2007

Bundling Information Goods: The Case of E-Journals

Yong TAN
Singapore Management University, yong.tan.2006@smu.edu.sg

Bundling Information Goods: The Case of E-Journals

Tan Yong

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN ECONOMICS

Singapore Management University

School of Economics

2007

Abstract

Bundling Information Goods: The Case of E-Journals

Tan Yong

With the development of the Internet, e-business has become popular. Increasingly, e-journals are being sold via the Internet. E-Journals have two main characteristics: one is the low marginal cost associated with access; the other is the large number of items. For the commercially motivated seller, the issue of bundling a large number of low marginal cost items so as to maximize profits needs to be dealt with. In this thesis, a solution by way of an intermediate bundle is proposed. It is found that the profit obtained under the proposed procedure is 4% to 5% higher than that under the Chuang-Sirbu procedure, which is currently adopted by many sellers. Furthermore, when the number of products involved is not extremely large, the proposed procedure yields a profit level that is closer to the first price discrimination profit level than the Armstrong two-part tariff procedure. In this thesis, a heuristic rule to facilitate the determination of the optimal intermediate bundle size is also proposed. This is designed to avoid the lengthy simulation procedure that will be needed otherwise.

Contents

Acknowledgements

My foremost thank goes to my thesis adviser Professor Vincent Chua. I appreciate him for his patience and encouragement that carried me on through difficult times, and for suggestions that helped to finished the thesis. His valuable feedback contributed greatly to this dissertation.

I am grateful to my examiner Prof Gea M. Lee, his valuable feedback helped me to improve the dissertation in many ways.

I also thank all the Professors that taught at SMU and WISE, Xiamen University.

It is them who helped me build up my knowledge in Finance, and Economics.

I thank all the classmates at SMU whose presences and fun-loving spirits made the otherwise grueling experience tolerable. I also thank all my classmates at WISE, Xiamen University.

Last but not least, I thank those who love me for always being there when I needed them most, and for supporting me through all these years.

Chapter 1

Introduction

Product bundling refers to the business practice whereby a seller sells two or more heterogeneous products or services in a bundle while charging a single price. Practical examples of bundling include Microsoft Office, which contains a set of stand-alone products such as MS-Word, Powerpoint, and Excel; Yahoo! Music, to which a consumer may listen to any item of music tracks within a month by subscribing for a fee of $4.99; and, in the more traditional setting, Pepboys Auto’s Maintenance service, in which an automobile owner can have both oil change and tire rotation done at a single discount price.

The bundling literature offers three major reasons for a seller to bundle products. First, there is cost saving argument. For instance, it has been noted in Nalebuff (2004), that “(*i)*n a larger sense, almost everything is a bundle product. A car is a bundle of seats, engine, steering wheel, gas pedal, cup holders, and much more. An obvious explanation for many bundles is that the company can integrate the products better than its customers can.”

In addition to reducing the integration cost, bundling may also reduce transaction and distribution costs involved in the selling process whenever seller-side bundling simplifies the shopping and shipping processes [See, for instance, Salinger (1995), Bakos and Brynjolfsson (2000)]. Whenever cost-saving via bundling is feasible, the bundle in question has an intrinsic cost-advantage over its individual items, thus the practice of bundling here may result in the seller enjoys a higher margin.

Though often mentioned in the literature, the cost-saving argument received little attention in theoretical expositions due to its straightforwardness. Instead, the theoretical literature has focused attention on those cases where bundling itself does not offer any cost advantage. In particular, the focus has been on the use of bundling as a tool for price discrimination and as a tool in competition, the former typically addressed under a monopoly setting while the later in a duopoly setting.

Earlier research on the use of bundling for price discrimination argues that bundling reduces heterogeneity in buyers' valuations which in turn enables a monopolist to better capture the consumer surplus when marginal production cost is not high [See, for instance, Stigler (1963), Adams and Yellen (1976)]. This logic is well illustrated in instances where buyers' valuations over two products are negatively correlated.

While earlier studies on bundling two products are more concerned with *when* the monopolist should bundle, a number of recent papers shift the focus to *how* to bundle—namely, what bundling strategy should a seller adopt (unbundling, pure bundling, or mixed bundling), and the structure of prices the seller should choose under each of these strategies. When dealing with business practices on the e-platform, owing to the large number of items involved in the typical situation, it has all the more become necessary to deal with this problem of *how* to bundle. Many papers have attempted to shed light on this issue. These include Hanson and Martin (1990), Armstrong (1999), Bakos and Brynjolfsson (1999) and Chang and Sirbu (1999) and different strategies have been proposed. This thesis adds to the literature by proposing a different albeit simple bundling strategy through which a seller may be able to enhance its business performance. In particular, a three-bundle strategy is proposed.

In terms of exposition, the thesis is organized as follow. In the main, Chapter 2 discusses to some detail the papers just mentioned. While the emphasis here is on bundling as a tool for price discrimination, a brief outline of bundling as a competitive tool is also provided. Chapter 3 outlines the basic Chuang-Sirbu model and the extension that this thesis proposes. In Chapter 4, in addition to reporting the main results of the study, a comparison of the business performance of the proposed strategy and other existing procedures is also carried out. Chapter 5 concludes with further research directions.

Chapter 2

Review of Selected Literature

In this review, the main focus is on bundling as a tool for price discrimination. The emphasis here is especially on those papers that relate to the bundling of a large number of products. This survey also briefly covers bundling as a tool for competition.

First of all, as mentioned, there are three strategies frequently cited and compared in the literature: pure unbundling, pure bundling; and mixed bundling. Pure unbundling refers to the business practice of selling all goods or items separately in the market; pure bundling, on the other hand, refers to the selling of all goods/items in a single package; and mixed bundling refers to the business practice of doing both, that is, offering customers the opportunity to purchase single items as well as the opportunity to purchase the entire bundled package. This obviously refers to the two-product case. If there are more than two products, the situation will obviously be more complicated. Potentially, mixed bundling in this other instance could involve 2^N-1 bundles altogether.

2.1 Bundling as a Tool for Competition

When rivals exist, the reason for a seller to bundle products is often to use bundling as a competition tool. Indeed, this role of bundling is widely publicized in both business and in academic community following the trials on U.S. vs. Microsoft, where Microsoft is alleged to have use its monopoly power (and later judged so in several cases) in personal computer (PC) operating systems to unfairly compete with rivals in other software markets, such as the web browser market.

There are two major research strands on using bundling as a competition tool. The first strand relates to what is termed as "leverage theory" and is typically discussed in a setting involving two products and two *ex ante* asymmetric sellers: one seller has monopoly power over one product and competes with the other seller in the second product market. The research is on whether the first seller can leverage its monopoly power in the first product market to gain competitive advantage over (or even foreclose) its rival in the second product market. In this strand of research, the term "tying" is frequently used in place of "bundling".

Ward S. Bowman, Jr (1957) and Burstein (1960) examine both litigated and non-litigated industrial examples and conclude that leveraging is an implausible explanation for most of them. Even in those cases where leverage is not implausible, they conclude that price discrimination is a more plausible explanation.

In recent years there has been a dramatic shift in opinion as to whether leveraging is effective for the first seller. Beginning with the seminal paper by Whinston (1990), however, there has been a series of papers that shifted the setting of the second product market from a competitive one to a duopoly one. As pointed out by Whinston, "… tying may be an effective (and profitable) means for a monopolist to affect the market structure of the tied good market by making continued operation unprofitable for tied product rivals". Whinston points out that the reason why in the previous papers, those authors think there is no leverage effect of tying is because of an implicit assumption: there is a *monopoly market plus a competitive market*. He says: "with this assumption, the use of leverage to affect the market structure of the tied good market is actually impossible." But, in contrast to the previous paper, if the assumption is changed to one involving a monopoly market and a duopoly market, tying sales can affect the market structure.

