model always make good on their loan obligations, it is assumed that in every period, there is a time-varying probability that the government will confiscate any payments made by local borrowers to foreign lenders[1].[2] This makes loans to the economy risky, creating two volatility sources for the interest rate:

1. Interest rates can change as the world expectations of default risk in the economy change, and

[1] This simplistic way of modeling default risk is also used in Kehoe & Perri (2004).
[2] Domestic lenders (the household) will always receive full payment for their loaned funds.

2. Risk preferences of international lenders may change.

To capture these two sources, the interest rate faced by an emerging country is decomposed into two components:

$$R(s^t) = R^*(s^t)D(s^t), \tag{2.34}$$

or in linearized form,

$$\hat{R}(s^t) = \hat{R}^*(s^t) + \hat{D}(s^t) \tag{2.35}$$

where R^* is the international rate for risky assets, which is not specific for any particular economy, and D is the country risk spread, which is assumed to be driven by underlying fluctuations in the government's confiscation probability.

R^* is identified as the US rate for risky assets, and is completely independent of conditions in the local economy. Fluctuations in D, on the other hand, is modeled in two very simple, alternative ways.

In the first approach, the *independent country risk case*, factors independent of domestic productivity shocks, such as politics or foreign events, determine country risk. Then both R^* and D are unaffected by local economic fundamentals. Both components are modeled as AR(1) processes with independent innovations, given by

$$\hat{R}^*(s^t) = \rho_1 \hat{R}^*(s^{t-1}) + \varepsilon_{R\ t}, \quad \varepsilon_{R\ t} \sim \mathcal{N}(0, \sigma_R) \tag{2.36}$$

and

$$\hat{D}(s^t) = \rho_2 \hat{D}(s^{t-1}) + \varepsilon_{D\ t}, \quad \varepsilon_{D\ t} \sim \mathcal{N}(0, \sigma_D). \tag{2.37}$$

The second approach, called the *induced country risk case*, proposes that fundamental TFP shocks drive both the business cycle and country risk. It utilizes theoretical results from models of default and incomplete markets by Eaton & Gersovitz (1981) and Arellano (2008) to develop a reduced form specification in which default risk is higher when expected productivity (one period ahead) is lower. In particular,

$$D(s^t) = \eta(\mathbb{E}_t A(s^{t+1})), \tag{2.38}$$

where $\eta(\cdot)$ is a decreasing function. In linearized form, this becomes

$$\hat{D}(s^t) = -\eta \mathbb{E}_t[\hat{A}(s^{t+1})] + \varepsilon_{I\ t}, \quad \varepsilon_{I\ t} \sim \mathcal{N}(0, \sigma_I) \tag{2.39}$$

Note that in this case, combined with the dependence of labor and output on interest rates (due to the working capital constraint), the effects of productivity shocks are amplified by the fact that they also affect country risk.

2.6 Other Equations

Although the model is already closed, it is useful to keep track of equilibrium equations for two more variables on which the literature usually places a lot of importance. For investment, it is as it was defined in (2.15). Net exports are goods produced in the country that are not used for consumption, investment or bond holding costs, divided by output.

$$\begin{aligned} NX(s^t) &= \frac{\tilde{nx}(s^t)}{\tilde{y}(s^t)} \\ &= \frac{\tilde{y}(s^t) - \tilde{c}(s^t) - \tilde{x}(s^t) - \frac{\kappa}{2}\tilde{y}(s^t)\left[\frac{\tilde{b}(s^{t-1})}{\tilde{y}(s^t)} - \frac{b}{y}\right]^2}{\tilde{y}(s^t)}. \end{aligned} \tag{2.40}$$

Also, the country's net foreign asset position for period t, despite not being used in the empirical analysis, is useful when solving for the steady state. It is defined as the household's net foreign asset position $b(s^{t-1})$ minus the firm's working capital debt, $\theta w(s^t)l(s^t)$.

2.7 Steady State Solution

In this section, the nonstochastic steady state of each variable is calculated as a function of the structural model parameters, so that the model solution can be linearized around these given the parameter values. To do so, set all shocks to their unconditional mean of zero, and assume that TFP shocks are transitory, so that the steady state level of TFP, A, is unity. Then all variables will remain constant, so we denote steady states by the corresponding symbols without time subscripts.

The steady state equations for labor under the GHH and CD specifications, respectively, are

1. GHH Preferences:

$$\psi \nu l^{\nu} = \frac{(1-\alpha)y}{1+\theta(R-1)} \tag{2.41}$$

$$\psi \nu l^{\nu-1} = \frac{(1-\alpha)y}{l[1+\theta(R-1)]} \tag{2.42}$$

2. Cobb-Douglas Preferences:

$$\frac{1-\mu}{\mu}\frac{l}{1-l} = \frac{(1-\alpha)y}{c[1+\theta(R-1)]} \tag{2.43}$$

$$\frac{1-\mu}{\mu}\frac{c}{1-l} = \frac{(1-\alpha)y}{l[1+\theta(R-1)]} \tag{2.44}$$

For bonds, we have

$$\frac{1+\gamma}{\beta^*} = R. \tag{2.45}$$

For capital,

$$1+\gamma = \beta^*\left[\frac{\alpha y}{k} + 1 - \delta\right] \tag{2.46}$$

$$\Longrightarrow \frac{1+\gamma}{\beta^*} + \delta - 1 = \frac{\alpha y}{k}.$$

Note how this constitutes a long run no-arbitrage condition when combined with (2.45). Steady state output is given by

$$y = k^{\alpha} l^{1-\alpha}. \tag{2.47}$$

The budget constraint is

$$c = wl + (r - \gamma - \delta)k + (R - 1 - \gamma)b. \tag{2.48}$$

Investment and net exports over GDP are given by

$$\frac{x}{y} = (\gamma + \delta)\frac{k}{y} \tag{2.49}$$

and

$$NX = \frac{nx}{y} = 1 - \frac{c}{y} - \frac{x}{y}. \tag{2.50}$$

The steady state values can be obtained by the following steps. From (2.45), the steady state interest rate is given by $R = \frac{1+\gamma}{\beta^*}$. Then from (2.46), the capital-output ratio is

$$\frac{k}{y} = \frac{\alpha}{\frac{1+\gamma}{\beta^*} + \delta - 1}. \tag{2.51}$$

l/y can be obtained from the output equation as

$$1 = \left(\frac{k}{y}\right)^{\alpha}\left(\frac{l}{y}\right)^{1-\alpha} \tag{2.52}$$

$$\Longrightarrow \frac{l}{y} = \left[\frac{1}{(k/y)^{\alpha}}\right]^{\frac{1}{1-\alpha}} \tag{2.53}$$

From (2.42), steady state wage is

$$w = \psi \nu l^{\nu-1}. \tag{2.54}$$

Then the household's net foreign asset position, b/y can be calculated given the long run average of Argentina's net foreign asset position of -0.42, which Neumeyer and Perri calculated from cumulated capital flows from 1983 to 1998. The data for these was taken from Lane &

Milesi-Ferretti (2001). Specifically,

$$-0.42 = \frac{\theta wl}{y} - \frac{b}{y}. \tag{2.55}$$

After all of these ratios have been calculated, c/y can be obtained from

$$\begin{aligned} \frac{c}{y} &= w\frac{l}{y} + r\frac{k}{y} + \frac{b}{y}(R - 1 - \gamma) - (\gamma + \delta)\frac{k}{y} \\ \Longrightarrow \frac{c}{y} &= \text{labor share} + \alpha + \frac{b}{y}(R - 1 - \gamma) - \frac{x}{y}. \end{aligned} \tag{2.56}$$

Once we have all these ratios, if we can solve for steady state output, all of the other variables can already be solved for. To do this, each preference specification must be considered in turn. For GHH preferences, use (2.41):

$$\begin{aligned} \frac{\psi \nu l^{\nu}}{y} &= \frac{(1-\alpha)}{1+\theta(R-1)} \\ \Longrightarrow \frac{1-\alpha}{\psi\nu[1+\theta(R-1)]} &= \left(\frac{l}{y}\right)^{\nu}\left(\frac{1}{y}\right)^{1-\nu} \\ \Longrightarrow y &= \left[\frac{\psi\nu[1+\theta(R-1)](l/y)^{\nu}}{1-\alpha}\right]^{\frac{1}{1-\nu}}. \end{aligned} \tag{2.57}$$

For CD, use (2.44) and average hours worked (steady state l) from the data to get

$$c = \frac{1-\alpha}{1+\theta(R-1)}\left(\frac{y}{l}\right)\left(\frac{\mu}{1-\mu}\right)(1-l) \tag{2.58}$$

which yields

$$y = \frac{c}{c/y} \tag{2.59}$$

2.8 Calibration

The calculations in the previous section solve for steady state values as functions of model parameters. To calibrate these parameters, they are divided into three groups - those set beforehand, those set to match long run averages (steady states) of the data, and free parameters.

The parameters set beforehand are the curvature of the per-period utility function, σ, and the curvature of labor in the GHH preferences, ν. The former is set to 5, following Reinhart & Vegh (1995), who estimate Argentina's intertemporal elasticity of substitution (given by $\frac{1}{\sigma}$) to be 0.21. Both Neumeyer & Perri (2005) and Uribe & Yue (2006) indicate the lack of estimates for the wage elasticity of labor supply in emerging economies, given by $\frac{1}{\nu-1}$ in the model. See page 41 of del Negro & Schorfheide (2010) for more on the varied estimates for this elasticity in the US. In terms of GHH preferences, Mendoza (1991) sets ν to 1.455 while Correia et. al (1995) sets this to 1.7, implying an elasticity of 2.2 and 1.4, respectively. Neumeyer and Perri then set $\nu = 1.6$, the average between the two.

$\gamma, \beta, \psi, \mu, \alpha$, and δ are all set such that the model's steady states are consistent with long run averages in the data. γ was set to match the average growth rate of Argentine real output. Based on this, the average real quarterly interest rate R, and (2.45), β can be set using the following:

$$\beta = \frac{1+\gamma}{R(1+\gamma)^{1-\sigma}} \tag{2.60}$$

for GHH preferences and

$$\beta = \frac{1+\gamma}{R(1+\gamma)^{\mu(1-\sigma)}} \tag{2.61}$$

for Cobb-Douglas (note that the latter can only be calibrated after μ is determined). α is calibrated to match labor's share of income, given as 0.6 in NP. In the model, labor's share is $\frac{1-\alpha}{1+\theta(R-1)}$, which gives α as

$$\alpha = 1 - 0.6[1 + \theta(R-1)]. \tag{2.62}$$

δ is set to match the average investment to output ratio in the data, or x/y in the model. Then, combining (2.45), (2.46) and (2.49),

$$\frac{x}{y} = (\gamma + \delta)\frac{k}{y} \tag{2.63}$$

$$= (\gamma + \delta)\frac{\alpha}{\frac{1+\gamma}{\beta^*} + \delta - 1} \tag{2.64}$$

$$= (\gamma + \delta)\frac{\alpha}{R + \delta - 1} \tag{2.65}$$

which yields

$$\delta = \frac{\frac{x/y}{\alpha}(R-1) - \gamma}{1 - \frac{x/y}{\alpha}} \tag{2.66}$$

ψ (under GHH) and μ (under Cobb-Douglas) are set to match an average time spent working of around 20% of total time. Note, first, that once we know α, R and δ, we can use (2.46) and (2.47) to determine $\frac{l}{y}$ as

$$\frac{l}{y} = \left[\frac{1}{(k/y)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$$
$$= \left[\frac{1}{\left(\frac{\alpha}{R+\delta-1}\right)^{\alpha}}\right]^{\frac{1}{1-\alpha}}.$$

Now, with the fact that steady state labor share is 0.6, and that average time working is 0.2 (steady state l), we can use (2.42) to determine ψ:

$$\psi = \frac{0.6}{\nu l^{\nu-1}}\frac{y}{l}. \tag{2.67}$$

As for μ, begin by calculating the steady state consumption to output ratio:

$$\frac{c}{y} = \psi\nu l^{\nu-1}\frac{l}{y} + \alpha + \frac{b}{y}(R - 1 - \gamma) - (\gamma + \delta)\frac{k}{y}$$
$$= \text{labor share} + \alpha + \frac{b}{y}(R - 1 - \gamma) - \frac{x}{y}$$

Then, using (2.43) and $l = 0.2$, calculate μ as

$$\begin{aligned}
\frac{1-\mu}{\mu}\frac{l}{1-l} &= \frac{(1-\alpha)y}{c[1+\theta(R-1)]} \\
&\Longrightarrow \frac{1}{\mu} - 1 = 0.6\Big(\frac{1-l}{l}\Big)\frac{y}{c} \\
&\Longleftrightarrow \mu = \frac{1}{1+0.6\big(\frac{1-l}{l}\big)\frac{y}{c}} \qquad (2.68)
\end{aligned}$$

We turn now to the free parameters. Since ϕ mostly affects the volatility of investment relative to output, it is set to match this statistic in the data. κ is set to the minimum value that guarantees that the equilibrium solution is stationary (i.e. that the Blanchard-Kahn conditions are met). θ is arbitrarily set to 1.

As for the shock processes, the parameter ρ_A is set to be equal to the persistence of TFP commonly used in US studies, which is 0.95. Technically, this process should be calibrated using reliable estimates of the Solow residuals for Argentina. However, this may not be feasible given the unreliability of Argentine labor data.

Whenever productivity shocks are turned on in the simulations, the standard deviation σ_A is set to match the standard deviation of output in the data.

In the Neumeyer and Perri paper, the exact innovations to both components of the interest rate are fed into the independent country risk model. Here, I take the persistences and standard deviations of the estimated AR(1) processes for the US Tbill rate and the country spread data to calibrate ρ_1, ρ_2, σ_R, and σ_D.

