2010

Pricing and Lead Time Decisions in a Duopoly Common Retailer Channel

Xiuming NIU
Singapore Management University, niuxiuming.2008@smu.edu.sg

PRICING AND LEAD-TIME DECISIONS IN A DUOPOLY COMMON RETAILER CHANNEL

XIUMING NIU

SINGAPORE MANAGEMENT UNIVERSITY

2010

Pricing and Lead-Time Decisions in a Duopoly Common Retailer Channel

by

Xiuming Niu

Submitted to Lee Kong Chian School of Business in partial fulfillment of the requirements for the Degree of Master of Science in Operations Management

Supervisor: Prof. Zhengping Wu

Singapore Management University

2010

Pricing and Lead-Time Decisions in a Duopoly Common Retailer Channel

by Xiuming Niu

Abstract

This thesis studies a dual-level decentralized supply chain consisting of two suppliers and two retailers facing a price- and lead-time-sensitive demand. We model the suppliers' operations as M/M/1 queues and demand as a linear function of the retail prices and promised delivery lead-times offered to the customers. Three different kinds of games are constructed to analyze the pricing and lead-time decisions of the suppliers and retailers. We show the existence of a unique equilibrium in all games and provide the exact formulas to compute the optimal decisions for both the suppliers and retailers. We further present numerical examples to illustrate how the results of our model can be used to provide useful managerial insights for selecting the best strategies for suppliers and retailers under different market and operational environments.

Key Words: Pricing; Lead-time; Duopoly common retailer channel; Stackelberg game; Vertical Nash game

Table of Contents

Acknowledgements

I would like to express my gratitude to all those who helped me during the writing of this thesis.

My deepest gratitude goes first and foremost to Professor Zhengping Wu, my supervisor, for his constant encouragement and guidance. He has walked me through all the stages of the writing of this thesis. Without his consistent and illuminating instruction, this thesis could not have reached its present form.

Second, I would like to express my heartfelt gratitude to Professor Kum Khiong Yang and Kwan Eng Wee. They give me really useful suggestions during the writing process. And I also want to thank all the professors in my programme, who have instructed and helped me a lot in the past two years.

Last my thanks would go to my beloved family for their loving considerations and great confidence in me all through these years. I also owe my sincere gratitude to my friends and my fellow classmates who gave me their help and time in listening to me and helping me work out my problems during the difficult course of the thesis.

Chapter 1

Introduction

In recent time, the quality of many consumer products, such as washing machines and televisions, has become fairly similar across brands. Many companies thus compete increasingly on price and lead-time. An e-retailer, for example, can offer products from different brands on his website. A customer who wants to buy a television can visit the websites of e-retailers, such as www.51mjd.com and www.qhea.com, to compare the products, prices, and promised delivery lead-times of different brands. In some cases, a customer can even liaise with an e-retailer to customize a product according to his preference, such as adding his name or a logo on a jersey produced by a specific supplier. Once the order is placed, the e-retailer can then relay the order to the specific supplier to customize, assemble and deliver the product directly to the customer.

The above scenario is relatively common in many e-retailing platforms where a group of competing retailers serves as intermediaries or brand-agents for a group of competing suppliers. Each supplier offers its product to potential retailers at a fixed wholesale price with a promise to produce and deliver the orders to the end customers within a Promised Delivery Lead-Time (PDL). Each retailer then, in turn, adds on its own desired margin and offers the product to its customers at a fixed retail price and the PDL promised by the supplier. In most cases, we observe that each seller (both supplier and retailer) generally favors offering to all its clients a fixed price and PDL which may differ amongst different sellers.

In an environment where suppliers and retailers compete for consumers to maximize their own profits, setting the right price and PDL is a challenge. For example, a supplier who offers its product at a higher wholesale price and longer PDL might lose its customers to other suppliers. In contrast, a supplier who sets a lower wholesale price and shorter PDL might increase its sales, but face a greater load on its capacity and ability to meet its PDL. With many orders and limited capacity, the actual Realized Delivery Lead-Time (RDL) of a supplier might differ from its PDL. If RDL is larger than PDL, loss of goodwill and penalty for late delivery might occur. If RDL is smaller than PDL, storage cost for early order completions might occur. As suppliers are responsible for the delivery of their own products, each supplier is likely the sole party responsible for the cost of its early and late deliveries. To maximize its revenue and minimize its penalty for late delivery, a supplier must offer a short and competitive PDL with sufficient capacity to achieve a reliable RDL. Similarly, a retailer who offers a lower retail price may enjoy higher sales than a retailer who offers a higher retail price. The revenue of each supplier and retailer is thus a function of the retail prices and PDLs offered by different suppliers and retailers. The goal of each seller is to set a price and PDL that maximize its own profits.

This thesis studies the pricing and lead-time decisions of a dual-level decentralized supply chain (DSC) consisting of two suppliers and two retailers in a price- and lead-time-sensitive market. The retailers serve as brand-agents and collate orders for two competing products, each produced by a different supplier. Each supplier produces its own product and delivers the orders directly to the customers.

The two retailers sell the products of both suppliers such that competitions are involved in both horizontal and vertical levels.

This thesis is the first study to examine the pricing and lead-time decisions in the presence of vertical and horizontal interactions in a dual-level supply chain. Most of the past papers on the duopoly common retailer structure focus on price and/or quantity decisions. In this study, we introduce the PDL as another decision for the suppliers. As a result, the suppliers suffer a lead-time cost resulting from the different between PDL and RDL (which is first introduced by Liu et al. 2007). We find suppliers can gain significant advantages from their decisions on PDL, especially when their capacities are high. The performance of the retailers is significantly affected by the capacities of the suppliers, too.

When there is more than one level, members between different levels and members in the same level can play game with each other. Channel leadership, which describes whether suppliers or retailers have the channel power to exploit the others' reaction functions, will influence the results of equilibrium. Retailers normally have conflicting goals from those of the suppliers. Thus far, most channel studies have traditionally approached the problem from the supplier's perspective.

Presently, many retailers have caught up with the suppliers, some are even more powerful than the suppliers, and are gaining more influence on how products are distributed and at what price. There have been only a limited number of research papers that focus on channels with these powerful retailers. In order to fully analyze the effects of different channel leadership, we provide an analysis of this duopoly common retailer structure under three different decision scenarios. One is considering more powerful suppliers under which the suppliers act as the leaders and the retailers as the followers (which is often referred to as Suppliers Stakelberg game), the second one is considering more powerful retailers under which the retailers act as the leaders and the suppliers as the followers (which is called Retailers Stakelberg game), and at last we consider retailers and suppliers with similar power make decisions at the same time (which is so-called Vertical Nash game). We will examine the effect of different market and operational environments on the performance of the system under these different game rules.

This thesis is organized as follows. In Section 2, we provide a review of the literature on price and lead-time competition and on different channel structure. In Section 3, we introduce our model and assumptions. In section 4, we establish the Suppliers Stackelberg game, describe best reactions for retailers and get the equilibrium solution for suppliers. The Retailers Stackelberg game and theVertical Nash game are examined in sections 5 and 6 respectively. In section 7, we present numerical comparisons of supply chain's performance under different games and conduct several sensitivity analysis of the main marketing and operational parameters. Finally, we conclude in section 8 with future research directions.

Chapter 2

Literature Review

The importance of products' PDL has been widely recognized since 1980s. From then on, many papers have studied on price and PDL decisions together. Three main streams of literature are relevant to our study: the first stream consider a monopolist firm, which means there is only one decision maker. Palaka et al. (1998) study a firm, where customer demand is treated as linear in the quoted price and lead-time. Firm operations are modeled as an M/M/1 queue with first-come first-serve sequencing. The objective is to maximize revenues minus the total variable production costs, congestion-related costs and lateness penalty costs subject to a service-level constraint, which specifies the minimum probability of meeting the quoted lead-time. So and Song (1998) use the log-linear Cobb-Douglas demand function to model the demand in a similar setting, but do not include congestion or lateness penalty costs in the objective function. They developed a model to study the optimal selection of price, uniform delivery time and capacity expansion to maximize the overall profit, where demands were assumed to be sensitive to both the price and delivery time. Boyaci and Ray (2003) extend the previous two models to the case of two substitutable products within one firm for which dedicated capacities are allocated. They examine price, lead-time, and capacity decisions of two substitutable products for a firm with price- and time-sensitive demands. They develop insights into the relationship between the relative cost of capacity for the two products and the price or time differentiation that the firm offers to the market. Ray and Jewkes (2004) extend previous research by explicitly modeling a relationship between price and delivery time. The firm they investigate can invest in increasing capacity to guarantee a shorter delivery time but must be able to satisfy the guarantee according to a pre-specified reliability level. The model accounts for whether customers are "price sensitive" or "lead-time sensitive" by capturing the dependence of both price and demand on delivery time.

The second stream mainly focuses on competition among firms without consideration of upstream or downstream. So (2000) extends the work of So and Song (1998) to a competitive setting of N M/M/1 firms using a multiplicative competitive interaction model, where the market size is constant and shared among firms based on their "attraction" given their quoted prices and lead-times. Cachon and Harker (2002) present the option of outsourcing to a supplier for two competing firms. Two types of competition are analyzed: an M/M/1 queueing game with price and time sensitive demand and an EOQ game with fixed ordering costs and price sensitive demand. In their paper, they aggregate price and waiting time into a “full price”. For the queueing game, each firm’s demand rate is modeled as a function of the full prices of both firms with two forms: linear and truncated logit. Allon and Federgruen (2007, 2009) model N M/M/1 firms, the former for a single customer class and the latter for N customer classes. In Allon and Federgruen (2007), the authors use service level, which is defined as the difference between an upper bound benchmark for waiting time and the firm’s actual waiting time standard, and expressed in terms of the expected waiting time or the critical fractile of the waiting time distribution. A cost per unit time proportional to adopted capacity is included in the profit function. Three types of competition are studied: Two-stage games, where service level is set in the first stage while price is set in the second stage and vice versa, and simultaneous price and service competition. In Allon and Federgruen (2009), waiting time is explicitly incorporated into the demand model. A class dependent cost and a cost per unit time proportional to capacity are included in the profit function. Price only competition, waiting time only competition, and simultaneous competition are studied using dedicated or shared facilities for customer classes.

The third stream involves about two-level decision makers. Those papers consider a two-level supply chain and analyze the effects of intra-supply chain competition on the selection of optimal price and lead-time. Boyaci and Gallego (2004) consider two supply chains, each one consists of a supplier and a retailer, and compete on the basis of customer service. They use a queueing model with generic lead-time distribution. Three scenarios are analyzed: 1) Both supply chains are uncoordinated, i.e., each party selects their own decisions , 2) a hybrid scenario where only one supply chain is coordinated, and 3) both supply chains are coordinated. Bernstein and Federgruen (2007) study a multi-period setting , where there exist a common supplier and competing independent retailers. Customer demand depends on all of the firms' prices and a measure of service level. In these papers, the lead-time decision is formulated in the way of the service level, and they are taken as stable without considering the tardiness cost for late delivery. Liu et al. (2007) study a decentralized supply chain consisting of a supplier and a retailer facing price- and lead-time-sensitive demands. They examine the interaction between the suppliers' RDL and PDL closely, and demonstrate its strong impact on the decisions and on the performance in a DSC. However, this paper only consider the competition within the supply chain. We establish a more complex supply chain and take the inter-supply chain competition into account.

From the channel structure aspect, the issue of "power" in the supply chain channels for consumer products has received considerable attention in both academic and practitioner journals (e.g., Messinger and Narasimhan 1995, Johnson 1988, Business Week 1992). While most of previous models about channel structures are about single supplier and single retailer; McGuire and Staelin (1983) consider a single supplier being paired with a single retailer, but there are two (or more) such exclusive pairs. Gupta and Loulou (1998) extend the former work and find the optimal channel structure decision depends on interactions between two parameters: the degree of substitutability between products and the level of investements required to achieve production cost reduction; Ingene and Parry (1995) studies a two-part tariff problem using a multiple retailer model with a single supplier. It was shown that the (near-) optimal tariff from this multiple retailer model can be more profitable than the channel coordinating solution from dyadic models. A model with multiple suppliers and a single retailer has also been analyzed: Choi (1991) focuses on the effects of retailer power that stems from dealing multiple products. Vrinda et al. (2000) study under a similar situation, but they extend the game-theoretic literature by allowing for a continuum of possible channel interactions between suppliers and a retailer, they examine how channel power is related to demand conditions facing various brands and cost parameters of various suppliers. Choi (1996) merges these latter two structures to study multiple suppliers and retailers channel. He manages to study price decision in a duopoly common retailer channel and finds some new insights. From then on, the duopoly common retailer channel has been widely discussed, but almost all papers only focus on price and/or quantity decisions under this structure. In our model, we combine the pricing and PDL decisions into this channel, and it works out that the new decision variable we introduced, the supplier's PDL, will make the results different from the previous results obtained by considering the pricing decisions only under identical marketing conditions (such as Choi, 1996) .

Chapter 3

Problem Formulation

In this paper, we consider a duopoly common retailer channel structure, as depicted in Figure 1. Each of the two suppliers sells their partially differentiated products through both retailers to end customers, who are sensitive to both price and lead-time.

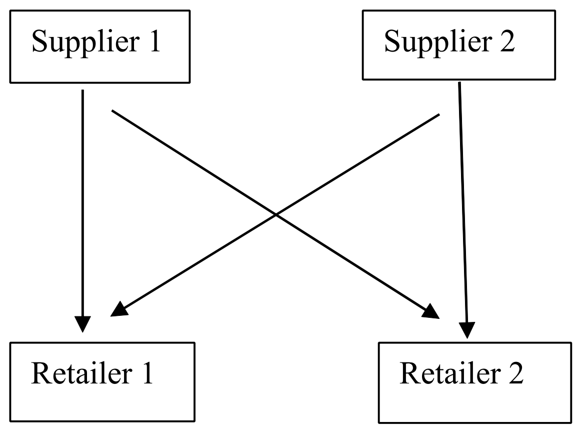

Figure 1 Duopoly Common Retailer Channel Structure

Upon receiving an order from the retailer, the supplier completes the product and delivers it to the customer directly. Since the product is not unique in the market (the other supplier produces a substitutable product) and potential customers for the product are sensitive to both price and PDL, the suppliers has to offer a competitive wholesale price and PDL. On the other hand, as the capacity of the supplier is finite and cannot be changed quickly, higher demands may cause longer customer RDL and late delivery. When this occurs, we assume that customers' additional waiting cost and inconvenience from late delivery will be covered exactly by the supplier with a generic penalty cost per unit per unit time late, and the possible delivery delay and the associated compensation are known to the customer . Since two retailers sell both products, they have to determine their individual retail price carefully to attract more customers and maximize own profit.

As independent decision makers, the suppliers and the retailers make their own decisions aiming at maximizing their individual profit rates. The suppliers know their production facility and how quickly they can respond to customer orders. Naturally then, the suppliers should determine the PDL to be quoted to customers in addition to the wholesale price. They will be fully responsible for any late delivery penalties. On the other hand, the retailers should determine the best retail prices so that the retailers' own profits rate are maximized. Because the retail prices together with the PDL will affect the level of demands and thus the suppliers' profits, the suppliers must consider the action of the retailers when making their decisions and this is the so-called intra supply chain effects. Meanwhile, the decisions of the other supplier will also affect its demand and profit and this is the so-called inter supply chain competition. We assume that the suppliers share with the retailers the RDL information fully and are informed by the retailers of how customers will react to the PDL and retail prices.