The second major research strand on using bundling as a competition tool considers two *ex ante* symmetric sellers [See, for instance, Matutes and Regibeau (1992), Chen (1997)]. Having one seller beating the other one is not the research focus. Instead, this strand focuses on two questions. First, will sellers adopt bundling in equilibrium? So far the results are mixed. Second, can the option of bundling (compared to unbundled selling only) increase profits of both sellers? In an insightful paper, Chen (1997) gives an affirmative answer—he shows that bundling can be an effective price differentiation mechanism, which helps to avoid price wars.

We now leave this brief review of the use of bundling as a tool for competition to focus attention on the aspect of bundling that is directly related to the thesis research, namely, the use of bundling as a tool for price discrimination.

2.2 Bundling as a Tool for Price Discrimination

This is another reason for a seller to engage in product bundling, even if there is no leverage effect or competitive advantage from a bundling strategy, a seller may still use bundling to enhance its profits. This is a form of price discrimination, which is distinct from traditional price discrimination. The ensuing elaborates on this idea.

2.2.1 The Two-Product Case

Stigler (1963) is one of the earliest professors who suggest the bundling problem in reality. He observed that the phenomenon of block-booking of movie—the offer of only a combined assortment of movies to an exhibitor—has been the subject of several antitrust cases at that time. And, he finds the explanation of the practice of block-booking is not because of leveraging market power, just a way to increase profit. Further more, he points that, negative correlation in reserve price can make bundling be a better than unbundling for seller.

Based on Stigler's work, Adams and Yellen (1976) consider a monopolist producing two goods with constant unit cost and facing buyers with diverse tastes. Further more, they assume the marginal utility of a second unit of either good to be zero for all buyers. Under these setups they find the conditions under which pure bundling is more profitable than unbundling are: (a) consumers' reserve prices for the two goods should be negatively correlated; and (b) the marginal cost of the goods should not be too high.

However, researchers soon discover that, for a monopolist, negatively correlation in reserve prices is not a necessary condition for bundling to be profitable. Schmalensee (1984) opts for a special form of joint valuation distribution—the bivariate normal distribution, that is analytically less daunting (yet numerical analysis is still needed to find results even in the case). The basic model adopt by Schmalensee (1984) is as follows. Let $F(v_1, v_2)$ be a symmetric bivariate normal distribution with mean μ , standard deviation σ , and correlation coefficient ρ . Where v_1, v_2 is consumer's reserve price to the product 1 and product 2 separately, here assume v_1, v_2 follows same distribution, so E(v_1) = E(v_2) = μ and $\text{var}(v_1) = \text{var}(v_2) = \sigma$. Then he calculates the maximized profit under the three different strategies—pure unbundling, pure bundling, and mixed bundling. Next step, he compares the maximized profit under different strategies to find the optimal strategy. And his conclusion is that: pure bundling can be better than unbundling for the seller even if the valuations of both goods are positively (yet not perfect) correlated. The reason is that, pure bundling can reduce the heterogeneity of the reserve price.

Complementary and substitutive goods are considered by many researchers, they try to find out which strategy is optimal under the situation of bundling complementary and substitutive products respectively. To answer the question, Venkatesh and Kamakura (2003) develop an analytical model of contingent valuations and address two questions of import to a monopolist: (1) should a given pair of complements or substitutes be sold separately (pure components), together (pure bundling), or both (mixed bundling), and at what prices? (2) How do optimal bundling and pricing strategies for complements and substitutes differ from those for independently valued products? In Venkatesh and Kamakura's model they only consider two goods, and they use θ to denote a consumer's degree of complementarity or substitutability. θ = (Reservation price for bundle 12-sum of stand-alone reservation prices for product 1 and 2).

$$v_{12} = (1+\theta)(v_1 + v_2)$$

Where v_1 and v_2 are the reserve price for separate good 1 and good 2; and v_{12} is the reserve price for the bundle includes good 1 and good 2 in it. For complementary goods 1 and 2, θ should be positive, and for substitutive goods θ should be negative. Then he use the same way with Schmalensee: calculates the maximized profit under the three strategies, then they compare them to find which strategy is the optimal strategy with the changing of θ.

2.2.2 The *N*-Product Case

While earlier studies on bundling two products are more concerned with *when* to bundle, a number of recent papers have shifted focus to *how* to bundle—namely, what bundling strategy a seller should pick (unbundling, pure bundling, or mixed bundling), and what price the seller should choose for each sub-bundle or individual product. Answering these questions requires the researcher to deal with more than two products, which is the norm, rather than the exception, in practice. The literature takes two routes in dealing with the “how” question. One strand of research tries to provide numerical solutions to optimal mixed-bundling prices using integer programming approaches [See, for instance, Hanson and Martin (1990), Armstrong (1999), Hitt and Chen (2005)]. Computational complexity is a major challenge to this strand of research. The other strand focuses on getting analytical results for pure bundling or very simple forms of mixed bundling [See, for instance, Bakos and Brynjolfsson (1999), (2000a, 2000b)]. One important result for this strand of research is that bundling can be surprisingly profitable and at the same time extremely simple.

Integer Programming (IP) and Customized Bundling

For the first strand of research *a la* Hanson and Martin (1990), the details are as follows. Let N be a finite number and let there be at most M buyer types, M finite. Let β_j denote the percentage of type j buyers in the whole buyer population, where $j \in M$. Any type j buyer has a constant valuation vector ($v_{1j}, v_{2j}, ...v_{Nj}$), where v_{ij} is the reserve price of consumer j to product i . Let B be the set of all possible sub-bundles under mixed bundling. The size of B is 2^N-1. A type j buyer’s valuation of sub-bundle $b \in B$ is $v_j^b = \sum_{i \in b} v_{ji}$. The marginal cost for the seller to provide sub-bundle $b \in B$ is $c^b = \sum_{i \in b} c_i$, where c_i is the marginal cost for product i. Let $x_j^b = 1$ if a buyer of type j buys sub-bundle b, and 0 otherwise. Now the optimal bundling pricing problem can be formulated as the following IP problem: (M consistent)

IP problem: $$\max_{\{p^b : b \in B\}} \sum_{j=1}^{M} \sum_{b \in B} x_j^b (p^b - c^b) \beta_j$$

Constraints:

(No arbitrage) $p^{b1} + p^{b2} \geq p^{b3}$, where $b_1, b_2, b_3 \in B$, $b_1 \bigcap b_2 = \Phi$, and $b_1 \bigcup b_2 = b_3$

(If no arbitrage condition is hold given $b_1 \bigcap b_2 = \Phi$, it is easy to show it will be still hold under $b_1 \bigcap b_2 \neq \Phi$.)

(Unit demand) $x_j^b \in \{0,1\}$, where $j \in M, b \in B$

(Buys at most one bundle) $\sum_{b \in B} X_j^b \leq 1$, where $j \in M$

(IR) $x_j^b = 0$ if $\left(v_j^b - p^b\right) < \max\{0, \max\{(v_j^{\hat{b}} - p^{\hat{b}}) \mid \hat{b} \in B\}\}$, otherwise $x_j^b = 1$ where $j \in M, b \in B$

(non-negative price) $p^b \geq 0$, where $b \in B$

Solving this IP problem gives the seller the optimal prices for every possible sub-bundle. The main difficulty with this approach is its computational complexity. Notice that there are *M* times (2^N-1) IR constraints, the number of which explodes when either the number of buyer types *M* or the number of products *N* increases. Hanson and Martin (1990) consider the case where a buyer's payment depends only on how many products, not what products, she buys. Therefore the seller can only have at most *N* different bundle prices. They further assume that the number of buyer types *M* is much smaller than *N*. Finally, Hanson and Martin only solve the problem when *N* is less or equal to 21. When *N* continually increases, it is time-consuming to get the result with IP approach.

