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Hian Teck HOON
Singapore Management University, hthoon@smu.edu.sg

Edmund S. PHELPS
Columbia University

CENTER ON CAPITALISM AND SOCIETY

COLUMBIA UNIVERSITY

http://www.capitalism.columbia.edu/

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Hian Teck Hoon and Edmund Phelps [†]

[†] Affiliations on following page

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Hian Teck Hoon, Singapore Management University*

Edmund S. Phelps, Columbia University†

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Abstract

Is employment higher in an economy that has a higher rate of innovation? In Hoon and Phelps (1997), we study this question in the small open, and closed, economy under the assumption that the rate of technological progress is exogenous to the economic system. In this paper, we reexamine this question in the context of a model with endogenous product innovation (and thus endogenous technological progress) and endogenous labor supply first in a small open economy taking the world interest rate as given and then in a closed economy that determines the whole term structure of the interest rate. In our present model, creating a given flow of new ideas per unit time directly generates labor demand. Indirectly, as later innovations can benefit from a larger stock of ideas ("the standing on the shoulders of giant" effect), thus requiring less R&D labor input to create a new idea, an economy that has been growing more rapidly in the past demands less labor input now to create a given flow of new ideas. Another source of labor demand in our model economy comes from the production of a variety of intermediate inputs whose product designs have already been discovered through past R&D activity. We find that in both the small open economy and the closed economy, a policy shock that leads to

*Professor of Economics, School of Economics, Singapore Management University, 90, Stamford Road, Singapore 178903. Email: hthoon@smu.edu.sg; Tel: 65-6828-0248; Fax: 65-6828-0833.

†McVickar Professor of Political Economy and Director, Center on Capitalism and Society, 1134 International Affairs Building, 420 West 118th Street, Columbia University, New York, NY 10027. Email: esp2@columbia.edu; Tel: 1-212-854-2060; Fax: 1-212-854-3735.

a higher rate of innovation also increases aggregate labor demand. When a shock leads the model economy to transit to a higher steady-growth path, the transition path is characterized unambiguously by rising stock market capitalization (taken as a ratio to GDP) in the closed economy. In the small open economy, there is the possibility that higher growth is accompanied by declining stock market capitalization (taken as a ratio to GDP) as GDP races ahead of the stock market on the transition path. In both cases employment is unambiguously rising with the possibility of overshooting in the small open economy. In terms of labor supply, the transition to a higher steady-growth path leads agents to plan a rising path of employment until they reach the steady state where employment stays constant at a permanently higher level absent another shock. What induces agents to keep postponing their leisure along such a transition path is the falling consumption-growth-adjusted real rate of interest and *accelerating* wage increases. The prospect of a permanently higher debt-to-GDP ratio in the future requiring a higher payroll tax rate to ensure fiscal solvency leads to a transient period of slower innovation, declining or rising stock market capitalization (taken as a ratio to GDP), and reduced employment in the small open economy. In the closed economy, the same shock has permanent effects as the economy settles on a lower steady-growth path with reduced employment and lower stock market capitalization (taken as a ratio to GDP).

JEL classification: E13, E22, E24, J20, O30

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1. Introduction

When the rate of technological progress is exogenous to the market system, Hoon and Phelps (1997) show in a labor-turnover model that a growth slowdown contracts employment in a small open economy that takes the world interest rate as given.[1] The growth-adjusted rate of interest, $r^* - \lambda$, where r^* is the given world interest rate and λ is the Harrod-neutral rate of technological progress, is *raised* with a growth slowdown and that leads firms to cut down their firm-specific investment in new workers with the result that the unemployment rate is raised. Furthermore, the growth slowdown causes wages to fall behind wealth and

[1]Other papers studying the relationship between growth and unemployment include Aghion and Howitt (1994), Manning (1992) and Pissarides (2000).

that leads to increased labor turnover or quitting that swells the pool of unemployed. An increase in the rate of technological progress works in reverse to reduce the natural rate of unemployment. In the closed economy where the term structure of interest rate adjusts to attain intertemporal equilibrium, they show that, in the limit, as the steady-growth path is approached, an increase in the rate of technological progress is neutral for the natural rate of unemployment. Its effects are completely offset by the equal increase in the real interest rate that it induces.

An important feature of a capitalist economy, however, is that the incentive to engage in the creation of new ideas is affected by variables determined endogenously within the market system. In this paper, we take the next step of developing an aggregative general-equilibrium model within which both the rate of innovation (which determines the rate of technological progress) as well as aggregate employment are simultaneously determined in intertemporal equilibrium. Since both the rate of technological progress as well as aggregate employment are endogenously determined, we introduce exogenous policy shocks that cause the dynamic shifts of both variables. As in Hoon and Phelps (1997), we find that shocks that lead to a transition to a higher steady-growth path involve a steady increase in the rate of innovation along with increased employment in the small open economy. In contrast to our earlier finding that an increase in the exogenous rate of technological progress is neutral for employment for the closed economy, we find that a policy shock such as the prospective introduction of an R&D subsidy financed by a consumption tax permanently raises both the rate of innovation as well as aggregate employment.