3.1 Notation

The notation used in our model is summarized as follows. Whenever two subscripts are used, the first one refers to the suppliers, while the second one to the retailers.

λ_{ij} : demand rate for product i sold by retailer j;

λ_{S_i} : total demand for product i, i.e., demand rate for supplier i, $\lambda_{S_i} = \lambda_{ii} + \lambda_{ij}$;

λ_{R_i} : demand rate for retailer i, $\lambda_{R_i} = \lambda_{ii} + \lambda_{ji}$;

λ_0 : market potential for a single product sold by each retailer (this is the total demand if all decisions are zero);

α : price sensitivity of demand;

β : lead-time sensitivity of demand;

θ_p : sensitivity of switchovers toward price difference;

θ_l : sensitivity of switchovers toward promised lead-time difference;

θ_r : sensitivity of switchovers toward retailer difference;

p_{ij}: price of product i sold by retailer j;

l_i : promised lead-time by supplier i;

All these parameters are assumed to be nonnegative.

3.2 Demand Function

In this thesis, we choose to employ a linear demand function that:

$$\lambda_{11} = \lambda_0 - \alpha p_{11} - \beta l_1 + \theta_p (p_{21} - p_{11}) + \theta_l (l_2 - l_1) + \theta_r (p_{12} - p_{11})$$
$$\lambda_{12} = \lambda_0 - \alpha p_{12} - \beta l_1 + \theta_p (p_{22} - p_{12}) + \theta_l (l_2 - l_1) + \theta_r (p_{11} - p_{12})$$
$$\lambda_{21} = \lambda_0 - \alpha p_{21} - \beta l_2 + \theta_p (p_{11} - p_{21}) + \theta_l (l_1 - l_2) + \theta_r (p_{22} - p_{21})$$
$$\lambda_{22} = \lambda_0 - \alpha p_{22} - \beta l_2 + \theta_p (p_{12} - p_{22}) + \theta_l (l_1 - l_2) + \theta_r (p_{21} - p_{22})$$

From where we can get:

$$\lambda_{S_1} = 2\lambda_0 - \alpha(p_{11} + p_{12}) - 2\beta l_1 + 2\theta_l(l_2 - l_1) + \theta_p(p_{21} + p_{22} - p_{11} - p_{12})$$
$$\lambda_{S_2} = 2\lambda_0 - \alpha(p_{11} + p_{12}) - 2\beta l_2 + 2\theta_l(l_1 - l_2) + \theta_p(p_{11} + p_{12} - p_{21} - p_{22})$$

This kind of demand function is very common in economic literatures, such as Choi (1996) and Boyaci and Ray (2003). We derive the demand function for two suppliers by using a method mentioned by Singh and Vives (1984). We assume there is a continuum of consumers of the same type with a utility function separable and linear in the particular product. From the perspective of two products (or two suppliers), the representative consumer maximizes:

$$U(\lambda_{s_1},\lambda_{s_2})-(p_{11}q_1+p_{12}q_2)\lambda_{s_1}-(p_{21}q_3+p_{22}q_4)\lambda_{s_2}-c_w l_1\lambda_{s_1}-c_w l_2\lambda_{s_2}$$

Where $q_1=\Pr(p_{11}<p_{12}), q_2=\Pr(p_{11}\geq p_{12})$, $q_3=\Pr(p_{21}<p_{22}), q_4=\Pr(p_{21}\geq p_{22})$, and c_w is the waiting cost per unit time. The utility function $U(\lambda_{s_1},\lambda_{s_2})$ is assumed to be quadratic and strictly concave, which can be expressed as follows:

$$U(\lambda_{s_1},\lambda_{s_2})=x(\lambda_{s_1}+\lambda_{s_2})-y(\lambda_{S_1}^2+z\lambda_{s_1}\lambda_{s_2}+\lambda_{S_2}^2)/2$$

Here x,y,z are positive parameters. This utility function gives rise to a linear demand structure. Direct demands are given by:

$$\lambda_{s_1}=\frac{xy-xz}{y^2-z^2}+\frac{z(p_{21}q_3+p_{22}q_4)}{y^2-z^2}-\frac{y(p_{11}q_1+p_{12}q_2)}{y^2-z^2}+\frac{c_w(zl_2-yl_1)}{y^2-z^2}$$

$$\lambda_{s_2}=\frac{xy-xz}{y^2-z^2}+\frac{z(p_{11}q_1+p_{12}q_2)}{y^2-z^2}-\frac{y(p_{21}q_3+p_{22}q_4)}{y^2-z^2}+\frac{c_w(zl_1-yl_2)}{y^2-z^2}$$

We can get the demand function for each supplier exactly as we mentioned above by letting $q_1=q_2=q_3=q_4=1/2$, $\lambda_0=x/(y+z)$, $\alpha=1/[2(y+z)]$ and $\beta=\alpha c_w$.

3.3 Assumptions

We make some reasonable assumptions to simplify our model without loss of generality.

1. Assume no competition between different products in different retailers because this influence is quite small, we can just ignore it. If needed, we can include

this factor into our demand function by just add a new parameter and the results will not be affected (further explanation can be found in Appendix A).

2. Assume each retail selects the same level of margin for both products, which denoted by m_j; so we have $p_{ij} = w_i + m_j$. We make this assumption because that due to the assumptions of symmetric demand function and horizontal differentiations of products and stores, we have a prior knowledge that the nontrivial equilibrium retail margins for the two products are the same (we take one of our games as an example to show this in the Appendix A). We make it also for analytical tractability and reduce the dimensions of the problem. So the demand function can rewrite as follows:

$$\lambda_{R_1} = 2\lambda_0 - \alpha(2m_1 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_2 - m_1) \tag{1}$$

$$\lambda_{R_2} = 2\lambda_0 - \alpha(2m_2 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_1 - m_2) \tag{2}$$

$$\lambda_{S_1} = 2\lambda_0 - \alpha(2w_1 + m_1 + m_2) - 2\beta l_1 + 2\theta_p(w_2 - w_1) + 2\theta_l(l_2 - l_1) \tag{3}$$

$$\lambda_{S_2} = 2\lambda_0 - \alpha(2w_2 + m_1 + m_2) - 2\beta l_2 + 2\theta_p(w_1 - w_2) + 2\theta_l(l_1 - l_2) \tag{4}$$

3. Assume symmetric manufacturing costs, holding and tardiness cost for both suppliers to simplify our analysis without distracting from main insights.

Let b be the tardiness cost per unit per unit time and h be the holding cost per unit per unit time. Let F_{λ_i} and f_{λ_i} be the PDF and CDF of the RDL for a given demand rate λ_i, then the expected lead-time cost for supplier i is:

$$C(l_i, f_{\lambda_i}(t)) = h\int_0^{l_i}(l_i - t)f_{\lambda_i}(t)dt + b\int_{l_i}^{\infty}(t - l_i)f_{\lambda_i}(t)dt$$

We can easiy show that $C(l_i, f_{\lambda_i}(t))$ is convex in l_i. Note that if early delivery is allowed without penalty, the holding cost h can be set to zero and the above model is still completely valid. Also, for practical systems, we should have $b > h \geq 0$;

otherwise, suppliers will always set PDL to be 0.

Let c be the manufacturing cost per unit, let w_i be the wholesale prices given by supplier i, we can get the supplier's optimization problem(SOP)should be given by:

$$SOP\text{: max } \Pi_{S_i} = (w_i - c - C(l_i, f_{\lambda_i}(t)))\lambda_{S_i} \tag{5}$$

4. Normalize the processing costs to retailers to be 0 . So the retailer's optimization problem (ROP) is given by:

$$ROP\text{: max } \Pi_{R_j} = m_j \lambda_{R_j} \tag{6}$$

Under this basic model, the final decision variables are: suppliers' wholesale prices, suppliers' PDL and retailers' margins. As we have discussed, the supply chain members play games in making decisions, then different game rules may result in different results. We present three different kinds of games to cover all the possible power structures in this two-level competition in next three sections, to see the effects of different games on the players' decisions.

Chapter 4

The Suppliers Stackelberg Game

In this section, we consider the scenario that the four members of the supply chain interact in a Stackelberg game with the suppliers as the leaders and the retailers as the followers. In this Suppliers Stackelberg (SS) game, each manufacturer chooses his wholesale price and lead-time using the retailers' reaction functions and the wholesale price of the competitor's product. Given these wholesale prices, each retailer determines its margins. We present the retailers best response to the suppliers decisions and determine the suppliers PDL strategy. We propose one common RDL model (the M/M/1 queuing model, which has been often used in the supply chain literature, e.g., So, 2000) and present the corresponding equilibrium strategies of the players under this model and other two games.

4.1 Retailers' Best Response

The retailers' best response for the suppliers' w and l are given in the following lemma. The proof of this lemma as well as all the other proofs in this thesis is given in the appendix B.

Lemma 1. *The payoff function for each retailer is concave in its own strategy. And there exists a unique Nash equilibrium between two retailers.*

Due to the concavity of the payoff function and the uniqueness of equilibrium, the retailers best pricing strategy is given by:

$$\begin{cases} \dfrac{\partial \Pi_{R_1}}{\partial m_1} = 2\lambda_0 - \beta(l_1 + l_2) + 2\theta_r m_2 - \alpha(w_1 + w_2) - 4m_1(\alpha + \theta_r) = 0 \\ \dfrac{\partial \Pi_{R2}}{\partial m_2} = 2\lambda_0 - \beta(l_1 + l_2) + 2\theta_r m_1 - \alpha(w_1 + w_2) - 4m_2(\alpha + \theta_r) = 0 \end{cases}$$

Solving this, we can get that:

$$m_1^* = m_2^* = \frac{2\lambda_0 - \beta(l_1 + l_2) - \alpha(w_1 + w_2)}{2(2\alpha + \theta_r)} \tag{7}$$

As a rational decision maker, we should have $m_1^* = m_2^* \geq 0$, which means that we should have $[\,2\lambda_0 - \beta(l_1 + l_2) - \alpha(w_1 + w_2)\,]/2(2\alpha + \theta_r) \geq 0$, since every parameter and decision variable should be positive, we need to have the following relationships $0 \leq w_1 + w_2 \leq 2\lambda_0 / \alpha - \beta(l_1 + l_2)/\alpha$ and $2\lambda_0 \geq \beta(l_1 + l_2)$.

4.2 Suppliers' Decisions

Substituting m_i^* in Eq. (7) back to Eq. (3) and (4), we can get the demand function only in terms of w_i and l_i. For two suppliers, since each supplier must determine two variables: the wholesales price w_i and the promised lead-time l_i, we can make a simple operation on these two equations to get the supplier's wholesales price w_i in terms of l_i and λ_{S_i} (from now on, we call it λ_i for short) to simplify the calculation procedures.

Add Eq.(3) to Eq.(4), we can get:

$$w_2 + w_1 = \frac{2\lambda_0}{\alpha} - \frac{\beta(l_1 + l_2)}{\alpha} - \frac{(\lambda_1 + \lambda_2)(2\alpha + \theta_r)}{2\alpha(\alpha + \theta_r)} = k_1$$

Here, we need to have $w_1 + w_2 \geq 0$ to ensure the decision makers are rational, which means we should have $2\lambda_0 \geq \beta(l_1 + l_2) + (\lambda_1 + \lambda_2)(2\alpha + \theta_r)/[2(\alpha + \theta_r)]$.

Subtract Eq.(4) from Eq.(3), we can get:

$$w_2 - w_1 = \frac{(\lambda_1 - \lambda_2) + (2\beta + 4\theta_l)(l_1 - l_2)}{2(\alpha + 2\theta_p)} = k_2$$

From where we can get that $w_1 = (k_1 - k_2)/2$ and $w_2 = (k_1 + k_2)/2$. Using these two equations, we can express the suppliers' problem in terms of λ_i and l_i.

$$SOP: \max \Pi_{s_1} = \left(\frac{k_1 - k_2}{2} - c - C(l_1, f_{\lambda_1}(t)) \right) \lambda_1$$

$$\max \Pi_{s_2} = \left(\frac{k_1 + k_2}{2} - c - C(l_2, f_{\lambda_2}(t)) \right) \lambda_2$$

We use a sequential solution procedure to solve this problem (similar to Liu et al., 2007): for a given λ_i, we first obtain $l^*(\lambda_i)$ as a function of λ_i; we then substitute $l^*(\lambda_i)$ into Π_{S_i} and change the supplier's decision problem to a single-variable problem.

Lemma 2. *For a given λ_i, the supplier's payoff function is concave in l_i.*

Due to the concavity of supplier's payoff function, the suppliers best PDL is uniquely given by setting:

$$\begin{cases} \dfrac{\partial \Pi_{S_1}}{\partial l_1} = \lambda_1 [b - (b+h) F_{\lambda_1}(l_1) - \dfrac{1}{2}(\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p})] = 0 \\ \dfrac{\partial \Pi_{S_2}}{\partial l_2} = \lambda_2 [b - (b+h) F_{\lambda_2}(l_2) - \dfrac{1}{2}(\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p})] = 0 \end{cases}$$

Solving this, we can get the suppliers' best l_i as follows:

$$l_i^* = F_{\lambda_i}^{-1}\left[\frac{b - \frac{1}{2}(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p})}{b + h}\right]$$

The above two equations are similar to the optimal order quantity formula in the standard newsvendor problem, with the demand distribution being replaced by the RDL distribution. The term $b - [\beta/\alpha + (\beta + 2\theta_l)/(\alpha + 2\theta_p)]/2$ is a reflection of the cost structure and the market factors. Moreover, if the suppliers tardiness cost $b \le [\beta/\alpha + (\beta + 2\theta_l)/(\alpha + 2\theta_p)]/2$, then it is optimal for the supplier to always quote a zero lead-time regardless of the demand rate. In order to avoid this situation, we assume that $b > [\beta/\alpha + (\beta + 2\theta_l)/(\alpha + 2\theta_p)]/2$ to exclude this trivial solution.

4.3 The RDL Model

It is usually very difficult to derive the RDL distribution for a real supply chain. Even though for some simple systems we can derive the exact RDL distributions, they would be too complicated to be useful in our optimization. We assume that there exists an inherent (uncapacitated or the RDL without any waiting) RDL X_i and propose a model to modify this X_i so as to characterize the actual system RDL Y_i.

Let ϕ and Φ denote the density function and distribution function of X_i respectively. Assuming no workload influence, then, the supplier should quote

$$\bar{l}_i = \Phi_i^{-1}\left[\frac{b - \frac{1}{2}(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p})}{b + h}\right]$$

We note that $\bar{l}_i$ depends on the system cost structure, the marketing conditions and the property of the inherent RDL, but is independent of the external demands, so in a particular market, for a given system this parameter is a constant. We call $\bar{l}_i$ the system's configuration lead-time.

4.3.1 The M/M/1 Model

Suppose that X_i is the exponential service time of a single-server queue with a service rate μ_i. Assume that the demand process is Poisson with rate λ_i (where $\mu_i \geq \lambda_i$). Then, naturally, Y_i is the steady-state customer sojourn time in the M/M/1 queue. From standard queuing results, we will have $\Phi_i^{-1}(x_i) = \mu_i^{-1} \ln[1/(1-x_i)]$ and $F_{\lambda_i}^{-1}(x_i) = (\mu_i - \lambda_i)^{-1} \ln[1/(1-x_i)]$.