For the induced country risk model, the process for the interest rate is given by

$$\begin{aligned}
\hat{R}(s^t) &= \hat{R}^*(s^t) - \eta\mathbb{E}_t\{\hat{A}(s^{t+1})\} + \varepsilon_{I\ t}, \quad \eta > 0 \\
&= \hat{R}^*(s^t) - \eta\rho_A\hat{A}(s^{t+1}) + \varepsilon_{I\ t}. \qquad (2.69)
\end{aligned}$$

Given the estimated AR(1) processes for TFP and the Tbill rates, η and σ_I are set such that the model-generated interest rates have exactly the same persistence and standard deviation as the data. In particular, using (2.69), the variance of $\hat{R}$ is

$$Var(\hat{R}) = Var(\hat{R}^*) + \eta^2\rho_A^2 Var(\hat{A}) + \sigma_I^2. \tag{2.70}$$

As for the persistence of the interest rate, we have

$$\begin{aligned}\mathbb{E}[\hat{R}(s^t)\hat{R}(s^{t-1})] = \rho(\hat{R})Var(\hat{R}) &= \mathbb{E}[\hat{R}^*(s^{t-1})\hat{R}^*(s^{t-2})] + \eta^2\rho_A^2\mathbb{E}[\hat{A}(s^t)\hat{A}(s^{t-1})] \\ &= \rho_1 Var(\hat{R}^*) + \eta^2\rho_A^3 Var(\hat{A}) \end{aligned} \tag{2.71}$$

so that η^2 is given by

$$\eta^2 = \frac{\rho(\hat{R})Var(\hat{R}) - \rho_1 Var(\hat{R}^*)}{\rho_A^3 Var(\hat{A})}\ ,^3 \tag{2.72}$$

where $\rho(\hat{R})$, and $Var(\hat{R})$ are the estimates of serial correlation and variance of HP-filtered data on interest rates.

[3] $Var(\hat{R}^*) = \frac{\sigma_R^2}{1-\rho_1^2}$. Similarly for $Var(\hat{A})$.

Chapter 3

Replication

In this chapter, the simulation results for the model are presented and analyzed. In the first section, the published results in Neumeyer & Perri (2005) are discussed, while the second section presents the calibration and results obtained from simulating the model using the data described in Chapter 1.4.2.

3.1 Published Results

3.1.1 Calibration

To calibrate the parameters which are set to match long run averages, we need average growth rate of output, the average real interest rate, average labor share of income, average investment/output ratio, and average time spent working. In the Neumeyer and Perri paper, real output grew by an average of 2.5% per year, or 0.62% on a quarterly basis. Average real interest rate was 14.8% per annum or 3.5% per quarter. As previously mentioned, average time spent working and labor share were 20% and 60%, respectively. Finally, the investment to output ratio was reported as 0.21.

The reported persistence parameters for $\hat{R}^*$ and $\hat{D}$ were 0.81 and 0.78, respectively. The corresponding standard deviations of the innovations were 0.0063 and 0.0259.

The published calibration is given in Table 3.1. Note that many of these parameters do not correspond to the implied values if we plugged the given averages above into the calibration relations in the previous chapter. The correct values, at least for those set to match steady state relations, are also reported in Table 3.1.

However, I still choose to report the results as published in the paper, and defer using correct calibration restrictions until the replication I perform using my own data. The reason for this is that data provided by Neumeyer and Perri give only the interest rate, not its decomposition into the international and country risk components. This makes calibration of the shock processes based on the original dataset impossible.

3.1.2 Computational Experiments

To determine whether the model can explain the effects of interest rates and country risk on Argentine business cycles, Neumeyer and Perri conduct five computational experiments. The first simulates the model with only the international risky rate shock active (R^*). They feed actual innovations of their international rate data into the model to achieve the actual cyclical behavior of this series. In the second simulation, they add productivity shocks. In both experiments, ϕ is set such that the volatility of investment relative to output of the model with both R^* and A shocks exactly matches the data. σ_A in the second experiment is set so that the model's relative standard deviation of output matches the actual statistic in the data. The results for these two are not presented here. Note only that the model with only international risky rate shocks produces an output series that is only 30% as volatile as actual GDP. However, when productivity shocks are added, the model fails to produce countercyclical interest rates.

The third and fourth experiments correspond to the model of independent country risk, with TFP shocks turned off and on, respectively. As before, Neumeyer and Perri feed the exact

Table 3.1: Baseline Parameter Values

Shocks	NP Values		New Data	
Productivity	$\rho_A = 0.95$	σ_A = Varies	$\rho_A = 0.95$	σ_A = Varies[a]
International rate	$\rho_1 = 0.81$	$\sigma_R = 0.006$	$\rho_1 = 0.85$	$\sigma_R = 0.001$
Independent country risk	$\rho_2 = 0.78$	$\sigma_D = 0.026$	$\rho_2 = 0.76$	$\sigma_D = 0.020$
Induced country risk	$\eta = 1.04$	$\sigma_I = 0.017$	$\eta = 0.35$	$\sigma_I = 0.014$

Preference parameters	Symbol	GHH		CD	
NP		Published	Corrected	Published	Corrected
Discount factor	β	0.93	0.996	0.98	0.98
Utility curvature	σ	5	5	5	5
Labor curvature	ν	1.6	1.6	-	-
Labor weight	ψ	2.48	2.89	-	-
Consumption share	μ	-	-	0.24	0.26

Preference parameters	Symbol	GHH	CD
New Data			
Discount factor	β	0.988	0.97
Utility curvature	σ	5	5
Labor curvature	ν	1.6	-
Labor weight	ψ	2.465	-
Consumption share	μ	-	0.255

Technology parameters	Symbol	NP calibration		New Data
		Published	Corrected	
Technological progress	γ	0.0062	0.0062	0.0065
Exponent on k in production	α	0.38	0.38	0.3727
Depreciation rate	δ	0.044	0.030	0.034
% labor paid in advance	θ	1	1	1
Bond holding cost	κ	10^{-5}	10^{-5}	10^{-10}
Capital adjustment cost	ϕ	Varies	-	Varies [b]

[a] Set to match volatility of output in experiments when productivity shocks are turned on. In the replication, this is set to 0.0251 in the independent country risk case and 0.0235 in the induced country risk case. In NP, these are 0.0175 and 0.0147, respectively.

[b] Set to match the volatility of investment relative to output. In the replication, this is is set to 12.5 and 19 in the independent and induced country risk cases, respectively. In NP, these are 25.5 and 40.

innovations from the data for the components of the interest rate into the model. ϕ in both models is set to match the volatility of investment relative to output produced by the model with R^*, D and A shocks, and σ_A is set to match actual volatility of output in the data. Lines (a1) and (b1) in Table 3.2 present the published results for the third and fourth simulation, respectively. From the lines of the table labeled (a1), we see that the % standard deviation of output generated by the model is roughly half of that in the data, while it overpredicts the relative volatility of consumption, investment, and hours. Moreover, it generates correlations of GDP with net exports and investment that are too small in magnitude compared to the data, and a correlation of output with hours that is too big. It also produces correlations of R with the other variables that are too big in absolute value compared to the data. It is promising, though, that even without TFP shocks, the model can already produce countercyclical interest rates and net exports.

From the lines of the table labeled (b1), we see that although the model can now reproduce the volatility of consumption relative to output, the generated correlations with GDP are now worse than the previous simulation. Both interest rates and net exports are much less countercyclical, while improvements in the rest of the reported correlations are marginal. Neumeyer and Perri propose that these discrepancies are probably due to the fact that in the current experiment, country risk affects business cycles, but not the other way around. Allowing business cycles to influence country risk should generate more countercyclical interest rates. Also, as discussed in Chapter 1, an increase in interest rates will tend to generate a large change in net exports in this model. Hence, with a strong negative correlation between interest rates and output, the model would be able to generate strongly countercyclical net exports as well.

The fifth computational experiment does exactly this. It uses the model of induced country risk, that is, with shocks to R^* and A. Once again the exact innovations from the data of R^* are fed into the model, but D is now generated by (2.38). ϕ and σ_A are set in the same way as in the previous simulations. The published results for this model are in the lines labeled as (c1) on the table. From this we see that the model with induced country risk is able to reproduce most of the correlations with output and interest rates of the main macroeconomic aggregates. In particular, the countercyclicality of interest rates and net exports are now close to their counterparts in the data. The three major discrepancies that remain are that the model overpredicts the volatility of hours, consumption and net exports relative to output. To explain the first of these, Neumeyer and Perri reiterate their doubt regarding the quality of labor data in Argentina. To explain the volatility of consumption and net exports, they note that in the model, the household directly faces the volatile interest rate. As large fluctuations happen in R, the household substitutes consumption intertemporally, leading to high volatility of consumption and domestic absorption. One way they deal with this is by decreasing the intertemoral elasticity of substitution (increasing σ from 5 to 50), the results for which are presented in the sensitivity analysis below.

Nonetheless, the model with induced country risk already does a decent job of accounting for most of the Argentine business cycle facts, so they use this to quantitatively estimate the importance of interest rate shocks to the volatility of output cycles. If shocks to the international risky rate R^* are shut down, the percentage standard deviation of GDP becomes 4.10, a mere 3% decrease from the corresponding statistic in the model with all shocks. The reason for this small change is that the volatility of R^* innovations is much smaller relative to that of innovations of the country risk component, D.

Similarly, to estimate the contribution of fluctuations in D, the model is simulated with η and σ_I set to zero. In this case, the amplification of productivity shocks due to country risk is absent in the model. This simulation gives the percentage standard deviation of GDP as 3.06, a 27% drop from the model with the full set of shocks.

Neumeyer and Perri conclude that large fluctuations in country risk seem to be related to fluctuations in economic activity in developing markets. Both the amplification effect of country risk on economic fluctuations through the working capital constraint and the effect of economic fundamentals on the country risk (as given by (2.38)) need to be investigated further.

Table 3.2: Simulated and Actual Argentine Business Cycles[1]

	% std dev			$\frac{\text{\% std dev of x}}{\text{\% std dev of GDP}}$		
	GDP	R	NX	CONS	INV	HRS
NP data	4.22	3.87	1.42	1.17	2.95	0.57
Independent country risk (NP)						
(a1) R^* and D shocks	2.33	3.87	2.06	1.69	5.26	1.41
(b1) R^*, D and A shocks	4.22	3.87	2.12	1.13	2.95	0.9
Induced country risk (NP)						
(c1) R^* and A shocks	4.22	3.87	1.95	1.54	2.95	0.89
New Data	5.74	2.87	1.80	1.06	2.87	1.47
Independent country risk (New data)						
(a2) R^* and D shocks	1.99	3.04	3.61	1.12	7.68	1.28
(b2) R^*, D and A shocks	5.76	3.04	3.71	0.82	2.88	0.74
Induced country risk (New data)						
(c2) R^* and A shocks	5.72	2.88	2.17	0.96	2.88	0.74

	Correlation of GDP with				
	R	NX	TC	INV	HRS
NP data	-0.63	-0.89	0.97	0.94	0.52
Independent country risk (NP)					
(a1) R^* and D shocks	-0.54	-0.48	0.88	0.57	0.97
(b1) R^*, D and A shocks	-0.29	-0.08	0.87	0.44	0.90
Induced country risk (NP)					
(c1) R^* and A shocks	-0.54	-0.80	0.97	0.90	0.98
New data	-0.46	-0.54	0.99	0.91	0.88
Independent country risk (New data)					
(a2) R^* and D shocks	-0.38	-0.19	0.85	0.40	0.96
(b2) R^*, D and A shocks	-0.13	0.15	0.97	0.49	0.95
Induced country risk (New data)					
(c2) R^* and A shocks	-0.57	-0.77	0.99	0.95	0.98

	Correlation of R with			
	NX	TC	INV	HRS
NP data	0.71	-0.67	-0.59	-0.58
Independent country risk (NP)				
(a1) R^* and D shocks	0.99	-0.86	-0.99	-0.78
(b1) R^*, D and A shocks	0.96	-0.70	-0.97	-0.62
Induced country risk (NP)				
(c1) R^* and A shocks	0.65	-0.60	-0.66	-0.69
New data	0.54	-0.52	-0.55	-0.48
Independent country risk (New data)				
(a2) R^* and D shocks	0.97	-0.81	-0.99	-0.53
(b2) R^*, D and A shocks	0.95	-0.36	-0.92	-0.31
Induced country risk (New data)				
(c2) R^* and A shocks	0.73	-0.60	-0.69	-0.53

[1] All reported moments are taken over HP-filtered data with $\lambda = 1600$.

3.1.3 Sensitivity Analysis

Three elements of the model are crucial for the results presented in the previous subsection: the GHH specification of preferences, its elasticity of labor supply determined by ν, and the presence of working capital. Neumeyer and Perri perform a number of experiments to try to determine the importance of these elements in quantitatively reproducing Argentine business cycle facts. In particular, they consider how changing the specification or the parameter values affects two key statistics in the model: the % standard deviation of output in the model relative to that in the data, and the correlation of output with interest rates. In these quantitative assessments, all parameters are kept to their baseline values except for the following:

1. ν is set to imply a high and low value of elasticity of labor supply. $\nu = 1.2$ implies an elasticity of 5, while $\nu = 4$ implies an elasticity of $\frac{1}{3}$,
2. for Cobb-Douglas preferences, a high (5) and low (50) value of intertemporal substitution are used,
3. θ is varied between 0, 0.5 and 1.

Since they want to quantify the effects of interest rates on business cycles, they focus on the model with only interest rate shocks (the independent country risk model with no productivity shocks). The capital adjustment parameter ϕ is set to its baseline value of 25.5 in all experiments. The published results for these are presented in top panel of Table 3.3.

For the baseline GHH preferences, when all of the labor costs have to be paid in advance ($\theta = 1$), the model generates interest rates that match the data's negative correlation with output. However, output volatility in the model is only half of what is found in the data. As θ is decreased, both these statistics move further away from the corresponding values in the data. In particular, interest rates become less countercyclical, and even procyclical (when $\theta = 0$), since the negative impact of interest rates on labor demand is reduced when less of the firm's wage bill has to be paid for with borrowed funds. In fact, the case $\theta = 0$ reduces the model to that in Mendoza (1991), which is unable to explain the cyclicality of interest rates in emerging economies.

Increasing the labor supply elasticity while holding θ fixed increases the volatility of output and the magnitude of the correlation between output and interest rates. This is because by increasing this elasticity, the labor supply curve gets flatter so that for the same changes in labor demand, one can observe larger changes in equilibrium hours, thus affecting output.

The model results' sensitivity to the preference specification are shown in the last four columns. Using the baseline CD preferences with $\sigma = 5$, the model generates an output series with volatility very close to the actual data. However, interest rates in this model are highly procyclical. With the CD utility function, changes in consumption due to interest rates also increase the labor supply, which offsets the movement of labor demand in the opposite direction. As θ is decreased to zero, output volatility and the output-interest rate correlation both go up since the effect of interest rates on labor demand soften, so that the consumption effect on labor supply is more pronounced in equilibrium.

To generate countercyclical interest rates in the Cobb-Douglas model, one needs to weaken the effect of interest rates on consumption, which works through the intertemporal elasticity of substitution, given by $\frac{1}{\sigma}$. Hence, by increasing σ to 50, the model generates an output-interest rate correlation of -0.18. However, the output fluctuations are nowhere near as volatile as actual GDP's. As θ is reduced from 1, the intertemporal effect starts to dominate again.