In what follows, section 2 develops the basic model covering both the case of a small open economy that takes the world interest rate as given as well as the case of a closed economy. We look for conditions under which a positive rate of innovation exists. In section 3, we study two policy shocks: how the model economies respond to a policy of implementing a consumption tax to finance an R&D subsidy, and the effects on the rate of innovation, the stock market capitalization (relative to GDP), and aggregate employment of a prospective future increase in the debt-to-GDP ratio requiring an increase in payroll taxes to attain fiscal solvency. Section 4 concludes.

2. The Basic Model

We proceed by first setting out the labor supply decisions of individuals and aggregating to obtain the economy-wide supply of labor and then identifying the economy-wide sources of labor demand. For simplicity we take the case of a neoclassical labor market so there is market clearing in labor-market equilibrium. Capital-market equilibrium requires that the rate of return to holding a share giving the stockholder a claim to the stream of monopoly profits from the use of a newly invented idea be equal to the parametrically given world interest rate in the small open economy. In the closed economy, the whole path of the instantaneous real rate of interest, to which the equity rate of return must be equal in capital-market equilibrium, is endogenously determined.

2.1. Supply of labor

The economy comprises of individuals who all face a constant and equal probability of death per unit time denoted θ (see Blanchard [1985]).[2] A new cohort is born at each instant of time with individuals facing the same probability of death, θ. The cohort size is sufficiently large that the size of each cohort declines deterministically through time. By normalizing the size of each new cohort to equal θ, total population (also equal to the total size of the labor force) at time t is given by $\int_{-\infty}^{t} \theta \exp^{-\theta(t-s)} ds = 1$. With this normalization, aggregate variables are also interpretable as per-capita variables.

Individuals purchase insurance policies that take the form of receiving payments from the insurance company when alive and turning over their estate to the company upon death. With an actuarially fair scheme, a member who is alive receives the payment θ in exchange for the insurance company's claim of one unit of good contingent on death.

Let the period-t utility function of an individual born at time s be given by $A^{-1} \log(\bar{L} - l(s,t)) + \log c(s,t) + B$. Here, A and B are positive parameters, $\bar{L}$ is total time endowment, $l(s,t)$ is individual labor supply, and $c(s,t)$ is consumption of a homogeneous non-durable final market good. We note that individuals in this model are not bequeathed with nonhuman wealth at birth and thus start life with a positive supply of labor. As worker-savers, they

[2] See also Obstfeld (1989) and Weil (1989).

accumulate nonhuman wealth through their lifetime leaving the possibility that there would exist individuals who become so wealthy that they would use up all their time endowment for leisure. This would make it difficult to derive an aggregate labor supply schedule.[3] To avoid this aggregation problem, we assume that while additional hours spent at market work leads directly to a loss of utility as time available for leisure declines in the traditional way, spending a positive amount of time in market work also gives a positive fixed amount of utility that is subsumed in the term B that the activity of home production does not give. This positive fixed utility from being in the workforce captures the value obtainable from social interactions with colleagues at the workplace as well as mental stimulus from solving problems not normally found from homework. We relegate to the Appendix a formal derivation of the utility function used above as a reduced-form expression from a more basic problem involving utility derivable from consuming both a market good and a non-market good.[4]

To ensure that every living person in the economy spends a positive amount of time working in the market in order to facilitate aggregation, we make the following assumption:

Assumption 1: $B > A^{-1}[\log \bar{L} - \log(\bar{L} - 0^{+})]$.

Under Assumption 1, a very wealthy individual who might have chosen to retire in a model without a positive utility value from market work spends a very small positive amount of time working in the market ($l_m = 0^{+} > 0$) given the positive utility value of market work compared to housework in our model.

At time 0, the individual's intertemporal optimization problem can be written as

$$\begin{aligned} &\text{Maximize} \int_0^{\infty} \{A^{-1}[\log(\bar{L} - l(s,t))] + \log c(s,t) + B\} \exp^{-(\theta+\rho)t}\, dt \\ &\text{subject to } \dot{w}(s,t) = [r(t) + \theta] w(s,t) + v^h(t) l(s,t) - c(s,t), \\ &\text{given } w(s,0). \end{aligned}$$

Here $w(s,t)$ is the individual's nonhuman wealth, $r(t)$ is the interest rate, θ is the actuarially fair premium rate, ρ is the subjective rate of time preference, and $v^h(t)$ is the wage received by the individual. We choose the homogeneous market good as the numeraire.

[3] See Hoon (2011) for a discussion of this problem.

[4] For a model using an incentive-wage labor market, see Hoon and Phelps (1996).