Combining these, we can get the configuration lead-time and best PDL become:

$$\bar{l}_i = \mu_i^{-1} \ln \left[\frac{b+h}{h + \frac{1}{2}\left(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)} \right]$$

$$l_i^* = \left(\frac{\lambda_i}{\mu_i - \lambda_i} + 1\right) \bar{l}_i \tag{10}$$

Note that the configuration lead-time $\bar{l}_i$ is decreasing in μ_i and the optimal lead-time decision l_i^* is independent of other player's strategy. This independence enables us to reduce the dimension of the game, thus facilitate the proof of the existence of a unique equilibrium. Further more, Eq. (10) has an interesting interpretation: PDL can be treated as the product of the system configuration lead-time and the average number of outstanding orders plus one. The configuration lead-time is the product of the expected service time $1/\mu$ and the safety factor. This interpretation has an intriguing implication: The configuration lead-time is associated with a single outstanding order, i.e., it is the PDL we quote to an order when no other order is on hand. This shows that the configuration lead-time is the key in determining the PDL. Furthermore, we note that l^* will never be smaller than $\bar{l}$.

Under this M/M/1 queue assumption, the optimal PDL should be as follows:

$$l_i^* = \frac{\mu_i}{\mu_i - \lambda_i}\bar{l}_i$$

$$= \frac{1}{\mu_i - \lambda_i}\ln\left(\frac{b+h}{h + \frac{1}{2}(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p})}\right)$$

And from where we can further simplify our lead-time cost function as follows:

$$C(l_i, f_\lambda(t)) = h\int_0^{l_i}(l_i - t)f_\lambda(t)dt + b\int_{l_i}^{\infty}(t - l_i)f_\lambda(t)dt$$

$$= h\int_0^{\bar{l}_i}\left(\frac{\mu_i}{\mu_i - \lambda_i}\bar{l}_i - \frac{\mu_i}{\mu_i - \lambda_i}t\right)\mu_i e^{-\mu_i t}dt + b\int_{\bar{l}_i}^{\infty}(\frac{\mu_i}{\mu_i - \lambda_i}t - \frac{\mu_i}{\mu_i - \lambda_i}\bar{l}_i)\mu_i e^{-\mu_i t}dt$$

$$= \frac{\mu_i}{\mu_i - \lambda_i}C(\bar{l}_i, \Phi_i)$$

Where we have:

$$C(\bar{l}_i, \Phi_i) = h\int_0^{\bar{l}_i}\left(\bar{l}_i - t\right)\mu_i e^{-\mu_i t}dt + b\int_{\bar{l}_i}^{\infty}(t - \bar{l}_i)\mu_i e^{-\mu_i t}dt$$

$$= \frac{h}{\mu_i}\ln\left(\frac{b+h}{h + \frac{1}{2}(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p})}\right) + \frac{1}{2}\left(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)$$

We can see that, $C(\bar{l}_i, \Phi_i)$ which is the system's configuration lead-time cost, depends on the system cost structure, the marketing conditions and the supplier's capacity μ_i, but is independent in external demands or other players' strategies, so in a particular market, for a given system this parameter will be a constant.

In above analysis, we need $2\lambda_0 \geq \beta(l_1 + l_2) + (\lambda_1 + \lambda_2)(2\alpha + \theta_r)/[2(\alpha + \theta_r)]$ to make sure $w_1 + w_2 \geq 0$, and in this M/M/1 model, in order to ensure a positive wholesale price for each supplier, we need to have the following relationship that $2\lambda_0 / \beta - (\lambda_1 + \lambda_2)(2\alpha + \theta_r)/[2\beta(\alpha + \theta_r)] \geq \mu_1 \bar{l}_1(\mu_1 - \lambda_1) + \mu_2 \bar{l}_2(\mu_2 - \lambda_2)$.

4.3.2 The Stackelberg Equilibrium

Substituting the suppliers' optimal PDL given by Eq.(10) and the new lead-time cost function into the profit function given by Eq.(5), we can obtain a new profit function with only one variable λ_i:

$$\Pi_{s_1} = \left(\frac{k_1(\lambda_i) - k_2(\lambda_i)}{2} - c - \frac{\mu_1}{\mu_1 - \lambda_1} C(\bar{l}_1, \Phi_1) \right) \lambda_1$$
$$\Pi_{s_2} = \left(\frac{k_1(\lambda_i) + k_2(\lambda_i)}{2} - c - \frac{\mu_2}{\mu_2 - \lambda_2} C(\bar{l}_2, \Phi_2) \right) \lambda_2$$

Lemma 3. *The payoff function for each supplier is concave in its own strategy. And there exists a unique Nash equilibrium between two suppliers.*

Due to the concavity of the payoff function and the uniqueness of equilibrium, the suppliers best demand rate strategy λ_i^* is the unique solution to :

$$\begin{cases} \dfrac{\partial \Pi_{S_1}}{\partial \lambda_1} = \dfrac{k_1(\lambda_i) - k_2(\lambda_i)}{2} - c - \dfrac{\mu_1}{\mu_1 - \lambda_1} C(\bar{l}_1, \Phi_1) + \lambda_1 \left[\dfrac{1}{2}\left(\dfrac{\partial k_1}{\partial \lambda_1} - \dfrac{\partial k_2}{\partial \lambda_1} \right) - \dfrac{\mu_1}{(\mu_1 - \lambda_1)^2} C(\bar{l}_1, \Phi_1) \right] = 0 \\ \dfrac{\partial \Pi_{S_2}}{\partial \lambda_2} = \dfrac{k_1(\lambda_i) + k_2(\lambda_i)}{2} - c - \dfrac{\mu_2}{\mu_2 - \lambda_2} C(\bar{l}_2, \Phi_2) + \lambda_2 \left[\dfrac{1}{2}\left(\dfrac{\partial k_1}{\partial \lambda_2} + \dfrac{\partial k_2}{\partial \lambda_2} \right) - \dfrac{\mu_2}{(\mu_2 - \lambda_2)^2} C(\bar{l}_2, \Phi_2) \right] = 0 \end{cases}$$

Once we get the optimal demand rate strategies λ_i^* , we can get l_i^* from Eq. (10) and get w_i^* from Eq. (8) and (9).

Further more, we can simplify Eq. (7) and (1), (2) as follows:

$$m_1^* = m_2^* = \frac{\lambda_1^* + \lambda_2^*}{4(\alpha + \theta_r)}$$

$$\lambda_{R_1}^* = \lambda_{R_2}^* = \frac{\lambda_1^* + \lambda_2^*}{2}$$

Then we can know that:

$$\Pi_{R_1} = \Pi_{R_2} = \frac{(\lambda_1^* + \lambda_2^*)^2}{8(\alpha + \theta_r)}$$

Chapter 5

The Retailers Stackelberg Game

In this section, we consider the scenario that the four members of the supply chain interact in a Stackelberg game with the retailers as the leaders and the suppliers as the followers. In this Retailer Stackelberg (RS) game, each retailer chooses its margins using the suppliers' reaction functions and the other retailer's margins . Each supplier sets his wholesale price and PDL, conditional on these retailer margins and the competing product's wholesale price and PDL.

5.1 Suppliers' Best Response

To the suppliers, the solution step is similar to former section, the only difference is that here the retailers margin m_i is given , then suppliers' wholesales price w_i can be expressed in terms of m_i, l_i and w_i . Add Eq. (3) to Eq.(4), we can get:

$$w_2 + w_1 = \frac{2\lambda_0}{\alpha} - \frac{\beta(l_1 + l_2)}{\alpha} - \frac{(\lambda_1 + \lambda_2)}{2\alpha} - (m_1 + m_2) = k_3$$

Here, we need to have $w_1 + w_2 \geq 0$ to ensure the decision makers are rational, which means we should have $2\lambda_0 / \alpha \geq \beta(l_1 + l_2)/\alpha - (\lambda_1 + \lambda_2)/2\alpha + (m_1 + m_2)$.

Subtract Eq.(4) from Eq.(3), we can get:

$$w_2 - w_1 = \frac{(\lambda_1 - \lambda_2) + (2\beta + 4\theta_l)(l_1 - l_2)}{2(\alpha + 2\theta_p)} = k_4$$

Once more we can get that $w_1 = (k_3 - k_4)/2$ and $w_2 = (k_3 + k_4)/2$ and we can express the suppliers' problem in terms of λ_i and l_i.

$$SOP: \max \Pi_{s_1} = \left(\frac{k_3 - k_4}{2} - c - C(l_1, f_{\lambda_1}(t)) \right) \lambda_1$$

$$\max \Pi_{s_2} = \left(\frac{k_3 + k_4}{2} - c - C(l_2, f_{\lambda_2}(t)) \right) \lambda_2$$

We use a similar sequential solution procedure to solve this problem as we have shown in Section 4.2: for a given λ_i, we first obtain $l^*(\lambda_i)$ as a function of λ_i; we then substitute $l^*(\lambda_i)$ into Π_{S_i} and change the supplier's decision problem to a single-variable problem.

Lemma 4. *For a given* λ_i *, the supplier's payoff function is concave in* l_i.

Due to the concavity of supplier's payoff function, the suppliers best demand rate λ_i^* is uniquely given by setting:

$$\begin{cases} \dfrac{\partial \Pi_{S_1}}{\partial \lambda_1} = \dfrac{k_3 - k_4}{2} - c - \dfrac{\mu_1}{\mu_1 - \lambda_1} C(\bar{l}_1, \Phi_1) + \lambda_1 \left[\dfrac{1}{2} \left(\dfrac{\partial k_3}{\partial \lambda_1} - \dfrac{\partial k_4}{\partial \lambda_1} \right) - \dfrac{\mu_1}{(\mu_1 - \lambda_1)^2} C(\bar{l}_1, \Phi_1) \right] = 0 \\ \dfrac{\partial \Pi_{S_2}}{\partial \lambda_2} = \dfrac{k_3 + k_4}{2} - c - \dfrac{\mu_2}{\mu_2 - \lambda_2} C(\bar{l}_2, \Phi_2) + \lambda_2 \left[\dfrac{1}{2} \left(\dfrac{\partial k_3}{\partial \lambda_2} + \dfrac{\partial k_4}{\partial \lambda_2} \right) - \dfrac{\mu_2}{(\mu_2 - \lambda_2)^2} C(\bar{l}_2, \Phi_2) \right] = 0 \end{cases}$$

Once we get the optimal demand rate strategies λ_i^* , we can get l_i^* from Eq. (10) and get w_i^* from Eq.(11) and (12).

5.2 Retailers' Decisions

Since the supplier's decision variable becomes only λ_i, so after we know the best response actions of the suppliers, the demand rate for the retailers, which is obtained by Eq. (1) and (2), can be simplified to only contain λ_i^* and m_i as follows:

$$\lambda_{R_1}^* = \frac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_2 - m_1)$$

$$\lambda_{R_2}^* = \frac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_1 - m_2)$$

Lemma 5. *The payoff function for each retailer is concave in its own strategy. And there exists a unique Nash equilibrium between two retailers.*

Due to the concavity of the payoff function and the uniqueness of equilibrium, the retailer's best strategy m_i^* is the unique solution to:

$$\begin{cases} \dfrac{\partial \Pi_{R_1}}{\partial m_1} = m_1 \left[\dfrac{1}{2} \left(\dfrac{\partial \lambda_1^*}{\partial m_1} + \dfrac{\partial \lambda_2^*}{\partial m_1} \right) - (\alpha + 2\theta_r) \right] + \dfrac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_2 - m_1) = 0 \\ \dfrac{\partial \Pi_{R2}}{\partial m_2} = m_2 \left[\dfrac{1}{2} \left(\dfrac{\partial \lambda_1^*}{\partial m_2} + \dfrac{\partial \lambda_2^*}{\partial m_2} \right) - (\alpha + 2\theta_r) \right] + \dfrac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_1 - m_2) = 0 \end{cases}$$

Since $\partial \lambda_1^* / \partial m_1^* = \partial \lambda_1^* / \partial m_2^*$ and $\partial \lambda_1^* / \partial m_1^* = \partial \lambda_1^* / \partial m_2^*$ (see appendix: proof of Lemma 5), we can derive that $m_1^* = m_2^*$, then we have:

$$m_1^* = m_2^* = \frac{\lambda_1^* + \lambda_2^*}{2(\alpha + 2\theta_r) - \left(\dfrac{\partial \lambda_1^*}{\partial m_1^*} + \dfrac{\partial \lambda_2^*}{\partial m_2^*} \right)}$$

$$\lambda_{R_1}^* = \lambda_{R_2}^* = \frac{\lambda_1^* + \lambda_2^*}{2}$$

Then we can know that:

$$\Pi_{R_1} = \Pi_{R_2} = \frac{(\lambda_1^* + \lambda_2^*)^2}{4(\alpha + \theta_r) - 2\left(\dfrac{\partial \lambda_1^*}{\partial m_1^*} + \dfrac{\partial \lambda_2^*}{\partial m_2^*} \right)}$$

Chapter 6

The Vertical Nash Game

In this Vertical Nash(VN) game, both suppliers decide their wholesales price and PDL, both retailers decide their margins at the same time conditioning on the decisions of all other members are given to maximize individual combined profit from both products. We can describe the players' profits under this game as follows:

$$\begin{cases} \Pi_{R_1} = m_1[2\lambda_0 - \alpha(2m_1 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_2 - m_1)] \\ \Pi_{R_2} = m_2[2\lambda_0 - \alpha(2m_2 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_1 - m_2)] \\ \Pi_{S_1} = [w_1 - c - C(l_1, f_{\lambda_1}(t))][2\lambda_0 - \alpha(2w_1 + m_1 + m_2) - 2\beta l_1 + 2\theta_p(w_2 - w_1) + 2\theta_l(l_2 - l_1)] \\ \Pi_{S_2} = [w_2 - c - C(l_2, f_{\lambda_2}(t))][2\lambda_0 - \alpha(2w_2 + m_1 + m_2) - 2\beta l_2 + 2\theta_p(w_1 - w_2) + 2\theta_l(l_1 - l_2)] \end{cases}$$

From the perspective of retailers, they try to maximize their profits as w_i and l_i are given, the solution steps are quite similar as in section 4 when they are followers. For the suppliers, from section 5, we can see that when they are followers, we can always transform the two-dimension decision problem of w_i and l_i into one dimension of demand λ_i only. So the solution steps for suppliers are quite similar with that in section 5 , we will not describe them again.