3.2 Results Using New Data

The results for model simulations using new data are presented in this section. The data used are real GDP from 1980:1 to 2009:4, real gross capital formation, total consumption, net exports over GDP (simply referred to as net exports), Tbill rates and EMBI+ spread from 1994:1 to 2009:4. Labor hours are from 1997:1 to 2008:1. All series are logged and HP-filtered using

Table 3.3: Sensitivity Analysis

Published										
	GHH, $\nu = 1.2$		GHH, baseline		GHH, $\nu = 4$		CD, $\sigma = 5$		CD, $\sigma = 50$	
	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)
$\theta = 1$	0.89	-0.57	0.55	-0.54	0.22	-0.34	0.97	0.86	0.36	-0.18
$\theta = 0.5$	0.52	-0.44	0.34	-0.38	0.18	-0.16	1.02	0.94	0.26	0.20
$\theta = 0$	0.27	0.16	0.21	0.14	0.15	0.13	1.12	0.97	0.24	0.69

New Data										
	GHH, $\nu = 1.2$		GHH, baseline		GHH, $\nu = 4$		CD, $\sigma = 5$		CD, $\sigma = 50$	
	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)	$\frac{\sigma(y)}{\sigma y_{DATA}}$	Corr(Y,R)
$\theta = 1$	0.53	-0.41	0.35	-0.38	0.16	-0.22	0.48	0.70	0.30	-0.29
$\theta = 0.5$	0.37	-0.28	0.25	-0.24	0.14	-0.10	0.48	0.82	0.23	-0.17
$\theta = 0$	0.23	0.12	0.17	0.09	0.12	0.07	0.51	0.90	0.19	0.05

λ = 1600. GDP, investment, consumption and net exports are in constant 1993 prices. Real rates are obtained by subtracting expected US inflation as outlined in the Data section in the introduction.

3.2.1 Calibration

Average GDP growth over the sample is 0.65 percent per quarter, while the average investment to GDP ratio is 0.19. The average real interest rate is 4.55 percent per quarter, and average time spent working is 20 percent. Labor share and the average net foreign asset position are kept to their values in NP of 0.6 and -0.42, respectively. The values of the parameters that NP set in advance are kept as well (σ and ν).

The persistence parameters for the two components of the interest rate are obtained by running AR(1) models without constants on the corresponding data series. These are 0.85 and 0.76 for the international rate and the country spread, respectively. Their corresponding standard deviations are 0.001163 and 0.019605. The persistence of the productivity shock process is kept at 0.95, as in the original calibration.

The parameter values implied by these averages are reported in the parts of Table 3.1 labeled "New Data".

3.2.2 Computational Experiments

The last three of the original five computational experiments described in the previous section are reproduced here. These are: the case with independent country risk and no productivity shocks, the case with independent country risk and productivity shocks, and the case with induced country risk. The results are presented in Table 3.2, in the lines labeled (a2), (b2), and (c2), respectively. As with the original simulations, ϕ for the first two cases is set to match the volatility of investment relative to output in model (b2), and ϕ for model (c2) is set in the same way. For both cases, σ_A is set to match the data's percent standard deviation of output. Instead of directly feeding the data's innovations for Tbills and country spreads into the model, I merely use the calibrated persistence parameters and standard deviations above, and generate interest rates using (2.36), (2.37) and (2.38).

The first thing to notice is that the models are no longer forced to produce interest rates that are exactly as in the data. However, for the independent country risk models, the percent standard deviation of interest rates is still decently close to the actual statistic, while for the induced country risk case, the statistic is almost exactly duplicated by the model.

Most of the results presented in NP are qualitatively matched by the simulations reported here. For the independent country risk case with no productivity shocks, the simulated output fluctuations are only about one third of actual data. Consumption and investment are more volatile than output, with the latter much more so than what is seen in actuality. The volatility of net exports is also overpredicted, even more so than in the NP simulations. Real interest rates are not as countercyclical as they are in the data, but quite close. The correlation between output and net exports is negative, but much less so than the actual number. Both total consumption and investment are procyclical, but less so than what is observed in data.

In the independent country risk case with productivity shocks, there is, as in the corresponding NP experiment, a drop in the magnitude of the correlation between output and interest rates. While in the NP results, net exports become only marginally countercyclical, with the new calibration net exports become procyclical. Also unlike the NP simulation, consumption becomes less volatile compared to output, but not by too much.

Finally, for the induced country risk case, the improvement over the previous model in terms of reproducing crucial moments of the data is similar to those in the NP experiments. Both interest rates and net exports are strongly countercyclical. In fact, there is now an overprediction of this negative correlation for net exports. The correlations of both consumption and investment with GDP are both very close to those in the data. Consumption is still less volatile than output, but the standard deviation is close to that found in the data. The excess volatility of net exports generated by the model is less than for the previous two models.

One last point worth noting is that unlike the NP simulations, labor hours' volatility and correlation with GDP are not overpredicted by an absurd amount. This is mainly due to the improvement in the dataset brought about by the addition of the CEIC time series for employment and hours.

Taking model (c2) to be the best representation of the economy, the analysis of the effects of different components of interest rates on output volatility is redone here. Shutting down the $\hat{R}^*$ shock does not reduce output volatility at all, unlike the published results. This is due to the much smaller volatility of the Tbill rate in New Data compared to that reported in the NP data. Moreover, a variance decomposition of the influences of each shock on output reveals that only 0.04 percent of output fluctuations can be attributed to movements in the international rate.

On the other hand, shutting down induced country risk shocks reduces output volatility by 11 percent, in contrast to the published 27 percent. Once again, this is due to the smaller volatility of the corresponding shock in New Data, and to the small contribution of σ_I (2.13 percent) to the variance decomposition of output.

3.2.3 Sensitivity Analysis

The exact same sensitivity analysis is carried out on model (a2). The results are presented in Table 3.3. Qualitatively, the same conclusions can be reached as before. For instance, for fixed specifications of preferences and elasticities, decreasing θ reduces the model's output volatility and countercyclicality of interest rates. Having $\theta = 0$ makes the correlation between interest rates and GDP switch from negative to positive.

In the GHH specification, for a fixed θ, increasing ν also reduces the volatility of output and $\mathrm{Corr}(Y, R)$ generated by the model. In the CD specification, the correlation between output and R switches from positive to negative as σ is changed from 5 to 50, except for when there is no working capital constraint.

Given that this calibration replicates the published results well, we can now move into a verification of this same model using Bayesian estimation methods, with the calibrated values guiding the selection of prior distributions. Note in particular how θ has been calibrated arbitrarily to be 1, when the main empirical regularity that the model is supposed to explain - that of countercyclical interest rates - seems to depend so much on it. Hence, it is beneficial to use time series data to explicitly provide an estimate of the importance of the working capital constraint. Moreover, Bayesian methods will allow a more informed decision to be made on the other crucial elements of the model, which are the preference specification and the elasticities of labor supply and intertemporal substitution.

Chapter 4

Bayesian Estimation

4.1 Preliminaries

Lubik & Schorfheide (2005) pose the following dilemma in conducting structural empirical analysis of macroeconomic models:

1. Small, stylized models such as the one in this paper lead to misspecification
2. Large scale models with many exogenous shocks introduce identification and computational problems.

Bayesian analysis is able to cope with both.

With regards to the first problem, an obvious form of misspecification is one caused by stochastic singularity, which can be easily fixed in the Bayesian framework by adding either rational expectations shocks or measurement errors. In general, the number of observables used to estimate the model should be no more than the number of shocks.

A more difficult form of misspecification to correct for is the presence of incorrect cross-coefficient restrictions on the time series generated by the model, leading to poor out-of-sample fit. For instance, del Negro et al. (2007) find that even an elaborate DSGE model fails to achieve the fit attained by fitting a well-formulated VAR model, when estimated by likelihood methods. Maximum likelihood estimation of DSGE models tend to produce absurd parameter estimates, considering additional information that the researcher has that is not contained in the estimation sample. Bayesian methods can cope with this due to its acknowledgement that the DSGE model provides a mere approximation to the law of motion of the time series, and so there need not be a "true" vector of parameters. Information from previous studies or an additional dataset can be incorporated into the prior, and this serves to down-weigh the likelihood function at areas of the parameter space that are at odds with micro-level studies or widely held beliefs. Thus the change from prior to posterior can signal tensions between different sources of information. If the likelihood peaks at a value at odds with information in the prior, then the marginal data density of the DSGE model (the marginal distribution of the data series), defined as

$$p(\boldsymbol{Y}) = \int \mathcal{L}(\boldsymbol{\theta}\,|\boldsymbol{Y})p(\boldsymbol{\theta})\,d\boldsymbol{\theta} \tag{4.1}$$

will automatically be penalized in model comparisons (An & Schorfheide, 2007). Here, $\boldsymbol{\theta}$ is the vector of parameters, $\mathcal{L}(\boldsymbol{\theta}\,|\boldsymbol{Y})$ is the likelihood, and $p(\boldsymbol{\theta})$ is the prior distribution.

Identification problems, on the other hand, arise from models that are observationally equivalent to other, completely different specifications. Likelihood functions of observationally equivalent models have ridges that can cause difficulties in numerical optimization, since the likelihood is flat in certain directions of the parameter space. In Bayesian inference, priors introduce enough curvature into the posterior to facilitate numerical optimization and numerical sampling procedures, since the prior distribution is not updated in directions of the parameter space in which the likelihood function is flat (An & Schorfheide, 2007). Hence, if a prior is essentially not updated, it may indicate lack of information in the data regarding certain parameters.

There are many other advantages of using Bayesian estimation over other empirical methods such as GMM, Vector autoregressions, and likelihood estimation. These are documented in detail in An & Schorfheide (2007) and Lubik & Schorfheide (2005). In this paper, the focus is on the advantages of Bayesian methods over calibration. These will be discussed next, followed by an explanation of the algorithms that make up Bayesian analysis of DSGE models. These will be drawing heavily upon the two papers mentioned above, as well as del Negro & Schorfheide (2010).

4.2 Comparison of Bayesian Estimation and Calibration

Traditionally, DSGE models have been brought to data using calibration, where model parameters are selected to match a certain subset of stylized facts, such as long run averages of key ratios and micro-level evidence. Researchers "accept" the model if the simulations based on this parameterization match the rest of the stylized facts about the economy that were not used in calibration. This approach does not treat the model as a data generating process, acknowledging potential misspecification. Thus, it cannot provide probabilistic justifications for the quantitative statements that are made based on model simulations. In fact, calibration does not acknowledge parameter uncertainty satisfactorily, if at all.

An advantage of the Bayesian method over calibration is that it employs a full econometric approach, which enables the researcher to use statistical methods to assess the model's fit with the data. Also, it enables the researcher to attach probabilistic statements to model predictions, fully acknowledging parameter uncertainty.

At the same time, prior distributions play an important role in the Bayesian framework. This enables the researcher to bring to bear information that is not available in the estimation sample, such as micro-level evidence, to sharpen inference. In this regard, it is similar to calibration. Moreover, non-degenerate priors can be used to incorporate inconclusive evidence on the values of certain parameters, a feature that is not available in the calibration paradigm. The resulting posterior can guide the researcher regarding parameter and model uncertainty.

In the absence of model misspecification and in the presence of abundant out-of-sample evidence, likelihood-based methods should generate the same parameter values as calibration, and vice versa. Unfortunately, this has not been the case. There is neither enough information in previous studies to unambiguously pin down parameters, while values from micro-level studies have not necessarily led to large values of likelihood functions. The Bayesian framework allows the researcher to introduce priors that weigh the information about different parameters according to reliability. For instance, a tight prior signals that evidence regarding a certain parameter is particularly strong (Lubik & Schorfheide, 2005).

4.3 Priors

As mentioned in the previous sections, priors down-weigh regions of the parameter space that are at odds with observations not contained in the sample, and they add curvature to a likelihood function that is nearly flat in some dimensions of the parameter space, influencing the shape of the posterior density.

del Negro & Schorfheide (2010) give three sources of information for prior densities:

1. other time series that are not included in the sample,
2. micro-level studies that are informative about some parameters (e.g., estimates of Argentina's intertemporal elasticity of substitution) and
3. presample estimates drawn from the same observed variables used in estimation.

Usually, all parameters are assumed *a priori* independent. In applications in which the independence assumption is unreasonable, one could derive parameter transformations, such as steady state ratios, and specify independent priors on the transformed parameters, which induce dependent priors for the original parameters (An & Schorfheide, 2007). This is not done in the estimation conducted in the next chapter.

To develop the likelihood function to be used in estimating DSGE models, we take advantage of a state space representation composed of a state equation and a set of measurement equations.

4.4 Model Solution: The State Equation

In general, a DSGE model will consist of a set of equilibrium first order conditions, some economic identities and specifications of each shock process. These equations form a non-linear rational expectations system in the endogenous variables that is driven by the vector of innovations. The solution of the rational expectations system takes the form

$$\boldsymbol{s}_t = \boldsymbol{\Phi}(\boldsymbol{s}_{t-1}, \boldsymbol{\varepsilon}; \boldsymbol{\theta}). \tag{4.2}$$

From an econometric perspective, $\mathbf{s}_t$ can be viewed as a partially latent state vector in a non-linear state space model, and the above is the state transition equation.

For likelihood-based estimation of DSGE models, linear approximations of model solutions have been popular since, along with the assumption that innovations are normally distributed, they lead to a linear Gaussian state space representation that can be analyzed using the Kalman filter. In linearized form, the state equation will be

$$\mathbf{s}_t = \boldsymbol{\Phi}_{\mathbf{s}}\mathbf{s}_{t-1} + \boldsymbol{\Phi}_{\varepsilon}\boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}). \tag{4.3}$$

Here, the elements of the matrices $\boldsymbol{\Phi}_{\mathbf{s}}$ and $\boldsymbol{\Phi}_{\varepsilon}$ are non-linear functions of the deep structural parameters of the model.

Note, however, that depending on the parameterization of the model, there are three possibilities: no stable solution exists, the stable solution is unique (determinacy), or there are multiple stable solutions (indeterminacy). We focus only on the determinacy case in this paper.

4.5 Measurement Equations

We will have access to a vector of observable variables, $\mathbf{y}_t$ from time $t = 1$ to T. The measurement equations essentially select the relevant variables from $\mathbf{s}_t$ and apply appropriate transformations to map these into those in $\mathbf{y}_t$. In general, we will have

$$\mathbf{y}_t = \mathbf{A} + \boldsymbol{\Psi}_{\mathbf{s}}\mathbf{s}_t + \boldsymbol{\Psi}_{\nu}\boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\nu}) \tag{4.4}$$

Here, $\mathbf{A}$ captures the mean (steady states) of $\mathbf{y}_t$ which is related to underlying structural parameters, and as such, depends on $\boldsymbol{\theta}$. On the other hand, $\boldsymbol{\Psi}_{\mathbf{s}}$ does not depend on $\boldsymbol{\theta}$ since it merely selects elements out of $\mathbf{s}_t$. $\boldsymbol{\nu}_t$, in this paper, is a vector of measurement errors, to be discussed later.

