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by

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PRICING AND INVENTORY CONTROL IN DUAL-CHANNEL NETWORK WITH ONE MANUFACTURER AND ONE RETAILER

Abstract

by Zhicong Pan

The study on multi-channel problems has been one of the most active research fields in recent years. In this paper, we consider a dual-channel network problem with one manufacture and one retailer. The manufacturer, acting as the Stackelberg leader, sells a single type of product through a traditional channel to the retailer and/or through a direct channel to customers. The retailer, acting as the follower, operates a Newsvendor model, ordering from the manufacturer and selling to the customers. We study the problem with the deterministic demand.

We develop an efficient algorithm to find the joint optimal policy for three prices: the wholesale price, the retail price in the traditional channel and the selling price in the direct channel. Our framework involves four different operational scenarios: the dual-channel scenario, the traditional-channel-only scenario, the direct-channel-only scenario, and the "equal pricing" scenario in which the wholesale price is equal to the selling price in the direct channel. We provide some criteria to identify different operational scenarios, and compare the performance of the four operational scenarios through numerical analysis. The scenario using dual channel possesses much more complementary effect between two channels than the performance in the "equal pricing" scenario. This observation calibrates some arguments based on the references only considering the "equal pricing" scenario. In addition, we have also examined a vertically integrated firm that operates a dual-channel supply chain. This vertically integrated firm is a centralized decision maker that decides two selling prices for the dual channels simultaneously. We have also compared the performance of the four scenarios with the performance of the integrated firm through numerical analysis.

We also consider stochastic demands for the dual-channel problem with one manufacturer and one retailer. In addition to pricing decisions, the manufacturer and the retailer also make inventory decisions (The retailer decides order quantity.) in the stochastic-demand problem. In our model, we consider exogenous wholesale price. There are four decision variables in our model: the retailer price, the direct channel price, the production capacity of the manufacturer, and the order quantity of retailer. We have developed a mechanism based on the chain rule to obtain the solutions one by one for these four decision variables. Given the wholesale price and the selling price in direct channel, we have obtained the retailer's order quantity and the retail price in the traditional channel. We have also obtained the optimal inventory capacity and the optimal direct price for the manufacturer given the retailer's best response for its order quantity and retail price. we also describe the optimal policy and compare the performance with regards to the retailer's order quantity through numerical analysis. We find that the manufacturer's profit is convex over the retailer's safety stock (order quantity), which indicates that an unique optimal wholesale price may not exist to maximize manufacturer's profit.

Key words: pricing; inventory control; dual channels; manufacturer; retailer

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Chapter 1

Deterministic Case

1.1 Introduction

Companies use the Internet as a new avenue to directly sell products to their customers. While the Internet provides an opportunity to increase sales by attracting more customers, it could also be a threat to the existing, traditional channel. The problem of introducing a new direct channel to customers so that the overall sales of a company is increased is called *the dual-channel problem*. In this paper, we study the dual-channel problem in the manufacturing industry.

Dual-channel distribution systems are widely used in various industries. Manufacturers like Sony Ellectronics, Apple Computers, Dell, etc. sell products to the consumers through independent retailers like Best Buy, Circuit City, etc. as well as through their respective e-commerce web-site (direct channel). The sales volume from the direct channel can be significant, especially when companies like Dell or Apple are well-known to most customers and internet is accessible for more and more consumers. More and more customers tend to buy their products from their web-site not from the traditional store. For a company that operates two distribution channels, the first decision to make would be the "pricing" decision. That is, what prices would be optimal for them to sell products through the two channels?

Different companies use different pricing strategies. For example, a Dell *Inspiron 1525* laptop can be obtained for 1050SGD at Dell's web-site. This price matches exactly the non-sale price at Dell's traditional retailers such as Suntec City. In this case, Dell prices its products in such a way that the direct channel price matches the retail price, which means the price charged from customers who order products from the direct channel, e.g. Dell's web-site, is the same as the retail price retailers charge customers when customers order products from the retailer. We call this pricing strategy 'Price Matching' strategy. 'Price Matching' strategy is often adopted by many companies because it can alleviate channel conflicts when those companies operate dual-channel supply chain.

Many companies operate a direct channel, 'not to obtain a larger share of the channel profit, but rather to induce the existing channel to expand sales volume and profits to a more efficient level' (*Chiang (2003)*). Aside from the 'Matching Pricing' strategy, some companies price their products in such a way that the wholesale price manufacturers charge retailers is equal to the online price or direct price. We call such pricing strategy 'Equal Pricing' strategy. By using the 'Equal Pricing' strategy, there may not be any sales occur in the direct channel. However, companies can still get more profits because 'the direct channel indirectly increases the flow of profits through retail channel' (*Chiang (2003)*). These interesting results are obtained and examined by Chiang in 2003.

The 'Matching Pricing' strategy and 'Equal Pricing' strategy can be efficient and useful when it comes to alleviating channel conflicts and expanding the existing channel's sales volume and profit. However, are the two pricing strategies always optimal for the manufacturers? Will retailers and customers always favor those pricing strategies? Except for the 'Matching Pricing' strategy and 'Equal Pricing' strategy, are there some other pricing strategies that may be more efficient under some circumstances? This paper tries to answer such questions and come up with some other pricing strategies that may be more favorable for manufacturers.

Balasubramania(1998) did some early research on the dual-channel problem through modeling "the competition in the multiple-channel environment from a strategic viewpoint" and marked "the early attempt to analyze this issue" (direct Versus retail competition). After Balasubramanian's early move on researching this multiple-channel problem, a lot of papers regarding this area have been published. Most of them are dealing with the "pricing" problem and the effects of direct marketing on the manufacturer and the retailer (Chiang et al. 3003; Viswanathan 2005; Swaminathan et al., 2006 and 2009).

In this paper, we solve such dual-channel problem in the manufacturing industry with one manufacturer and one retailer considered. We use a stylish demand model to solve the pricing problems facing manufacturers operating dual channels and answer questions raised in the above. Our analysis characterizes the equilibrium of the Stackelberg game where the manufacturer, as the leader in the game, knows the pricing decision taken by the retailer and decides its wholesale price to the retailer and direct price for the direct channel.

Our work contributes to the operations management literature by attempting to solve the manufacturer's pricing problem and the retailer's pricing problem under different scenarios. We have also designed an efficient algorithm for manufacturers to use when they are selecting their pricing strategies. We have developed some criteria under which it is optimal for the manufacturer to operates dual channels or it is optimal for the manufacturer to operate only one channel, either traditional channel or direct channel. Our results show that 'Equal Pricing' strategy and 'Match Pricing' strategy may not always be optimal for manufacturers. In some cases, it would be optimal for manufacturers price their products at a higher price in direct channel than their wholesale price offered to retailers. In addition, most of the time, it is optimal for manufacturers operate dual channels even when the direct channel has become much more convenient than the retail channel, as long as there are sufficient customers to buy from retailers.

The remainder of this paper is organized as follows. The next section provides a review of the related literature. Section 1.3 presents problem analysis, assumptions and our model. Section 1.4 presents solutions and analysis. Section 1.5 provides some insights to the results, structure results and sensitivity analysis. Section 1.6 provides some numerical study to illustrate the different channel strategies of the manufacturer. Finally section 1.7 summarizes and concludes the paper.

1.2 Related literature

Multi-channel problem has been extensively researched in the literature. Some of them focus on the pricing problem with competition, while some of them focus on demand forecasting and mixed-channel strategy with value-adding retailer.

Balasubramanian (1998) analyzed the competition between direct marketers and conventional retailers through using the spatial setting of the circular market, which considered the role of information as a strategic lever in the multiple-channel market. Direct sellers can regulate the level of consumer information and control the competitive flavor of the market. Tsay et al. (1999) and Frazier (1999) survey channel structure and incentive design for performance enhancement, but not channel conflict. Rhee and Park (2000) study a hybrid channel design problem, assuming that there are two consumer segments: a price sensitive segment and a service sensitive segment. Chiang et al. (2003) examine a price-competition game in a dual channel supply chain. Their results show that a direct channel strategy makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer, even though the equilibrium sales volume in the direct channel is zero. Their results however depend on the assumption that customer's acceptance of online channel is homogeneous.

Boyaci (2004) studies stocking decisions for both the manufacturer and retailer and assumes that all the prices are exogenous and demand is stochastic. Tsay and

Agrawal (2004) provide an excellent review of recent work in the area and examine different ways to adjust the manufacturer-retailer relationship. Viswanathan (2005) studies the competition across online, traditional and hybrid channels using a variant of circular city model. His focus is on understanding the impact of differences in channel flexibility, network externalities, and switching costs. Cattani et al.(2006) study coordination of pricing on Internet and traditional channels by modeling micro-level consumer behavior for demand generation. In their model, customers are at a random physical distance from traditional retailers, and at a random virtual distance from the direct marketer, independent of the physical distance. The market then is segmented according to the utility each customer attains from either the direct channel or the traditional channel. Customers are not excluded from a specific market; thus both markets have a chance to compete for all customers. Aussadavut et al.(2006) studied a dual channel supply chain in which a manufacturer sells to a retailer as well as to consumers directly. Consumers choose the purchase channel based on price and service qualities. The manufacturer decides the price of the direct channel and the retailer decides both price and order quantity. They developed conditions under which manufacturer the manufacturer and the retailer share the market in equilibrium. They also showed that the difference in marginal costs of the two channels plays an important role in determining the existence of dual channels in equilibrium.

Another two related papers are published in 2009 by Swaminathan et al.(2009) and Hu et al. (2009). Swaminathan (2009) studied the optimal pricing strategies when a product is sold on two channels. They provided theoretical bounds for the four prevalent pricing strategies proposed in the paper. Hu et al. (2009) discussed the revenue management for a service supply chain with two streams of customers, with the supplier having limited capacity of a perishable product. Monotone properties for the revenue functions and pricing strategies have been derived in this paper.

Our model differs from the prior studies in the following areas: (i) We focus on

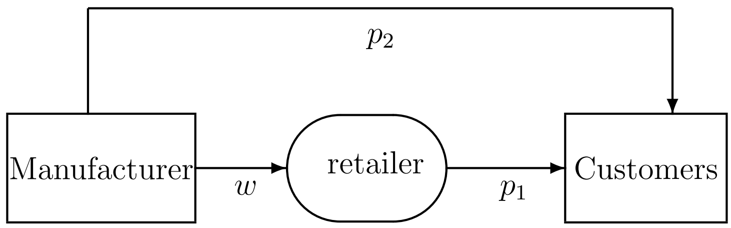

Figure 1.1: **Dual channels**. A manufacturer sells its products to customers through an retailer and through a direct channel. For each unit of product sold through the retailer, the retailer charges the customer a price p_1 and pays the manufacturer a wholesale price $w \leq p_1$. For each unit of product that is sold directly from the manufacturer, the manufacturer charges the customer a direct price p_2.

a stylish demand model to model the pricing problem for manufacturer and the retailer. (ii) We study the optimal pricing decisions of the manufacturer and the retailer under different conditions. Contributions of our work include: we develop optimal pricing strategy for the retailer and the manufacturer under different conditions and develop some interesting insights.

1.3 Assumptions and problem formulation

Consider a manufacturer that sells its products to customers through an retailer. For each product that is sold through the retailer, the customer pays p_1 to the retailer, who in turn pays a wholesale price $w \leq p_1$ to the manufacturer. Alternatively, the manufacturer can sells its products to customers through a direct channel (such as the manufacturer's web page) with a direct price p_2. The goal of the manufacturer is to maximize its own profit from both channels by properly setting the prices w and p_2, while the goal of the retailer is to maximize his own profit by properly choosing the price p_1. Figure 2.1 shows the dual channels of the manufacturer.

The problem can be further divided into two sub-problems: the retailer's problem and the manufacturer's problem. Below, we first describe the retailer's problem and its modeling. Then after that, we will describe the manufacturer's problem and its modeling. In the following section, which is section 4, we will focus on the solutions analysis. We list the notations used in this paper as below. where in table 1.1, $i = 1$

Table 1.1: Notations (in order of appearance)

D_i	demand function for channel i $(i = 1, 2)$
a_i	base demand for channel i $(i = 1, 2)$
b_{ij}	price sensitivity coefficients $(i, j = 1, 2)$
p_i	retail price for channel i $(i = 1, 2)$
w	wholesale price for one unit of product
V_m	manufacturer's profit
V_r	retailer's profit
$p_1(p_2, w)$	retailer's pricing decision as function of p_2 and w

represents the traditional channel and $i = 2$ represents direct channel, respectively. Notice that in the above notations, a_i, b_{ij} and c are all non-negative.

1.3.1 Demand model and assumptions

Let p_1 denote the retail price for one unit of product sold from the retailer to the customer via the traditional channel. Let w denote the wholesale price for one unit of product sold from the manufacturer to the retailer. Let p_2 denote the direct price for one unit of product sold from the manufacturer to the customer via the direct channel.

Given that the prices are p_1, p_2, and w, we assume that the demand is deterministic and only consider the cases that the demands are non-negative. Define $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ as the basic demand function for traditional channel and direct channel, respectively. Then D_1 and D_2 are defined as below (note that in following we use D_1 and D_2 to represent $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ in future).

$$D_1 = a_1 - b_{11}p_1 + b_{12}p_2 \tag{1.1}$$

and similarly, D_2 can be expressed as

$$D_2 = a_2 - b_{22}p_2 + b_{21}p_1 \tag{1.2}$$

a_i represent the market potential for each channel and both are positive, while b_{ii} and b_{ij} represent the price and cross-price sensitivity parameters ($i = 1, 2$ and $i \neq j$). In general, both a_i and b_{ij} are all positive. Notice that in the above definition, D_1 and D_2 are basic demand functions. The actual demand functions are limited by the boundary conditions. From the demand's definitions and non-negativity condition, we can obtain the upper bound for the retail price p_1 and direct price p_2, namely $\bar{p}_1$ and $\bar{p}_2$, as below.

$$\bar{p}_1 = \frac{a_1 b_{22} + a_2 b_{12}}{b_{22} b_{11} - b_{21} b_{12}} \tag{1.3}$$

$$\bar{p}_2 = \frac{a_2 b_{11} + a_1 b_{21}}{b_{22} b_{11} - b_{21} b_{12}} \tag{1.4}$$

We will discuss the problem with p_1 and p_2 within their upper bounds. We then define the actual demand functions for the problem as below.

$$D_1 = \begin{cases} a_1 - b_{11} p_1 + b_{12} p_2 & if p_1 \leq \frac{a_1 + b_{12} p_2}{b_{11}}, \\ 0 & otherwise. \end{cases}$$

$$D_2 = \begin{cases} a_2 - b_{22} p_2 + b_{21} p_1 & if p_1 \geq \frac{b_{22} p_2 - a_2}{b_{21}}, \\ 0 & otherwise. \end{cases}$$

To keep the retailer from buying through the direct channel or other arbitrators with a lower price, the wholesale price should not be higher than the direct channel price, that is $w \leq p_2$. We assume that the wholesale price is bounded and its upper bound equals to the minimum of upper bound of the retailer's retail price and the upper bound of the direct price.

Assumption 1.1. *: (Price Constraint assumption) we assume that p_1, p_2, and w are all non-negative and bounded. Let* $P_1 = \{0 \leq p_1 \leq \bar{p}_1\}, P_2 = \{0 \leq p_2 \leq$

$\bar{p}_2\}, W = \{0 \le w \le \bar{w}\}$, *then* P_1, P_2, W *denote the price ranges. Note that we define* $\bar{w} = \min\{\bar{p}_1, \bar{p}_2\}$. *We call this assumption as Price Constraint assumption.*

Assumption 1.2. *: (Dominance assumption) the price and cross-price sensitivity parameters have some relationships that are treated as common constraints in the literatures.*

$$b_{ii} \ge b_{ij}, \quad where \quad i, j = 1, 2 (i \ne j). \tag{1.5}$$

Assumption 1.2 says that demand for each product i is more sensitive to a change in its own price than it is to a simultaneous change in the prices of all other products. Assumption 1.2 is commonly used in the literature. (*Horn and Johnson 1994; and C. Maglaras and J. Meissner 2006*)

1.3.2 Problem formulation

In this section, we model the retailer and the manufacturer problem individually, while in the next section, we focus on the problem analysis and solutions.