Lemma 6. *The payoff function for each game player is concave in its own strategy. And there exists a unique Nash equilibrium between these four supply chain members.*

Due to the concavity of the payoff function and the uniqueness of equilibrium, the supplier's best strategy λ_i^* and the retailer's best margin m_i^* are the unique solution to:

$$
\begin{cases}
\dfrac{\partial \Pi_{R_1}}{\partial m_1} = 2\lambda_0 - 2m_1(\alpha+\theta_r) - \alpha(2m_1 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_2 - m_1) = 0 \\
\dfrac{\partial \Pi_{R2}}{\partial m_2} = 2\lambda_0 - 2m_2(\alpha+\theta_r) - \alpha(2m_2 + w_1 + w_2) - \beta(l_1 + l_2) + 2\theta_r(m_1 - m_2) = 0 \\
\dfrac{\partial \Pi_{S_1}}{\partial \lambda_1} = \dfrac{M' - N'}{2} - c - \dfrac{\mu_1}{\mu_1 - \lambda_1} C(\bar{l}_1, \Phi_1) + \lambda_1 \left[\dfrac{1}{2}\left(\dfrac{\partial M'}{\partial \lambda_1'} - \dfrac{\partial N'}{\partial \lambda_1'} \right) - \dfrac{\mu_1}{(\mu_1 - \lambda_1)^2} C(\bar{l}_1, \Phi_1) \right] = 0 \\
\dfrac{\partial \Pi_{S_2}}{\partial \lambda_2} = \dfrac{M' + N'}{2} - c - \dfrac{\mu_2}{\mu_2 - \lambda_2} C(\bar{l}_2, \Phi_2) + \lambda_2 \left[\dfrac{1}{2}\left(\dfrac{\partial M'}{\partial \lambda_2'} + \dfrac{\partial N'}{\partial \lambda_2'} \right) - \dfrac{\mu_2}{(\mu_2 - \lambda_2)^2} C(\bar{l}_2, \Phi_2) \right] = 0
\end{cases}
$$

Due to the transformation of the suppliers' decision variables as in section 5, the demand rate λ_{R_i} for retailers can also be simplified as follows:

$$
\lambda_{R_1} = \frac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_2 - m_1)
$$
$$
\lambda_{R_2} = \frac{\lambda_1^* + \lambda_2^*}{2} + (\alpha + 2\theta_r)(m_1 - m_2)
$$

From above equations, we can obtain that:

$$
m_1^* = m_2^* = \frac{\lambda_1^* + \lambda_2^*}{4(\alpha + \theta_r)}
$$
$$
\lambda_{R_1}^* = \lambda_{R_2}^* = \frac{\lambda_1^* + \lambda_2^*}{2}
$$

Then we can know that:

$$
\Pi_{R_1} = \Pi_{R_2} = \frac{(\lambda_1^* + \lambda_2^*)^2}{8(\alpha + \theta_r)}
$$

Chapter 7

Numerical Analysis and Discussion of Results

In this section, we examine the impact of various input parameters on the optimal decisions, and discuss main managerial insights offered by our model. Due to the complexity of our problem, no closed-form solutions to the equilibrium strategies could be obtained, although they could be completely characterized. Therefore, in order to gain more concrete understanding of our results and the underlying intuition, we resort to extensive numerical experiments, whose results are reported below with interpretations.

In what follows, we first perform a sensitivity analysis to look at how equilibrium decisions are affected by various parameters in section 7.1. We then proceed to highlight main managerial insights along with intuitive explanation in section 7.2. To facilitate our study, throughout the following text, we limit our attention to the symmetric case only, namely, the two suppliers are assumed to have the same capacity ($\mu_1 = \mu_2 = \mu$). This symmetry assumption is commonly made in many papers involving competition (see McGuire and Staelin 1983, Ha and Tong 2008). As a result, the two suppliers' configuration lead-times are equal, and their configuration lead-time costs are equal as well. In other words, the asymmetry assumption implies that $\bar{l}_1 = \bar{l}_2 = \bar{l}$ and $C(\bar{l}_1, \Phi_1) = C(\bar{l}_2, \Phi_2) = \bar{C}$.

7.1 Sensitivity Analysis

In this section, we focus on how the equilibrium results such as the wholesale price, PDL, retail price and profits change with model parameters. The following Table 1 summarizes some of the important results of our sensitivity analysis (explanations are provided in the following several subsections).

7.1.1 Effect of Price Sensitivity Factor α

In this part, we examine the influence of price sensitivity factor α . From previous discussion (Eq. 10), we know that in order to keep the configuration lead-time greater than 0, we should have $b+h > h+\beta/2\alpha+(\beta+2\theta_l)/2(\alpha+2\theta_p)$. Under our given value of parameters, we need α > 0.2115, so our numerical example is conducted when $\alpha \in [0.25,1.5]$ with step size of 0.05 (results are shown in Appendix C, Figure 2).

Understandably, the effect of α is more related to the relationship between decision variables, but less related to the competition effect. That is, the effect of α in the absence of competition is essentially the same as that in the presence of competition. Consequently, our interpretation below will place an emphasis in the intrinsic relationship among all factors but competition, as if competition were not existent, as it only plays a secondary role in driving the effect of α .

A smaller value of α implies demand for a particular product is less effected by its price change. Therefore, as α decreases, it is natural to see higher wholesale price and retail price. Our results, as can be seen in Figure 2, agree with this convention.

Table 1. Comparative Statics

	Demand	PDL (l_i)	Wholesale Price (w_i)	Retailer Margin (m_{ij})	Retail Price	Supplier Profit (Π_{S_i})	Retailer Profit (Π_{R_j})	Total Profit
Price Sensitivity factor α	↓	↑	↓	↓	↓	↓	↓	↓
PDL sensitivity factor β	↓	↓	↑	↓	↑	↓	↓	↓
Product Differentiation θ_p	↑	↑	↓	↑	↓	↓	↑	↑
Retailer Differentiation θ_r	↑	↑	↑	↓	↓	↑	↓	↑

The impact of α on PDL is more intricate. In the traditional monopolist model without consideration of lead-time, demand is specified by $\lambda = \lambda_0 - \alpha p$, and the optimal price and demand are $\lambda_0 / 2\alpha$ and $\lambda_0 / 2$, respectively. Therefore, as α decreases, the optimal price increases, whereas the optimal demand is kept constant. This suggests that as the price sensitivity factor α decreases, it is more important to increase the profit margin in order to maximize profit. Similar logic applies in our problem when lead-time comes into the picture. In our model, the suppliers' profit margin is $w - c - C$. Since gaining deeper profit margin is the direction to maximize profit, the lead-time cost C should decrease at the optimum. Note that the lead-time cost and the optimal lead-time quotation always move in the same direction (both are $\mu / (\mu - \lambda)$ times a constant, as can be seen from Eq. (10) and the immediately following equations). Therefore, the optimal lead-time quotation decreases.

Now we turn to the optimal demand. As noted above, in the absence of lead-time, the optimal demand is independent of price sensitivity factor α. In the presence of lead-time, the demand is augmented by an additional term $-\beta l$. Since we know from above the PDL increases in α, it follows that a decrease in α will cause the optimal demand to increase.

A direct consequence of the increased demand and profit margin is that both the suppliers and the retailers enjoy higher profits as α decreases, which is straightforward.

7.1.2 Effect of Lead-Time Sensitivity Factor β

In this part, we examine the influence of lead-time sensitivity factor β on the performance of the supply chain.

Figure 3 (in Appendix 3) illustrate the results and indicates that as β increases, both the suppliers and the retailers are worse off, which is easy to understand because more demanding consumers never benefit any firm. Less obvious are the changes of the optimal decisions and the induced demand, which we now explain as follows. By definition, a larger value of β implies demand for a particular product is more effected by a lead-time change. Therefore, as β increases, which means the customers are more sensitive to lead-time, the suppliers have to quote a shorter PDL, which in turn stimulates more demand. Since the capacity of suppliers does not change, in order to avoid overloading the system, the suppliers have to raise their wholesale prices and this in turn leads to an decrease of demand rate. The perplexing question is how the equilibrium demand compares to that before the change of β . Our results show the answer is that it is lower than before, because quoting a shorter PDL only makes sense when the random lead-time becomes stochastically smaller. By queuing analogy, given the same service rate, the only way to achieve a stochastically smaller lead-time is to reduce the average arrival rate. Hence the lower equilibrium demand rate.

7.1.3 Effects of Price and Retail Store Competition Intensities θ_p and θ_r

As a measure of price competition intensity, the parameter θ_p indicates the sensitivity of customers to price difference between two products. Put it in another way, it can be considered as the degree of product differentiation or substitutability – the more differentiated the two products, the lower the price competition, and vice versa. The effect of θ_p is illustrated in Figure 4 (Appendix 3) and is explained as follows.

As the two products become more differentiated, i.e., as θ_p increases, suppliers will naturally raise the wholesale price due to the waning price competition pressure. On the other hand, the retailers' best reaction is to absorb a portion of the wholesale price increase and pass the rest to consumers. Consequently, their retail margins decrease while the retail prices increase.

As a result of the higher retail price, the induced demand will decrease, which in turn results in a stochastically smaller lead-time. Therefore the PDL tends to decreases. The feedback effect of the PDL on demand implies that the shorter PDL will then recover demand to certain extent. Nonetheless, the equilibrium demand will be definitely smaller.

As for the effect of θ_p on different parties' profits, obviously the suppliers will be better off, because they have more monopolistic power as θ_p decreases. The retailers, however, will suffer because both their profit margin and the demand decrease. To summarize, lower price competition (alternatively, higher product differentiation) benefits the suppliers at the expense of the retailers.

The effect of θ_r is shown in Figure 5, and can be explained in a similar fashion. To elaborate, a decreasing θ_r implies higher degree of retail stores differentiation. Facing less competition pressure, the retailers will demand a higher profit margin. The rational response of the suppliers is to absorb part of the shock to reduce the negative impact on their demand by decreasing wholesale price, and pass the rest to consumers, leading to an increase in retail price.

The inflated retail price in turn shrinks the demand, making it sensible to quote a shorter PDL. As for the profits, the retailers benefit from more monopolistic power, whereas the suppliers suffer due to their falling wholesale price and demand.

We remark that the above findings are consistent with Choi (1996), which not only validates our results (our model reduces to Choi's if all lead-time related parameters are set to 0, i.e., $b = h = \beta = \theta_l = 0$), but also implies that the presence of lead-time and the competition centering around it do not change the effect of product or store differentiation qualitatively.

7.2 Discussion of Managerial Insights

In this subsection, we discuss our main managerial insights and explain the underlying intuition. In particular, we first investigate the implication of channel leadership, then study the role of channel structure, finally examine some counterintuitive findings pertaining to the presence of lead-time and its competition.

7.2.1 Effect of Different Channel Leaderships

Figure 6 presents the equilibrium results in the three leadership structures as the capacity of supplier changes. Under the vertical strategic substitute-type demand functions, previous studies have shown that retail prices are generally higher when there is a channel leadership by either a supplier (Jeuland and Shugan, 1988) or a retailer (Choi, 1991). Our study reveals similar results:

$$p^{SS} > p^{RS} > p^{VN}.$$

This implies a channel leadership of either form is not socially desirable as consumers are worse off (i.e., pay more and buy less).

The impact of channel leadership on equilibrium profits for both suppliers and retailers is reflected in the following inequalities:

$$\Pi_S^{SS} > \Pi_S^{VN} > \Pi_S^{RS}$$
$$\Pi_R^{RS} > \Pi_R^{VN} > \Pi_R^{SS}$$

which implies that a channel member always prefers Stackelberg leadership, because of the first mover advantage.

Finally, we compare total channel profits of the three leadership structures. Similar to Choi (1996), we find the total channel profit is the largest in the VN game where there is no channel leadership, which implies that in either Stackelberg game, by moving away from the Nash game, the leaders can increase their profits but the followers' loss outweighs the leaders' gain.

It is worth noting that all the above results are reminiscent of existing findings reported in the literature when lead-time is not taken into account. This means that the consideration of lead-time does not change the effect of channel leadership qualitatively.

7.2.2 Effect of Different Channel Structures

If one supplier and one retailer are removed from our duopoly common retailers market structure, then our model boils down to the one studied by Liu et al. (2007). We now draw a comparison with the Liu et al. (2007) to examine the effect of different channel structures. For fair comparison, we set the potential market for the single product in Liu et al. (2007) to be twice as large as that in our model (i.e., λ_0 = 20), and the other parameters are the same in both models.

Figure 7 shows the effect of the suppliers' capacity μ on equilibrium results of Liu et al. (2007) model. It indicates similar effect of channel leadership on equilibrium profits: channel leadership is still preferred, and the VN games still yields the best system performance. As the main difference between Liu et al. (2007) and our work is the horizontal competition between counterparts at the same supply chain level, the above observation means that the horizontal competition does not alter the nature of the channel leadership effect.

A closer look at Figures 6 and 7 exhibits that the presence of horizontal competition brings down the price and profits noticeably, as the monopolistic power of both suppliers and retailers is significantly weakened.

7.2.3 Effect of Suppliers' Capacity μ

The distinctive feature of our model is to endogenize the lead-time decision, which is inherently driven by the suppliers' capacity. We now look at the impact of capacity on the optimal decision making process.

If we look at the extreme case where capacity is infinite, then our problem reduces to the Choi (1996) model, in which lead-time is not a concern at all (i.e., everything is made instantaneously so lead-time is 0, and of course there is no lead-time competition). Figure 6 confirms the above claim, as we can see that when μ increases, all curves in the figure become flat and converge to the Choi (1996) results.

We next look at how capacity μ affects the equilibrium results. As capacity increases, the supplier demand increases, while PDL decreases. This is intuitive because the production time and waiting time decrease with higher capacity, and this in turn leads to a shorter PDL and higher demand. It is a little surprising that the supplier's wholesale price w decreases as μ increases. From previous discussion in section 4.2, we know that the wholesale price w is a function of the demand and PDL and it increases as the PDL and demand decrease. While the decrease in the PDL is smaller than the increase in the demand as capacity μ increases, which leads to the decrease of the wholesale price w.

As μ increases, the retail price for the customers decreases, suggesting that customers benefit from an increase in supplier capacity. The profits of both retailers and entire system also increase as μ increases. This makes sense since as μ increases, the PDL decreases and hence the demand increases. In addition, an increase in retail price and a decrease in wholesale price result in an increase in the margins for both products, which together lead to the increase of retailers' profits. Due to the increase of both supplier and retailer profit, the profit for the entire system also increases.

7.2.4 Effect of Lead-Time Competition Intensity θ_l

The lead-time competition intensity θ_l in effect measures the competition pressure the suppliers face. Using the logic behind the effect of θ_p and θ_r, one would expect a larger θ_l to hurt the suppliers while benefiting retailers. Surprisingly, our results show the opposite (Figure 8).

To understand the driver of this counterintuitive result, note that as θ_l increases, PDL will naturally decrease. On the other hand, wholesale price will increase, because the suppliers now focus on the more intense lead-time competition and leverage less on price competition. As a result, demand will fall. To reduce the negative impact of price on demand, the retailers' rational response is to absorb some of the price increase by demanding a lower margin, and pass the rest to customers. As both the margin and demand for the retailers go down, their profits decrease accordingly. As for the suppliers, their increase in profit margin more than compensate the decrease in demand, therefore they turn out to be better off.

Chapter 8

Conclusion and Future Research

This thesis extends the growing literature of pricing and lead-time studies by analyzing competitive pricing and PDL strategies of duopoly suppliers who produce differentiated but substitutable products and duopoly retailers who sell both products. A major contribution of this thesis is to incorporate the inter-supply chain competition in the decision model. Independent players make decisions about prices and the PDL, knowing that their decisions will affect the decision of their rival and the demand rate, which in turn will affect their final profits. Fluctuations in the RDL affect the delivery time service performance, hence the PDL decision is intimately related to the pricing decision; together they affect the profitability of the firms and the whole supply chain system. Formulating the decentralized decision problem under three different games, we have obtained the unique equilibrium analytically and provided exact formulas to compute the optimal prices and PDL for suppliers and retailers.

Some results here are very similar to those of previous studies which consider pricing decisions only in a common retailer channel structure (see Choi, 1996), such as price differentiation helps suppliers while hurting retailers, whereas retailer differentiation helps retailers while hurting suppliers. When using multiple common retailers, therefore, suppliers are better off by recruiting homogeneous retailers. On the other hand, common retailers are better off by selling a relatively less differentiated set of products.