4.6 Bayesian Estimation

4.6.1 Bayes' Theorem

Let $\mathbf{Y}$ be the matrix of observable data, $p(\boldsymbol{\theta})$ the prior, $p(\boldsymbol{\theta}, \mathbf{Y})$ the joint density, $p(\mathbf{Y})$ the marginal data density, $\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y}) = p(\mathbf{Y}| \, \boldsymbol{\theta})$ the likelihood, and $p(\boldsymbol{\theta} \mid \mathbf{Y})$ the posterior. By Bayes' Theorem, we have

$$p(\boldsymbol{\theta} \mid \mathbf{Y}) = \frac{p(\boldsymbol{\theta}, \mathbf{Y})}{p(\mathbf{Y})} = \frac{p(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y})}{p(\mathbf{Y})} \propto p(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y}) \tag{4.5}$$

since $p(\mathbf{Y})$ is constant with respect to $\boldsymbol{\theta}$.

With a diffuse/uninformative prior, the analysis coincides with maximum likelihood. With degenerate priors, it coincides with calibration.

4.6.2 Evaluating the Likelihood

Taking advantage of the state space representation above, we can use the Kalman filter to evaluate the likelihood function. If we add the additional notation of letting $\mathbf{Y} = \mathbf{Y}^T$ and $\mathbf{Y}^t$ be the collection of observable vectors $\mathbf{y}_t$ until time t, then the likelihood function is given by

$$\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y}^T) = p(\mathbf{y}_T | \mathbf{Y}^{T-1}, \boldsymbol{\theta}) p(\mathbf{y}_{T-1} | \mathbf{Y}^{T-2}, \boldsymbol{\theta}) \ldots p(\mathbf{y}_2 | \mathbf{Y}^1, \boldsymbol{\theta}) p(\mathbf{y}_1 | \boldsymbol{\theta}) \tag{4.6}$$

where

$$p(\mathbf{y}_1 | \boldsymbol{\theta}) = \int p(\mathbf{y}_1 | \mathbf{s}_1, \boldsymbol{\theta}) p(\mathbf{s}_1 | \boldsymbol{\theta})\, d\mathbf{s}_1 \tag{4.7}$$

$$= \int p(\mathbf{y}_1 | \mathbf{s}_1, \boldsymbol{\theta}) \left(\int p(\mathbf{s}_1 | \mathbf{s}_0, \boldsymbol{\theta}) p(\mathbf{s}_0 | \boldsymbol{\theta})\, d\mathbf{s}_0 \right) d\mathbf{s}_1 \tag{4.8}$$

Therefore, the log-likelihood is given by

$$\ln \mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y}^T) = \ln p(\mathbf{y}_1 | \boldsymbol{\theta}) + \sum_{t=2}^{T} \ln p(\mathbf{y}_t | \mathbf{Y}^{t-1}, \boldsymbol{\theta}) \tag{4.9}$$

These densities can be given by the Kalman filter using the following iteration. Note that since all densities are Gaussian, one need only specify the mean and variance of the random variable to determine the density. Let $\hat{\boldsymbol{s}}_{t+1|t} = \mathbb{E}(\boldsymbol{s}_{t+1} | \boldsymbol{s}_t, \boldsymbol{Y}^t)$, the linear projection of $\boldsymbol{s}_{t+1}$ on $\boldsymbol{Y}^t$ and $\boldsymbol{s}_t$, and $\Sigma_{t+1|t} = \mathbb{E}(\boldsymbol{s}_{t+1} - \hat{\boldsymbol{s}}_{t+1|t})^2$ its associated mean squared error. The goal is to obtain a series of expressions for the conditional means and variances of the densities, $p(\mathbf{y}_t | \mathbf{Y}^{t-1}, \boldsymbol{\theta})$.

1. Initialization - set the parameters of the distribution to their unconditional values to get $p(\mathbf{s}_0 | \boldsymbol{\theta}) = p(\mathbf{s}_0 | \mathbf{y}_0, \boldsymbol{\theta})$. This is the best guess we have of the state variable when there is no data available yet.

$$\begin{cases} \hat{\boldsymbol{s}}_{0|0} & = \mathbb{E}(\boldsymbol{s}_0) \\ \boldsymbol{\Sigma}_{0|0} & = \mathbb{E}[\boldsymbol{s}_0 - \mathbb{E}(\boldsymbol{s}_0)]^2 = Var(\boldsymbol{s}_0) \end{cases} \tag{4.10}$$

Then iterate of the next steps for $t = 1$ to T.

2. Prediction - note that $p(\mathbf{s}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta}) = p(\mathbf{s}_t|\ \boldsymbol{\theta})$, since from the state equation we see that the current state does not depend on past observables except through their effects on the past state.

$$p(\mathbf{s}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta}) = \int p(\mathbf{s}_t|\ \mathbf{s}_{t-1}, \mathbf{Y}^{t-1}, \boldsymbol{\theta}) p(\mathbf{s}_{t-1}|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})\, d\mathbf{s}_{t-1} \tag{4.11}$$

Here, $p(\mathbf{s}_t|\ \mathbf{s}_{t-1}, \mathbf{Y}^{t-1}, \boldsymbol{\theta}) = p(\mathbf{s}_t|\ \mathbf{s}_{t-1}, \boldsymbol{\theta})$ can be obtained from the state equation which provides a first order Markov process for the state variable. This distribution is determined by

$$\begin{cases} \hat{\boldsymbol{s}}_{t|t-1} & = \mathbb{E}(\boldsymbol{s}_t|\boldsymbol{s}_{t-1}) = \boldsymbol{\Phi}_s \hat{\boldsymbol{s}}_{t-1|t-1} \\ \boldsymbol{\Sigma}_{t|t-1} & = \boldsymbol{\Phi}_s \boldsymbol{\Sigma}_{t-1|t-1} \boldsymbol{\Phi}_s' + \boldsymbol{\Phi}_\varepsilon \boldsymbol{\Sigma}_\nu \boldsymbol{\Phi}_\varepsilon' \end{cases} \tag{4.12}$$

$p(\mathbf{s}_{t-1}|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})$, $\hat{\boldsymbol{s}}_{t-1|t-1}$ and $\boldsymbol{\Sigma}_{t-1|t-1}$ are known from the previous iteration, or in the first case, the initialization. Then from the measurement equation, we can get $p(\mathbf{y}_t|\ \mathbf{s}_t, \mathbf{Y}^{t-1}, \boldsymbol{\theta}) = p(\mathbf{y}_t|\ \mathbf{s}_t, \boldsymbol{\theta})$, determined by the first two moments

$$\begin{cases} \mathbb{E}(\boldsymbol{y}_t|\boldsymbol{s}_t) & = \boldsymbol{\Psi}_s \hat{\boldsymbol{s}}_{t|t-1} \\ Var(\boldsymbol{y}_t|\boldsymbol{s}_t) & = \boldsymbol{\Psi}_s \boldsymbol{\Sigma}_{t|t-1} \boldsymbol{\Psi}_s' + \boldsymbol{\Psi}_\varepsilon \boldsymbol{\Sigma}_\nu \boldsymbol{\Psi}_\varepsilon' \end{cases} \tag{4.13}$$

We can then combine these to obtain

$$p(\mathbf{y}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta}) = \int p(\mathbf{y}_t, \mathbf{s}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})\, d\mathbf{s}_t \tag{4.14}$$

$$= \int p(\mathbf{y}_t|\ \mathbf{s}_t, \mathbf{Y}^{t-1}, \boldsymbol{\theta}) p(\mathbf{s}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})\, d\mathbf{s}_t \tag{4.15}$$

which is used in the evaluation of the log-likelihood.

3. Filtering/Updating

$$p(\mathbf{s}_t|\ \mathbf{Y}^t, \boldsymbol{\theta}) = \frac{p(\mathbf{y}_t, \mathbf{s}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})}{p(\mathbf{y}_t|\ \mathbf{Y}^{t-1}, \boldsymbol{\theta})} \tag{4.16}$$

which will be used in the prediction step of the next iteration.

Note that for values of $\boldsymbol{\theta}$ that imply indeterminacy or non-existence of a stable rational expectations solution, the likelihood function is set to zero instead. The Kalman filter is only used to evaluate the likelihood for values of the parameter vector that imply a unique stable solution.

4.6.3 Posterior Computations

We now have $p(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y})$ which determines the kernel density of the posterior. However, we still do not have an expression for the normalizing constant, $p(\mathbf{Y})$, and so we cannot evaluate $p(\boldsymbol{\theta} \mid \mathbf{Y})$. Therefore, we need to resort to sampling algorithms such as Markov-Chain Monte Carlo methods to generate random draws from the posterior.

MCMC Methods - Random Walk Metropolis

The goal is to generate a sequence of random draws $\{\boldsymbol{\theta}^{(1)}, \ldots, \boldsymbol{\theta}^{(N_{sim})}\}$ from a target distribution for which direct sampling is difficult. The Random Walk Metropolis algorithm does this by generating Markov chains with stationary distributions that correspond to the posterior distributions of interest. The RWM algorithm works as follows:

1. Use a numerical optimization procedure to maximize $\ln \mathcal{L}(\boldsymbol{\theta} \mid \mathbf{Y}^T) + \ln p(\boldsymbol{\theta})$. Denote the posterior mode by $\tilde{\boldsymbol{\theta}}$.

2. Let $\tilde{\boldsymbol{\Sigma}}$ be the inverse of the Hessian computed at the posterior mode $\tilde{\boldsymbol{\theta}}$.

3. Draw $\boldsymbol{\theta}^{(0)}$ from $\mathcal{N}(\tilde{\boldsymbol{\theta}}, c^2\tilde{\boldsymbol{\Sigma}})$ or directly specify an initial value.

4. For $s = 1, \ldots, N_{sim}$, draw $\boldsymbol{\theta}^*$ from the proposal distribution $\mathcal{N}(\boldsymbol{\theta}^{(s-1)}, c^2\tilde{\boldsymbol{\Sigma}})$. In particular,

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}^{(s-1)} + c\tilde{\boldsymbol{\Sigma}}^{\frac{1}{2}} u_t$$

The jump from $\boldsymbol{\theta}^{(s-1)}$ is accepted ($\boldsymbol{\theta}^{(s)} = \boldsymbol{\theta}^*$) with probability $min\{1, r(\boldsymbol{\theta}^{(s-1)}, \boldsymbol{\theta}^* \mid \mathbf{Y})\}$ and rejected ($\boldsymbol{\theta}^{(s)} = \boldsymbol{\theta}^{(s-1)}$) otherwise. Here,

$$r(\boldsymbol{\theta}^{(s-1)}, \boldsymbol{\theta}^{*} | \, \mathbf{Y}) = \frac{\mathcal{L}(\boldsymbol{\theta}^{*} | \, \mathbf{Y}) p(\boldsymbol{\theta}^{*})}{\mathcal{L}(\boldsymbol{\theta}^{(s-1)} | \, \mathbf{Y}) p(\boldsymbol{\theta}^{(s-1)})}. \tag{4.17}$$

That is, when $r > 1$, the new candidate is more likely than the previous one, meaning we have achieved a higher point on the posterior density, so we accept it. If $r < 1$, the candidate achieves a lower point on the posterior density, but we do not reject it outright. Instead, we accept it with a probability less than 1 to allow for the possibility that we are merely near a local maximum. Note, too, that the ratio r is the quotient of the unknown posterior density evaluated at the candidate and the previous values. Thus, the normalizing constant is canceled out, leaving us with the need to use only the kernel density, which is known.

5. Approximate the moments, $h(\boldsymbol{\theta})$, of the posterior using Monte Carlo integration:

$$\begin{aligned} \mathbb{E}[h(\boldsymbol{\theta})] &= \int h(\boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \mathbf{Y}) \, d\boldsymbol{\theta} \\ &\approx \frac{1}{N_{sim}} \sum_{s=1}^{N_{sim}} h(\boldsymbol{\theta}^{(s)}) \end{aligned} \tag{4.18}$$

Here, we have used equal weighting for each s since the assumption that each $\boldsymbol{\theta}^{(s)}$ was drawn from the same distribution already incorporates weights through the frequencies of their values.

del Negro & Schorfheide (2010) suggest that c should be chosen such that the rejection rate is around 50%. For the specific details on how numerical optimization is carried out and on the conditions for convergence of the posterior expectations, see An & Schorfheide (2007). For now, note that under fairly general conditions, the posterior distribution of $\boldsymbol{\theta}$ is asymptotically normal. The RWM constructs a Gaussian approximation around the posterior mode and uses a scaled version of the asymptotic covariance matrix as the covariance matrix for the proposal distribution. This allows for efficient sampling from the posterior distribution at least in the vicinity of the mode.

Once the posterior distribution and its moments have been determined, the optimal estimates of the parameters under a quadratic loss function are given by the posterior means. Uncertainty about these estimates can be inferred from the posterior standard deviations (Hamilton, 1994).

4.7 Convergence Diagnostics

Although many statistical tests for convergence of simulated Markov chains exist, the DSGE literature has mainly used graphical methods. In this paper, the one proposed by Brooks & Gelman (1998) are employed. The method requires more than one chain of the RWM algorithm to be run in parallel with each other. Then consider a scalar summary of the parameter draws obtained up to some point in the Markov chain. Here, these scalar summaries will be taken to be the mean and variance. The algorithm recursively constructs the variation of these summary statistics in between chains and within chains after each draw, as well as a pooled variation, which is a weighted average of the previous two. The details of the method are outside the scope of this paper. The Markov chains are said to have converged approximately if both the pooled and within variations have stabilized at the same value.

Chapter 5

Results for Bayesian Estimation

In this chapter, four versions of the NP model are estimated:

1. the independent country risk case with TFP shocks (model b1/b2),
2. the induced country risk case (model c1/c2),
3. the independent country risk case using CD preferences, and
4. the induced country risk case using CD preferences.

For all the estimated models, four observable variables are used: output growth (log-differenced real GDP), the investment to output ratio, the Tbill rate and the EMBI+ country spread. The estimation sample covers 1994:2 to 2009:4. The latter two are HP-filtered with $\lambda = 1600$. Output growth is included to inform γ, which in turn feeds into β, δ, and most of the steady state relations. The investment to output ratio informs the parameter ϕ. Lastly, since interest rates are the main issue of interest in this paper, its two components are also included in the measurement equations.