Retailer's problem formulation. The retailer has only one decision variable to control to maximize its profit, i.e. the retail price p_1. The profit function $V_r(w, p_2)$ represents the maximum expected profit of retailer. Define a function $f_r(p_1)$ as below

$$\begin{aligned} f_r(p_1) &= (p_1 - w) D_1(p_1, p_2) \\ &= (p_1 - w)(a_1 - b_{11} p_1 + b_{12} p_2) \end{aligned} \tag{1.6}$$

Then we can obtain the retailer's maximum profit

$$V_r(p_2, w) = \max_{p_1} \left\{ f_r(p_1) \right\} \tag{1.7}$$

$$\textbf{s.t.} \qquad p_1 \leq \frac{a_1 + b_{12}p_2}{b_{11}}$$

When solving the retailer's problem, we assume that p_2 and w are fixed and known to the retailer.

Manufacturer's problem formulation. There are two decision variables for the manufacturer to control over to maximize its profit, i.e. the direct price p_2 and the wholesale price w. Assuming that p_1 is the retailer's best response given p_2 and w, then we can obtain V_m as the maximum expected profit of manufacturer, which is a function of p_2 and w. Note that we assume the retailer and the manufacturer are playing a Stackelberg game with the manufacturer acting the Stackelberg leader and the retailer as follower. We define $f_m(p_2, w)$ as below

$$\begin{aligned} f_m(p_2, w) &= (w - c)D_1(p_1, p_2) + (p_1 - c)D_2(p_1, p_2) \\ &= (w - c)(a_1 - b_{11}p_1 + b_{12}p_2) + (p_1 - c)(a_2 - b_{22}p_2 + b_{21}p_1) \quad (1.8) \end{aligned}$$

Note that in the above, we assume p_1 is the retailer's best response given p_2 and w, which means p_1 denotes $p_1(p_2, w)$. Thus, $f_m(p_2, w)$ is a quadratic function of p_2 and w. Before defining the manufacturer's problem, we first discuss the constraints for manufacturer's problem. According to assumption 1.1, we have defined that the wholesale price w and direct price p_2 are bounded and non-negative. Thus, we can define $R_m = \{p_2 \in P_2, w \in W, p_2 \geq w, D_1 \geq 0, D_2 \geq 0\}$ as the feasible area for the manufacturer's problem.

Then manufacturer's problem can be formulated as below.

$$V_m = \max_{(p_2,w)\in R_m} \{f_m(p_2, w)\} \qquad (1.9)$$

In the above formulation, we assume the non-negativity of all prices and manufacturing cost c. We also assume that the prices are bounded. In the next section, we focus on the solution and analysis of the problem.

1.4 Solutions and analysis

In this section, we focus on obtaining the optimal solutions for the retailer's problem and manufacturer's problem. Sub-section 1.4.1 introduces the framework. Sub-section 1.4.2 solves the retailer's problem, while Sub-section 1.4.3 discusses the manufacturer's problem.

1.4.1 Introduction of the framework and sequential decision

When solving the problem, we use sequential decisions procedure. First we assume that the direct price p_2 and w are known and given for the retailer, under which we solve the retailer's problem and obtain optimal retail price p_1^*. Note that p_1^* is a function of p_2 and w. We then solve the manufacturer's problem and obtain the optimal solutions p_2^* and w^* for the manufacturer. Plugging p_2^* and w^* back into p_1^*, we can obtain the optimal solution for the retailer. Finally, we can obtain the profits for retailer and manufacturer using the optimal prices obtained.

1.4.2 Solutions for retailer's problem

In this subsection, the retailer's problem is analyzed and solved. The retailer, acting as Stackelberg follower, decides its retail price first given the manufacturer's wholesale price and direct channel price.

Given p_2 and w, let $f_r(p_1) = (p_1 - w)D_1(p_1, p_2)$, we can obtain the retailer's maximum profit V_r as below.

$$V_r(p_2, w) = \max_{p_1 \in P_1} \left\{ f_r(p_1) \right\} \tag{1.10}$$

$$\textbf{s.t.} \quad p_1 \le \frac{a_1 + b_{12} p_2}{b_{11}}.$$

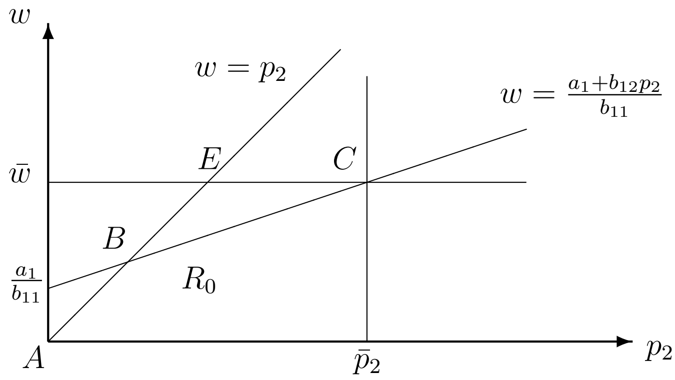

Figure 1.2: **Feasible region for retailer.** Region R_0: $D_1 > 0$, $w \leq p_2$, $p_2 \leq \bar{p}_2$; Line segment $\overline{BC}$: $D_1 = 0$.

Maximizing the profit V_r is equivalent to maximize f_r over p_1 subjective to the constraints of $p_1 \leq \frac{a_1+b_{12}p_2}{b_{11}}$. We can easily see that f_r is concave over p_1, given that p_2 and w are fixed and known. Set the first derivative to be zero, we can obtain the optimal retail price as a function of p_2 and w. That is $p_1^* = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}}$.

Lemma 1.1. *Given w and p_2, $\hat{p}_1$ maximizes f_r*

$$\hat{p}_1 = \frac{a_1 + b_{11}w + b_{12}p_2}{2b_{11}} \tag{1.11}$$

Lemma 1.1 illustrates the optimal pricing decision for the retailer without considering any constraints. That is $\hat{p}_1$ is the retailer's optimal solution only when $\hat{p}_1 \leq \frac{a_1+b_{12}p_2}{b_{11}}$ satisfies. When $p_1 > \frac{a_1+b_{12}p_2}{b_{11}}$, the retailer's optimal solution would be $\frac{a_1+b_{12}p_2}{b_{11}}$.

Area R_0 is defined as $R_0 = \left\{0 \leq p_1 \leq \bar{p}_1, 0 \leq p_2 \leq \bar{p}_2, 0 \leq w \leq \bar{w}, w \leq \frac{a_1+b_{12}p_2}{b_{11}}, w \leq p_2\right\}$. It's easy to verify that $\hat{p}_1 \leq \bar{p}_1$ as long as $w \in W$ and $p_2 \in P_2$. Thus, R_0 represents the feasible area of wholesale price w and direct price p_d for the retailer's problem. Figure 2 illustrates the feasible area R_0 for the retailer's problem. Notice that R_0 includes all the boundaries of R_0. If the manufacturer sets its wholesale price too high such that $D_1 < 0$, then the retailer's response would be $p_1^* = \frac{a_1+b_{12}p_2}{b_{11}}$, which means the optimal retail price would fall on line segment $\overline{BC}$. In fact, this "optimal" retail price does not make any sense for the retailer because it does not generate any profits for the retailer. However, we define this price because we want to use it to obtain the manufacturer's optimal solutions when the solutions are on the boundary $D_1 = 0$. We formally state the retailer's best pricing strategy in the following theorem.

Theorem 1.1. *(Retailer's Optimal Pricing Decision) Given the hotel's decision of wholesale price w and direct channel price p_2, the optimal retail price p_1^* for the retailer is*

$$p_1^* = \begin{cases} \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}} & if(p_2, w) \in R_0, \\ No feasible solution & otherwise. \end{cases}$$

Next, we solve the manufacturer's problem by first identifying the manufacturer's feasible area R_h.

1.4.3 Manufacturer's problem

Knowing the retailer's best responses, the manufacturer's problem is to maximize its total profits by choosing a proper wholesale price w and direct market price p_2. Define $f_m(p_2, w)$ as below

$$f_m(p_2, w) = (w - c)D_1 + (p_2 - c)D_2 \tag{1.12}$$

where D_1 and D_2 are as defined in section 3.1. The manufacturer's profit V_m can then be obtained as below.

$$V_m = \max_{p_2, w} \{f_m(p_2, w)\} \tag{1.13}$$

$$\textbf{s.t.} \quad D_1 \geq 0, D_2 \geq 0, p_2 \geq w, p_2 \leq \bar{p}_2, w \leq \bar{p}_1.$$

Thus manufacturer's problem is to maximize $f_m(p_2, w)$ under the constraints listed above. Notice that p_1 represents function $p_1(p_2, w)$ here. At first, we ignore all the conditions and maximize $f_m(p_2, w)$.

Lemma 1.2. *Under assumption 2, $f_m(p_2, w)$ is joint concave over the wholesale price w and direct price p_2. An unique solution p_2^* and w^* can be obtained to maximize $f_m(p_2, w)$.*

Proof and optimal prices are listed in Appendix A. The results in Lemma 1.2 maximize f_m if we don't consider any constraints. However, there are several constraints to be considered when solving the manufacturer's problem. There are five constraints for the manufacturer's problem: $D_1 \geq 0$, $D_2 \geq 0$, $p_2 \geq w$, $p_2 \leq \bar{p}_2$, $w \leq \bar{p}_1$.

Under these constraints, we can divide the manufacturer's problem into four different cases. Different constraints correspond to different problems. If one of the constraints is violated, then the solutions will be on the boundaries. In this case, the problem becomes a different problem with only either direct channel or traditional channel exists. Note that we define $p_2 = w$ as one of the boundary constraints for manufacturer's problem and discuss the problem separately. This is different from the literature. In the literature, people often treat $p_2 \leq w$ as a constraint for manufacturer (*Chiang (2003)*). However, in our paper, we discuss the manufacturer's problem separately when $p_2 < w$ and $p_2 = w$.

We define the manufacturer's feasible area R_h, as is illustrated in figure 3 based on the constraints. Given that the retailer's best responding price as $p_1 = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}}$, we can obtain the manufacturer's feasible area R_h based on wholesale price w and direct price p_2. The first constraint to be considered is $p_1^* \leq \bar{p}_1$. That is p_1^* must be within the boundary of p_1, i.e. $p_1^* = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}} < \bar{p}_1$. However, as long as $(p_2, w) \in R_1$, $p_1^* = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}} < \bar{p}_1$ satisfies.

The demand for the traditional channel must be non-negative, i.e. $D_1 \geq 0$, from which we can obtain $p_1 \leq \frac{a_1+b_{12}p_2}{b_{11}}$. And from $p_1^* = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}}$, we can obtain

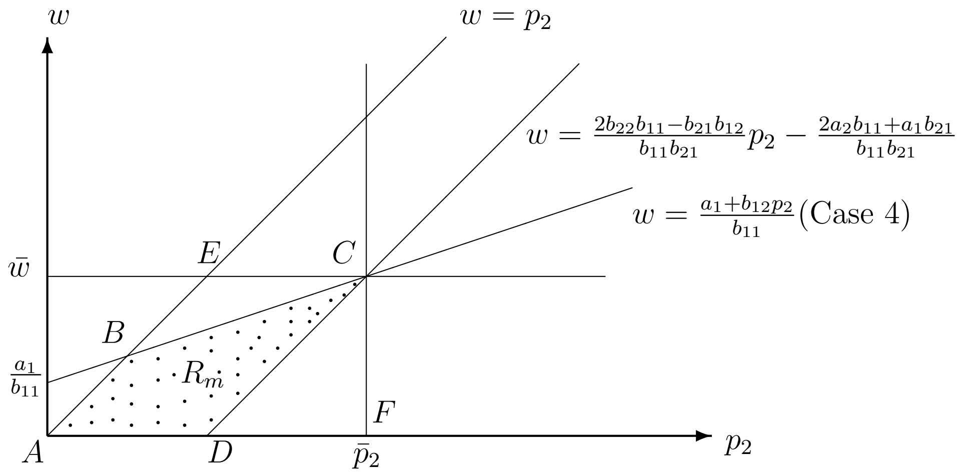

Figure 1.3: **Feasible region for manufacturer's problem.** Region R_1: $D_1 > 0$, $D_2 > 0$, $w < p_2$; Line segment $\overline{AB}$: $D_1 \geq 0$, $D_2 > 0$, $w = p_2$; Line segment $\overline{BC}$: $D_1 = 0$, $D_2 \geq 0$, $w \leq p_2$; Line segment $\overline{CD}$: $D_1 \geq 0$, $D_2 = 0$, $w < p_2$.

$b_{11}w \leq a_1 + b_{12}p_2$. Another constraint, $D_2 \geq 0$, together with $p_1^* = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}}$, we can obtain $(2b_{22}b_{11} - b_{21}b_{12})p_2 \leq 2a_2b_{11} + a_1b_{21} + b_{11}b_{21}w$. In other words, this retail price is optimal only when the manufacturer sets the direct channel price p_2 and wholesale price w in region R_m, where $R_m = \{(p_2, w)|b_{11}w \leq a_1 + b_{12}p_2, w \leq p_2, (2b_{22}b_{11} - b_{21}b_{12})p_2 \leq 2a_2b_{11} + a_1b_{21} + b_{11}b_{21}w, w \leq \bar{p}_1, p_2 \leq \bar{p}_2\}$. Notice that when the values of wholesale price w and direct price p_2 fall on the triangle area of $\triangle\, BCE$ and $\triangle\, CDF$, the solutions fall on the line segments $\overline{BC}$ and $\overline{CD}$.

Here, we can then divide the manufacturer's problem into four different cases based on the boundary conditions or the constraints:

1. Consider the problem in the region R_h that does not include the boundaries, i.e. the open area of region R_h. We define this area as $R_1 = \{(p_2, w)|b_{11}w < a_1 + b_{12}p_2, w < p_2, (2b_{22}b_{11} - b_{21}b_{12})p_2 < 2a_2b_{11} + a_1b_{21} + b_{11}b_{21}w, w < \bar{p}_1, p_2 < \bar{p}_2\}$. The solutions in this area are interior solutions of the problem. This is defined as Case 1 below. We will discuss this case later in this sub-section;

2. Consider the optimal prices that lies on line segments $\overline{AB}$, $\overline{BC}$ and $\overline{DE}$. These correspond to three special cases. These three cases are discussed in sub-section 4.3. We redefine and solve the problem separately when the solutions are on the boundaries.

We define the four cases for the manufacturer's problem as below:

Case 1.1. *: (Regular case)* $(p_2, w) \in R_1$. *When we solve the manufacturer's dual channel problem below, we assume the condition* $(p_2, w) \in R_1$ *hold. We call this case as Regular case.*

The manufacturer's problem can be formulated as below:

$$V_m = \max_{p_2, w} \{f_m(p_2, w)\} \tag{1.14}$$

$$\textbf{s.t.} \quad D_1 > 0, D_2 > 0, p_2 > w > 0, p_2 < \bar{p}_2, w < \bar{p}_1.$$

Proposition below solves the manufacturer's problem when $(p_2, w) \in R_1$ hold.

Proposition 1.1. p_1^*, p_2^* *and* w^* *in Lemma 1.2 are optimal for the manufacturer when* $(p_2^*, w^*) \in R_1$ *are satisfied.*

Proposition 1.1 shows that when there is an interior solution, p_1^*, p_2^* and w^* in Lemma 1.2 are the optimal solutions for the manufacturer and thus maximize the manufacturer's profit.
When one of these constraints is violated, we redefine the problem and obtain its optimal solutions. If the manufacturer's optimal prices w^* and direct price p_2^* is not in R_1, then the solutions must be on the boundaries. We define another three special cases under which one of these constraints is violated.

Case 1.2. *: (Equal pricing)* $w = p_2$. *This case happens when the manufacturer forces its pricing strategy to let the wholesale price equals to its direct channel price. This case corresponding to the line segment* $\overline{AB}$ *in figure 3.*

Case 1.3. *: (Single traditional channel)* $D_1 > 0, D_2 = 0$. *This case happens when the manufacturer control the direct channel price* p_2 *to ensure that there is no sales for the direct channel. This case corresponding to the line segment* $\overline{DE}$ *in figure 3.*

Case 1.4. *: (Single direct channel)* $D_1 = 0, D_2 > 0$. *This case happens when the manufacturer sets its wholesale price to sufficient high (higher than the retailer's retail price) so that there is no demand for the traditional channel. This case corresponding to the line segment* $\overline{BC}$ *in figure 3.*

Notice that the retailer's optimal solutions for case 1, case 2 and case 3 are the same and is illustrated in Theorem **??**. However, the optimal solutions for the manufacturer for the three special cases listed in the above are different. We will discuss these three special cases in sub-section 1.4.3. The manufacturer's profit can be obtained by plugging p_1^*, p_2^* and w^* in Lemma 1.2 into the manufacturer's profit function:

$$V_m = (w^* - c)(a_1 - b_{11}p_1^* + b_{12}p_2^*) + (p_2^* - c)n \tag{1.15}$$

Similarly, the retailer's profit can be obtained:

$$V_r = (p_1^* - w^*)(a_1 - b_{11}p_1^* + b_{12}p_2^*) \tag{1.16}$$

The results of V_r and V_m are listed in the *Appendix A*.