Some results provide really fresh new insights and may need further investigation. In common sense, the profits of channel leaders' are always higher than those of the followers' or the Nash players, but we find that when suppliers' capacities are high enough, retailers' profits will be highest in the VN game, which counters intuition that they should be able to earn more if they act as Stackelberg-leaders. We think this is mainly because the PDL variable, higher capacity makes the suppliers can charge a much shorter PDL, which makes the suppliers more powerful in making decisions since they can switch between wholesale price and PDL to avoid disadvantage. This hurts the retailers no matter when they are leaders or followers, but when the suppliers and retailers need to make their decisions at the same time, namely, in the VN game, the retailers can suffer the smallest disadvantages incurred by introducing the PDL decision variable to the suppliers. This shows that as retailers, before struggling for the channel leaders, they should first find out their suppliers' real capacity levels, if suppliers' capacities are low, they will be most beneficial by becoming the channel leaders, otherwise, it will be better for them to just catch up with the suppliers' powers.

From our analysis in the last part of section 7, we can see that the monopoly supplier and monopoly retailer channel is worst for customers, since it results in highest retail price and lowest demand. If the suppliers want to lower down the retail price to attract more demand, they will choose to introduce more homogeneous retailers while the retailers will want to carry more relatively homogeneous products. There will be a conflict between them. Similar conflict exists other aspects, such as the suppliers will always struggle for channel leadership to earn highest profit while the retailers will always try to catch up with the suppliers or even become the Stackelberg leaders to protect their own profits.

As a potential future research direction, one can compare the results of the linear demand model with a constant elasticity model to see how the decisions and the performance of the supply chain change. Another extension could be considering asymmetric conditions. In our model, retailers are assumed to set same margins for both products they sell and the cost for two suppliers are completely identical. It is left for future research to allow for asymmetric demand functions and strategies. Furthermore, we assume that the cost parameters and demand rate are common knowledge. In practice, they are often private information, unknown to other parties. We must then make decisions under asymmetric information and may need to design a scheme to induce a supply chain partner to reveal his or her private information. Further research is also needed to investigate these issues. Finally, competition under dynamic price and lead-time quotations would also be of interest for future work.

References

Allon, G. and A. Federgruen. 2007, "Competition in service industries", *Operations Research,* 55(1), 37–55.

Allon, G.and A. Federgruen.2009, "Competition in service industries with segmented markets", *Management Science,* Published online in articles in advance, January 5, 2009.

Bernstein, F. and A. Federgruen. 2007, "Coordination mechanisms for supply chains under price and service competition" *Manufacturing & Service Operations Management,* 9(3), 242–262.

Boyaci, T. and S. Ray. 2003, "Product differentiation and capacity cost interaction in time and price sensitive markets", *Manufacturing & Service Operations Management,* 5(1), 18–36.

Boyaci, T. and G. Gallego. 2004, "Supply chain coordination in a market with customer service competition", *Production and Operations Management,* 13(1), 3-22.

Cachon, G.P. and P. T. Harker. 2002, "Competition and outsourcing with scale economies", *Management Science*, 48(10), 1314 -1333.

Choi, S. C. 1991, "Price Competition in a channel structure with a common retailer", *Marketing Science*, 10, 271–296.

Choi, S. C. 1996, "Price competition in a duopoly common retailer channel", *Journal of Retailing,* 72(2), 117–134.

Gupta, S. and R. Loulou. 1998, "Process innovation, product differentiation, and channel structure: strategic incentives in a duopoly", *Marketing Science,* 17(4), 301 – 316.

Ha, A. Y. and S. L. Tong. 2008, "Contracting and information sharing under supply chain competition" , *Management Science*, 54(4), 701-715.

Ingene. C. A. and M. E. Parry. 1995, "Channel coordination when retailers compete", *Marketing Science,* 14(4), 360–377.

Jeuland, A.P., and S.M. Shugan. 1988, "Channels of distribution profits when channel members form conjectures", *Marketing Science,* 7(2**)**, 202–210.

Johnson, W. C. 1988, "Sales promotions: it comes down to push marketing", *Marketing News,* February 8.

Kopel M. and C. Loffler. 2008, "Commitment, first-mover-, and second-mover advantage", *Journal of Economics,* 94, 143–166.

Liu, L.M., M. Parlar and S. X. Zhu. 2007, "Pricing and lead time decisions in decentralized supply chains", *Management Science,* 53(5), 713–725.

Maltz, A., E. Rabinovich and R. Sinha. 2004, "Logistics: the key to e-retail success", *Supply Chain Management Review,* 8, 56–63.

McGuire, T.W. and R. Staelin. 1983, "An industry equilibrium analysis of downstream vertical integration", *Marketing Science,* 2**,** 161–190.

Messinger, P.R. and C. Narasimhan. 995, "Has power shifted in the grocery channel", *Marketing Science,* 14(2), 189–223.

Palaka, K., S. Erlebacher and D. H. Kropp. 1998, "Lead time setting, capacity utilization, and pricing decisions under lead time dependent demand", *IIE Transaction*, 30(2), 151–163.

Ray, S. and E. Jewkes. 2004, "Customer lead time management when both demand

and price are lead time sensitive", *European Journal of Operational Research,* 153(3), 769–781.

Reinganum J. F. 1985, "A two-stage model of research and development with endogenous second-mover advantages", *International Journal of Industrial Organization*, 3, 275-292.

So, K. C. and J. Song. 1998, "Price, delivery time guarantees and capacity selection", *European journal of operational research*, 111, 28–49.

So, K. C. 2000, "Price and time competition for service delivery", *Manufacturing & Service Operations Management*, 2(4), 392–409.

Spengler, J.J. 1950, "Vertical integration and antitrust policy", *The Journal of Political Economy*, 58(4), 347–352.

Urban, G., F. Sultan and W. Qualls. 2000, "Placing trust at the center of your Internet strategy", *Sloan Management Review,* 42, 39–48.

Vrinda, K., P. Chintagunta and N. Vilcassim. 2000, "Manufacturer-retailer channel interactions and implications for channel power: an empirical investigation of pricing in a local market", *Marketing Science*, 19(2), 127–148.

Appendix A. Discussion of Assumptions

Discussion of Assumption 1

If we consider the influence between product i sold by retailer j and product j sold by retailer i(denoted by θ_{r2}, and the former θ_r is replaced by θ_{r1}), the demand function will become as follows:

$$\lambda_{11} = \lambda_0 - \alpha p_{11} - \beta l_1 + \theta_p(p_{21} - p_{11}) + \theta_l(l_2 - l_1) + \theta_{r1}(p_{12} - p_{11}) + \theta_{r2}(p_{22} - p_{11})$$

$$\lambda_{12} = \lambda_0 - \alpha p_{12} - \beta l_1 + \theta_p(p_{22} - p_{12}) + \theta_l(l_2 - l_1) + \theta_{r1}(p_{11} - p_{12}) + \theta_{r2}(p_{21} - p_{12})$$

$$\lambda_{21} = \lambda_0 - \alpha p_{21} - \beta l_2 + \theta_p(p_{11} - p_{21}) + \theta_l(l_1 - l_2) + \theta_{r1}(p_{22} - p_{21}) + \theta_{r2}(p_{12} - p_{21})$$

$$\lambda_{22} = \lambda_0 - \alpha p_{22} - \beta l_2 + \theta_p(p_{12} - p_{22}) + \theta_l(l_1 - l_2) + \theta_{r1}(p_{21} - p_{22}) + \theta_{r2}(p_{11} - p_{22})$$

and the demand rate for each supplier and each retailer will become :

$$\lambda_{R_1} = 2\lambda_0 - \alpha(p_{11} + p_{21}) - \beta(l_1 + l_2) + (\theta_{r1} + \theta_{r2})(p_{22} + p_{12} - p_{11} - p_{21})$$

$$\lambda_{R_2} = 2\lambda_0 - \alpha(p_{12} + p_{22}) - \beta(l_1 + l_2) + (\theta_{r1} + \theta_{r2})(p_{11} + p_{21} - p_{12} - p_{22})$$

$$\lambda_{S_1} = 2\lambda_0 - \alpha(p_{11} + p_{12}) - 2\beta l_1 + 2\theta_l(l_2 - l_1) + (\theta_p + \theta_{r2})(p_{21} + p_{22} - p_{11} - p_{12})$$

$$\lambda_{S_2} = 2\lambda_0 - \alpha(p_{11} + p_{12}) - 2\beta l_2 + 2\theta_l(l_1 - l_2) + (\theta_p + \theta_{r2})(p_{11} + p_{12} - p_{21} - p_{22})$$

From where we can see that ifr we add one more parameter θ_{r2} (swithover of demand toward price differenciation of different product sold in different store), the demand function do not change a lot, for retailers' demand rate, the only change is former θ_r becomes $\theta_{r1} + \theta_{r2}$, for two suppliers, former θ_p becomes $\theta_p + \theta_{r2}$, so it would not change the finally results we obtained.

Discussion of Assumption 2

Here we will take Supplier Stackelberg game as an example to illustrate that the equilibrium retail margins for two products are the same.

Assume that m_{ij} is the margin retailer i determines toward product j, and the demand function can be rewrite as follows:

$$\lambda_{11} = \lambda_0 - \alpha(m_{11} + w_1) - \beta l_1 + \theta_p(m_{21} - m_{11}) + \theta_p(w_2 - w_1) + \theta_l(l_2 - l_1) + \theta_r(m_{12} - m_{11})$$

$$\lambda_{12} = \lambda_0 - \alpha(m_{12} + w_1) - \beta l_1 + \theta_p(m_{22} - m_{12}) + \theta_p(w_2 - w_1) + \theta_l(l_2 - l_1) + \theta_r(m_{11} - m_{12})$$

$$\lambda_{21} = \lambda_0 - \alpha(m_{21} + w_2) - \beta l_2 + \theta_p(m_{11} - m_{21}) + \theta_p(w_1 - w_2) + \theta_l(l_1 - l_2) + \theta_r(m_{22} - m_{21})$$

$$\lambda_{22} = \lambda_0 - \alpha(m_{22} + w_2) - \beta l_2 + \theta_p(m_{12} - m_{22}) + \theta_p(w_1 - w_2) + \theta_l(l_1 - l_2) + \theta_r(m_{21} - m_{22})$$

So we have:

$$\lambda_{R_1} = 2\lambda_0 - \alpha(m_{11} + m_{21}) - \alpha(w_1 + w_2) - \beta(l_1 + l_2) + \theta_r(m_{22} + m_{12} - m_{11} - m_{21}) \quad (13)$$

$$\lambda_{R_2} = 2\lambda_0 - \alpha(m_{12} + m_{22}) - \alpha(w_1 + w_2) - \beta(l_1 + l_2) + \theta_r(m_{11} + m_{21} - m_{12} - m_{22}) \quad (14)$$

$$\lambda_{S_1} = 2\lambda_0 - \alpha(m_{11} + m_{12}) - 2\alpha w_1 - 2\beta l_1 + \theta_p(m_{22} + m_{21} - m_{11} - m_{12}) + 2\theta_p(w_2 - w_1) + 2\theta_l(l_2 - l_1) \quad (15)$$

$$\lambda_{S_2} = 2\lambda_0 - \alpha(m_{21} + m_{22}) - 2\alpha w_2 - 2\beta l_2 + \theta_p(m_{11} + m_{12} - m_{21} - m_{22}) + 2\theta_p(w_1 - w_2) + 2\theta_l(l_1 - l_2) \quad (16)$$

(1) Retailers' best response

The profit function for each retailer should be:

$$\Pi_{R_1} = m_{11}\lambda_{11} + m_{21}\lambda_{21}$$

$$\Pi_{R_2} = m_{12}\lambda_{12} + m_{22}\lambda_{22}$$

Take retail 1 as an example

$$\frac{\partial \Pi_{R_1}}{\partial m_{11}} = \lambda_{11} - m_{11}(\alpha + \theta_p + \theta_r) + m_{21}\theta_p$$

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_{11}{}^2} = -2(\alpha + \theta_p + \theta_r) < 0$$

$$\frac{\partial \Pi_{R_1}}{\partial m_{21}} = \lambda_{21} - m_{21}(\alpha + \theta_p + \theta_r) + m_{11}\theta_p$$

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_{21}{}^2} = -2(\alpha + \theta_p + \theta_r) < 0$$

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_{11}\partial m_{21}} = \frac{\partial^2 \Pi_{R_1}}{\partial m_{21}\partial m_{11}} = -(\alpha + \theta_r)$$

From where we can see that the expected profit function is jointly concave in m_{11} and m_{21}, and for retailer 2, the expected function is also jointly concave in m_{12} and m_{22}; then the optimal solutions can be obtained by setting the first order conditions equal to 0:

$$\begin{cases} \dfrac{\partial \Pi_{R_1}}{\partial m_{11}} = \lambda_{11} - m_{11}(\alpha + \theta_p + \theta_r) + m_{21}\theta_p = 0 \\ \dfrac{\partial \Pi_{R_1}}{\partial m_{21}} = \lambda_{21} - m_{21}(\alpha + \theta_p + \theta_r) + m_{11}\theta_p = 0 \\ \dfrac{\partial \Pi_{R_2}}{\partial m_{12}} = \lambda_{12} - m_{12}(\alpha + \theta_p + \theta_r) + m_{22}\theta_p = 0 \\ \dfrac{\partial \Pi_{R_2}}{\partial m_{22}} = \lambda_{22} - m_{22}(\alpha + \theta_p + \theta_r) + m_{12}\theta_p = 0 \end{cases}$$

From where we can get the conclusion that we have

$$m_{11} = m_{12} = -\frac{(B_1 + XA_2 + XB_2 - YA_1 + A_1 - YB_1)}{X^2 - Y^2 + 2Y - 1}$$

$$m_{21} = m_{22} = -\frac{(B_2 + XA_1 + XB_1 - YA_2 + A_2 - YB_2)}{X^2 - Y^2 + 2Y - 1}$$

Where

$$X = \frac{\theta_p}{\alpha + \theta_p + \theta_r}$$

$$Y = \frac{\theta_r}{2(\alpha + \theta_p + \theta_r)}$$

$$A_1 = \frac{\lambda_0 - \alpha w_1 - \beta l_1}{2(\alpha + \theta_p + \theta_r)}$$

$$A_2 = \frac{\lambda_0 - \alpha w_2 - \beta l_2}{2(\alpha + \theta_p + \theta_r)}$$

$$B_1 = \frac{\theta_p(w_2 - w_1) + \theta_l(l_2 - l_1)}{2(\alpha + \theta_p + \theta_r)}$$

$$B_2 = -B_1 = \frac{\theta_p(w_1 - w_2) + \theta_l(l_1 - l_2)}{2(\alpha + \theta_p + \theta_r)}$$

And

$$m_{11} + m_{21} = \frac{2\lambda_0 - \alpha(w_1 + w_2) - \beta(l_1 + l_2)}{2\alpha + \theta_r}$$

$$m_{11} - m_{21} = \frac{(\alpha + 2\theta_p)(w_2 - w_1) + (\beta + 2\theta_l)(l_2 - l_1)}{2\alpha + 4\theta_p + \theta_r}$$

(2)Suppliers' decision

For suppliers, from Eq. (15)+(16), we have:

$$w_1 + w_2 = \frac{2\lambda_0}{\alpha} - (m_{11} + m_{21}) - \frac{\beta}{\alpha}(l_1 + l_2) - \frac{\lambda_{S_1} + \lambda_{S_2}}{2\alpha}$$

Substituting $m_{11} + m_{21}$ into this equation and we will find the result is just the same as in part 4.2, so does the following calculation steps and the final equilibrium results. So if the retailer charge different margins for different product, the initial calculation steps may change, but it would not affect the final equilibrium results, which means the retails will always charge same margin for both products.

Appendix B. Mathematical Proofs

Proof of Lemma 1

(1) The concavity of retailer's payoff function and the existence of Nash equilibrium between two retailers.