The customized bundling method is proposed to simplify the IP approach. Customized bundling is a bundling strategy which gives consumers the right to choose any bundle of products of size *q* (*q* running from 1 through *N*) from a large pool of *N* different goods, the price of which will depend only on the size of the bundle. Such an approach will be reasonable in the case of information goods where the marginal cost of providing each of the differentiated goods is roughly similar. The advantage of a customized bundling strategy is that it can increase profit for seller without the need to offer too many sub-bundles. In fact a seller only needs to consider N potential bundles instead of offering all (2^N-1) sub-bundles. Furthermore, Hitt and Chen (2005) prove that, in some situations, the full bundling problem can be reduced to a customized bundle problem, and even if the problem cannot be reduced to a customized bundle problem, the profit the seller can get by using customized bundling will be closed to that which could be extracted under a full bundling strategy. This greatly simplifies the complexity of the problem.

Restricting the Choice of Bundles

To overcome the computational intractability, the customized bundling method is not good enough when N is large. In fact, when N is very large, it is still as time-demanding to get to the computational results when the customized bundling method is considered. Further restriction on the choice of bundles is proposed here to overcome the computational intractability when N is large. For this method, a seller only offers some sub-bundles, and not all N bundles; consumers only choose to buy from these offered bundles. This method can significantly help overcome the computational intractability problem since fewer sub-bundles are now being offered. The two-part tariff procedure proposed by Armstrong (1999) is one such procedure.

The Armstrong Two-Part Tariff Procedure

Armstrong (1999) suggests a two-part tariff procedure to avoid intractable computation, which can be considered as a method of dealing with the large number of products' case.

Specifically speaking, two-part tariff is that a buyer can choose any product she wants; if a buyer decides to buy at least one product, the seller charges her a fixed fee, *p*, plus the sum of marginal costs of all products this buyer purchases. For example, if a buyer buys products 1, 4, and 5, the price she pays is $p + c_1 + c_4 + c_5$. Where c_i is cost of product i. Therefore this two-part tariff is a specific pricing schedule for mixed bundling.

Given such a pricing plan and if a buyer decides to pay *p*, she will buy product *i* if and only if $v_i > c_i$. Where v_i is the reserve price for product *i*. For convenience define $\hat{v}_i = \max\{v_i - c_i, 0\}$. Then, a buyer will pay the fixed fee p if and only if $\sum_{i=1}^{N} \hat{v}_i \geq p$.

Denote $Y = \sum_{i=1}^{N} \hat{v}_i$, where *N* refers to the number of products. Let the mean and variance of Y be μ_Y and σ_Y^2, respectively. Note that μ_Y is the upper bound of the profit the seller can get (i.e. profit under first-degree price discrimination. Under first-degree price discrimination, seller can extracts all consumers' surplus, and the maximum of profit the seller can get from each consumer is $\mu_Y = E(Y)$ on average. Because the seller cannot clearly observe each consumer's exact reserve price, instead of that, the seller only knows the distribution of the con sumers' reserve price, so the profit the seller can extract is π, which satisfies $\pi < \mu_Y$). If the seller sets $p = (1-\varepsilon)\mu_Y$, then he can get a profit of

$$\begin{aligned}\Pi &= (1-\varepsilon)\mu_Y \bullet \Pr ob(Y > (1-\varepsilon)\mu_Y) \\ &\geq (1-\varepsilon)\mu_Y \bullet \Pr ob(|Y-\mu_Y| > \mu_Y) \\ &\geq (1-\varepsilon)\mu_Y \bullet [1-(\sigma_Y / \varepsilon\mu_Y)^2] \\ &\geq \left[1-\varepsilon-(\sigma_Y / \varepsilon\mu_Y)^2\right]\mu_Y\end{aligned}$$

Set $\varepsilon = (\sigma_Y / \mu_Y)^{\frac{2}{3}}$, the above inequality becomes

$$1 \geq \Pi / \mu_Y \geq 1-2(\sigma_Y / \mu_Y)^{\frac{2}{3}} \quad (1)$$

Therefore, the seller's profit from this two-part tariff bundling can come close to μ_Y if σ_Y / μ_Y is small enough. For any given number of products, N, the seller can calculate σ_Y / μ_Y to determine the effectiveness of this bundling approach (in fact there is a potential assumption: the taste parameter v_i should be independent distribute across different products). This bundling approach is especially promising when the number of products is very large.

When $N < \infty$, let $\underline{\mu} = \min\{E\left(\hat{v}_i\right) | i \in N\}$ and $\bar{\sigma}^2 = \max\{Var(\hat{v}_i) | i \in N\}$, where $\bar{\sigma}$ >0. Then, $(\sigma_Y / \mu_Y)^2 \leq \left(N \bar{\sigma}^2\right) / (N^2 \underline{\mu}^2) = (\bar{\sigma} / \underline{\mu})^2 / N$ and from equation (1):

$$1 \geq \Pi / \mu_Y \geq 1-2(\bar{\sigma} / \underline{\mu})^{\frac{2}{3}} / N^{\frac{1}{3}} \quad (2)$$

When $N \to \infty$, equation (2) implies $\Pi \to \mu_Y$. In other words, when the number of products is very large, this simple two-part tariff enables the seller to get approximately the first-degree price discrimination profit.

This two-part tariff procedure, nevertheless, has one shortcoming: seller's profit converges to the first price discrimination profit level at a rate of $N^{\frac{1}{3}}$ which is very slow.

The proportion of first-best profits obtainable using Armstrong's procedure is as reported in Armstrong (1999):

Table 1: The Percentage Profit Extraction under Varying N

N = 10	N = 100	N = 1000	N = 10,000
40%	72%	87%	94%

These results suggest that, in general, it is better to apply the two-part tariff strategy only when the number of products for sale is very large. An example of such instances is the case of Yahoo! Music that has in excess of one million songs on its server. When the number of products is small there is a significant discrepancy between the profits under the two-part tariff procedure and under first-price discrimination.

A special case of the two-part tariff procedure is when all products have zero marginal costs [See Bakos and Brynjolfsson (1999, 2000a, 2000b), Geng et al. (2005)]. This would be a reasonable assertion when, for instance, information goods such as online news are considered where the cost of duplicating or accessing information is virtually zero. In this special case, the simple mixed-bundling pricing degenerates to an even simpler pure bundling pricing, and the two-part tariff degenerates to a single fixed fee, p, which can be viewed as the bundle price for the bundle of all products.

Specifically, Bakos and Brynjolfsson (1999a) consider a single seller providing N information goods to a set of consumers Ω. Each consumer demands either 0 or 1 units of each information good, and resale of these goods is not permitted (or is prohibitively costly for consumers). Valuations for each good are heterogeneous among consumers, and for each consumer $\omega \in \Omega$, they use $v_{Ni}(\omega)$ to denote the valuation of good i when a total of N goods are purchased. It is allowed that $v_{Ni}(\omega)$ depends on N so that the distributions of valuations for individual goods can change as the number of goods purchased change. The practical example is that: these goods are complementary or substitutive, when a consumer buys different quantity goods, the reserve price for a specific item will change, for complementarity, the reserve price for a specific item will increase with the increasing of the purchasing of other goods; and for substitute, the case is reverse.

Let $x_N = \frac{1}{N}\sum_{k=1}^{N} v_{Nk}$ be the mean (per-good) valuation of the bundle of N information goods. Let p_N^*, q_N^* and π_N^* denote the profit-maximizing price per good for a bundle of N goods, the corresponding sales as a fraction of the population, and the seller's resulting profits per good. Assuming the following conditions hold:

"A1: The marginal cost for copies of all information goods is zero to the seller.

A2: For all N, consumer's valuation v_{Ni} is independent and uniformly bounded, with continuous density functions, non-negative support, mean μ_{Ni} and variance σ_{Ni}^2.

A3: Consumers have free disposal. In particular, for all $N > 1$, $\sum_{k=1}^{N} v_{Nk} > \sum_{k=1}^{N-1} v_{(N-1)k}$."