Note that for each of these models there are only three shocks, while there are four observables. Hence, to avoid misspecification due to stochastic singularity, a measurement error is added to the measurement equation for country risk. Recall that the data for interest rates decomposed in this way actually measures the rate at which the sovereign government, not the private sector, can borrow from the international markets. Though Neumeyer and Perri claim that a separate dataset of interest rates faced by firms closely matches their sovereign rates, a measurement error could help the model fit the data better.

The measurement equations used for all models are as follows:

1. Output Growth:

$$\mathrm{YGR}_t = \ln(1+\gamma) + \tilde{y}_t - \tilde{y}_{t-1} \tag{5.1}$$

2. Investment-Output Ratio

$$\mathrm{INV\_Y}_t = \frac{\tilde{x}_t}{\tilde{y}_t} \tag{5.2}$$

3. Tbill Rate

$$\mathrm{Tbill}_t = \hat{R^*}_t \tag{5.3}$$

4. EMBI+ (Risk)

$$\mathrm{Risk}_t = \hat{D}_t + \varepsilon_M, \quad \varepsilon_M \sim \mathcal{N}(0, \sigma_M) \tag{5.4}$$

5.1 Priors

The prior distributions are centered at, or close to the calibrated parameter values based on New Data, except for the parameters of the shock processes for the international rate and country risk components. The priors' functional forms, means and standard deviations are reported in Tables 5.1 and 5.2. As a general guideline, parameters with admissible values between 0 to 1 are set to beta distributions and those which can take positive values are given gamma distributions, except for shock standard deviations, which are given inverse gamma distributions. Parameters such as the average growth rate, which can take both positive and negative values are set to Gaussian priors. These functional forms are standard in the Bayesian DSGE literature.

In the calibration exercise, $\phi > 0$ is set to match the standard deviation of investment relative to output in the data. Based on New Data, this value is between 10 to 20. In the published calibration, however, this was 25.5 and 40 for the independent and induced country risk cases, respectively. To incorporate this large discrepancy between the published results and replication, the prior is set to a gamma distribution with mean and standard deviation of 15 and 5. This puts most of the density between 5 and 20, but gives a non-trivial density to the range of values beyond that range.

γ is set to a normal distribution with mean 0.0065 and standard deviation 0.002. β is set to a tight beta distribution centered at 0.98 with standard deviation 0.014, to place most of the density between 0.95 and 1. α is given a beta distribution with mean 0.38. Since calibrated studies generally agree that this parameter should be somewhere between 0.35 to 0.4, the standard deviation is set to 0.0085 to put most of the prior density between these two values.

The depreciation rate, δ is set to 0.034 based on New Data, and to 0.0297 in the corrected NP calibration. The prior is therefore set to be a beta distribution with mean 0.03. However, the published value in the NP paper is 0.044, while in del Negro & Schorfheide (2010), it is 0.014. To cover this range, the prior is given a standard deviation of 0.009.

The wage elasticity of labor supply is given by $\frac{1}{\nu-1}$. As previously mentioned, there are no well-known empirical studies that pin down this parameter for Argentina. Even for the US, del Negro & Schorfheide (2010) note the wide variation of estimates for these parameters. Hence, a dispersed gamma prior is set for this parameter, covering values from 0 to 5. The mean and standard deviation are 2 and 1, respectively.

Reinhart & Vegh (1995) provide a micro-level estimate of Argentina's intertemporal elasticity of substitution, $\frac{1}{\sigma} = 0.2$. Neumeyer and Perri and the replication in this paper use this to set $\sigma = 5$. Uribe & Yue (2006), on the other hand, use the value given by Mendoza (1991) for Canada of 0.5, implying $\sigma = 2$. Following Smets & Wouters (2003), the prior is set to a normal distribution, and is centered about the NP value, but with a standard deviation of 1.5 to give a non-trivial density around the Uribe and Yue value.

There are not many studies that use the parameter ψ in the GHH preferences. This was calibrated based on average percentage of time spent working of 0.2, implying that $\psi = 2.47$. However, it has already been noted that hours data for Argentina is not as dependable as the other series. Hence, a range of values is considered. If steady state labor were 0.1, then $\psi = 4.38$, whereas if it had been 1, $\psi = 1.1$. To cover this range, ψ is set to a gamma distribution with mean 2.5 and standard deviation 0.75.

κ is arbitrarily set to as small a value as possible in the calibration. From the perspective of setting Bayesian priors, this signals that there is almost no prior information about this parameter, except that it should be close to zero. Hence, a diffuse gamma prior should be set, centered at a value near zero. To achieve this, the mean and standard deviation are 0.002 and 0.001 for the cases which use the GHH preferences, and 0.003 and 0.0015 for the cases using the CD preferences. The difference is due to the consideration of having to make sure that the Hessian matrix of the posterior kernel at the mode is positive definite.

As has been discussed, θ, the portion of labor costs that the firm needs to set aside before production, is one of the most crucial parameters driving the NP results. However, it is arbitrarily calibrated to a value of 1, when the researcher actually has no information at all regarding what this portion is. Therefore, a very diffuse beta prior is used for this parameter, giving nontrivial density to values slightly above zero, and steadily increasing until it peaks at values closer to 1. The mean is 0.7 and the standard deviation is 0.2

The persistence parameters and standard deviations of shocks should ideally come from a presample of observations that are not used in the Bayesian estimation. However, the estimation sample is already small as it is, with only 63 observations. Hence, priors for these are centered near the published NP values instead. For the interest rate processes, the beta priors are centered around 0.8 and 0.78 for ρ_1 and ρ_2 respectively, with standard deviations of 0.04. These put most of the density between 0.7 and 0.9, which are consistent with what is found in the estimation sample. A slightly more dispersed prior is given to ρ_A, since in the calibration, it was merely set to be equal to the persistence of TFP shocks in the US. The mean and standard deviation for this parameter are given by 0.9 and 0.07, putting most of the density between 0.6 and 1. $\eta > 0$ in the induced country risk case is calibrated to 0.35 based on New Data, but the published value in NP is 1.04. To incorporate this discrepancy, the prior is set to a gamma distribution with mean 0.38 and standard deviation 0.15, to put most of the density between 0 to 1.

The volatilities of the shocks are all set to inverse gamma priors with infinite standard deviations, as is common in the DSGE literature (see for instance, Adolfson et al. (2007)). The priors for the structural shocks' standard deviations are set close to the published NP values, while that of the measurement error shock is set to a small value of 0.01, as this parameter preferably should not drive the model results too much. Note that for σ_A under CD preferences, the standard deviation is set to 0.025 for the independent country risk case and 0.05 for the induced country risk case. Once again, this is to ensure that the Hessian matrix evaluated at the mode is positive definite.

5.2 Estimation Results

For each of the four models estimated, two independent Markov chains, each with 500 000 draws, are generated by the Random Walk Metropolis algorithm. The scale factor c is set to 0.18 and 0.17 for the independent and induced country risk cases with GHH preferences. These lead to rejection rates of 0.46 for both chains of the independent country risk case, and 0.45 and 0.46 for the two chains of the induced country risk case. The scale factors used for the independent and induced country risk cases with CD preferences are 0.23 and 0.18, respectively. These lead to rejection rates of 0.42 for both chains of the former, and rejection rates of 0.47 and 0.46 for the two chains of the latter. To calculate the posterior moments, the first 250 000 draws are thrown out. Monte Carlo integration to find these moments are based on the modified harmonic mean given in Geweke (1999).

The estimation results for the models with GHH preferences are given in Table 5.1. For the model with CD preferences, the results are in Table 5.2. The mean, mode and standard deviation of the posterior distribution for each parameter are given for each method of modeling country risk. Graphical comparisons of priors and posteriors under GHH preferences are given by Figures 5.1 and 5.2 for the independent and induced country risk cases, respectively. For CD preferences, these graphs are presented in Figures 5.3 and 5.4. In these figures, the dashed lines represent prior distributions while the solid lines trace out the posterior distributions. Results across the two preference specifications are very similar, so only those for the GHH case are discussed in detail here.

For the independent country risk case, the posteriors of γ, α, σ, and θ are essentially not updated from their respective priors, suggesting that no information can be gained for these parameters using data on GDP growth, interest rates and the investment-output ratio besides what is already contained in the priors. The data is informative, however, on the standard deviations of the shocks and the persistence of TFP shocks. In particular, the estimate of the standard deviation of TFP shocks is a lot larger than the prior mean, while those for the shocks to the components of the interest rate are much smaller. The standard deviation of the interest rate's components actually closely match the corresponding statistics in the estimation sample, as the measurement equations directly relate the theoretical variables to their data counterparts. That the posterior means for these are smaller than the priors captures the fact that the published NP values for these are bigger than they are in New Data. The mean of the distribution for ρ_A moves from 0.9 in the prior to 0.96 in the posterior. This parameter does not converge properly in the metropolis hastings algorithm, though, as will be discussed in the next section. The rest of the parameter estimates are sufficiently updated, but not too different from their calibrated values.

For the induced country risk case, there seems to be more information gained from the data regarding the parameters, although the posteriors for α and κ are not updated. Notice how once the interaction between default risk and TFP shocks is included in the model, the posterior distribution of ρ_A is pulled to become less than 0.95. Recall that the prior for ρ_A was based merely on the persistence of TFP in the US. Therefore, one will expect to see an estimate for this parameter that is quite different from the prior once the model links this process to more variables in the model. The differences between the priors and posteriors for the standard deviations of the shocks are similar to those found in the previous estimation. Other variables that have notable differences from their prior means are δ (from 0.03 to 0.0155), σ (from 5 to 3.7) and ψ (from 2.47 to 3.54).

The responsiveness of default risk to TFP shocks, determined by η, is also not as strong in the estimation as it is under the calibrated study. Moreover, the posterior for θ is also pulled much closer to 0, even though the prior has been set to be very diffuse but still leaning closer to 1. Recall that η was calibrated using (2.72). Given that the standard deviations of interest rate components are smaller (these enter into the numerator) and the standard deviation of the TFP process is much larger, the decrease in the estimate of η relative to its value in the prior is to be expected. Since this effect feeds into the interest rate, which in turn determines the firm's labor demand decision, there is now sufficient information regarding θ in the likelihood function generated by the model above that which is in the prior, and this causes the update to the lower value.

Overall, we see that taking into account the feedback between country risk and productivity seems to help in terms of identification of parameters. That is, as country risk is given more structure in the model, the data seems to be able to provide more information about parameters of interest that is not already contained in the priors. Note that even in the induced country risk model, the probability of default is still given in reduced form. It would be interesting to see, then, how much more information can be obtained by making this purely structural.

Of particular interest is the fact that in both cases of country risk, θ is not pulled any closer to 1 from its prior centered at 0.7. In fact, the data pulls it very far away from 1 in the induced country risk case. Given the sensitivity of the calibrated model's results to this parameter, it would be interesting to see whether the model can still quantitatively reproduce the counter-cyclicality of interest rates once it is confronted with data using Bayesian techniques.

Table 5.1: Priors and Posteriors for the Models with GHH Preferences

			Prior		Posterior (Indep. Country Risk)			Posterior (Induced Country Risk)		
Parameters	Domain	Density	Mean	Std Dev	Mode	Mean	Std Dev	Mode	Mean	Std Dev
ϕ	$\mathbb{R}^+$	Gamma	15	5	14.98	14.64	3.15	13.71	11.99	2.66
γ	$\mathbb{R}$	Normal	0.0065	0.002	0.0067	0.0063	0.0018	0.0058	0.0056	0.0011
β	[0,1)	Beta	0.98	0.014	0.99	0.985	0.0081	0.995	0.993	0.0052
α	[0,1)	Beta	0.38	0.0085	0.38	0.38	0.0085	0.377	0.379	0.0084
δ	[0,1)	Beta	0.03	0.009	0.025	0.0264	0.0056	0.0125	0.0155	0.003
$\frac{1}{\nu-1}$	$\mathbb{R}^+$	Gamma	2	1	1.87	2.086	0.87	1.51	1.94	0.7587
σ	$\mathbb{R}$	Normal	5	1.5	4.94	4.93	1.35	3.45	3.70	0.92
ψ	$\mathbb{R}^+$	Gamma	2.5	0.75	2.63	2.89	0.59	3.54	3.54	0.5288
κ	$\mathbb{R}^+$	Gamma	0.002	0.001	0.0019	0.0024	0.001	0.0013	0.0021	0.00097
θ	[0,1)	Beta	0.7	0.2	0.80	0.65	0.3255	0.1798	0.2571	0.1275
ρ_A	[0,1)	Beta	0.9	0.07	0.97	0.96	0.02	0.83	0.83	0.0391
σ_{ε_A}	$\mathbb{R}^+$	InvGamma	0.025	∞	0.0696	0.0915	0.03	0.0346	0.0504	0.0125
ρ_1	[0,1)	Beta	0.8	0.04	0.82	0.81	0.03	0.76	0.76	0.0312
σ_{ε_R}	$\mathbb{R}^+$	InvGammma	0.005	∞	0.0012	0.0013	0.00097	0.0013	0.0013	0.0001
ρ_2	[0,1)	Beta	0.78	0.04	0.79	0.79	0.03	-	-	-
σ_{ε_D}	$\mathbb{R}^+$	InvGamma	0.025	∞	0.0091	0.0097	0.0017	-	-	-
$\bar{\eta}$	$\mathbb{R}^+$	Gamma	0.38	0.15	-	-	-	0.2554	0.2049	0.0913
σ_{ε_I}	$\mathbb{R}^+$	InvGamma	0.015	∞	-	-	-	0.0216	0.0224	0.0029
σ_{ε_M}	$\mathbb{R}^+$	InvGamma	0.01	∞	0.0231	0.0237	0.0021	0.0232	0.0238	0.0022