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} = -4(\alpha + \theta_r) < 0 \\ \dfrac{\partial^2 \Pi_{R2}}{\partial m_2^2} = -4(\alpha + \theta_r) < 0 \end{cases}$$

The payoff function is continuous and concave in each retailer's own strategy. Then, there exists at least one pure strategy Nash Equilibrium in the game.

(2) The uniqueness of this equilibrium. We use index theory approach. In order to prove the uniqueness of the equilibrium, we need to show that:

$$\frac{\partial m_1^*}{\partial m_2} \frac{\partial m_2^*}{\partial m_1} < 1$$

Since we have:

$$\frac{\partial m_i^*}{\partial m_j} = \frac{\dfrac{\partial^2 \Pi_{R_i}}{\partial m_i \partial m_j}}{\dfrac{\partial^2 \Pi_{R_i}}{\partial m_i^2}}$$

And:

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial m_2} = 2\theta_r \\ \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} = 2\theta_r \end{cases}$$

So:

$$\frac{\partial m_1^*}{\partial m_2}\frac{\partial m_2^*}{\partial m_1}=\frac{4\theta_r^2}{16(\alpha+\theta_r)^2}$$

This equation is obviously < 1, so we know that there is a unique equilibrium.

Proof of Lemma 2

$$\begin{cases}\dfrac{\partial \Pi_{S_1}^2}{\partial l_1^2}=-\lambda_1(b+h)f_{\lambda_1}(l_1)<0\\ \dfrac{\partial \Pi_{S_2}^2}{\partial l_2^2}=-\lambda_2(b+h)f_{\lambda_2}(l_2)<0\end{cases}$$

So we get the conclusion that for any give λ_i, the suppliers' payoff functions are concave in l_i.

Proof of Lemma 3

(1) The concavity of supplier's payoff function and the existence of a Nash Equilibrium between two suppliers.

$$\frac{\partial^2\Pi_{S_1}}{\partial\lambda_1^2}=\left(\frac{\partial k_1}{\partial\lambda_1}-\frac{\partial k_2}{\partial\lambda_1}\right)+\frac{\lambda_1}{2}\left[\left(\frac{\partial^2 k_1}{\partial\lambda_1^2}-\frac{\partial^2 k_2}{\partial\lambda_1^2}\right)-\frac{2\mu_1^2}{(\mu_1-\lambda_1)^3}C(\bar{l}_1,\Phi_1)\right]$$

$$=-\left(\frac{\beta}{\alpha}+\frac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\frac{\mu_1^2}{(\mu_1-\lambda_1)^3}\bar{l}_1-\frac{2\mu_1^2}{(\mu_1-\lambda_1)^3}C(\bar{l}_1,\Phi_1)-\left(\frac{1}{2(\alpha+2\theta_p)}+\frac{2\alpha+\theta_r}{2\alpha(\alpha+\theta_r)}\right)$$

$$<0$$

$$\frac{\partial^2\Pi_{S_2}}{\partial\lambda_2^2}=\left(\frac{\partial k_1}{\partial\lambda_2}-\frac{\partial k_2}{\partial\lambda_2}\right)+\frac{\lambda_2}{2}\left[\left(\frac{\partial^2 k_1}{\partial\lambda_2^2}-\frac{\partial^2 k_2}{\partial\lambda_2^2}\right)-\frac{2\mu_2^2}{(\mu_2-\lambda_2)^3}C(\bar{l}_2,\Phi_2)\right]$$

$$=-\left(\frac{\beta}{\alpha}+\frac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\frac{\mu_2^2}{(\mu_2-\lambda_2)^3}\bar{l}_2-\frac{2\mu_2^2}{(\mu_2-\lambda_2)^3}C(\bar{l}_2,\Phi_2)-\left(\frac{1}{2(\alpha+2\theta_p)}+\frac{2\alpha+\theta_r}{2\alpha(\alpha+\theta_r)}\right)$$

$$<0$$

The payoff function is continuous and concave in each supplier's own strategy. Then, there exists at least one pure strategy Nash Equilibrium in the game.

(2) Uniqueness of equilibrium. We still use index theory approach. In order to prove the uniqueness of the equilibrium, we need to show that:

$$\frac{\partial \lambda_1^*}{\partial \lambda_2}\frac{\partial \lambda_2^*}{\partial \lambda_1} < 1$$

Since we have:

$$\frac{\partial \lambda_i^*}{\partial \lambda_j} = \frac{\dfrac{\partial^2 \Pi_{S_i}}{\partial \lambda_i \partial \lambda_j}}{\dfrac{\partial^2 \Pi_{S_i}}{\partial \lambda_i^2}}$$

And:

$$\begin{cases} \dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} = \dfrac{1}{2}\left(\dfrac{\partial k_1}{\partial \lambda_2} - \dfrac{\partial k_2}{\partial \lambda_2}\right) = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\dfrac{\mu_2}{(\mu_2 - \lambda_2)^2}\bar{l}_2 + \left(\dfrac{1}{2(\alpha + 2\theta_p)} - \dfrac{2\alpha + \theta_r}{2\alpha(\alpha + \theta_r)}\right)\right] \\ \dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} = \dfrac{1}{2}\left(\dfrac{\partial k_1}{\partial \lambda_1} - \dfrac{\partial k_2}{\partial \lambda_1}\right) = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\dfrac{\mu_1}{(\mu_1 - \lambda_1)^2}\bar{l}_1 + \left(\dfrac{1}{2(\alpha + 2\theta_p)} - \dfrac{2\alpha + \theta_r}{2\alpha(\alpha + \theta_r)}\right)\right] \end{cases}$$

Furthermore, we can know $\partial^2 \Pi_{S_i} / \partial \lambda_i^2$ from the first part of this proof. It is straightforward that:

$$\left|\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2}\right| < \left|\frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2}\right|$$

$$\left|\frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1}\right| < \left|\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2}\right|$$

So we can get our conclusion that there is a unique equilibrium.

Proof of Lemma 4. The proof of Lemma 4 is the same as that of Lemma 2.

Proof of Lemma 5

(1)The concavity of two suppliers' payoff functions and the existence of a Nash Equilibrium between them.

$$\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} = \left(\frac{\partial k_3}{\partial \lambda_1} - \frac{\partial k_4}{\partial \lambda_1}\right) + \frac{\lambda_1}{2}\left[\left(\frac{\partial^2 k_3}{\partial \lambda_1^2} - \frac{\partial^2 k_4}{\partial \lambda_1^2}\right) - \frac{2\mu_1^2}{(\mu_1-\lambda_1)^3} C(\bar{l}_1, \Phi_1)\right]$$
$$= -\left(\frac{\beta}{\alpha} + \frac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\frac{\mu_1^2}{(\mu_1-\lambda_1)^3}\bar{l}_1 - \frac{2\mu_1^2}{(\mu_1-\lambda_1)^3} C(\bar{l}_1, \Phi_1) - \left(\frac{1}{2(\alpha+2\theta_p)} + \frac{1}{2\alpha}\right)$$
$$< 0$$

$$\frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} = \left(\frac{\partial M'^*}{\partial \lambda_2} - \frac{\partial N'^*}{\partial \lambda_2}\right) + \frac{\lambda_2}{2}\left[\left(\frac{\partial^2 M'^*}{\partial \lambda_2^2} - \frac{\partial^2 N'^*}{\partial \lambda_2^2}\right) - \frac{2\mu_2^2}{(\mu_2-\lambda_2)^3} C(\bar{l}_2, \Phi_2)\right]$$
$$= -\left(\frac{\beta}{\alpha} + \frac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\frac{\mu_2^2}{(\mu_2-\lambda_2)^3}\bar{l}_2 - \frac{2\mu_2^2}{(\mu_2-\lambda_2)^3} C(\bar{l}_2, \Phi_2) - \left(\frac{1}{2(\alpha+2\theta_p)} + \frac{1}{2\alpha}\right)$$
$$< 0$$

The payoff function is continuous and concave in each supplier's own strategy. Then, there exists at least one pure strategy Nash Equilibrium in the game.

(2) Uniqueness of equilibrium (index theory approach), the steps are quite similar with that in the proof of Lemma 3. Since we have:

$$\begin{cases} \dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} = \dfrac{1}{2}\left(\dfrac{\partial k_3}{\partial \lambda_2} - \dfrac{\partial k_4}{\partial \lambda_2}\right) = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha} + \dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_2}{(\mu_2-\lambda_2)^2}\bar{l}_2 + \left(\dfrac{1}{2(\alpha+2\theta_p)} - \dfrac{1}{2\alpha 1}\right)\right] \\ \dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} = \dfrac{1}{2}\left(\dfrac{\partial k_3}{\partial \lambda_1} - \dfrac{\partial k_4}{\partial \lambda_1}\right) = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha} + \dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_1}{(\mu_1-\lambda_1)^2}\bar{l}_1 + \left(\dfrac{1}{2(\alpha+2\theta_p)} - \dfrac{1}{2\alpha}\right)\right] \end{cases}$$

Furthermore, we can know $\partial^2 \Pi_{S_i} / \partial \lambda_i^2$ from the first part of this proof. It is straightforward that:

$$\left|\frac{\partial^2 \Pi_{S_1}}{\partial\lambda_1\partial\lambda_2}\right| < \left|\frac{\partial^2 \Pi_{S_2}}{\partial\lambda_2^2}\right|$$

$$\left|\frac{\partial^2 \Pi_{S_2}}{\partial\lambda_2\partial\lambda_1}\right| < \left|\frac{\partial^2 \Pi_{S_1}}{\partial\lambda_1^2}\right|$$

So we can get our conclusion that there is a unique equilibrium.

Proof of Lemma 6

(1) The concavity of two retailers' payoff functions and the existence of a Nash Equilibrium between them.

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} = \dfrac{\partial \lambda_1^*}{\partial m_1} + \dfrac{\partial \lambda_2^*}{\partial m_1} - 2(\alpha + 2\theta_r) + \dfrac{m_1}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_1^2} + \dfrac{\partial^2 \lambda_2^*}{\partial m_1^2}\right) \\ \dfrac{\partial^2 \Pi_{R2}}{\partial m_2^2} = \dfrac{\partial \lambda_1^*}{\partial m_2} + \dfrac{\partial \lambda_2^*}{\partial m_2} - 2(\alpha + 2\theta_r) + \dfrac{m_2}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_2^2} + \dfrac{\partial^2 \lambda_2^*}{\partial m_2^2}\right) \end{cases}$$

Letting $\partial\Pi_{S_i} / \partial\lambda_1 = F(\lambda_1, \lambda_2, m1, m2)$ and $\partial\Pi_{S_i} / \partial\lambda_2 = G(\lambda_1, \lambda_2, m1, m2)$, we consider λ_i^* as a function of m_i. Using the implicit function theorem, we can get:

$$\frac{\partial \lambda_1^*}{\partial m_1} = -\begin{vmatrix} \dfrac{\partial F}{\partial m_1} & \dfrac{\partial F}{\partial \lambda_2} \\ \dfrac{\partial G}{\partial m_1} & \dfrac{\partial G}{\partial \lambda_2} \end{vmatrix} \Bigg/ \begin{vmatrix} \dfrac{\partial F}{\partial \lambda_1} & \dfrac{\partial F}{\partial \lambda_2} \\ \dfrac{\partial G}{\partial \lambda_1} & \dfrac{\partial G}{\partial \lambda_2} \end{vmatrix}$$

Since we have:

$$\frac{\partial F}{\partial m_1} = \frac{\partial G}{\partial m_1} = -\frac{1}{2}$$

$$\frac{\partial F}{\partial \lambda_1} = -\left(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\frac{\mu_1^2}{(\mu_1 - \lambda_1)^3}\bar{l}_1 - \frac{2\mu_1^2}{(\mu_1 - \lambda_1)^3}C(\bar{l}_1, \Phi_1) - \left(\frac{1}{2(\alpha + 2\theta_p)} + \frac{1}{2\alpha}\right)$$

$$\frac{\partial G}{\partial \lambda_1} = \frac{1}{2}\left[\left(-\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\frac{\mu_1}{(\mu_1 - \lambda_1)^2}\bar{l}_1 + \left(\frac{1}{2(\alpha + 2\theta_p)} - \frac{1}{2\alpha}\right)\right]$$

$$\frac{\partial F}{\partial \lambda_2} = \frac{1}{2}\left[\left(-\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\frac{\mu_2}{(\mu_2 - \lambda_2)^2}\bar{l}_2 + \left(\frac{1}{2(\alpha + 2\theta_p)} - \frac{1}{2\alpha}\right)\right]$$

$$\frac{\partial G}{\partial \lambda_2} = -\left(\frac{\beta}{\alpha} + \frac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\frac{\mu_2^2}{(\mu_2 - \lambda_2)^3}\bar{l}_2 - \frac{2\mu_2^2}{(\mu_2 - \lambda_2)^3}C(\bar{l}_2, \Phi_2) - \left(\frac{1}{2(\alpha + 2\theta_p)} + \frac{1}{2\alpha}\right)$$

We can find that:

$$\frac{\partial F}{\partial \lambda_1}\frac{\partial G}{\partial \lambda_2} - \frac{\partial G}{\partial \lambda_1}\frac{\partial F}{\partial \lambda_2} > 0$$

$$\frac{\partial F}{\partial m_1}\frac{\partial G}{\partial \lambda_2} - \frac{\partial G}{\partial m_1}\frac{\partial F}{\partial \lambda_2} > 0$$

We can get the conclusion that $\partial \lambda_1^* / \partial m_1 < 0$ and it is independent of m_j. Using the same method, we can see that $\partial \lambda_2^* / \partial m_1, \partial \lambda_1^* / \partial m_2$ and $\partial \lambda_2^* / \partial m_2$ are also smaller than 0 and are independent of . So we can get that:

$$\frac{\partial^2 \lambda_1^*}{\partial m_1^2} = \frac{\partial^2 \lambda_2^*}{\partial m_1^2} = \frac{\partial^2 \lambda_1^*}{\partial m_2^2} = \frac{\partial^2 \lambda_1^*}{\partial m_2^2} = 0$$

And now we can get the conclusion that:

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} < 0 \\ \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2^2} < 0 \end{cases}$$

The payoff function is continuous and concave in each retailer's own strategy. Then, there exists at least one pure strategy Nash Equilibrium in the game.

(2) Uniqueness of equilibrium (Index theory approach). Since we have:

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial m_2} = \alpha + 2\theta_r + \dfrac{m_1}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_1 \partial m_2} + \dfrac{\partial^2 \lambda_2^*}{\partial m_1 \partial m_2}\right) + \dfrac{1}{2}\left(\dfrac{\partial \lambda_1^*}{\partial m_2} + \dfrac{\partial \lambda_2^*}{\partial m_2}\right) \\ \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} = \alpha + 2\theta_r + \dfrac{m_2}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_2 \partial m_1} + \dfrac{\partial^2 \lambda_2^*}{\partial m_2 \partial m_1}\right) + \dfrac{1}{2}\left(\dfrac{\partial \lambda_1^*}{\partial m_1} + \dfrac{\partial \lambda_2^*}{\partial m_1}\right) \\ \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} = \dfrac{\partial \lambda_1^*}{\partial m_1} + \dfrac{\partial \lambda_2^*}{\partial m_1} - 2(\alpha + 2\theta_r) + \dfrac{m_1}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_1^2} + \dfrac{\partial^2 \lambda_2^*}{\partial m_1^2}\right) \\ \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2^2} = \dfrac{\partial \lambda_1^*}{\partial m_2} + \dfrac{\partial \lambda_2^*}{\partial m_2} - 2(\alpha + 2\theta_r) + \dfrac{m_2}{2}\left(\dfrac{\partial^2 \lambda_1^*}{\partial m_2^2} + \dfrac{\partial^2 \lambda_2^*}{\partial m_2^2}\right) \end{cases}$$

From where we can easily find that:

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial m_2} \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} < \frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} \frac{\partial^2 \Pi_{R_2}}{\partial m_2^2}$$

So we can get our conclusion that there is a unique equilibrium.