Solving the individual's intertemporal problem gives the following optimal conditions:

$$\frac{A^{-1}}{\bar{L}-l(s,t)} = v^h(t)\mu(s,t), \tag{1}$$

$$\frac{1}{c(s,t)} = \mu(s,t), \tag{2}$$

$$\frac{\dot{\mu}(s,t)}{\mu(s,t)} = -[r(t)-\rho], \tag{3}$$

$$\lim_{T\to\infty}\mu(s,T)w(s,T)\exp^{-(\theta+\rho)T} = 0, \tag{4}$$

where $\mu(s,t)$ is the co-state variable. Combining (1) and (2), and eliminating $\mu(s,t)$, we obtain

$$c(s,t) = Av^h(t)[\bar{L}-l(s,t)]. \tag{5}$$

Using (5) in the individual's dynamic budget constraint, we obtain

$$\dot{w}(s,t) = [r(t)+\theta]w(s,t) + v^h(t)\bar{L} - \left(\frac{A+1}{A}\right)c(s,t). \tag{6}$$

Using a no-Ponzi game condition that, conditional on being alive at time T, we require

$$\lim_{T\to\infty}\exp^{-\int_t^T[r(\kappa)+\theta]d\kappa}w(s,T) = 0,$$

we can integrate (6) forward in time to obtain

$$\int_0^\infty\left(\frac{A+1}{A}\right)c(s,t)\exp^{-\int_0^t[r(\kappa)+\theta]d\kappa}dt = w(s,0) + \int_0^\infty v^h(t)\bar{L}\exp^{-\int_0^t[r(\kappa)+\theta]d\kappa}dt. \tag{7}$$

The RHS of (7) gives the sum of nonhuman wealth and what may be called full human wealth.

From (2) and (3), we obtain

$$\dot{c}(s,t) = [r(t)-\rho]c(s,t). \tag{8}$$

Integrating (8) to obtain

$$c(s,t) = c(s,0)\exp^{\int_0^t[r(\kappa)-\rho]d\kappa},$$

and replacing in (7) gives an individual consumption function:

$$c(s,t) = (\theta+\rho)\left(\frac{A}{A+1}\right)\left[w(s,t) + \int_t^\infty v^h(\kappa)\bar{L}\exp^{-\int_t^\kappa[r(\nu)+\theta]d\nu}d\kappa\right]. \tag{9}$$

Denoting aggregate consumption by $C(t)$ and aggregate nonhuman wealth by $W(t)$, we obtain the aggregates by integrating over the generations:

$$C(t) = \int_{-\infty}^{t} c(s,t)\theta \exp^{-\theta(t-s)} ds, \tag{10}$$

$$W(t) = \int_{-\infty}^{t} w(s,t)\theta \exp^{-\theta(t-s)} ds. \tag{11}$$

Since the wage rate paid is independent of the cohort, aggregate full human wealth, denoted W^{fh}, is given by

$$W^{fh}(t) = \int_{t}^{\infty} v^h(\kappa)\bar{L} \exp^{-\int_t^\kappa [r(\nu)+\theta] d\nu} d\kappa. \tag{12}$$

From (9) then, we obtain the aggregate consumption function:

$$C(t) = (\theta+\rho)\left(\frac{A}{A+1}\right)[W(t) + W^{fh}(t)]. \tag{13}$$

Differentiating $W(t)$ in (11) with respect to time, we obtain

$$\dot{W}(t) = \theta w(t,t) + \int_{-\infty}^{t} \left[\theta \exp^{-\theta(t-s)} \dot{w}(s,t) - \theta^2 w(s,t) \exp^{-\theta(t-s)}\right] ds.$$

Taking note that $w(t,t) = 0$ and using (6), this equation can be simplified to give

$$\dot{W}(t) = r(t)W(t) + v^h(t)\bar{L} - \left(\frac{A+1}{A}\right) C(t). \tag{14}$$

Differentiating $W^{fh}(t)$ in (12) with respect to time, we obtain

$$\dot{W}^{fh}(t) = -v^h(t)\bar{L} + [r(t)+\theta]W^{fh}(t), \tag{15}$$

and

$$\lim_{T\to\infty} W^{fh}(T) \exp^{-\int_t^T [r(\kappa)+\theta] d\kappa} = 0.$$

Using (13), (14) and (15), we obtain

$$\frac{\dot{C}(t)}{C(t)} = [r(t)-\rho] - \theta(\theta+\rho)\left(\frac{A}{A+1}\right)\left[\frac{W(t)}{C(t)}\right]. \tag{16}$$

We also note that aggregating across generations in (5) gives

$$C(t) = Av^h(t)C^L(t), \tag{17}$$

where we define the aggregate demand for leisure, $C^L(t)$, as $C^L(t) \equiv \bar{L} - L(t)$. Using (16) and (17), we obtain

$$\frac{\dot{C}^L(t)}{C^L(t)} = \left[r(t) - \rho - \frac{\dot{v}^h(t)}{v^h(t)}\right] - \frac{\theta(\theta+\rho)}{A+1}\left[\frac{W(t)}{v^h(t)C^L(t)}\right]. \tag{18}$$

Aggregate labor supply, $L^s(t)$, is given by $L^s(t) \equiv \bar{L} - C^L(t)$.