Under these conditions, it can be shown that selling a bundle of all N information goods can be remarkably superior to selling the N goods respectively. For the distribution of valuations underlying most common demand functions, bundling substantially reduces the average deadweight loss and leads to higher average profits for the seller. As N

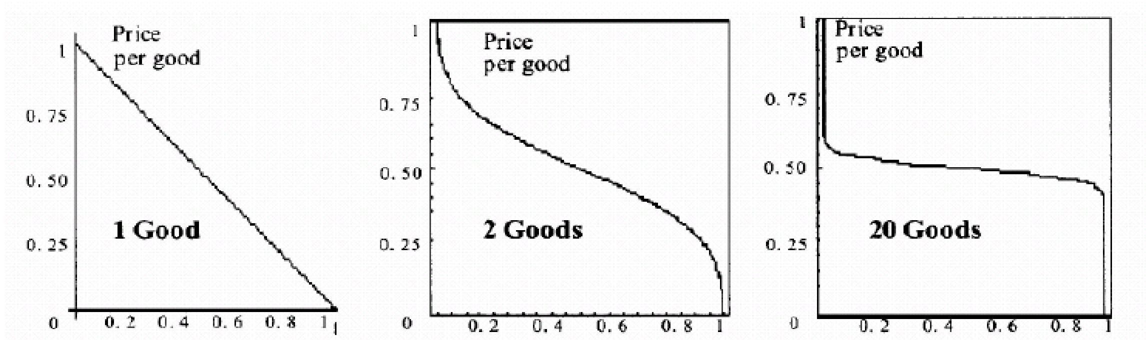
increases, the seller captures an increasing fraction of the total area under the demand curve, correspondingly reducing both the deadweight loss and consumers' surplus relative to selling the goods separately.

Since the demand curve is derived from the cumulative distribution function for consumer valuations, it becomes more elastic near the mean, and less elastic away from the mean. Figure 1 which is reproduced from Bakos and Brynjolfsson (1999) illustrates this for the case of linear demand for the individual goods, showing, for instance, that combining two goods each with a linear demand produces a bundle with an s-shaped demand curve. As a result, the demand function (adjusted for the number of goods in the bundle) becomes more "square" as the number of goods increases. The seller is able to extract as profits an increasing fraction of the total area under this demand curve, while selling to an increasing fraction of consumers.

Though Bakos and Brynjolfsson (1999) started this strand of research, their analysis is significantly flawed, as pointed out by Geng et al. (2005). Gent et al. 2005 consider bundling of information goods with decreasing values, such as when buyers consume products sequentially along the timeline and when a discount factor exists.

> Geng *et al*. point out that, when the number of products, *N*, goes to infinity, it is conceivable that μ_Y / N converges to zero—otherwise the bundle will be infinitely valuable and pricey, which is never the case in reality. In this case all average measures converge to zero and thus are not useful in deriving pricing suggestions. Instead, Geng et al. argues that the correct measures are the ones on the complete bundle, such as P and Π in the discussion above.

Figure 1: Quantity for Bundle as a Fraction of Total Population



The chapter reviews the literature on bundling as a tool for competition and a tool for price discrimination, especially on the bundling strategies for a large number of items. Of course there are other reasons for a seller to engage in bundling but these are of peripheral interest to the present work and thus not reviewed here. To understand how the bundling strategy is applied in reality, the next chapter describes a basic model which is an application of bundling strategy in e-journals.

Chapter 3

The Chuang–Sirbu (CS) Model and An Extension

Academic journals have traditionally been sold in the form of hard copies and by way of subscriptions. Individual articles are bundled into journal issues; issues are bundled into subscriptions. This aggregation approach has worked well in the paper-based environment because there exists strong economies of scale in the production, distribution and transaction of journals. However, with the global expansion and rapid commercialization of the Internet, the economics of journal publishing has been rapidly transformed. Many publishers are experimenting with various forms of on-line access to their journals. It is now technically feasible for the publisher to electronically deliver, and charge for, individual journal articles requested by a customer. The deployment of micropayment services, in particular, will dramatically lower the cost of purchasing digital information goods over the Internet. From the scholars' perspective, this form of access is instantaneous, on-demand, and can avoid the cost associated with traditional library access, such as travel to the library, physical duplication of the article, and congestion due to shared use of journals. But in order to make network-delivery of journal articles become a reality, economic incentives must exist for publishers to unbundle their journals, or offer some sub-bundles whose size are not necessarily equal to the total number of articles.

3.1 The Basic Model

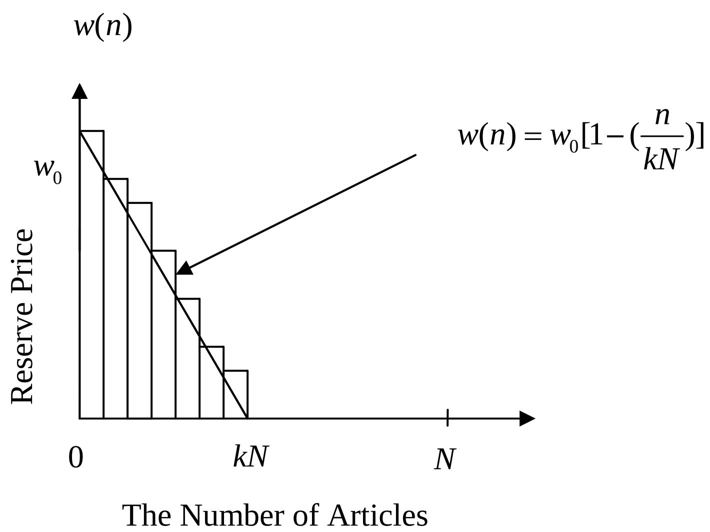
In Chuang and Sirbu (1999), or CS for short, they consider an N-good bundling model with multi-dimensional consumer preferences in order to study the key factors that determine the optimal bundling strategy. In this paper the N-dimensional goods are academic journals.

By developing an N-goods bundling model with multi-dimensional consumer preferences, CS seek to demonstrate the existence of such incentive, and quantify how this incentive to unbundle is affected by readers' journal-reading behavior. They employ two variables, w_0 and k, to describe the N-dimensional consumer preference. They allow each journal reader to rank all the N articles in the journal in such a decreasing order of preference as his/her favorite article is ranked first, the least favorite is ranked last. And weak monotonicity is observed. The reader may place zero value on any number of the N articles. By assuming a linear demand function for all positive-valued articles, they plot an individual reader's valuation of all the articles in the journal axis. The individual's most highly valued article has $n = 0$, and so its intercept, w_0 on y-axis, represents the willingness-to-pay for his/her favorite article. The valuation for the subsequently ranked articles is assumed to fall off at a constant rate until it reaches zero at $n = kN$. No articles have negative value on the assumption of free disposal—readers are free to discard unwanted articles at zero cost. The variable k dictates the slope of the demand curve, and it also indicates the fraction of articles in the journal that has non-zero value to the individual. If an individual's k is greater than unity, that means he/she places positive value on all N articles in the journal.

Formally, an individual's valuation for the nth article can be expressed as:

$$w(n) = \max\left\{0, w_0[1-(\frac{n}{kN})]\right\} \ , \quad 0 \le n \le N-1 \qquad (3)$$

Figure 2: The Ranked Valuations



Assuming that w_0 follows the uniform distribution in the unit interval [0, 1] and k follows the exponential distribution, then the survey conducted by Griffiths in 1995 is used to fit the critical variable in exponential distribution. Until now, CS have used two specific variables to control the heterogeneity of consumers. The last step is to get the optimal profit under pure bundling, pure unbundling and mixed bundling strategies respectively with numeric method (here, pure bundling means all articles be sold in a journal; pure unbundling means all articles are sold individually; mixed bundling means both journal and individual article are sold at the same time). Then the conclusion is that mixed bundling is superior to pure bundling and pure unbundling.

3.2 An Extension of the CS Model

In CS (1999), they only resolve the optimal problem for the two-bundle situation. However, an important question that is often difficult to answer is: how does the maximized profit change if the seller of the articles can offer more bundles? In this thesis, the situation of the seller employing a three-bundle strategy is considered, that is, in addition to individualized selling (the first kind of bundle) and selling the entire journal (the second kind of bundle), the seller offers an intermediate bundle of size *n* (the third kind of bundle), where *n* is an integer that lies between 1 and *N* (Note that *N* refers to the total number of articles).