Proof of Lemma 7

(1) The concavity of four players payoff functions and the existence of a Nash Equilibrium between them. From the proof of Lemma 1 and 4, we know that:

$$\begin{cases} \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} = -4(\alpha + \theta_r) < 0 \\ \dfrac{\partial^2 \Pi_{R2}}{\partial m_2^2} = -4(\alpha + \theta_r) < 0 \\ \dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} = -\left(\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\dfrac{\mu_1^2}{(\mu_1 - \lambda_1)^3}\bar{l}_1 - \dfrac{2\mu_1^2}{(\mu_1 - \lambda_1)^3} C(\bar{l}_1, \Phi_1) - \left(\dfrac{1}{2(\alpha + 2\theta_p)} + \dfrac{1}{2\alpha}\right) < 0 \\ \dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} = -\left(\dfrac{\beta}{\alpha} + \dfrac{\beta + 2\theta_l}{\alpha + 2\theta_p}\right)\dfrac{\mu_2^2}{(\mu_2 - \lambda_2)^3}\bar{l}_2 - \dfrac{2\mu_2^2}{(\mu_2 - \lambda_2)^3} C(\bar{l}_2, \Phi_2) - \left(\dfrac{1}{2(\alpha + 2\theta_p)} + \dfrac{1}{2\alpha}\right) < 0 \end{cases}$$

The payoff function is continuous and concave in each player's own strategy. Then, there exists at least one pure strategy Nash Equilibrium in the game.

(2) Uniqueness of equilibrium (Index theory approach).

$$H = \begin{vmatrix} \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} & \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} & \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial m_1} & \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial m_2} \\ \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} & \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} & \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial m_1} & \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial m_2} \\ \frac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial \lambda_1} & \frac{\partial^2 \Pi_{R_1}}{\partial \lambda_1 \partial m_2} & \frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} & \frac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial m_2} \\ \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial \lambda_1} & \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial \lambda_2} & \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} & \frac{\partial^2 \Pi_{R_2}}{\partial m_2^2} \end{vmatrix}$$

Let $det(H)$ denote the determinant of matrix H and M_{ij} denote the minor of a determinant of matrix H, which is obtained by deleting the i-th row and j-th column from matrix H. According to the expansion theorem of a determinant, we can get that:

$$\det(H) = \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2}(-1)^{1+1} M_{11} + \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1}(-1)^{2+1} M_{21} + \frac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial \lambda_1}(-1)^{3+1} M_{31} + \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial \lambda_1}(-1)^{4+1} M_{41}$$

Using the same theorem, we can get $\det(M_{11})$, $\det(M_{21})$, $\det(M_{31})$ and $\det(M_{41})$, then we can simplify the determinant of matrix H as follows:

$$\begin{aligned} \det(H) = & \left(\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} - \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} \right) \left(\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} - \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} \right) \left(\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} + \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} \right) \\ & + \left(\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} + \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} - \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} - \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} \right) \left(\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} - \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} \right) \end{aligned}$$

Under Vertical Nash game, we have:

$$
\begin{cases}
\dfrac{\partial^2 \Pi_{R_1}}{\partial m_1^2} = -4(\alpha+\theta_r) \\
\dfrac{\partial^2 \Pi_{R2}}{\partial m_2^2} = -4(\alpha+\theta_r) \\
\dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} = -\left(\dfrac{\beta}{\alpha}+\dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_1^2}{(\mu_1-\lambda_1)^3}\bar{l}_1 - \dfrac{2\mu_1^2}{(\mu_1-\lambda_1)^3}C(\bar{l}_1,\Phi_1) - \left(\dfrac{1}{2(\alpha+2\theta_p)}+\dfrac{1}{2\alpha}\right) \\
\dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} = -\left(\dfrac{\beta}{\alpha}+\dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_2^2}{(\mu_2-\lambda_2)^3}\bar{l}_2 - \dfrac{2\mu_2^2}{(\mu_2-\lambda_2)^3}C(\bar{l}_2,\Phi_2) - \left(\dfrac{1}{2(\alpha+2\theta_p)}+\dfrac{1}{2\alpha}\right) \\
\dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha}+\dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_2}{(\mu_2-\lambda_2)^2}\bar{l}_2 + \left(\dfrac{1}{2(\alpha+2\theta_p)}-\dfrac{1}{2\alpha}\right)\right] \\
\dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} = \dfrac{1}{2}\left[\left(-\dfrac{\beta}{\alpha}+\dfrac{\beta+2\theta_l}{\alpha+2\theta_p}\right)\dfrac{\mu_1}{(\mu_1-\lambda_1)^2}\bar{l}_1 + \left(\dfrac{1}{2(\alpha+2\theta_p)}-\dfrac{1}{2\alpha}\right)\right] \\
\dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial m_1} = \dfrac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial m_2} = \dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial m_1} = \dfrac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial m_2} = -1/2 \\
\dfrac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial m_2} = \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} = 2\theta_r \\
\dfrac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial \lambda_1} = \dfrac{\partial^2 \Pi_{R_1}}{\partial m_1 \partial \lambda_2} = \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial \lambda_1} = \dfrac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial \lambda_2} = 0
\end{cases}
$$

Where we can find that:

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} - \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} = -4(\alpha+\theta_r) - 2\theta_r = -4\alpha - 6\theta_r < 0$$

$$\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} + \frac{\partial^2 \Pi_{R_2}}{\partial m_2 \partial m_1} = -4(\alpha+\theta_r) + 2\theta_r = -4\alpha - 2\theta_r < 0$$

$$\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2} + \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} - \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2} - \frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} < 0$$

$$\frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1^2}\frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2^2} - \frac{\partial^2 \Pi_{S_1}}{\partial \lambda_1 \partial \lambda_2}\frac{\partial^2 \Pi_{S_2}}{\partial \lambda_2 \partial \lambda_1} > 0$$

So we can conclude that $det(H) > 0$, and this make sure the uniqueness of a equilibrium.

Appendix C. Figures for Sensitivity Analysis

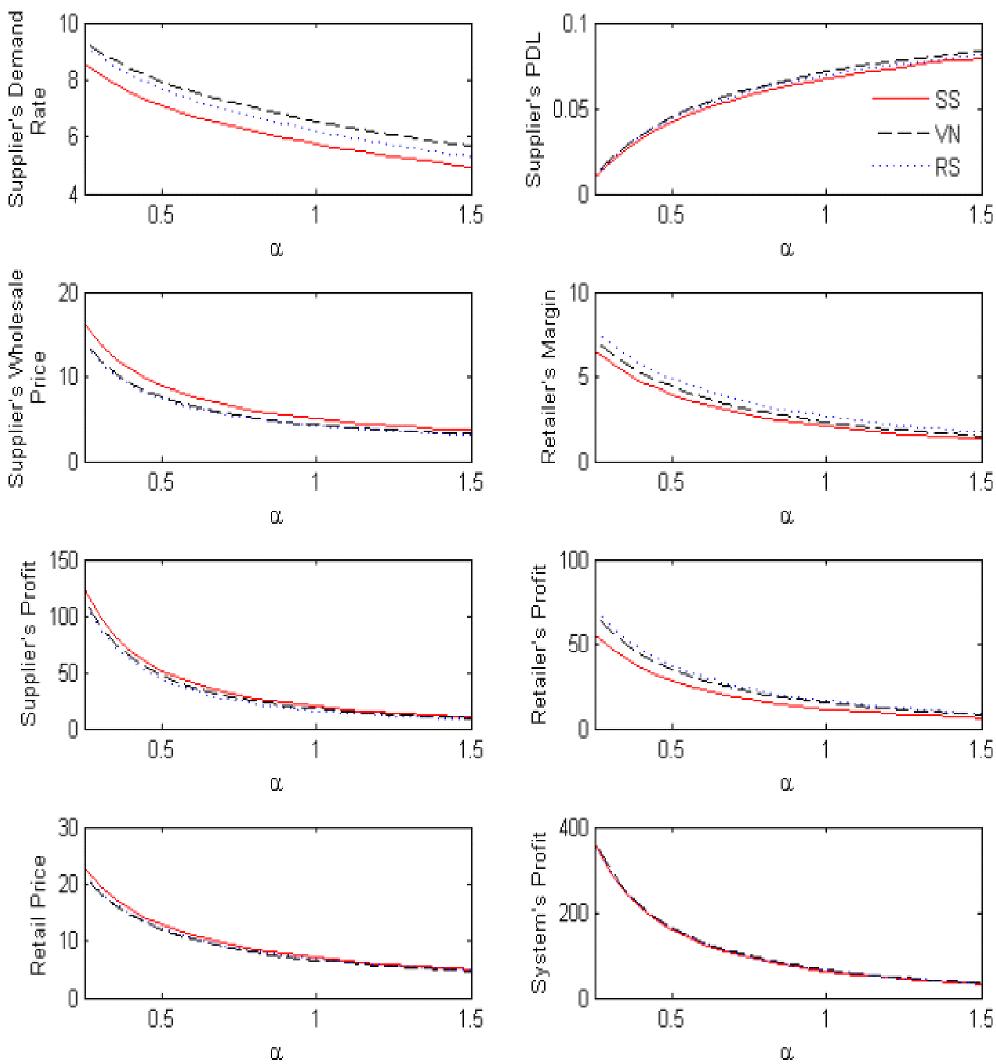

Figure 2: Influence of price sensitivity factor α when $\mu = 20$ ($\lambda_0 = 10, b = 3, h = 0.3, \beta = 0.8, \theta_p = 0.3, \theta_l = 0.5, \theta_r = 0.4, c = 1.5$ and $\alpha \in [0,1.3]$ with step size of 0.05)

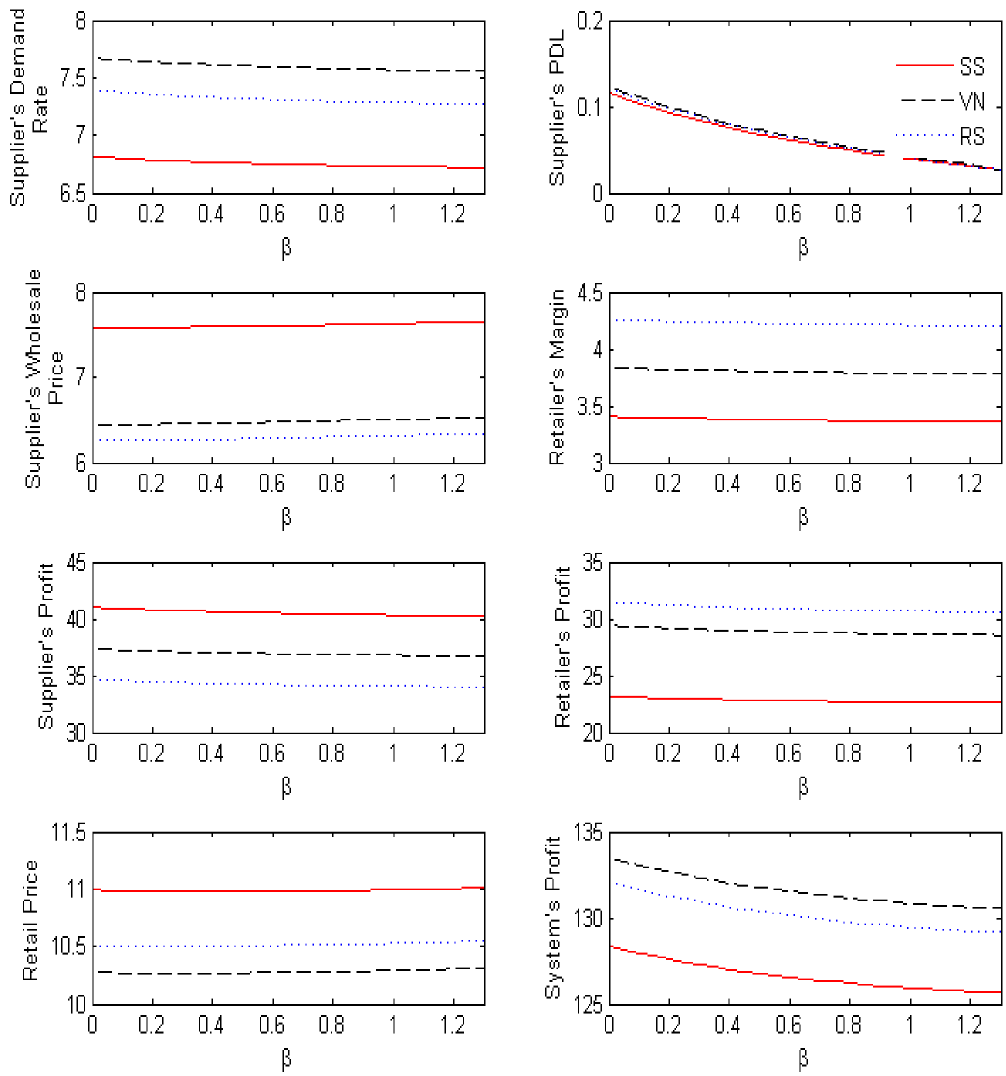

Figure 3: Influence of lead-time sensitivity factor β when $\mu = 20$ ($\lambda_0 = 10, b = 3, h = 0.3, \alpha = 0.6, \theta_p = 0.3, \theta_l = 0.5, \theta_r = 0.4, c = 1.5$ and $\beta \in [0,1.3]$ with step size of 0.05)