Table 5.2: Priors and Posteriors for the Models with CD Preferences

			Prior		Posterior (Indep. Country Risk)			Posterior (Induced Country Risk)		
Parameters	Domain	Density	Mean	Std Dev	Mode	Mean	Std Dev	Mode	Mean	Std Dev
ϕ	$\mathbb{R}^+$	Gamma	15	5	15.01	14.92	2.95	14.49	14.41	5.75
γ	$\mathbb{R}$	Normal	0.0065	0.002	0.0065	0.0062	0.0021	0.0060	0.0059	0.0013
β	[0,1)	Beta	0.98	0.014	0.97	0.973	0.0065	0.982	0.983	0.0092
α	[0,1)	Beta	0.38	0.0085	0.38	0.38	0.0085	0.380	0.38	0.0089
δ	[0,1)	Beta	0.03	0.009	0.025	0.0260	0.0055	0.0146	0.0151	0.0073
σ	$\mathbb{R}$	Normal	5	1.5	5.04	5.92	1.42	3.08	3.99	1.57
μ	[0,1)	Beta	0.26	0.07	0.25	0.2529	0.06	0.2532	0.2647	0.0721
κ	$\mathbb{R}^+$	Gamma	0.003	0.0015	0.0024	0.0034	0.0012	0.0023	0.003	0.0032
θ	[0,1)	Beta	0.7	0.2	0.81	0.64	0.3074	0.2276	0.3042	0.1582
ρ_A	[0,1)	Beta	0.9	0.07	0.98	0.98	0.0122	0.87	0.88	0.0331
σ_{ε_A}	$\mathbb{R}^+$	InvGamma	0.025/0.05	∞	0.1055	0.1136	0.0289	0.0613	0.0632	0.0278
ρ_1	[0,1)	Beta	0.8	0.04	0.82	0.81	0.0327	0.76	0.76	0.0348
σ_{ε_R}	$\mathbb{R}^+$	InvGammma	0.005	∞	0.0012	0.0013	0.0001	0.0013	0.0013	0.0001
ρ_2	[0,1)	Beta	0.78	0.04	0.78	0.78	0.034	-	-	-
σ_{ε_D}	$\mathbb{R}^+$	InvGamma	0.025	∞	0.0082	0.0086	0.0013	-	-	-
$\bar{\eta}$	$\mathbb{R}^+$	Gamma	0.16	0.07	-	-	-	0.1307	0.1319	0.0615
σ_{ε_I}	$\mathbb{R}^+$	InvGamma	0.01	∞	-	-	-	0.0194	0.0208	0.0027
σ_{ε_M}	$\mathbb{R}^+$	InvGamma	0.01	∞	0.0232	0.0238	0.0021	0.0230	0.0239	0.0021

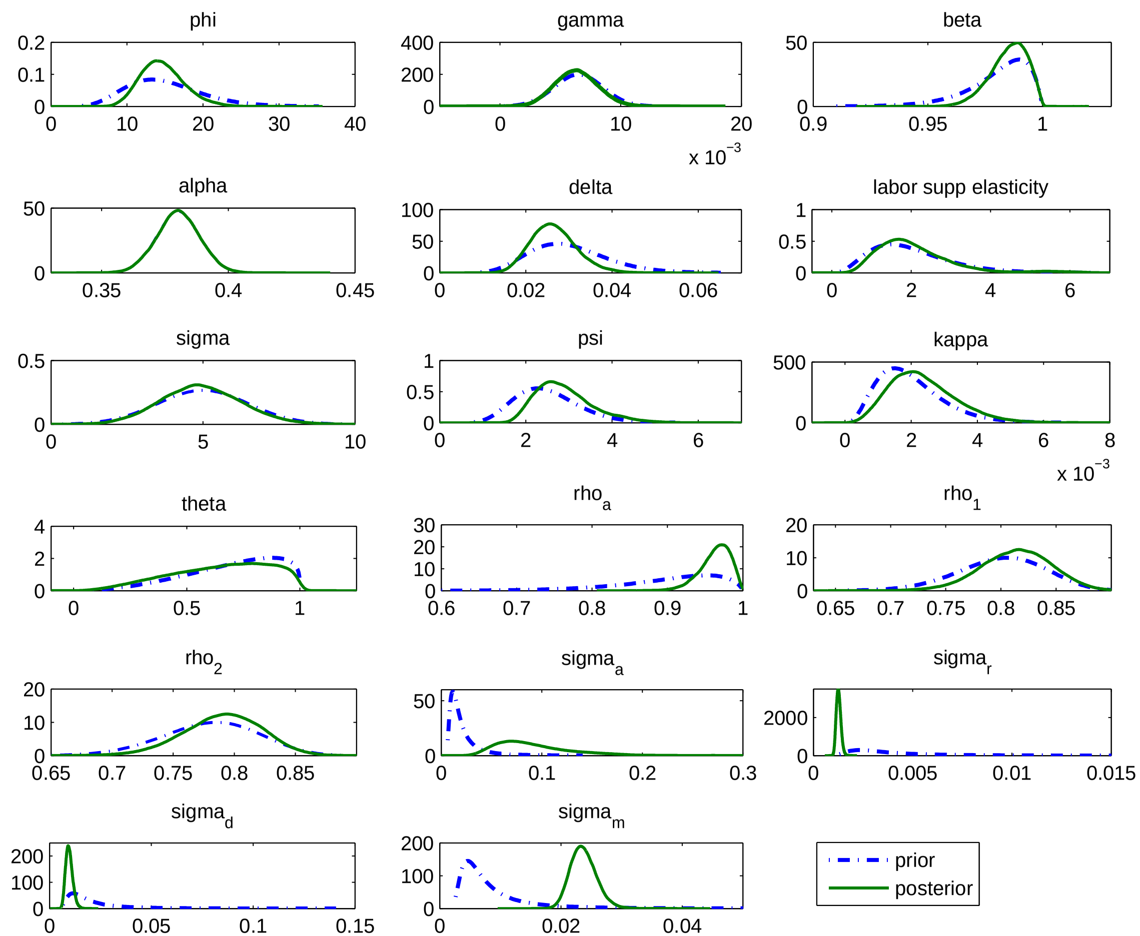

Figure 5.1: Prior and Posterior Comparisons for Independent Country Risk with GHH Preferences

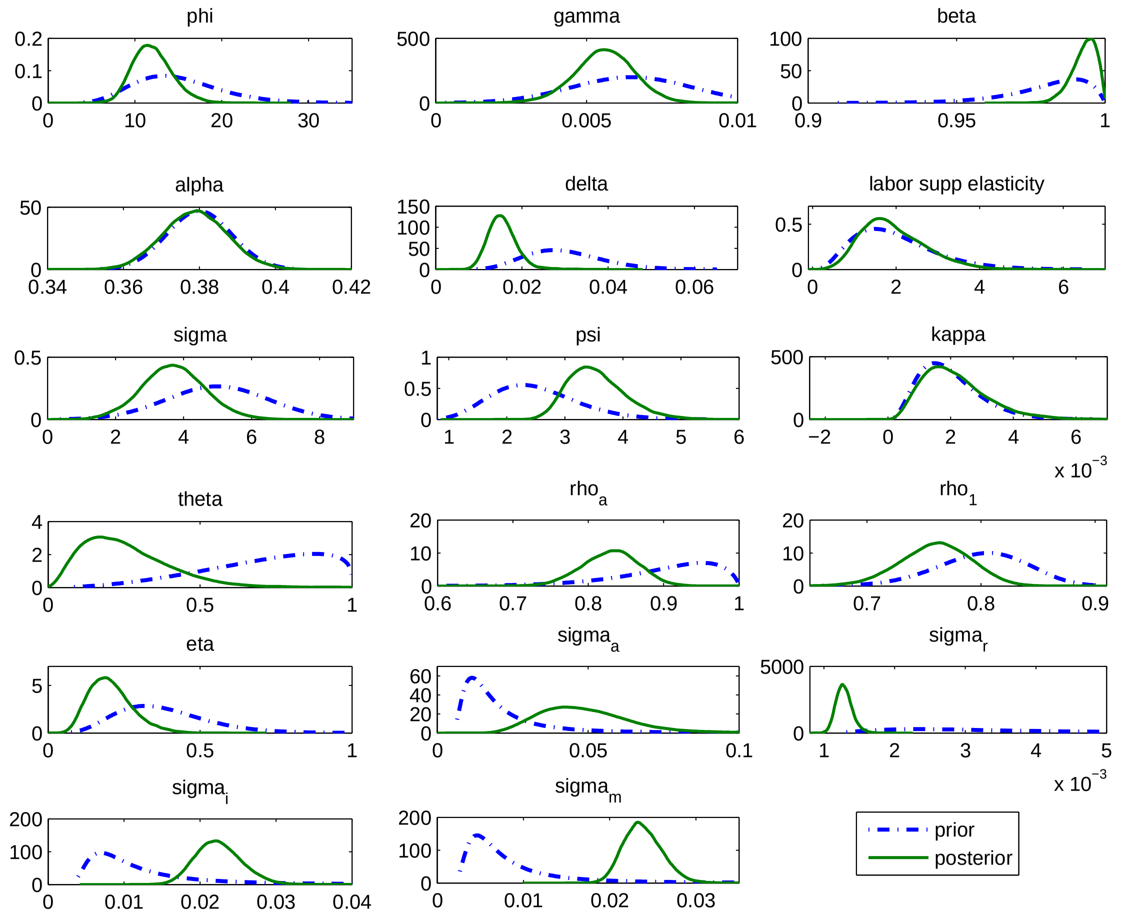

Figure 5.2: Prior and Posterior Comparisons for Induced Country Risk with GHH Preferences

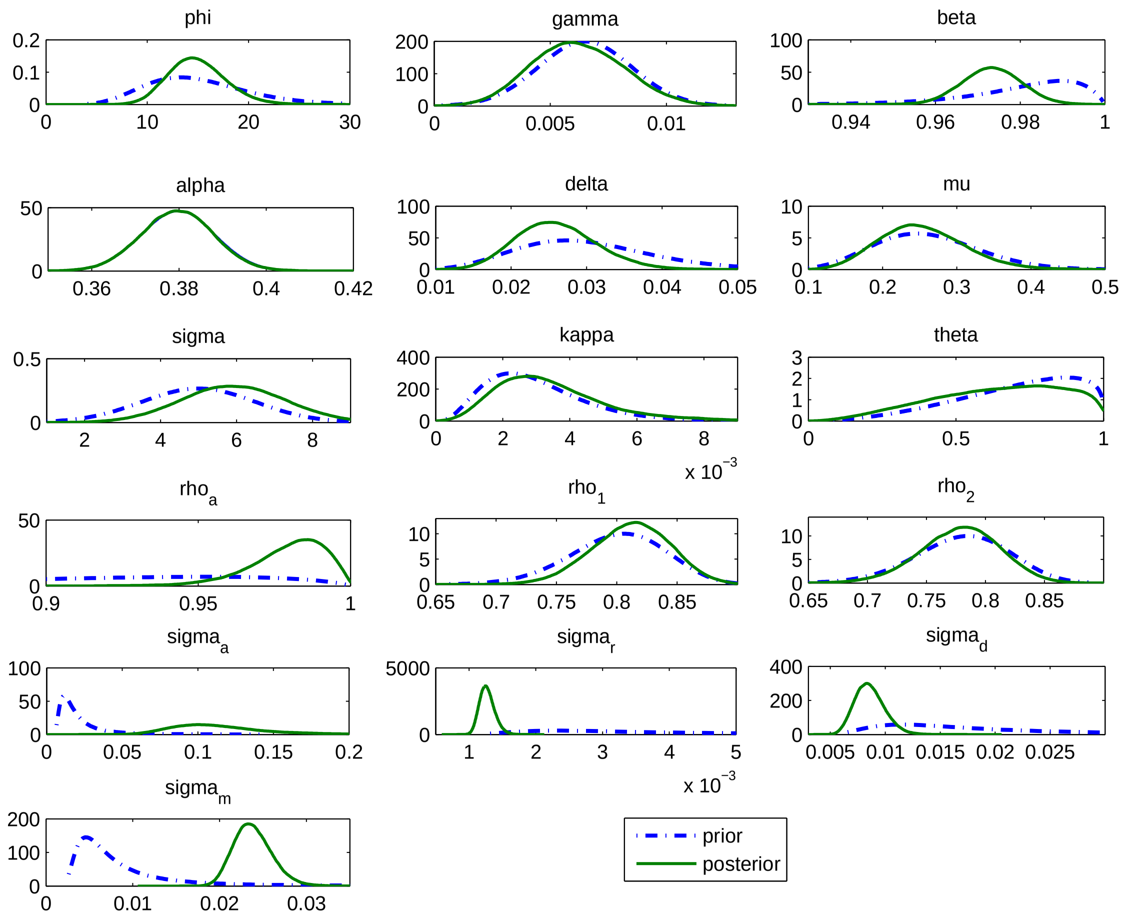

Figure 5.3: Prior and Posterior Comparisons for Independent Country Risk with CD Preferences

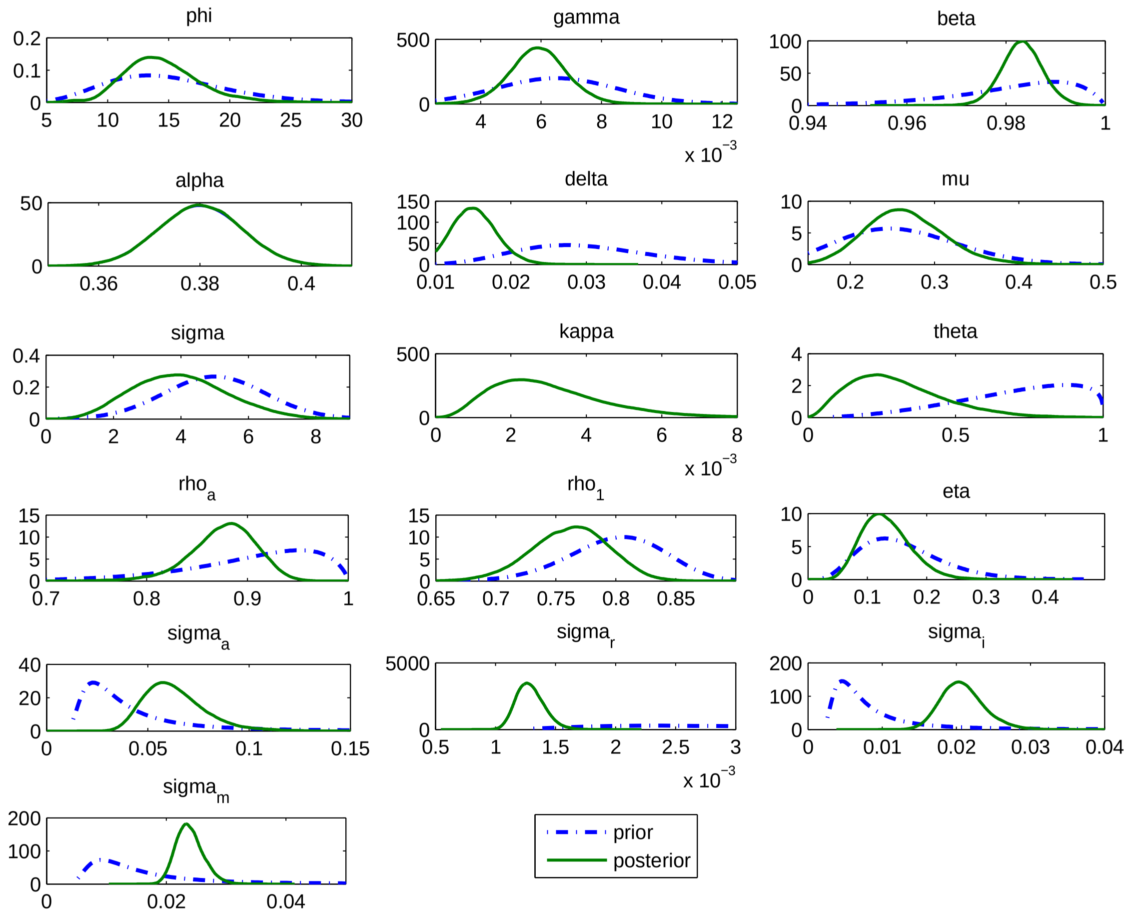

Figure 5.4: Prior and Posterior Comparisons for Induced Country Risk with CD Preferences

5.3 Convergence Checks

The convergence checks from Brooks & Gelman (1998), both the multivariate and univariate cases, are presented here for the model with induced country risk and GHH preferences. Convergence diagnostics for the rest of the models are not presented in detail. It is worth mentioning, though, that for the model with independent country risk and GHH preferences, the mean and standard deviations of ρ_A and σ_A do not converge properly (convergence diagnostics for these are shown in Figures 5.5 and 5.6), while all other parameters have roughly converged after the 250 000th draw. Nonetheless, the estimates for these parameters are kept since they are not too different from the corresponding ones in the independent risk CD case. For both models using CD preferences, the means and standard deviations for all parameters converge halfway through the Random Walk Metropolis.