When employing this proposed strategy, several considerations are relevant. To maximize profits, the optimal intermediate bundle size n^* and the price structure [the price for a single article P_1, the price for the intermediate bundle (n^*articles) P_{n*}, and the price for the entire journal P_N] need to be known. Additionally, when the seller offers three bundles, the question as to how profits will compare with that under he CS procedure and with the situation under first price discrimination profit becomes relevant and whether there is an efficient and simple way to find the optimal $n*$ also have to be considered.

3.3 Description of the Proposed Method

It is impossible to find the optimal $n*$ and price structure analytically. Consequently, this thesis resorts to simulation for solutions. This simulation exercise involves the following:

(a) Choice of marginal cost for each article: the empirical marginal cost is between $0.05 and $0.5, so the marginal cost I choose is 0.3—nearly the mean of the empirical cost.

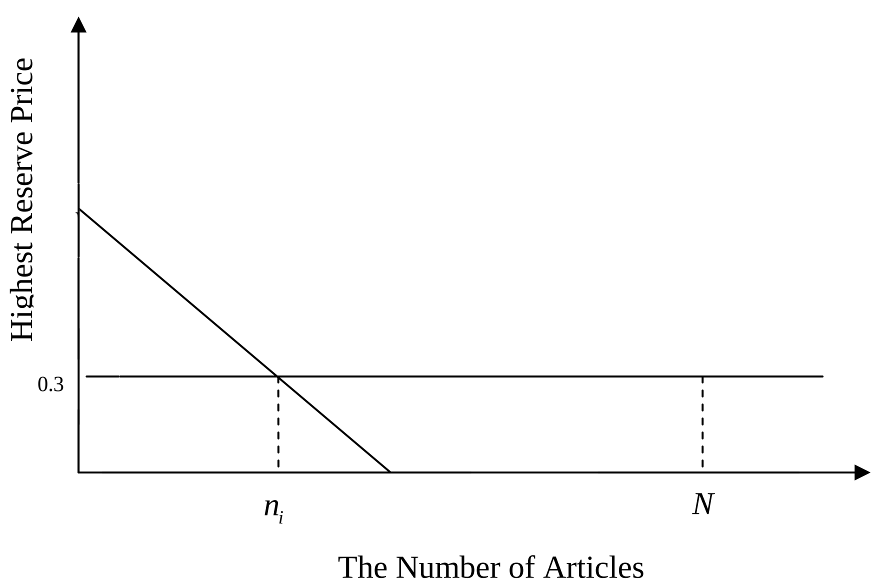
(b) Random selection of highest reserve price, w_{0i} (w_{0i} is the *i*-th consumer's highest reserve price): selecting w_{0i} independently, with a uniform distribution [0.3, 1.3]. Thus for each i = 1, 2, … , *M*,

$$\begin{aligned} f(w_{0i} = m) &= 1, m \in [0.3,1.3] \\ f(w_{0i} = m) &= 0, m \notin [0.3,1.3] \end{aligned} \quad (4)$$

Where $f(\bullet)$ is a density function of w_{0i}. The reason for the choice of 0.3 as the bottom value for the reserve price is that the seller always takes account of those consumers whose highest reserve price is higher than the marginal cost of the article. Because it is unprofitable for a seller to sell his articles to a consumer with a price less than 0.3 no matter in individual selling or bundling selling, They would never set a price for any individual article lower than 0.3, no matter in individual selling or in bundle selling. It means no articles will be sold to a consumer whose highest reserve price is less than 0. Therefore, it is redundant for the seller to consider such consumers whose highest reserve price is lower than 0.3.

(c) Random selection of k_i, but k_i is not generated directly. Instead, n_i is selected independently from a uniform distribution [0, N], where n_i is the number of articles that the i-th consumer wants to buy, given the individual price equals to the marginal cost (given there is only individual selling), and N is the number of total articles in a journal. (See Figure 3 below)

Figure 3: The Correlated Relationship Between k_i and n_i



Since $w_{0i}(1-\frac{n}{k_i N}) = 0.3$, therefore $k_i = \frac{n \bullet w_{0i}}{N \bullet (w_{0i} - 0.3)}$.

(d) Choice of N, the number of total articles in a journal: Let N = 20, 21 ……, 39. The reason for these choices of N is that: in Hanson and Martin's paper they only deal with the cases where the number of products $N \leq 21$, and the number of consumers is finite, M; while Armstrong's way can work well only when the number of products is extremely large. So, there exists a gap between these two procedures. The proposed method is an attempt to fill this gap.

(e) Choice of M, the number of consumers: For this exercise, the value of M is set at 1000.

The steps of the simulation procedure are summarized below:

Step 1: Generating a set of heterogeneous consumers (generate w_{0i} and k_i).

Step 2: Set n^*=1 and prices for all the three bundles equal to their marginal costs and, let consumers choose their favorite bundle, then calculate the profit the seller can get.

Step 3: Let $n^* = n^*$+1 and increase the prices for all bundles gradually by a small increment, then calculate the profit the seller can get again.

Step 4: Repeat step 3 until $n^* = N$ and at this time prices for all bundles are too high for consumers to buy (for instance, we only need to increase the individual price up to 1.3). Save all the results of the profits that can be got each time.

Step 5: Compare all the profits, and find out the optimal n^* and optimal price structure.

For information, the MATLAB codes use to implement the procedure is included as Appendix 2.

Chapter 4
The Main Results

4.1 The Optimal Intermediate Bundle Size *n**

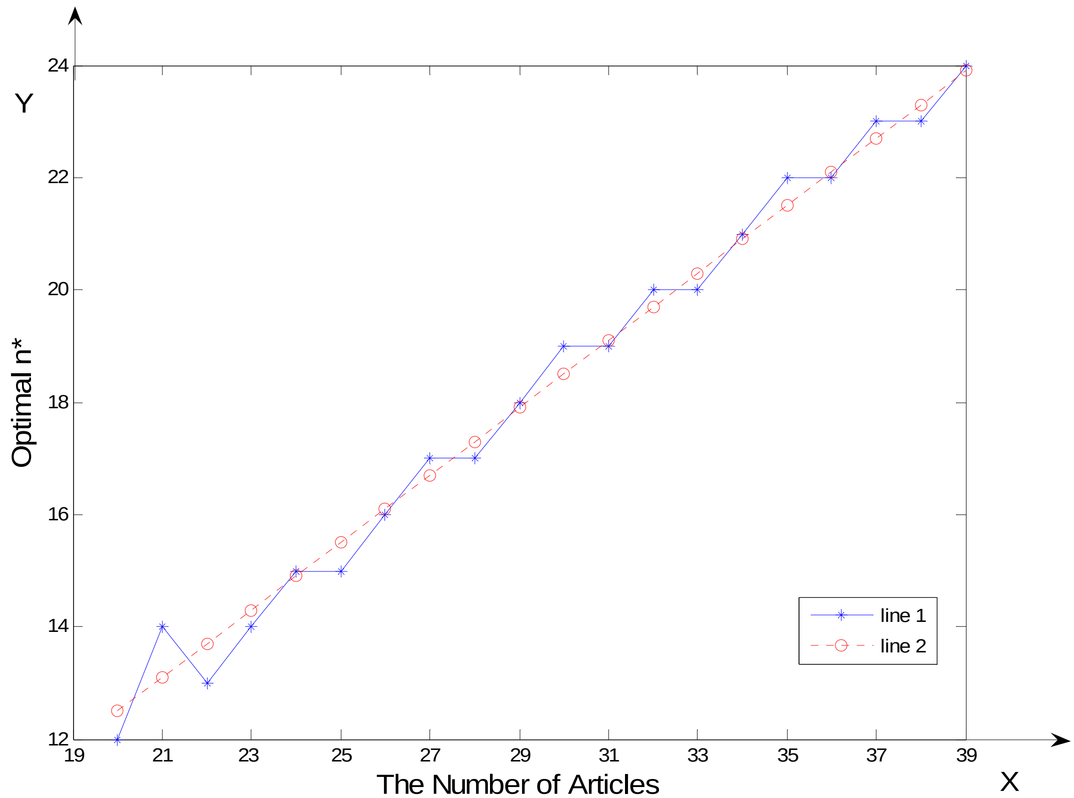
Although in the proposed procedure, the seller offers three bundles, what he only needs to do is to find out the optimal bundle size *n** for his intermediate bundle. Since the other two bundles' sizes are already given, namely, one bundle with individual article in it and one bundle with all the articles in it. According to our intuition, the optimal intermediate bundle size should be that: given the optimal number of articles that consumers want to buy follows a uniform distribution in the interval $[0, N]$, and the other two bundles, individual article's bundle and all articles' bundle, the optimal intermediate bundle size should be $n^* = 0.5N$, where N is the number of total articles. It seems that: the individual article's bundle serves consumers whose n_i is small, where n_i is defined as before; the intermediate bundle serves consumers whose n_i is moderately large; and the all articles' bundle serves consumers whose n_i is very large. However, with simulations, the result is different from our original intuition:

Table 2: The Optimal Intermediate Bundle Size with varying *N*

Total Number of Articles	*n**
20	12
25	15
30	19
35	22

The optimal intermediate bundle size is nearly $0.6N$ (more numerical details are shown in Annex Table 1 of Appendix 1) and this can be easily discerned in Figure 4 below.

Figure 4: The Optimal Intermediate Bundle Size (*n)**



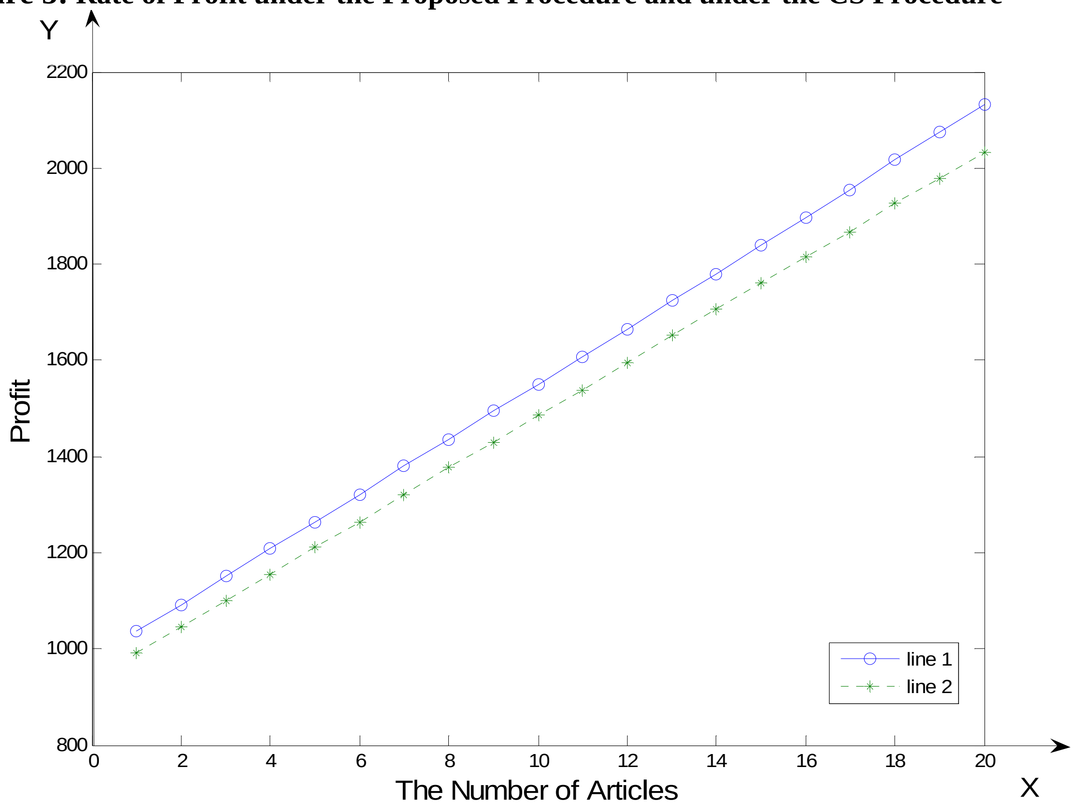
In figure 4, the x-axis label is N, the total number of articles in a journal and the label for the y-axis is n^*, the optimal bundle size for different values of N. Line 1 is the true optimal bundle size n^* obtained from simulations. And line 2 is the approximate calculation of n^* by using the formula $n^* = 0.6N$. Two points may be discerned in the results. First, the true optimal bundle size fluctuates around $0.6N$. Second, the amplitude of the fluctuations tapers off as the value of N increases. This suggests that setting the intermediate bundle size equal to $0.6N$ may be a reasonable practical approximation to the optimal bundle size n^*, that is:

$$n^* \approx 0.6N \qquad (4)$$

4.2 Comparison with the CS Procedure

With repeated simulation, in which different consumers are generated each time, the rate of profit is 4% to 5% higher than that under CS procedure. This result is not surprising and is illustrated in figure 5 below:

Figure 5: Rate of Profit under the Proposed Procedure and under the CS Procedure



In Figure 5, the y-axis indicates the rate of profit while the x-axis indicates the number of articles. Line 1 is the profit level under the proposed procedure, and line 2 is the profit under the CS procedure. It can be noticed that when the number of articles varies, the profit under my way will be always higher than that under the CS way.

Profit1 is used to denote the profit under the proposed procedure, and *Profit2* the profit under the CS procedure.

$$(profit1 - profit2)/\,profit2 \approx 4\% - 5\%.$$

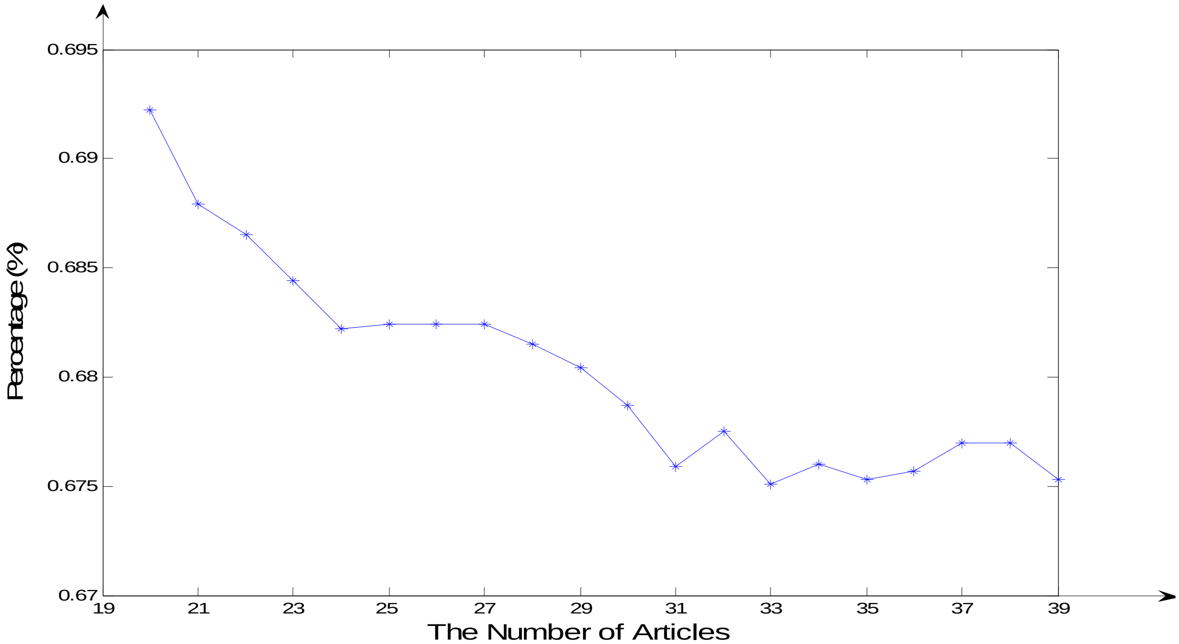
The reason is direct: In the proposed procedure the seller offers three bundles, and the profit is not less than that under the CS procedure, since if offering the intermediate bundle is not profitable, the seller would set the price for the intermediate bundle extremely high so that no one would buy it, that would make the situation the same with that under the CS procedure.