2.2. Sources of labor demand

The homogeneous non-durable final good is produced competitively by assembling a range of produced intermediate inputs according to the production function $Q = [\int_0^n x_i^\alpha]^{1/\alpha}$, $0 < \alpha < 1$, where Q is the output of the final good, x_i is the quantity of intermediate input i, and n is the number indicating how many types of intermediate inputs are actually produced. We assume that no labor is directly required to produce the final good. There is another sector, characterized by monopolistic competition, which produces a range of intermediate inputs using only labor.[5]

To develop a new variety of the intermediate input requires R&D costs to be incurred in the form of wages paid to labor engaged in the development of new ideas. Due to knowledge spillover, the effective number of units of labor required to come up with a new design is a/K_n, where K_n is a measure of the pool of knowledge acquired from past discoveries. With a suitable normalization, we write $K_n \equiv n$, where n is the current stock of varieties already developed. Hence, the R&D cost required to develop a new product variety is given by $v^f a/n$, where v^f denotes the wage paid by the firm per unit of labor. Given free entry and exit in the research activity, we have, for a positive rate of innovation,

$$\frac{v^f a}{n} = \nu, \tag{19}$$

where ν gives the value of the firm. Expressing $V \equiv n\nu$, so that V gives the total stock market value, we can rewrite (19) as

$$v^f a = V. \tag{20}$$

Once the design of an intermediate input has been developed, the firm faces a constant marginal cost of production. The dividend per firm, π, is given by $\pi = p_x x - v^f x$, where

[5]See Romer (1990) and Grossman and Helpman (1991) for the model of product innovation used here.

we assume that to produce one unit of the input requires one unit of labor. With symmetry assumed, p_x and x are the price and quantity per unit, respectively, of the typical input produced. Letting $X \equiv nx$, and noting that profit maximization implies $v^f = \alpha p_x$, we can write $\pi = (1-\alpha)p_x X/n$. We note that with perfect competition in the final good market, the value of final output is equal to the total value of inputs assembled in the final good sector, that is, $Q = p_x X$, recalling that we choose the final good as the numeraire. Thus, we can also write

$$\pi = (1-\alpha)Q/n. \tag{21}$$

With symmetry, we can write $Q = n^{(1-\alpha)/\alpha}X$. Since $Q = p_x X$, we have $p_x = n^{(1-\alpha)/\alpha}$. Noting that $v^f = \alpha p_x$, we then obtain

$$v^f = \alpha n^{\frac{(1-\alpha)}{\alpha}}. \tag{22}$$

Using (22) in (20), we have

$$V = a\alpha n^{\frac{(1-\alpha)}{\alpha}}. \tag{23}$$

Rewriting (23) as $n^{(1-\alpha)/\alpha} = V/(a\alpha)$ and using $Q = n^{(1-\alpha)/\alpha}X$, we obtain

$$X = a\alpha \left(\frac{V}{Q}\right)^{-1}, \tag{24}$$

where V/Q gives stock market capitalization (taken as a ratio to GDP). Demand for labor in the monopolistically competitive sector is, therefore, inversely related to the stock market capitalization (taken as a ratio to GDP).

Demand for labor comes both from the manufacturing activity as well as the R&D activity. The latter is given by $(a/n)\dot{n}$. Letting g denote $g \equiv \dot{n}/n$, aggregate labor demand is given by

$$\text{Aggregate labor demand} = ag + a\alpha \left(\frac{V}{Q}\right)^{-1}. \tag{25}$$

2.3. Labor-market equilibrium and capital-market equilibrium

With our assumption of a neoclassical labor market, equilibrium in the labor market involves market clearing:

$$ag(t) + a\alpha \left(\frac{V(t)}{Q(t)}\right)^{-1} = \bar{L} - C^L(t). \tag{26}$$

We assume that there exists a bond market offering a riskless rate of return of $r(t)$. The rate of return to holding equity is given by $(\pi/\nu) + (\dot{\nu}/\nu)$. Arbitrage ensures

$$r(t) = \frac{\pi(t)}{\nu(t)} + \frac{\dot{\nu}(t)}{\nu(t)} = \frac{(1-\alpha)Q(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} - \frac{\dot{n}(t)}{n(t)}, \tag{27}$$

where $\dot{V}/V \equiv \dot{\nu}/\nu + \dot{n}/n$. Differentiating through (23), we can rewrite (27) as

$$r(t) = (1-\alpha) \left. \frac{V(t)}{Q(t)} \right)^{-1} + \left(\frac{1-\alpha}{\alpha}\right) g(t) - g(t), \tag{28}$$

where $g \equiv \dot{n}/n$.

2.4. General equilibrium of the small open economy

In the small open economy case, the domestic interest rate is pinned down by the parametrically given world rate of interest so $r(t) = r^*$. We assume that the final good is a tradable good produced with the use of non-traded intermediate inputs. In the absence of a policy shock, we write $v^h \equiv v^f$. Using (14) and (17), and $v^f = V/a$, the following system of three dynamic equations summarizes the general-equilibrium evolution of the economy given initial conditions and given $g(t)$:

$$\dot{W}(t) = r^* W(t) + \frac{V(t)\bar{L}}{a} - \left. \frac{(A+1)V(t)C^L(t)}{a} \right), \tag{29}$$

$$\dot{V}(t) = \left(\frac{1-\alpha}{\alpha}\right) g(t)V(t), \tag{30}$$

$$\dot{C}^L(t) = \left[r^* - \rho - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] C^L(t) - \frac{\theta(\theta+\rho)a}{A+1}\left[\frac{W(t)}{V(t)}\right]. \tag{31}$$