1.4.4 Solutions for boundary cases

In this subsection, we solve the dual channel problem when there is no interior solution existing.

First, consider special case 3, where the wholesale price is very low compared with the direct price and at the same time, the manufacturer shuts down the direct channel. This case corresponds to the line segment $\overline{CD}$.

Lemma 1.3. *The prices on line segment* $\overline{CD}$ *are not optimal for the manufacturer.*

The proof of Lemma 1.3 is given in the Appendix and the intuition is as follows.

Because the direct price p_2 has a dominant effect on the direct channel and its effect on the direct channel is greater than that on traditional channel, thus when we decrease the direct price, it brings more customers to the direct channel which more than offsets the number of customers decreases from the traditional channel. In addition, from our assumption know that the direct price is no less than the wholesale price, which means $p_2 \geq w$. Thus when we decrease the direct price a little bit, the profit generated from the direct channel would be able to offset the profit lost from the traditional channel. Therefore, it is never optimal for the manufacturer to shut down the direct channel and only keep the traditional channel. It's quite common for the manufacturer to set its wholesale price equal to its direct channel's retail price and this strategy has been discussed in many literatures and proved to be an optimal pricing strategy for the manufacturer (*Chiang et al. (2003) and Cattani et al. (2006)*). This strategy is also widely used in the industry, for example Dell.

Retailer's problem. Let $f_r(p_1) = (p_1 - w)D_1$, then maximizing the retailer's profit is equivalent to maximize $f_r(p_1)$ over p_1.

$$V_r(p_2, w) = \max_{p_1} \left\{ f_r(p_1) \right\} \tag{1.17}$$

$$\textbf{s.t.} \qquad D_1 > 0, D_2 > 0, w = p_2$$

Lemma 1.4 below gives us the optimal response of the retailer given wholesale price w and direct price p_2.

Lemma 1.4. *Given b_{11} is non-negative, $f_r(p_1)$ is concave over p_1. Given that*

$(p_2, w) \in \overline{AB}$, *the optimal retail price* p_1^* *can be obtained as below.*

$$\begin{aligned} p_1(w, p_2) &= \frac{a_1 + b_{11}w + b_{12}p_2}{2b_{11}} \\ &= \frac{a_1 + (b_{11} + b_{12})w}{2b_{11}} \end{aligned} \tag{1.18}$$

Proof. Setting the first order derivative of f_r over p_1 to zero, we can obtain the optimal p_1^*. □

From Lemma 1.4, we can see that, the solution for the retailer is the same to the solution obtained from dual channel problem with $w < p_2$. That means the manufacturer's decision of whether or not to discriminately price the two channels does not affect the retailer's pricing decision. Next, we solve the manufacturer's problem.

Manufacturer's problem. Define $f_m(p_2, w) = (w - c)D_1 + (p_2 - c)D_2$. Then, maximizing manufacturer's profit is equivalent to maximize $f_m(p_2, w)$. The manufacturer's maximum profit can be obtained as

$$V_m = \max_{p_2, w} \left\{ f_m(p_2, w) \right\} \tag{1.19}$$

$$\textbf{s.t.} \qquad D_1 > 0, D_2 > 0, w = p_2.$$

Given the retailer's pricing response in lemma 1.4, Lemma 1.5 below gives us the optimal pricing decision for the manufacturer.

Lemma 1.5. *Under Assumption 2,* f_m *is concave on* p_2 *and* w*. An unique optimal* w^* *can be obtained when the conditions of* $D_1 > 0$ *and* $D_2 >$ *hold. The optimal wholesale price* w^* *and direct price* p_2^* *is*

$$w^* = p_2^* = \frac{a_1 b_{11} + 2a_2 b_{11} + a_1 b_{21}}{2(b_{11}^2 + 2b_{22}b_{11} - b_{11}b_{21} - b_{11}b_{12} - b_{21}b_{12})} + \frac{1}{2}c \tag{1.20}$$

The corresponding retail price for the retailer is

$$p_1^* = \frac{a_1 + (b_{11} + b_{12})w^*}{2b_{11}} \tag{1.21}$$

Please see the proof in the Appendix.

Next, we solve the problem with positive demand for the direct channel only, i.e. $D_1 = 0$ and $D_2 > 0$. In order to ensure zero demand for the traditional channel, we set the wholesale price $p_1 \leq w$ and $p_2 \leq \bar{p}_2$. In this case, we solve the problem for the manufacturer, while the retailer will not have any profits. Notice that this problem corresponds to the line segment $\overline{BC}$ of Figure 2.

Manufacturer's problem. Let $f_m(p_2) = (p_2 - c)D_2$ and $D_2 = a_2 - b_{22}p_2 + b_{21}p_1$, then the manufacturer's maximum profit can be obtained

$$V_m = \max_{p_2} \left\{ f_m(p_2) \right\} \tag{1.22}$$

$$\textbf{s.t.} \qquad D_2 > 0, D_1 = 0.$$

Lemma 1.6 below shows us the optimal pricing decision for the manufacturer when the manufacturer operates only one direct channel.

Lemma 1.6. *Under Assumption 2, f_m is concave over p_2. Under the condition of $\bar{p}_1 < 2\bar{p}_2 + c$, the optimal direct channel price can be obtained*

$$p_2^* = \frac{1}{2}\bar{p}_2 + \frac{1}{2}c. \tag{1.23}$$

The corresponding retail price is $p_1^* = \frac{1}{2}\bar{p}_1 + \frac{a_1 + b_{12}c}{2b_{11}}$.

Plugging p_2^* and p_1^* into V_m, we can obtain the optimal profit of the manufacturer as $V_m = (p_2^* - c)(a_2 - b_{22}p_2^* + b_{21}p_1^*)$.

1.4.5 Optimal solutions for the manufacturer's problem

In this subsection, we obtain an optimal solution for the manufacturer's problem and propose an algorithm to solve the manufacturer's problem based on the discussions of section 1.2 and 1.3.
The retailer's problem can be solved by Theorem **??**, given manufacturer's decisions of wholesale price w and direct price p_2. From Theorem **??**, the retailer's optimal solution depends solely on the manufacturer's pricing decisions and is not affected by the allocation decision for the direct channel. we summarize the manufacturer's optimal decision under different scenarios. From subsection 1.4.2 and 1.4.3, we can obtain an algorithm to solve the manufacturer's problem under different scenarios.

Algorithm 1.1. *The manufacturer's problem can be solved by taking the following steps:*

1. *Solve the manufacturer's problem according to the retailer's best response without considering any constraints.*
2. *Examine the solutions obtained in step 1 to see if the solutions satisfy the constraints of the manufacturer's problem, i.e. examine interior solutions existing or not.*
3. *If there are no interior solutions, then re-solve the problem using boundary conditions.*
4. *If the parameters satisfy the constraints of more than two cases, use the solutions that generates most profits for the manufacturer.*

The manufacturer's problem can be solved by Algorithm 1.1. We formally state the manufacturer's optimal pricing decision in the following theorem.

Theorem 1.2. *The manufacturer's pricing decision can be solved using Algorithm 1.1.*

We have solved the four different cases for the retailer and the manufacturer and obtained solutions under different scenarios. However, notice that there are two special cases: the first case is that the wholesale price w equals to zero, i.e. $w = 0$; the second case is that the direct price p_2 equals to its upper bound price $\bar{p}_2$, i.e. $p_2 = \bar{p}_2$. we can easily justify that these two cases are not optimal for the manufacturer. For the first case, if the manufacturer sets wholesale price $w = 0$, then the manufacturer would gain no profit from the traditional channel. Thus it would be more profitable for the manufacturer to set positive wholesale price, which will benefit the manufacturer's direct channel due to channel competition. For the second case, from sub-section 1.4.4, we have solved the case with zero demand for direct channel, which actually identical to setting $p_2 = \bar{p}_2$.

1.5 Price matching policy and centralized decision making

When opening a direct channel, it is common for the manufacturer to set its direct price matching with retailer's retail price. For example, Dell company sells its computers online at the same price as its retailer's retail price, i.e. $p_1 = p_2$. Such pricing strategy is also discussed in the literature (*Cattani et al (2006)*). In addition, there may cases that a company acts as a centralized decision maker and decides its selling prices simultaneously for the direct channel and traditional channel without caring the wholesale price. We call such kind of company "Integrated Firm". The integrated firm's performance is often used as a performance bench mark to compare the performance of other pricing policies, i.e. dual channel, single channel, price matching, etc. We solve these two cases in this section.

1.5.1 Manufacturer matches its direct price with retail price

In this subsection, we will show how does the manufacturer's profits perform under this strategy compared with the manufacturer setting its prices freely. When setting $p_1 = p_2 = p$, we assume the format of demand function maintains same as D_1 and D_2 as we define at the beginning: $D_1 = a_1 - b_{11}p + b_{12}p$ and $D_2 = a_2 - b_{22}p + b_{21}p$. There are two decision variables: one is the retail price p and the other is wholesale price w. As the manufacturer commits to price-matching with the retail price, p is decided by the retailer, while the manufacturer still optimally decides its wholesale price w.

There is an upper bound for the retail price in order to satisfy the non-negativity condition of demands, namely $D_1 \geq 0$ and $D_2 \geq 0$. The upper bound is defined by $\bar{p} = \max\{\frac{a_1}{b_{11}-b_{12}}, \frac{a_2}{b_{22}-b_{21}}\}$. Thus we define the feasible area for price p as $P = \{p \leq \bar{p}\}$. Next, we solve the problem following the above procedure. That is, we solve the problem with the manufacturer and the retail playing a Stackelberg game: the manufacturer acts as a Stackelberg leader, while the retailer acts as a follower.

Retailer's problem. Defining $f_{mtr}(p) = (p-w)D_1$, we model the retailer's problem as below.

$$V_{mtr} \quad = \quad \max_{p \in P} \left\{ f_{mtr}(p) \right\} \tag{1.24}$$

Notice that in the above formulation, we have assume the positivity of demand D_1 and D_2. If the either demand equals to zero, i.e $D_1 = 0$ or $D_2 = 0$, we would have $p = \frac{a_1}{b_{11}-b_{12}}$ or $p = \frac{a_2}{b_{22}-b_{21}}\}$, under each case we say the solutions of the problem lay on the boundary. If $p = \frac{a_2}{b_{22}-b_{21}}$, we know that the retailer sets her retial price such that there is zero demand for the manufacturer's direct channel. In this case, there is an implicit condition, i.e $\frac{a_1}{b_{11}-b_{12}} > \frac{a_2}{b_{22}-b_{21}}$. If such condition does not satisfy, there would not be any sales for both channels. We may imagine that this case happens because the manufacturer wants to operate only retail channel, thus the

manufacturer sets its wholesale price larger than the upper bound price of the direct channel, i.e. $w \geq \frac{a_2}{b_{22}-b_{21}}$.
If the retailer sets its retail price $p = \frac{a_1}{b_{11}-b_{12}}$, it means that the manufacturer wants to shut down the retail channel by setting a proper wholesale price w to force the retailer to price the upper bound of the retail channel, i.e. $w \geq \frac{a_2}{b_{22}-b_{21}}$. That also means the manufacturer wants to gain his profit solely from the direct channel.
For the case that the manufacturer wants to operate only single direct channel, it does no longer make any sense for the manufacturer to 'match' its price. Instead, the manufacturer would set its direct price optimally to maximize its own profits. However, under such case, the direct price set by the manufacturer must be greater than the upper bound price for the retail channel, i.e. $p \geq \frac{a_1}{b_{11}-b_{12}}$. In this case, there is also an implicit condition, namely $\frac{a_1}{b_{11}-b_{12}} < \frac{a_2}{b_{22}-b_{21}}$. We would solve the the problem later when the two special cases happen.
Solving the retailer's problem according to the retailer's objective function 1.24, we can obtain the retailer's optimal retail price as below.

$$\hat{p} = \frac{a_1}{2(b_{11} - b_{12})} + \frac{1}{2}w \tag{1.25}$$

Manufacturer's problem. Defining $f_{mtm} = (p - c)D_2 + (w - c)D_1$, the manufacturer's problem can be modeled as below.

$$V_{mtm} = \max_{w} \left\{ f_{mtm}(w) \right\} \tag{1.26}$$

$$\textbf{s.t.} \qquad D_1 > 0, D_2 > 0.$$

Solving the manufacturer's problem, we can obtain manufacturer's optimal wholesale price as below.

$$w^* = \frac{a_1 + a_2 - \frac{a_1(b_{22}-b_{21})}{b_{11}-b_{12}} + (b_{11} - b_{12} + b_{22} - b_{21})c}{b_{11} - b_{12} + 2(b_{22} - b_{21})} \tag{1.27}$$

The corresponding retail price can be obtained as

$$p^* = \frac{3a_1 + a_2 + (b_{11} - b_{12} + b_{22} - b_{21})c}{2(b_{11} - b_{12} + 2(b_{22} - b_{21}))} \tag{1.28}$$

The conditions under which the above solutions are optimal can be obtained as below.

Constraint 1.1. $(a_1 - b_{11}c + b_{12}c) \geq \frac{a_2(b_{11}-b_{12})-a_1(b_{22}-b_{21})}{b_{11}-b_{12}+2(b_{22}-b_{21})}$. *This constraint can guarantee the non-negativity of* D_1*.*

Constraint 1.2. $p^* < \min\{\frac{a_1}{b_{11}-b_{12}}, \frac{a_2}{b_{22}-b_{21}}\}$ *This constraint means that the prices are bounded. This constraint also guarantee the non-negativity of the demand of the direct channel.*

If constraint 1.1 is violated, then we must have $D_1 = 0$ which means $w = \frac{a_1}{b_{11}-b_{12}}$. In this case, the manufacturer sets its wholesale price high enough to shut down the retail channel but optimally sets its direct price to maximize its profit from direct channel. The manufacturer's profit can be obtained as below.

$$V_{mtm} = \max_p\{(p - c)(a_2 - b_{22}p + b_{21}p)\} \tag{1.29}$$

$$\textbf{s.t.} \qquad \frac{a_2}{b_{22} - b_{21}} > p \geq \frac{a_1}{b_{11} - b_{12}}.$$

The optimal direct price can be obtained as $p^* = \frac{a_2+b_{22}c-b_{21}c}{2(b_{22}-b_{21})}$. The constraint for this solution is $\frac{2a_1}{b_{11}-b_{12}} - \frac{a_2}{b_{22}-b_{21}} \leq c < \frac{a_2}{b_{22}-b_{21}}$. Notice that in this case, the upper bound price of the direct channel must be greater than that of traditional channel, i.e. $\frac{a_2}{b_{22}-b_{21}} > \frac{a_1}{b_{11}-b_{12}}$.

When the manufacturer sets its wholesale price greater than the upper bound price of the direct channel, i.e. $w \geq \frac{a_2}{b_{22}-b_{21}}$, the retailer is forced to set its retail price greater than $\frac{a_2}{b_{22}-b_{21}}$ in order to be profitable. Under this case, the manufacturer operates only traditional channel. In this case, the retailer's optimal response to the manufacturer's wholesale price is still $p = \frac{a_1}{2(b_{11}-b_{12})} + \frac{1}{2}w$. The manufacturer's profit in this case can be obtained as

$$\begin{aligned} V_{mtm} &= (w-c)(a_1 - b_{11}p + b_{12}p) \\ &= \frac{1}{2}[-(b_{11}-b_{12})w^2 + (a_1 + b_{11}c - b_{12}c)w - a_1 c] \end{aligned} \tag{1.30}$$

V_{mtm} is concave over w and we can obtain the optimal wholesale price as $w^* = \frac{a_1}{2(b_{11}-b_{12})} + \frac{1}{2}c$. The corresponding retail price would be $p^* = \frac{3a_1}{4(b_{11}-b_{12})} + \frac{1}{4}c$. The condition for this case is $\frac{2a_2}{b_{22}-b_{21}} - \frac{a_1}{b_{11}-b_{12}} \leq c < \frac{a_1}{b_{11}-b_{12}}$. Notice that in this case, the upper bound price of the direct channel must be less than that of traditional channel, i.e. $\frac{a_2}{b_{22}-b_{21}} < \frac{a_1}{b_{11}-b_{12}}$

1.5.2 Should a vertically integrated firm use the direct channel?

In many cases, we can expect to see some manufacturers selling through dual channels but having a centralized decision maker. In this case, we call the firm as a vertically integrated firm (*Chang et al (2003)*). Obviously, the manufacturer cases about its profits by deciding the retail price p_1 and direct price p_2 simultaneously. The profit for an integrated firm can be formulated as below.