The multivariate diagnostic for the model with induced country risk and GHH preferences is given in Figure 5.7. The univariate diagnostics are presented in Figures 5.8 to 5.13. In these figures, the solid and dashed lines indicate the measures of within and pooled variation, respectively. For each parameter, the first graph plots the variation for the means and associated confidence interval calculated at each step of the metropolis algorithm. The second plot graphs the variation for the corresponding standard deviations.

These graphs show that the model parameters mostly converge after the first half of the metropolis draws. There still seem to be some convergence problems for the standard deviation of the posteriors of β, ψ and η, but these are not too serious.

5.4 Model Simulation Using Estimated Parameters

Finally, the model is simulated using the estimated parameters for the four models in the previous section. To do this, the simulated parameter vectors from the Random Walk Metropolis Algorithm are used. Every hundredth draw (after the first 250 000) from the posterior distri-

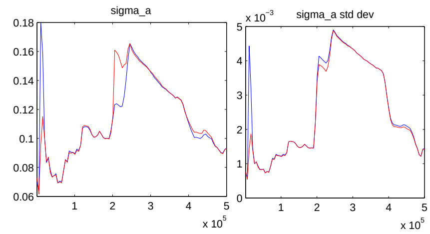

Figure 5.5: Univariate Convergence Diagnostic for ρ_A (Indep. Risk)

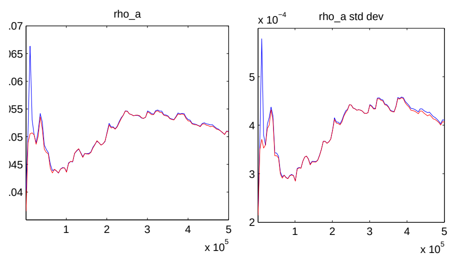

Figure 5.6: Univariate Convergence Diagnostic for σ_A (Indep. Risk)

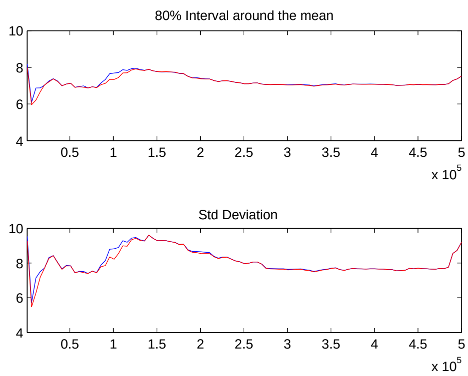

Figure 5.7: Multivariate Convergence Diagnostic

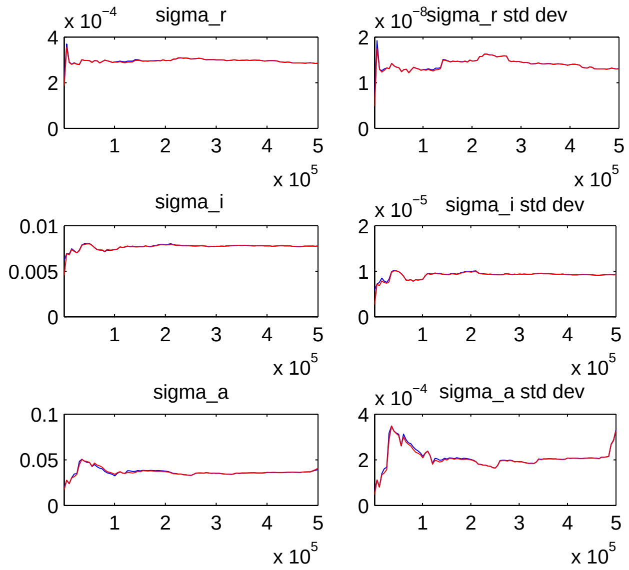

Figure 5.8: Univariate Convergence Diagnostic I

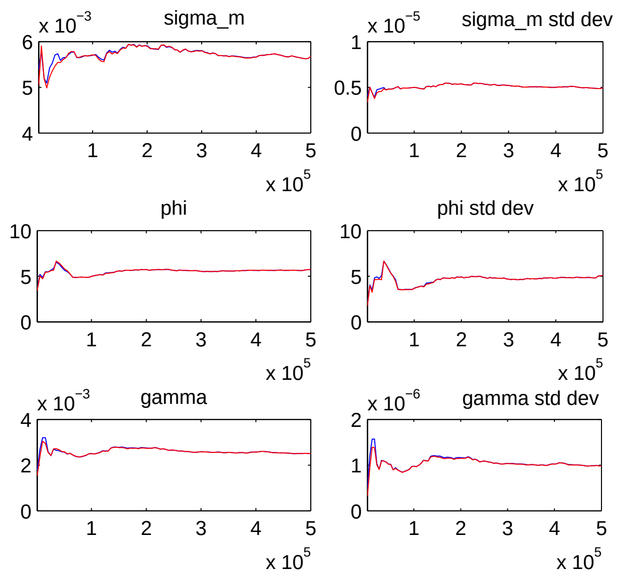

Figure 5.9: Univariate Convergence Diagnostic II

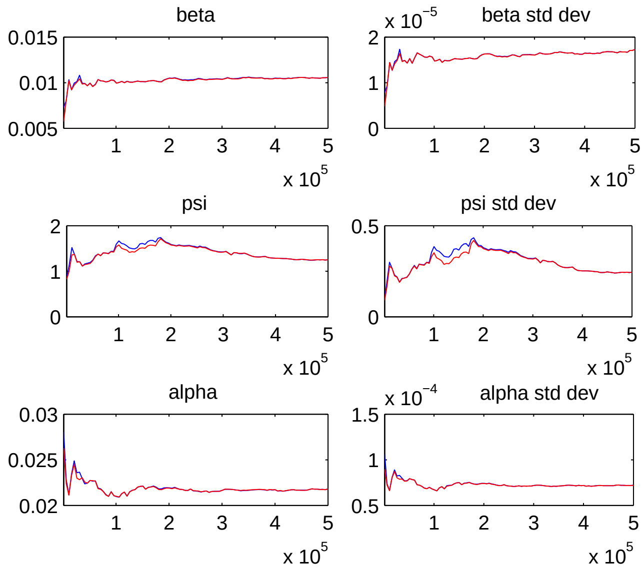

Figure 5.10: Univariate Convergence Diagnostic III

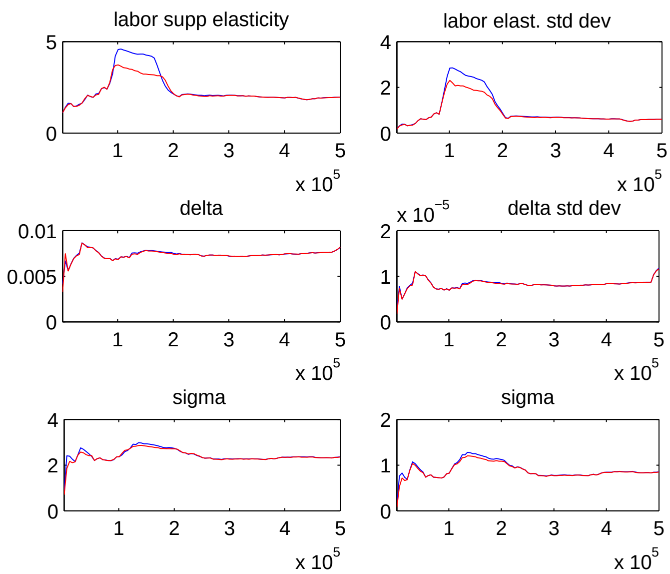

Figure 5.11: Univariate Convergence Diagnostic IV

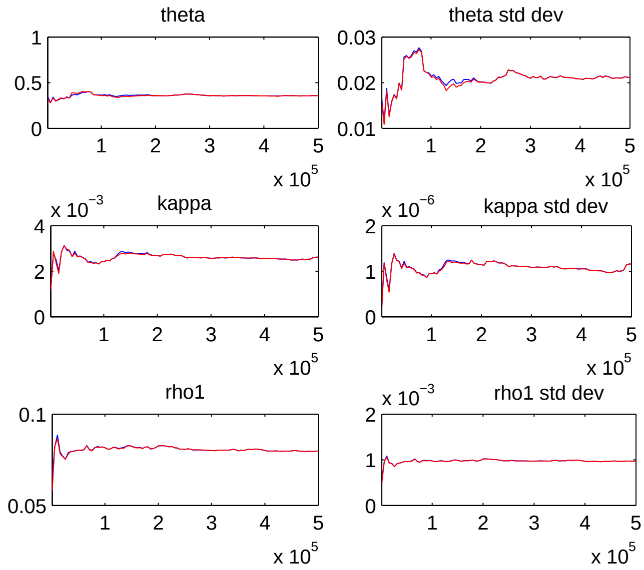

Figure 5.12: Univariate Convergence Diagnostic V

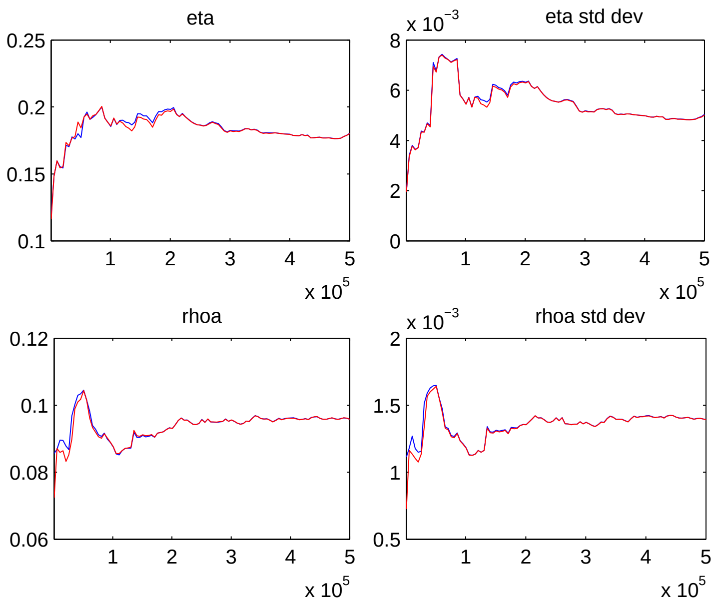

Figure 5.13: Univariate Convergence Diagnostic VI

bution is fed into the model simulation to calculate the key business cycle statistics that the parameter vector implies. These statistics are then averaged over the 2 500 draws. The results for these are then compared to the key business cycle facts of Argentine data in Tables 5.3 and 5.4 for the GHH and CD cases, with the numbers in parenthesis representing the standard deviations of the corresponding simulated statistics.

The results generally show that the model cannot reproduce most of the Argentine economy's cyclical characteristics when estimated parameter values are used. This is most clearly seen from the volatilities of the variables generated by the simulation, most notably the percentage standard deviation of output. This discrepancy is probably due mostly to the difference between the estimated value of σ_A and its prior mean. The posterior mean for this parameter ranges from 0.05 to 0.11, which are about two to four times their corresponding calibrated values. This leads to the severe overstatement of output volatility in the model. Percentage standard deviations for interest rates are also understated, while those for net exports over GDP are overstated.

The model underpredicts the relative volatility of consumption, more so under induced country risk than under independent country risk. This difference is mainly due to the much larger overstatement of output volatility under independent country risk, causing slightly bigger fluctuations in consumption's response to productivity shocks compared to the induced country risk case. The larger fluctuations of output under the first case, in turn, may be due to the fact that there is not as much information in the model or the data regarding the parameter σ_A. This could in principle lead to the estimation overcompensating for this given the highly diffuse prior, relative to the latter case, which gives some kind of relationship between observables (country risk) and the model variables concerning TFP through (2.38) to inform the parameter estimate.

The relative volatility of investment is also understated, but this time, it is more so under independent country risk. This difference is probably due to capital adjustment costs being higher relative to the calibrated value under independent country risk, and vice versa for induced country risk, leading investment to be less volatile in the former case. Note, however, that the actual relative volatility of investment in the data falls within the 90% confidence interval of the Bayesian estimate.

Labor volatility is much smaller in the model compared to the data for the GHH models, and even smaller under CD preferences. This can be explained by the offsetting effect of labor supply movements to those of labor demand, as explained in Chapter 1.

A very surprising result is that the estimated model is still able to reproduce the countercyclicality of interest rates very well. As with the calibration exercise, the independent country risk case produces mildly countercyclical interest rates, while the induced country risk case produces strongly countercyclical ones (the exception is the independent country risk case with CD preferences, which produces mildly procyclical interest rates). These go against the intuition laid out by the sensitivity analysis in Neumeyer & Perri (2005) and the replication in this paper, where the correlation of output and interest rates decreases in magnitude as the portion of labor costs that the firm needs to pay in advance decreases. θ is estimated to be somewhere between 0.25 and 0.3 under the four models presented here, none of which are near 1. Also, this correlation should decrease as the labor supply elasticity goes down (in the GHH case), or as the inverse of the intertemporal elasticity of substitution, σ, is lessened. Though the estimated values for the labor supply elasticity are very close to the calibrated values, estimates for σ are not (from 5 to 3.7 and 3.99 in the induced country risk cases with GHH and CD preferences, respectively). Despite these, the models with induced country risk are able to reproduce the output-interest rate correlation quite closely.