Defining $\omega \equiv W/V$, we can reduce the system of equations (29) to (31) to a two-equation system in $\omega(t)$ and $C^L(t)$, given $g(t)$:

$$\dot{\omega}(t) = \left[r^* - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] \omega(t) + \frac{\bar{L}}{a} - \left(\frac{A+1}{a}\right) c^L(t), \tag{32}$$

$$\dot{C}^L(t) = \left[r^* - \rho - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] C^L(t) - \left[\frac{\theta(\theta+\rho)a}{A+1}\right] \omega(t). \tag{33}$$

In turn, with $r(t) = r^*$, (28) becomes

$$r^* = (1-\alpha) \left. \frac{V(t)}{Q(t)} \right)^{-1} + \left(\frac{1-\alpha}{\alpha}\right) g(t) - g(t). \tag{34}$$

Substituting out for V/Q in (26) using (34), we obtain an expression for $g(t)$ in terms of $C^L(t)$:

$$g(t) = \left[\frac{1-\alpha}{2(1-\alpha)-\alpha}\right]\left[\frac{\bar{L}-C^L(t)}{a} + \left(\frac{1-\alpha}{\alpha}\right)r^*\right]. \tag{35}$$

From (35), we obtain the following lemma:

Lemma 1: If the world real interest rate, r^*, is nonnegative, an equilibrium with a positive rate of innovation requires that $0 < \alpha < 2/3$. When $2/3 < \alpha < 1$, no new innovation occurs.

Focusing on the economy in its steady-state, we set $\dot{\omega} = 0$ and $\dot{C}^L = 0$. Then, substitution gives us an expression for C^L_{ss} (using the subscripts ss to denote steady state):

$$C^L_{ss} = \frac{\bar{L}}{(A+1)\left\{1 - \frac{[r^*-\rho-\left(\frac{1-\alpha}{\alpha}\right)g(t)][r^*-\left(\frac{1-\alpha}{\alpha}\right)g(t)]}{\theta(\theta+\rho)}\right\}}. \tag{36}$$

From (36), we obtain the following result:

Result 1: Between two small open economies facing the same world real rate of interest, the one in which no new innovation takes place (where $2/3 < \alpha < 1$ and $g(t) = 0$) has the lower employment rate compared to the innovative economy where $0 < \alpha < 2/3$ and $g(t) > 0$.

As we will focus in what follows on an equilibrium with a positive rate of innovation, we make another assumption:

Assumption 2: $0 < \alpha < \frac{2}{3}$.

Under Assumption 2, (35) allows us to write $g(t)$ as a reduced function of $C^L(t)$, a, and r^*. We have the following lemma:

Lemma 2: $g(t) = G(C^L(t); a, r^*);\ G_1 < 0;\ G_2 < 0;\ G_3 > 0.$

Using Lemma 2 in (32) and (33), we obtain

$$\dot{\omega}(t) = \left[r^* - \left(\frac{1-\alpha}{\alpha}\right)G(C^L(t); a, r^*)\right]\omega(t) + \frac{\bar{L}}{a} - \left(\frac{A+1}{a}\right)C^L(t), \tag{37}$$

$$\dot{C}^L(t) = \left[r^* - \rho - \left(\frac{1-\alpha}{\alpha}\right)G(C^L(t); a, r^*)\right]C^L(t) - \left[\frac{\theta(\theta+\rho)a}{A+1}\right]\omega(t). \tag{38}$$

Focusing on an economy initially in steady state that is neither a net creditor nor net debtor, $(W/V)_{ss} = 1$. We make the following assumption that will ensure saddle-path stability:

$$\textbf{Assumption 3: } r^* - \left(\frac{1-\alpha}{\alpha}\right) g(t) > 0 \text{ and } \frac{(1-\alpha)^2}{\alpha[2(1-\alpha)-\alpha]} - (A+1) > 0.$$

Under Assumption 3, the pair of equations (37) and (38) give us a stationary-C^L locus that is positively-sloped, a stationary-ω locus that is negatively-sloped, and a saddle path that is positively-sloped but gentler than the stationary-C^L locus. Given an initial value of ω, say, that is below steady-state ω_{ss}, the economy travels along a path of declining employment and rising ω. Along such a path, (35) tells us that the rate of innovation $g(t)$ is also declining. When $0 < \alpha < 1/2$, stock market capitalization (taken as a ratio to GDP) is declining along the adjustment path. We find that, in this case, even as the demand for manufacturing workers is rising, the fall in demand for workers in the R&D sector as the rate of innovation declines dominates so that overall, for the economy, aggregate employment is declining. When $1/2 < \alpha < 2/3$, both manufacturing demand for labor and labor demand for R&D activity are declining along the adjustment path.