Let $f_{vi} = (p_1 - c)D_1 + (p_2 - c)D_2$, then the firm's profits equal

$$V_{vi} = \max_{(p_1,p_1)\in(P_1,P_2)} \left\{ f_{vi}(p_1, p_2) \right\} \tag{1.31}$$

$$\textbf{s.t.} \qquad D_1 \geq 0, D_2 \geq 0$$

A vertically integrated firm controls both traditional retailing and direct sales. Given the formulation of 1.31, the manufacturer sets its retail price p_1 and direct price p_2 to maximize its own profits V_{vi}.

Maximizing V_{vi} with respect to p_1 and p_2 gives

$$p_1 = \frac{a_2b_{12} + a_2b_{21} + 2a_1b_{22}}{4b_{11}b_{22} - b_{12}^2 - 2b_{12}b_{21} - b_{21}^2} + \frac{-b_{12}^2 + b_{12}b_{22} - b_{21}b_{22} + 2b_{11}b_{22} - b_{12}b_{21}}{4b_{11}b_{22} - b_{12}^2 - 2b_{12}b_{21} - b_{21}^2} \tag{1.32}$$

$$p_2 = \frac{a_1b_{21} + a_1b_{12} + 2a_2b_{11}}{4b_{11}b_{22} - b_{12}^2 - 2b_{12}b_{21} - b_{21}^2} + \frac{-b_{21}^2 + b_{21}b_{11} - b_{12}b_{21} + 2b_{11}b_{22} - b_{11}b_{12}}{4b_{11}b_{22} - b_{12}^2 - 2b_{12}b_{21} - b_{21}^2} \tag{1.33}$$

This solution satisfies only when the demands for both channels are positive, i.e. $D_1 \geq 0$ and $D_2 > 0$.

If either channel's demand is negative, we must have zero demand for that channel and the problem becomes a different one. When the demand for the traditional channel is not positive, we will have

$$V_{vi} = \max_{(p_1,p_1)\in(P_1,P_2)} \{f_{vi}(p_1,p_2)\} \tag{1.34}$$

$$\textbf{s.t.} \qquad D_1 = 0, D_2 \geq 0$$

where $f_{vi} = (p_2 - c)D_2$.

With $D_1 = 0$, we can reduce two variables into one variable p_2 (we plug $p_1 = \frac{a_1+b_{12}p_2}{b_{11}}$

into V_{vi}). Maximizing V_{vi} with respect to p_2 gives

$$p_2 = \frac{1}{2}\bar{p}_2 + \frac{1}{2}c \tag{1.35}$$

This solution satisfies only when the demand for the direct channel is positive, i.e. $D_2 > 0$. From the solution, we can obtain $D_2 = \frac{b_{11}b_{22} - b12b_{21}}{2b_{11}}(\bar{p}_2 - c)$, which is positive given $\bar{p}_2 > c$. Alternatively, there may be only traditional channel having positive demand while zero demand for direct channel. In such case, we can obtain the optimal retail price p_1 using $p_2 = \frac{a_2 + b_{21}p_1}{b_{22}}$ to maximize V_{vi}.

$$p_1 = \frac{1}{2}\bar{p}_1 + \frac{1}{2}c \tag{1.36}$$

This solution satisfies only when the demand for the traditional channel is positive, i.e. $D_1 > 0$. From the solution, we can obtain $D_1 = \frac{b_{11}b_{22} - b_{12}b_{21}}{2b_{22}}(\bar{p}_1 - c)$. If the parameters satisfy both dual channel setting as well as single channel setting, then it is optimal for the manufacturer to select the strategy that generates most profits.

1.6 Numerical results and managerial insights

In this section, we provide some numerical study to illustrate some main results. We first observe how the manufacturer and the retail's profits change when the parameters a_1, a_2, b_{11}, b_{12}, b_{22}, b_{21} change. Meanwhile, we also present how the corresponding prices change with regards to the parameters. When examining the market potential's effects, i.e. the effects of a_1 and a_2, we maintain the values of b_{11}, b_{12}, b_{22}, b_{21} unchanged and let $b_{11} = b_{12}$ and $b_{22} = b_{21}$. When we examine the effects of b_{12} and b_{21} (or b_{11} and b_{22}), we let $a_2 >> a_1$. We set the market potential for direct channel much greater than that of traditional channel because we want to see whether the manufacturer would abandon the traditional channel or not when the direct channel is much more attractive for the customers.

our results show that it's optimal for the manufacturer to operate dual channel most of the time, even when the direct market is much greater than that of traditional market. Our result shows some inconsistence with some results from some literatures. Some literatures have shown that when the direct channel becomes a lot more convenient than that of traditional channel, the manufacturer would abandon the traditional channel and only operate single direct channel (*Chiang et al. (2003) and Cattani et al. (2006)*).

Our results also show that when traditional market is greater than that of direct market, it is optimal for the manufacturer use dual channel with $p_2 > w$. This pricing policy means that the manufacturer tends to give some pricing advantage to the retailer in order to avoid channel conflicts. However, when the direct market is greater than that of traditional channel, the manufacturer will price the direct channel more aggressively and let the direct price equals to the wholesale price, i.e. $p_2 = w$. When the traditional market is extremely unprofitable for the manufacturer (even the manufacturer sets its wholesale price close to its manufacturing cost, there is still very few customer buying from traditional channel), the manufacturer would prefer to operate single direct channel.

1.6.1 Numerical study for a_1 and a_2

In this section, we present numerical study for market potential a_1 and a_2 under the three cases. We set the parameters as below: $b_{11} = b_{22} = 65$, $b_{12} = b_{21} = 25$ and $c = 1$. In order to simplify the illustration, we use case 1 to case 4 to denote the following channel strategies for the following explanation:

Case 1 - Dual channel strategy for the manufacturer;

Case 2 - Dual channel strategy with $p_d = w$;

Case 4 - Single direct channel strategy.

Remark 3: From Figure 4 to 5, we can see that, as a_1 (from 0 to 300) and a_2

change (increase from 100 to 400), it is optimal for the manufacturer to choose different channel strategies.

It's optimal for the manufacturer to operate dual channels, unless the market base for one channel is very small compared with the other channel. Even the direct channel becomes very convenient for the customers, the manufacturer still betters off if he operates dual channels (not bandon the retail channel). This is consistent with the industry and it may due to the brand awareness and advertisement effect of the direct channel.

The manufacturer can gain more profit operating dual channel with $p_2 > w$ than using 'equal pricing' strategy ($p_2 = w$) when $a_1 > a_2$. This is because the manufacturer needs to alleviate the channel conflicts between the two channels through giving some advantage to the retailer regarding pricing. However, when the market size of the direct channel is much greater than that of direct channel, i.e.$a_2 > a_1$, the manufacturer would not give such pricing advantage to the retailer and choose to price more aggressively ($p_2 = w$), which in turn would benefit the retailer.

In addition, the manufacturer's and the retailer's profit increase as a_1 and a_2 increase which is easy to understand and intuitive.

1.6.2 Optimal profit compared with the pricing matching

Table 1.2 shows how manufacturer's profit changes comparing with the Pricing Matching strategy and the centralized system when the market size for the traditional channel changes, i.e. a_1 changes. Table 1.3 shows how manufacturer's profit changes comparing with the Pricing Matching strategy and the centralized system when the market size for the traditional channel changes, i.e. a_2 changes. Table 1.4 shows how manufacturer's profit changes comparing with the Pricing Matching strategy and the centralized system when b_{11} changes. Table 1.5 shows how manufacturer's profit changes comparing with the Pricing Matching strategy and the centralized system when b_{22} changes. Table 1.6 shows how manufacturer's profit

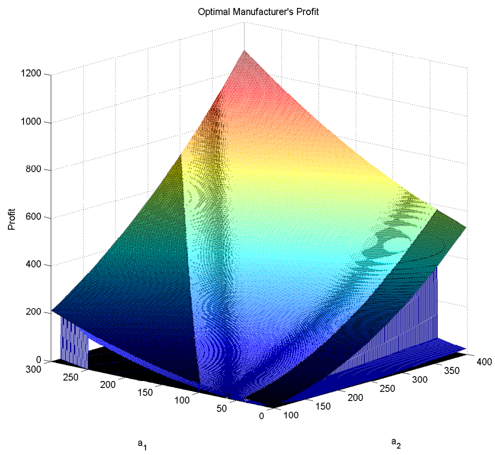

Figure 1.4: Manufacturer's profit

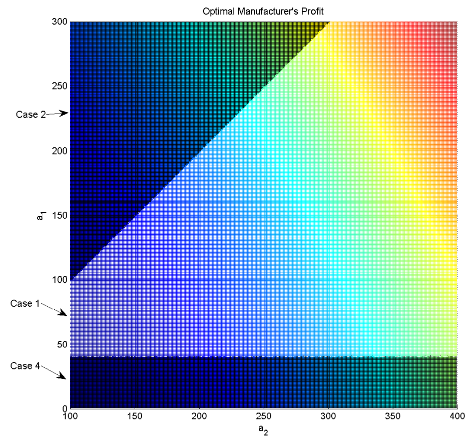

Figure 1.5: Manufacturer's profit

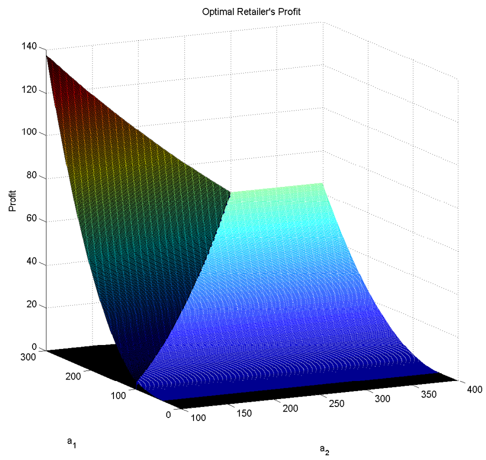

Figure 1.6: Retailer's profit

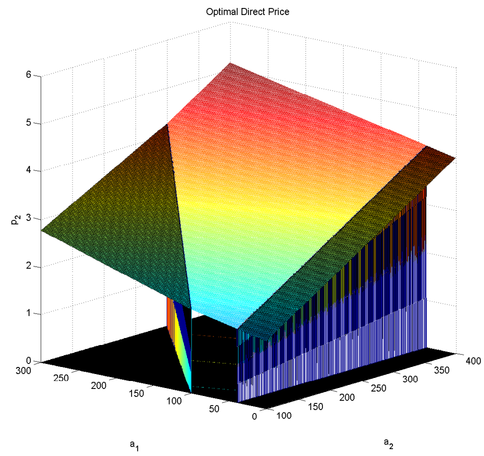

Figure 1.7: Direct price

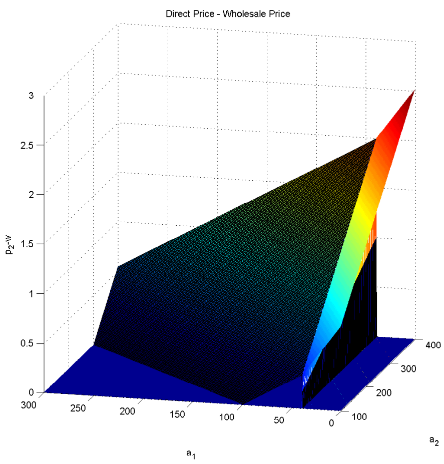

Figure 1.8: Direct price - Wholesale price

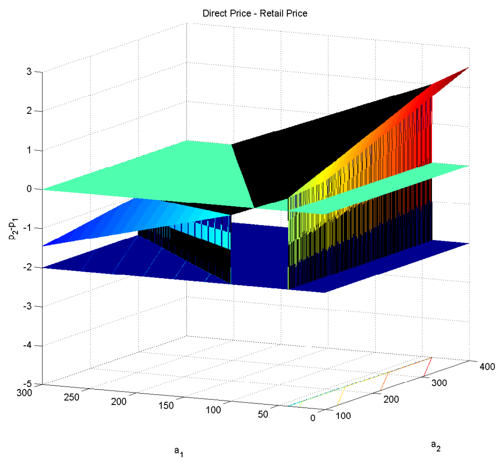

Figure 1.9: Direct price - Retail price

Table 1.2: Profits against a_1 (with $a_2 = 400, b_{11} = b_{22} = 65, b_{12} = b_{21} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
a_1	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
180	810.78	771.06	750.00	18.85	0.10	30.00	848.47	0.96
190	830.79	794.60	765.00	21.63	0.65	45.00	874.06	0.95
200	851.32	818.50	780.00	24.62	1.68	60.00	900.56	0.95
210	872.37	842.75	795.00	27.79	3.20	75.00	927.95	0.94
220	893.94	867.35	810.00	31.15	5.21	90.00	956.25	0.93
230	916.03	892.31	825.00	34.71	7.71	105.00	985.45	0.93
240	938.63	917.62	840.00	38.46	10.69	120.00	1015.56	0.92
250	961.75	943.29	855.00	42.40	14.16	135.00	1046.56	0.92
260	985.40	969.31	870.00	46.54	18.12	150.00	1078.47	0.91
270	1009.55	995.68	885.00	50.87	22.56	165.00	1111.28	0.91
280	1034.23	1022.41	900.00	55.38	27.49	180.00	1145.00	0.90
290	1059.43	1049.50	915.00	60.10	32.91	195.00	1179.62	0.90
300	1085.14	1076.93	930.00	65.00	38.81	210.00	1215.14	0.89
310	1111.37	1104.72	945.00	70.10	45.20	225.00	1251.56	0.89
320	1138.12	1132.87	960.00	75.38	52.08	240.00	1288.89	0.88
330	1165.39	1161.37	975.00	80.87	59.44	255.00	1327.12	0.88
340	1193.17	1190.22	990.00	86.54	67.29	270.00	1366.25	0.87
350	1221.48	1219.43	1005.00	92.40	75.63	285.00	1406.28	0.87
360	1250.30	1248.99	1020.00	98.46	84.45	300.00	1447.22	0.86
370	1279.64	1278.90	1035.00	104.71	93.76	315.00	1489.06	0.86

Table 1.3: Profits against a_2 (with $a_1 = 200, b_{11} = b_{22} = 65, b_{12} = b_{21} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
a_2	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
150	180.002	180.002	123.75	34.546	34.55	56.528	231.2847222	0.78
160	196.678	196.678	140	32.425	32.43	60	247.2222222	0.79
170	214.093	214.093	157.083	30.372	30.37	63.194	264.0625	0.81
180	232.247	232.247	175	28.386	28.39	66.111	281.8055556	0.82
190	251.139	251.139	193.75	26.467	26.47	68.75	300.4513889	0.84
200	270.769	270.769	213.333	24.615	24.62	71.111	320	0.85
210	291.221	291.139	233.75	24.615	22.83	73.194	340.4513889	0.86
220	312.575	312.247	255	24.615	21.11	75	361.8055556	0.86
230	334.832	334.093	277.083	24.615	19.46	76.528	384.0625	0.87
240	357.991	356.678	300	24.615	17.88	77.778	407.2222222	0.88
250	382.054	380.002	323.75	24.615	16.36	78.75	431.2847222	0.88
260	407.019	404.065	348.333	24.615	14.91	79.444	456.25	0.89
270	432.887	428.866	373.75	24.615	13.53	79.861	482.1180556	0.90
280	459.658	454.406	400	24.615	12.22	80	508.8888889	0.90
290	487.332	480.684	427.083	24.615	10.97	79.861	536.5625	0.91
300	515.908	507.701	455	24.615	9.791	79.444	565.1388889	0.91
310	545.387	535.457	483.75	24.615	8.678	78.75	594.6180556	0.92
320	575.769	563.951	513.333	24.615	7.632	77.778	625	0.92
330	607.054	593.184	543.75	24.615	6.653	76.528	656.2847222	0.92
340	639.241	623.156	575	24.615	5.741	75	688.4722222	0.93
350	672.332	653.866	607.083	24.615	4.897	73.194	721.5625	0.93