4.3 Comparison with the Armstrong Procedure

The two-part tariff procedure as proposed by Armstrong does not work well when the number of products is not too large. Specifically, when the number of products is small, there is a large gap between the profit under the two-part tariff procedure and the first price discrimination profit. Through simulations, it is found that the proportion of the profit divided by first price discrimination profit under the proposed procedure is higher than that under Armstrong's procedure in this "not too large" number case. The details provided in Annex Table 2 of Appendix 1verify this point. However, there is a decreasing trend in the rate of profit extraction as may be seen from Figure 6 below. Note that in Figure 6, the x-axis gives the total number of articles in a journal (from 20 to 39); y-axis is the proportion of the profit under three bundles' strategy divided by first price discrimination profit.

The declining trend indicates the inadequacy of using only three bundles when there are more articles in a journal, a fact that should be intuitive. In other words, if there are more goods, in order to extract the same or a higher rate of the consumers' surplus, the seller will need to offer more bundles. On the contrary, there is an increasing trend in the rate of extraction under the Armstrong procedure. Therefore, it can be expected that when the number of goods steadily increases, Armstrong's procedure will be the natural choice.

Figure 6: **The Rate of Profit Extraction under the Proposed Procedure**



4.4 The Price Structure under the Three-Bundle Strategy

With each simulation, the price structure under the three-bundle strategy has also been calculated. Information on the price structure is detailed in Annex Table 3 of Appendix 1.

The average individual price in the bundle with all articles, which is denoted by $\bar{p}_N$; and the average individual price in the bundle with n^* articles, which is denoted by $\bar{p}_{n*}$, can be calculated. Regardless of the value of *N*, the total number of articles, the following relationship holds:

$$\bar{p}_N < \bar{p}_{n*} < p_1$$

It means there is no arbitrage. Simply out, it will not be profitable for anyone to reconstruct the bundles that are offered by purchasing the individualized products. And this relationship is compatible with the reality.

4.5 A Robustness Test

There is a need to emphasize that the optimal bundle size that is obtained for each value of N is derived from one simulation only. This raise the question as to the robustness of the results elucidated above. Put different, if the simulation is repeated many times, will the result be different? Since testing all of the results is time-consuming and impractical, a compromise approach is to perform tests for specific values of N. Here N= 20 and N = 30 were selected. The results which are reported in Annex Table 4 of Appendix 1 suggest that robustness is not likely to be an issue.

Chapter 5
Directions for Further Research

First of all, in the thesis, only the case of a seller offering three bundles in a journal market is considered. However, the optimal way to apply customized bundling to the journal market is to offer N bundles, where N is the number of total articles. However, it is impractical to find the optimal price structure for all the N bundles when N is large. Alternatively, the problem can be approached by focusing on the question of how profit changes when the number of bundles is increased. If the profit rises slowly even if the number of bundles continues to increase, when the number of bundles offered in the market reaches some N^* ($N^* < N$), the seller only needs to offer N^* bundles instead of N bundles.

Secondly, in the basic model of this paper, the articles in the journal are assumed to be independent. Specifically, in the model, when a consumer buys a different article, his valuation of a specific article would not change, since his valuation of a specific article would only be decided by his own highest reserve price w_{0i}, the decrease rate k_i and his rank of the specific article. If the articles are considered as complements or substitutes (that means when a consumer buys a different article, his valuation of a specific article will change). For instance, when a consumer reads an article that is related to a specific article, it will be help him to have a better understanding of the specific article. Thus, his valuation of the specific article will change when he buys another related article. In this case, the articles are complimentary. This will have impact on results of the proposed model. This is a potential area for further research.

Thirdly, in this thesis, the seller of a journal is treated as a monopolist in the journal market. The customized bundle is then used as a tool for price discrimination for the seller. It is reasonable that in the market for journals two different journals will not contain articles that are exactly the same. Thus, two articles from different journals have nothing to do with each other due to the difference between journals. However, this case may not be applied to other Internet goods, such as mp3 music, since theses kinds of Internet goods' sellers cannot be treated as monopolists. Mp3 music is taken as an example here. Different websites may have the same Mp3 music most of the time, which means that the music seller has to compete with other music sellers in the mp3 music market. Therefore both the bundling strategy and the price structure should be different from those in the situation of a monopoly market. It is interesting and important to study how the bundle strategy and the price structure can change in a duopoly market compared to those in a monopoly market.

References

Adams, W.J. and J. L. Yellen. 1976. Commodity bundling and the burden of monopoly. *Quarterly Journal of Economics,* Vol. 90, No. 3, pp. 475-498.

Armstrong, M. 1999. Price discrimination by a many-product firm. *Review of Economic Studies,* Vol. 66, Issue. 226, pp. 151-168.

Bakos, Y. and E. Brynjolfsson. 2000a. Bundling and competition on the internet. *Marketing Science,* Vol. 19, No. 1, pp. 63-82.

Bakos, Y. and E. Brynjolfsson. 2000b. Aggregation and disaggregation of information goods: implications for bundling, site licensing and micropayment systems. *http://www.gsm.uci.edu/~bakos/aig.pdf.*

Chen, Y. 1997. Equilibrium product bundling. *The Journal of Business,* Vol. 70, No. 1, pp. 85-103.

Chuang, C. I. and M. A. Sirbu. 1999. Optimal bundle strategy for digital information goods: Network delivery of articles and subscriptions. *Information Economics and Policy*, Vol. 11, No. 2, pp. 147-176.

Geng, X., M. B. Stinchcombe and A. B. Whinston. 2005. Bundling information goods of decreasing value**.** *Management Science,* Vol. 51, No. 4, pp. 662-667.

Hanson, W. A. and R. K. Martin. 1990. Optimal bundle price. *Management Science,* Vol. 36, No. 2, pp. 155-174.

Hitt, L. and P.-Y. Chen. 2005. Bundling with customer self-selection: a simple approach to bundling low marginal cost goods. *Management Science,* Vol. 51, No. 10, pp. 1481-1481.

McAfee, R. P., J. Mcmillan and M. D. Whinston. 1989. Multiproduct monopoly, commodity bundling, and correlation of values. *Quarterly Journal of Economics,* Vol. 104, No. 2, pp. 371-383.

Burstein. M. L. 1960. The economics of tie-in sale. *The Review of Economics and Statistics*, Vol. 42, No. 1, pp. 68-73.

Matutes, C. and P. Regibeau. 1992. Compatibility and bundling of complementary goods in a duopoly. *The Journal of Industrial Economics*, Vol. 40, No. 1 pp. 37-35.

Nalebuff, B. 2004. Bundling as an entry barrier. *Quarterly Journal of Economics,* Vol. 119, No.1, pp. 159-187.

Salinger, M. A. 1995. A graphical analysis of bundling. *Journal of Business,* Vol. 68, No.

1, pp. 85-98.

Schmalensee, R. 1982. Commodity bundling by a single-product monopolist. *Journal of Law and Economics,* Vol. 25, No. 1, pp. 67-71.

Schmalensee, R. 1984. Gaussian demand and commodity bundling. *Journal of Business* Vol. 57, No. 1, pp. 211-230.

Stigler, G. J. 1963. United States v. Loew's Inc.: A note on block booking. *Supreme Court Review*, Vol. 152, No. 1, pp. 52–57.

Venkatesh, R. and W. Kamakura. 2003. Optimal bundling and pricing under a monopoly: contrasting complements and substitutes from independently valued products. *Journal of Business,* Vol. 76, No. 2, pp. 211-231.

Bowman, W. S. 1957. Tying arrangement and the leverage problem. *The Yale Law Journal*, Vol. 67, No. 1, pp. 19-36.

Whinston, M. D. 1990. Tying, foreclosure, and exclusion. *American Economic Review,* Vol. 80, No. 4, pp. 837-859.