Upon a reexamination of the model, it becomes apparent that the model with induced country risk can produce countercyclical interest rates independent of the parameters mentioned above. This is because of the way country risk is modeled in (2.38). Any increase in productivity would increase output and decrease country risk, thereby decreasing interest rates. Hence, the Bayesian analysis highlights the most crucial factor that was driving the results - namely, the functional form of default probabilities. The Bayesian estimation conducted here makes this clear. Regardless of the labor supply elasticity,the working capital constraint, the intertemporal elasticity of substitution, or the the functional form of preferences, this model can give countercyclical interest rates as long as Equation (2.38) holds. This actually makes sense, given the fact that these interest rates are measures of prices of the sovereign government's borrowing. Though the likelihood of the government's default would definitely have an impact on the real economy and vice versa, its cyclical properties are not as strongly dependent on the equilibrium interactions of the economy's private agents as the model presumes.

In contrast, none of the estimated models are able to produce countercyclical net exports. This is not surprising, given that the volatilities of consumption and investment generated by each of the estimated models are much less than the corresponding volatility of output. Contrast this with the calibrated version of the induced country risk model with GHH preferences, which is able to match the output-net exports correlation closely. In the model, an increase in interest rates would produce a large dip in output, so both consumption and investment decrease as well, but under the estimated version, this dip is nowhere near as large as that of output, so net exports decrease with GDP.

In all, the discrepancies of the estimated model results with the data come from two main factors: the large volatility of TFP shocks, which are inconsistent with output volatility in the data, and values of the capital adjustment cost parameter that are inconsistent with investment volatility in the data. The latter is less serious, as this parameter is not heavily updated in any of the estimations conducted in this chapter. Hence, setting tighter priors may help solve this. The bigger problem is the former, because there is no reliable series for Solow residuals available for Argentina and most developing countries due to low-quality labor data. Alternatively, one could calibrate these two parameters as they were in Chapter 3, given the values implied by estimates of the rest of the parameters.

The results for this latter approach are given in Table 5.5 for the model with induced country risk and GHH preferences. 250 simulations are performed using every 1000th draw of the parameter vector to run the model. For each run, σ_A and ϕ are changed such that the model

Table 5.3: Bayesian Simulation of Argentine Business Cycles (GHH)[1]

	% std dev			$\frac{\text{\% std dev of x}}{\text{\% std dev of GDP}}$		
	GDP	R	NX	CONS	INV	HRS
New Data	5.74	2.87	1.80	1.06	2.87	1.47
Independent Country Risk	21.14	1.60	3.32	0.84	1.55	0.64
	[10.98, 40.97]	[1.29, 2.06]	[1.96, 7.30]	[0.74, 1.01]	[1.28, 2.03]	[0.52, 0.77]
Induced Country Risk	11.07	2.67	2.24	0.65	2.71	0.65
	[6.25, 21.71]	[2.27, 3.28]	[1.76, 3.66]	[0.59, 0.72]	[1.94, 4.08]	[0.53, 0.78]

	Correlation of GDP with				
	R	NX	TC	INV	HRS
New data	-0.46	-0.54	0.99	0.91	0.88
Independent Country Risk	-0.01	0.41	0.998	0.94	0.998
	[-0.02, 0.00]	[-0.37, 0.95]	[0.996,0.999]	[0.90, 0.98]	[0.997, 0.999]
Induced Country Risk	-0.40	0.18	0.997	0.93	0.998
	[-0.51, -0.25]	[-0.35, 0.86]	[0.994, 0.999]	[0.90, 0.97]	[0.995, 0.999]

	Correlation of R with			
	NX	TC	INV	HRS
New data	0.54	-0.52	-0.55	-0.48
Independent Country Risk	0.67	-0.05	-0.32	-0.04
	[0.32, 0.94]	[-0.08, -0.02]	[-0.45, -0.18]	[-0.06, -0.01]
Induced Country Risk	0.69	-0.44	-0.68	-0.39
	[0.23, 0.97]	[-0.55, -0.30]	[-0.75, -0.57]	[-0.51, -0.25]

[1] Reported numbers are averages of the corresponding statistics over 2500 simulations using every 100th posterior draw of parameter vectors. Numbers in parenthesis represent 90% confidence intervals for these statistics assuming approximately symmetric posterior distributions.

Table 5.4: Bayesian Simulation of Argentine Business Cycles (CD)[1]

	% std dev			$\frac{\text{\% std dev of x}}{\text{\% std dev of GDP}}$		
	GDP	R	NX	CONS	INV	HRS
New Data	5.74	2.87	1.80	1.06	2.87	1.47
Independent Country Risk	16.49	1.39	3.83	0.81	1.56	0.20
	[11.45, 25.70]	[1.15, 1.73]	[2.82, 5.87]	[0.67, 1.02]	[1.37, 1.86]	[0.12, 0.32]
Induced Country Risk	9.78	2.58	2.97	0.65	2.61	0.34
	[7.03, 14.39]	[2.18, 3.17]	[2.32, 4.12]	[0.55, 0.84]	[2.16, 3.31]	[0.28, 0.42]

	Correlation of GDP with				
	R	NX	TC	INV	HRS
New data	-0.46	-0.54	0.99	0.91	0.88
Independent Country Risk	-0.05	0.36	0.988	0.93	0.75
	[0.03, 0.07]	[-0.21, 0.85]	[0.981,0.995]	[0.89, 0.97]	[0.31, 0.97]
Induced Country Risk	-0.32	0.17	0.97	0.93	0.89
	[-0.43, -0.16]	[-0.21, 0.66]	[0.96, 0.99]	[0.90, 0.96]	[0.77, 0.98]

	Correlation of R with			
	NX	TC	INV	HRS
New data	0.54	-0.52	-0.55	-0.48
Independent Country Risk	0.80	-0.09	-0.30	0.42
	[0.60, 0.95]	[-0.11, -0.06]	[-0.39, -0.18]	[0.20, 0.78]
Induced Country Risk	0.79	-0.50	-0.63	0.04
	[0.56, 0.96]	[-0.61, -0.36]	[-0.71, -0.51]	[-0.22, 0.51]

[1] Reported numbers are averages of the corresponding statistics over 2500 simulations using every 100th posterior draw of parameter vectors. Numbers in parenthesis represent 90% confidence intervals for these statistics assuming approximately symmetric posterior distributions.

matches output and investment volatility. Each statistic is calculated as the average across these model runs.

With the model now matching the two crucial volatilities, the roles of the amplification of shocks through working capital and the interaction of country risk and productivity become apparent. This model no longer underpredicts the volatility of interest rates. However, because of the small reaction of interest rates to productivity (small estimate for η) and the small fraction of costs that the firm has to pay in advance, interest rates are only half as countercyclical as they are in the data. Moreover, the smaller amplification of productivity shocks in the estimated model relative to the calibrated model produces consumption that is far less volatile than output, so that once again, the model fails to produce countercyclical net exports.

This shows that although their roles are secondary, the mechanisms by which the model reproduces business cycle facts as given by Neumeyer & Perri (2005) still matter. The poor fit that the model achieves under all the simulations in this chapter, though, indicate that these mechanisms are not enough to be able to explain country risk and business cycles in a developing small open economy. Two possible reasons for this inability are:

1. The model's results depend on parameters that have no solid grounding due to the lack of availability of studies regarding these, or inadequate data. These parameters include σ_A, ρ_A, θ, and ν. Thus, priors for these parameters are highly uninformed, making strong estimation results harder to come by.

2. The model's success most heavily relies on the interaction between country risk (risk of default) and productivity shocks. However, it treats the risk premium as an object that is faced directly by private agents, while these rates measure probabilities of default by a sovereign government. Hence, the mechanisms by which interest rate shocks affect the economy may not be fully captured by using interest rates on sovereign bonds.

Table 5.5: Bayesian Simulation of Argentine Business Cycles with Calibrated ϕ and σ_A [1]

	% std dev				$\frac{\text{\% std dev of x}}{\text{\% std dev of GDP}}$	
	GDP	R	NX	CONS	INV	HRS
New Data	5.74	2.87	1.80	1.06	2.87	1.47
Simulation	5.74	2.40	1.90	0.66	2.87	0.66
	[5.74, 5.74]	[1.91, 2.99]	[1.34, 2.53]	[0.57, 0.73]	[2.87, 2.87]	[0.46, 0.80]

	Correlation of GDP with				
	R	NX	TC	INV	HRS
New data	-0.46	-0.54	0.99	0.91	0.88
Simulation	-0.24	0.09	0.994	0.83	0.995
	[-0.42, -0.10]	[-0.15, 0.30]	[0.987, 0.998]	[0.70, 0.93]	[0.984, 0.999]

	Correlation of R with			
	NX	TC	INV	HRS
New data	0.54	-0.52	-0.55	-0.48
Simulation	0.91	-0.31	-0.71	-0.23
	[0.77, 0.97]	[-0.52, -0.16]	[-0.79, -0.62]	[-0.41, -0.09]

[1] Each number is the average of the corresponding statistics over 250 simulations using every 1000th posterior draw of parameter vectors. For each iteration, ϕ and σ_A are set to match the volatilities of investment and GDP, respectively, given Bayesian estimates of the rest of the parameters. Numbers in brackets represent 90% confidence intervalsfor these statistics assuming approximately symmetric posterior distributions.

Chapter 6

Conclusions

This thesis analyzes in great detail a model presented by Neumeyer & Perri (2005), which aims to explain the strong countercyclicality of interest rates and net exports in emerging market economies, among other things. The model accomplishes this by decomposing interest rates into an international rate and a country risk component, and by making labor demand sensitive to movements in these rates via a working capital constraint imposed on the representative firm. Moreover, it proposes two approaches to determining the stochastic processes for these interest rates: the independent country risk case and the induced country risk case.

The original calibrated results and the subsequent replication conducted in this paper find that the model in which country risk depends on productivity and vice versa successfully reproduces the key business cycle facts of the Argentine economy. These results are found to be sensitive to the specification of preferences, the labor supply elasticity, the intertemporal elasticity of substitution, and the working capital constraint. However, a calibration approach is unable to confront some of these key parameters with Argentine data. Instead, the first and last of these are calibrated to developed economy values.

This paper then attempts to verify the results of the model by estimating it using Bayesian DSGE methods. Using estimated parameters from this exercise to simulate the model, a number of facts regarding the model's mechanisms are highlighted.

First, the countercyclicality of interest rates depends most heavily on how country risk is modeled, more than the parameters mentioned previously. The induced country risk model has a built in negative correlation between interest rates and output.

Second, there is very little quality information regarding the productivity process. When this process is treated as part of the structural Bayesian estimation instead of calibrating the parameters of the TFP process to US values, the posterior estimates of the standard deviation of this shock are absurdly huge.

However, even after taking all other parameters at their estimated values while changing this standard deviation to match the volatility of output in the data, the model still fails to quantitatively reproduce the key cyclical properties of the Argentine economy. The output-interest rate correlation is only half of the magnitude found in the data, while net exports generated by the model are procyclical.

The results presented in this paper provide evidence that the mechanisms through which the NP model tries to explain the data need to be augmented somehow in order to capture the dynamics of output and interest rate more fully. Two plausible reasons for this are proposed. One is the lack of informative data to guide certain important parameters, such as the standard deviation and persistence of TFP shocks. The other is the way in which the interest rate's effects on the real economy are modeled. Instead of treating default risk as a sovereign government's decision, the model takes this as given and forces the private agents to face this rate on the market. This possibly misses the true effects of these rates on the economy. Instead, a fully structural approach in determining default and country risk premia may be able to perform better.

Bibliography

Adolfson, M., Laseen, S., Linde, J., & Villani, M. (2007). Bayesian estimation of an open economy dsge model with incomplete pass-through. *Journal of International Economics*, 72, 481–511.

Aguiar, M. & Gopinath, G. (2007). Emerging market business cycles: the cycle is the trend. *Journal of Political Economy*, 115, 69–101.

An, S. & Schorfheide, F. (2007). Bayesian analysis of dsge models. *Econometric Reviews*, 26, 113–172.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*, 98, 690–712.

Brooks, S. P. & Gelman, A. (1998). General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics*, 7, 434–455.

Christiano, L. J., Gust, C., & Roldos, J. (2004). Monetary policy in a financial crisis. *Journal of Economic Theory*, 119, 64–103.

Correia, I., Neves, J. C., & Rebelo, S. (1995). Business cycles in a small open economy. *European Economic Review*, 39, 1089–1113.

del Negro, M. & Schorfheide, F. (Forthcoming 2010). Bayesian macroeconometrics. In J. Geweke, G. Koop, & H. van Dijk (Eds.), *Handbook of Bayesian Econometrics*. Oxford University Press.

del Negro, M., Schorfheide, F., Smets, F., & Wouters, R. (2007). On the fit and forecasting performance of new keynesian models. *Journal of Business and Economic Statistics*, 25, 123–162.

Eaton, J. & Gersovitz, M. (1981). Debt with potential repudation:theoretical and empirical analysis. *Review of Economic Studies*, 48, 289–309.

Geweke, J. (1999). Using simulation methods for bayesian econometric models: inference, development and communication. *Econometric Reviews*, 18, 1–126.

Greenwood, J., Hercowitz, Z., & w. Huffman, G. (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review*, 78, 402–417.

Hamilton, J. D. (1994). *Time series analysis*. New Jersey: Princeton University Press.

Kehoe, P. J. & Perri, F. (2004). Competitive equilibria with limited enforcement. *Journal of Economic Theory*, 119, 184–206.

Kydland, F. E. & Zarazaga, C. (2002). Argentina's lost decade. *Review of Economic Dynamics*, 5, 152–165.

Lane, P. & Milesi-Ferretti, G. (2001). The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. *Journal of International Economics*, 55, 263–294.

Lubik, T. & Schorfheide, F. (2005). A bayesian look at new open economy macroeconomics. In M. Gertler & K. Rogoff (Eds.), *NBER Macroeconomics Annual 2005*, volume 20 (pp. 313–366). The University of Chicago Press.

Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review*, 81, 797–818.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, 36, 101–137.

Neumeyer, P. A. & Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics*, 52, 345–380.

Reinhart, C. & Vegh, C. (1995). Nominal interest rates, consumption booms, and lack of credibility: a quantitative examination. *Journal of Development Economics*, 46, 357–378.

Schmitt-Grohe, S. & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61, 163–185.

Smets, F. & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1, 1123–1175.

Uribe, M. & Yue, V. Z. (2006). Country spreads and emerging countries: who drives whom? *Journal of International Economics*, 69, 6–36.