Table 1.4: Profits against b_{11} (with $a_1 = a_2 = 600, b_{22} = 65, b_{12} = b_{21} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
b_{11}	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
26	8032.5	8032.5	0	3290.6	3291	0	11325.74296	0.71
31	6618.04	6618.04	0	2193.9	2194	0	8864.73741	0.75
36	5684.93	5684.93	0	1539.9	1540	0	7336.948251	0.77
41	5016.26	5016.26	0	1117.8	1118	0	6296.352941	0.80
46	4508.91	4508.91	5050.36	830.5	830.5	58.414	5542.099894	0.81
51	4107.64	4107.64	3930.97	627.34	627.3	665.1	4970.403346	0.83
56	3780.18	3780.18	3264.64	479.57	479.6	886.14	4522.227612	0.84
61	3506.38	3506.38	2843.83	369.73	369.7	917.74	4161.51497	0.84
66	3273.18	3273	2566.16	295.91	286.7	853.24	3864.997613	0.85
71	3076.64	3070.93	2376.37	270.17	223	740.62	3616.988722	0.85
76	2910.8	2893.72	2242.61	247.86	173.8	605.79	3406.52752	0.85
81	2769.03	2736.64	2145.62	228.35	135.3	463.15	3225.724138	0.86
86	2646.49	2596.12	2073.32	211.13	105.1	320.78	3068.754532	0.86
91	2539.53	2469.46	2017.88	195.85	81.23	183.14	2931.225898	0.87
96	2445.39	2354.51	1974.13	182.19	62.37	52.615	2809.762467	0.87
101	2361.91	2249.58	1959.56	169.91	47.44	52.615	2701.727273	0.87
106	2287.39	2153.31	1945.95	158.82	35.63	52.615	2605.030527	0.88
111	2220.48	2064.58	1918.12	148.76	26.32	52.615	2517.994689	0.88
116	2160.08	1982.48	1880.86	139.59	19.03	52.615	2439.257592	0.88
121	2105.29	1906.23	1837.5	131.21	13.36	52.615	2367.701657	0.90
126	2055.38	1835.18	1790.35	123.51	9.026	52.615	2302.401355	0.89
131	2009.72	1768.79	1741.05	116.43	5.769	52.615	2242.58365	0.90
136	1967.82	1706.58	1690.72	109.89	3.397	52.615	2187.597839	0.90
141	1929.22	1648.15	1640.19	103.84	1.753	52.615	2136.892272	0.90
146	1893.56	1593.14	1590.01	98.22	0.706	52.615	2089.996193	0.91
151	1860.52	1541.24	1540.59	92.995	0.151	52.615	2046.505441	0.91
156	1829.82	0	1492.19	88.125	0	52.615	2006.071072	0.91
161	1801.23	0	1444.98	83.578	0	52.615	1968.390244	0.92
166	1774.55	0	1399.09	79.323	0	52.615	1933.198844	0.92
171	1749.6	0	1354.59	75.335	0	52.615	1900.265491	0.92

Table 1.5: Profits against b_{22} (with $a_1 = a_2 = 600, b_{11} = 65, b_{12} = b_{21} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
b_{22}	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
26	10722.67	0.00	8190.00	301.54	0.00	0.00	11325.74	0.95
31	8261.66	0.00	7140.00	301.54	0.00	0.00	8864.74	0.93
36	6733.87	6163.55	6090.00	301.54	11.70	0.00	7336.95	0.92
41	5693.28	5402.47	5040.00	301.54	51.05	0.00	6296.35	0.90
46	4939.02	4797.96	5040.00	301.54	102.11	0.00	5542.10	0.89
51	4367.33	4306.26	3367.70	301.54	156.59	544.65	4970.40	0.88
56	3919.15	3898.55	2981.17	301.54	210.60	797.11	4522.23	0.87
61	3558.44	3555.05	2739.08	301.54	262.32	879.49	4161.51	0.86
66	3261.74	3261.74	2588.92	311.02	311.02	861.83	3865.00	0.84
71	3008.40	3008.40	2498.85	356.44	356.44	785.05	3616.99	0.83
76	2787.42	2787.42	2448.88	398.62	398.62	673.77	3406.53	0.82
81	2592.98	2592.98	2426.06	437.72	437.72	543.16	3225.72	0.80
86	2420.61	2420.61	2421.81	473.94	473.94	402.69	3068.75	0.79
91	2266.77	2266.77	2430.32	507.51	507.51	258.37	2931.23	0.77
96	2128.65	2128.65	2447.60	538.65	538.65	113.96	2809.76	0.76
101	2003.96	2003.96	2447.60	567.59	567.59	113.96	2701.73	0.74
106	1890.87	1890.87	2447.60	594.53	594.53	113.96	2605.03	0.73
111	1787.83	1787.83	2447.60	619.64	619.64	113.96	2517.99	0.71
116	1693.57	1693.57	2447.60	643.09	643.09	113.96	2439.26	0.69
121	1607.04	1607.04	2447.60	665.03	665.03	113.96	2367.70	0.68
126	1527.33	1527.33	2447.60	685.59	685.59	113.96	2302.40	0.66
131	1453.68	1453.68	2447.60	704.89	704.89	113.96	2242.58	0.65
136	1385.42	1385.42	2447.60	723.04	723.04	113.96	2187.60	0.63
141	1322.00	1322.00	2447.60	740.13	740.13	113.96	2136.89	0.62
146	1262.94	1262.94	2447.60	756.25	756.25	113.96	2090.00	0.60
151	1207.79	1207.79	2447.60	771.48	771.48	113.96	2046.51	0.59
156	1156.20	1156.20	2447.60	785.89	785.89	113.96	2006.07	0.58
161	1107.84	1107.84	2447.60	799.54	799.54	113.96	1968.39	0.56
166	1062.42	1062.42	2447.60	812.49	812.49	113.96	1933.20	0.55
171	1019.69	1019.69	2447.60	824.79	824.79	113.96	1900.27	0.54

Table 1.6: Profits against b_{12} (with $a_1 = a_2 = 600, b_{11} = b_{22} = 65, b_{21} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
b_{12}	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
0	2549.96	2545.93	2027.89	150.06	123.60	210.15	2855.83	0.89
3	2624.55	2621.36	2061.08	163.11	137.98	292.77	2955.68	0.89
6	2703.13	2700.69	2099.85	177.26	153.84	377.35	3061.67	0.88
9	2786.01	2784.24	2145.62	192.62	171.34	463.15	3174.39	0.88
12	2873.54	2872.33	2200.20	209.33	190.66	549.12	3294.47	0.87
15	2966.10	2965.36	2265.93	227.53	212.03	633.73	3422.65	0.87
18	3064.12	3063.75	2345.89	247.41	235.70	714.85	3559.75	0.86
21	3168.09	3167.97	2444.15	269.18	261.94	789.44	3706.74	0.85
24	3278.56	3278.55	2566.16	293.06	291.10	853.24	3864.70	0.85
27	3396.09	3396.09	2719.39	323.57	323.57	900.11	4034.89	0.84
30	3521.26	3521.26	2914.16	359.79	359.79	921.23	4218.78	0.83
33	3654.84	3654.84	3165.09	400.31	400.31	903.67	4418.09	0.83
36	3797.68	3797.68	3493.43	445.74	445.74	828.15	4634.80	0.82
39	3950.80	3950.80	3930.97	496.84	496.84	665.10	4871.30	0.81
42	4115.32	4115.32	4527.09	554.50	554.50	367.67	5130.40	0.80
45	4292.58	4292.58	4527.09	619.79	619.79	367.67	5415.50	0.79
48	4484.10	4484.10	4527.09	694.00	694.00	367.67	5730.69	0.78
51	4691.68	4691.68	4527.09	778.73	778.73	367.67	6080.99	0.77
54	4917.41	4917.41	4527.09	875.90	875.90	367.67	6472.58	0.76
57	5163.78	5163.78	4527.09	987.91	987.91	367.67	6913.21	0.75

changes comparing with the Pricing Matching strategy and the centralized system when b_{12} changes. Table 1.7 shows how manufacturer's profit changes comparing with the Pricing Matching strategy and the centralized system when b_{21} changes.

1.7 Conclusions and future research

In this paper, we have modeled a dual-channel problem with only one manufacturer and one retailer considered. We have solved the manufacturer's pricing problem as well as the retailer's pricing problem. Our results show that it is optimal for the manufacturer to operate dual channels under some conditions, while it is optimal for the manufacturer to sell its products only through one single channel, either direct or traditional channel only, under some circumstances.

Table 1.7: Profits against b_{21} (with $a_1 = a_2 = 600, b_{11} = b_{22} = 65, b_{12} = 25, c = 1$)

Value	Manufacturer's Profits			Retailer's Profits			Centralized	%
b_{21}	Our Model	Equal Pricing	Price Matching	Our Model	Equal Pricing	Price Matching	Company's Profit	Opt/ Cent
0	1953.60	1953.60	2427.81	521.53	521.53	287.34	2855.83	0.68
3	2081.55	2081.55	2422.64	497.33	497.33	373.99	2955.68	0.70
6	2217.63	2217.63	2421.70	472.51	472.51	459.62	3061.67	0.72
9	2362.50	2362.50	2426.06	447.07	447.07	543.16	3174.39	0.74
12	2516.88	2516.88	2437.05	421.01	421.01	623.22	3294.47	0.76
15	2681.54	2681.54	2456.36	394.35	394.35	697.96	3422.65	0.78
18	2857.38	2857.38	2486.14	367.12	367.12	764.96	3559.75	0.80
21	3045.39	3045.39	2529.14	339.34	339.34	821.02	3706.74	0.82
24	3246.67	3246.67	2588.92	311.06	311.06	861.83	3864.70	0.84
27	3462.50	3462.47	2670.11	286.15	282.36	881.66	4034.89	0.86
30	3694.38	3694.20	2778.84	261.97	253.34	872.73	4218.78	0.88
33	3943.88	3943.46	2923.28	236.54	224.10	824.38	4418.09	0.89
36	4212.78	4212.06	3114.50	210.05	194.83	721.90	4634.80	0.91
39	4503.12	4502.09	3367.70	182.69	165.74	544.65	4871.30	0.92
42	4817.27	4815.92	3704.12	154.76	137.10	263.23	5130.40	0.94
45	5157.94	5156.32	4200.00	126.67	109.29	0.00	5415.50	0.95
48	5528.31	5526.48	4830.00	98.93	82.79	0.00	5730.69	0.96
51	5932.07	5930.13	5460.00	72.26	58.22	0.00	6080.99	0.98
54	6373.59	6371.66	6090.00	47.62	36.40	0.00	6472.58	0.98
57	6858.04	6856.25	6720.00	26.26	18.41	0.00	6913.21	0.99

Different channel settings for the manufacturer corresponds to different pricing decisions. There is always a feasible area for the pricing decisions made by the manufacturer and the retailer which consists of several boundaries. Our theoretical results show that the optimal prices may fall on the boundaries under some conditions, while there are interior solutions as well depending on the parameters of the model. Our numerical results have also illustrated that the manufacturer optimizes its profit under different channel setting and different pricing decisions with different values of the parameters of our model.

Compared with other models used in the literature, our model is more general and thus can explain most of the results obtained by other researchers. For example Cattani et al.(2006) study a dual channel problem where a manufacturer with a traditional channel partner opens up a direct channel in competition with the traditional channel, whose results show that equal-pricing strategy is optimal for the manufacturer as long as the direct channel is significantly less convenient than the traditional channel. Such equal pricing strategy can also been seen in our model under specific value of the parameters. Note that in Cattani's paper, equal-pricing is optimal for the manufacturer ($p_d = w$), while in our paper, the direct price p_d can be greater than wholesale price w, i.e. $p_d > w$.

In addition, we have also discussed some interesting properties that have not been examined in the literature. For example, we have showed the relationship between the prices and the direct channel's demand n. Our results show that the prices are linear with the direct channel's demand n, while the manufacturer's profit is concave over n and the retailer's profit is convex over n. There exists an unique optimal n^* that can optimize the manufacturer's profits. We have also obtained some structure results based on n.

There are quite a few directions that can be extended to based on our models in this paper. One possible direction would be to consider multiple retailers competing with one manufacturer and multiple retailers competing with multiple manufacturers.

Chapter 2

Stochastic Case

2.1 Introduction

Companies use the Internet as a new avenue to directly sell products to their customers. While the Internet provides an opportunity to increase sales by attracting more customers, it could also be a threat to the existing, traditional channel. The problem of introducing a new direct channel to customers so that the overall sales of a company is increased is called *the dual-channel problem*. In this paper, we study the dual-channel problem in the manufacturer industry.

Dual-channel distribution systems are widely used in various industries. Manufacturers like Sony Ellectronics, Apple Computers, Dell, etc. sell products to the consumers through independent retailers like Best Buy, Circuit City, etc. as well as through their respective e-commerce web-site (direct channel). The sales volume from the direct channel can be significant, especially when companies like Dell or Apple are well-known to most customers and internet is accessible for more and more consumers. More and more customers tend to buy their products from their web-site not from the traditional store. For a company that operates two distribution channels, the first decision to make would be the "pricing" decision. That is, what prices would be optimal for them to sell products through the two channels?

Except for the "pricing" decision, inventory decision is another decision facing the companies that operates two distribution channels. Inventory competition between a manufacturer and its channel partner is inevitable under dual channel scenario. How does the manufacturer allocate inventory to the competing channel members? Does a manufacturer always favor its own channel? Some manufacturers, like Dell and Apple maintain web-sites that can accept customer orders while selling through retailers. For these companies, they need to decide how many should be allocated to each channel.

Our motivation for this research came from our literature review of recent research on dual channel problems. Balasubramanian (1998) modeled "the competition in the multiple-channel environment from a strategic viewpoint" and marked "the early attempt to analyze this issue" (direct versus retail competition). After Balasubramanian's early move on researching this multiple-channel problem, a lot of papers regarding this area have been published. Most of them are dealing with the "pricing" problem and the effects of direct marketing on the manufacturer and the retailer (Chiang et al. 3003; Viswanathan 2005; Swaminathan et al., 2006 and 2009).

Aside from the "pricing" strategies for the manufacturer, the allocation problem (Allocation here means the number of units allocated to the direct channel.) is also important for the manufacturer. However, only a few papers address the pricing and allocation problem at the same time (Tsay and Agrawal, 2004b; Mallik et al. 2006; Yao et al. 2009.).

In the manufacturing industry, more and more manufacturers selling through retailers as well as its web-site. In this paper, we try to solve such dual-channel problem in the manufacturing industry with one manufacturer and one retailer considered. Our analysis characterizes the equilibrium of the Stackelberg game where the manufacturer, as the leader in the game, knows the pricing decision taken by the retailer and decides its wholesale price to the retailer and direct price for the direct channel. The demand we consider is stochastic.

Our work contributes to the operations management literature by attempting to solve the manufacturer's pricing problem and the retailer's pricing problem under stochastic demand case. We are also trying to obtain the optimal inventory level for the manufacturer and optimal order quantity for the retailer, which has not been solve in the literature under such a general model like ours. Our results so far show that the prices (wholesale price w and direct price p_d) are linear decreasing with n. We are trying to obtain the optimal inventory level, which is n^*, and optimal order quantity, which is z_0^* for the manufacturer and the retailer.

The remainder of this paper is organized as follows. The next section provides a review of the related literature. Section 2.3 presents problem analysis, assumptions and our model as well as our main results. Section 2.4 provides some numerical study, while section 2.5 summarizes and concludes the paper.

2.2 Literature review

Our work relates to two streams of literature in operations management: channel conflict and capacity allocation. We provide a brief review of the literature for each of these two areas. As for channel conflict, there are quite a lot of papers that are closely related to our work.

Multi-channel problem has been extensively researched in the literature. Some of them focus on the pricing problem with competition, while some of them focus on demand forecasting and mixed-channel strategy with value-adding retailer.

Balasubramanian (1998) analyzed the competition between direct marketers and conventional retailers through using the spatial setting of the circular market, which considered the role of information as a strategic lever in the multiple-channel market. Direct sellers can regulate the level of consumer information and control the competitive flavor of the market. Tsay et al. (1999) and Frazier (1999) survey channel structure and incentive design for performance enhancement, but not channel conflict. Rhee and Park (2000) study a hybrid channel design problem, assuming that there are two consumer segments: a price sensitive segment and a service sensitive segment. Chiang et al. (2003) examine a price-competition game in a dual channel supply chain. Their results show that a direct channel strategy makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer, even though the equilibrium sales volume in the direct channel is zero. Their results however depend on the assumption that customer's acceptance of online channel is homogeneous.