2.4. General equilibrium of the closed economy

In the closed economy, we have, for given $g(t)$ and $r(t)$, the system of equations

$$\dot{\omega}(t) = \left[r(t) - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] \omega(t) + \frac{\bar{L}}{a} - \left(\frac{A+1}{a}\right) C^L(t), \tag{39}$$

$$\dot{C}^L(t) = \left[r(t) - \rho - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] C^L(t) - \left[\frac{\theta(\theta+\rho)a}{A+1}\right] \omega(t). \tag{40}$$

In the absence of public debt, $W(t) \equiv V(t)$ so $\omega(t) \equiv 1$. Using this in (39), we get

$$r(t) - \left(\frac{1-\alpha}{\alpha}\right) g(t) = \left(\frac{A+1}{a}\right) c^L(t) - \frac{\bar{L}}{a}. \tag{41}$$

Using (41) in (40) along with $\omega(t) \equiv 1$, we obtain the key dynamic equation summarizing the dynamic behavior of the economy in general equilibrium:

$$\frac{\dot{C}^L(t)}{C^L(t)} = \left[\frac{A+1}{a}\right] C^L(t) - \frac{\bar{L}}{a} - \rho - \frac{\theta(\theta+\rho)a}{(A+1)C^L(t)}. \tag{42}$$

The economically meaningful solution to the quadratic equation in the variable $C^L(t)$ with $\dot{C}^L(t) = 0$ in (42) is given by

$$C^L_{ss} = \frac{\left(\frac{\bar{L}}{a} + \rho\right) + \sqrt{\left(\frac{\bar{L}}{a} + \rho\right)^2 + 4\theta(\theta + \rho)}}{2}. \tag{43}$$

Noting that in the closed economy, the market-clearing condition for the final good is $Q(t) = C(t)$, we can write capital-market equilibrium condition as

$$r(t) = \frac{(1-\alpha)C(t)}{V(t)} + \left(\frac{1-\alpha}{\alpha}\right) g(t) - g(t). \tag{44}$$

In the absence of a policy shock, $v^h \equiv v^f$. Using $v^f = V/a$ and (17), we can rewrite (44) as

$$r(t) - \left(\frac{1-\alpha}{\alpha}\right) g(t) = \frac{(1-\alpha)AC^L(t)}{a} - g(t). \tag{45}$$

Using (41) and (45), we obtain

$$g(t) = \frac{\bar{L}}{a} - \frac{(1+\alpha)C^L(t)}{a}. \tag{46}$$

We now obtain the following lemma in the closed economy:

Lemma 3: If the economy jumps immediately to take on the value of C^L_{ss} given by (43) and

$$\frac{2\bar{L}}{1+\alpha} > \frac{\bar{L}}{a} + \rho\Big) + \sqrt{\frac{\bar{L}}{a} + \rho\Big)^2 + 4\theta(\theta + \rho)},$$

the economy ends up on a steady-growth path with a positive rate of innovation. If the inequality is reversed, the economy's equilibrium is one of zero innovation.

If the inequality in Lemma 3 is satisfied, rational expectations require that the economy jump immediately to the steady-state value of C^L_{ss} given by (43). If C^L exceeds this value, then $C^L(t)$ must grow without bound. On the other hand, if $C^L(t)$ is less than this value, at some point $C(t)/V(t)$ would drop to zero whence the product innovation rate is maximal at $g = \bar{L}/a$, and $V(t)$ grows at $[(1-\alpha)/\alpha]\bar{L}/a$. Desired consumption would be positive but since $X = 0$, $Q = 0$. Thus, there would be unfulfilled expectations.

We make an assumption that will ensure a positive rate of innovation in the closed economy:

$$\textbf{Assumption 4:} \quad \frac{2\bar{L}}{1+\alpha} > \quad \frac{\bar{L}}{a} + \rho\Big) + \sqrt{\quad \frac{\bar{L}}{a} + \rho\Big)^2 + 4\theta(\theta+\rho)}.$$

We have the following result:

Result 2: Under Assumption 4, the closed economy jumps immediately to a balanced-growth path with a constant positive rate of innovation, constant employment rate, and a constant stock market capitalization (taken as a ratio to GDP).

3. Policy Shocks

We study two policy shocks. The first is the prospective future introduction of an R&D subsidy financed by a consumption tax. The second is the future prospect of maintaining a higher debt-to-GDP ratio that requires raising payroll taxes to attain fiscal solvency.

3.1. Introduction of R&D subsidy financed by consumption tax

With an R&D subsidy introduced, the R&D cost is reduced to $(1 - s_R)v^f a/n$. Free entry into the R&D activity now gives $(1 - s_R)v^f a/n = \nu$ with a positive rate of innovation. Rearranging, we obtain

$$v^f = \frac{V}{a(1 - s_R)}. \tag{47}$$

The total cost of the R&D subsidy program is equal to $(s_R v^f a/n)\dot{n} = s_R v^f ag$. Using (47), we obtain

$$\text{Total cost of R\&D subsidy program} = \frac{s_R V g}{1 - s_R}.$$

In turn,

$$\text{Total tax revenue from consumption tax} = \frac{\tau_C C}{1 - \tau_C}.$$

Balanced budget requires

$$\frac{\tau_C}{1 - \tau_C} = \left(\frac{s_R}{1 - s_R}\right)\left(\frac{V}{C}\right) g. \tag{48}$$

With the policy parameters introduced, we have

$$\dot{\omega}(t) \quad = \quad \left[r^* - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right](1 - s_R)\omega(t) + \frac{\bar{L}}{a} - \left(\frac{A+1}{a}\right) c^L(t), \tag{49}$$

$$\dot{C}^L(t) = \left[r^* - \rho - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] C^L(t) - \left[\frac{\theta(\theta+\rho)a}{A+1}\right](1-s_R)(1-\tau_C)\omega(t). \qquad (50)$$