Appendix 1

Tables

Annex Table 1: The Intermediate Bundle Size (n^*) [+]

Scenario	Total Number of Articles	n^*
1	20	12
2	21	14
3	22	13
4	23	14
5	24	15
6	25	15
7	26	16
8	27	17
9	28	17
10	29	18
11	30	19
12	31	19
13	32	20
14	33	20
15	34	21
16	35	22
17	36	22
18	37	23
19	38	23
20	39	24

[+] The second column is the number of total articles in a journal, and the third column is the optimal bundle size under different number of articles. We can see that: the optimal bundle size will always be larger than the median value of N (N is the number of articles in a journal).

Annex Table 2: **Rate of Profit Extraction under the Proposed Procedure with under the Armstrong Procedure**

The Number of Articles (N)	Proposed Procedure	Armstrong's Procedure
20	69.22%	52.48%
21	68.79%	53.24%
22	68.65%	53.96%
23	68.44%	54.64%
24	68.22%	55.28%
25	68.24%	55.88%
26	68.24%	56.46%
27	68.24%	57%
28	68.15%	57.52%
29	68.04%	58.01%
31	67.59%	58.94%
32	67.75%	59.37%
33	67.51%	59.78%
34	67.60%	60.18%
35	67.53%	60.56%
36	67.57%	60.93%
37	67.70%	61.29%
38	67.70%	61.63%
39	67.53%	61.96%

Annex Table 3: The Price Structure under the Proposed Procedure[+]

Total	Intermediate Bundle	Profit Level	p_N	p_{n^*}	p_1
20	12	1037.5	9.6	6.84	0.66
21	14	1091.6	10.8	7.7	0.67
22	13	1151.2	10.56	7.54	0.66
23	14	1207.8	11.04	7.98	0.67
24	15	1262.5	11.52	8.55	0.68
25	15	1321	12.25	8.7	0.68
26	16	1379.8	13	9.28	0.69
27	17	1435.5	12.96	9.52	0.66
28	17	1494.1	14	9.86	0.67
29	18	1550.3	14.5	10.44	0.68
30	19	1605.9	15	11.02	0.71
31	19	1663	15.5	11.02	0.69
32	20	1725.4	16	11.6	0.69
33	20	1778	16.5	11.6	0.67
34	21	1840.4	17	12.18	0.69
35	22	1897.2	17.5	12.76	0.69
36	22	1955.4	18	12.76	0.67
37	23	2016.5	18.5	13.34	0.68
38	23	2073.4	19	13.57	6.9
39	24	2131	19.5	14.16	0.7

[+] p_N is the price for the journal (all articles' bundle); p_{n^*} is the price for the intermediate bundle n^* articles; and p_1 is the individual price.

Annex table 4: Optimal Bundle Size under Different Volume of N [+] (Robustness Test with Varying N)

N=20, M=1000

Trial Number	Optimal Bundle Size (n*)
1	12
2	12
3	13
4	12
5	13
6	12
7	12
8	12
9	11
10	12

N=30, M=1000

Trial Number	Optimal Bundle Size (n*)
1	18
2	19
3	20
4	19
5	19
6	21
7	19
8	20
9	19
10	19

[+] N is the total number of articles

Appendix 2

MATLAB Codes for the Simulation Program

```
% generate the heterogeneous consumers;$
q=1000;   % the number of consumers
c1=0.3;   % marginal cost
N=20;    % N is the total number of articles, which can be changed in each case
rand('state',sum(100*clock));            % control the state, each time we can pick different
random                  sample from a specific distribution
w=rand(q,1)+c1;                          % generate w0i
intersect=N*rand(q,1);
for i=1:q;
    k(i)=intersect(i)/N*w(i)/(w(i)-c1);
end;
c=zeros(q,2);
for i=1:q;
c(i,:)=[w(i),k(i)];
end;
maxprofit=0;
 step 2
a1=zeros(N,1);
for j0=1:101;
       pN=N*c1+(j0-1)/100*N;
tic
 for n=1:N;
 profit1=zeros(101,101);
   for j1=1:101;
       p1=c1+(j1-1)/100;
       for j2=1:101;
         pn=n*c1+(j2-1)/100*n;
    for i=1:q;
    n1=(1-p1/c(i,1))*N*c(i,2);
    if n1>0;
        n1=n1;
    else n1=0;
    end;
    n1=fix(n1);        %if a consumer choose to buy individual article, this is the number of
articles he will buy;
     p=pn/n;        % average price of article in the bundle;
  n2=(1-p/c(i,1))*c(i,2)*N;
  if n2<0;
      n2=0;
  else if n2>=0;
         n2=fix(n2);        %n2 is the number of article that consumers want to buy by bundle
buying;
      end;
   end;
```

```
a=fix(n2/n);        % if a consumer chooses to buy intermediate bundle, this is the number of intermediate bundles he will buy;
    if a*n<=N
        a=a;
    else if a*n>N;
            a=fix(N/n);      %gurantee a*n<=N;
        end;
    end;
    %calculate that after a consumer buys several bundles, how many individual articles the consumer will buy;
     if n1-a*n>=0&n1<N;
        n4=n1-a*n;
     else if n1>N;
            n4=N-a*n;
    else if n1-a*n<0;
            n4=0;
        end;
    end;
     end;

n2=a*n+n4;             %the number of articles being bought under intermediate bundle buying;

    n3=(a+1)*n;
    n5=c(i,2)*N;             % w(1-n5/k/N)=0;
    % 1 category: buying individual article;
    if n1<=N;
        n1=n1;
        else if n1>N;
            n1=N;
        end;
    end;
    b1=c(i,1)*(n1-n1*(n1+1)/2/c(i,2)/N)-n1*p1;
    % 2 category: buying intermediate bundles with individual article;
    b2=c(i,1)*(n2-n2*(n2+1)/2/c(i,2)/N)-a*pn-n4*p1; % consumer surplus by buying intermediate bundles and the additional individual articles;
    % 3 category: buying intermediate bundles without individual article;
    if n3<=N&n5>n3;
        b3=c(i,1)*(n3-n3*(n3+1)/2/c(i,2)/N)-(a+1)*pn;
    elseif n3<=N&n5<=n3;
            b3=c(i,1)*(n5-n5*(n5+1)/2/c(i,2)/N)-(a+1)*pn;
    elseif n3>N&n5>=N;
            b3=c(i,1)*(N-N*(N+1)/2/c(i,2)/N)-(a+1)*pn;
    elseif n3>N&n5<N;
                b3=c(i,1)*(n5-n5*(n5+1)/2/c(i,2)/N)-(a+1)*pn;
        end;
    %4 category: buying all articles as a whole bundle;
    if n5>=N;
        b4=c(i,1)*(N-N*(N+1)/2/c(i,2)/N)-pN;
    elseif n5<N;
        b4=c(i,1)*(n5-n5*(n5+1)/2/c(i,2)/N)-pN;
```

```
    end;
 if b1>=b2&b1>=b3&b1>=b4;
     profit1(j1,j2)=profit1(j1,j2)+n1*p1-n1*c1;
 elseif b2>b1&b2>=b3&b2>=b4;
         profit1(j1,j2)=profit1(j1,j2)+a*pn+n4*p1-n2*c1;
 elseif b3>b1&b3>b2&b3>=b4;
             profit1(j1,j2)=profit1(j1,j2)+(a+1)*pn-(a+1)*n*c1;
         elseif b4>b1&b4>b2&b4>b3;
             profit1(j1,j2)=profit1(j1,j2)+pN-N*c1;
             else profit1(j1,j2)=profit1(j1,j2);
         end;
     end;
 end;
    end;

        optimal(j0,n)=max(max(profit1));
end;
 toc
   sprintf('j0=%d',j0)
end;
% for different bundle size, find the maximum profit;
maxprofit=max(max(optimal));
 % find the optimal intermediate bundle size n*;
a2=find(optimal==maxprofit);
a2=fix(a2/101)+1;                              % a2 is the optimal n*
```