Boyaci (2004) studies stocking decisions for both the manufacturer and retailer and assumes that all the prices are exogenous and demand is stochastic. Tsay and Agrawal (2004) provide an excellent review of recent work in the area and examine different ways to adjust the manufacturer-retailer relationship. Viswanathan (2005) studies the competition across online, traditional and hybrid channels using a variant of circular city model. His focus is on understanding the impact of differences in channel flexibility, network externalities, and switching costs. Cattani et al.(2006) study coordination of pricing on Internet and traditional channels by modeling micro-level consumer behavior for demand generation. In their model, customers are at a random physical distance from traditional retailers, and at a random virtual distance from the direct marketer, independent of the physical distance. The market then is segmented according to the utility each customer attains from either the direct channel or the traditional channel. Customers are not excluded from a specific market; thus both markets have a chance to compete for all customers. Aussadavut et al.(2006) studied a dual channel supply chain in which a manufacturer sells to a retailer as well as to consumers directly. Consumers choose the purchase channel based on price and service qualities. The manufacturer decides the price of the direct channel and the retailer decides both price and order quantity. They developed conditions under which manufacturer the manufacturer and the retailer share the market in equilibrium. They also showed that the difference in marginal costs of the two channels plays an important role in determining the existence of dual channels in equilibrium.

Another two related papers are published in 2009 by Swaminathan et al.(2009) and Hu et al. (2009). Swaminathan (2009) studied the optimal pricing strategies when a product is sold on two channels. They provided theoretical bounds for the four prevalent pricing strategies proposed in the paper. Hu et al. (2009) discussed the revenue management for a service supply chain with two streams of customers, with the supplier having limited capacity of a perishable product. Monotone properties for the revenue functions and pricing strategies have been derived in this paper.

Another stream of literature that relates to our work is capacity allocation. Cachon and Lariviere (1999b) consider a single supplier with limited capacity selling to several retailers who are privately informed of their optimal stocking levels. They find that supply chain might be better off under an allocation mechanism that induces retailers to inflate orders. Deshpande and Schwarts (2002) consider a generalization of the above model using both pricing and allocation mechanisms. Geng and Mallik (2007) consider a supply chain involving one manufacturer and one independent retailer. The manufacturer distributes her product to the end consumer through the retailer as well as through her direct channel. Each of the two channels faces a stochastic demand. They establish the necessary condition for a manufacturer to undercut a retailer's order and show that a manufacturer may deny the retailer of inventory even when the capacity is ample. Yao et al. (2009) study the strategic inventory deployment for retail and e-tail stores. They also consider a supply chain consisting of one manufacturer and one retailer. Customers can purchase either from the retailer or directly from the manufacturer via an e-tail channel. They study three different inventory strategies, namely centralized inventory strategy, a Stackelberg inventory strategy, and a strategy where the e-tail operation is out sourced to a third party logistics provider. Optimal inventory levels in retail and e-tail stores and the respective expected profits have been obtained.

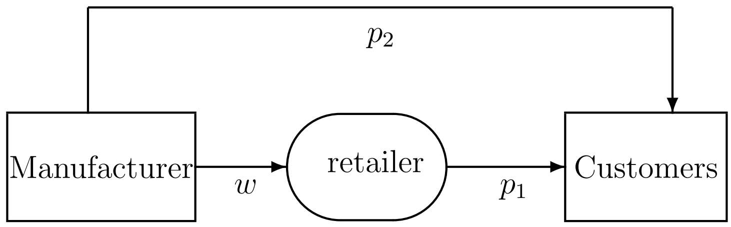

Figure 2.1: **Dual channels stochastic demand**. A manufacturer sells its products to customers through an retailer and through a direct channel. For each unit of product sold through the retailer, the retailer charges the customer a price p_1 and pays the manufacturer a wholesale price $w \leq p_1$. For each unit of product that is sold directly from the manufacturer, the manufacturer charges the customer a direct price p_2.

Our model differs from the prior studies in the following areas: (i) We focus on a general demand model to model the pricing problem for manufacturer and the retailer. (ii) We study the optimal pricing decisions of the manufacturer and the retailer under stochastic demand. In addition, we also try to obtain the optimal inventory level for the manufacturer and optimal order quantity for the retailer.

2.3 Problem formulation

We introduce the assumptions and the model in this section. We divide the problem into two sub-problems: the manufacturer's problem and the retailer's problem. Assuming that the manufacturer and the retailer are playing a Stackelberg game with the manufacturer being the leader and the retailer being the follower. We solve the problem backwards. That is, we solve the retailer's problem first, after which the manufacturer's problem is solved after obtaining the retailer's optimal response.
Figure 2.1 shows the dual channels of the manufacturer.
The problem can be further divided into two sub-problems: the retailer's problem and the manufacturer's problem. Below, we first describe the retailer's problem and its modeling. Then after that, we will describe the manufacturer's problem and its modeling. In the following section, which is section 2.4, we will focus on the

Table 2.1: Notations for stochastic case

a_i	base demand for channel i ($i = 1, 2$)
b_{ij}	price sensitivity coefficients ($i, j = 1, 2$)
c	manufacturing cost for one unit of product
p_i	retail price for channel i ($i = 1, 2$)
$p_1(p_2, w)$	retailer's pricing decision as functions of p_2 and w
w	wholesale price for one unit of product
D_i	demand function for channel i ($i = 1, 2$)
Π_m	manufacturer's profit
Π_r	retailer's profit
N	the manufacturer's inventory level
q_1	the retailer's order quantity
f_1	the density function for the traditional channel's demand
F_1	the cumulated function for the traditional channel's demand
f_2	the density function for the direct channel's demand
F_2	the cumulated function for the direct channel's demand

solutions analysis and numerical study. We list the notations used in this paper as below. where in above table, $i = 1$ represents the traditional channel and $i = 2$ represents direct channel, respectively. Notice that in the above notations, a_i, b_{ij} and c are all non-negative.

2.3.1 Assumptions and modeling

Demand functions are modeled as below. D_1 and D_2 denote the demand for traditional channel and direct channel respectively.

$$D_1 = a_1 - b_{11}p_1 + b_{12}p_2 + \epsilon_1 \tag{2.1}$$

and similarly, D_2 can be expressed as

$$D_2 = a_2 - b_{22}p_2 + b_{21}p_1 + \epsilon_2 \tag{2.2}$$

Assumption 2.1. *: we assume that p_1, p_2, and w are all non-negative and bounded. Let $P_1 = \{0 \le p_1 \le \bar{p}_1\}, P_2 = \{0 \le p_2 \le \bar{p}_2\}, W = \{0 \le w \le \bar{w}\}$, then P_1, R_d, W*

denote the price ranges. Note that we define $\bar{w} = \min\{\bar{p}_1, \bar{p}_2\}$.

Assumption 2.2. *: the price and cross-price sensitivity parameters have some relationships that are viewed as common constraints in the literatures.*

$$b_{ii} \geq b_{ij}, \quad where \quad i, j = 1, 2 (i \neq j). \tag{2.3}$$

2.3.2 Retailer's problem formulation and solutions

In this section, we are going to discuss the problem when the demand is stochastic. We define the Demand function for the retailer as: $D_1(p_1, p_2, \epsilon_1) = y_1(p_1, p_2) + \epsilon_1$. Alternatively, we define the demand function for the manufacturer from direct channel as : $D_2(p_1, p_2, \epsilon_2) = y_2(p_1, p_2) + \epsilon_2$. Specifically, y_1 and y_2 are defined as $y_1(p_1, p_2) = a_1 - b_{11}p_1 + b_{12}p_2$ and $y_2(p_1, p_2) = a_2 - b_{22}p_2 + b_{21}p_1$. We define q_1 as the order quantity for the retailer. We assume that there is no salvage value for the unsold rooms and the shortage cost incurred for the retailer is s. Also, we use $f_1(\cdot)$ and $F_1(\cdot)$ to denote the density function and cumulative distribution function of ϵ_1, while $f_2(\cdot)$ and $F_2(\cdot)$ are used to represent the density function and cumulative distribution function of ϵ_2.

Then the profit function for the retailer is as below:

$$\Pi_r(q_1, p_1) = \begin{cases} p_1 D_1(p_1, p_2, \epsilon_1) - wq_1, & D_1(p_1, p_2, \epsilon_1) \leq q_1, \\ p_1 q_1 - wq_1 - s[D_1(p_1, p_2, \epsilon_1) - q_1], & D(p_1, p_2, \epsilon_1) > q_1 \end{cases}$$

In order to make it more convenient to solve, we can change the expression by defining $z_1 = q_1 - y_1(p_1, p_2)$ and substituting $D_1(p_1, p_2, \epsilon_1) = y_1(p_1, p_2) + \epsilon_1$ into the objective function of the above. (Ernst(1970), Thowsen(1975) and Data et al.(1997)).

$$\Pi_r(z_1, p_1) = \begin{cases} p_1[y_1(p_1,p_2)+\epsilon_1] - w[y_1(p_1,p_2)+z_1], & \epsilon_1 \leq z_1, \\ p_1[y_1(p_1,p_2)+z_1] - w[y_1(p_1,p_2)+z_1] - s[\epsilon_1 - z_1], & \epsilon > z_1 \end{cases}$$

This transformation of variables provides an alternative interpretation of the stocking decisions: if the choice of z_1 is larger than the realized value of ϵ_1, then leftovers occur; if the choice of z_1 is smaller than the realized value of ϵ_1, then the shortages occur. However, leftovers here have no value and thus not formulated. The corresponding optimal stocking and pricing policy is to stock $q_1^* = y_1(p_1^*) + z_1$ units to sell at the unit price p_1^*, where z_1^* and p_1^* maximize expected profit. See Data et al. (1997).

The retailer's expected profit is:

$$\begin{aligned} E[\Pi_r(z_1, p_1)] &= \int_A^{z_1} (p_1[y_1(p_1,p_2)+u])f_1(u)du \\ &\quad + \int_{z_1}^{B} (p_1[y_1(p_1,p_2)+z_1] - s[u-z_1])f_1(u)du \\ &\quad - w[y_1(p_1,p_2)+z_1]. \end{aligned} \tag{2.4}$$

Defining $\Lambda(z_1) = \int_A^{z_1}(z_1 - u)f_1(u)du$ and $\Theta(z_1) = \int_{z_1}^{B}(u - z_1)f_1(u)du$,
we can write:

$$E[\Pi_r(z_1, p_1)] = \psi(p_1) - L(z_1, p_1), \tag{2.5}$$

where $\psi(p_1) = (p_1 - w)[y_1(p_1,p_2) + \mu_1]$, and $L(z_1, p_1) = w\Lambda(z_1) + (p_1 + s - w)\Theta(z_1)$.
The objective is to maximize the retailer's expected profit:

$$\max_{(z_1,p_1)\in(\infty,P_1)} \left\{ E[\Pi_r(z_1, p_1)] \right\} \tag{2.6}$$

We can get the first and second partial derivatives of $E[\Pi_r(z_1, p_1)]$ taken with respect to z_1 and p_1:

$$\frac{\partial E[\Pi_r(z_1,p_1)]}{\partial z_1} = -w + (p_1 + s)[1 - F_1(z_1)], \tag{2.7}$$

$$\frac{\partial^2 E[\Pi_r(z_1,p_1)]}{\partial z_1^2} = -(p_1 + s) f_1(z_1), \tag{2.8}$$

$$\frac{\partial E[\Pi_r(z_1,p_1)]}{\partial p_1} = 2b_{11}(p^0 - p_1) - \Theta(z_1), \tag{2.9}$$

$$\frac{\partial^2 E[\Pi_r(z_1,p_1)]}{\partial p_1^2} = -2b_{11}, \tag{2.10}$$

where $p^0 = \frac{a_1+b_{11}w+\mu_1+b_{12}p_2}{2b_{11}}$. The term p^0 denotes the optimal risk-less price, which is the price that maximizes $\Pi(p_1)$.

Lemma 2.1. *For a fixed z_1, the optimal price is determined uniquely as a function of z_1: $p_1^* \equiv p_1(z_1) = p^0 - \frac{\Theta(z_1)}{2b_{11}}$*

Then we can solve for optimal z_1 by substituting $p_1^* = p_1(z_1)$ into the profit function, and the optimization problem becomes a maximization over the single variable z_1: $\max_{z_1} E[\Pi_r(z_1, p_1(z_1))]$.
From the first derivative of profit function $E[\Pi_r(z_1, p_1(z_1))]$ over z_1, we can get the below equation.

$$\begin{aligned}\frac{\partial E[\Pi_r(z_1,p_1)]}{\partial z_1} &= -w + (p^0 - \frac{\Theta(z_1)}{2b_{11}} + s)[1 - F_1(z_1)] \\ &= -w + (\frac{a_1 + b_{11}w + \mu_1 + b_{12}p_2}{2b_{11}} - \frac{\Theta(z_1)}{2b_{11}} + s)[1 - F_1(z_1)] \\ &= -\frac{1 + F_1(z_1)}{2} w + \frac{b_{12}}{2b_{11}}[1 - F_1(z_1)]p_2 \\ &\quad + [\frac{a_1 + \mu_1 - \Theta(z_1)}{2b_{11}} + s][1 - F_1(z_1)]\end{aligned} \tag{2.11}$$

Set the above equation into zero, we can solve for the optimal z_1. However, similar to what was introduced by Nicholas C. Petruzzi and Maqbool Dada 1999, demonstrates, $E[\Pi_r(z_1, p_1(z_1))]$ might have multiple points that satisfy the first-order optimality condition, depending on the parameters of the problem. See *Data (1997)* for details. Thus we have the following Theorem 2.

Theorem 2.1. *: Given the manufacturer's direct price* p_2 *and wholesale price* w*, the single-period optimal stocking and pricing policy for the retailer is to stock* $q_1^* = y_1(p_1^*) + z_1^*$ *units and sell at the unit price* p_1^**, where* p_1^* *is specified by Lemma 1 and* z_1^* *is determined according to the following:*

1. *If* $F_1(\cdot)$ *is an arbitrary distribution function, then search exhaustively over all values of* z_1 *in the region* $[A, B]$ *will determine* z_1^**.*

2. *If* $F_1(\cdot)$ *is a distribution function satisfying the condition* $2r_1(z_1)^2 + cr_1(z_1) > 0$ *for* $A \leq z_1 \leq B$*, where* $r_1(\cdot) \equiv \frac{f_1(\cdot)}{1-F_1(\cdot)}$ *is the hazard rate, then* z_1^* *is the largest* z_1 *in the region* $[A, B]$ *that satisfies* $\frac{dE[\Pi_r(z_1,p_1(z_1))]}{dz_1} = 0$*.*

3. *If the condition in (2) is satisfied, and* $a_1 - b_1(c_1 - 2s) + A > 0$*, then* z_1^* *is the unique* z_1 *in the region* $[A, B]$ *that satisfies* $\frac{dE[\Pi_r(z_1,p_1(z_1))]}{dz_1} = 0$*.*

Proof. *See the appendix B.*

2.3.3 Manufacturer's problem formulation

In this section, we are going to solve the manufacturer's problem. In the above section, we have assumed that p_2 and w are fixed and known to the retailer, based on which the retailer solves for its optimal ordering quantity q_1^* and set its optimal price p_1^*. Here we use their value to solve the manufacturer's problem and decide the optimal price p_2 and inventory capacity N.

The manufacturer's capacity is fixed and defined as N. The manufacturer first determines its optimal inventory capacity N^*. After that, the manufacturer determines its optimal direct channel price p_2^* for direct channel. However, we know that the inventory capacity for the manufacturer equals to the total inventory allocated to the retail channel and direct channel. Assuming that he manufacturer allocates q_2 number of units to the direct channel, while selling number of q_1 units to the retailer, then we have $N = q_1 + q_2$. Thus, after obtaining total inventory capacity N^*, we can obtain the optimal inventory allocation q_2^* allocated to the direct channel using $q_2^* = N^* - q_1^*$.

The manufacturer's profit consists of two parts: the profits from the retailer, and the profits obtained through selling products directly to the customers. We can obtain manufacturer's profit as below:

$$\Pi_m(N, p_2) = \begin{cases} p_2 D_2(p_2, \epsilon_2) - cq_2 + (w - c)q_1(p_2), & D_2(p_2, \epsilon_2) \leq q_2, \\ p_2 q_2 - cq_2 + (w - c)q_1(p_2), & D_2(p_2, \epsilon_2) > q_2 \end{cases}$$

where $q_2 = N - q_1(p_2)$. Notice that in the above, we use $q_1(p_2)$ to represent retailer's optimal order quantity obtained in Theorem 1.1 given manufacturer's direct price p_2. Then we can get the expected total profit for the manufacturer as below:

$$E[\Pi_m(p_2, w)] = -Nc + wq_1 + p_2 E[\min\{q_2, D_2\}] \tag{2.12}$$

Where,

$$\begin{aligned} E[\min\{q_2, D_2\}] &= \int_0^\infty (\min\{q_2, D_2\}) f_2(u) du \\ &= \int_0^{q_2} D_2 f_2(u) du + \int_{q_2}^{B} q_2 f_2(u) du \\ &= y_2(p_2) \int_0^{q_2} f_2(u) du + q_2[1 - F_2(q_2)] + \int_0^{q_2} u f_2(u) du \\ &= (a_2 - b_{22} p_2 + b_{21} p_1) F_2(q_2) + q_2[1 - F_2(q_2)] + \int_0^{q_2} u f_2(u) du \end{aligned} \tag{2.13}$$

The objective function thus can be transformed into:

$$
\begin{aligned}
E[\Pi_m(p_2, w)] &= -Nc + wq_1 + p_2E[\min\{q_2, D_2\}] \\
&= (w-c)q_1 - cq_2 + p_2E[\min\{q_2, D_2\}] \\
&= -Nc + wq_1 + p_2(a_2 - b_{22}p_2 + b_{21}p_1)F_2(q_2) + p_2q_2[1 - F_2(q_2)] \\
&\quad + p_2\int_0^{q_2} uf_2(u)du \qquad (2.14)
\end{aligned}
$$

From the discussion of retailer's problem, we know that the objective function of retailer's profit $E[\Pi_r(z_1, p_1)]$ satisfies first order condition over z_1. Thus, we can obtain the following corollary.