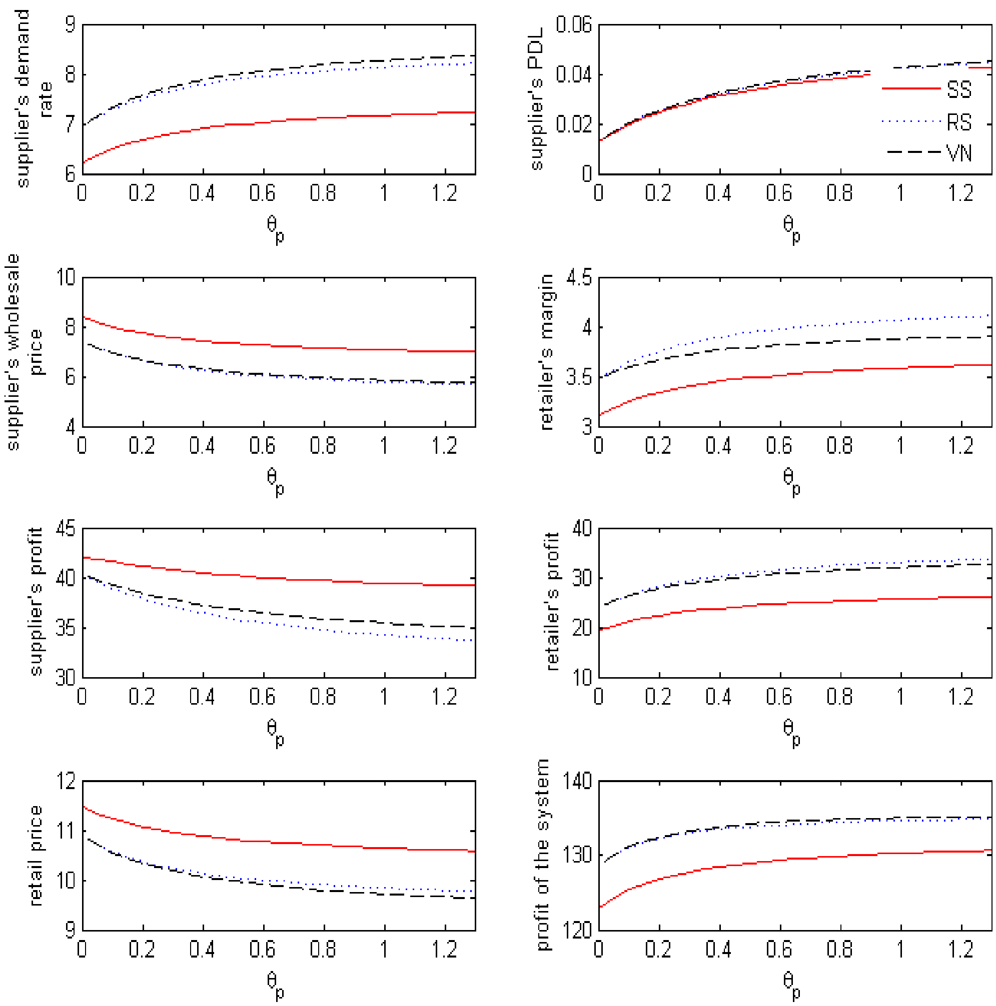

Figure 4: Influence of price competition intensity θ_p when $\mu = 20$ ($\lambda_0 = 10, b = 3, h = 0.3, \alpha = 0.6, \beta = 0.8, \theta_l = 0.5, \theta_r = 0.4, c = 1.5$ and $\theta_p \in [0,1.3]$ with step size of 0.05)

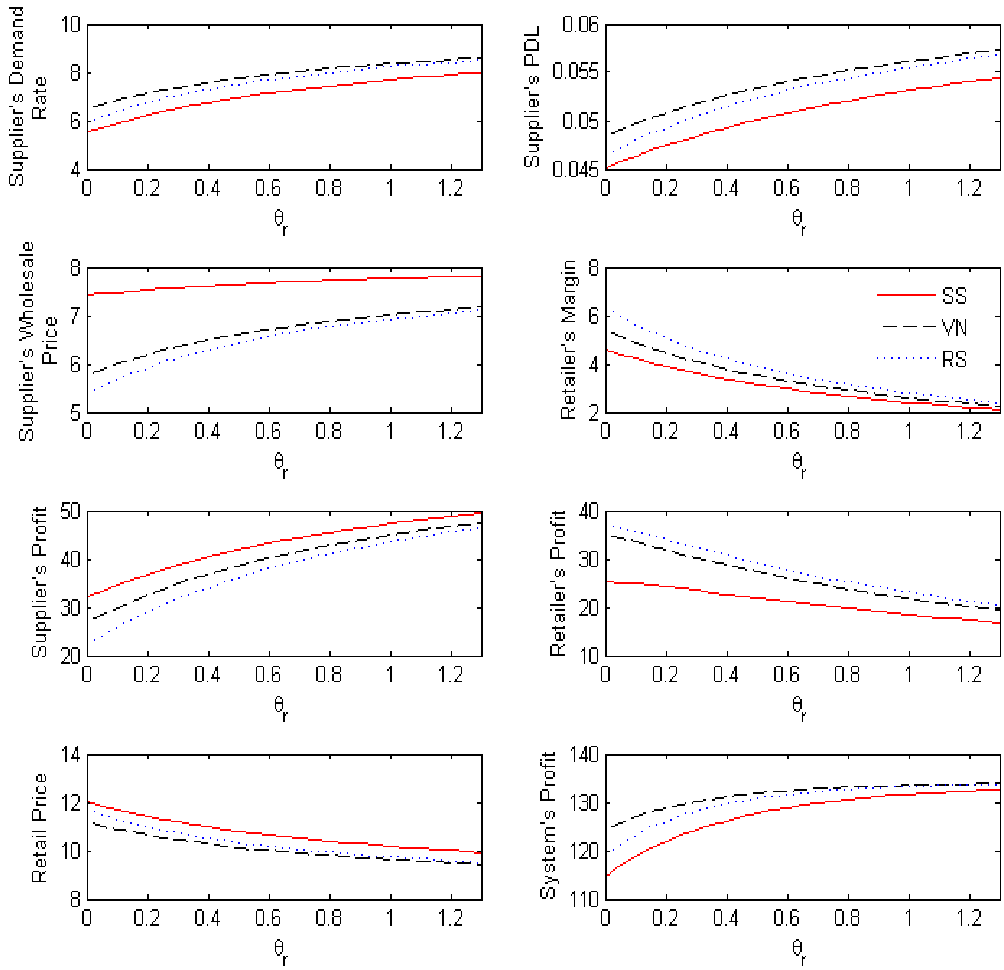

Figure 5: Influence of retail store competition intensity θ_r when $\mu = 20$ ($\lambda_0 = 10, b = 3, h = 0.3, \alpha = 0.6, \beta = 0.8, \theta_p = 0.3, \theta_l = 0.5, c = 1.5$ and $\theta_r \in [0,1.3]$ with step size of 0.05)

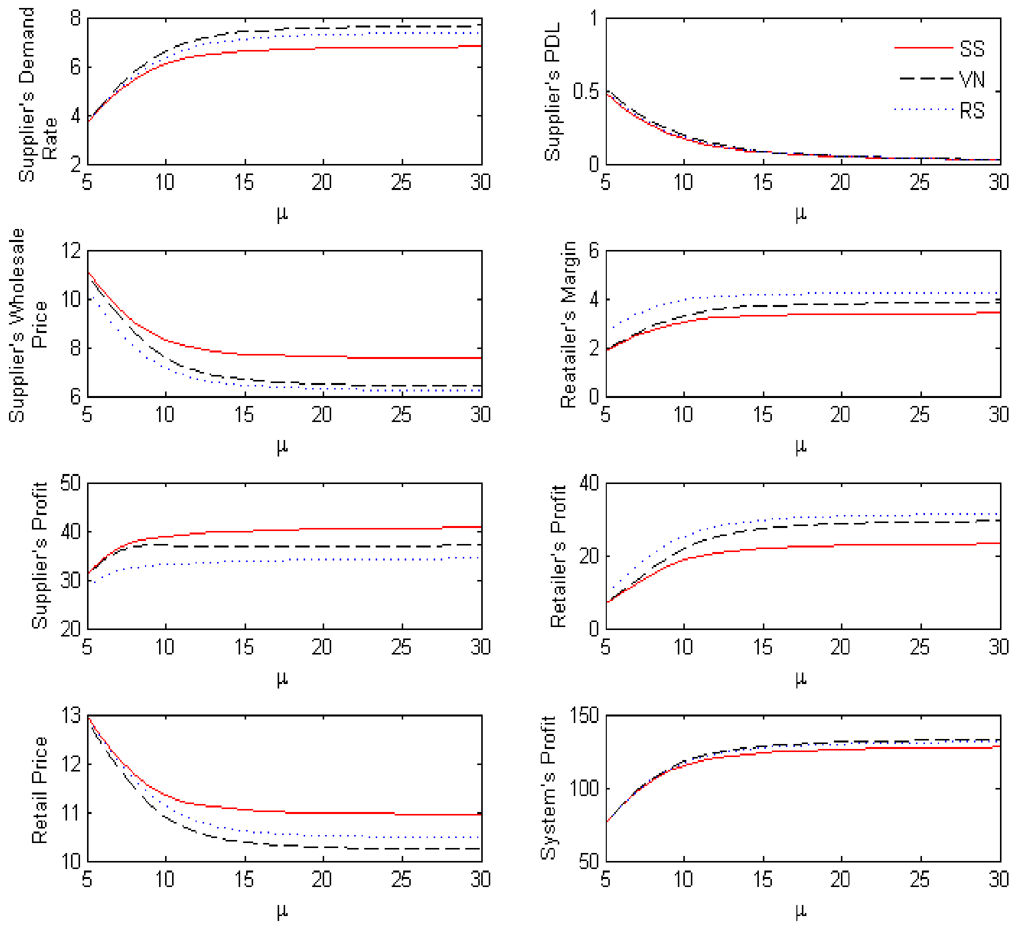

Figure 6: Influence of supplier's capacity μ on the performance of the supply chain ($\lambda_0 = 10, b = 3, h = 0.3, \alpha = 0.6, \beta = 0.8, \theta_p = 0.3, \theta_l = 0.5, \theta_r = 0.4, c = 1.5$ and $\mu \in [5,30]$ with step size of 1)

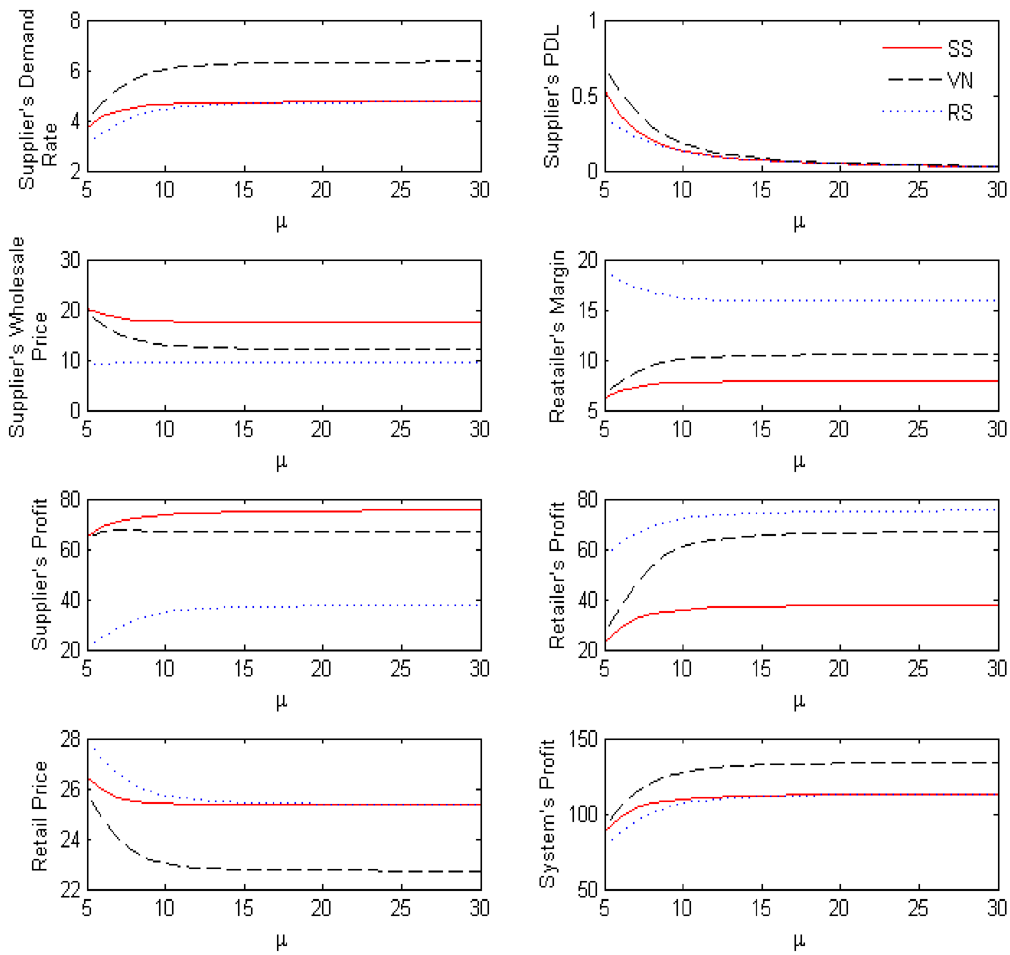

Figure 7: Influence of supplier's capacity μ on the performance of monopoly supplier and retailer channel structure ($\lambda_0 = 20, b = 3, h = 0.3, \alpha = 0.6, \beta = 0.8, c = 1.5$ and $\mu \in [5,30]$ with step size of 1)

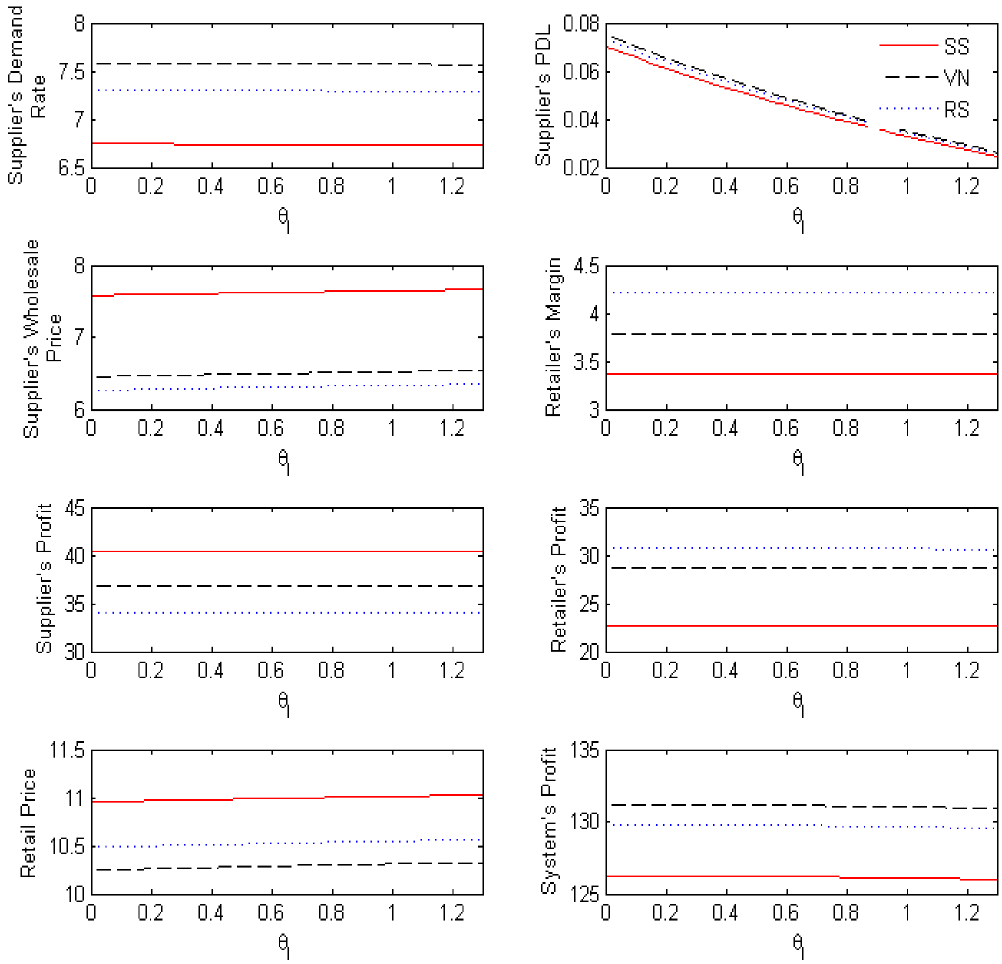

Figure 8: Influence of lead-time competition intensity θ_l when $\mu = 20$ ($\lambda_0 = 10, b = 3, h = 0.3, \alpha = 0.6, \beta = 0.8, \theta_p = 0.3, \theta_r = 0.4, c = 1.5$ and $\theta_l \in [0,1.3]$ with step size of 0.05)