Suppose that we start off with an initial steady state with $s_R = \tau_C = 0$. There is then, at current time t_0, an expectation formed that at future time t_1, a consumption tax will be raised to finance an R&D subsidy program. It is then straightforward to obtain the following result:

Result 3: Suppose that at current time t_0, an expectation is formed that at future time t_1, a consumption tax will be raised to finance an R&D subsidy program in the small open economy. There is an immediate increase in employment accompanied by a sudden increase in the rate of innovation. Between time t_0 and time t_1, employment steadily rises along with a rising growth rate. At time t_1, employment and growth rate begin to decline. The steady state, however, is characterized by a permanently higher rate of innovation and aggregate employment. The response of the economy exhibits overshooting.

What induces agents to keep postponing their leisure along such a transition path is the falling consumption-growth-adjusted real rate of interest and *accelerating* wage increases.

A similar result is obtained in the closed economy. Here, the equation that summarizes the general-equilibrium behavior of the economy with the policy parameters included is

$$\frac{\dot{C}^L(t)}{C^L(t)} = \left[\frac{\frac{A}{1-s_R}+1}{a}\right] C^L(t) - \frac{\bar{L}}{a} - \rho - \frac{\theta(\theta+\rho)a(1-\tau_C)(1-s_R)}{(A+1)C^L(t)}. \qquad (51)$$

Result 4: Suppose that at current time t_0, an expectation is formed that at future time t_1, a consumption tax will be raised to finance an R&D subsidy program in the closed economy. There is an immediate increase in employment accompanied by a sudden increase in the rate of innovation. Between time t_0 and time t_1, employment steadily rises along with a rising growth rate. At time t_1, the economy reaches its new steady state characterized by a permanently higher rate of innovation and aggregate employment. There is no overshooting.

3.2. Introduction of payroll tax to maintain higher debt-to-GDP ratio

With payroll tax rate, τ, we now have a wedge between the wage paid by firms and the wage received by individuals, $v^f = (1+\tau)v^h$. With $v^f a = V$, we now have

$$v^h = \frac{v^f}{1+\tau} = \frac{V}{a(1+\tau)}.$$

Budget balance to maintain a given debt-to-GDP ratio, $\bar{d}$, requires

$$r(t)\bar{d} = \left. \frac{\tau(\bar{L} - C^L(t))}{(1+\tau)a} \quad \frac{V(t)}{Q(t)} \right). \tag{52}$$

With the policy parameters introduced, we have

$$\dot{\omega}(t) = \left[r^* - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right](1+\tau)\omega(t) + \frac{\bar{L}}{a} - \left(\frac{A+1}{a}\right) c^L(t), \tag{53}$$

$$\dot{C}^L(t) = \left[r^* - \rho - \left(\frac{1-\alpha}{\alpha}\right) g(t)\right] C^L(t) - \left[\frac{\theta(\theta+\rho)a}{A+1}\right](1+\tau)\omega(t). \tag{54}$$

A striking result we obtain in the small open economy is the following:

Result 5: Suppose that at current time t_0, an expectation is formed that at future time t_1, a payroll tax will be raised to maintain a higher debt-to-GDP ratio in the small open economy. There is an immediate decrease in employment accompanied by a sudden decrease in the rate of innovation. Between time t_0 and time t_1, employment steadily declines along with a falling growth rate. At time t_1, employment and growth rate begin to rise and return to the original rate of growth and employment level.

This result can be contrasted to that in the closed economy:

Result 6: Suppose that at current time t_0, an expectation is formed that at future time t_1, a payroll tax will be raised to maintain a higher debt-to-GDP ratio in the closed economy. There is an immediate decrease in employment accompanied by a sudden decrease in the rate of innovation. Between time t_0 and time t_1, employment steadily declines along with a falling growth rate. At time t_1, the economy reaches its new steady state characterized by a permanently lower rate of innovation and reduced aggregate employment.

4. Concluding Remarks

We recap some of our main results. We showed that between two small open economies facing the same world real rate of interest, the one in which no new innovation takes place has the lower employment rate compared to the innovative economy where $g(t) > 0$. Policy has strong effects on growth and aggregate employment. Suppose that at current time t_0, an expectation is formed that at future time t_1, a consumption tax will be raised to finance an R&D subsidy program in the small open economy. We showed that there is an immediate increase in employment accompanied by a sudden increase in the rate of innovation. Between time t_0 and time t_1, employment steadily rises along with a rising growth rate. At time t_1, employment and growth rate begin to decline. The steady state, however, is characterized by a permanently higher rate of innovation and aggregate employment. The response of the economy exhibits overshooting. A similar result is obtained in the closed economy although in that case there is no overshooting.

Suppose that at current time t_0, an expectation is formed that at future time t_1, a payroll tax will be raised to maintain a higher debt-to-GDP ratio in the small open economy. There is an immediate decrease in employment accompanied by a sudden decrease in the rate of innovation. Between time t_0 and time t_1, employment steadily declines along with a falling growth rate. At time t_1, employment and growth rate begin to rise and return to the original rate of growth and employment level. Hence the negative effects on growth and employment of worries about the budget situation is transient. In contrast, in the closed economy, a higher debt-to-GDP ratio permanently depresses growth and aggregate employment.