Corollary 2.1. *From the discussion of retailer's problem, given z_1 as the optimal solution to the retailer's problem, then we can obtain*

$$
w = \frac{b_{12}[1 - F_1(z_1)]}{b_{11}[1 + F_1(z_1)]}p_2 + \frac{[1 - F_1(z_1)]}{b_{11}[1 + F_1(z_1)]}[a_1 + \mu_1 - \Theta(z_1)] \qquad (2.15)
$$

From Lemma 2.1 and 2.15, assuming $\mu_1 = 0$, that we can obtain $w = [1 - F_1(z_1)]p_1$. Plugging it back to $p_1(z_1)$ of Lemma1, we can obtain

$$
p_1 = \frac{a_1 + b_{12}p_2 - \Theta(z_1)}{b_{11}[1 + F_1(z_1)]} \qquad (2.16)
$$

2.4 Solutions for manufacturer's problem

In this section, we will analyze the dual channel problem and solve the manufacturer's problem. For the previous section, we have solved the retailer's pricing problem, which is p_1, given the manufacturer's pricing decisions, i.e. wholesale price w and direct price p_2. After that, in Theorem 2.1, we have proposed a solution to solve for the retailer's optimal order quantity given w and p_2.

Next, we solve the manufacturer's problem following the below procedure. There are two decision variables left in the manufacturer's problem, i.e. the direct price p_2 and manufacturer's inventory capacity N. Notice that in our paper, we use a trick to transform the problem and change these two decision variables into direct price p_2 and manufacturer's capacity level N according to Corollary 2.1. According to Corollary 2.1, we can use z_1 to represent wholesale price w. Plug w, obtained in Corollary 2.1, into the manufacturer's objective function, we can now consider z_1 as fixed and known and then solve for manufacturer's optimal capacity N^* and direct price p_2^*. We use sequential decision making procedure to solve for optimal N^* and p_2^* in this section.

First, we solve for the optimal N^* for the manufacturer given direct channel price p_2. After we obtain the optimal value N^* as a function of p_2, we then solve for the optimal direct price p_2^*.

2.4.1 Obtaining the optimal inventory capacity N^*

We have obtained the simplified objective function for the manufacturer as below.

$$\begin{aligned} E[\Pi_m(N,p_2)] &= -Nc + wq_1 + p_2E[\min\{q_2, D_2\}] \\ &= (w-c)q_1 - c(N-q_1) + p_2E[\min\{N-q_1, D_2\}] \qquad (2.17) \end{aligned}$$

Observing the above objective function, we can find out that the manufacturer's profit consists of two parts: the first part is the profit from selling through the traditional channel, which is $(w-c)q_1$; the second part of profit is from selling through the direct channel, which is $-c(N-q_1) + p_2E[\min\{N-q_1, D_2\}]$. We use $E\Pi_t$ and $E\Pi_d$ to denote the two parts of profits, respectively, as below.

$$E\Pi_t = (w-c)q_1 \qquad (2.18)$$

$$E\Pi_d = -(N - q_1)c + p_2 E[\min\{N - q_1, D_2\}] \tag{2.19}$$

It's easy to verify that $E\Pi_t$ is convex in p_2 given z_1. And $E\Pi_d$ is actually a joint decision News-vendor problem with decision variables of $N - q_1$ and p_2, which is very similar to the retailer's problem. We thus use sequential decision to obtain the optimal solutions.

We first obtain manufacturer's optimal capacity N^*, which is a function of p_2. After that, we plug N^* into the manufacturer's objective function to obtain optimal direct price p_2. This procedure to solve the problem is different from solving the retailer's problem. The method in solving the retailer's problem is introduced by Zabel (1970) and is used by Data (1998). The method of first solving for the optimal value of N^* as a function of p_2 and then substituting the result back to $E\Pi_d$ is introduced by Whitin (1955). Both sequential procedures yield the same results. However, in order to simplify the problem, we use the Whitin's method to solve the problem.

Given wholesale price w and direct price p_2, we can find that the profit generated from the first part is not affected by N. The second part of the profit is actually a News-vendor problem for the direct channel. We can then obtain the first order condition as below.

$$\frac{\partial E[\Pi_m(N, p_2)]}{\partial N} = -c + p_2 Pro(D_2 > N - q_1) \tag{2.20}$$

From the first order condition, we can obtain the optimal N^* as below.

Theorem 2.2. *Given p_2, the manufacturer's optimal inventory level N^* can be obtained as below*

$$N^* = F_2^{-1}\left(\frac{p_2 - c}{p_2}\right) + q_1 + y_2. \tag{2.21}$$

Notice that in the above, F_2^{-1} denotes the inverse cumulative distribution function of D_2, which is defined as $F_2(\cdot)$ in the beginning of this section.

2.4.2 Obtaining the optimal direct price p_2^*

we use Whitin's (1955) method obtain the optimal direct price p_2^* in this subsection. However, due to complexity of the problem, there is no closed form solution of p_2. Defining z_2 as $z_2 = N^* - (z_1 + y_1) - y_2$, then we can obtain optimal direct price p_2^* in the following proposition.

Proposition 2.1. *Given z_1, the optimal direct price p_2^* can be determined according to the following:*

1. *If $F_2(\cdot)$ is an arbitrary distribution function, then p_2^* can be obtained by exhaustively searching the region $[A, B]$.*
2. *If $F(\cdot)$ is a distribution function satisfying the condition $2r_2(z_2)^2 + \frac{dr_2(z_2)}{dz_2} > 0$ for $A \leq z_2 \leq B$, where $r_2(\cdot) = f_2(\cdot)/[1 - F_2(\cdot)]$ is hazard rate, then p_2^* is the largest p_2 in the region $[A, B]$ that satisfies $\partial E[\Pi_m(p_2)]/\partial p_2 = 0$.*

Notice that in the above solution, we have obtained all the prices, i.e. retail price p_1 and direct price p_2, as functions of z_1. In addition, according to Lemma 2.21, we can also obtain optimal N^* as a function of z_1. And from Corollary 2.1, we know that z_1 is corresponding one-to-one with wholesale price w. Thus, given any wholesale price w, we can obtain unique optimal direct price p_2^* and manufacturer's inventory capacity N^*. After that, we can then obtain optimal z_1^* and optimal retail price p_1^*.

From the first-order condition of direct channel price p_2, we can obtain the following corollary.

Corollary 2.2. *According to Proposition 2.1, we have $\partial E[\Pi_m(p_2)]/\partial p_2 = 0$. Thus* *we have the following equation.*

$$\begin{aligned}\frac{\partial E[\Pi_m(p_2)]}{\partial p_2} &= y_2 + \frac{(p_2-c)(b_{12}b_{21} - b_{11}b_{22}[1+F_1(z_1)])}{b_{11}[1+F_1(z_1)]} + \frac{b_{12}[1-F_1(z_1)](y_1+z_1)}{b_{11}[1+F_1(z_1)]} \\ &\quad + \frac{b_{12}F_1(z_1)(w-c)}{1+F_1(z_1)} - \Theta_2(z_2) \\ &= 0 \qquad (2.22)\end{aligned}$$

where $\Theta_2(z_2) = \int_{z_2}^{B}(u-z_2)f_2(u)du$.

2.5 Numerical analysis for z_1

We will include some numerical experiments here for the stochastic case. Due to the complexity of the problem, we cannot obtain the optimal safety stock for the retailer (note that we have changed decision variable wholesale price w into z_1 using Corollary 2.1). However, we know that the wholesale price is exogenous in a lot of industries. In these industries, for example mining industry, manufacturers cannot change their wholesale price too much. The wholesale price is given according to the market. Thus, from this perspective, we have solved the whole problem for these industries with exogenous wholesale price.

From figure 2.2 and figure 2.3, we can see that manufacturer's profit is convex over z_1, which means an unique optimal z_1^* that can maximize manufacturer's profit may not exist. Thus, we use numerical analysis to observe the behavior of z_1 and how manufacturer's profit changes with regards to retailer's safety stock z_1.

For Figure 2.2, the parameters are as below: $a_1 = 200, a_2 = 450, b_{11} = b_{22} = 6.5, b_{12} = b_{21} = 2.5, c = 1, \sigma_1 = 10$. For Figure 2.3 parameters are: $a_1 = 200, a_2 = 50, b_{11} = b_{22} = 6.5, b_{12} = b_{21} = 2.5, c = 1, \sigma_1 = 100$.

From figure 2.2 and figure 2.3, we can see that manufacturer's profit is convex over retailer's safety stock z_1.

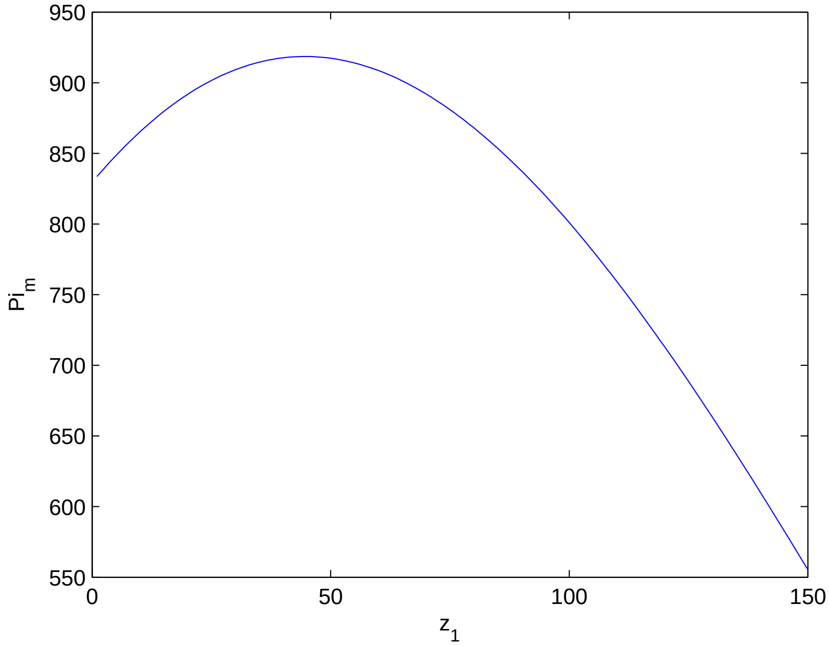

Figure 2.2: Manufacturer's profit against z_1

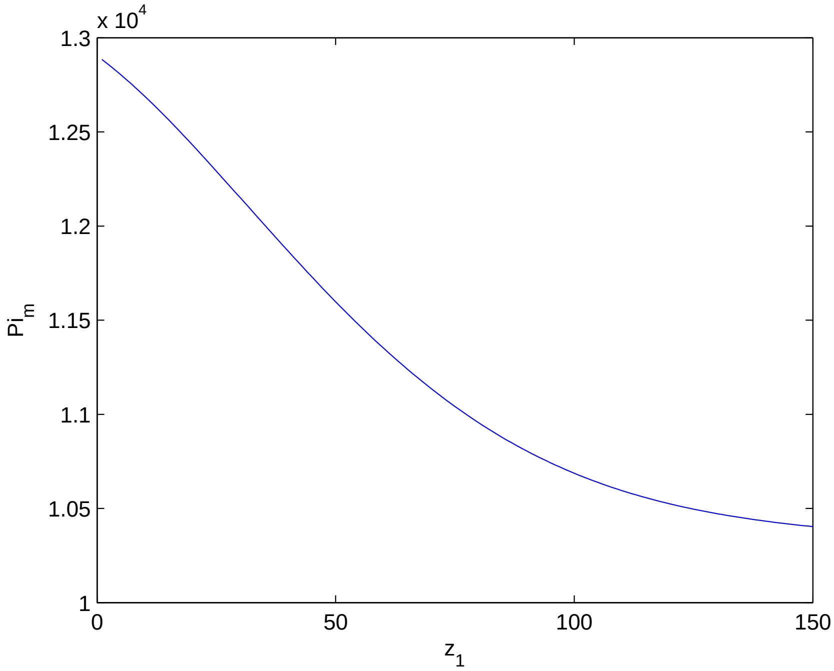

Figure 2.3: Manufacturer's profit against z_1

2.6 Conclusions and future research

We conclude the dual channel problem considering stochastic demands in this section. For the stochastic demand problem, we not only need to consider the pricing problem faced by the manufacturer and the retailer, but also need to consider the inventory control problem face by the manufacturer and the retailer. There are four decision variables in our model: the production capacity of manufacturer, the order quantity of retailer, the retail price offered by the retailer, and the direct channel price offered by the manufacturer. We have developed a mechanism based on the chain rule to obtain the solutions one by one for these variables. Notice that we consider the wholesale price as exogenous.

Given the selling price in direct channel, the retailer can decide the order quantity and the selling price in the traditional channel, which is similar to the News-vendor problem (Petruzzi and Dada 1998). Meanwhile, The manufacturer can determine the capacity for the direct channel which is similar to the News-vendor solution. Given the retailer's pricing and order quantity decisions as well as the manufacturer's capacity decision, we have obtained the selling price for the direct channel.

In the second part of this thesis, I have solved the joint pricing and inventory control problem in dual-channel network with one manufacturer and one retailer, considering wholesale price as exogenous. To the best knowledge of mine, there is no papers talking about the joint pricing and inventory control decisions in a dual channel network. I have also done some numerical analysis to see how manufacturer's profit changes with regards to the retailer's safety stock z_1. In the numerical analysis, we can see that the manufacturer's profit is convex over retailer's safety stock z_1, which indicates that an unique z_1^* that can optimize the manufacturer's profit.

For future research, there are several directions. For example, our model can be extended to multiple retailers ordering from multiple manufacturers while manufacturers selling directly simultaneously.

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Appendix A

Proof of deterministic case

Proof of dual-channel case with positive demands for both channels

Below, we prove the results obtained in Lemma 1.1, Lemma 1.2, Theorem 1.1, and Proposition 1.1.

Proof. The retailer's problem:

$$\begin{aligned} f_r(p_1) &= (p_1 - w)(a_1 - b_{11}p_1 + b_{12}p_2) \\ &= -b_{11}p_1^2 + (a_1 + b_{11}w + b_{12}p_2)p_1 - (a_1 + b_{12}p_2)w \end{aligned} \tag{A.1}$$

Thus, f_r is concave over p_1 given $b_{11} > 0$. Setting the first order derivative to zero, we can obtain

$$p_1^* = \frac{a_1 + b_{11}w + b_{12}p_2}{2b_{11}} \tag{A.2}$$

Proposition 1 follows.

Manufacturer's problem:

$$\begin{aligned} f_m(p_2, w) &= (w-c)(a_1 - b_{11}p_1 + b) + (p_2 - c)(a_2 - b_{22}p_2 + b_{21}p_1) \\ &= -\frac{1}{2}b_{11}w^2 + \frac{1}{2}(a_1 + b_{11}c - b_{21}c)w + \frac{1}{2}(b_{12} + b_{21})wp_2 \\ &\quad + \frac{2a_2b_{11} + 2b_{11}b_{22}a_1b_{21} - b_{11}b_{12}c - b_{12}b_{21}c}{2b_{11}}p_2 - \frac{(a_1b_{11} + 2a_2b_{11} + a_{21})c}{2b_{11}} \end{aligned} \tag{A.3}$$

We can obtain $\Delta = \frac{\partial^2 f_m}{\partial w^2} \cdot \frac{\partial^2 f_m}{\partial p_2^2} - [\frac{\partial^2 V_m}{\partial p_2 \partial w}]^2 \leq 0$ under assumption 2. Thus f_m is concave over w and p_2. Setting the first derivative of p_2 and w, respectively, to zero, we can obtain the optimal wholesale price w^* and p_2^* as below.

$$\begin{aligned} w^* &= \frac{-b_{12}^2b_{21} + 2b_{12}b_{11}b_{22} + b_{21}^2b_{12} - b_{12}^2b_{11} - 2b_{21}b_{11}b_{22} - 3b_{12}b_{21}b_{11} + 4b_{11}^2b_{22}}{b_{11}(8b_{11}b_{22} - b_{12}^2 - 6b_{21}b_{12} - b_{21}^2)}c \\ &\quad + \frac{2a_2b_{11}b_{21} + 2a_2b_{11}b_{12} + 4a_1b_{11}b_{22} + b_{21}^2a_1 - a_1b_{12}b_{21}}{b_{11}(8b_{11}b_{22} - b_{12}^2 - 6b_{21}b_{12} - b_{21}^2)} \end{aligned} \tag{A.4}$$

$$\begin{aligned} p_2^* &= \frac{a_1b_{21} + 2b_{11}a_2}{2b_{11}b_{22} - b_{12}b_{21}} - \frac{2b_{11}}{2b_{11}b_{22} - b_{12}b_{21}}n^* + \frac{b_{11}b_{21}}{2b_{11}b_{22} - b_{12}b_{21}}w^* \\ &= \frac{(b_{11}b_{21} + 4b_{11}b_{22} - 3b_{21}b_{12} - b_{12}b_{11} - b_{21}^2)c + 4a_2b_{11} + 3a_1b_{21} + a_1b_{12}}{8b_{11}b_{22} - b_{12}^2 - 6b_{21}b_{12} - b_{21}^2} \end{aligned} \tag{A.5}$$

The corresponding retail price for the traditional channel can be obtained

$$\begin{aligned} p_1^* &= \frac{2b_{11}^2b_{22} - b_{11}b_{12}b_{21} - b_{11}b_{22}b_{21} + 3b_{11}b_{22}b_{12} - 2b_{12}^2b_{21} - b_{11}b_{12}^2}{b_{11}(8b_{11}b_{22} - b_{12}^2 - 6b_{21}b_{12} - b_{21}^2)}c \\ &\quad + \frac{6a_1b_{11}b_{22} + a_2b_{11}b_{21} + 3a_2b_{11}b_{12} - 2a_1b_{12}b_{21}}{b_{11}(8b_{11}b_{22} - b_{12}^2 - 6b_{21}b_{12} - b_{21}^2)} \end{aligned} \tag{A.6}$$

However, in order to make sure that both the traditional channel and direct channel have positive demand, we must have $D_1 > 0$ and $D_2 > 0$, which in turn can be obtained as the constraints for the dual-channel problem. □

Proof of Lemma 1.3

Proof. For case 3, we know that $D_2 = a_2 - b_{22}p_2 + b_{21}p_1 = 0$. Assuming that w and p_2 are on line segment $\overline{CD}$, then we have $D_1 > 0$. From Theorem 1 we can obtain retailer's best response as $p_1 = \frac{a_1+b_{11}w+b_{12}p_2}{2b_{11}}$ and the manufacturer's profit as $V_m = (w-c)D_1$.

If the manufacturer maintains wholesale price w unchange but reduces the direct price by δ, then we have: $p_2' = p_2 - \delta$, and $p_1' = p_1 - \frac{b_{12}}{2b_{11}}\delta$. Thus, we can obtain $D_1' = a_1 - b_{11}p_1' + b_{12}p_2' = D_1 - \frac{b_{12}}{2}\delta$ and $D_2' = a_2 - b_{22}p_2' + b_{21}p_1' = D_2 + (b_{22} - \frac{b_{12}b_{21}}{2b_{11}})\delta$. Thus we can obtain the manufacturer's profit as below:

$$\begin{aligned}
V_m' &= (w-c)D_1' + (p_2' - c)D_2' \\
&= (w-c)(D_1 - \frac{b_{12}}{2}\delta) + (p_2 - c - \delta)(D_2 + (b_{22} - \frac{b_{12}b_{21}}{2b_{11}})\delta) \\
&= (w-c)D_1 + (p_2 - c)D_2 - \delta D_2 + (p_2 - c)(b_{22} - \frac{b_{12}b_{21}}{2b_{11}})\delta \\
&\quad -\frac{b_{12}}{2}(w-c)\delta - o(\delta^2) \qquad \text{(A.7)}
\end{aligned}$$

Because $D_2 = 0$ and $o(\delta^2) = 0$, we have $V_m' = V_m + (p_2 - c)(b_{22} - \frac{b_{12}b_{21}}{2b_{11}})\delta - \frac{b_{12}}{2}(w-c)\delta$ It's easily to obtain $V_m' \geq V_m$ from assumption 2 (the dominance assumption) and $p_2 \geq w$. □

Proof of Lemma 1.5

Proof. Here, we prove the results obtained for the case with $p_d = w$. Let

$$\begin{aligned}
f_r(p_1) &= (p_1 - w)D_1(p_1, p_2) \\
&= -b_{11}p_r^2 + (a_1 + b_{11}w + b_{12}p_2)p_1 - (a_1 + b_{12}p_2)w \qquad \text{(A.8)}
\end{aligned}$$

Thus, f_r is concave over p_1 given $b_{11} > 0$. Setting the first order derivative to zero, we can obtain the optimal retail price as below

$$p_1^* = \frac{a_1 + b_{11}w + b_{12}p_2}{2b_{11}} \tag{A.9}$$

Let

$$\begin{aligned} f_m(w) &= (w-c)D_1 + (p_2 - c)D_2 \\ &= (w-c)(a_1 - b_{11}p_1 + b_{12}p_2) + (p_2 - c)(a_2 - b_{22}p_2 + b_{21}p_1) \end{aligned} \tag{A.10}$$

Taking the second order derivative of f_m over w and using $w = p_2$, we can obtain $f_m(w)$ is concave over w under assumption 2. Taking the first order derivative over w and setting it equal to zero, we can obtain the optimal wholesale price w^* and the corresponding p_1^* as below.

$$\begin{aligned} p_1^* &= \frac{a_1 + b_{11}w^* + b_{12}w^*}{2b_{11}} \\ &= \frac{-b_{11}b_{12}^2 - 2b_{12}b_{21}b_{11} + 2b_{12}b_{11}b_{22} - b_{21}b_{11}^2 + 2b_{11}^2b_{22} + b_{11}^3 - b_{21}b_{12}^2}{4b_{11}(2b_{11}b_{22} - b_{12}b_{11} + b_{11}^2 - b_{21}b_{12} - b_{21}b_{11})} \\ &+ \frac{4a_1b_{11}b_{22} + 2b_{12}b_{11}a_2 - b_{12}b_{21}a_1 - b_{12}b_{11}a_1 - b_{11}b_{21}a_1 + 3b_{11}^2a_1 + 2b_{11}^2a_2}{4b_{11}(2b_{11}b_{22} - b_{12}b_{11} + b_{11}^2 - b_{21}b_{12} - b_{21}b_{11})} \end{aligned} \tag{A.11}$$

□

Appendix B

Proof of stochastic case

Proof of Theorem 2.1

Proof. From the equation 1.7, we have:

$$\frac{\partial E[\Pi_r(z_1, p_1)]}{\partial z_1} = -w + p_1[1 - F_1(z_1)]. \quad \text{(B.1)}$$

To obtain the values of z_1 that satisfy this first-order optimality condition, we define: $R_1(z_1) \equiv dE[\Pi(z_1, p_1(z_1))]/dz_1$ and consider the zero points of $R_1(z_1)$:

$$\begin{aligned} \frac{dR_1(z_1)}{dz_1} &= \frac{d}{dz_1}[\frac{dE[\Pi_r(z_1, p_1(z_1))]}{dz_1}] \\ &= -\frac{f_1(z_1)}{2b_{11}} 2b_{11}p^0 - \Theta_1(z_1) - \frac{1 - F_1(z_1)}{r_1(z_1)} \end{aligned} \quad \text{(B.2)}$$

where $r_1(\cdot) \equiv f_1(\cdot)/[1 - F_1(\cdot)]$ denotes the hazard rate.

$$\begin{aligned} \frac{d^2R_1(z_1)}{dz_1^2} &= [\frac{dR_1(z_1)}{dz_1}]\frac{df_1(z_1)}{dz_1} - \frac{f_1(z_1)}{2b_{11}} \\ &\cdot\{[1 - F_1(z_1)] + \frac{f_1(z_1)}{r_1(z_1)} + \frac{[1 - F_1(z_1)[dr_1(z_1)/dz_1]]}{r_1(z_1)^2}\} \end{aligned} \quad \text{(B.3)}$$

Thus, we can obtain,

$$\frac{d^2R_1(z_1)}{dz_1^2}|_{dR_1(z_1)/dz_1=0} = -\frac{f_1(z_1)[1-F_1(z_1)]}{2b_{11}r_1(z_1)^2}\{2r_1(z_1)^2 + \frac{dr_1(z_1)}{dz_1}\} \quad \text{(B.4)}$$

If $F(\cdot)$ is a distribution satisfying the condition $2r_1(z_1)^2+\frac{dr_1(z_1)}{dz_1} > 0$, then if follows that $R_1(z_1)$ is monotone or unimodal, implying that $dE[\Pi(z_1, p_1(z_1))]/dz_1 = 0$ has at most two roots. □

Proof of Corollary 2.1

Proof. From Theorem 2.1, we know that the retailer's profit function satisfies first order condition, i.e. $\frac{\partial E[\Pi_r(z_1,p_1)]}{\partial z_1} = 0$. Thus we can obtain:

$$-\frac{1+F_1(z_1)}{2}w + [b_{12}p_2 + a_1 + \mu_1 - \Theta(z_1)]\frac{[1-F_1(z_1)]}{2b_{11}} = 0 \quad \text{(B.5)}$$

Re-arrange the terms in the above, we can obtain the equation in Corollary. □

Proof of Theorem 2.21

Proof. From the equation 2.1, we have:

$$\begin{aligned}\partial E[\Pi_m(N,p_2)] &= (p_2-c)y_2 - c\int_A^{z_2}(z_2-u)f_2(u)du \\ &\quad -(p_2-c)\int_{z_2}^{B}(u-z_2)f_2(u)du + (w-c)(y_1+z_1) \quad \text{(B.6)}\end{aligned}$$

where $y_2 = a_2 - b_{22}p_2 + b_{21}p_1$, $y_1 = a_1 - b_{11}p_1 + b_{12}p_2$ and $z_2 = q_2 - y_2$.

Define $N = q_1 + q_2$, then instead of solving for optimal N^*, we can solve for optimal z_2^*. Given w and p_2, we can obtain,

$$\begin{aligned}\frac{\partial E[\Pi_m(N,p_2)]}{\partial z_2} &= -cF_2(z_2)] + (p_2 - c)[1 - F_2(z_2)] \\ &= -c + p_2[1 - F_2(z_2)]. \end{aligned} \tag{B.7}$$

$$\frac{\partial^2 E[\Pi_m(N,p_2)]}{\partial z_2^2} = -p_2 f_2(z_2). \tag{B.8}$$

Given z_2, we can obtain,

$$\begin{aligned}\frac{\partial E[\Pi_m(N,p_2)]}{\partial p_2} &= y_2 + (p_2 - c)(-b_{22} + \frac{b_{12}b_{21}}{b_{11}[1 + F_1(z_1)]}) - \int_{z_2}^{B} (u - z_2) f_2(u) du \\ &+ (y_1 + z_1) \cdot \frac{b_{12}[1 - F_1(z_1)]}{b_{11}[1 + F_1(z_1)]} + (w - c)(b_{12} - \frac{b_{12}}{1 + F_1(z_1)}) \end{aligned} \tag{B.9}$$

$$\begin{aligned}\frac{\partial^2 E[\Pi_m(N,p_2)]}{\partial p_2^2} &= \frac{2b_{12}}{b_{11}[1 + F_1(z_1)]}(b_{21} - b_{11}) \\ &- \frac{2b_{12}F_1(z_1)}{b_{11}[1 + F_1(z_1)]}[b_{11} - b_{12} + b_{12}F_1(z_1)] \end{aligned} \tag{B.10}$$

Thus, we can see that given z_2, $E[\Pi_m(N,p_2)]$ is concave in p_2 and vice versa. Given p_2, setting the first order condition equal to zero, we can obtain,

$$z_2^* = F_2^{-1}(\frac{p_2 - c}{p_2}) \tag{B.11}$$

Thus, we can obtain $N^* = y_2 + y_1 + z_1 + F_2^{-1}(\frac{p_2 - c}{p_2})$. □

Proof of Proposition 2.1

Proof. From the equation 2.1, we have:

$$\begin{aligned}\partial E[\Pi_m(N,p_2)] &= (p_2-c)y_2 - c\int_A^{z_2}(z_2-u)f_2(u)du \\ &\quad -(p_2-c)\int_{z_2}^{B}(u-z_2)f_2(u)du + (w-c)(y_1+z_1)\end{aligned} \tag{B.12}$$

where $y_2 = a_2 - b_{22}p_2 + b_{21}p_1$, $y_1 = a_1 - b_{11}p_1 + b_{12}p_2$ and $z_2 = q_2 - y_2$.

From the proof of Theorem 2.3, given z_2, we can have,

$$\begin{aligned}\frac{\partial E[\Pi_m(N,p_2)]}{\partial p_2} &= y_2 + (p_2-c)(-b_{22} + \frac{b_{12}b_{21}}{b_{11}[1+F_1(z_1)]}) - \int_{z_2}^{B}(u-z_2)f_2(u)du \\ &\quad +(y_1+z_1)\cdot\frac{b_{12}[1-F_1(z_1)]}{b_{11}[1+F_1(z_1)]} + (w-c)(b_{12} - \frac{b_{12}}{1+F_1(z_1)})\end{aligned} \tag{B.13}$$

From Theorem 2.3, we have $N^* = y_2 + y_1 + z_1 + F_2^{-1}(\frac{p_2-c}{p_2})$. Plugging N^* into the manufacturer's profit function, we can reduce two decision variables into single decision variable p_2.

Define $R_2(p_2) = \frac{\partial E\Pi_m}{\partial p_2}$, we can obtain,

$$\begin{aligned}\frac{\partial R_2(p_2)}{\partial p_2} &= (-b_{22} + \frac{b_{12}b_{21}}{b_{11}[1+F_1(z_1)]}) + \frac{b_{12}b_{21} - b_{11}b_{22}[1+F_1(z_1)]}{b_{11}[1+F_1(z_1)]} \\ &\quad + \frac{b_{12}[1-F_1(z_1)]}{b_{11}[1+F_1(z_1)]}(b_{12} - \frac{b_{12}}{1+F_1(z_1)}) + \frac{b_{12}^2F_1(z_1)[1+F_1(z_1)]}{b_{11}[1+F_1(z_1)]^2} \\ &\quad +(1-\frac{p_2-c}{p_2})[F_2^{-1}(\frac{p_2-c}{p_2})]' \\ &= \frac{2b_{12}b_{21} - 2b_{11}b_{22}[1+F_1(z_1)]}{b_{11}[1+F_1(z_1)]} + \frac{2b_{12}^2F_1(z_1)[1-F_1(z_1)]}{b_{11}[1+F_1(z_1)]} + \frac{c^2}{p_2^3 f_2(\frac{p_2-c}{p_2})}\end{aligned} \tag{B.14}$$

Define $r_2(\cdot) = \frac{f_2(\cdot)}{1-F_2(\cdot)}$, we can obtain,

$$\begin{aligned}
\frac{\partial^2 R_2(p_2)}{\partial p_2^2} &= c^2 \cdot [p_2^3 f_2(\frac{p_2 - c}{p_2})]' \\
&= c^2 \cdot [p_2^3 f_2(z_2)]' \\
&= \frac{c^2}{[p_2^2 f_2(z_2)]^2}[2f_2(z_2) + \frac{cf_2'(z_2)}{p_2(f_2(z_2))}] \\
&= -\frac{c^2}{p_2^4 f_2(z_2) r_2^2(z_2)}[2r_2^2(z_2) + \frac{dr_2(z_2)}{dz_2}]
\end{aligned} \tag{B.15}$$

Thus, we can see that $R_2(z_2)$ is unimodal in z_2, first increasing then decreasing. Therefore, given that $2r_2^2(z_2) + \frac{dr_2(z_2)}{dz_2} > 0$, $E\Pi_m(p_2, N(p_2))$ reaches its maximum at the unique value of p_2 that satisfies $\frac{dE[\Pi_m(N(p_2),p_2)]}{dp_2} = 0$. □