Appendix

We first focus on an individual's choice of his time spent in market work, non-market housework, and time for leisure. We explicitly model the choice of time spent in three activities: the market sector, non-market housework, and leisure. Building upon Benhabib, Rogerson and Wright (1991), we suppose that the period individual utility function is given by

$$\begin{aligned} U &= \log \hat{c} + A' \log[\bar{L} - l_m - l_n] + B', \quad \text{if } l_m > 0 \\ &= \log \hat{c} + A' \log[\bar{L} - l_m - l_n], \qquad \text{if } l_m = 0, \end{aligned}$$

where A', $B' > 0$ and $\hat{c} \equiv c_m^{\mu} c_n^{1-\mu}$, $0 < \mu < 1$. Here, $\bar{L}$ is time endowment, l_m is time spent working in the market sector, l_n is time spent in non-market housework, c_m is consumption of the market good, and c_n is consumption of the home produced non-market good. We assume that the non-market good is produced according to $c_n = s_n l_n;\ s_n > 0$. Notice that as in Benhabib, Rogerson and Wright (1991), we suppose that working in the market sector gives positive direct utility, presumably because one enjoys certain social interactions and types of mental stimulation at the work place that one does not get by devoting all of one's time to leisure and homework. We assume that there is a fixed positive utility value from working in the market sector (given by B') that is independent of the actual number of hours worked. In contrast, the utility value derived from housework comes indirectly from consuming the home-produced good generated by the time input into the non-market sector.

To ensure that every living person in the economy spends a positive amount of time working in the market in order to facilitate aggregation, we make the assumption that the direct utility value from spending a positive amount of time in the market is sufficiently large. (See Assumption 1 below.)

The agent maximizes

$$\int_t^\infty \{\log[(c_m(s,\kappa))^\mu (c_n(s,\kappa))^{1-\mu}] + A' \log[\bar{L} - l_n(s,\kappa) - l_m(s,\kappa)] + B'\} \exp^{-(\theta+\rho)(\kappa-t)}\, d\kappa$$

subject to

$$\begin{aligned} c_n(s,t) &= s_n l_n(s,t), \\ \frac{dw(s,t)}{dt} &= [r(t)+\theta]w(s,t) + v(t) l_m(s,t) - c_m(s,t), \end{aligned}$$

and a transversality condition that prevents agents from going indefinitely into debt. (The homogeneous market good is used as the numeraire.) As in Blanchard (1985), agents save or dissave by buying or selling actuarial bonds, that is, bonds that are cancelled by death. Here, ρ is the subjective rate of time discount, θ is the constant instantaneous probability of death so θ^{-1} is the expected remaining life, $w(s,t)$ is non-human wealth at time t of an agent born at time s, and $v(t)$ is wage rate.[6] The rate of interest on actuarial bonds is $r(t) + \theta$.

From the optimal choice of c_m, c_n, l_m, and l_n, we obtain, after some manipulation, the following two relationships:

$$\frac{\mu v}{c_m} = \frac{A'}{\bar{L} - l_n - l_m}, \tag{55}$$

$$\frac{(1-\mu)s_n}{c_n} = \frac{A'}{\bar{L} - l_n - l_m}. \tag{56}$$

[6] We assume that the wage per hour worked in the market is independent of the age of the agent.

Using these two equations to get $c_n/c_m = (1-\mu)s_n(\bar{L}-l_m)[(A'+(1-\mu))h]^{-1}$, and using $c_n = s_n l_n$, we further obtain $l_n = (1-\mu)(A')^{-1}[\bar{L}-l_n-l_m]$. We can then eliminate l_n and c_n and write the individual's intertemporal optimization problem simply as

$$\text{Maximize} \quad \int_t^{\infty} \{\log c_m(s,\kappa) + A^{-1}\log[\bar{L}-l_m(s,\kappa)] + B\} \exp^{-(\theta+\rho)(\kappa-t)}\, d\kappa$$

subject to

$$\frac{dw(s,t)}{dt} = [r(t)+\theta]w(s,t) + v(t)l_m(s,t) - c_m(s,t), \tag{57}$$

where

$$\begin{aligned} A^{-1} &\equiv \mu^{-1}[A'+(1-\mu)], \\ B &\equiv \mu^{-1}(1-\mu)\log\left[\frac{(1-\mu)s_n}{A'+(1-\mu)}\right] + \mu^{-1}A'\log\left[\frac{A'}{A'+(1-\mu)}\right] + B'\mu^{-1}. \end{aligned}$$

We make a notational change in main text and drop the "m" subscript to denote market work. Thus, $l(s,t)$ replaces $l_m(s,t)$ with the understanding that $l(s,t)$ is the individual's supply of labor to the market sector.

We make the following assumption:

Assumption 1: $B > A^{-1}[\log \bar{L} - \log(\bar{L} - 0^+)]$.

Under Assumption 1, a very wealthy individual who might have chosen to retire in a model without a positive utility value from market work spends a very small positive amount of time working in the market ($l_m = 0^+ > 0$) given the positive utility value of market work compared to housework in our model.

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